Computer algebra independent integration tests

5-Inverse-trig-functions/5.3-Inverse-tangent/5.3.7-Inverse-tangent-functions

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May 24, 2020 Compiled on May 24, 2020 at 10:45am

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [153]. This is test number [153].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (153)	% 0. (0)
Mathematica	% 100. (153)	% 0. (0)
Maple	% 86.93 (133)	% 13.07 (20)
Maxima	% 56.21 (86)	% 43.79 (67)
Fricas	% 85.62 (131)	% 14.38 (22)
Sympy	% 27.45 (42)	% 72.55 (111)
Giac	% 43.79 (67)	% 56.21 (86)

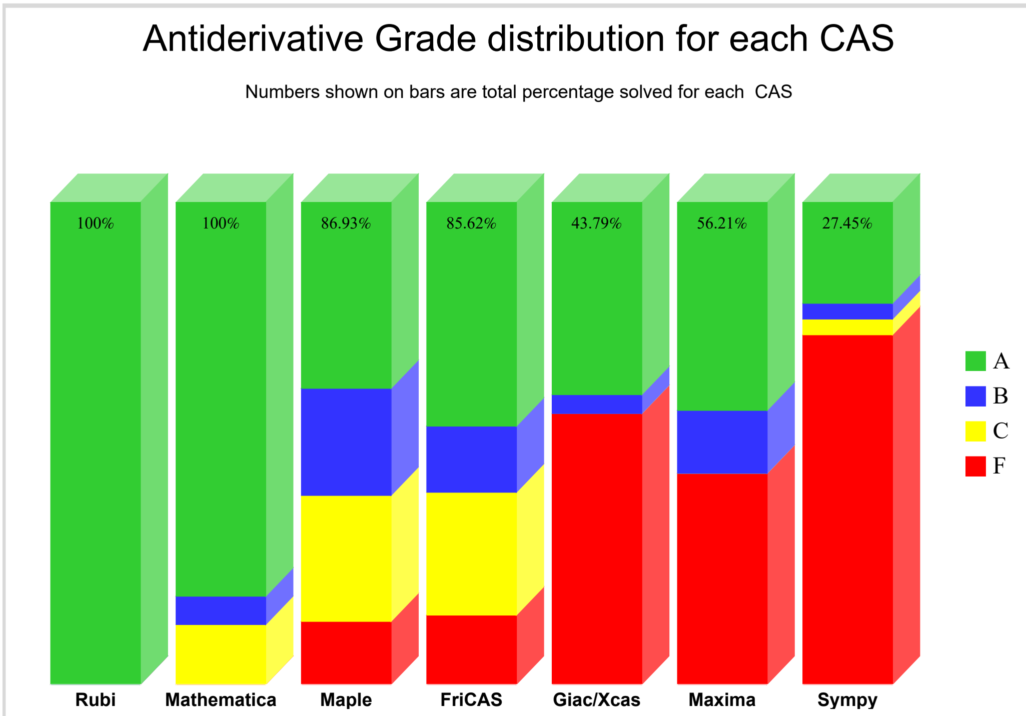
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

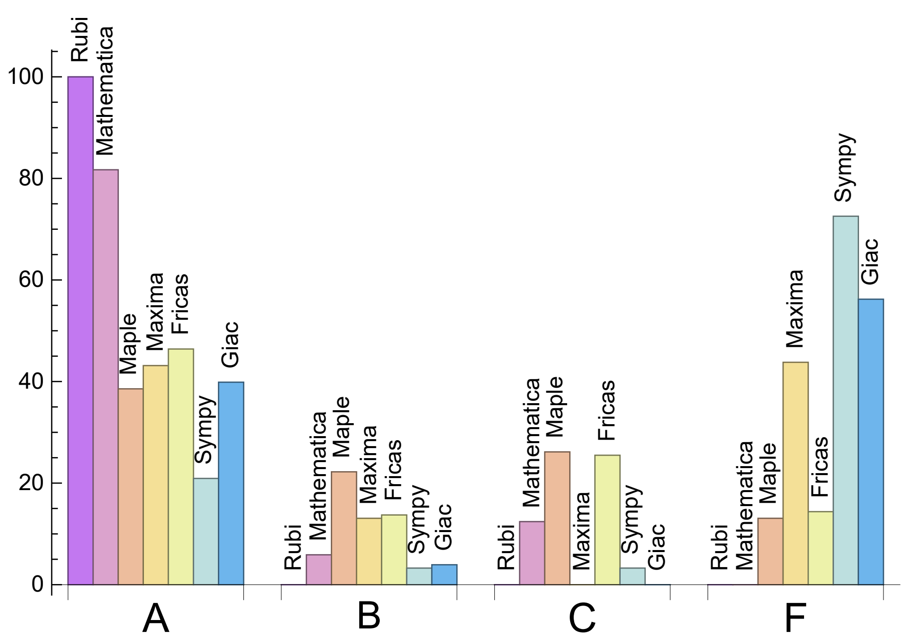
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	81.7	5.88	12.42	0.
Maple	38.56	22.22	26.14	13.07
Maxima	43.14	13.07	0.	43.79
Fricas	46.41	13.73	25.49	14.38
Sympy	20.92	3.27	3.27	72.55
Giac	39.87	3.92	0.	56.21

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	111.39	0.89	85.	1.
Mathematica	1.52	142.71	1.21	81.	0.9
Maple	2.73	1110.77	6.56	163.	2.01
Maxima	1.79	142.36	1.62	75.	1.25
Fricas	2.	699.83	4.57	219.	3.27
Sympy	7.48	81.38	1.4	62.5	0.96
Giac	0.86	72.79	1.12	38.	1.13

1.4 list of integrals that has no closed form antiderivative

$\{31, 35, 36, 51, 55, 59, 64, 68, 72, 80, 84, 88, 92, 97, 101, 105, 109\}$

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{117, 118\}$

Mathematica $\{6, 50, 54, 58, 63, 67, 71, 117, 118, 147, 148, 149, 150, 151, 152\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

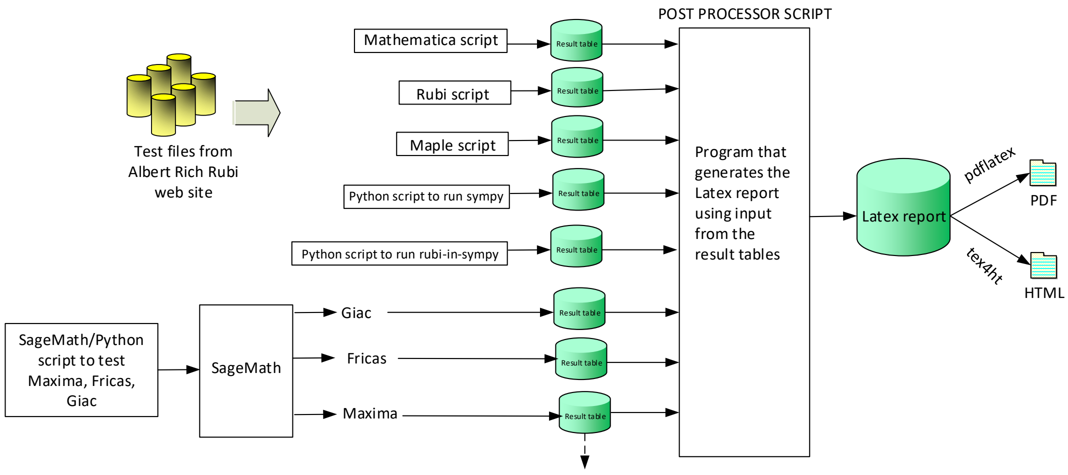
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 55, 56, 57, 59, 60, 61, 62, 64, 65, 66, 68, 69, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 152 }

B grade: { 50, 54, 58, 63, 67, 71, 75, 76, 93 }

C grade: { 15, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 121, 148, 149, 150, 151, 153 }

F grade: { }

2.1.3 Maple

A grade: { 1, 3, 4, 5, 7, 8, 9, 10, 15, 16, 17, 30, 31, 35, 36, 37, 38, 39, 40, 41, 42, 46, 47, 51, 55, 59, 64, 68, 72, 80, 84, 88, 92, 97, 101, 105, 109, 111, 112, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { 11, 12, 13, 14, 32, 33, 34, 43, 44, 45, 50, 54, 58, 60, 63, 67, 71, 73, 79, 83, 87, 91, 96, 100, 104, 108, 110, 113, 114, 115, 117, 118, 130, 131 }

C grade: { 2, 48, 49, 52, 53, 56, 57, 61, 62, 65, 66, 69, 70, 74, 75, 76, 77, 78, 81, 82, 85, 86, 89, 90, 93, 94, 95, 98, 99, 102, 103, 106, 107, 147, 148, 149, 150, 151, 152, 153 }

F grade: { 6, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 140, 141, 142, 143, 144, 145, 146 }

2.1.4 Maxima

A grade: { 1, 2, 7, 8, 9, 10, 11, 12, 13, 30, 31, 35, 36, 40, 43, 44, 45, 46, 47, 60, 80, 84, 85, 86, 87, 88, 89, 90, 91, 92, 97, 101, 102, 103, 104, 105, 106, 107, 108, 109, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 138, 139, 144, 145, 147, 148, 149, 150, 151, 152

B grade: { 14, 38, 39, 41, 50, 52, 53, 54, 56, 57, 58, 63, 65, 66, 67, 69, 70, 71, 110, 113 }

C grade: { }

F grade: { 3, 4, 5, 6, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 37, 42, 48, 49, 51, 55, 59, 61, 62, 64, 68, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 93, 94, 95, 96, 98, 99, 100, 111, 112, 114, 115, 117, 118, 134, 135, 136, 137, 140, 141, 142, 143, 146, 153 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 30, 31, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 51, 55, 59, 60, 64, 67, 68, 71, 72, 80, 84, 88, 92, 97, 101, 105, 109, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 137, 138, 139, 140, 143, 144, 145, 147 }

B grade: { 50, 54, 58, 63, 73, 79, 83, 87, 91, 96, 100, 104, 108, 110, 113, 134, 148, 149, 150, 151, 152 }

C grade: { 48, 49, 52, 53, 56, 57, 61, 62, 65, 66, 69, 70, 74, 75, 76, 77, 78, 81, 82, 85, 86, 89, 90, 93, 94, 95, 98, 99, 102, 103, 106, 107, 111, 112, 114, 115, 117, 118, 153 }

F grade: { 6, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 130, 135, 136, 141, 142, 146 }

2.1.6 Sympy

A grade: { 1, 3, 4, 5, 7, 8, 11, 12, 13, 14, 15, 16, 17, 30, 31, 35, 36, 38, 39, 40, 41, 43, 44, 45, 47, 60, 80, 119, 122, 123, 124, 129 }

B grade: { 9, 10, 120, 121, 125 }

C grade: { 20, 21, 26, 27, 28 }

F grade: { 2, 6, 18, 19, 22, 23, 24, 25, 29, 32, 33, 34, 37, 42, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 11, 12, 13, 14, 15, 16, 17, 20, 30, 31, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 51, 55, 59, 60, 64, 68, 72, 80, 84, 88, 92, 97, 101, 105, 109, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 131, 132, 133, 147, 148, 149, 150, 151, 152 }

B grade: { 7, 8, 9, 10, 37, 125 }

C grade: { }

F grade: { 6, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 48, 49, 50, 52, 53, 54, 56, 57, 58, 61, 62, 63, 65, 66, 67, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 102, 103, 104, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 130, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 153 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	46	50	105	60	50
normalized size	1	1.	0.88	1.1	1.19	2.5	1.43	1.19
time (sec)	N/A	0.041	0.015	0.035	0.991	2.365	6.029	1.133

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	40	140	54	140	0	54
normalized size	1	1.	0.89	3.11	1.2	3.11	0.	1.2
time (sec)	N/A	0.042	0.036	0.115	0.975	2.338	0.	1.1

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	86	211	0	182	129	119
normalized size	1	1.	0.6	1.47	0.	1.26	0.9	0.83
time (sec)	N/A	0.08	0.075	0.043	0.	2.23	8.857	1.22

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	74	163	0	153	102	101
normalized size	1	1.	0.64	1.41	0.	1.32	0.88	0.87
time (sec)	N/A	0.042	0.053	0.041	0.	2.279	2.479	1.17

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	59	116	0	119	71	84
normalized size	1	1.	0.67	1.32	0.	1.35	0.81	0.95
time (sec)	N/A	0.029	0.04	0.04	0.	2.355	0.833	1.183

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	288	288	171	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.167	2.462	0.316	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	54	67	78	111	53	142
normalized size	1	1.	0.95	1.18	1.37	1.95	0.93	2.49
time (sec)	N/A	0.019	0.036	0.039	1.143	2.355	3.246	1.236

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	67	69	92	139	83	267
normalized size	1	1.	0.79	0.81	1.08	1.64	0.98	3.14
time (sec)	N/A	0.028	0.044	0.039	1.012	2.524	5.535	1.206

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	78	117	147	165	352	381
normalized size	1	1.	0.69	1.04	1.3	1.46	3.12	3.37
time (sec)	N/A	0.039	0.057	0.042	1.018	2.689	12.426	1.212

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	89	167	178	189	575	487
normalized size	1	1.	0.63	1.18	1.26	1.34	4.08	3.45
time (sec)	N/A	0.05	0.068	0.043	1.002	3.186	31.126	1.213

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	83	230	225	182	124	185
normalized size	1	1.	0.67	1.85	1.81	1.47	1.	1.49
time (sec)	N/A	0.067	0.104	0.046	1.007	2.33	15.824	1.19

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	72	180	188	158	97	138
normalized size	1	1.	0.73	1.82	1.9	1.6	0.98	1.39
time (sec)	N/A	0.051	0.089	0.043	0.999	2.218	4.501	1.203

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	60	132	150	131	70	88
normalized size	1	1.	0.81	1.78	2.03	1.77	0.95	1.19
time (sec)	N/A	0.036	0.083	0.042	1.01	2.364	1.327	1.162

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	84	104	96	39	55
normalized size	1	1.	1.	1.95	2.42	2.23	0.91	1.28
time (sec)	N/A	0.01	0.02	0.038	1.002	2.361	0.768	1.139

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	86	57	0	339	60	73
normalized size	1	1.	1.46	0.97	0.	5.75	1.02	1.24
time (sec)	N/A	0.033	0.06	0.039	0.	2.607	4.084	1.209

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	101	100	0	460	82	112
normalized size	1	1.	1.11	1.1	0.	5.05	0.9	1.23
time (sec)	N/A	0.046	0.111	0.039	0.	2.711	8.67	1.239

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	114	150	0	529	148	143
normalized size	1	1.	0.96	1.26	0.	4.45	1.24	1.2
time (sec)	N/A	0.06	0.141	0.04	0.	2.849	17.144	1.184

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	170	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	0.588	0.297	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	158	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.416	0.401	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	147	0	0	0	75	1
normalized size	1	1.	0.96	0.	0.	0.	0.49	0.01
time (sec)	N/A	0.074	0.285	0.289	0.	0.	3.615	1.268

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	115	0	0	0	71	0
normalized size	1	1.	0.94	0.	0.	0.	0.58	0.
time (sec)	N/A	0.061	0.122	0.293	0.	0.	5.202	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	150	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.267	0.29	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	162	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.366	0.29	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	171	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.578	0.285	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	139	0	0	0	0	0
normalized size	1	1.	0.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	0.138	0.29	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	119	0	0	0	75	0
normalized size	1	1.	0.4	0.	0.	0.	0.25	0.
time (sec)	N/A	0.161	0.109	0.296	0.	0.	40.537	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	89	0	0	0	71	0
normalized size	1	1.	0.34	0.	0.	0.	0.27	0.
time (sec)	N/A	0.139	0.106	0.283	0.	0.	7.209	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	121	0	0	0	78	0
normalized size	1	1.	0.41	0.	0.	0.	0.26	0.
time (sec)	N/A	0.166	0.133	0.296	0.	0.	37.529	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	137	0	0	0	0	0
normalized size	1	1.	0.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.194	0.118	0.313	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	43	57	143	41	58
normalized size	1	1.	1.	0.86	1.14	2.86	0.82	1.16
time (sec)	N/A	0.146	0.015	0.043	1.48	2.032	1.077	1.126

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.089	1.224	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	530	1745	0	0	0	0
normalized size	1	1.	1.23	4.05	0.	0.	0.	0.
time (sec)	N/A	0.485	0.177	2.559	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	354	947	0	0	0	0
normalized size	1	1.	1.25	3.35	0.	0.	0.	0.
time (sec)	N/A	0.295	0.107	1.218	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	93	371	0	0	0	0
normalized size	1	1.	0.95	3.79	0.	0.	0.	0.
time (sec)	N/A	0.067	0.028	0.717	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.087	1.135	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.778	1.051	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	41	0	72	0	108
normalized size	1	1.	0.92	1.11	0.	1.95	0.	2.92
time (sec)	N/A	0.024	0.056	0.056	0.	1.726	0.	1.126

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	109	31	109	36
normalized size	1	1.	0.87	0.87	4.74	1.35	4.74	1.57
time (sec)	N/A	0.009	0.016	0.041	0.989	1.708	0.775	1.152

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	77	31	32	36
normalized size	1	1.	0.87	0.87	3.35	1.35	1.39	1.57
time (sec)	N/A	0.007	0.015	0.048	1.001	1.749	0.241	1.117

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	16	23	42	35
normalized size	1	1.	1.12	0.94	1.	1.44	2.62	2.19
time (sec)	N/A	0.003	0.007	0.034	0.972	1.751	0.173	1.079

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	21	57	22	34	31
normalized size	1	1.	0.9	1.	2.71	1.05	1.62	1.48
time (sec)	N/A	0.031	0.014	0.045	1.468	1.804	0.893	1.108

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	31	56	0	101	0	84
normalized size	1	1.	0.86	1.56	0.	2.81	0.	2.33
time (sec)	N/A	0.021	0.048	0.052	0.	2.072	0.	1.094

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	65	23	45	68	26
normalized size	1	1.	0.87	2.83	1.	1.96	2.96	1.13
time (sec)	N/A	0.008	0.015	0.061	0.964	2.074	0.782	1.151

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	54	23	45	65	26
normalized size	1	1.	0.87	2.35	1.	1.96	2.83	1.13
time (sec)	N/A	0.007	0.014	0.057	0.992	2.048	0.382	1.11

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	51	20	42	24	20
normalized size	1	1.	1.12	3.19	1.25	2.62	1.5	1.25
time (sec)	N/A	0.003	0.007	0.056	0.951	1.978	0.151	1.085

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	35	19	41	0	20
normalized size	1	1.	1.	1.84	1.	2.16	0.	1.05
time (sec)	N/A	0.033	0.015	0.059	0.95	2.021	0.	1.111

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	16	23	42	35
normalized size	1	1.	1.12	0.94	1.	1.44	2.62	2.19
time (sec)	N/A	0.003	0.	0.034	0.964	1.933	0.168	1.102

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	363	8076	0	5146	0	0
normalized size	1	1.	0.9	20.04	0.	12.77	0.	0.
time (sec)	N/A	0.517	0.91	8.128	0.	2.952	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	272	7660	0	4035	0	0
normalized size	1	1.	0.89	25.11	0.	13.23	0.	0.
time (sec)	N/A	0.405	0.569	24.112	0.	2.728	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	198	198	555	1002	585	2894	0	0
normalized size	1	1.	2.8	5.06	2.95	14.62	0.	0.
time (sec)	N/A	0.234	6.765	0.384	1.881	2.707	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	4.666	0.354	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	140	1532	417	907	0	0
normalized size	1	1.	0.91	9.95	2.71	5.89	0.	0.
time (sec)	N/A	0.238	0.424	17.733	1.149	2.2	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	110	1497	294	755	0	0
normalized size	1	1.	0.89	12.17	2.39	6.14	0.	0.
time (sec)	N/A	0.206	0.311	9.421	1.076	2.217	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	967	1489	605	547	0	0
normalized size	1	1.	11.38	17.52	7.12	6.44	0.	0.
time (sec)	N/A	0.125	14.265	0.132	1.59	2.304	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	0.59	0.408	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	137	1533	419	915	0	0
normalized size	1	1.	0.88	9.89	2.7	5.9	0.	0.
time (sec)	N/A	0.244	0.44	19.335	1.135	1.985	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	111	1498	296	760	0	0
normalized size	1	1.	0.9	12.08	2.39	6.13	0.	0.
time (sec)	N/A	0.208	0.306	8.505	1.065	1.943	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	847	1681	605	549	0	0
normalized size	1	1.	9.85	19.55	7.03	6.38	0.	0.
time (sec)	N/A	0.129	13.618	0.143	1.581	1.729	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	0.932	0.428	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	51	20	42	24	20
normalized size	1	1.	1.12	3.19	1.25	2.62	1.5	1.25
time (sec)	N/A	0.003	0.007	0.043	0.986	1.692	0.165	1.085

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	359	7924	0	4058	0	0
normalized size	1	1.	0.9	19.86	0.	10.17	0.	0.
time (sec)	N/A	0.511	0.856	6.152	0.	3.217	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	270	7556	0	3298	0	0
normalized size	1	1.	0.89	24.94	0.	10.88	0.	0.
time (sec)	N/A	0.4	0.538	24.825	0.	2.883	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	198	198	1648	1159	710	2508	0	0
normalized size	1	1.	8.32	5.85	3.59	12.67	0.	0.
time (sec)	N/A	0.25	21.684	0.38	1.912	2.967	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.134	4.493	0.405	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	136	1532	417	459	0	0
normalized size	1	1.	0.88	9.95	2.71	2.98	0.	0.
time (sec)	N/A	0.256	0.326	18.458	1.136	1.944	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	110	1497	294	394	0	0
normalized size	1	1.	0.89	12.17	2.39	3.2	0.	0.
time (sec)	N/A	0.222	0.206	9.3	1.086	1.905	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	929	1495	616	309	0	0
normalized size	1	1.	10.93	17.59	7.25	3.64	0.	0.
time (sec)	N/A	0.131	15.84	0.137	1.573	1.968	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	0.629	0.408	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	140	1533	419	466	0	0
normalized size	1	1.	0.9	9.89	2.7	3.01	0.	0.
time (sec)	N/A	0.261	0.321	18.921	1.126	1.933	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	110	1498	296	400	0	0
normalized size	1	1.	0.89	12.08	2.39	3.23	0.	0.
time (sec)	N/A	0.224	0.196	9.099	1.077	1.93	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	872	1753	616	313	0	0
normalized size	1	1.	10.14	20.38	7.16	3.64	0.	0.
time (sec)	N/A	0.134	12.737	0.136	1.595	1.984	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.645	0.431	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	64	142	0	217	0	0
normalized size	1	1.	1.64	3.64	0.	5.56	0.	0.
time (sec)	N/A	0.032	0.031	0.051	0.	1.96	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	105	732	0	351	0	0
normalized size	1	1.	1.42	9.89	0.	4.74	0.	0.
time (sec)	N/A	0.061	0.025	0.388	0.	1.979	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	356	758	0	477	0	0
normalized size	1	1.	3.3	7.02	0.	4.42	0.	0.
time (sec)	N/A	0.088	0.107	0.227	0.	1.886	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	600	7275	0	4070	0	0
normalized size	1	1.	2.01	24.33	0.	13.61	0.	0.
time (sec)	N/A	0.209	4.936	6.493	0.	3.1	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	375	5425	0	2903	0	0
normalized size	1	1.	1.64	23.69	0.	12.68	0.	0.
time (sec)	N/A	0.152	2.622	8.241	0.	2.675	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	278	2414	0	1894	0	0
normalized size	1	1.	1.75	15.18	0.	11.91	0.	0.
time (sec)	N/A	0.097	1.706	7.638	0.	2.322	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	132	440	0	1098	0	0
normalized size	1	1.	1.78	5.95	0.	14.84	0.	0.
time (sec)	N/A	0.042	0.062	0.121	0.	2.102	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	6.441	0.606	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	305	6981	0	3641	0	0
normalized size	1	1.	0.86	19.66	0.	10.26	0.	0.
time (sec)	N/A	0.461	5.225	6.736	0.	2.924	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	229	6640	0	2985	0	0
normalized size	1	1.	0.86	24.87	0.	11.18	0.	0.
time (sec)	N/A	0.373	3.992	16.382	0.	2.637	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	288	350	0	2288	0	0
normalized size	1	1.	1.66	2.01	0.	13.15	0.	0.
time (sec)	N/A	0.231	3.907	0.085	0.	5.626	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	8.14	0.358	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	128	1555	174	868	0	0
normalized size	1	1.	0.9	10.95	1.23	6.11	0.	0.
time (sec)	N/A	0.228	5.128	11.773	5.932	1.99	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	102	1519	143	718	0	0
normalized size	1	1.	0.9	13.44	1.27	6.35	0.	0.
time (sec)	N/A	0.195	5.235	6.903	5.797	1.885	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	1381	108	518	0	0
normalized size	1	1.	0.9	17.48	1.37	6.56	0.	0.
time (sec)	N/A	0.118	1.701	0.145	5.864	1.905	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	3.598	0.428	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	128	1570	174	857	0	0
normalized size	1	1.	0.88	10.83	1.2	5.91	0.	0.
time (sec)	N/A	0.22	5.103	10.954	5.898	2.017	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	102	1534	144	710	0	0
normalized size	1	1.	0.88	13.22	1.24	6.12	0.	0.
time (sec)	N/A	0.197	4.981	3.744	5.922	1.958	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	71	1351	108	513	0	0
normalized size	1	1.	0.87	16.48	1.32	6.26	0.	0.
time (sec)	N/A	0.119	1.665	0.143	5.946	1.985	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	3.602	0.439	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	600	7275	0	4072	0	0
normalized size	1	1.	2.01	24.33	0.	13.62	0.	0.
time (sec)	N/A	0.209	4.815	4.787	0.	2.998	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	375	5425	0	2901	0	0
normalized size	1	1.	1.64	23.69	0.	12.67	0.	0.
time (sec)	N/A	0.153	2.656	7.741	0.	2.621	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	278	2415	0	1894	0	0
normalized size	1	1.	1.75	15.19	0.	11.91	0.	0.
time (sec)	N/A	0.1	1.773	7.496	0.	2.618	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	132	440	0	1098	0	0
normalized size	1	1.	1.81	6.03	0.	15.04	0.	0.
time (sec)	N/A	0.043	0.055	0.118	0.	2.446	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	4.329	0.872	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	299	6909	0	3614	0	0
normalized size	1	1.	0.85	19.68	0.	10.3	0.	0.
time (sec)	N/A	0.465	5.477	5.885	0.	3.268	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	225	6529	0	2963	0	0
normalized size	1	1.	0.85	24.64	0.	11.18	0.	0.
time (sec)	N/A	0.37	4.073	16.575	0.	3.021	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	287	350	0	2272	0	0
normalized size	1	1.	1.65	2.01	0.	13.06	0.	0.
time (sec)	N/A	0.228	3.703	0.091	0.	6.505	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	8.399	0.387	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	128	1554	174	855	0	0
normalized size	1	1.	0.9	10.94	1.23	6.02	0.	0.
time (sec)	N/A	0.236	1.572	12.993	5.847	2.292	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	102	1518	143	707	0	0
normalized size	1	1.	0.9	13.43	1.27	6.26	0.	0.
time (sec)	N/A	0.206	1.475	4.582	5.892	2.264	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	1381	108	510	0	0
normalized size	1	1.	0.9	17.48	1.37	6.46	0.	0.
time (sec)	N/A	0.122	0.661	0.152	5.924	2.247	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	3.594	0.438	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	128	1571	174	871	0	0
normalized size	1	1.	0.88	10.83	1.2	6.01	0.	0.
time (sec)	N/A	0.231	1.575	12.849	5.856	2.263	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	102	1535	144	721	0	0
normalized size	1	1.	0.88	13.23	1.24	6.22	0.	0.
time (sec)	N/A	0.201	1.495	7.276	5.759	2.235	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	71	1351	108	521	0	0
normalized size	1	1.	0.87	16.48	1.32	6.35	0.	0.
time (sec)	N/A	0.121	0.662	0.144	5.829	2.239	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	3.629	0.447	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	59	59	46	147	0	0
normalized size	1	1.	1.9	1.9	1.48	4.74	0.	0.
time (sec)	N/A	0.026	0.044	0.049	1.51	2.168	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	50	44	0	238	0	0
normalized size	1	1.	0.79	0.7	0.	3.78	0.	0.
time (sec)	N/A	0.043	0.009	0.089	0.	2.163	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	70	0	298	0	0
normalized size	1	1.	1.	0.77	0.	3.27	0.	0.
time (sec)	N/A	0.066	0.008	0.079	0.	2.282	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	83	106	85	297	0	0
normalized size	1	1.	1.84	2.36	1.89	6.6	0.	0.
time (sec)	N/A	0.028	0.101	0.049	1.499	2.232	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	71	349	0	431	0	0
normalized size	1	1.	0.78	3.84	0.	4.74	0.	0.
time (sec)	N/A	0.058	0.013	0.171	0.	2.336	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	133	407	0	540	0	0
normalized size	1	1.	1.	3.06	0.	4.06	0.	0.
time (sec)	N/A	0.09	0.01	0.182	0.	2.351	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	167	186	302	554	0	0
normalized size	1	1.	0.85	0.95	1.54	2.83	0.	0.
time (sec)	N/A	0.148	0.181	0.053	1.7	2.413	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	232	250	236	672	0	778	0	0
normalized size	1	1.08	1.02	2.9	0.	3.35	0.	0.
time (sec)	N/A	0.151	0.087	0.242	0.	2.476	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	302	313	334	758	0	967	0	0
normalized size	1	1.04	1.11	2.51	0.	3.2	0.	0.
time (sec)	N/A	0.201	0.014	0.217	0.	2.473	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	26	82	19	27
normalized size	1	1.	1.	0.92	1.04	3.28	0.76	1.08
time (sec)	N/A	0.021	0.017	0.04	0.946	2.148	11.452	1.099

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	35	32	42	144	153	43
normalized size	1	1.	0.78	0.71	0.93	3.2	3.4	0.96
time (sec)	N/A	0.032	0.033	0.043	1.442	2.392	0.652	1.104

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	81	54	73	219	223	74
normalized size	1	1.	1.27	0.84	1.14	3.42	3.48	1.16
time (sec)	N/A	0.067	0.044	0.044	1.448	2.421	0.78	1.102

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	22	25	32	58	26	32
normalized size	1	1.	0.73	0.83	1.07	1.93	0.87	1.07
time (sec)	N/A	0.011	0.01	0.038	1.438	2.395	0.256	1.112

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	27	5	7
normalized size	1	1.	1.	1.2	1.4	5.4	1.	1.4
time (sec)	N/A	0.025	0.029	0.036	0.954	1.985	0.35	1.109

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	22	45	14	22
normalized size	1	1.	1.	1.12	1.29	2.65	0.82	1.29
time (sec)	N/A	0.039	0.045	0.073	0.969	1.796	0.705	1.095

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	84	53	45
normalized size	1	1.	1.	0.94	1.22	4.67	2.94	2.5
time (sec)	N/A	0.151	0.031	0.045	1.912	1.804	0.856	1.131

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	58	45	59	124	0	59
normalized size	1	1.	0.85	0.66	0.87	1.82	0.	0.87
time (sec)	N/A	0.028	0.059	0.056	1.589	1.968	0.	1.129

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	40	53	107	0	53
normalized size	1	1.	0.9	0.68	0.9	1.81	0.	0.9
time (sec)	N/A	0.026	0.032	0.051	1.595	1.967	0.	1.132

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	35	46	92	0	46
normalized size	1	1.	0.96	0.7	0.92	1.84	0.	0.92
time (sec)	N/A	0.018	0.029	0.054	1.57	1.877	0.	1.113

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	39	28	35	72	29	36
normalized size	1	1.	1.05	0.76	0.95	1.95	0.78	0.97
time (sec)	N/A	0.01	0.397	0.054	1.596	1.915	77.464	1.111

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	84	374	58	0	0	0
normalized size	1	1.	2.	8.9	1.38	0.	0.	0.
time (sec)	N/A	0.042	0.185	1.115	1.55	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	40	57	39	81	0	38
normalized size	1	1.	0.98	1.39	0.95	1.98	0.	0.93
time (sec)	N/A	0.022	0.032	0.055	1.621	2.025	0.	1.115

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	35	46	100	0	46
normalized size	1	1.	0.96	0.7	0.92	2.	0.	0.92
time (sec)	N/A	0.024	0.029	0.07	1.607	1.961	0.	1.108

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	51	40	53	115	0	53
normalized size	1	1.	0.86	0.68	0.9	1.95	0.	0.9
time (sec)	N/A	0.025	0.041	0.048	1.625	2.069	0.	1.131

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	73	0	252	0	0
normalized size	1	1.	1.	1.16	0.	4.	0.	0.
time (sec)	N/A	0.107	0.06	0.756	0.	2.432	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	72	0	0	0	0
normalized size	1	1.	1.	1.22	0.	0.	0.	0.
time (sec)	N/A	0.098	0.03	0.68	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	72	0	0	0	0
normalized size	1	1.	1.	1.22	0.	0.	0.	0.
time (sec)	N/A	0.062	0.023	0.675	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	71	0	165	0	0
normalized size	1	1.	1.	1.29	0.	3.	0.	0.
time (sec)	N/A	0.108	0.055	1.108	0.	1.808	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	71	39	158	0	0
normalized size	1	1.	1.	1.25	0.68	2.77	0.	0.
time (sec)	N/A	0.099	0.026	0.672	1.538	1.851	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	72	39	165	0	0
normalized size	1	1.	1.	1.22	0.66	2.8	0.	0.
time (sec)	N/A	0.101	0.026	0.675	1.693	1.881	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	66	0	0	270	0	0
normalized size	1	1.	0.92	0.	0.	3.75	0.	0.
time (sec)	N/A	0.108	0.222	0.9	0.	2.138	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.125	0.722	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.06	0.717	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	58	0	0	170	0	0
normalized size	1	1.	0.91	0.	0.	2.66	0.	0.
time (sec)	N/A	0.108	0.104	0.715	0.	1.765	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	60	0	62	163	0	0
normalized size	1	1.	0.91	0.	0.94	2.47	0.	0.
time (sec)	N/A	0.101	0.091	0.728	1.516	1.845	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	80	173	0	0
normalized size	1	1.	0.91	0.	1.18	2.54	0.	0.
time (sec)	N/A	0.1	0.088	0.711	1.711	1.919	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	79	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.266	0.137	1.936	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	48	48	61	1299	65	192	0	88
normalized size	1	1.	1.27	27.06	1.35	4.	0.	1.83
time (sec)	N/A	0.075	0.103	0.671	1.516	1.927	0.	1.116

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	146	1375	177	591	0	208
normalized size	1	1.	1.42	13.35	1.72	5.74	0.	2.02
time (sec)	N/A	0.155	0.144	0.801	1.532	1.997	0.	1.125

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	89	1355	225	1152	0	344
normalized size	1	1.	0.49	7.53	1.25	6.4	0.	1.91
time (sec)	N/A	0.182	0.117	0.73	1.523	2.236	0.	1.418

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	89	1355	225	1153	0	344
normalized size	1	1.	0.49	7.53	1.25	6.41	0.	1.91
time (sec)	N/A	0.179	0.111	0.591	1.546	2.232	0.	1.364

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	145	842	228	741	0	208
normalized size	1	1.	1.41	8.17	2.21	7.19	0.	2.02
time (sec)	N/A	0.145	0.148	0.609	1.575	2.068	0.	1.13

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	47	47	57	885	63	343	0	89
normalized size	1	1.	1.21	18.83	1.34	7.3	0.	1.89
time (sec)	N/A	0.079	0.107	0.513	1.519	1.905	0.	1.151

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	116	896	0	711	0	0
normalized size	1	1.	0.71	5.5	0.	4.36	0.	0.
time (sec)	N/A	0.572	0.324	0.422	0.	2.174	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [73] had the largest ratio of [1.333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.	12	0.333
2	A	4	4	1.	14	0.286
3	A	6	4	1.	25	0.16
4	A	5	4	1.	25	0.16
5	A	4	4	1.	23	0.174
6	A	8	8	1.	25	0.32
7	A	2	2	1.	25	0.08
8	A	3	3	1.	25	0.12
9	A	4	3	1.	25	0.12
10	A	5	3	1.	25	0.12
11	A	4	3	1.	25	0.12
12	A	4	3	1.	25	0.12
13	A	4	3	1.	25	0.12
14	A	2	2	1.	21	0.095
15	A	4	4	1.	25	0.16
16	A	5	5	1.	25	0.2
17	A	6	5	1.	25	0.2
18	A	6	4	1.	27	0.148
19	A	5	4	1.	27	0.148
20	A	4	4	1.	27	0.148
21	A	3	3	1.	27	0.111
22	A	4	4	1.	27	0.148
23	A	5	4	1.	27	0.148
24	A	6	4	1.	27	0.148
25	A	7	6	1.	27	0.222
26	A	6	6	1.	27	0.222
27	A	5	5	1.	27	0.185

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	6	6	1.	27	0.222
29	A	7	6	1.	27	0.222
30	A	8	7	1.	11	0.636
31	A	0	0	0.	0	0.
32	A	9	7	1.	40	0.175
33	A	7	6	1.	40	0.15
34	A	4	4	1.	38	0.105
35	A	0	0	0.	0	0.
36	A	0	0	0.	0	0.
37	A	2	2	1.	11	0.182
38	A	2	2	1.	11	0.182
39	A	2	2	1.	9	0.222
40	A	2	2	1.	7	0.286
41	A	2	2	1.	11	0.182
42	A	2	2	1.	11	0.182
43	A	2	2	1.	11	0.182
44	A	2	2	1.	9	0.222
45	A	2	2	1.	7	0.286
46	A	2	2	1.	11	0.182
47	A	2	2	1.	7	0.286
48	A	11	6	1.	15	0.4
49	A	9	5	1.	13	0.385
50	A	7	4	1.	11	0.364
51	A	0	0	0.	0	0.
52	A	7	7	1.	21	0.333
53	A	6	6	1.	19	0.316
54	A	5	5	1.	17	0.294
55	A	0	0	0.	0	0.
56	A	7	7	1.	21	0.333
57	A	6	6	1.	19	0.316
58	A	5	5	1.	17	0.294
59	A	0	0	0.	0	0.
60	A	2	2	1.	7	0.286
61	A	11	6	1.	15	0.4
62	A	9	5	1.	13	0.385
63	A	7	4	1.	11	0.364
64	A	0	0	0.	0	0.
65	A	7	7	1.	21	0.333
66	A	6	6	1.	19	0.316
67	A	5	5	1.	17	0.294
68	A	0	0	0.	0	0.
69	A	7	7	1.	21	0.333
70	A	6	6	1.	19	0.316
71	A	5	5	1.	17	0.294

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	0	0	0.	0	0.
73	A	6	4	1.	3	1.333
74	A	8	5	1.	5	1.
75	A	10	6	1.	7	0.857
76	A	12	6	1.	15	0.4
77	A	10	6	1.	15	0.4
78	A	8	5	1.	13	0.385
79	A	6	4	1.	7	0.571
80	A	0	0	0.	0	0.
81	A	11	6	1.	15	0.4
82	A	9	5	1.	13	0.385
83	A	7	4	1.	11	0.364
84	A	0	0	0.	0	0.
85	A	7	7	1.	19	0.368
86	A	6	6	1.	17	0.353
87	A	5	5	1.	15	0.333
88	A	0	0	0.	0	0.
89	A	7	7	1.	22	0.318
90	A	6	6	1.	20	0.3
91	A	5	5	1.	18	0.278
92	A	0	0	0.	0	0.
93	A	12	6	1.	15	0.4
94	A	10	6	1.	15	0.4
95	A	8	5	1.	13	0.385
96	A	6	4	1.	7	0.571
97	A	0	0	0.	0	0.
98	A	11	6	1.	15	0.4
99	A	9	5	1.	13	0.385
100	A	7	4	1.	11	0.364
101	A	0	0	0.	0	0.
102	A	7	7	1.	19	0.368
103	A	6	6	1.	17	0.353
104	A	5	5	1.	15	0.333
105	A	0	0	0.	0	0.
106	A	7	7	1.	22	0.318
107	A	6	6	1.	20	0.3
108	A	5	5	1.	18	0.278
109	A	0	0	0.	0	0.
110	A	4	3	1.	4	0.75
111	A	7	4	1.	6	0.667
112	A	9	5	1.	8	0.625
113	A	4	3	1.	8	0.375
114	A	7	4	1.	10	0.4
115	A	9	5	1.	12	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	6	6	1.	12	0.5
117	A	9	5	1.08	14	0.357
118	A	11	6	1.04	16	0.375
119	A	5	6	1.	10	0.6
120	A	5	4	1.	8	0.5
121	A	9	8	1.	14	0.571
122	A	4	4	1.	8	0.5
123	A	1	1	1.	14	0.071
124	A	1	1	1.	19	0.053
125	A	5	3	1.	26	0.115
126	A	9	6	1.	21	0.286
127	A	8	6	1.	21	0.286
128	A	7	6	1.	19	0.316
129	A	6	6	1.	18	0.333
130	A	6	5	1.	21	0.238
131	A	6	6	1.	21	0.286
132	A	7	6	1.	21	0.286
133	A	8	6	1.	21	0.286
134	A	2	2	1.	39	0.051
135	A	2	2	1.	39	0.051
136	A	2	2	1.	37	0.054
137	A	2	2	1.	39	0.051
138	A	2	2	1.	39	0.051
139	A	2	2	1.	39	0.051
140	A	2	2	1.	40	0.05
141	A	2	2	1.	40	0.05
142	A	2	2	1.	38	0.053
143	A	2	2	1.	40	0.05
144	A	2	2	1.	40	0.05
145	A	2	2	1.	40	0.05
146	A	9	7	1.	24	0.292
147	A	5	5	1.	20	0.25
148	A	8	7	1.	20	0.35
149	A	13	10	1.	20	0.5
150	A	13	10	1.	20	0.5
151	A	8	7	1.	20	0.35
152	A	5	5	1.	20	0.25
153	A	13	9	1.	24	0.375

Chapter 3

Listing of integrals

3.1 $\int x^3 \tan^{-1}\left(a+bx^4\right) dx$

Optimal. Leaf size=42

$$\frac{\left(a+bx^4\right)\tan^{-1}\left(a+bx^4\right)}{4b}-\frac{\log\left(\left(a+bx^4\right)^2+1\right)}{8b}$$

```
[Out] ((a + b*x^4)*ArcTan[a + b*x^4])/(4*b) - Log[1 + (a + b*x^4)^2]/(8*b)
```

Rubi [A] time = 0.0408917, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6715, 5039, 4846, 260}

$$\frac{\left(a+bx^4\right)\tan^{-1}\left(a+bx^4\right)}{4b}-\frac{\log\left(\left(a+bx^4\right)^2+1\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTan[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcTan[a + b*x^4])/(4*b) - Log[1 + (a + b*x^4)^2]/(8*b)
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 5039

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int x^3 \tan^{-1}\left(a+bx^4\right) dx &= \frac{1}{4}\operatorname{Subst}\left(\int \tan^{-1}(a+bx)\,dx, x, x^4\right)\\
&= \frac{\operatorname{Subst}\left(\int \tan^{-1}(x)\,dx, x, a+bx^4\right)}{4b}\\
&= \frac{\left(a+bx^4\right)\tan^{-1}\left(a+bx^4\right)}{4b} - \frac{\operatorname{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx^4\right)}{4b}\\
&= \frac{\left(a+bx^4\right)\tan^{-1}\left(a+bx^4\right)}{4b} - \frac{\log\left(1+\left(a+bx^4\right)^2\right)}{8b}
\end{aligned}$$

Mathematica [A] time = 0.0147019, size = 37, normalized size = 0.88

$$-\frac{\log\left(\left(a+bx^4\right)^2+1\right)-2\left(a+bx^4\right)\tan^{-1}\left(a+bx^4\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTan[a + b*x^4],x]
```

```
[Out] -(-2*(a + b*x^4)*ArcTan[a + b*x^4] + Log[1 + (a + b*x^4)^2])/(8*b)
```

Maple [A] time = 0.035, size = 46, normalized size = 1.1

$$\frac{\arctan\left(bx^4+a\right)x^4}{4}+\frac{\arctan\left(bx^4+a\right)a}{4b}-\frac{\ln\left(1+\left(bx^4+a\right)^2\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(b*x^4+a),x)
```

```
[Out] 1/4*arctan(b*x^4+a)*x^4+1/4/b*arctan(b*x^4+a)*a-1/8*ln(1+(b*x^4+a)^2)/b
```

Maxima [A] time = 0.991128, size = 50, normalized size = 1.19

$$\frac{2\left(bx^4+a\right)\arctan\left(bx^4+a\right)-\log\left(\left(bx^4+a\right)^2+1\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arctan(b*x^4 + a) - log((b*x^4 + a)^2 + 1))/b
```

Fricas [A] time = 2.36511, size = 105, normalized size = 2.5

$$\frac{2\left(bx^4+a\right)\arctan\left(bx^4+a\right)-\log\left(b^2x^8+2\,abx^4+a^2+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arctan(b*x^4 + a) - log(b^2*x^8 + 2*a*b*x^4 + a^2 + 1))/
b
```

Sympy [A] time = 6.02931, size = 60, normalized size = 1.43

$$\begin{cases} \frac{a\,\mathrm{atan}\left(a+bx^4\right)}{4b}+\frac{x^4\,\mathrm{atan}\left(a+bx^4\right)}{4}-\frac{\log\left(a^2+2abx^4+b^2x^8+1\right)}{8b} & \text{for}\ b\neq 0\\ \frac{x^4\,\mathrm{atan}\,(a)}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(b*x**4+a),x)
```

```
[Out] Piecewise((a*atan(a + b*x**4)/(4*b) + x**4*atan(a + b*x**4)/4 - log(a**2 +
2*a*b*x**4 + b**2*x**8 + 1)/(8*b), Ne(b, 0)), (x**4*atan(a)/4, True))
```

Giac [A] time = 1.13338, size = 50, normalized size = 1.19

$$\frac{2\left(bx^4+a\right)\arctan\left(bx^4+a\right)-\log\left(\left(bx^4+a\right)^2+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arctan(b*x^4 + a) - log((b*x^4 + a)^2 + 1))/b
```

3.2 $\int x^{-1+n} \tan^{-1}(a+bx^n)\,dx$

Optimal. Leaf size=45

$$\frac{(a+bx^n)\tan^{-1}(a+bx^n)}{bn}-\frac{\log\left((a+bx^n)^2+1\right)}{2bn}$$

```
[Out] ((a + b*x^n)*ArcTan[a + b*x^n])/(b*n) - Log[1 + (a + b*x^n)^2]/(2*b*n)
```

Rubi [A] time = 0.0421264, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6715, 5039, 4846, 260}

$$\frac{(a+bx^n)\tan^{-1}(a+bx^n)}{bn}-\frac{\log\left((a+bx^n)^2+1\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcTan[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcTan[a + b*x^n])/(b*n) - Log[1 + (a + b*x^n)^2]/(2*b*n)
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]
```

Rule 5039

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*ArcTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x^{-1+n}\tan^{-1}(a+bx^n)\,dx = \frac{\text{Subst}\left(\int \tan^{-1}(a+bx)\,dx,x,x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \tan^{-1}(x)\,dx,x,a+bx^n\right)}{bn}$$
$$= \frac{(a+bx^n)\tan^{-1}(a+bx^n)}{bn} - \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,a+bx^n\right)}{bn}$$
$$= \frac{(a+bx^n)\tan^{-1}(a+bx^n)}{bn} - \frac{\log\left(1+(a+bx^n)^2\right)}{2bn}$$

Mathematica [A] time = 0.0361654, size = 40, normalized size = 0.89

$$-\frac{\log\left((a+bx^n)^2+1\right)-2(a+bx^n)\tan^{-1}(a+bx^n)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcTan[a + b*x^n],x]
```

```
[Out] -(-2*(a + b*x^n)*ArcTan[a + b*x^n] + Log[1 + (a + b*x^n)^2])/(2*b*n)
```

Maple [C] time = 0.115, size = 140, normalized size = 3.1

$$\frac{-\frac{i}{2}x^n\ln(1+i(a+bx^n))}{n} + \frac{\frac{i}{2}x^n\ln(1-i(a+bx^n))}{n} - \frac{1}{2bn}\ln\left(\frac{i+a}{b}+x^n\right) - \frac{1}{2bn}\ln\left(x^n-\frac{i-a}{b}\right) + \frac{\frac{i}{2}a}{bn}\ln\left(\frac{i+a}{b}+x^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*arctan(a+b*x^n),x)
```

```
[Out] -1/2*I/n*x^n*ln(1+I*(a+b*x^n))+1/2*I/n*x^n*ln(1-I*(a+b*x^n))-1/2/b/n*ln((I+
a)/b+x^n)-1/2/b/n*ln(x^n-(I-a)/b)+1/2*I/b/n*ln((I+a)/b+x^n)*a-1/2*I/b/n*ln(
x^n-(I-a)/b)*a
```

Maxima [A] time = 0.974863, size = 54, normalized size = 1.2

$$\frac{2(bx^n+a)\arctan(bx^n+a)-\log\left((bx^n+a)^2+1\right)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctan(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x^n + a)*arctan(b*x^n + a) - log((b*x^n + a)^2 + 1))/(b*n)
```

Fricas [A] time = 2.3383, size = 140, normalized size = 3.11

$$\frac{2bx^n\arctan(bx^n+a)+2a\arctan(bx^n+a)-\log\left(b^2x^{2n}+2abx^n+a^2+1\right)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctan(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^n*arctan(b*x^n + a) + 2*a*arctan(b*x^n + a) - log(b^2*x^(2*n) +
2*a*b*x^n + a^2 + 1))/(b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*atan(a+b*x**n),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09973, size = 54, normalized size = 1.2

$$\frac{2\,(bx^n + a)\arctan\left(bx^n + a\right) - \log\left((bx^n + a)^2 + 1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctan(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/2*(2*(b*x^n + a)*arctan(b*x^n + a) - log((b*x^n + a)^2 + 1))/(b*n)
```

3.3 $\int x^5 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=144

$$\frac{5d^3\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{96e^{7/2}} + \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{1}{6}x^6\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] (5*d^2*x*Sqrt[d + e*x^2])/(96*(-e)^(5/2)) + (5*d*x^3*Sqrt[d + e*x^2])/(144*
(-e)^(3/2)) + (x^5*Sqrt[d + e*x^2])/(36*Sqrt[-e]) + (x^6*ArcTan[(Sqrt[-e]*x
)/Sqrt[d + e*x^2]])/6 + (5*d^3*Sqrt[-e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]
])/(96*e^(7/2))
```

Rubi [A] time = 0.0801685, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {5151, 321, 217, 206}

$$\frac{5d^3\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{96e^{7/2}} + \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{1}{6}x^6\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (5*d^2*x*Sqrt[d + e*x^2])/(96*(-e)^(5/2)) + (5*d*x^3*Sqrt[d + e*x^2])/(144*
(-e)^(3/2)) + (x^5*Sqrt[d + e*x^2])/(36*Sqrt[-e]) + (x^6*ArcTan[(Sqrt[-e]*x
)/Sqrt[d + e*x^2]])/6 + (5*d^3*Sqrt[-e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]
])/(96*e^(7/2))
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^5 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{6}x^6 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{-e}\int \frac{x^6}{\sqrt{d+ex^2}} dx$$
$$= \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(5d)\int \frac{x^4}{\sqrt{d+ex^2}} dx}{36\sqrt{-e}}$$
$$= \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(5d^2)\int \frac{x^2}{\sqrt{d+ex^2}} dx}{48(-e)^{3/2}}$$
$$= \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(5d^3)\int \frac{1}{\sqrt{d+ex^2}} d}{96(-e)^{5/2}}$$
$$= \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(5d^3)\,\mathrm{Subst}\left(\int\right.}{96(}$$
$$= \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{5d^3 \tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{d+}}\right.}{96(-e)^{5/2}\sqrt{e}}$$

Mathematica [A] time = 0.0754222, size = 86, normalized size = 0.6

$$\frac{\sqrt{-e}x\sqrt{d+ex^2}\left(-15d^2+10dex^2-8e^2x^4\right)+3\left(5d^3+16e^3x^6\right)\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{288e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[-e]*x*Sqrt[d + e*x^2]*(-15*d^2 + 10*d*e*x^2 - 8*e^2*x^4) + 3*(5*d^3 +
 16*e^3*x^6)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(288*e^3)
```

Maple [A] time = 0.043, size = 211, normalized size = 1.5

$$\frac{x^6}{6}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right) + \frac{x^7}{48\,d}\sqrt{-e}\sqrt{ex^2+d} - \frac{7\,x^5}{288\,e}\sqrt{-e}\sqrt{ex^2+d} + \frac{35\,dx^3}{1152\,e^2}\sqrt{-e}\sqrt{ex^2+d} - \frac{5\,d^2x}{128\,e^3}\sqrt{-e}\sqrt{ex^2+d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/6*x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/48*(-e)^(1/2)/d*x^7*(e*x^2+d
)^(1/2)-7/288*(-e)^(1/2)/e*x^5*(e*x^2+d)^(1/2)+35/1152*(-e)^(1/2)/e^2*d*x^3
*(e*x^2+d)^(1/2)-5/128*(-e)^(1/2)/e^3*d^2*x*(e*x^2+d)^(1/2)+5/96*(-e)^(1/2)
/e^(7/2)*d^3*ln(x*e^(1/2)+(e*x^2+d)^(1/2))-1/48*(-e)^(1/2)/d*x^5*(e*x^2+d)^
(3/2)/e+5/288*(-e)^(1/2)/e^2*x^3*(e*x^2+d)^(3/2)-5/384*(-e)^(1/2)*d/e^3*x*(
e*x^2+d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}x^6 \arctan\left(\sqrt{-e}x, \sqrt{ex^2+d}\right) - d\sqrt{-e}\int -\frac{\sqrt{ex^2+d}x^6}{6\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - d*sqrt(-e)*integrate(-1/6*sq
rt(e*x^2 + d)*x^6/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

Fricas [A] time = 2.2305, size = 182, normalized size = 1.26

$$-\frac{\left(8 e^{2} x^{5}-10 d e x^{3}+15 d^{2} x\right) \sqrt{e x^{2}+d} \sqrt{-e}-3\left(16 e^{3} x^{6}+5 d^{3}\right) \arctan \left(\frac{\sqrt{-e} x}{\sqrt{e x^{2}+d}}\right)}{288 e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/288*((8*e^2*x^5 - 10*d*e*x^3 + 15*d^2*x)*sqrt(e*x^2 + d)*sqrt(-e) - 3*(1
6*e^3*x^6 + 5*d^3)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/e^3
```

Sympy [A] time = 8.85741, size = 129, normalized size = 0.9

$$\begin{cases} \frac{5 i d^{3} \operatorname{atanh}\left(\frac{\sqrt{e} x}{\sqrt{d+e x^{2}}}\right)}{96 e^{3}}-\frac{5 i d^{2} x \sqrt{d+e x^{2}}}{96 e^{\frac{5}{2}}}+\frac{5 i d x^{3} \sqrt{d+e x^{2}}}{144 e^{\frac{3}{2}}}+\frac{i x^{6} \operatorname{atanh}\left(\frac{\sqrt{e} x}{\sqrt{d+e x^{2}}}\right)}{6}-\frac{i x^{5} \sqrt{d+e x^{2}}}{36 \sqrt{e}} & \text{for}\ e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((5*I*d**3*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(96*e**3) - 5*I*d**2*
x*sqrt(d + e*x**2)/(96*e**(5/2)) + 5*I*d*x**3*sqrt(d + e*x**2)/(144*e**(3/2
)) + I*x**6*atanh(sqrt(e)*x/sqrt(d + e*x**2))/6 - I*x**5*sqrt(d + e*x**2)/(
36*sqrt(e)), Ne(e, 0)), (0, True))
```

Giac [A] time = 1.21964, size = 119, normalized size = 0.83

$$\frac{1}{6} x^{6} \arctan \left(\frac{x \sqrt{-e}}{\sqrt{x^{2} e+d}}\right)-\frac{5}{96} d^{3} \arcsin \left(\frac{x e}{\sqrt{-d e}}\right) e^{(-3)}-\frac{1}{288}\left(2\left(4 x^{2} e^{(-1)}-5 d e^{(-2)}\right) x^{2}+15 d^{2} e^{(-3)}\right) \sqrt{-x^{2} e^{2}-d e x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/6*x^6*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) - 5/96*d^3*arcsin(x*e/sqrt(-d*e)
)*e^(-3) - 1/288*(2*(4*x^2*e^(-1) - 5*d*e^(-2))*x^2 + 15*d^2*e^(-3))*sqrt(-
x^2*e^2 - d*e)*x
```

3.4 $\int x^3 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=116

$$-\frac{3d^2\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32e^{5/2}}+\frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}}+\frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}}+\frac{1}{4}x^4\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] (3*d*x*Sqrt[d + e*x^2])/(32*(-e)^(3/2)) + (x^3*Sqrt[d + e*x^2])/(16*Sqrt[-e
]) + (x^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/4 - (3*d^2*Sqrt[-e]*ArcTanh
[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(32*e^(5/2))
```

Rubi [A] time = 0.0419833, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {5151, 321, 217, 206}

$$-\frac{3d^2\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32e^{5/2}}+\frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}}+\frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}}+\frac{1}{4}x^4\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (3*d*x*Sqrt[d + e*x^2])/(32*(-e)^(3/2)) + (x^3*Sqrt[d + e*x^2])/(16*Sqrt[-e
]) + (x^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/4 - (3*d^2*Sqrt[-e]*ArcTanh
[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(32*e^(5/2))
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{4}x^4 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{4}\sqrt{-e}\int \frac{x^4}{\sqrt{d+ex^2}} dx$$

$$= \frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}} + \frac{1}{4}x^4 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(3d)\int \frac{x^2}{\sqrt{d+ex^2}} dx}{16\sqrt{-e}}$$

$$= \frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}} + \frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}} + \frac{1}{4}x^4 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(3d^2)\int \frac{1}{\sqrt{d+ex^2}} dx}{32(-e)^{3/2}}$$

$$= \frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}} + \frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}} + \frac{1}{4}x^4 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(3d^2)\,\text{Subst}\left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+\cdots}}\right.}{32(-e)^{3/2}}$$

$$= \frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}} + \frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}} + \frac{1}{4}x^4 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{3d^2\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32e^{5/2}}$$

Mathematica [A] time = 0.0528404, size = 74, normalized size = 0.64

$$\frac{(8e^2x^4 - 3d^2)\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + \sqrt{-e}x\sqrt{d+ex^2}\,(3d - 2ex^2)}{32e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[-e]*x*(3*d - 2*e*x^2)*Sqrt[d + e*x^2] + (-3*d^2 + 8*e^2*x^4)*ArcTan[(
Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(32*e^2)
```

Maple [A] time = 0.041, size = 163, normalized size = 1.4

$$\frac{x^4}{4}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right) + \frac{x^5}{24\,d}\sqrt{-e}\sqrt{ex^2+d} - \frac{5\,x^3}{96\,e}\sqrt{-e}\sqrt{ex^2+d} + \frac{dx}{16\,e^2}\sqrt{-e}\sqrt{ex^2+d} - \frac{3\,d^2}{32}\sqrt{-e}\ln\left(x\sqrt{e} + \sqrt{ex\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/4*x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/24*(-e)^(1/2)/d*x^5*(e*x^2+d
)^(1/2)-5/96*(-e)^(1/2)/e*x^3*(e*x^2+d)^(1/2)+1/16*(-e)^(1/2)/e^2*d*x*(e*x^
2+d)^(1/2)-3/32*(-e)^(1/2)/e^(5/2)*d^2*ln(x*e^(1/2)+(e*x^2+d)^(1/2))-1/24*(
-e)^(1/2)/d*x^3*(e*x^2+d)^(3/2)/e+1/32*(-e)^(1/2)/e^2*x*(e*x^2+d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}x^4\arctan\left(\sqrt{-e}x, \sqrt{ex^2+d}\right) - d\sqrt{-e}\int -\frac{\sqrt{ex^2+dx^4}}{4\left(e^2x^4 + dex^2 - (ex^2+d)^2\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - d*sqrt(-e)*integrate(-1/4*sq
rt(e*x^2 + d)*x^4/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

Fricas [A] time = 2.27933, size = 153, normalized size = 1.32

$$-\frac{\left(2\,ex^3-3\,dx\right)\sqrt{ex^2+d}\sqrt{-e}-\left(8\,e^2x^4-3\,d^2\right)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{32\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/32*((2*e*x^3 - 3*d*x)*sqrt(e*x^2 + d)*sqrt(-e) - (8*e^2*x^4 - 3*d^2)*arc
tan(sqrt(-e)*x/sqrt(e*x^2 + d)))/e^2
```

Sympy [A] time = 2.47892, size = 102, normalized size = 0.88

$$\begin{cases} -\frac{3id^2\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32e^2}+\frac{3idx\sqrt{d+ex^2}}{32e^{\frac{3}{2}}}+\frac{ix^4\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4}-\frac{ix^3\sqrt{d+ex^2}}{16\sqrt{e}} & \text{for } e\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((-3*I*d**2*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(32*e**2) + 3*I*d*x*
sqrt(d + e*x**2)/(32*e**(3/2)) + I*x**4*atanh(sqrt(e)*x/sqrt(d + e*x**2))/4
 - I*x**3*sqrt(d + e*x**2)/(16*sqrt(e)), Ne(e, 0)), (0, True))
```

Giac [A] time = 1.16984, size = 101, normalized size = 0.87

$$\frac{1}{4}\,x^4\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)+\frac{3}{32}\,d^2\arcsin\left(\frac{xe}{\sqrt{-de}}\right)e^{(-2)}-\frac{1}{32}\,\sqrt{-x^2e^2-de}\left(2\,x^2e^{(-1)}-3\,de^{(-2)}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*x^4*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) + 3/32*d^2*arcsin(x*e/sqrt(-d*e)
)*e^(-2) - 1/32*sqrt(-x^2*e^2 - d*e)*(2*x^2*e^(-1) - 3*d*e^(-2))*x
```

3.5 $\int x\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=88

$$\frac{d\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4e^{3/2}}+\frac{x\sqrt{d+ex^2}}{4\sqrt{-e}}+\frac{1}{2}x^2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] (x*Sqrt[d + e*x^2])/(4*Sqrt[-e]) + (x^2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]
])/2 + (d*Sqrt[-e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(4*e^(3/2))
```

Rubi [A] time = 0.0286042, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5151, 321, 217, 206}

$$\frac{d\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4e^{3/2}}+\frac{x\sqrt{d+ex^2}}{4\sqrt{-e}}+\frac{1}{2}x^2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(4*Sqrt[-e]) + (x^2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]
])/2 + (d*Sqrt[-e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(4*e^(3/2))
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{1}{2}x^2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{2}\sqrt{-e}\int\frac{x^2}{\sqrt{d+ex^2}}dx$$
$$= \frac{x\sqrt{d+ex^2}}{4\sqrt{-e}} + \frac{1}{2}x^2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{d\int\frac{1}{\sqrt{d+ex^2}}dx}{4\sqrt{-e}}$$
$$= \frac{x\sqrt{d+ex^2}}{4\sqrt{-e}} + \frac{1}{2}x^2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{d\,\text{Subst}\left(\int\frac{1}{1-ex^2}dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{4\sqrt{-e}}$$
$$= \frac{x\sqrt{d+ex^2}}{4\sqrt{-e}} + \frac{1}{2}x^2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{-e^2}}$$

Mathematica [A] time = 0.0397558, size = 59, normalized size = 0.67

$$\frac{\left(d+2ex^2\right)\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \sqrt{-e}x\sqrt{d+ex^2}}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (-(Sqrt[-e]*x*Sqrt[d + e*x^2]) + (d + 2*e*x^2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d +
 e*x^2]])/(4*e)
```

Maple [A] time = 0.04, size = 116, normalized size = 1.3

$$\frac{x^2}{2}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right) + \frac{x^3}{8d}\sqrt{-e}\sqrt{ex^2+d} - \frac{x}{8e}\sqrt{-e}\sqrt{ex^2+d} + \frac{d}{4}\sqrt{-e}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{3}{2}} - \frac{x}{8de}\sqrt{-e}\left(ex^2+d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/2*x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/8*(-e)^(1/2)/d*x^3*(e*x^2+d)
^(1/2)-1/8*(-e)^(1/2)/e*x*(e*x^2+d)^(1/2)+1/4*(-e)^(1/2)/e^(3/2)*d*ln(x*e^(
1/2)+(e*x^2+d)^(1/2))-1/8*(-e)^(1/2)/d*x*(e*x^2+d)^(3/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2\arctan\left(\sqrt{-e}x, \sqrt{ex^2+d}\right) - d\sqrt{-e}\int -\frac{\sqrt{ex^2+dx^2}}{2\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - d*sqrt(-e)*integrate(-1/2*sq
rt(e*x^2 + d)*x^2/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

Fricas [A] time = 2.35485, size = 119, normalized size = 1.35

$$-\frac{\sqrt{ex^2+d}\sqrt{-e}x - \left(2\,ex^2+d\right)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(e*x^2 + d)*sqrt(-e)*x - (2*e*x^2 + d)*arctan(sqrt(-e)*x/sqrt(e*x
^2 + d)))/e
```

Sympy [A] time = 0.832721, size = 71, normalized size = 0.81

$$\begin{cases} \frac{id\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4e} + \frac{ix^2\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2} - \frac{ix\sqrt{d+ex^2}}{4\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((I*d*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(4*e) + I*x**2*atanh(sqrt(
e)*x/sqrt(d + e*x**2))/2 - I*x*sqrt(d + e*x**2)/(4*sqrt(e)), Ne(e, 0)), (0,
 True))
```

Giac [A] time = 1.1828, size = 84, normalized size = 0.95

$$\frac{1}{2}\,x^2\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right) - \frac{1}{4}\,d\arcsin\left(\frac{xe}{\sqrt{-de}}\right)e^{(-1)} - \frac{1}{4}\,\sqrt{-x^2e^2-de}xe^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) - 1/4*d*arcsin(x*e/sqrt(-d*e))*e
^(-1) - 1/4*sqrt(-x^2*e^2 - d*e)*x*e^(-1)
```

3.6 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx$

Optimal. Leaf size=288

$$\frac{\sqrt{d}\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\sqrt{d+ex^2}}-\frac{\sqrt{d}\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}}+\log(x)\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)+\frac{\sqrt{d}\sqrt{-e}\sqrt{\frac{ex^2}{d}\cdots}}{\cdots}$$

```
[Out] -(Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/(2*S
qrt[e]*Sqrt[d + e*x^2]) + (Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sq
rt[e]*x)/Sqrt[d]]*Log[1 - E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(Sqrt[e]*Sqr
t[d + e*x^2]) - (Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/S
qrt[d]]*Log[x])/(Sqrt[e]*Sqrt[d + e*x^2]) + ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*
x^2]]*Log[x] + (Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*PolyLog[2, E^(2*ArcSin
h[(Sqrt[e]*x)/Sqrt[d]])])/(2*Sqrt[e]*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.166801, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5149, 2327, 2325, 5659, 3716, 2190, 2279, 2391}

$$\frac{\sqrt{d}\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\sqrt{d+ex^2}}-\frac{\sqrt{d}\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}}+\log(x)\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)+\frac{\sqrt{d}\sqrt{-e}\sqrt{\frac{ex^2}{d}\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x,x]
```

```
[Out] -(Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/(2*S
qrt[e]*Sqrt[d + e*x^2]) + (Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sq
rt[e]*x)/Sqrt[d]]*Log[1 - E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(Sqrt[e]*Sqr
t[d + e*x^2]) - (Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/S
qrt[d]]*Log[x])/(Sqrt[e]*Sqrt[d + e*x^2]) + ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*
x^2]]*Log[x] + (Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*PolyLog[2, E^(2*ArcSin
h[(Sqrt[e]*x)/Sqrt[d]])])/(2*Sqrt[e]*Sqrt[d + e*x^2])
```

Rule 5149

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]/(x_), x_Symbol] :> Simp
[ArcTan[(c*x)/Sqrt[a + b*x^2]]*Log[x], x] - Dist[c, Int[Log[x]/Sqrt[a + b*x
^2], x], x] /; FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

Rule 2327

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Dist[Sqrt[1 + (e*x^2)/d]/Sqrt[d + e*x^2], Int[(a + b*Log[c*x^n])/Sqr
t[1 + (e*x^2)/d], x], x] /; FreeQ[{a, b, c, d, e, n}, x] &&  !GtQ[d, 0]
```

Rule 2325

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSinh[(Rt[e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[e, 2], x]
 - Dist[(b*n)/Rt[e, 2], Int[ArcSinh[(Rt[e, 2]*x)/Sqrt[d]]/x, x], x] /; Free
```

```
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log(x) - \sqrt{-e}\int \frac{\log(x)}{\sqrt{d+ex^2}}\,dx$$

$$= \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log(x) - \frac{\left(\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{\log(x)}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{e}\sqrt{d+ex^2}} + \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{\left(\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{\sinh^{-1}(}{x}}{\sqrt{e}\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{e}\sqrt{d+ex^2}} + \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{\left(\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}(}{\sqrt{e}}$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{e}\sqrt{d+ex^2}} + \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{e}\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{}}{}$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{e}\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{}}{}$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{e}\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{}}{}$$

Mathematica [A] time = 2.4616, size = 171, normalized size = 0.59

$$\frac{\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\left(-\text{PolyLog}\left(2,e^{-2\sinh^{-1}\left(x\sqrt{\frac{e}{d}}\right)}\right) - 2\log(x)\log\left(\sqrt{\frac{ex^2}{d}+1}+x\sqrt{\frac{e}{d}}\right) + \sinh^{-1}\left(x\sqrt{\frac{e}{d}}\right)^2 + 2\sinh^{-1}\left(x\sqrt{\frac{e}{d}}\right)\log(1}{2\sqrt{\frac{e}{d}}\sqrt{d+ex^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x,x]
```

```
[Out] ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]*Log[x] + (Sqrt[-e]*Sqrt[1 + (e*x^2)/d]
*(ArcSinh[Sqrt[e/d]*x]^2 + 2*ArcSinh[Sqrt[e/d]*x]*Log[1 - E^(-2*ArcSinh[Sqr
t[e/d]*x])] - 2*Log[x]*Log[Sqrt[e/d]*x + Sqrt[1 + (e*x^2)/d]] - PolyLog[2,
E^(-2*ArcSinh[Sqrt[e/d]*x])]))/(2*Sqrt[e/d]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.316, size = 0, normalized size = 0.

$$\int \frac{1}{x}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x,x)
```

```
[Out] Integral(atan(x*sqrt(-e)/sqrt(d + e*x**2))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x, x)
```

3.7 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3} dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{-e}\sqrt{d+ex^2}}{2dx}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(2*d*x) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/
(2*x^2)
```

Rubi [A] time = 0.0187291, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {5151, 264}

$$-\frac{\sqrt{-e}\sqrt{d+ex^2}}{2dx}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^3,x]
```

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(2*d*x) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/
(2*x^2)
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3} dx &= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{2x^2}+\frac{1}{2}\sqrt{-e}\int \frac{1}{x^2\sqrt{d+ex^2}} dx\\ &= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{2dx}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0363215, size = 54, normalized size = 0.95

$$-\frac{\sqrt{-e}x\sqrt{d+ex^2}+d\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^3,x]
```

```
[Out] -(Sqrt[-e]*x*Sqrt[d + e*x^2] + d*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(2*d
*x^2)
```

Maple [A] time = 0.039, size = 67, normalized size = 1.2

$$-\frac{1}{2x^2}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)-\frac{1}{2d^2x}\sqrt{-e}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{ex}{2d^2}\sqrt{-e}\sqrt{ex^2+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x)
```

```
[Out] -1/2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2-1/2*(-e)^(1/2)/d^2/x*(e*x^2+d
)^(3/2)+1/2*(-e)^(1/2)/d^2*e*x*(e*x^2+d)^(1/2)
```

Maxima [A] time = 1.14299, size = 78, normalized size = 1.37

$$-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{2x^2}-\frac{\sqrt{-e}ex^2+d\sqrt{-e}}{2\sqrt{ex^2+d}dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^2 - 1/2*(sqrt(-e)*e*x^2 + d*sqrt(
-e))/(sqrt(e*x^2 + d)*d*x)
```

Fricas [A] time = 2.35543, size = 111, normalized size = 1.95

$$-\frac{\sqrt{ex^2+d}\sqrt{-e}x+d\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(e*x^2 + d)*sqrt(-e)*x + d*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(d
*x^2)
```

Sympy [A] time = 3.24586, size = 53, normalized size = 0.93

$$-\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{2x^2}-\frac{\sqrt{e}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**3,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/(2*x**2) - sqrt(e)*sqrt(-e)*sqrt(d/(e*x*
*2) + 1)/(2*d)
```

Giac [B] time = 1.2364, size = 142, normalized size = 2.49

$$\frac{xe^3}{4\left(\sqrt{-x^2e^2 - dee} - \sqrt{-dee}\right)d} - \frac{\left(\sqrt{-x^2e^2 - dee} - \sqrt{-dee}\right)e^{(-1)}}{4\,dx} - \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 1/4*x*e^3/((sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)*d) - 1/4*(sqrt(-x^2*e^2
- d*e)*e - sqrt(-d*e)*e)*e^(-1)/(d*x) - 1/2*arctan(x*sqrt(-e)/sqrt(x^2*e +
d))/x^2
```

3.8 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx$

Optimal. Leaf size=85

$$-\frac{(-e)^{3/2}\sqrt{d+ex^2}}{6d^2x}-\frac{\sqrt{-e}\sqrt{d+ex^2}}{12dx^3}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(12*d*x^3) - ((-e)^(3/2)*Sqrt[d + e*x^2])/(6*d^
2*x) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(4*x^4)
```

Rubi [A] time = 0.0281823, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5151, 271, 264}

$$-\frac{(-e)^{3/2}\sqrt{d+ex^2}}{6d^2x}-\frac{\sqrt{-e}\sqrt{d+ex^2}}{12dx^3}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(12*d*x^3) - ((-e)^(3/2)*Sqrt[d + e*x^2])/(6*d^
2*x) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(4*x^4)
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}dx = -\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{4x^4} + \frac{1}{4}\sqrt{-e}\int \frac{1}{x^4\sqrt{d+ex^2}}dx$$
$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{12dx^3} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{4x^4} + \frac{(-e)^{3/2}\int \frac{1}{x^2\sqrt{d+ex^2}}dx}{6d}$$
$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{12dx^3} - \frac{(-e)^{3/2}\sqrt{d+ex^2}}{6d^2x} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

Mathematica [A] time = 0.0441768, size = 67, normalized size = 0.79

$$\frac{\sqrt{-e}x\sqrt{d+ex^2}\left(2ex^2-d\right)-3d^2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{12d^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

```
[Out] (Sqrt[-e]*x*Sqrt[d + e*x^2]*(-d + 2*e*x^2) - 3*d^2*ArcTan[(Sqrt[-e]*x)/Sqrt
[d + e*x^2]])/(12*d^2*x^4)
```

Maple [A] time = 0.039, size = 69, normalized size = 0.8

$$-\frac{1}{4x^4}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)+\frac{e}{4d^2x}\sqrt{-e}\sqrt{ex^2+d}-\frac{1}{12d^2x^3}\sqrt{-e}\left(ex^2+d\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x)
```

```
[Out] -1/4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4+1/4*(-e)^(1/2)*e/d^2/x*(e*x^2
+d)^(1/2)-1/12*(-e)^(1/2)/d^2/x^3*(e*x^2+d)^(3/2)
```

Maxima [A] time = 1.01178, size = 92, normalized size = 1.08

$$\frac{\sqrt{ex^2+d}\sqrt{-e}e}{4d^2x}-\frac{\left(ex^2+d\right)^{\frac{3}{2}}\sqrt{-e}}{12d^2x^3}-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(e*x^2 + d)*sqrt(-e)*e/(d^2*x) - 1/12*(e*x^2 + d)^(3/2)*sqrt(-e)/(d
^2*x^3) - 1/4*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^4
```

Fricas [A] time = 2.5242, size = 139, normalized size = 1.64

$$-\frac{3d^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\left(2ex^3-dx\right)\sqrt{ex^2+d}\sqrt{-e}}{12d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(3*d^2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*e*x^3 - d*x)*sqrt(e*x^
2 + d)*sqrt(-e))/(d^2*x^4)
```

Sympy [A] time = 5.53503, size = 83, normalized size = 0.98

$$-\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{4x^4}-\frac{\sqrt{e}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{12dx^2}+\frac{e^{\frac{3}{2}}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**5,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/(4*x**4) - sqrt(e)*sqrt(-e)*sqrt(d/(e*x*
*2) + 1)/(12*d*x**2) + e**(3/2)*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(6*d**2)
```

Giac [B] time = 1.20594, size = 267, normalized size = 3.14

$$-\frac{x^3\left(\frac{9\left(\sqrt{-x^2e^2-dee}-\sqrt{-dee}\right)^2e^{(-2)}}{x^2}+e^2\right)e^6}{96\left(\sqrt{-x^2e^2-dee}-\sqrt{-dee}\right)^3d^2}-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{4\,x^4}+\frac{\left(\frac{9\left(\sqrt{-x^2e^2-dee}-\sqrt{-dee}\right)d^4e^6}{x}+\frac{\left(\sqrt{-x^2e^2-dee}-\sqrt{-dee}\right)^3d^4e^2}{x^3}\right)e^{(-6)}}{96\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] -1/96*x^3*(9*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^2*e^(-2)/x^2 + e^2)*e^
6/((sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^3*d^2) - 1/4*arctan(x*sqrt(-e)/s
qrt(x^2*e + d))/x^4 + 1/96*(9*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)*d^4*e
^6/x + (sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^3*d^4*e^2/x^3)*e^(-6)/d^6
```

3.9 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx$

Optimal. Leaf size=113

$$-\frac{4(-e)^{5/2}\sqrt{d+ex^2}}{45d^3x}-\frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{\sqrt{-e}\sqrt{d+ex^2}}{30dx^5}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(30*d*x^5) - (2*(-e)^(3/2)*Sqrt[d + e*x^2])/(45
*d^2*x^3) - (4*(-e)^(5/2)*Sqrt[d + e*x^2])/(45*d^3*x) - ArcTan[(Sqrt[-e]*x)
/Sqrt[d + e*x^2]]/(6*x^6)
```

Rubi [A] time = 0.0385326, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5151, 271, 264}

$$-\frac{4(-e)^{5/2}\sqrt{d+ex^2}}{45d^3x}-\frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{\sqrt{-e}\sqrt{d+ex^2}}{30dx^5}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(30*d*x^5) - (2*(-e)^(3/2)*Sqrt[d + e*x^2])/(45
*d^2*x^3) - (4*(-e)^(5/2)*Sqrt[d + e*x^2])/(45*d^3*x) - ArcTan[(Sqrt[-e]*x)
/Sqrt[d + e*x^2]]/(6*x^6)
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6} + \frac{1}{6}\sqrt{-e}\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx$$

$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{30dx^5} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6} + \frac{\left(2(-e)^{3/2}\right)\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{15d}$$

$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{30dx^5} - \frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6} + \frac{\left(4(-e)^{5/2}\right)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{45d^2}$$

$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{30dx^5} - \frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3} - \frac{4(-e)^{5/2}\sqrt{d+ex^2}}{45d^3x} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

Mathematica [A] time = 0.0571997, size = 78, normalized size = 0.69

$$\frac{\sqrt{-e}x\sqrt{d+ex^2}\left(-3d^2+4dex^2-8e^2x^4\right)-15d^3\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{90d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

```
[Out] (Sqrt[-e]*x*Sqrt[d + e*x^2]*(-3*d^2 + 4*d*e*x^2 - 8*e^2*x^4) - 15*d^3*ArcTa
n[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(90*d^3*x^6)
```

Maple [A] time = 0.042, size = 117, normalized size = 1.

$$-\frac{1}{6x^6}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)+\frac{e}{18d^2x^3}\sqrt{-e}\sqrt{ex^2+d}-\frac{e^2}{9d^3x}\sqrt{-e}\sqrt{ex^2+d}-\frac{1}{30d^2x^5}\sqrt{-e}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{e}{45d^3x^3}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x)
```

```
[Out] -1/6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6+1/18*(-e)^(1/2)*e/d^2/x^3*(e*
x^2+d)^(1/2)-1/9*(-e)^(1/2)*e^2/d^3/x*(e*x^2+d)^(1/2)-1/30*(-e)^(1/2)/d^2/x
^5*(e*x^2+d)^(3/2)+1/45*(-e)^(1/2)/d^3*e/x^3*(e*x^2+d)^(3/2)
```

Maxima [A] time = 1.01803, size = 147, normalized size = 1.3

$$-\frac{\left(2e^2x^4+dex^2-d^2\right)\sqrt{-e}e}{18\sqrt{ex^2+d}d^3x^3}-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{6x^6}+\frac{\left(2e^2x^4-dex^2-3d^2\right)\sqrt{ex^2+d}\sqrt{-e}}{90d^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="maxima")
```

```
[Out] -1/18*(2*e^2*x^4 + d*e*x^2 - d^2)*sqrt(-e)*e/(sqrt(e*x^2 + d)*d^3*x^3) - 1/
6*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^6 + 1/90*(2*e^2*x^4 - d*e*x^2 - 3*d^
2)*sqrt(e*x^2 + d)*sqrt(-e)/(d^3*x^5)
```

Fricas [A] time = 2.68875, size = 165, normalized size = 1.46

$$-\frac{15\,d^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+\left(8\,e^2x^5-4\,dex^3+3\,d^2x\right)\sqrt{ex^2+d}\sqrt{-e}}{90\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="fricas")
```

```
[Out] -1/90*(15*d^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (8*e^2*x^5 - 4*d*e*x^3 +
 3*d^2*x)*sqrt(e*x^2 + d)*sqrt(-e))/(d^3*x^6)
```

Sympy [B] time = 12.4261, size = 352, normalized size = 3.12

$$-\frac{d^4e^{\frac{9}{2}}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{2\left(15d^5e^4x^4+30d^4e^5x^6+15d^3e^6x^8\right)}-\frac{d^3e^{\frac{11}{2}}x^2\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{3\left(15d^5e^4x^4+30d^4e^5x^6+15d^3e^6x^8\right)}-\frac{d^2e^{\frac{13}{2}}x^4\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{2\left(15d^5e^4x^4+30d^4e^5x^6+15d^3e^6x^8\right)}-15$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**7,x)
```

```
[Out] -d**4*e**(9/2)*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(2*(15*d**5*e**4*x**4 + 30*d**
4*e**5*x**6 + 15*d**3*e**6*x**8)) - d**3*e**(11/2)*x**2*sqrt(-e)*sqrt(d/(e*
x**2) + 1)/(3*(15*d**5*e**4*x**4 + 30*d**4*e**5*x**6 + 15*d**3*e**6*x**8))
- d**2*e**(13/2)*x**4*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(2*(15*d**5*e**4*x**4 +
 30*d**4*e**5*x**6 + 15*d**3*e**6*x**8)) - 2*d*e**(15/2)*x**6*sqrt(-e)*sqrt
(d/(e*x**2) + 1)/(15*d**5*e**4*x**4 + 30*d**4*e**5*x**6 + 15*d**3*e**6*x**8
) - 4*e**(17/2)*x**8*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(3*(15*d**5*e**4*x**4 +
30*d**4*e**5*x**6 + 15*d**3*e**6*x**8)) - atan(x*sqrt(-e)/sqrt(d + e*x**2))
/(6*x**6)
```

Giac [B] time = 1.2122, size = 381, normalized size = 3.37

$$\frac{x^5\left(\frac{25\left(\sqrt{-x^2e^2-dee}-\sqrt{-dee}\right)^2e^{(-1)}}{x^2}+\frac{150\left(\sqrt{-x^2e^2-dee}-\sqrt{-dee}\right)^4e^{(-5)}}{x^4}+3\,e^3\right)e^{10}}{2880\left(\sqrt{-x^2e^2-dee}-\sqrt{-dee}\right)^5d^3}-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{6\,x^6}-\frac{\left(\frac{150\left(\sqrt{-x^2e^2-dee}-\sqrt{-dee}\right)d^{12}e^{16}}{x}+\frac{25(\sqrt{}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="giac")
```

```
[Out] 1/2880*x^5*(25*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^2*e^(-1)/x^2 + 150*(
sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^4*e^(-5)/x^4 + 3*e^3)*e^10/((sqrt(-x
^2*e^2 - d*e)*e - sqrt(-d*e)*e)^5*d^3) - 1/6*arctan(x*sqrt(-e)/sqrt(x^2*e +
 d))/x^6 - 1/2880*(150*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)*d^12*e^16/x
+ 25*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^3*d^12*e^12/x^3 + 3*(sqrt(-x^2
*e^2 - d*e)*e - sqrt(-d*e)*e)^5*d^12*e^8/x^5)*e^(-15)/d^15
```

3.10 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx$

Optimal. Leaf size=141

$$-\frac{2(-e)^{7/2}\sqrt{d+ex^2}}{35d^4x}-\frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3}-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5}-\frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(56*d*x^7) - (3*(-e)^(3/2)*Sqrt[d + e*x^2])/(14
0*d^2*x^5) - ((-e)^(5/2)*Sqrt[d + e*x^2])/(35*d^3*x^3) - (2*(-e)^(7/2)*Sqrt
[d + e*x^2])/(35*d^4*x) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(8*x^8)
```

Rubi [A] time = 0.0495252, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5151, 271, 264}

$$-\frac{2(-e)^{7/2}\sqrt{d+ex^2}}{35d^4x}-\frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3}-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5}-\frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(56*d*x^7) - (3*(-e)^(3/2)*Sqrt[d + e*x^2])/(14
0*d^2*x^5) - ((-e)^(5/2)*Sqrt[d + e*x^2])/(35*d^3*x^3) - (2*(-e)^(7/2)*Sqrt
[d + e*x^2])/(35*d^4*x) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(8*x^8)
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{1}{8}\sqrt{-e}\int \frac{1}{x^8\sqrt{d+ex^2}}\,dx$$

$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{\left(3(-e)^{3/2}\right)\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx}{28d}$$

$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{\left(3(-e)^{5/2}\right)\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{35d^2}$$

$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{\left(2(-e)^{7/2}\right)\int \frac{1}{x^2\sqrt{d+ex^2}}}{35d^3}$$

$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3} - \frac{2(-e)^{7/2}\sqrt{d+ex^2}}{35d^4x} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

Mathematica [A] time = 0.0682556, size = 89, normalized size = 0.63

$$\frac{\sqrt{-e}x\sqrt{d+ex^2}\left(6d^2ex^2-5d^3-8de^2x^4+16e^3x^6\right)-35d^4\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{280d^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

```
[Out] (Sqrt[-e]*x*Sqrt[d + e*x^2]*(-5*d^3 + 6*d^2*e*x^2 - 8*d*e^2*x^4 + 16*e^3*x^
6) - 35*d^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(280*d^4*x^8)
```

Maple [A] time = 0.043, size = 167, normalized size = 1.2

$$-\frac{1}{8x^8}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)+\frac{e}{40d^2x^5}\sqrt{-e}\sqrt{ex^2+d}-\frac{e^2}{30d^3x^3}\sqrt{-e}\sqrt{ex^2+d}+\frac{e^3}{15d^4x}\sqrt{-e}\sqrt{ex^2+d}-\frac{1}{56d^2x^7}\sqrt{-e}\,(e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x)
```

```
[Out] -1/8*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^8+1/40*(-e)^(1/2)*e/d^2/x^5*(e*
x^2+d)^(1/2)-1/30*(-e)^(1/2)*e^2/d^3/x^3*(e*x^2+d)^(1/2)+1/15*(-e)^(1/2)*e^
3/d^4/x*(e*x^2+d)^(1/2)-1/56*(-e)^(1/2)/d^2/x^7*(e*x^2+d)^(3/2)+1/70*(-e)^(
1/2)/d^3*e/x^5*(e*x^2+d)^(3/2)-1/105*(-e)^(1/2)/d^4*e^2/x^3*(e*x^2+d)^(3/2)
```

Maxima [A] time = 1.0016, size = 178, normalized size = 1.26

$$\frac{\left(8e^3x^6+4de^2x^4-d^2ex^2+3d^3\right)\sqrt{-e}e}{120\sqrt{ex^2+d}d^4x^5}-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{8x^8}-\frac{\left(8e^3x^6-4de^2x^4+3d^2ex^2+15d^3\right)\sqrt{ex^2+d}\sqrt{-e}}{840d^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="maxima")
```

```
[Out] 1/120*(8*e^3*x^6 + 4*d*e^2*x^4 - d^2*e*x^2 + 3*d^3)*sqrt(-e)*e/(sqrt(e*x^2
+ d)*d^4*x^5) - 1/8*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^8 - 1/840*(8*e^3*x
^6 - 4*d*e^2*x^4 + 3*d^2*e*x^2 + 15*d^3)*sqrt(e*x^2 + d)*sqrt(-e)/(d^4*x^7)
```

Fricas [A] time = 3.18562, size = 189, normalized size = 1.34

$$-\frac{35\,d^4\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\left(16\,e^3x^7-8\,de^2x^5+6\,d^2ex^3-5\,d^3x\right)\sqrt{ex^2+d}\sqrt{-e}}{280\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="fricas")
```

```
[Out] -1/280*(35*d^4*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (16*e^3*x^7 - 8*d*e^2*x
^5 + 6*d^2*e*x^3 - 5*d^3*x)*sqrt(e*x^2 + d)*sqrt(-e))/(d^4*x^8)
```

Sympy [B] time = 31.1258, size = 575, normalized size = 4.08

$$-\frac{5d^6e^{\frac{19}{2}}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{8\left(35d^7e^9x^6+105d^6e^{10}x^8+105d^5e^{11}x^{10}+35d^4e^{12}x^{12}\right)}-\frac{9d^5e^{\frac{21}{2}}x^2\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{8\left(35d^7e^9x^6+105d^6e^{10}x^8+105d^5e^{11}x^{10}+35d^4e^{12}x^{12}\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**9,x)
```

```
[Out] -5*d**6*e**(19/2)*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(8*(35*d**7*e**9*x**6 + 105
*d**6*e**10*x**8 + 105*d**5*e**11*x**10 + 35*d**4*e**12*x**12)) - 9*d**5*e*
*(21/2)*x**2*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(8*(35*d**7*e**9*x**6 + 105*d**6
*e**10*x**8 + 105*d**5*e**11*x**10 + 35*d**4*e**12*x**12)) - 5*d**4*e**(23/
2)*x**4*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(8*(35*d**7*e**9*x**6 + 105*d**6*e**1
0*x**8 + 105*d**5*e**11*x**10 + 35*d**4*e**12*x**12)) + 5*d**3*e**(25/2)*x*
*6*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(8*(35*d**7*e**9*x**6 + 105*d**6*e**10*x**
8 + 105*d**5*e**11*x**10 + 35*d**4*e**12*x**12)) + 15*d**2*e**(27/2)*x**8*s
qrt(-e)*sqrt(d/(e*x**2) + 1)/(4*(35*d**7*e**9*x**6 + 105*d**6*e**10*x**8 +
105*d**5*e**11*x**10 + 35*d**4*e**12*x**12)) + 5*d*e**(29/2)*x**10*sqrt(-e)
*sqrt(d/(e*x**2) + 1)/(35*d**7*e**9*x**6 + 105*d**6*e**10*x**8 + 105*d**5*e
**11*x**10 + 35*d**4*e**12*x**12) + 2*e**(31/2)*x**12*sqrt(-e)*sqrt(d/(e*x*
*2) + 1)/(35*d**7*e**9*x**6 + 105*d**6*e**10*x**8 + 105*d**5*e**11*x**10 +
35*d**4*e**12*x**12) - atan(x*sqrt(-e)/sqrt(d + e*x**2))/(8*x**8)
```

Giac [B] time = 1.21304, size = 487, normalized size = 3.45

$$-\frac{x^7\left(\frac{245\left(\sqrt{-x^2e^2-dee}-\sqrt{-dee}\right)^4e^{(-4)}}{x^4}+\frac{1225\left(\sqrt{-x^2e^2-dee}-\sqrt{-dee}\right)^6e^{(-8)}}{x^6}+\frac{49\left(\sqrt{-x^2e^2-dee}-\sqrt{-dee}\right)^2}{x^2}+5\,e^4\right)e^{14}}{35840\left(\sqrt{-x^2e^2-dee}-\sqrt{-dee}\right)^7d^4}-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{8\,x^8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="giac")
```

```
[Out] -1/35840*x^7*(245*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^4*e^(-4)/x^4 + 12
25*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^6*e^(-8)/x^6 + 49*(sqrt(-x^2*e^2
 - d*e)*e - sqrt(-d*e)*e)^2/x^2 + 5*e^4)*e^14/((sqrt(-x^2*e^2 - d*e)*e - sq
rt(-d*e)*e)^7*d^4) - 1/8*arctan(x*sqrt(-e)/sqrt(x^2*e + d))/x^8 + 1/35840*(
1225*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)*d^24*e^30/x + 245*(sqrt(-x^2*e
^2 - d*e)*e - sqrt(-d*e)*e)^3*d^24*e^26/x^3 + 49*(sqrt(-x^2*e^2 - d*e)*e -
sqrt(-d*e)*e)^5*d^24*e^22/x^5 + 5*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^7
*d^24*e^18/x^7)*e^(-28)/d^28
```

3.11 $\int x^6 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=124

$$-\frac{d^2\left(d+ex^2\right)^{3/2}}{7(-e)^{7/2}}+\frac{d^3\sqrt{d+ex^2}}{7(-e)^{7/2}}-\frac{\left(d+ex^2\right)^{7/2}}{49(-e)^{7/2}}+\frac{3d\left(d+ex^2\right)^{5/2}}{35(-e)^{7/2}}+\frac{1}{7}x^7\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] (d^3*Sqrt[d + e*x^2])/(7*(-e)^(7/2)) - (d^2*(d + e*x^2)^(3/2))/(7*(-e)^(7/2
)) + (3*d*(d + e*x^2)^(5/2))/(35*(-e)^(7/2)) - (d + e*x^2)^(7/2)/(49*(-e)^(
7/2)) + (x^7*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7
```

Rubi [A] time = 0.0670544, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5151, 266, 43}

$$-\frac{d^2\left(d+ex^2\right)^{3/2}}{7(-e)^{7/2}}+\frac{d^3\sqrt{d+ex^2}}{7(-e)^{7/2}}-\frac{\left(d+ex^2\right)^{7/2}}{49(-e)^{7/2}}+\frac{3d\left(d+ex^2\right)^{5/2}}{35(-e)^{7/2}}+\frac{1}{7}x^7\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d^3*Sqrt[d + e*x^2])/(7*(-e)^(7/2)) - (d^2*(d + e*x^2)^(3/2))/(7*(-e)^(7/2
)) + (3*d*(d + e*x^2)^(5/2))/(35*(-e)^(7/2)) - (d + e*x^2)^(7/2)/(49*(-e)^(
7/2)) + (x^7*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^6 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{7}x^7 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{7}\sqrt{-e}\int \frac{x^7}{\sqrt{d+ex^2}} dx$$
$$= \frac{1}{7}x^7 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{14}\sqrt{-e}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{d+ex}} dx, x, x^2\right)$$
$$= \frac{1}{7}x^7 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{14}\sqrt{-e}\,\text{Subst}\left(\int \left(-\frac{d^3}{e^3\sqrt{d+ex}} + \frac{3d^2\sqrt{d+ex}}{e^3} - \frac{3d(d+ex)^{3/2}}{e^3} + \right.\right.$$
$$= \frac{d^3\sqrt{d+ex^2}}{7(-e)^{7/2}} - \frac{d^2\left(d+ex^2\right)^{3/2}}{7(-e)^{7/2}} + \frac{3d\left(d+ex^2\right)^{5/2}}{35(-e)^{7/2}} - \frac{\left(d+ex^2\right)^{7/2}}{49(-e)^{7/2}} + \frac{1}{7}x^7 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

Mathematica [A] time = 0.104445, size = 83, normalized size = 0.67

$$\frac{\sqrt{d+ex^2}\left(-8d^2ex^2+16d^3+6de^2x^4-5e^3x^6\right)}{245(-e)^{7/2}} + \frac{1}{7}x^7 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[d + e*x^2]*(16*d^3 - 8*d^2*e*x^2 + 6*d*e^2*x^4 - 5*e^3*x^6))/(245*(-e
)^(7/2)) + (x^7*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7
```

Maple [B] time = 0.046, size = 230, normalized size = 1.9

$$\frac{x^7}{7}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right) + \frac{x^8}{63\,d}\sqrt{-e}\sqrt{ex^2+d} - \frac{8\,x^6}{441\,e}\sqrt{-e}\sqrt{ex^2+d} + \frac{16\,dx^4}{735\,e^2}\sqrt{-e}\sqrt{ex^2+d} - \frac{64\,d^2x^2}{2205\,e^3}\sqrt{-e}\sqrt{ex^2+d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/7*x^7*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/63*(-e)^(1/2)/d*x^8*(e*x^2+d
)^(1/2)-8/441*(-e)^(1/2)/e*x^6*(e*x^2+d)^(1/2)+16/735*(-e)^(1/2)/e^2*d*x^4*
(e*x^2+d)^(1/2)-64/2205*(-e)^(1/2)/e^3*d^2*x^2*(e*x^2+d)^(1/2)+128/2205*(-e
)^(1/2)/e^4*d^3*(e*x^2+d)^(1/2)-1/63*(-e)^(1/2)/d*x^6*(e*x^2+d)^(3/2)/e+2/1
47*(-e)^(1/2)/e^2*x^4*(e*x^2+d)^(3/2)-8/735*(-e)^(1/2)*d/e^3*x^2*(e*x^2+d)^
(3/2)+16/2205*(-e)^(1/2)*d^2/e^4*(e*x^2+d)^(3/2)
```

Maxima [A] time = 1.00712, size = 225, normalized size = 1.81

$$\frac{1}{7}x^7\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{\left(35\left(ex^2+d\right)^{\frac{9}{2}} - 135\left(ex^2+d\right)^{\frac{7}{2}}d + 189\left(ex^2+d\right)^{\frac{5}{2}}d^2 - 105\left(ex^2+d\right)^{\frac{3}{2}}d^3\right)\sqrt{-e}}{2205\,de^4} + \frac{\left(35\left(ex^2+d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/7*x^7*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/2205*(35*(e*x^2 + d)^(9/2) -
 135*(e*x^2 + d)^(7/2)*d + 189*(e*x^2 + d)^(5/2)*d^2 - 105*(e*x^2 + d)^(3/2
```

```
)*d^3)*sqrt(-e)/(d*e^4) + 1/2205*(35*(e*x^2 + d)^(9/2) - 180*(e*x^2 + d)^(7
/2)*d + 378*(e*x^2 + d)^(5/2)*d^2 - 420*(e*x^2 + d)^(3/2)*d^3 + 315*sqrt(e*
x^2 + d)*d^4)*sqrt(-e)/(d*e^4)
```

Fricas [A] time = 2.3296, size = 182, normalized size = 1.47

$$\frac{35\,e^4x^7\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\left(5\,e^3x^6-6\,de^2x^4+8\,d^2ex^2-16\,d^3\right)\sqrt{ex^2+d}\sqrt{-e}}{245\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/245*(35*e^4*x^7*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (5*e^3*x^6 - 6*d*e^2
*x^4 + 8*d^2*e*x^2 - 16*d^3)*sqrt(e*x^2 + d)*sqrt(-e))/e^4
```

Sympy [A] time = 15.8235, size = 124, normalized size = 1.

$$\begin{cases} \frac{16id^3\sqrt{d+ex^2}}{245e^{\frac{7}{2}}}-\frac{8id^2x^2\sqrt{d+ex^2}}{245e^{\frac{5}{2}}}+\frac{6idx^4\sqrt{d+ex^2}}{245e^{\frac{3}{2}}}+\frac{ix^7\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7}-\frac{ix^6\sqrt{d+ex^2}}{49\sqrt{e}} & \text{for } e\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((16*I*d**3*sqrt(d + e*x**2)/(245*e**(7/2)) - 8*I*d**2*x**2*sqrt(d
 + e*x**2)/(245*e**(5/2)) + 6*I*d*x**4*sqrt(d + e*x**2)/(245*e**(3/2)) + I*
x**7*atanh(sqrt(e)*x/sqrt(d + e*x**2))/7 - I*x**6*sqrt(d + e*x**2)/(49*sqrt
(e)), Ne(e, 0)), (0, True))
```

Giac [A] time = 1.18974, size = 185, normalized size = 1.49

$$\frac{1}{7}\,x^7\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)+\frac{1}{245}\left(35\,\sqrt{-x^2e^2-de}d^3e^3+35\left(-x^2e^2-de\right)^{\frac{3}{2}}d^2e^2+21\left(x^2e^2+de\right)^2\sqrt{-x^2e^2-de}de-5\left(x^2e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/7*x^7*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) + 1/245*(35*sqrt(-x^2*e^2 - d*e)
*d^3*e^3 + 35*(-x^2*e^2 - d*e)^(3/2)*d^2*e^2 + 21*(x^2*e^2 + d*e)^2*sqrt(-x
^2*e^2 - d*e)*d*e - 5*(x^2*e^2 + d*e)^3*sqrt(-x^2*e^2 - d*e))*e^(-7)
```

3.12 $\int x^4 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=99

$$\frac{d^2\sqrt{d+ex^2}}{5(-e)^{5/2}}+\frac{\left(d+ex^2\right)^{5/2}}{25(-e)^{5/2}}-\frac{2d\left(d+ex^2\right)^{3/2}}{15(-e)^{5/2}}+\frac{1}{5}x^5\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] (d^2*Sqrt[d + e*x^2])/(5*(-e)^(5/2)) - (2*d*(d + e*x^2)^(3/2))/(15*(-e)^(5/
2)) + (d + e*x^2)^(5/2)/(25*(-e)^(5/2)) + (x^5*ArcTan[(Sqrt[-e]*x)/Sqrt[d +
 e*x^2]])/5
```

Rubi [A] time = 0.0505896, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5151, 266, 43}

$$\frac{d^2\sqrt{d+ex^2}}{5(-e)^{5/2}}+\frac{\left(d+ex^2\right)^{5/2}}{25(-e)^{5/2}}-\frac{2d\left(d+ex^2\right)^{3/2}}{15(-e)^{5/2}}+\frac{1}{5}x^5\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d^2*Sqrt[d + e*x^2])/(5*(-e)^(5/2)) - (2*d*(d + e*x^2)^(3/2))/(15*(-e)^(5/
2)) + (d + e*x^2)^(5/2)/(25*(-e)^(5/2)) + (x^5*ArcTan[(Sqrt[-e]*x)/Sqrt[d +
 e*x^2]])/5
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^4 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{5}x^5 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{5}\sqrt{-e}\int \frac{x^5}{\sqrt{d+ex^2}} dx$$
$$= \frac{1}{5}x^5 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{10}\sqrt{-e}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex}} dx, x, x^2\right)$$
$$= \frac{1}{5}x^5 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{10}\sqrt{-e}\,\text{Subst}\left(\int \left(\frac{d^2}{e^2\sqrt{d+ex}} - \frac{2d\sqrt{d+ex}}{e^2} + \frac{(d+ex)^{3/2}}{e^2}\right) dx,$$
$$= \frac{d^2\sqrt{d+ex^2}}{5(-e)^{5/2}} - \frac{2d\left(d+ex^2\right)^{3/2}}{15(-e)^{5/2}} + \frac{\left(d+ex^2\right)^{5/2}}{25(-e)^{5/2}} + \frac{1}{5}x^5 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

Mathematica [A] time = 0.0887031, size = 72, normalized size = 0.73

$$\frac{\sqrt{d+ex^2}\left(8d^2-4dex^2+3e^2x^4\right)}{75(-e)^{5/2}} + \frac{1}{5}x^5 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4))/(75*(-e)^(5/2)) + (x^5*Ar
cTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/5
```

Maple [B] time = 0.043, size = 180, normalized size = 1.8

$$\frac{x^5}{5}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right) + \frac{x^6}{35\,d}\sqrt{-e}\sqrt{ex^2+d} - \frac{6\,x^4}{175\,e}\sqrt{-e}\sqrt{ex^2+d} + \frac{8\,dx^2}{175\,e^2}\sqrt{-e}\sqrt{ex^2+d} - \frac{16\,d^2}{175\,e^3}\sqrt{-e}\sqrt{ex^2+d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/5*x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/35*(-e)^(1/2)/d*x^6*(e*x^2+d
)^(1/2)-6/175*(-e)^(1/2)/e*x^4*(e*x^2+d)^(1/2)+8/175*(-e)^(1/2)/e^2*d*x^2*(
e*x^2+d)^(1/2)-16/175*(-e)^(1/2)/e^3*d^2*(e*x^2+d)^(1/2)-1/35*(-e)^(1/2)/d*
x^4*(e*x^2+d)^(3/2)/e+4/175*(-e)^(1/2)/e^2*x^2*(e*x^2+d)^(3/2)-8/525*(-e)^(
1/2)*d/e^3*(e*x^2+d)^(3/2)
```

Maxima [A] time = 0.999257, size = 188, normalized size = 1.9

$$\frac{1}{5}x^5\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{\left(15\left(ex^2+d\right)^{\frac{7}{2}} - 42\left(ex^2+d\right)^{\frac{5}{2}}d + 35\left(ex^2+d\right)^{\frac{3}{2}}d^2\right)\sqrt{-e}}{525\,de^3} + \frac{\left(5\left(ex^2+d\right)^{\frac{7}{2}} - 21\left(ex^2+d\right)^{\frac{5}{2}}d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/525*(15*(e*x^2 + d)^(7/2) -
42*(e*x^2 + d)^(5/2)*d + 35*(e*x^2 + d)^(3/2)*d^2)*sqrt(-e)/(d*e^3) + 1/175
```

```
*(5*(e*x^2 + d)^(7/2) - 21*(e*x^2 + d)^(5/2)*d + 35*(e*x^2 + d)^(3/2)*d^2 -
 35*sqrt(e*x^2 + d)*d^3)*sqrt(-e)/(d*e^3)
```

Fricas [A] time = 2.21825, size = 158, normalized size = 1.6

$$\frac{15\,e^3x^5\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\left(3\,e^2x^4-4\,dex^2+8\,d^2\right)\sqrt{ex^2+d}\sqrt{-e}}{75\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*e^3*x^5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (3*e^2*x^4 - 4*d*e*x^
2 + 8*d^2)*sqrt(e*x^2 + d)*sqrt(-e))/e^3
```

Sympy [A] time = 4.50107, size = 97, normalized size = 0.98

$$\begin{cases} -\frac{8id^2\sqrt{d+ex^2}}{75e^{\frac{5}{2}}}+\frac{4idx^2\sqrt{d+ex^2}}{75e^{\frac{3}{2}}}+\frac{ix^5\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5}-\frac{ix^4\sqrt{d+ex^2}}{25\sqrt{e}} & \text{for } e\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((-8*I*d**2*sqrt(d + e*x**2)/(75*e**(5/2)) + 4*I*d*x**2*sqrt(d + e
*x**2)/(75*e**(3/2)) + I*x**5*atanh(sqrt(e)*x/sqrt(d + e*x**2))/5 - I*x**4*
sqrt(d + e*x**2)/(25*sqrt(e)), Ne(e, 0)), (0, True))
```

Giac [A] time = 1.20283, size = 138, normalized size = 1.39

$$\frac{1}{5}\,x^5\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)-\frac{1}{75}\left(15\,\sqrt{-x^2e^2-de}d^2e^2+10\left(-x^2e^2-de\right)^{\frac{3}{2}}de+3\left(x^2e^2+de\right)^2\sqrt{-x^2e^2-de}\right)e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/5*x^5*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) - 1/75*(15*sqrt(-x^2*e^2 - d*e)*
d^2*e^2 + 10*(-x^2*e^2 - d*e)^(3/2)*d*e + 3*(x^2*e^2 + d*e)^2*sqrt(-x^2*e^2
 - d*e))*e^(-5)
```

3.13 $\int x^2 \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=74

$$-\frac{\left(d+ex^2\right)^{3/2}}{9(-e)^{3/2}}+\frac{d\sqrt{d+ex^2}}{3(-e)^{3/2}}+\frac{1}{3}x^3\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] (d*Sqrt[d + e*x^2])/(3*(-e)^(3/2)) - (d + e*x^2)^(3/2)/(9*(-e)^(3/2)) + (x^
3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/3
```

Rubi [A] time = 0.0362828, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5151, 266, 43}

$$-\frac{\left(d+ex^2\right)^{3/2}}{9(-e)^{3/2}}+\frac{d\sqrt{d+ex^2}}{3(-e)^{3/2}}+\frac{1}{3}x^3\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d*Sqrt[d + e*x^2])/(3*(-e)^(3/2)) - (d + e*x^2)^(3/2)/(9*(-e)^(3/2)) + (x^
3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/3
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx &= \frac{1}{3}x^3\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{3}\sqrt{-e}\int\frac{x^3}{\sqrt{d+ex^2}}dx\\ &= \frac{1}{3}x^3\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{-e}\,\text{Subst}\left(\int\frac{x}{\sqrt{d+ex}}dx,x,x^2\right)\\ &= \frac{1}{3}x^3\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{-e}\,\text{Subst}\left(\int\left(-\frac{d}{e\sqrt{d+ex}}+\frac{\sqrt{d+ex}}{e}\right)dx,x,x^2\right)\\ &= \frac{d\sqrt{d+ex^2}}{3(-e)^{3/2}}-\frac{\left(d+ex^2\right)^{3/2}}{9(-e)^{3/2}}+\frac{1}{3}x^3\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0826092, size = 60, normalized size = 0.81

$$\frac{1}{9}\left(\frac{\left(2d-ex^2\right)\sqrt{d+ex^2}}{(-e)^{3/2}}+3x^3\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (((2*d - e*x^2)*Sqrt[d + e*x^2])/(-e)^(3/2) + 3*x^3*ArcTan[(Sqrt[-e]*x)/Sqr
t[d + e*x^2]])/9
```

Maple [B] time = 0.042, size = 132, normalized size = 1.8

$$\frac{x^3}{3}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)+\frac{x^4}{15d}\sqrt{-e}\sqrt{ex^2+d}-\frac{4x^2}{45e}\sqrt{-e}\sqrt{ex^2+d}+\frac{8d}{45e^2}\sqrt{-e}\sqrt{ex^2+d}-\frac{x^2}{15de}\sqrt{-e}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{2}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/3*x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/15*(-e)^(1/2)/d*x^4*(e*x^2+d
)^(1/2)-4/45*(-e)^(1/2)/e*x^2*(e*x^2+d)^(1/2)+8/45*(-e)^(1/2)/e^2*d*(e*x^2+
d)^(1/2)-1/15*(-e)^(1/2)/d*x^2*(e*x^2+d)^(3/2)/e+2/45*(-e)^(1/2)/e^2*(e*x^2
+d)^(3/2)
```

Maxima [A] time = 1.00957, size = 150, normalized size = 2.03

$$\frac{1}{3}x^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\frac{\left(3\left(ex^2+d\right)^{\frac{5}{2}}-5\left(ex^2+d\right)^{\frac{3}{2}}d\right)\sqrt{-e}}{45\,de^2}+\frac{\left(3\left(ex^2+d\right)^{\frac{5}{2}}-10\left(ex^2+d\right)^{\frac{3}{2}}d+15\sqrt{ex^2+d}d^2\right)\sqrt{-e}}{45\,de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/45*(3*(e*x^2 + d)^(5/2) - 5*
(e*x^2 + d)^(3/2)*d)*sqrt(-e)/(d*e^2) + 1/45*(3*(e*x^2 + d)^(5/2) - 10*(e*x
^2 + d)^(3/2)*d + 15*sqrt(e*x^2 + d)*d^2)*sqrt(-e)/(d*e^2)
```

Fricas [A] time = 2.36367, size = 131, normalized size = 1.77

$$\frac{3\,e^2x^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\sqrt{ex^2+d}\left(ex^2-2\,d\right)\sqrt{-e}}{9\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*e^2*x^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - sqrt(e*x^2 + d)*(e*x^2
- 2*d)*sqrt(-e))/e^2
```

Sympy [A] time = 1.32682, size = 70, normalized size = 0.95

$$\begin{cases} \frac{2id\sqrt{d+ex^2}}{9e^{\frac{3}{2}}}+\frac{ix^3\,\mathrm{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3}-\frac{ix^2\sqrt{d+ex^2}}{9\sqrt{e}} & \text{for } e\neq 0\\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((2*I*d*sqrt(d + e*x**2)/(9*e**(3/2)) + I*x**3*atanh(sqrt(e)*x/sqr
t(d + e*x**2))/3 - I*x**2*sqrt(d + e*x**2)/(9*sqrt(e)), Ne(e, 0)), (0, True
))
```

Giac [A] time = 1.16226, size = 88, normalized size = 1.19

$$\frac{1}{3}\,x^3\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)+\frac{1}{9}\left(3\,\sqrt{-x^2e^2-de}de+\left(-x^2e^2-de\right)^{\frac{3}{2}}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) + 1/9*(3*sqrt(-x^2*e^2 - d*e)*d*
e + (-x^2*e^2 - d*e)^(3/2))*e^(-3)
```

3.14 $\int \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=43

$$\frac{\sqrt{d+ex^2}}{\sqrt{-e}}+x\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] Sqrt[d + e*x^2]/Sqrt[-e] + x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]
```

Rubi [A] time = 0.0101861, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5147, 261}

$$\frac{\sqrt{d+ex^2}}{\sqrt{-e}}+x\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] Sqrt[d + e*x^2]/Sqrt[-e] + x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]
```

Rule 5147

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]], x_Symbol] :> Simp[x*Ar
cTan[(c*x)/Sqrt[a + b*x^2]], x] - Dist[c, Int[x/Sqrt[a + b*x^2], x], x] /;
FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx &= x\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\sqrt{-e}\int \frac{x}{\sqrt{d+ex^2}}\,dx\\ &= \frac{\sqrt{d+ex^2}}{\sqrt{-e}}+x\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0195625, size = 43, normalized size = 1.

$$\frac{\sqrt{d+ex^2}}{\sqrt{-e}}+x\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] Sqrt[d + e*x^2]/Sqrt[-e] + x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]
```

Maple [B] time = 0.038, size = 84, normalized size = 2.

$$x\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)+\frac{x^2}{3d}\sqrt{-e}\sqrt{ex^2+d}-\frac{2}{3e}\sqrt{-e}\sqrt{ex^2+d}-\frac{1}{3de}\sqrt{-e}\left(ex^2+d\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/3*(-e)^(1/2)/d*x^2*(e*x^2+d)^(1/2)
-2/3*(-e)^(1/2)/e*(e*x^2+d)^(1/2)-1/3*(-e)^(1/2)/d/e*(e*x^2+d)^(3/2)
```

Maxima [B] time = 1.00229, size = 104, normalized size = 2.42

$$x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\frac{\left(ex^2+d\right)^{\frac{3}{2}}\sqrt{-e}}{3de}+\frac{\left(\left(ex^2+d\right)^{\frac{3}{2}}-3\sqrt{ex^2+d}d\right)\sqrt{-e}}{3de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/3*(e*x^2 + d)^(3/2)*sqrt(-e)/(d*e)
 + 1/3*((e*x^2 + d)^(3/2) - 3*sqrt(e*x^2 + d)*d)*sqrt(-e)/(d*e)
```

Fricas [A] time = 2.36114, size = 96, normalized size = 2.23

$$\frac{ex\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\sqrt{ex^2+d}\sqrt{-e}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] (e*x*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - sqrt(e*x^2 + d)*sqrt(-e))/e
```

Sympy [A] time = 0.768269, size = 39, normalized size = 0.91

$$\begin{cases} ix\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{i\sqrt{d+ex^2}}{\sqrt{e}} & \text{for } e\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((I*x*atanh(sqrt(e)*x/sqrt(d + e*x**2)) - I*sqrt(d + e*x**2)/sqrt(
e), Ne(e, 0)), (0, True))
```

Giac [A] time = 1.13894, size = 55, normalized size = 1.28

$$x \arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right) - \sqrt{-x^2e^2 - de}e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] x*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) - sqrt(-x^2*e^2 - d*e)*e^(-1)
```

3.15 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx$

Optimal. Leaf size=59

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

```
[Out] -(ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x) - (Sqrt[-e]*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]])/Sqrt[d]
```

Rubi [A] time = 0.0329614, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {5151, 266, 63, 208}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^2,x]
```

```
[Out] -(ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x) - (Sqrt[-e]*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]])/Sqrt[d]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x} + \sqrt{-e}\int \frac{1}{x\sqrt{d+ex^2}}\,dx$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x} + \frac{1}{2}\sqrt{-e}\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x} - \frac{\mathrm{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{\sqrt{-e}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x} - \frac{\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

Mathematica [C] time = 0.0603867, size = 86, normalized size = 1.46

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x} + \frac{i\sqrt{e}\log\left(-\frac{2\sqrt{-e}\sqrt{d+ex^2}}{ex} + \frac{2i\sqrt{d}}{\sqrt{e}x}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^2,x]
```

```
[Out] -(ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x) + (I*Sqrt[e]*Log[((2*I)*Sqrt[d])/
(Sqrt[e]*x) - (2*Sqrt[-e]*Sqrt[d + e*x^2])/(e*x)])/Sqrt[d]
```

Maple [A] time = 0.039, size = 57, normalized size = 1.

$$-\frac{1}{x}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right) - \sqrt{-e}\ln\left(\frac{1}{x}\left(2d + 2\sqrt{d}\sqrt{ex^2+d}\right)\right)\frac{1}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x)
```

```
[Out] -arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x-(-e)^(1/2)/d^(1/2)*ln((2*d+2*d^(1/2
)*(e*x^2+d)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{-d\sqrt{-e}x\int \frac{\sqrt{ex^2+d}}{e^2x^5+dex^3-(ex^3+dx)(ex^2+d)}\,dx - \arctan\left(\sqrt{-e}x, \sqrt{ex^2+d}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] (d*sqrt(-e)*x*integrate(-sqrt(e*x^2 + d)/(e^2*x^5 + d*e*x^3 - (e*x^3 + d*x)
*(e*x^2 + d)), x) - arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)))/x
```

Fricas [A] time = 2.60711, size = 339, normalized size = 5.75

$$\left[\frac{x\sqrt{-\frac{e}{d}}\log\left(-\frac{e^2x^2+2\sqrt{ex^2+d}d\sqrt{-e}\sqrt{-\frac{e}{d}}+2de}{x^2}\right)-2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{2x}, -\frac{x\sqrt{\frac{e}{d}}\arctan\left(\frac{\sqrt{ex^2+d}d\sqrt{-e}\sqrt{\frac{e}{d}}}{e^2x^2+de}\right)+\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-e/d)*log(-(e^2*x^2 + 2*sqrt(e*x^2 + d)*d*sqrt(-e)*sqrt(-e/d)
+ 2*d*e)/x^2) - 2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/x, -(x*sqrt(e/d)*arct
an(sqrt(e*x^2 + d)*d*sqrt(-e)*sqrt(e/d)/(e^2*x^2 + d*e)) + arctan(sqrt(-e)*
x/sqrt(e*x^2 + d)))/x]
```

Sympy [A] time = 4.0837, size = 60, normalized size = 1.02

$$-\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{x}+\frac{\sqrt{-e}\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{d}}\sqrt{d+ex^2}}\right)}{d\sqrt{-\frac{1}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**2,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/x + sqrt(-e)*atan(1/(sqrt(-1/d)*sqrt(d +
 e*x**2)))/(d*sqrt(-1/d))
```

Giac [A] time = 1.20885, size = 73, normalized size = 1.24

$$-\frac{\arctan\left(\frac{\sqrt{-x^2e^2-de}e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{\sqrt{d}}-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] -arctan(sqrt(-x^2*e^2 - d*e)*e^(-1/2)/sqrt(d))*e^(1/2)/sqrt(d) - arctan(x*s
qrt(-e)/sqrt(x^2*e + d))/x
```

3.16 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4} dx$

Optimal. Leaf size=91

$$-\frac{(-e)^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}-\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(6*d*x^2) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]
]/(3*x^3) - ((-e)^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(6*d^(3/2))
```

Rubi [A] time = 0.0460897, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5151, 266, 51, 63, 208}

$$-\frac{(-e)^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}-\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(6*d*x^2) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]
]/(3*x^3) - ((-e)^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(6*d^(3/2))
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx &= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{1}{3}\sqrt{-e}\int \frac{1}{x^3\sqrt{d+ex^2}}\,dx \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{1}{6}\sqrt{-e}\,\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}\,dx,x,x^2\right) \\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{(-e)^{3/2}\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{12d} \\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{\sqrt{-e}\,\mathrm{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{6d} \\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{(-e)^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.111211, size = 101, normalized size = 1.11

$$
\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{-e}}{\sqrt{e}\sqrt{d+ex^2}}\right)}{6d^{3/2}}-\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(6*d*x^2) + (e^(3/2)*ArcTan[(Sqrt[d]*Sqrt[-e])/
(Sqrt[e]*Sqrt[d + e*x^2])])/(6*d^(3/2)) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^
2]]/(3*x^3)
```

Maple [A] time = 0.039, size = 100, normalized size = 1.1

$$
-\frac{1}{3x^3}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)+\frac{e}{6}\sqrt{-e}\ln\left(\frac{1}{x}\left(2d+2\sqrt{d}\sqrt{ex^2+d}\right)\right)d^{-\frac{3}{2}}-\frac{1}{6d^2x^2}\sqrt{-e}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{e}{6d^2}\sqrt{-e}\sqrt{ex^2+}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x)
```

```
[Out] -1/3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3+1/6*(-e)^(1/2)*e/d^(3/2)*ln((
2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)-1/6*(-e)^(1/2)/d^2/x^2*(e*x^2+d)^(3/2)+1/
6*(-e)^(1/2)/d^2*e*(e*x^2+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{-d\sqrt{-e}x^3 \int \frac{\sqrt{ex^2+d}}{e^2x^7+dex^5-(ex^5+dx^3)(ex^2+d)}\,dx - \arctan\left(\sqrt{-e}x, \sqrt{ex^2+d}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*d*sqrt(-e)*x^3*integrate(-1/3*sqrt(e*x^2 + d)/(e^2*x^7 + d*e*x^5 - (
e*x^5 + d*x^3)*(e*x^2 + d)), x) - arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)))/x^3
```

Fricas [A] time = 2.71131, size = 460, normalized size = 5.05

$$\left[\frac{ex^3\sqrt{-\frac{e}{d}}\log\left(-\frac{e^2x^2-2\sqrt{ex^2+d}d\sqrt{-e}\sqrt{-\frac{e}{d}}+2\,de}{x^2}\right)-2\sqrt{ex^2+d}\sqrt{-e}x-4\,d\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{12\,dx^3}, \frac{ex^3\sqrt{\frac{e}{d}}\arctan\left(\frac{\sqrt{ex^2+d}d\sqrt{-e}\sqrt{\frac{e}{d}}}{e^2x^2+de}\right)-\sqrt{e\text{:}}}{6\,dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(e*x^3*sqrt(-e/d)*log(-(e^2*x^2 - 2*sqrt(e*x^2 + d)*d*sqrt(-e)*sqrt(-
e/d) + 2*d*e)/x^2) - 2*sqrt(e*x^2 + d)*sqrt(-e)*x - 4*d*arctan(sqrt(-e)*x/s
qrt(e*x^2 + d)))/(d*x^3), 1/6*(e*x^3*sqrt(e/d)*arctan(sqrt(e*x^2 + d)*d*sqr
t(-e)*sqrt(e/d)/(e^2*x^2 + d*e)) - sqrt(e*x^2 + d)*sqrt(-e)*x - 2*d*arctan(
sqrt(-e)*x/sqrt(e*x^2 + d)))/(d*x^3)]
```

Sympy [A] time = 8.6699, size = 82, normalized size = 0.9

$$-\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{\sqrt{e}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{6dx}+\frac{e\sqrt{-e}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{6d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**4,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/(3*x**3) - sqrt(e)*sqrt(-e)*sqrt(d/(e*x*
*2) + 1)/(6*d*x) + e*sqrt(-e)*asinh(sqrt(d)/(sqrt(e)*x))/(6*d**(3/2))
```

Giac [A] time = 1.23854, size = 112, normalized size = 1.23

$$\frac{1}{6}\left(\frac{\arctan\left(\frac{\sqrt{-x^2e^2-dee^{\left(-\frac{1}{2}\right)}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{d^{\frac{3}{2}}}-\frac{\sqrt{-x^2e^2-dee^{(-3)}}}{dx^2}\right)e^3-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] 1/6*(arctan(sqrt(-x^2*e^2 - d*e)*e^(-1/2)/sqrt(d))*e^(-3/2)/d^(3/2) - sqrt(
-x^2*e^2 - d*e)*e^(-3)/(d*x^2))*e^3 - 1/3*arctan(x*sqrt(-e)/sqrt(x^2*e + d)
)/x^3
```

3.17 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6} dx$

Optimal. Leaf size=119

$$-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{3(-e)^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}-\frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(20*d*x^4) - (3*(-e)^(3/2)*Sqrt[d + e*x^2])/(40
*d^2*x^2) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(5*x^5) - (3*(-e)^(5/2)*Ar
cTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(40*d^(5/2))
```

Rubi [A] time = 0.0599371, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5151, 266, 51, 63, 208}

$$-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{3(-e)^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}-\frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(20*d*x^4) - (3*(-e)^(3/2)*Sqrt[d + e*x^2])/(40
*d^2*x^2) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(5*x^5) - (3*(-e)^(5/2)*Ar
cTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(40*d^(5/2))
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx &= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{1}{5}\sqrt{-e}\int \frac{1}{x^5\sqrt{d+ex^2}}\,dx \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{1}{10}\sqrt{-e}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{d+ex}}\,dx,x,x^2\right) \\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{\left(3(-e)^{3/2}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}\,dx,x,x^2\right)}{40d} \\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4}-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{\left(3(-e)^{5/2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{80d^2} \\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4}-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}-\frac{\left(3(-e)^{3/2}\right)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d}\right.}{40d^2} \\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4}-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}-\frac{3(-e)^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.141426, size = 114, normalized size = 0.96

$$
-\frac{3e^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{-e}}{\sqrt{e}\sqrt{d+ex^2}}\right)}{40d^{5/2}}+\sqrt{-e}\left(\frac{3e}{40d^2x^2}-\frac{1}{20dx^4}\right)\sqrt{d+ex^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

```
[Out] Sqrt[-e]*(-1/(20*d*x^4) + (3*e)/(40*d^2*x^2))*Sqrt[d + e*x^2] - (3*e^(5/2)*
ArcTan[(Sqrt[d]*Sqrt[-e])/(Sqrt[e]*Sqrt[d + e*x^2])])/(40*d^(5/2)) - ArcTan
[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(5*x^5)
```

Maple [A] time = 0.04, size = 150, normalized size = 1.3

$$
-\frac{1}{5x^5}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)+\frac{e}{10d^2x^2}\sqrt{-e}\sqrt{ex^2+d}-\frac{3e^2}{40}\sqrt{-e}\ln\left(\frac{1}{x}\left(2d+2\sqrt{d}\sqrt{ex^2+d}\right)\right)d^{-\frac{5}{2}}-\frac{1}{20d^2x^4}\sqrt{-e}\,(e
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x)
```

```
[Out] -1/5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5+1/10*(-e)^(1/2)*e/d^2/x^2*(e*
x^2+d)^(1/2)-3/40*(-e)^(1/2)*e^2/d^(5/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))
```

```
/x)-1/20*(-e)^(1/2)/d^2/x^4*(e*x^2+d)^(3/2)+1/40*(-e)^(1/2)/d^3*e/x^2*(e*x^
2+d)^(3/2)-1/40*(-e)^(1/2)/d^3*e^2*(e*x^2+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{-d\sqrt{-e}x^5 \int \frac{\sqrt{ex^2+d}}{e^2x^9+dex^7-(ex^7+dx^5)(ex^2+d)}\,dx - \arctan\left(\sqrt{-e}x, \sqrt{ex^2+d}\right)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*(5*d*sqrt(-e)*x^5*integrate(-1/5*sqrt(e*x^2 + d)/(e^2*x^9 + d*e*x^7 - (
e*x^7 + d*x^5)*(e*x^2 + d)), x) - arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)))/x^5
```

Fricas [A] time = 2.84853, size = 529, normalized size = 4.45

$$\left[\frac{3\,e^2x^5\sqrt{-\frac{e}{d}}\log\left(-\frac{e^2x^2+2\,\sqrt{ex^2+d}d\sqrt{-e}\sqrt{-\frac{e}{d}}+2\,de}{x^2}\right) - 16\,d^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) + 2\left(3\,ex^3-2\,dx\right)\sqrt{ex^2+d}\sqrt{-e}}{80\,d^2x^5}, -\frac{3\,e^2x^5\sqrt{\frac{e}{d}}\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="fricas")
```

```
[Out] [1/80*(3*e^2*x^5*sqrt(-e/d)*log(-(e^2*x^2 + 2*sqrt(e*x^2 + d)*d*sqrt(-e)*sq
rt(-e/d) + 2*d*e)/x^2) - 16*d^2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*(3*e
*x^3 - 2*d*x)*sqrt(e*x^2 + d)*sqrt(-e))/(d^2*x^5), -1/40*(3*e^2*x^5*sqrt(e/
d)*arctan(sqrt(e*x^2 + d)*d*sqrt(-e)*sqrt(e/d)/(e^2*x^2 + d*e)) + 8*d^2*arc
tan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (3*e*x^3 - 2*d*x)*sqrt(e*x^2 + d)*sqrt(-e
))/(d^2*x^5)]
```

Sympy [A] time = 17.1436, size = 148, normalized size = 1.24

$$-\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{5x^5} - \frac{\sqrt{-e}}{20\sqrt{e}x^5\sqrt{\frac{d}{ex^2}+1}} + \frac{\sqrt{e}\sqrt{-e}}{40dx^3\sqrt{\frac{d}{ex^2}+1}} + \frac{3e^{\frac{3}{2}}\sqrt{-e}}{40d^2x\sqrt{\frac{d}{ex^2}+1}} - \frac{3e^2\sqrt{-e}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{40d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**6,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/(5*x**5) - sqrt(-e)/(20*sqrt(e)*x**5*sqr
t(d/(e*x**2) + 1)) + sqrt(e)*sqrt(-e)/(40*d*x**3*sqrt(d/(e*x**2) + 1)) + 3*
e**(3/2)*sqrt(-e)/(40*d**2*x*sqrt(d/(e*x**2) + 1)) - 3*e**2*sqrt(-e)*asinh(
sqrt(d)/(sqrt(e)*x))/(40*d**(5/2))
```

Giac [A] time = 1.18388, size = 143, normalized size = 1.2

$$-\frac{1}{40}\left(\frac{3\arctan\left(\frac{\sqrt{-x^2e^2-de}e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{d^{\frac{5}{2}}}+\frac{\left(5\sqrt{-x^2e^2-de}de+3\left(-x^2e^2-de\right)^{\frac{3}{2}}\right)e^{(-6)}}{d^2x^4}\right)e^5-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="giac")
```

```
[Out] -1/40*(3*arctan(sqrt(-x^2*e^2 - d*e)*e^(-1/2)/sqrt(d))*e^(-5/2)/d^(5/2) + (
5*sqrt(-x^2*e^2 - d*e)*d*e + 3*(-x^2*e^2 - d*e)^(3/2))*e^(-6)/(d^2*x^4))*e^
5 - 1/5*arctan(x*sqrt(-e)/sqrt(x^2*e + d))/x^5
```

3.18 $\int x^{9/2} \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=211

$$\frac{30d^{11/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{847e^{13/4}\sqrt{d+ex^2}}+\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847(-e)^{5/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{36dx^{5/2}\sqrt{d+}}{847(-e)^{3/2}}$$

```
[Out] (60*d^2*Sqrt[x]*Sqrt[d + e*x^2])/(847*(-e)^(5/2)) + (36*d*x^(5/2)*Sqrt[d +
e*x^2])/(847*(-e)^(3/2)) + (4*x^(9/2)*Sqrt[d + e*x^2])/(121*Sqrt[-e]) + (2*
x^(11/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/11 + (30*d^(11/4)*Sqrt[-e]*(
Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*
ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(847*e^(13/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.120293, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5151, 321, 329, 220}

$$\frac{30d^{11/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{847e^{13/4}\sqrt{d+ex^2}}+\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847(-e)^{5/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{36dx^{5/2}\sqrt{d+}}{847(-e)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (60*d^2*Sqrt[x]*Sqrt[d + e*x^2])/(847*(-e)^(5/2)) + (36*d*x^(5/2)*Sqrt[d +
e*x^2])/(847*(-e)^(3/2)) + (4*x^(9/2)*Sqrt[d + e*x^2])/(121*Sqrt[-e]) + (2*
x^(11/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/11 + (30*d^(11/4)*Sqrt[-e]*(
Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*
ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(847*e^(13/4)*Sqrt[d + e*x^2])
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{11}\left(2\sqrt{-e}\right)\int\frac{x^{11/2}}{\sqrt{d+ex^2}}\,dx$$

$$=\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(18d)\int\frac{x^{7/2}}{\sqrt{d+ex^2}}\,dx}{121\sqrt{-e}}$$

$$=\frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\left(90d^2\right)\int\frac{x^{3/2}}{\sqrt{d+ex^2}}\,dx}{847(-e)^{3/2}}$$

$$=\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847(-e)^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-$$

$$=\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847(-e)^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-$$

$$=\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847(-e)^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-$$

Mathematica [C] time = 0.587725, size = 170, normalized size = 0.81

$$-\frac{60id^3x\sqrt{\frac{d}{ex^2}+1}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{847(-e)^{5/2}\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}+\frac{4\sqrt{x}\sqrt{d+ex^2}\left(15d^2-9dex^2+7e^2x^4\right)}{847(-e)^{5/2}}+\frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (4*Sqrt[x]*Sqrt[d + e*x^2]*(15*d^2 - 9*d*e*x^2 + 7*e^2*x^4))/(847*(-e)^(5/2
)) + (2*x^(11/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/11 - (((60*I)/847)*d
^3*Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt
[x]], -1])/(Sqrt[(I*Sqrt[d])/Sqrt[e]]*(-e)^(5/2)*Sqrt[d + e*x^2])
```

Maple [F] time = 0.297, size = 0, normalized size = 0.

$$\int x^{\frac{9}{2}}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2}{11}x^{\frac{11}{2}}\arctan\left(\sqrt{-e}x,\sqrt{ex^2+d}\right)-2d\sqrt{-e}\int-\frac{xe^{\left(\frac{1}{2}\log\left(ex^2+d\right)+\frac{9}{2}\log(x)\right)}}{11\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] 2/11*x^(11/2)*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - 2*d*sqrt(-e)*integrate
(-1/11*x*e^(1/2*log(e*x^2 + d) + 9/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 +
d)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^{\frac{9}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] integral(x^(9/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\left[\text{undef},\text{undef},\text{undef},\text{undef},\text{undef},\text{undef},\text{undef},\text{undef},\text{undef},2e^{\frac{1}{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, undef, undef, 2*e^(1/2)]
```

3.19 $\int x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=181

$$-\frac{10d^{7/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{147e^{9/4}\sqrt{d+ex^2}}+\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{20d\sqrt{x}\sqrt{d+ex^2}}{147(-e)^{3/2}}+\frac{2}{7}x^{7/2}\tan^{-}$$

```
[Out] (20*d*Sqrt[x]*Sqrt[d + e*x^2])/(147*(-e)^(3/2)) + (4*x^(5/2)*Sqrt[d + e*x^2
])/(49*Sqrt[-e]) + (2*x^(7/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7 - (10
*d^(7/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]
*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(147*e^(9/4)*Sq
rt[d + e*x^2])
```

Rubi [A] time = 0.0883095, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5151, 321, 329, 220}

$$-\frac{10d^{7/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{147e^{9/4}\sqrt{d+ex^2}}+\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{20d\sqrt{x}\sqrt{d+ex^2}}{147(-e)^{3/2}}+\frac{2}{7}x^{7/2}\tan^{-}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (20*d*Sqrt[x]*Sqrt[d + e*x^2])/(147*(-e)^(3/2)) + (4*x^(5/2)*Sqrt[d + e*x^2
])/(49*Sqrt[-e]) + (2*x^(7/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7 - (10
*d^(7/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]
*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(147*e^(9/4)*Sq
rt[d + e*x^2])
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx=\frac{2}{7}x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{7}\left(2\sqrt{-e}\right)\int\frac{x^{7/2}}{\sqrt{d+ex^2}}dx$$
$$=\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{2}{7}x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(10d)\int\frac{x^{3/2}}{\sqrt{d+ex^2}}dx}{49\sqrt{-e}}$$
$$=\frac{20d\sqrt{x}\sqrt{d+ex^2}}{147(-e)^{3/2}}+\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{2}{7}x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\left(10d^2\right)\int\frac{1}{\sqrt{x}\sqrt{d+ex^2}}dx}{147(-e)^{3/2}}$$
$$=\frac{20d\sqrt{x}\sqrt{d+ex^2}}{147(-e)^{3/2}}+\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{2}{7}x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\left(20d^2\right)\text{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}d\right.}{147(-e)^{3/2}}$$
$$=\frac{20d\sqrt{x}\sqrt{d+ex^2}}{147(-e)^{3/2}}+\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{2}{7}x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{10d^{7/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{}}{147e^9}$$

Mathematica [C] time = 0.415558, size = 158, normalized size = 0.87

$$\frac{2}{147}\sqrt{x}\left(\frac{2\left(5d-3ex^2\right)\sqrt{d+ex^2}}{(-e)^{3/2}}+21x^3\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\right)-\frac{20id^2x\sqrt{\frac{d}{ex^2}+1}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{147(-e)^{3/2}\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*Sqrt[x]*((2*(5*d - 3*e*x^2)*Sqrt[d + e*x^2])/(-e)^(3/2) + 21*x^3*ArcTan[
(Sqrt[-e]*x)/Sqrt[d + e*x^2]]))/147 - (((20*I)/147)*d^2*Sqrt[1 + d/(e*x^2)]
*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(Sqrt[(I*Sq
rt[d])/Sqrt[e]]*(-e)^(3/2)*Sqrt[d + e*x^2])
```

Maple [F] time = 0.401, size = 0, normalized size = 0.

$$\int x^{\frac{5}{2}}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2}{7}x^{\frac{7}{2}}\arctan\left(\sqrt{-e}x, \sqrt{ex^2+d}\right) - 2d\sqrt{-e}\int -\frac{xe^{\left(\frac{1}{2}\log\left(ex^2+d\right)+\frac{5}{2}\log(x)\right)}}{7\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] 2/7*x^(7/2)*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - 2*d*sqrt(-e)*integrate(-
1/7*x*e^(1/2*log(e*x^2 + d) + 5/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^
2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] integral(x^(5/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\left[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 2e^{\frac{1}{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 2*e^(1/2)]
```

3.20 $\int \sqrt{x} \tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=153

$$\frac{2d^{3/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{9e^{5/4}\sqrt{d+ex^2}}+\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] (4*Sqrt[x]*Sqrt[d + e*x^2])/(9*Sqrt[-e]) + (2*x^(3/2)*ArcTan[(Sqrt[-e]*x)/S
qrt[d + e*x^2]])/3 + (2*d^(3/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*
x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)]
, 1/2])/(9*e^(5/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.0743981, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5151, 321, 329, 220}

$$\frac{2d^{3/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{9e^{5/4}\sqrt{d+ex^2}}+\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (4*Sqrt[x]*Sqrt[d + e*x^2])/(9*Sqrt[-e]) + (2*x^(3/2)*ArcTan[(Sqrt[-e]*x)/S
qrt[d + e*x^2]])/3 + (2*d^(3/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*
x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)]
, 1/2])/(9*e^(5/4)*Sqrt[d + e*x^2])
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx &= \frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{3}\left(2\sqrt{-e}\right)\int\frac{x^{3/2}}{\sqrt{d+ex^2}}\,dx\\
&= \frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(2d)\int\frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx}{9\sqrt{-e}}\\
&= \frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(4d)\,\text{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)}{9\sqrt{-e}}\\
&= \frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2d^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\cdots\right.\right.}{9\sqrt{-e}\sqrt[4]{e}\sqrt{d+ex^2}}
\end{aligned}$$

Mathematica [C] time = 0.284602, size = 147, normalized size = 0.96

$$-\frac{4idx\sqrt{\frac{d}{ex^2}+1}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{9\sqrt{-e}\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}+\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (4*Sqrt[x]*Sqrt[d + e*x^2])/(9*Sqrt[-e]) + (2*x^(3/2)*ArcTan[(Sqrt[-e]*x)/S
qrt[d + e*x^2]])/3 - (((4*I)/9)*d*Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh
[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[-
e]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.289, size = 0, normalized size = 0.

$$\int\sqrt{x}\arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2}{3}x^{\frac{3}{2}}\arctan\left(\sqrt{-e}x,\sqrt{ex^2+d}\right)-2d\sqrt{-e}\int-\frac{xe^{\left(\frac{1}{2}\log\left(ex^2+d\right)+\frac{1}{2}\log(x)\right)}}{3\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] 2/3*x^(3/2)*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - 2*d*sqrt(-e)*integrate(-
1/3*x*e^(1/2*log(e*x^2 + d) + 1/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^
2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(x)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

Sympy [C] time = 3.61452, size = 75, normalized size = 0.49

$$\frac{2x^{\frac{3}{2}}\,\text{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3}-\frac{x^{\frac{5}{2}}\sqrt{-e}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] 2*x**(3/2)*atan(x*sqrt(-e)/sqrt(d + e*x**2))/3 - x**(5/2)*sqrt(-e)*gamma(5/
4)*hyper((1/2, 5/4), (9/4,), e*x**2*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(9/4
))
```

Giac [A] time = 1.26849, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] +Infinity
```

3.21 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=122

$$\frac{2\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt[4]{e}\sqrt{d+ex^2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

```
[Out] (-2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + (2*Sqrt[-e]*(Sqrt[d] +
Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^
(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(d^(1/4)*e^(1/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.0607988, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5151, 329, 220}

$$\frac{2\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt[4]{e}\sqrt{d+ex^2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
```


```
[Out] (-2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + (2*Sqrt[-e]*(Sqrt[d] +
Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^
(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(d^(1/4)*e^(1/4)*Sqrt[d + e*x^2])
```


Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \left(2\sqrt{-e}\right)\int \frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \left(4\sqrt{-e}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{2\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt[4]{e}\sqrt{d+ex^2}}$$

Mathematica [C] time = 0.121743, size = 115, normalized size = 0.94

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{4i\sqrt{-e}x\sqrt{\frac{d}{ex^2}+1}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + ((4*I)*Sqrt[-e]*Sqrt[1
+ d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])
/(Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.293, size = 0, normalized size = 0.

$$\int \arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)x^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(-d\sqrt{-e}\sqrt{x}\int -\frac{\sqrt{ex^2+dx}}{\left(ex^2+d\right)^2x^{\frac{3}{2}}-\left(e^2x^4+dex^2\right)x^{\frac{3}{2}}}\,dx - \arctan\left(\sqrt{-e}x, \sqrt{ex^2+d}\right)\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="maxima
")
```

```
[Out] 2*(d*sqrt(-e)*sqrt(x)*integrate(-sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(
3/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 3/2*log(x))), x) - arctan2(sqrt(-e)*
```

```
x, sqrt(e*x^2 + d)))/sqrt(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(3/2), x)
```

Sympy [C] time = 5.20209, size = 71, normalized size = 0.58

$$-\frac{2\,\text{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{\sqrt{x}\sqrt{-e}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{ex^2e^{i\pi}}{d}\right)}{\sqrt{d}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(3/2),x)
```

```
[Out] -2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/sqrt(x) + sqrt(x)*sqrt(-e)*gamma(1/4)*
hyper((1/4, 1/2), (5/4,), e*x**2*exp_polar(I*pi)/d)/(sqrt(d)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(3/2), x)
```

3.22 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}} dx$

Optimal. Leaf size=156

$$-\frac{2\sqrt{-e}e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{15d^{5/4}\sqrt{d+ex^2}}-\frac{4\sqrt{-e}\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(15*d*x^(3/2)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[
d + e*x^2]])/(5*x^(5/2)) - (2*Sqrt[-e]*e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(
d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^
(1/4)], 1/2])/(15*d^(5/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.0749551, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5151, 325, 329, 220}

$$-\frac{2\sqrt{-e}e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{15d^{5/4}\sqrt{d+ex^2}}-\frac{4\sqrt{-e}\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(15*d*x^(3/2)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[
d + e*x^2]])/(5*x^(5/2)) - (2*Sqrt[-e]*e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(
d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^
(1/4)], 1/2])/(15*d^(5/4)*Sqrt[d + e*x^2])
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx &= -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}+\frac{1}{5}\left(2\sqrt{-e}\right)\int \frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx\\ &= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}+\frac{\left(2(-e)^{3/2}\right)\int \frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx}{15d}\\ &= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}+\frac{\left(4(-e)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)}{15d}\\ &= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}+\frac{2(-e)^{3/2}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{15d^{5/4}\sqrt[4]{e}\sqrt{d+ex^2}}\end{aligned}$$

Mathematica [C] time = 0.266727, size = 150, normalized size = 0.96

$$-\frac{2\left(2\sqrt{-e}x\sqrt{d+ex^2}+3d\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\right)}{15dx^{5/2}}+\frac{4i(-e)^{3/2}x\sqrt{\frac{d}{ex^2}+1}\,\mathrm{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{15d\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

```
[Out] (-2*(2*Sqrt[-e]*x*Sqrt[d + e*x^2] + 3*d*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]
]))/(15*d*x^(5/2)) + (((4*I)/15)*(-e)^(3/2)*Sqrt[1 + d/(e*x^2)]*x*EllipticF
[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(d*Sqrt[(I*Sqrt[d])/Sqr
t[e]]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.29, size = 0, normalized size = 0.

$$\int \arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)x^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(-d\sqrt{-e}x^{\frac{5}{2}}\int -\frac{\sqrt{ex^2+dx}}{\left(ex^2+d\right)^2x^{\frac{7}{2}}-\left(e^2x^4+dex^2\right)x^{\frac{7}{2}}}\,dx-\arctan\left(\sqrt{-e}x,\sqrt{ex^2+d}\right)\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="maxima
")
```

```
[Out] 2/5*(5*d*sqrt(-e)*x^(5/2)*integrate(-1/5*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*
x^2)*x^(7/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 7/2*log(x))), x) - arctan2(s
qrt(-e)*x, sqrt(e*x^2 + d)))/x^(5/2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="fricas
")
```

```
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(7/2), x)
```

3.23 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx$

Optimal. Leaf size=186

$$\frac{10\sqrt{-e}e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{189d^{9/4}\sqrt{d+ex^2}}-\frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}}-\frac{2\tan^{-1}\left(\cdots\right.}{9x^{9/\cdots}}$$

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(63*d*x^(7/2)) - (20*(-e)^(3/2)*Sqrt[d + e*x^
2])/(189*d^2*x^(3/2)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(9*x^(9/2)
) + (10*Sqrt[-e]*e^(7/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] +
Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(189*d^(
9/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.0903199, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5151, 325, 329, 220}

$$\frac{10\sqrt{-e}e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{189d^{9/4}\sqrt{d+ex^2}}-\frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}}-\frac{2\tan^{-1}\left(\cdots\right.}{9x^{9/\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(63*d*x^(7/2)) - (20*(-e)^(3/2)*Sqrt[d + e*x^
2])/(189*d^2*x^(3/2)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(9*x^(9/2)
) + (10*Sqrt[-e]*e^(7/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] +
Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(189*d^(
9/4)*Sqrt[d + e*x^2])
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{1}{9}\left(2\sqrt{-e}\right)\int \frac{1}{x^{9/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{\left(10(-e)^{3/2}\right)\int \frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx}{63d}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}} - \frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{\left(10(-e)^{5/2}\right)\int \frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx}{189d^2}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}} - \frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{\left(20(-e)^{5/2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{\right.}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}} - \frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{10\sqrt{-e}e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F}{189d^{9/4}\sqrt{d+ex^{\cdot}}}$$

Mathematica [C] time = 0.365501, size = 162, normalized size = 0.87

$$\frac{4\sqrt{-e}x\sqrt{d+ex^2}\left(5ex^2-3d\right)-42d^2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{189d^2x^{9/2}} + \frac{20i(-e)^{5/2}x\sqrt{\frac{d}{ex^2}+1}\,\mathrm{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{189d^2\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

```
[Out] (4*Sqrt[-e]*x*Sqrt[d + e*x^2]*(-3*d + 5*e*x^2) - 42*d^2*ArcTan[(Sqrt[-e]*x)
/Sqrt[d + e*x^2]])/(189*d^2*x^(9/2)) + (((20*I)/189)*(-e)^(5/2)*Sqrt[1 + d/
(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(d^
2*Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.29, size = 0, normalized size = 0.

$$\int \arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)x^{-\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(-d\sqrt{-e}x^{\frac{9}{2}}\int-\frac{\sqrt{ex^2+d}x}{\left(ex^2+d\right)^2x^{\frac{11}{2}}-\left(e^2x^4+dex^2\right)x^{\frac{11}{2}}}\,dx-\arctan\left(\sqrt{-e}x,\sqrt{ex^2+d}\right)\right)}{9\,x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="maxim
a")
```

```
[Out] 2/9*(9*d*sqrt(-e)*x^(9/2)*integrate(-1/9*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*
x^2)*x^(11/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 11/2*log(x))), x) - arctan2
(sqrt(-e)*x, sqrt(e*x^2 + d)))/x^(9/2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="frica
s")
```

```
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="giac"
)
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(11/2), x)
```

3.24 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx$

Optimal. Leaf size=216

$$-\frac{30\sqrt{-e}e^{11/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{1001d^{13/4}\sqrt{d+ex^2}}-\frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}}-\frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}-\frac{4\sqrt{-e}}{143d}$$

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(143*d*x^(11/2)) - (36*(-e)^(3/2)*Sqrt[d + e*
x^2])/(1001*d^2*x^(7/2)) - (60*(-e)^(5/2)*Sqrt[d + e*x^2])/(1001*d^3*x^(3/2
)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(13*x^(13/2)) - (30*Sqrt[-e]*
e^(11/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*El
lipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(1001*d^(13/4)*Sqrt[d +
e*x^2])
```

Rubi [A] time = 0.108009, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5151, 325, 329, 220}

$$-\frac{30\sqrt{-e}e^{11/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{1001d^{13/4}\sqrt{d+ex^2}}-\frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}}-\frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}-\frac{4\sqrt{-e}}{143d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(143*d*x^(11/2)) - (36*(-e)^(3/2)*Sqrt[d + e*
x^2])/(1001*d^2*x^(7/2)) - (60*(-e)^(5/2)*Sqrt[d + e*x^2])/(1001*d^3*x^(3/2
)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(13*x^(13/2)) - (30*Sqrt[-e]*
e^(11/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*El
lipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(1001*d^(13/4)*Sqrt[d +
e*x^2])
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{1}{13}\left(2\sqrt{-e}\right)\int \frac{1}{x^{13/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{\left(18(-e)^{3/2}\right)\int \frac{1}{x^{9/2}\sqrt{d+ex^2}}\,dx}{143d}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{\left(90(-e)^{5/2}\right)\int \frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx}{1001d^2}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{\left(30(-e)^{7/2}\right)}{10}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{\left(60(-e)^{7/2}\right)}{}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{30(-e)^{7/2}\left(\right.}{}$$

Mathematica [C] time = 0.577851, size = 171, normalized size = 0.79

$$\frac{2\left(\frac{30i(-e)^{7/2}x^{15/2}\sqrt{\frac{d}{ex^2}+1}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{d^3\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}} - \frac{2\sqrt{-e}\sqrt{d+ex^2}\left(7d^2x-9dex^3+15e^2x^5\right)}{d^3} - 77\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\right)}{1001x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

```
[Out] (2*((-2*Sqrt[-e]*Sqrt[d + e*x^2]*(7*d^2*x - 9*d*e*x^3 + 15*e^2*x^5))/d^3 -
77*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] + ((30*I)*(-e)^(7/2)*Sqrt[1 + d/(e*
x^2)]*x^(15/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])
/(d^3*Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])))/(1001*x^(13/2))
```

Maple [F] time = 0.285, size = 0, normalized size = 0.

$$\int \arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)x^{-\frac{15}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(-d\sqrt{-e}x^{\frac{13}{2}}\int-\frac{\sqrt{ex^2+d}x}{\left(ex^2+d\right)^2x^{\frac{15}{2}}-\left(e^2x^4+dex^2\right)x^{\frac{15}{2}}}\,dx-\arctan\left(\sqrt{-e}x,\sqrt{ex^2+d}\right)\right)}{13\,x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="maxim
a")
```

```
[Out] 2/13*(13*d*sqrt(-e)*x^(13/2)*integrate(-1/13*sqrt(e*x^2 + d)*x/((e^2*x^4 +
d*e*x^2)*x^(15/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 15/2*log(x))), x) - arc
tan2(sqrt(-e)*x, sqrt(e*x^2 + d)))/x^(13/2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{15}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="frica
s")
```

```
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(15/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="giac"
)
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(15/2), x)
```

3.25 $\int x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=326

$$-\frac{14d^{9/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{135e^{11/4}\sqrt{d+ex^2}}-\frac{28d^2\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{135e^{5/2}\left(\sqrt{d}+\sqrt{e}x\right)}+\frac{28d^{9/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{}}{135e^{11}}$$

```
[Out] (28*d*x^(3/2)*Sqrt[d + e*x^2])/(405*(-e)^(3/2)) + (4*x^(7/2)*Sqrt[d + e*x^2
])/(81*Sqrt[-e]) - (28*d^2*Sqrt[-e]*Sqrt[x]*Sqrt[d + e*x^2])/(135*e^(5/2)*(
Sqrt[d] + Sqrt[e]*x)) + (2*x^(9/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/9
+ (28*d^(9/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sq
rt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(135*e^(11
/4)*Sqrt[d + e*x^2]) - (14*d^(9/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d +
 e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/
4)], 1/2])/(135*e^(11/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.191851, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5151, 321, 329, 305, 220, 1196}

$$-\frac{28d^2\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{135e^{5/2}\left(\sqrt{d}+\sqrt{e}x\right)}-\frac{14d^{9/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{135e^{11/4}\sqrt{d+ex^2}}+\frac{28d^{9/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}}{135e^{11/4}\sqrt{d+e}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```


```
[Out] (28*d*x^(3/2)*Sqrt[d + e*x^2])/(405*(-e)^(3/2)) + (4*x^(7/2)*Sqrt[d + e*x^2
])/(81*Sqrt[-e]) - (28*d^2*Sqrt[-e]*Sqrt[x]*Sqrt[d + e*x^2])/(135*e^(5/2)*(
Sqrt[d] + Sqrt[e]*x)) + (2*x^(9/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/9
+ (28*d^(9/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sq
rt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(135*e^(11
/4)*Sqrt[d + e*x^2]) - (14*d^(9/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d +
 e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/
4)], 1/2])/(135*e^(11/4)*Sqrt[d + e*x^2])
```


Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx &= \frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{9}\left(2\sqrt{-e}\right)\int\frac{x^{9/2}}{\sqrt{d+ex^2}}dx \\
&= \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}+\frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(14d)\int\frac{x^{5/2}}{\sqrt{d+ex^2}}dx}{81\sqrt{-e}} \\
&= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}+\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}+\frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\left(14d^2\right)\int\frac{\sqrt{x}}{\sqrt{d+ex^2}}dx}{135(-e)^{3/2}} \\
&= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}+\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}+\frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\left(28d^2\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{d+e}}\right.}{135(-e)^{3/2}} \\
&= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}+\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}+\frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\left(28d^{5/2}\sqrt{-e}\right)\text{Subst}\left(\int\right.}{135e^5} \\
&= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}+\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}-\frac{28d^2\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{135e^{5/2}\left(\sqrt{d}+\sqrt{e}x\right)}+\frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.137955, size = 139, normalized size = 0.43

$$\frac{2x^{3/2}\left(2\sqrt{-e}\left(7d^2+2dex^2-5e^2x^4\right)-14d^2\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)+45e^2x^3\sqrt{d+ex^2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\right)}{405e^2\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*x^(3/2)*(2*Sqrt[-e]*(7*d^2 + 2*d*e*x^2 - 5*e^2*x^4) + 45*e^2*x^3*Sqrt[d
+ e*x^2]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] - 14*d^2*Sqrt[-e]*Sqrt[1 + (e
*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)]))/(405*e^2*Sqrt[d +
 e*x^2])
```

Maple [F] time = 0.29, size = 0, normalized size = 0.

$$\int x^{\frac{7}{2}} \arctan\left(x\sqrt{-e}\,\frac{1}{\sqrt{e x^2+d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2}{9}\, x^{\frac{9}{2}} \arctan\left(\sqrt{-e}x, \sqrt{e x^2+d}\right) - 2\, d\sqrt{-e} \int -\frac{x e^{\left(\frac{1}{2}\log\left(e x^2+d\right)+\frac{7}{2}\log(x)\right)}}{9\left(e^2 x^4 + d e x^2 - \left(e x^2+d\right)^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] 2/9*x^(9/2)*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - 2*d*sqrt(-e)*integrate(-
1/9*x*e^(1/2*log(e*x^2 + d) + 7/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^
2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^{\frac{7}{2}} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{e x^2+d}}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] integral(x^(7/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\left[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 2\,e^{\frac{1}{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 2*e^(1/2)]
```

3.26 $\int x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=296

$$\frac{6d^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{25e^{7/4}\sqrt{d+ex^2}}-\frac{12d^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right.\right.}{25e^{7/4}\sqrt{d+ex^2}}$$

```
[Out] (4*x^(3/2)*Sqrt[d + e*x^2])/(25*Sqrt[-e]) + (12*d*Sqrt[-e]*Sqrt[x]*Sqrt[d +
 e*x^2])/(25*e^(3/2)*(Sqrt[d] + Sqrt[e]*x)) + (2*x^(5/2)*ArcTan[(Sqrt[-e]*x
)/Sqrt[d + e*x^2]])/5 - (12*d^(5/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d
+ e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1
/4)], 1/2])/(25*e^(7/4)*Sqrt[d + e*x^2]) + (6*d^(5/4)*Sqrt[-e]*(Sqrt[d] + S
qrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(
1/4)*Sqrt[x])/d^(1/4)], 1/2])/(25*e^(7/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.161298, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5151, 321, 329, 305, 220, 1196}

$$\frac{6d^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{7/4}\sqrt{d+ex^2}}-\frac{12d^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{7/4}\sqrt{d+ex^2}}+\frac{12}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (4*x^(3/2)*Sqrt[d + e*x^2])/(25*Sqrt[-e]) + (12*d*Sqrt[-e]*Sqrt[x]*Sqrt[d +
 e*x^2])/(25*e^(3/2)*(Sqrt[d] + Sqrt[e]*x)) + (2*x^(5/2)*ArcTan[(Sqrt[-e]*x
)/Sqrt[d + e*x^2]])/5 - (12*d^(5/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d
+ e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1
/4)], 1/2])/(25*e^(7/4)*Sqrt[d + e*x^2]) + (6*d^(5/4)*Sqrt[-e]*(Sqrt[d] + S
qrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(
1/4)*Sqrt[x])/d^(1/4)], 1/2])/(25*e^(7/4)*Sqrt[d + e*x^2])
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{5}x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{5}\left(2\sqrt{-e}\right)\int\frac{x^{5/2}}{\sqrt{d+ex^2}}dx$$

$$=\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}}+\frac{2}{5}x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(6d)\int\frac{\sqrt{x}}{\sqrt{d+ex^2}}dx}{25\sqrt{-e}}$$

$$=\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}}+\frac{2}{5}x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(12d)\,\text{Subst}\left(\int\frac{x^2}{\sqrt{d+ex^4}}dx,x,\sqrt{x}\right)}{25\sqrt{-e}}$$

$$=\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}}+\frac{2}{5}x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\left(12d^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}dx,x,\sqrt{x}\right)}{25\sqrt{-e^2}}+\frac{(12\cdots}{\cdots}$$

$$=\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}}-\frac{12d\sqrt{x}\sqrt{d+ex^2}}{25\sqrt{-e^2}\left(\sqrt{d}+\sqrt{e}x\right)}+\frac{2}{5}x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)+\frac{12d^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\cdots}{25\cdots}$$

Mathematica [C] time = 0.109487, size = 119, normalized size = 0.4

$$\frac{2x^{3/2}\left(2d\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)-2\sqrt{-e}\left(d+ex^2\right)+5ex\sqrt{d+ex^2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\right)}{25e\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*x^(3/2)*(-2*Sqrt[-e]*(d + e*x^2) + 5*e*x*Sqrt[d + e*x^2]*ArcTan[(Sqrt[-e
]*x)/Sqrt[d + e*x^2]] + 2*d*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[
1/2, 3/4, 7/4, -((e*x^2)/d)]))/(25*e*Sqrt[d + e*x^2])
```

Maple [F] time = 0.296, size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}} \arctan\left(x\sqrt{-e}\frac{1}{\sqrt{e x^2+d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2}{5} x^{\frac{5}{2}} \arctan\left(\sqrt{-e}x, \sqrt{e x^2+d}\right) - 2 d\sqrt{-e} \int -\frac{x e^{\left(\frac{1}{2}\log\left(e x^2+d\right)+\frac{3}{2}\log(x)\right)}}{5\left(e^2 x^4 + d e x^2 - \left(e x^2+d\right)^2\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] 2/5*x^(5/2)*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - 2*d*sqrt(-e)*integrate(-
1/5*x*e^(1/2*log(e*x^2 + d) + 3/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^
2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^{\frac{3}{2}} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{e x^2+d}}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] integral(x^(3/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

Sympy [C] time = 40.5369, size = 75, normalized size = 0.25

$$\frac{2x^{\frac{5}{2}} \operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+e x^2}}\right)}{5} - \frac{x^{\frac{7}{2}}\sqrt{-e}\,\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{7}{4} \\ \frac{11}{4}\end{matrix}\right| \frac{e x^2 e^{i\pi}}{d}\right)}{5\sqrt{d}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] 2*x**(5/2)*atan(x*sqrt(-e)/sqrt(d + e*x**2))/5 - x**(7/2)*sqrt(-e)*gamma(7/
4)*hyper((1/2, 7/4), (11/4,), e*x**2*exp_polar(I*pi)/d)/(5*sqrt(d)*gamma(11
/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2 + d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

3.27 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} dx$

Optimal. Leaf size=260

$$-\frac{2\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{e^{3/4}\sqrt{d+ex^2}}+\frac{4\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right.\right.}{e^{3/4}\sqrt{d+ex^2}}$$

```
[Out] (-4*Sqrt[-e]*Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[e]*(Sqrt[d] + Sqrt[e]*x)) + 2*S
qrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] + (4*d^(1/4)*Sqrt[-e]*(Sqrt[d]
+ Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(
e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(3/4)*Sqrt[d + e*x^2]) - (2*d^(1/4)*Sqr
t[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*Ellip
ticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(3/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.138935, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5151, 329, 305, 220, 1196}

$$-\frac{2\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{e^{3/4}\sqrt{d+ex^2}}+\frac{4\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{e^{3/4}\sqrt{d+ex^2}}-\frac{4\sqrt{}}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```


```
[Out] (-4*Sqrt[-e]*Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[e]*(Sqrt[d] + Sqrt[e]*x)) + 2*S
qrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] + (4*d^(1/4)*Sqrt[-e]*(Sqrt[d]
+ Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(
e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(3/4)*Sqrt[d + e*x^2]) - (2*d^(1/4)*Sqr
t[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*Ellip
ticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(3/4)*Sqrt[d + e*x^2])
```


Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \left(2\sqrt{-e}\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx$$

$$= 2\sqrt{x}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \left(4\sqrt{-e}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)$$

$$= 2\sqrt{x}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{\left(4\sqrt{d}\sqrt{-e}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{e}} + \frac{\left(4\sqrt{d}\sqrt{-e}\right)\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt{e}x^2}{\sqrt{d}}}{\sqrt{d+ex^4}}\right.}{\sqrt{e}}$$

$$= -\frac{4\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{\sqrt{e}\left(\sqrt{d}+\sqrt{e}x\right)} + 2\sqrt{x}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + \frac{4\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\right.}{e^{3/4}\sqrt{d+ex^2}}$$

Mathematica [C] time = 0.106167, size = 89, normalized size = 0.34

$$2\sqrt{x}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{4\sqrt{-e}x^{3/2}\sqrt{\frac{ex^2}{d}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)}{3\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] - (4*Sqrt[-e]*x^(3/2)*Sqrt[1
 + (e*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(3*Sqrt[d + e
*x^2])
```

Maple [F] time = 0.283, size = 0, normalized size = 0.

$$\int \arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)\frac{1}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-2\,d\sqrt{-e}\int\frac{\sqrt{ex^2+d}x}{\left(ex^2+d\right)e^{\left(\log\left(ex^2+d\right)+\frac{1}{2}\log(x)\right)}-\left(e^2x^4+dex^2\right)\sqrt{x}}\,dx+2\sqrt{x}\arctan\left(\sqrt{-e}x,\sqrt{ex^2+d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="maxima
")
```

```
[Out] -2*d*sqrt(-e)*integrate(sqrt(e*x^2 + d)*x/((e*x^2 + d)*e^(log(e*x^2 + d) +
1/2*log(x)) - (e^2*x^4 + d*e*x^2)*sqrt(x)), x) + 2*sqrt(x)*arctan2(sqrt(-e)
*x, sqrt(e*x^2 + d))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{\sqrt{x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/sqrt(x), x)
```

Sympy [C] time = 7.20916, size = 71, normalized size = 0.27

$$2\sqrt{x}\,\text{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)-\frac{x^{\frac{3}{2}}\sqrt{-e}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{\sqrt{d}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(1/2),x)
```

```
[Out] 2*sqrt(x)*atan(x*sqrt(-e)/sqrt(d + e*x**2)) - x**(3/2)*sqrt(-e)*gamma(3/4)*
hyper((1/2, 3/4), (7/4,), e*x**2*exp_polar(I*pi)/d)/(sqrt(d)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/sqrt(x), x)
```

3.28 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=298

$$\frac{2\sqrt{-e}\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}-\frac{4\sqrt{-e}\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}$$

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(3*d*Sqrt[x]) + (4*Sqrt[-e^2]*Sqrt[x]*Sqrt[d
+ e*x^2])/(3*d*(Sqrt[d] + Sqrt[e]*x)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x
^2]])/(3*x^(3/2)) - (4*Sqrt[-e]*e^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x
^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)],
 1/2])/(3*d^(3/4)*Sqrt[d + e*x^2]) + (2*Sqrt[-e]*e^(1/4)*(Sqrt[d] + Sqrt[e]
*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*S
qrt[x])/d^(1/4)], 1/2])/(3*d^(3/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.166054, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5151, 325, 329, 305, 220, 1196}

$$\frac{2\sqrt{-e}\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}-\frac{4\sqrt{-e}\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}+\frac{4\sqrt{-}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(3*d*Sqrt[x]) + (4*Sqrt[-e^2]*Sqrt[x]*Sqrt[d
+ e*x^2])/(3*d*(Sqrt[d] + Sqrt[e]*x)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x
^2]])/(3*x^(3/2)) - (4*Sqrt[-e]*e^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x
^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)],
 1/2])/(3*d^(3/4)*Sqrt[d + e*x^2]) + (2*Sqrt[-e]*e^(1/4)*(Sqrt[d] + Sqrt[e]
*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*S
qrt[x])/d^(1/4)], 1/2])/(3*d^(3/4)*Sqrt[d + e*x^2])
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{1}{3}\left(2\sqrt{-e}\right)\int \frac{1}{x^{3/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} - \frac{\left(2(-e)^{3/2}\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{3d}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} - \frac{\left(4(-e)^{3/2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{3d}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} - \frac{\left(4(-e)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{3\sqrt{d}\sqrt{e}} + \frac{\left(4(-e)^{3/2}\right)\text{Su}}{}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{4(-e)^{3/2}\sqrt{x}\sqrt{d+ex^2}}{3d\sqrt{e}\left(\sqrt{d}+\sqrt{e}x\right)} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{4(-e)^{3/2}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{(\sqrt{d}+\sqrt{e}x}}}{3d^{3/4}e^{3/4}\sqrt{d}}$$

Mathematica [C] time = 0.132899, size = 121, normalized size = 0.41

$$-\frac{2\left(2(-e)^{3/2}x^3\sqrt{\frac{ex^2}{d}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right) + 6\sqrt{-e}x\left(d+ex^2\right) + 3d\sqrt{d+ex^2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\right)}{9dx^{3/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

```
[Out] (-2*(6*Sqrt[-e]*x*(d + e*x^2) + 3*d*Sqrt[d + e*x^2]*ArcTan[(Sqrt[-e]*x)/Sqr
t[d + e*x^2]] + 2*(-e)^(3/2)*x^3*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[1/2,
 3/4, 7/4, -((e*x^2)/d)]))/(9*d*x^(3/2)*Sqrt[d + e*x^2])
```

Maple [F] time = 0.296, size = 0, normalized size = 0.

$$\int \arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)x^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(-d\sqrt{-e}x^{\frac{3}{2}}\int-\frac{\sqrt{ex^2+d}x}{\left(ex^2+d\right)^2x^{\frac{5}{2}}-\left(e^2x^4+dex^2\right)x^{\frac{5}{2}}}\,dx-\arctan\left(\sqrt{-e}x,\sqrt{ex^2+d}\right)\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="maxima
")
```

```
[Out] 2/3*(3*d*sqrt(-e)*x^(3/2)*integrate(-1/3*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*
x^2)*x^(5/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 5/2*log(x))), x) - arctan2(s
qrt(-e)*x, sqrt(e*x^2 + d)))/x^(3/2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(5/2), x)
```

Sympy [C] time = 37.5289, size = 78, normalized size = 0.26

$$-\frac{2\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3x^{\frac{3}{2}}}+\frac{\sqrt{-e}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{3\sqrt{d}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(5/2),x)
```

```
[Out] -2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(3*x**(3/2)) + sqrt(-e)*gamma(-1/4)*hy
per((-1/4, 1/2), (3/4,), e*x**2*exp_polar(I*pi)/d)/(3*sqrt(d)*sqrt(x)*gamma
(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(5/2), x)
```

3.29 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{6e^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}-\frac{12e^{3/2}\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}x\right)}+\frac{12e^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\cdots}}{35d^{7/4}\cdots}$$

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(35*d*x^(5/2)) - (12*(-e)^(3/2)*Sqrt[d + e*x^
2])/(35*d^2*Sqrt[x]) - (12*Sqrt[-e]*e^(3/2)*Sqrt[x]*Sqrt[d + e*x^2])/(35*d^
2*(Sqrt[d] + Sqrt[e]*x)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(7*x^(7
/2)) + (12*Sqrt[-e]*e^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d]
 + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(35*d
^(7/4)*Sqrt[d + e*x^2]) - (6*Sqrt[-e]*e^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d
 + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(
1/4)], 1/2])/(35*d^(7/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.193637, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5151, 325, 329, 305, 220, 1196}

$$-\frac{12e^{3/2}\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}x\right)}-\frac{6e^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}+\frac{12e^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\cdots}{35d^{7/4}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(35*d*x^(5/2)) - (12*(-e)^(3/2)*Sqrt[d + e*x^
2])/(35*d^2*Sqrt[x]) - (12*Sqrt[-e]*e^(3/2)*Sqrt[x]*Sqrt[d + e*x^2])/(35*d^
2*(Sqrt[d] + Sqrt[e]*x)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(7*x^(7
/2)) + (12*Sqrt[-e]*e^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d]
 + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(35*d
^(7/4)*Sqrt[d + e*x^2]) - (6*Sqrt[-e]*e^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d
 + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(
1/4)], 1/2])/(35*d^(7/4)*Sqrt[d + e*x^2])
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} + \frac{1}{7}\left(2\sqrt{-e}\right)\int \frac{1}{x^{7/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} + \frac{\left(6(-e)^{3/2}\right)\int \frac{1}{x^{3/2}\sqrt{d+ex^2}}\,dx}{35d}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(6(-e)^{5/2}\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{35d^2}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(12(-e)^{5/2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex^4}}\,dx, x\right.}{35d^2}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(12\sqrt{-e}e^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx,\right.}{35d^{3/2}}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{12\sqrt{-e}e^{3/2}\sqrt{x}\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}x\right)} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} + \frac{12\sqrt{-e}}{}$$

Mathematica [C] time = 0.117839, size = 137, normalized size = 0.41

$$\frac{4\sqrt{-e}x\left(-d^2+2dex^2+3e^2x^4\right)-10d^2\sqrt{d+ex^2}\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-4(-e)^{5/2}x^5\sqrt{\frac{ex^2}{d}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)}{35d^2x^{7/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```

```
[Out] (4*Sqrt[-e]*x*(-d^2 + 2*d*e*x^2 + 3*e^2*x^4) - 10*d^2*Sqrt[d + e*x^2]*ArcTa
n[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] - 4*(-e)^(5/2)*x^5*Sqrt[1 + (e*x^2)/d]*Hype
rgeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(35*d^2*x^(7/2)*Sqrt[d + e*x^2]
)
```

Maple [F] time = 0.313, size = 0, normalized size = 0.

$$\int \arctan\left(x\sqrt{-e}\frac{1}{\sqrt{ex^2+d}}\right)x^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(-d\sqrt{-e}x^{\frac{7}{2}}\int -\frac{\sqrt{ex^2+d}x}{(ex^2+d)^2x^{\frac{9}{2}}-(e^2x^4+dex^2)x^{\frac{9}{2}}}\,dx - \arctan\left(\sqrt{-e}x, \sqrt{ex^2+d}\right)\right)}{7x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="maxima
")
```

```
[Out] 2/7*(7*d*sqrt(-e)*x^(7/2)*integrate(-1/7*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*
x^2)*x^(9/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 9/2*log(x))), x) - arctan2(s
qrt(-e)*x, sqrt(e*x^2 + d)))/x^(7/2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="fricas
")
```

```
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(9/2), x)
```

3.30 $\int \frac{\tan^{-1}(1+x+x^2)}{x^2} \, dx$

Optimal. Leaf size=50

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x^2+2x+2\right)-\frac{\tan^{-1}\left(x^2+x+1\right)}{x}+\frac{\log(x)}{2}+\frac{1}{2}\tan^{-1}(x+1)$$

```
[Out] ArcTan[1 + x]/2 - ArcTan[1 + x + x^2]/x + Log[x]/2 - Log[1 + x^2]/2 + Log[2
 + 2*x + x^2]/4
```

Rubi [A] time = 0.146014, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5205, 6742, 260, 634, 617, 204, 628}

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x^2+2x+2\right)-\frac{\tan^{-1}\left(x^2+x+1\right)}{x}+\frac{\log(x)}{2}+\frac{1}{2}\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[1 + x + x^2]/x^2,x]
```

```
[Out] ArcTan[1 + x]/2 - ArcTan[1 + x + x^2]/x + Log[x]/2 - Log[1 + x^2]/2 + Log[2
 + 2*x + x^2]/4
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}\left(1+x+x^2\right)}{x^2}\,dx &= -\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\int \frac{1+2x}{x\left(2+2x+3x^2+2x^3+x^4\right)}\,dx \\
&= -\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\int \left(\frac{1}{2x}-\frac{x}{1+x^2}+\frac{2+x}{2\left(2+2x+x^2\right)}\right)dx \\
&= -\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\frac{\log(x)}{2}+\frac{1}{2}\int \frac{2+x}{2+2x+x^2}\,dx-\int \frac{x}{1+x^2}\,dx \\
&= -\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\frac{\log(x)}{2}-\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{4}\int \frac{2+2x}{2+2x+x^2}\,dx+\frac{1}{2}\int \frac{1}{2+2x+x^2} \\
&= -\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\frac{\log(x)}{2}-\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{4}\log\left(2+2x+x^2\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-\right. \\
&= \frac{1}{2}\tan^{-1}(1+x)-\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\frac{\log(x)}{2}-\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{4}\log\left(2+2x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0149432, size = 50, normalized size = 1.

$$
-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x^2+2x+2\right)-\frac{\tan^{-1}\left(x^2+x+1\right)}{x}+\frac{\log(x)}{2}+\frac{1}{2}\tan^{-1}(x+1)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[1 + x + x^2]/x^2,x]
```

```
[Out] ArcTan[1 + x]/2 - ArcTan[1 + x + x^2]/x + Log[x]/2 - Log[1 + x^2]/2 + Log[2
 + 2*x + x^2]/4
```

Maple [A] time = 0.043, size = 43, normalized size = 0.9

$$
\frac{\arctan(x+1)}{2}-\frac{\arctan\left(x^2+x+1\right)}{x}+\frac{\ln(x)}{2}-\frac{\ln\left(x^2+1\right)}{2}+\frac{\ln\left(x^2+2x+2\right)}{4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^2+x+1)/x^2,x)
```

```
[Out] 1/2*arctan(x+1)-arctan(x^2+x+1)/x+1/2*ln(x)-1/2*ln(x^2+1)+1/4*ln(x^2+2*x+2)
```

Maxima [A] time = 1.47961, size = 57, normalized size = 1.14

$$-\frac{\arctan\left(x^2+x+1\right)}{x}+\frac{1}{2}\arctan\left(x+1\right)+\frac{1}{4}\log\left(x^2+2x+2\right)-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^2+x+1)/x^2,x, algorithm="maxima")
```

```
[Out] -arctan(x^2 + x + 1)/x + 1/2*arctan(x + 1) + 1/4*log(x^2 + 2*x + 2) - 1/2*l
og(x^2 + 1) + 1/2*log(x)
```

Fricas [A] time = 2.03198, size = 143, normalized size = 2.86

$$\frac{2x\arctan\left(x+1\right)+x\log\left(x^2+2x+2\right)-2x\log\left(x^2+1\right)+2x\log\left(x\right)-4\arctan\left(x^2+x+1\right)}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^2+x+1)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*x*arctan(x + 1) + x*log(x^2 + 2*x + 2) - 2*x*log(x^2 + 1) + 2*x*log(
x) - 4*arctan(x^2 + x + 1))/x
```

Sympy [A] time = 1.07726, size = 41, normalized size = 0.82

$$\frac{\log\left(x\right)}{2}-\frac{\log\left(x^2+1\right)}{2}+\frac{\log\left(x^2+2x+2\right)}{4}+\frac{\operatorname{atan}\left(x+1\right)}{2}-\frac{\operatorname{atan}\left(x^2+x+1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**2+x+1)/x**2,x)
```

```
[Out] log(x)/2 - log(x**2 + 1)/2 + log(x**2 + 2*x + 2)/4 + atan(x + 1)/2 - atan(x
**2 + x + 1)/x
```

Giac [A] time = 1.1258, size = 58, normalized size = 1.16

$$-\frac{\arctan\left(x^2+x+1\right)}{x}+\frac{1}{2}\arctan\left(x+1\right)+\frac{1}{4}\log\left(x^2+2x+2\right)-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^2+x+1)/x^2,x, algorithm="giac")
```

```
[Out] -arctan(x^2 + x + 1)/x + 1/2*arctan(x + 1) + 1/4*log(x^2 + 2*x + 2) - 1/2*l
og(x^2 + 1) + 1/2*log(abs(x))
```

3.31 $\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x
]
```

Rubi [A] time = 0.046276, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Defer[Int][(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [A] time = 0.0892483, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Maple [A] time = 1.224, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\arctan\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(b \arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^n}{c^2x^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] -integrate((b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b \arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^n}{c^2x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a + b \operatorname{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{c^2x^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] -Integral((a + b*atan(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b \arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^n}{c^2x^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

3.32 $\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=431

$$\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}-\frac{3b^2\text{PolyLog}\left(3,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3ib\text{PolyL}\cdots}{}$$

```
[Out] (-2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - 2/(1 + (I*Sqr
t[1 - c*x])/Sqrt[1 + c*x])])/c + (((3*I)/2)*b*(a + b*ArcTan[Sqrt[1 - c*x]/S
qrt[1 + c*x]])^2*PolyLog[2, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c
 - (((3*I)/2)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, -1
 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c + (3*b^2*(a + b*ArcTan[Sqrt[
1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c
*x])])/(2*c) - (3*b^2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3
, -1 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/(2*c) - (((3*I)/4)*b^3*Pol
yLog[4, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c + (((3*I)/4)*b^3*Po
lyLog[4, -1 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c
```

Rubi [A] time = 0.484625, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {6681, 4850, 4988, 4884, 4994, 4998, 6610}

$$\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}-\frac{3b^2\text{PolyLog}\left(3,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3ib\text{PolyL}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - 2/(1 + (I*Sqr
t[1 - c*x])/Sqrt[1 + c*x])])/c + (((3*I)/2)*b*(a + b*ArcTan[Sqrt[1 - c*x]/S
qrt[1 + c*x]])^2*PolyLog[2, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c
 - (((3*I)/2)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, -1
 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c + (3*b^2*(a + b*ArcTan[Sqrt[
1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c
*x])])/(2*c) - (3*b^2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3
, -1 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/(2*c) - (((3*I)/4)*b^3*Pol
yLog[4, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c + (((3*I)/4)*b^3*Po
lyLog[4, -1 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
```

```
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(6b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2\tanh^{-1}\left(1-\frac{}{1\cdot}\right.}{1+x^2}\right.}{c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2\log\left(2-\frac{2}{1+ix}\right)}{1+x^2}\right.}{c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3ib\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \text{Li}_2\left(1-\frac{}{1+}\right.}{2c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3ib\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \text{Li}_2\left(1-\frac{}{1+}\right.}{2c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3ib\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \text{Li}_2\left(1-\frac{}{1+}\right.}{2c}$$

Mathematica [A] time = 0.176544, size = 530, normalized size = 1.23

$$-\frac{6b^2\text{PolyLog}\left(3,-\frac{\sqrt{1-cx}+i\sqrt{cx+1}}{\sqrt{1-cx}-i\sqrt{cx+1}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-6b^2\text{PolyLog}\left(3,\frac{\sqrt{1-cx}+i\sqrt{cx+1}}{\sqrt{1-cx}-i\sqrt{cx+1}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)+6ib\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] -(8*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - (2*I)/(I - Sq
rt[1 - c*x]/Sqrt[1 + c*x])] + (6*I)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 +
c*x]])^2*PolyLog[2, -((Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*
Sqrt[1 + c*x]))] - (6*I)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*Po
lyLog[2, (Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[1 + c*x]
)] + 6*b^2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -((Sqrt[1
 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[1 + c*x]))] - 6*b^2*(a +
 b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, (Sqrt[1 - c*x] + I*Sqrt[
1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[1 + c*x])] - (3*I)*b^3*PolyLog[4, -((Sqrt
[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[1 + c*x]))] + (3*I)*b^
3*PolyLog[4, (Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[1 +
c*x])])/(4*c)
```

Maple [B] time = 2.559, size = 1745, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

```
[Out] -3*I*a*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(1+I*(-c*x+1)^
(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+6*I*a*b^2/c*arctan((-c*x+1)^(1
/2)/(c*x+1)^(1/2))*polylog(2,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(
c*x+1)+1)^(1/2))+6*I*a*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2
,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-1/2*a^3/c*
ln(c*x-1)+3*I*b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,(1+I*(
-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+3*I*b^3/c*arctan((
-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)
)/((-c*x+1)/(c*x+1)+1)^(1/2))-3/2*I*b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/
2))^2*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))
-3*a^2*b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(1+I*(-c*x+1)^(1/2)/(c
*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-3*a^2*b/c*arctan((-c*x+1)^(1/2)/(c
*x+1)^(1/2))*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(
1/2))+3*a^2*b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln((1+I*(-c*x+1)^(1/2)
/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)+3*I*a^2*b/c*polylog(2,-(1+I*(-c*x
+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+3*I*a^2*b/c*polylog(2,
(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-3/2*I*a^2*b/
c*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-3*a
*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x
+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-3*a*b^2/c*arctan((-c*x+1)^(1/2)/(c*x
+1)^(1/2))^2*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(
1/2))+3*a*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln((1+I*(-c*x+1)^(1/
2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)-6*b^3/c*arctan((-c*x+1)^(1/2)/(
c*x+1)^(1/2))*polylog(3,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1
)+1)^(1/2))-b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+(1+I*(-c*x+1)
^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-b^3/c*arctan((-c*x+1)^(1/
2)/(c*x+1)^(1/2))^3*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+
1)+1)^(1/2))+b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln((1+I*(-c*x+1)^
(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)+3/2*b^3/c*arctan((-c*x+1)^(1
/2)/(c*x+1)^(1/2))*polylog(3,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1
)/(c*x+1)+1))-6*I*b^3/c*polylog(4,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x
+1)/(c*x+1)+1)^(1/2))-6*I*b^3/c*polylog(4,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2
))/((-c*x+1)/(c*x+1)+1)^(1/2))+3/4*I*b^3/c*polylog(4,-(1+I*(-c*x+1)^(1/2)/(
c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-6*b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)
^(1/2))*polylog(3,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^
(1/2))-6*a*b^2/c*polylog(3,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*
x+1)+1)^(1/2))-6*a*b^2/c*polylog(3,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c
*x+1)/(c*x+1)+1)^(1/2))+3/2*a*b^2/c*polylog(3,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^
(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/2*a^3/c*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^3\left(\frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c}\right) + \frac{\frac{15}{2}\left(b^3\log(cx+1) - b^3\log(-cx+1)\right)\arctan\left(\sqrt{-cx+1}, \sqrt{cx+1}\right)^3 - \frac{45}{8}\left(b^3\log(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/64*(4*(b^3*log(c*x + 1) - b^3
*log(-c*x + 1))*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^3 - 3*(b^3*log(2)^2*
log(c*x + 1) - b^3*log(2)^2*log(-c*x + 1))*arctan2(sqrt(-c*x + 1), sqrt(c*x
 + 1)) - 64*c*integrate(1/128*(112*b^3*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1
))^3 + 384*a*b^2*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^2 - 3*(b^3*log(2)^2
*log(c*x + 1) - b^3*log(2)^2*log(-c*x + 1) - 4*(b^3*log(c*x + 1) - b^3*log(
```

```
-c*x + 1))*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^2)*sqrt(c*x + 1)*sqrt(-c*
x + 1) + 12*(b^3*log(2)^2 + 32*a^2*b)*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)
))/(c^2*x^2 - 1), x))/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^{3} \arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^{3} + 3\,ab^{2} \arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^{2} + 3\,a^{2}b \arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a^{3}}{c^{2}x^{2} - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^3*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arctan(sqrt
(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))
+ a^3)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b \arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^{3}}{c^{2}x^{2} - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x)
```

3.33 $\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=283

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{ib\text{PolyLog}\left(2,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{b^2\text{PolyLo}}{}$$

```
[Out] (-2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - 2/(1 + (I*Sqr
t[1 - c*x])/Sqrt[1 + c*x])])/c + (I*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 +
c*x]])*PolyLog[2, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c - (I*b*(a
 + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, -1 + 2/(1 + (I*Sqrt[1
- c*x])/Sqrt[1 + c*x])])/c + (b^2*PolyLog[3, 1 - 2/(1 + (I*Sqrt[1 - c*x])/S
qrt[1 + c*x])])/(2*c) - (b^2*PolyLog[3, -1 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[
1 + c*x])])/(2*c)
```

Rubi [A] time = 0.295276, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6681, 4850, 4988, 4884, 4994, 6610}

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{ib\text{PolyLog}\left(2,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{b^2\text{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - 2/(1 + (I*Sqr
t[1 - c*x])/Sqrt[1 + c*x])])/c + (I*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 +
c*x]])*PolyLog[2, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c - (I*b*(a
 + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, -1 + 2/(1 + (I*Sqrt[1
- c*x])/Sqrt[1 + c*x])])/c + (b^2*PolyLog[3, 1 - 2/(1 + (I*Sqrt[1 - c*x])/S
qrt[1 + c*x])])/(2*c) - (b^2*PolyLog[3, -1 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[
1 + c*x])])/(2*c)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(4b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)\tanh^{-1}\left(1-\frac{2}{1+i}\right.}{1+x^2}\right.}{c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(2b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)\log\left(2-\frac{2}{1+ix}\right)}{1+x^2}\right.}{c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{ib\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{Li}_2\left(1-\frac{2}{1+\frac{i\sqrt{}}{\sqrt{}}}\right.}{c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{ib\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{Li}_2\left(1-\frac{2}{1+\frac{i\sqrt{}}{\sqrt{}}}\right.}{c}$$

Mathematica [A] time = 0.106713, size = 354, normalized size = 1.25

$$-\frac{2ib\text{PolyLog}\left(2,-\frac{\sqrt{1-cx}+i\sqrt{cx+1}}{\sqrt{1-cx}-i\sqrt{cx+1}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-2ib\text{PolyLog}\left(2,\frac{\sqrt{1-cx}+i\sqrt{cx+1}}{\sqrt{1-cx}-i\sqrt{cx+1}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)+b^2\text{PolyL}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] -(4*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - (2*I)/(I - Sq
rt[1 - c*x]/Sqrt[1 + c*x])] + (2*I)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 +
c*x]])*PolyLog[2, -((Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sq
rt[1 + c*x]))] - (2*I)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLo
g[2, (Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[1 + c*x])] +
 b^2*PolyLog[3, -((Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt
[1 + c*x]))] - b^2*PolyLog[3, (Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c
*x] - I*Sqrt[1 + c*x])])/(2*c)
```

Maple [B] time = 1.218, size = 947, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

```
[Out] 1/2*a^2/c*ln(c*x+1)-1/2*a^2/c*ln(c*x-1)-b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)
^(1/2))^2*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2
))+2*I*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,(1+I*(-c*x+1)^(
1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-2*b^2/c*polylog(3,(1+I*(-c*
x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-b^2/c*arctan((-c*x+1)
^(1/2)/(c*x+1)^(1/2))^2*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(
c*x+1)+1)^(1/2))-I*a*b/c*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((
-c*x+1)/(c*x+1)+1))-2*b^2/c*polylog(3,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/(
(-c*x+1)/(c*x+1)+1)^(1/2))+b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(
(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)-I*b^2/c*arctan
((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)
)^2/((-c*x+1)/(c*x+1)+1))+1/2*b^2/c*polylog(3,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^
(1/2))^2/((-c*x+1)/(c*x+1)+1))-2*a*b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))
*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+2*I*a*
b/c*polylog(2,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2)
)-2*a*b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(1+I*(-c*x+1)^(1/2)/(c*
x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+2*I*b^2/c*arctan((-c*x+1)^(1/2)/(c*
x+1)^(1/2))*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)
+1)^(1/2))+2*a*b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln((1+I*(-c*x+1)^(1
/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)+2*I*a*b/c*polylog(2,-(1+I*(-c*
x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2\left(\frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c}\right) - \frac{b^2 \log(2)^2 \log(cx+1) - b^2 \log(2)^2 \log(-cx+1) - 4\left(b^2 \log(cx+1) - b^2 \log(-cx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/32*(b^2*log(2)^2*log(c*x + 1)
 - b^2*log(2)^2*log(-c*x + 1) - 4*(b^2*log(c*x + 1) - b^2*log(-c*x + 1))*ar
```

```
ctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^2 - (b^2*(log(c*x + 1)/c - log(c*x - 1
)/c)*log(2)^2 - 64*b^2*integrate(1/16*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(s
qrt(-c*x + 1)/sqrt(c*x + 1))*log(c*x + 1)/(c^2*x^2 - 1), x) + 64*b^2*integr
ate(1/16*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))*
log(-c*x + 1)/(c^2*x^2 - 1), x) - 384*b^2*integrate(1/16*arctan(sqrt(-c*x +
 1)/sqrt(c*x + 1))^2/(c^2*x^2 - 1), x) - 1024*a*b*integrate(1/16*arctan(sqr
t(-c*x + 1)/sqrt(c*x + 1))/(c^2*x^2 - 1), x))*c)/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^2\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 2\,ab\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a^2}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^2*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arctan(sqrt(-
c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^2}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x)
```

3.34 $\int \frac{a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=98

$$-\frac{ib\text{PolyLog}\left(2,-\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}+\frac{ib\text{PolyLog}\left(2,\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/c) - ((I/2)*b*PolyLog[2, ((-I)*Sqrt[
1 - c*x])/Sqrt[1 + c*x]])/c + ((I/2)*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1
+ c*x]])/c
```

Rubi [A] time = 0.0666915, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {206, 6681, 4848, 2391}

$$-\frac{ib\text{PolyLog}\left(2,-\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}+\frac{ib\text{PolyLog}\left(2,\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/c) - ((I/2)*b*PolyLog[2, ((-I)*Sqrt[
1 - c*x])/Sqrt[1 + c*x]])/c + ((I/2)*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1
+ c*x]])/c
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$
$$= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} - \frac{(ib)\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c} + \frac{(ib)\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}$$
$$= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} - \frac{ib\text{Li}_2\left(-\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c} + \frac{ib\text{Li}_2\left(\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}$$

Mathematica [A] time = 0.0277243, size = 93, normalized size = 0.95

$$-\frac{\frac{1}{2}ib\text{PolyLog}\left(2,-\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)-\frac{1}{2}ib\text{PolyLog}\left(2,\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)+a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]] + (I/2)*b*PolyLog[2, ((-I)*Sqrt[1 - c
*x])/Sqrt[1 + c*x]] - (I/2)*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/
c)
```

Maple [B] time = 0.717, size = 371, normalized size = 3.8

$$\frac{a\ln(cx+1)}{2c} - \frac{a\ln(cx-1)}{2c} - \frac{b}{c}\arctan\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\ln\left(1+\left(1+i\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\frac{1}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right) + \frac{ib}{c}\text{polylog}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

```
[Out] 1/2*a/c*ln(c*x+1)-1/2*a/c*ln(c*x-1)-b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)
)*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+I*b/c
*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-
b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(
1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+I*b/c*polylog(2,(1+I*(-c*x+1)^(1/2)/(c*x+
1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/
2))*ln((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)-1/2*I*b
/c*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a\left(\frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c}\right) + \frac{\Bigg((\log(cx+1)-\log(-cx+1))\arctan\left(\sqrt{-cx+1},\sqrt{cx+1}\right) - c\int \frac{e^{\left(\frac{1}{2}\log(cx+1)+\frac{1}{2}\log(-cx+1)\right)}}{(c^2x}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="maxima")
```

```
[Out] 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*((log(c*x + 1) - log(-c*x + 1
))*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)) - 2*c*integrate(1/2*(e^(1/2*log(c
*x + 1) + 1/2*log(-c*x + 1))*log(c*x + 1) - e^(1/2*log(c*x + 1) + 1/2*log(-
c*x + 1))*log(-c*x + 1))/((c^2*x^2 - 1)*(c*x + 1) - (c^2*x^2 - 1)*(c*x - 1)
), x))*b/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="fricas")
```

```
[Out] integral(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="giac")
```

```
[Out] integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

3.35 $\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)},x\right)$$

```
[Out] Unintegrable[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
 x]
```

Rubi [A] time = 0.0436831, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Mathematica [A] time = 0.0869805, size = 0, normalized size = 0.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x]
```

Maple [A] time = 1.135, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\arctan\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(c^2x^2-1)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="maxima")
```

```
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{ac^2x^2+(bc^2x^2-b)\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="fricas")
```

```
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1
)) - a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{ac^2x^2-a+bc^2x^2\operatorname{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b\operatorname{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] -Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*atan(sqrt(-c*x + 1)/sqrt(c*x + 1
)) - b*atan(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(c^2x^2-1)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
x)
```

3.36 $\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2
), x]
```

Rubi [A] time = 0.0411422, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]

[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [A] time = 0.778394, size = 0, normalized size = 0.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x
]

[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

Maple [A] time = 1.051, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\arctan\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(\left(b^2c^2\arctan\left(\sqrt{-cx+1},\sqrt{cx+1}\right)+abc^2\right)\sqrt{cx+1}\sqrt{-cx+1}\int\frac{x}{\left(abc^2x^2-ab+\left(b^2c^2x^2-b^2\right)\arctan\left(\sqrt{-cx+1},\sqrt{cx+1}\right)\right)\sqrt{cx+1}\sqrt{-cx+1}}\,dx+}{\left(b^2c\arctan\left(\sqrt{-cx+1},\sqrt{cx+1}\right)+abc\right)\sqrt{cx+1}\sqrt{-cx+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="maxima")
```

```
[Out] 2*(2*(b^2*c^2*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)) + a*b*c^2)*sqrt(c*x +
1)*sqrt(-c*x + 1)*integrate(1/2*x/((a*b*c^2*x^2 - a*b + (b^2*c^2*x^2 - b^2)
*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)
+ 1)/((b^2*c*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)) + a*b*c)*sqrt(c*x + 1)*
sqrt(-c*x + 1))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{a^2c^2x^2+\left(b^2c^2x^2-b^2\right)\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2-a^2+2\left(abc^2x^2-ab\right)\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arctan(sqrt(-c*x + 1)/sqrt(c
*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1
))), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{a^2c^2x^2-a^2+2abc^2x^2\,\mathrm{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-2ab\,\mathrm{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+b^2c^2x^2\,\mathrm{atan}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b^2\,\mathrm{atan}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x)
```

```
[Out] -Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*atan(sqrt(-c*x + 1)/sq
rt(c*x + 1)) - 2*a*b*atan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + b**2*c**2*x**2*at
an(sqrt(-c*x + 1)/sqrt(c*x + 1))**2 - b**2*atan(sqrt(-c*x + 1)/sqrt(c*x + 1
```

```
))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(c^2x^2-1\right)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2)
, x)
```

3.37 $\int x^m \tan^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=37

$$\frac{x^{m+1}\tan^{-1}(\tan(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcTan[Tan[a + b*x]])/(1 + m)
```

Rubi [A] time = 0.0242773, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{x^{m+1}\tan^{-1}(\tan(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTan[Tan[a + b*x]],x]
```

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcTan[Tan[a + b*x]])/(1 + m)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^m \tan^{-1}(\tan(a+bx))\,dx &= \frac{x^{1+m}\tan^{-1}(\tan(a+bx))}{1+m}-\frac{b\int x^{1+m}\,dx}{1+m}\\ &= -\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\tan^{-1}(\tan(a+bx))}{1+m}\end{aligned}$$

Mathematica [A] time = 0.0559663, size = 34, normalized size = 0.92

$$x^m\left(\frac{x\left(\tan^{-1}(\tan(a+bx))-bx\right)}{m+1}+\frac{bx^2}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTan[Tan[a + b*x]],x]
```

```
[Out] x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcTan[Tan[a + b*x]]))/(1 + m))
```

Maple [A] time = 0.056, size = 41, normalized size = 1.1

$$\frac{bx^2 e^{m\ln(x)}}{2+m} + \frac{(\arctan(\tan(bx+a)) - bx)\, x e^{m\ln(x)}}{1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(tan(b*x+a)),x)
```

```
[Out] b/(2+m)*x^2*exp(m*ln(x))+(arctan(tan(b*x+a))-b*x)/(1+m)*x*exp(m*ln(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72609, size = 72, normalized size = 1.95

$$\frac{\left((bm+b)x^2 + (am+2\,a)x\right)x^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(tan(b*x+a)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.1259, size = 108, normalized size = 2.92

$$\frac{bmx^2x^m - \pi mxx^m \left\lfloor \frac{\pi+2\,a}{2\,\pi} \right\rfloor + amxx^m + bx^2x^m - 2\,\pi xx^m \left\lfloor \frac{\pi+2\,a}{2\,\pi} \right\rfloor + 2\,axx^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] (b*m*x^2*x^m - pi*m*x*x^m*floor(1/2*(pi + 2*a)/pi) + a*m*x*x^m + b*x^2*x^m
- 2*pi*x*x^m*floor(1/2*(pi + 2*a)/pi) + 2*a*x*x^m)/(m^2 + 3*m + 2)
```

3.38 $\int x^2 \tan^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}x^3\tan^{-1}(\tan(a+bx))-\frac{bx^4}{12}$$

```
[Out] -(b*x^4)/12 + (x^3*ArcTan[Tan[a + b*x]])/3
```

Rubi [A] time = 0.008892, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{1}{3}x^3\tan^{-1}(\tan(a+bx))-\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[Tan[a + b*x]],x]
```

```
[Out] -(b*x^4)/12 + (x^3*ArcTan[Tan[a + b*x]])/3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}(\tan(a+bx))\,dx &= \frac{1}{3}x^3\tan^{-1}(\tan(a+bx))-\frac{1}{3}b\int x^3\,dx\\ &= -\frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(\tan(a+bx))\end{aligned}$$

Mathematica [A] time = 0.0162567, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3\left(bx-4\tan^{-1}(\tan(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[Tan[a + b*x]],x]
```

```
[Out] -(x^3*(b*x - 4*ArcTan[Tan[a + b*x]]))/12
```

Maple [A] time = 0.041, size = 20, normalized size = 0.9

$$-\frac{bx^4}{12}+\frac{x^3\arctan\left(\tan\left(bx+a\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(tan(b*x+a)),x)
```

```
[Out] -1/12*b*x^4+1/3*x^3*arctan(tan(b*x+a))
```

Maxima [B] time = 0.989099, size = 109, normalized size = 4.74

$$\frac{\frac{4\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan(\tan(bx+a))}{b^2}-\frac{(bx+a)^4-4\,(bx+a)^3a+6\,(bx+a)^2a^2}{b^2}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan(tan(b*x +
a))/b^2 - ((b*x + a)^4 - 4*(b*x + a)^3*a + 6*(b*x + a)^2*a^2)/b^2)/b
```

Fricas [A] time = 1.70805, size = 31, normalized size = 1.35

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A] time = 0.775372, size = 109, normalized size = 4.74

$$\begin{cases} \frac{x^2\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^2}{2b}-\frac{x\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^3}{3b^2}+\frac{\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^4}{12b^3} & \text{for}\ b\neq 0\\ \frac{x^3\left(\operatorname{atan}\left(\tan\left(a\right)\right)+\pi\left\lfloor\frac{a-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(tan(b*x+a)),x)
```

```
[Out] Piecewise((x**2*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**2/(2*
b) - x*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**3/(3*b**2) + (
atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**4/(12*b**3), Ne(b, 0))
```

```
, (x**3*(atan(tan(a)) + pi*floor((a - pi/2)/pi))/3, True))
```

Giac [A] time = 1.15182, size = 36, normalized size = 1.57

$$\frac{1}{4}bx^4 - \frac{1}{3}\pi x^3\left\lfloor\frac{a}{\pi} + \frac{1}{2}\right\rfloor + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 - 1/3*pi*x^3*floor(a/pi + 1/2) + 1/3*a*x^3
```

3.39 $\int x \tan^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}x^2\tan^{-1}(\tan(a+bx))-\frac{bx^3}{6}$$

```
[Out] -(b*x^3)/6 + (x^2*ArcTan[Tan[a + b*x]])/2
```

Rubi [A] time = 0.0070475, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5171, 30}

$$\frac{1}{2}x^2\tan^{-1}(\tan(a+bx))-\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[Tan[a + b*x]],x]

[Out] -(b*x^3)/6 + (x^2*ArcTan[Tan[a + b*x]])/2
```

Rule 5171

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tan[a + b*x]])/(f*(m +
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}(\tan(a+bx))\,dx&=\frac{1}{2}x^2\tan^{-1}(\tan(a+bx))-\frac{1}{2}b\int x^2\,dx\\&=-\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(\tan(a+bx))\end{aligned}$$

Mathematica [A] time = 0.0148217, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2\left(bx-3\tan^{-1}(\tan(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[Tan[a + b*x]],x]

[Out] -(x^2*(b*x - 3*ArcTan[Tan[a + b*x]]))/6
```

Maple [A] time = 0.048, size = 20, normalized size = 0.9

$$-\frac{bx^3}{6}+\frac{x^2\arctan\left(\tan\left(bx+a\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(tan(b*x+a)),x)
```

```
[Out] -1/6*b*x^3+1/2*x^2*arctan(tan(b*x+a))
```

Maxima [B] time = 1.00108, size = 77, normalized size = 3.35

$$\frac{\frac{3\left(\left(bx+a\right)^2-2\left(bx+a\right)a\right)\arctan\left(\tan\left(bx+a\right)\right)}{b}-\frac{\left(bx+a\right)^3-3\left(bx+a\right)^2a}{b}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/6*(3*((b*x + a)^2 - 2*(b*x + a)*a)*arctan(tan(b*x + a))/b - ((b*x + a)^3
- 3*(b*x + a)^2*a)/b)/b
```

Fricas [A] time = 1.74936, size = 31, normalized size = 1.35

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [A] time = 0.241458, size = 32, normalized size = 1.39

$$-\frac{bx^3}{6}+\frac{x^2\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(tan(b*x+a)),x)
```

```
[Out] -b*x**3/6 + x**2*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/2
```

Giac [A] time = 1.11709, size = 36, normalized size = 1.57

$$\frac{1}{3}\,bx^3-\frac{1}{2}\,\pi x^2\left\lfloor\frac{a}{\pi}+\frac{1}{2}\right\rfloor+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 - 1/2*pi*x^2*floor(a/pi + 1/2) + 1/2*a*x^2
```

3.40 $\int \tan^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\tan^{-1}(\tan(a+bx))^2}{2b}$$

```
[Out] ArcTan[Tan[a + b*x]]^2/(2*b)
```

Rubi [A] time = 0.002841, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2157, 30}

$$\frac{\tan^{-1}(\tan(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Tan[a + b*x]],x]

[Out] ArcTan[Tan[a + b*x]]^2/(2*b)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(\tan(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \tan^{-1}(\tan(a+bx))\right)}{b}\\ &= \frac{\tan^{-1}(\tan(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.0074467, size = 18, normalized size = 1.12

$$x\tan^{-1}(\tan(a+bx))-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Tan[a + b*x]],x]

[Out] -(b*x^2)/2 + x*ArcTan[Tan[a + b*x]]
```

Maple [A] time = 0.034, size = 15, normalized size = 0.9

$$\frac{(\arctan(\tan(bx+a)))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(tan(b*x+a)),x)
```

```
[Out] 1/2*arctan(tan(b*x+a))^2/b
```

Maxima [A] time = 0.972396, size = 16, normalized size = 1.

$$\frac{(bx+a)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x + a)^2/b
```

Fricas [A] time = 1.75121, size = 23, normalized size = 1.44

$$\frac{1}{2}bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

Sympy [A] time = 0.172969, size = 42, normalized size = 2.62

$$\begin{cases} \frac{\left(\operatorname{atan}(\tan(a+bx))+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^2}{2b} & \text{for } b\neq 0 \\ x\left(\operatorname{atan}(\tan(a))+\pi\left\lfloor\frac{a-\frac{\pi}{2}}{\pi}\right\rfloor\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(tan(b*x+a)),x)
```

```
[Out] Piecewise(((atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**2/(2*b), N
e(b, 0)), (x*(atan(tan(a)) + pi*floor((a - pi/2)/pi)), True))
```

Giac [A] time = 1.07878, size = 35, normalized size = 2.19

$$\frac{1}{2}bx^2-\pi x\left\lfloor\frac{bx+a}{\pi}+\frac{1}{2}\right\rfloor+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 - pi*x*floor((b*x + a)/pi + 1/2) + a*x
```

3.41 $\int \frac{\tan^{-1}(\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=21

$$bx - \log(x)\left(bx - \tan^{-1}(\tan(a+bx))\right)$$

```
[Out] b*x - (b*x - ArcTan[Tan[a + b*x]])*Log[x]
```

Rubi [A] time = 0.0313861, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2158, 29}

$$bx - \log(x)\left(bx - \tan^{-1}(\tan(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Tan[a + b*x]]/x,x]

[Out] b*x - (b*x - ArcTan[Tan[a + b*x]])*Log[x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(\tan(a+bx))}{x}\,dx &= bx - \left(bx - \tan^{-1}(\tan(a+bx))\right)\int \frac{1}{x}\,dx\\ &= bx - \left(bx - \tan^{-1}(\tan(a+bx))\right)\log(x)\end{aligned}$$

Mathematica [A] time = 0.0135539, size = 19, normalized size = 0.9

$$\log(x)\left(\tan^{-1}(\tan(a+bx)) - bx\right) + bx$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Tan[a + b*x]]/x,x]

[Out] b*x + (-(b*x) + ArcTan[Tan[a + b*x]])*Log[x]
```

Maple [A] time = 0.045, size = 21, normalized size = 1.

$$\ln(x)\arctan(\tan(bx+a)) - \ln(x)\,xb + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(tan(b*x+a))/x,x)
```

```
[Out] ln(x)*arctan(tan(b*x+a))-ln(x)*x*b+b*x
```

Maxima [B] time = 1.46779, size = 57, normalized size = 2.71

$$\frac{b \arctan\left(\tan\left(bx+a\right)\right) \log\left(bx\right) + \left(bx - \left(bx+a\right) \log\left(bx\right) + a \log\left(bx\right) + a\right) b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] (b*arctan(tan(b*x + a))*log(b*x) + (b*x - (b*x + a)*log(b*x) + a*log(b*x) +
 a)*b)/b
```

Fricas [A] time = 1.80399, size = 22, normalized size = 1.05

$$bx + a \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] b*x + a*log(x)
```

Sympy [A] time = 0.893141, size = 34, normalized size = 1.62

$$-bx \log\left(x\right) + bx + \left(\operatorname{atan}\left(\tan\left(a+bx\right)\right) + \pi \left\lfloor \frac{a + bx - \frac{\pi}{2}}{\pi} \right\rfloor\right) \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(tan(b*x+a))/x,x)
```

```
[Out] -b*x*log(x) + b*x + (atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))*lo
g(x)
```

Giac [A] time = 1.10772, size = 31, normalized size = 1.48

$$bx - \left(\pi \left\lfloor \frac{a}{\pi} + \frac{1}{2} \right\rfloor - a\right) \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] b*x - (pi*floor(a/pi + 1/2) - a)*log(abs(x))
```

3.42 $\int x^m \tan^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}\tan^{-1}(\cot(a+bx))}{m+1}+\frac{bx^{m+2}}{m^2+3m+2}$$

```
[Out] (b*x^(2 + m))/(2 + 3*m + m^2) + (x^(1 + m)*ArcTan[Cot[a + b*x]])/(1 + m)
```

Rubi [A] time = 0.0210559, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{x^{m+1}\tan^{-1}(\cot(a+bx))}{m+1}+\frac{bx^{m+2}}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTan[Cot[a + b*x]],x]
```

```
[Out] (b*x^(2 + m))/(2 + 3*m + m^2) + (x^(1 + m)*ArcTan[Cot[a + b*x]])/(1 + m)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^m \tan^{-1}(\cot(a+bx))\,dx &= \frac{x^{1+m}\tan^{-1}(\cot(a+bx))}{1+m}+\frac{b\int x^{1+m}\,dx}{1+m}\\ &= \frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\tan^{-1}(\cot(a+bx))}{1+m}\end{aligned}$$

Mathematica [A] time = 0.0479835, size = 31, normalized size = 0.86

$$\frac{x^{m+1}\left((m+2)\tan^{-1}(\cot(a+bx))+bx\right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTan[Cot[a + b*x]],x]
```

```
[Out] (x^(1 + m)*(b*x + (2 + m)*ArcTan[Cot[a + b*x]]))/((1 + m)*(2 + m))
```

Maple [A] time = 0.052, size = 56, normalized size = 1.6

$$\frac{\pi x^{1+m}}{2+2m}-\frac{bx^2 e^{m\ln(x)}}{2+m}-\frac{(\operatorname{arccot}(\cot(bx+a))-bx)\,x e^{m\ln(x)}}{1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(1/2*Pi-arccot(cot(b*x+a))),x)
```

```
[Out] 1/2*Pi*x^(1+m)/(1+m)-b/(2+m)*x^2*exp(m*ln(x))-(arccot(cot(b*x+a))-b*x)/(1+m
)*x*exp(m*ln(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07195, size = 101, normalized size = 2.81

$$-\frac{\left(2\,(bm+b)x^2-(\pi(m+2)-2\,am-4\,a)x\right)x^m}{2\left(m^2+3\,m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*m + b)*x^2 - (pi*(m + 2) - 2*a*m - 4*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(1/2*pi-acot(cot(b*x+a))),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.09397, size = 84, normalized size = 2.33

$$-\frac{2\,bmx^2x^m - \pi mxx^m + 2\,amxx^m + 2\,bx^2x^m - 2\,\pi xx^m + 4\,axx^m}{2\left(m^2 + 3\,m + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*m*x^2*x^m - pi*m*x*x^m + 2*a*m*x*x^m + 2*b*x^2*x^m - 2*pi*x*x^m +
 4*a*x*x^m)/(m^2 + 3*m + 2)
```

3.43 $\int x^2 \tan^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}x^3\tan^{-1}(\cot(a+bx))+\frac{bx^4}{12}$$

```
[Out] (b*x^4)/12 + (x^3*ArcTan[Cot[a + b*x]])/3
```

Rubi [A] time = 0.0083937, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{1}{3}x^3\tan^{-1}(\cot(a+bx))+\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[Cot[a + b*x]],x]

[Out] (b*x^4)/12 + (x^3*ArcTan[Cot[a + b*x]])/3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}(\cot(a+bx))\,dx&=\frac{1}{3}x^3\tan^{-1}(\cot(a+bx))+\frac{1}{3}b\int x^3\,dx\\&=\frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(\cot(a+bx))\end{aligned}$$

Mathematica [A] time = 0.0151806, size = 20, normalized size = 0.87

$$\frac{1}{12}x^3\left(4\tan^{-1}(\cot(a+bx))+bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[Cot[a + b*x]],x]

[Out] (x^3*(b*x + 4*ArcTan[Cot[a + b*x]]))/12
```

Maple [B] time = 0.061, size = 65, normalized size = 2.8

$$\frac{\pi x^3}{6} - \frac{x^3 \operatorname{arccot}(\cot(bx+a))}{3} - \frac{1}{3b^3}\left(-\frac{(bx+a)^4}{4} + a(bx+a)^3 - \frac{3a^2(bx+a)^2}{2} + (bx+a)a^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(1/2*Pi-arccot(cot(b*x+a))),x)
```

```
[Out] 1/6*Pi*x^3-1/3*x^3*arccot(cot(b*x+a))-1/3/b^3*(-1/4*(b*x+a)^4+a*(b*x+a)^3-3
/2*a^2*(b*x+a)^2+(b*x+a)*a^3)
```

Maxima [A] time = 0.964157, size = 23, normalized size = 1.

$$-\frac{1}{4}bx^4 + \frac{1}{6}(\pi - 2a)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="maxima")
```

```
[Out] -1/4*b*x^4 + 1/6*(pi - 2*a)*x^3
```

Fricas [A] time = 2.07415, size = 45, normalized size = 1.96

$$-\frac{1}{4}bx^4 + \frac{1}{6}(\pi - 2a)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] -1/4*b*x^4 + 1/6*(pi - 2*a)*x^3
```

Sympy [A] time = 0.782186, size = 68, normalized size = 2.96

$$\begin{cases} \frac{\pi x^3}{6} - \frac{x^2 \operatorname{acot}^2(\cot(a+bx))}{2b} + \frac{x \operatorname{acot}^3(\cot(a+bx))}{3b^2} - \frac{\operatorname{acot}^4(\cot(a+bx))}{12b^3} & \text{for } b \neq 0 \\ \frac{x^3\left(-\operatorname{acot}(\cot(a)) + \frac{\pi}{2}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1/2*pi-acot(cot(b*x+a))),x)
```

```
[Out] Piecewise((pi*x**3/6 - x**2*acot(cot(a + b*x))**2/(2*b) + x*acot(cot(a + b*
x))**3/(3*b**2) - acot(cot(a + b*x))**4/(12*b**3), Ne(b, 0)), (x**3*(-acot(
cot(a)) + pi/2)/3, True))
```

Giac [A] time = 1.15063, size = 26, normalized size = 1.13

$$-\frac{1}{4}bx^4 + \frac{1}{6}\pi x^3 - \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="giac")
```

```
[Out] -1/4*b*x^4 + 1/6*pi*x^3 - 1/3*a*x^3
```

3.44 $\int x \tan^{-1}(\cot(a + bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}x^2\tan^{-1}(\cot(a+bx))+\frac{bx^3}{6}$$

```
[Out] (b*x^3)/6 + (x^2*ArcTan[Cot[a + b*x]])/2
```

Rubi [A] time = 0.0071134, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5173, 30}

$$\frac{1}{2}x^2\tan^{-1}(\cot(a+bx))+\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[Cot[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTan[Cot[a + b*x]])/2
```

Rule 5173

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Cot[a + b*x]])/(f*(m +
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}(\cot(a+bx))\,dx &= \frac{1}{2}x^2\tan^{-1}(\cot(a+bx))+\frac{1}{2}b\int x^2\,dx\\ &= \frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(\cot(a+bx))\end{aligned}$$

Mathematica [A] time = 0.0142949, size = 20, normalized size = 0.87

$$\frac{1}{6}x^2\left(3\tan^{-1}(\cot(a+bx))+bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[Cot[a + b*x]],x]
```

```
[Out] (x^2*(b*x + 3*ArcTan[Cot[a + b*x]]))/6
```

Maple [B] time = 0.057, size = 54, normalized size = 2.4

$$\frac{\pi x^2}{4}-\frac{x^2\mathrm{arccot}\left(\cot\left(bx+a\right)\right)}{2}-\frac{1}{2b^2}\left(-\frac{\left(bx+a\right)^3}{3}+\left(bx+a\right)^2a-a^2\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1/2*Pi-arccot(cot(b*x+a))),x)
```

```
[Out] 1/4*Pi*x^2-1/2*x^2*arccot(cot(b*x+a))-1/2/b^2*(-1/3*(b*x+a)^3+(b*x+a)^2*a-a
^2*(b*x+a))
```

Maxima [A] time = 0.991631, size = 23, normalized size = 1.

$$-\frac{1}{3}bx^3+\frac{1}{4}(\pi-2a)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="maxima")
```

```
[Out] -1/3*b*x^3 + 1/4*(pi - 2*a)*x^2
```

Fricas [A] time = 2.04841, size = 45, normalized size = 1.96

$$-\frac{1}{3}bx^3+\frac{1}{4}(\pi-2a)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] -1/3*b*x^3 + 1/4*(pi - 2*a)*x^2
```

Sympy [A] time = 0.381749, size = 65, normalized size = 2.83

$$\begin{cases}\frac{bx^3}{6}-\frac{x^2\operatorname{acot}\left(\cot\left(a+bx\right)\right)}{2}+\frac{\pi x\operatorname{acot}\left(\cot\left(a+bx\right)\right)}{2b}-\frac{\pi\operatorname{acot}^2\left(\cot\left(a+bx\right)\right)}{4b^2} & \text{for } b\neq 0\\ \frac{x^2\left(-\operatorname{acot}\left(\cot\left(a\right)\right)+\frac{\pi}{2}\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1/2*pi-acot(cot(b*x+a))),x)
```

```
[Out] Piecewise((b*x**3/6 - x**2*acot(cot(a + b*x))/2 + pi*x*acot(cot(a + b*x))/(
2*b) - pi*acot(cot(a + b*x))**2/(4*b**2), Ne(b, 0)), (x**2*(-acot(cot(a)) +
 pi/2)/2, True))
```

Giac [A] time = 1.11006, size = 26, normalized size = 1.13

$$-\frac{1}{3}bx^3 + \frac{1}{4}\pi x^2 - \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="giac")
```

```
[Out] -1/3*b*x^3 + 1/4*pi*x^2 - 1/2*a*x^2
```

3.45 $\int \tan^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=16

$$-\frac{\tan^{-1}(\cot(a+bx))^2}{2b}$$

```
[Out] -ArcTan[Cot[a + b*x]]^2/(2*b)
```

Rubi [A] time = 0.0030273, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2157, 30}

$$-\frac{\tan^{-1}(\cot(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Cot[a + b*x]],x]

[Out] -ArcTan[Cot[a + b*x]]^2/(2*b)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(\cot(a+bx))\,dx &= -\frac{\text{Subst}\left(\int x\,dx, x, \tan^{-1}(\cot(a+bx))\right)}{b}\\ &= -\frac{\tan^{-1}(\cot(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.007294, size = 18, normalized size = 1.12

$$x\tan^{-1}(\cot(a+bx))+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Cot[a + b*x]],x]

[Out] (b*x^2)/2 + x*ArcTan[Cot[a + b*x]]
```

Maple [B] time = 0.056, size = 51, normalized size = 3.2

$$\frac{\pi x}{2}-\frac{1}{b}\left(-\left(\frac{\pi}{2}-\operatorname{arccot}\left(\cot\left(bx+a\right)\right)\right)\operatorname{arccot}\left(\cot\left(bx+a\right)\right)-\frac{1}{2}\left(\frac{\pi}{2}-\operatorname{arccot}\left(\cot\left(bx+a\right)\right)\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*Pi-arccot(cot(b*x+a)),x)
```

```
[Out] 1/2*Pi*x-1/b*(-(1/2*Pi-arccot(cot(b*x+a)))*arccot(cot(b*x+a))-1/2*(1/2*Pi-a
rccot(cot(b*x+a)))^2)
```

Maxima [A] time = 0.951275, size = 20, normalized size = 1.25

$$-\frac{1}{2}\,bx^2+\frac{1}{2}\,\pi x-ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*b*x^2 + 1/2*pi*x - a*x
```

Fricas [A] time = 1.97813, size = 42, normalized size = 2.62

$$-\frac{1}{2}\,bx^2+\frac{1}{2}\,(\pi-2\,a)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/2*b*x^2 + 1/2*(pi - 2*a)*x
```

Sympy [A] time = 0.150933, size = 24, normalized size = 1.5

$$\frac{\pi x}{2}-\begin{cases}\frac{\operatorname{acot}^{2}\left(\cot\left(a+bx\right)\right)}{2b} & \text{for}\: b\neq 0\\ x\operatorname{acot}\left(\cot\left(a\right)\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-acot(cot(b*x+a)),x)
```

```
[Out] pi*x/2 - Piecewise((acot(cot(a + b*x))**2/(2*b), Ne(b, 0)), (x*acot(cot(a))
, True))
```

Giac [A] time = 1.08506, size = 20, normalized size = 1.25

$$-\frac{1}{2}\,bx^2+\frac{1}{2}\,\pi x-ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/2*b*x^2 + 1/2*pi*x - a*x
```

3.46 $\int \frac{\tan^{-1}(\cot(a+bx))}{x} \, dx$

Optimal. Leaf size=19

$$\log(x)\left(\tan^{-1}(\cot(a+bx))+bx\right)-bx$$

```
[Out] -(b*x) + (b*x + ArcTan[Cot[a + b*x]])*Log[x]
```

Rubi [A] time = 0.0327329, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2158, 29}

$$\log(x)\left(\tan^{-1}(\cot(a+bx))+bx\right)-bx$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Cot[a + b*x]]/x,x]
```

```
[Out] -(b*x) + (b*x + ArcTan[Cot[a + b*x]])*Log[x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(\cot(a+bx))}{x} \, dx &= -bx-\left(-bx-\tan^{-1}(\cot(a+bx))\right)\int \frac{1}{x} \, dx\\ &= -bx+\left(bx+\tan^{-1}(\cot(a+bx))\right)\log(x)\end{aligned}$$

Mathematica [A] time = 0.0154947, size = 19, normalized size = 1.

$$\log(x)\left(\tan^{-1}(\cot(a+bx))+bx\right)-bx$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Cot[a + b*x]]/x,x]
```

```
[Out] -(b*x) + (b*x + ArcTan[Cot[a + b*x]])*Log[x]
```

Maple [A] time = 0.059, size = 35, normalized size = 1.8

$$\frac{\pi \ln(x)}{2}-bx-\ln(x)\,a-\ln(x)\left(\operatorname{arccot}(\cot(bx+a))-bx-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/2*Pi-arccot(cot(b*x+a)))/x,x)
```

```
[Out] 1/2*Pi*ln(x)-b*x-ln(x)*a-ln(x)*(arccot(cot(b*x+a))-b*x-a)
```

Maxima [A] time = 0.949681, size = 19, normalized size = 1.

$$-bx + \frac{1}{2}(\pi - 2a)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2*pi-arccot(cot(b*x+a)))/x,x, algorithm="maxima")
```

```
[Out] -b*x + 1/2*(pi - 2*a)*log(x)
```

Fricas [A] time = 2.02123, size = 41, normalized size = 2.16

$$-bx + \frac{1}{2}(\pi - 2a)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2*pi-arccot(cot(b*x+a)))/x,x, algorithm="fricas")
```

```
[Out] -b*x + 1/2*(pi - 2*a)*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{\pi}{x}\,dx + \int \frac{2\operatorname{acot}(\cot(a+bx))}{x}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2*pi-acot(cot(b*x+a)))/x,x)
```

```
[Out] -(Integral(-pi/x, x) + Integral(2*acot(cot(a + b*x))/x, x))/2
```

Giac [A] time = 1.11118, size = 20, normalized size = 1.05

$$-bx + \frac{1}{2}(\pi - 2a)\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2*pi-arccot(cot(b*x+a)))/x,x, algorithm="giac")
```

```
[Out] -b*x + 1/2*(pi - 2*a)*log(abs(x))
```

3.47 $\int \tan^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\tan^{-1}(\tan(a+bx))^2}{2b}$$

```
[Out] ArcTan[Tan[a + b*x]]^2/(2*b)
```

Rubi [A] time = 0.0028136, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2157, 30}

$$\frac{\tan^{-1}(\tan(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Tan[a + b*x]],x]

[Out] ArcTan[Tan[a + b*x]]^2/(2*b)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(\tan(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \tan^{-1}(\tan(a+bx))\right)}{b}\\ &= \frac{\tan^{-1}(\tan(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.0004024, size = 18, normalized size = 1.12

$$x\tan^{-1}(\tan(a+bx))-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Tan[a + b*x]],x]

[Out] -(b*x^2)/2 + x*ArcTan[Tan[a + b*x]]
```

Maple [A] time = 0.034, size = 15, normalized size = 0.9

$$\frac{(\arctan(\tan(bx+a)))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(tan(b*x+a)),x)
```

```
[Out] 1/2*arctan(tan(b*x+a))^2/b
```

Maxima [A] time = 0.964414, size = 16, normalized size = 1.

$$\frac{(bx+a)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x + a)^2/b
```

Fricas [A] time = 1.93273, size = 23, normalized size = 1.44

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

Sympy [A] time = 0.167606, size = 42, normalized size = 2.62

$$\begin{cases} \frac{\left(\operatorname{atan}(\tan(a+bx))+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^2}{2b} & \text{for } b \neq 0 \\ x\left(\operatorname{atan}(\tan(a)) + \pi\left\lfloor\frac{a-\frac{\pi}{2}}{\pi}\right\rfloor\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(tan(b*x+a)),x)
```

```
[Out] Piecewise(((atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**2/(2*b), N
e(b, 0)), (x*(atan(tan(a)) + pi*floor((a - pi/2)/pi)), True))
```

Giac [A] time = 1.10169, size = 35, normalized size = 2.19

$$\frac{1}{2}bx^2 - \pi x\left\lfloor\frac{bx+a}{\pi} + \frac{1}{2}\right\rfloor + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 - pi*x*floor((b*x + a)/pi + 1/2) + a*x
```

3.48 $\int x^2 \tan^{-1}(c + d\tan(a + bx))\,dx$

Optimal. Leaf size=403

$$\frac{ix\mathrm{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b^2}-\frac{ix\mathrm{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b^2}-\frac{\mathrm{PolyLog}\left(4,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{8b^3}+\frac{\mathrm{PolyLog}\left(4,-\right.}{8b}$$

```
[Out] (x^3*ArcTan[c + d*Tan[a + b*x]])/3 + (I/6)*x^3*Log[1 + ((1 + I*c + d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c - d)] - (I/6)*x^3*Log[1 + ((c + I*(1 - d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 + d))] + (x^2*PolyLog[2, -(((1 + I*c + d)*E
^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/(4*b) - (x^2*PolyLog[2, -(((c + I*
(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/(4*b) + ((I/4)*x*PolyL
og[3, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/b^2 - ((I/
4)*x*PolyLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d))
)])/b^2 - PolyLog[4, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d
))]/(8*b^3) + PolyLog[4, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I
*(1 + d)))]/(8*b^3)
```

Rubi [A] time = 0.516526, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5175, 2190, 2531, 6609, 2282, 6589}

$$\frac{ix\mathrm{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b^2}-\frac{ix\mathrm{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b^2}-\frac{\mathrm{PolyLog}\left(4,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{8b^3}+\frac{\mathrm{PolyLog}\left(4,-\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + d*Tan[a + b*x]],x]
```


```
[Out] (x^3*ArcTan[c + d*Tan[a + b*x]])/3 + (I/6)*x^3*Log[1 + ((1 + I*c + d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c - d)] - (I/6)*x^3*Log[1 + ((c + I*(1 - d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 + d))] + (x^2*PolyLog[2, -(((1 + I*c + d)*E
^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/(4*b) - (x^2*PolyLog[2, -(((c + I*
(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/(4*b) + ((I/4)*x*PolyL
og[3, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/b^2 - ((I/
4)*x*PolyLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d))
)])/b^2 - PolyLog[4, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d
))]/(8*b^3) + PolyLog[4, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I
*(1 + d)))]/(8*b^3)
```


Rule 5175

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tan[a + b*x]])/(f*(m +
 1)), x] + (Dist[(b*(1 - I*c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2
*I*a + 2*I*b*x))/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x)), x], x]
- Dist[(b*(1 + I*c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*a + 2*I
*b*x))/(1 + I*c - d + (1 + I*c + d)*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+d\tan(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{3}(b(1-ic-d))\int \frac{e^{2ia+2ibx}x^3}{1-ic+d+(1-ic-d)e^{2ia+2ibx}}\,dx$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{6}ix^3\log\left(1+\cdots\right)$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{6}ix^3\log\left(1+\cdots\right)$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{6}ix^3\log\left(1+\cdots\right)$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{6}ix^3\log\left(1+\cdots\right)$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{6}ix^3\log\left(1+\cdots\right)$$

Mathematica [A] time = 0.909663, size = 363, normalized size = 0.9

$$\frac{1}{3}x^3\tan^{-1}(d\tan(a+bx)+c) + \frac{6b^2x^2\text{PolyLog}\left(2,-\frac{(c-i(d+1))e^{2i(a+bx)}}{c+i(d-1)}\right) - 6b^2x^2\text{PolyLog}\left(2,-\frac{(c-id+i)e^{2i(a+bx)}}{c+i(d+1)}\right) + 6ibx\text{PolyLog}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Tan[a + b*x]])/3 + ((4*I)*b^3*x^3*Log[1 + ((c - I*(1 + d)
)*E^((2*I)*(a + b*x)))/(c + I*(-1 + d))] - (4*I)*b^3*x^3*Log[1 + ((I + c -
I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d))] + 6*b^2*x^2*PolyLog[2, -(((c - I
*(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] - 6*b^2*x^2*PolyLog[2, -(
((I + c - I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))] + (6*I)*b*x*PolyLog[3
, -(((c - I*(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] - (6*I)*b*x*Po
lyLog[3, -(((I + c - I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))] - 3*PolyLo
g[4, -(((c - I*(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] + 3*PolyLog
[4, -(((I + c - I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))])/(24*b^3)
```

Maple [C] time = 8.128, size = 8076, normalized size = 20.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+d*tan(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/6*x^3*arctan2(c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1)*
cos(2*b*x + 2*a) - c*sin(2*b*x + 2*a) - d + 1) + 1/6*x^3*arctan2(c*cos(2*b*
x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) + c*sin(
2*b*x + 2*a) + d + 1) + 4*b*d*integrate(-1/3*(2*(c^2 + d^2 + 1)*x^3*cos(2*b
*x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^3*sin(2*b*x
+ 2*a)^2 + (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*
a) - (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*co
s(2*b*x + 2*a) + (c^2 - d^2 + 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c
^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(
4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*
a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^
4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4
+ d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 + 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2
*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c
^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 + c)*d - 2*(c*
d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2
*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [C] time = 2.95211, size = 5146, normalized size = 12.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arctan(c+d*tan(b*x+a)),x, algorithm="fricas")

```
[Out] 1/48*(16*b^3*x^3*arctan(d*tan(b*x + a) + c) + 6*b^2*x^2*dilog((2*(I*c*d - d
^2 + d)*tan(b*x + a)^2 - 2*c^2 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*
I)*tan(b*x + a) + 2*d - 2)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^
2 - 2*d + 1) + 1) - 6*b^2*x^2*dilog((2*(I*c*d - d^2 - d)*tan(b*x + a)^2 - 2
*c^2 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 2*d - 2)
/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) + 6*b^2*
x^2*dilog((2*(-I*c*d - d^2 + d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2
 + 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a) + 2*d - 2)/((c^2 + d^2 - 2*d + 1)*ta
n(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1) - 6*b^2*x^2*dilog((2*(-I*c*d - d^2
 - d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2 + 4*c*d - 2*I*d^2 + 2*I)*
tan(b*x + a) - 2*d - 2)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 +
 2*d + 1) + 1) + 4*I*a^3*log(((I*c*d + d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*
d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d - 1)/(tan(b*x + a)^2 + 1))
 - 4*I*a^3*log(((I*c*d + d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I
*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x + a)^2 + 1)) + 4*I*a^3*log
(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d +
 I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1)) - 4*I*a^3*log(((I*c*d - d^2
 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x +
a) + d + 1)/(tan(b*x + a)^2 + 1)) + 6*I*b*x*polylog(3, ((c^2 + 2*I*c*d - d^
2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 + (2*I*c^2 - 4*c*d - 2*I*d^2 +
2*I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 +
2*d + 1)) - 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 -
c^2 + 2*I*c*d + d^2 + (-2*I*c^2 - 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 1)/
((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 6*I*b*x*pol
ylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 + (
2*I*c^2 - 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*t
an(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) + 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d
- d^2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 + (-2*I*c^2 - 4*c*d + 2*I*d
^2 - 2*I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d
^2 - 2*d + 1)) + (-4*I*b^3*x^3 - 4*I*a^3)*log(-(2*(I*c*d - d^2 + d)*tan(b*x
 + a)^2 - 2*c^2 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a)
 + 2*d - 2)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) +
 (4*I*b^3*x^3 + 4*I*a^3)*log(-(2*(I*c*d - d^2 - d)*tan(b*x + a)^2 - 2*c^2 -
 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 2*d - 2)/((c^2
 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + (4*I*b^3*x^3 + 4
*I*a^3)*log(-(2*(-I*c*d - d^2 + d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*I*
c^2 + 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a) + 2*d - 2)/((c^2 + d^2 - 2*d + 1)
*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) + (-4*I*b^3*x^3 - 4*I*a^3)*log(-(2*
(-I*c*d - d^2 - d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2 + 4*c*d - 2*
I*d^2 + 2*I)*tan(b*x + a) - 2*d - 2)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2
+ c^2 + d^2 + 2*d + 1)) + 3*polylog(4, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x +
 a)^2 - c^2 - 2*I*c*d + d^2 + (2*I*c^2 - 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a
) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + 3*po
lylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 +
(-2*I*c^2 - 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)
*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 3*polylog(4, ((c^2 + 2*I*c*d - d^
2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 + (2*I*c^2 - 4*c*d - 2*I*d^2 +
2*I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 -
2*d + 1)) - 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 +
2*I*c*d + d^2 + (-2*I*c^2 - 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 1)/((c^2
+ d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \arctan\left(d\tan\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(d*tan(b*x + a) + c), x)
```

3.49 $\int x\tan^{-1}(c+d\tan(a+bx))\,dx$

Optimal. Leaf size=305

$$\frac{i\text{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{8b^2}-\frac{i\text{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{8b^2}+\frac{x\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b}-\frac{x\text{PolyLog}\left(2,-\frac{(c+i(}{}\right.}{4b}$$

```
[Out] (x^2*ArcTan[c + d*Tan[a + b*x]])/2 + (I/4)*x^2*Log[1 + ((1 + I*c + d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c - d)] - (I/4)*x^2*Log[1 + ((c + I*(1 - d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 + d))] + (x*PolyLog[2, -(((1 + I*c + d)*E^(
(2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/(4*b) - (x*PolyLog[2, -(((c + I*(1 -
 d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/(4*b) + ((I/8)*PolyLog[3,
-(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/b^2 - ((I/8)*Pol
yLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/b^2
```

Rubi [A] time = 0.405354, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5175, 2190, 2531, 2282, 6589}

$$\frac{i\text{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{8b^2}-\frac{i\text{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{8b^2}+\frac{x\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b}-\frac{x\text{PolyLog}\left(2,-\frac{(c+i(}{}\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + d*Tan[a + b*x]],x]
```


```
[Out] (x^2*ArcTan[c + d*Tan[a + b*x]])/2 + (I/4)*x^2*Log[1 + ((1 + I*c + d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c - d)] - (I/4)*x^2*Log[1 + ((c + I*(1 - d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 + d))] + (x*PolyLog[2, -(((1 + I*c + d)*E^(
(2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/(4*b) - (x*PolyLog[2, -(((c + I*(1 -
 d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/(4*b) + ((I/8)*PolyLog[3,
-(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/b^2 - ((I/8)*Pol
yLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/b^2
```


Rule 5175

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tan[a + b*x]])/(f*(m +
 1)), x] + (Dist[(b*(1 - I*c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2
*I*a + 2*I*b*x))/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x)), x], x]
- Dist[(b*(1 + I*c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*a + 2*I
*b*x))/(1 + I*c - d + (1 + I*c + d)*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+d\tan(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{2}(b(1-ic-d))\int \frac{e^{2ia+2ibx}x^2}{1-ic+d+(1-ic-d)e^{2ia+2ibx}}$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{4}ix^2\log\left(1+\right.$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{4}ix^2\log\left(1+\right.$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{4}ix^2\log\left(1+\right.$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{4}ix^2\log\left(1+\right.$$

Mathematica [A] time = 0.569076, size = 272, normalized size = 0.89

$$\frac{1}{2}x^2\tan^{-1}(d\tan(a+bx)+c) + \frac{i\left(-2ibx\text{PolyLog}\left(2,-\frac{(c-i(d+1))e^{2i(a+bx)}}{c+i(d-1)}\right)+2ibx\text{PolyLog}\left(2,-\frac{(c-id+i)e^{2i(a+bx)}}{c+i(d+1)}\right)+\text{PolyLog}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Tan[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + ((c - I*(1 +
 d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d))] - 2*b^2*x^2*Log[1 + ((I + c - I
*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d))] - (2*I)*b*x*PolyLog[2, -(((c - I*
(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] + (2*I)*b*x*PolyLog[2, -((
(I + c - I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))] + PolyLog[3, -(((c - I
*(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] - PolyLog[3, -(((I + c -
I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))]))/b^2
```

Maple [C] time = 24.112, size = 7660, normalized size = 25.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+d*tan(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^2*arctan2(c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1)*
cos(2*b*x + 2*a) - c*sin(2*b*x + 2*a) - d + 1) + 1/4*x^2*arctan2(c*cos(2*b*
x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) + c*sin(
2*b*x + 2*a) + d + 1) + 2*b*d*integrate(-(2*(c^2 + d^2 + 1)*x^2*cos(2*b*x +
 2*a)^2 + 2*c*d*x^2*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^2*sin(2*b*x + 2*
a)^2 + (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) -
 (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*
b*x + 2*a) + (c^2 - d^2 + 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 +
 d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(4*b*
x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*a)^2
 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 +
d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4 + d^
4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 + 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a)
- 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 -
 d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 + c)*d - 2*(c*d^3
+ (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2*a))
*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [C] time = 2.72825, size = 4035, normalized size = 13.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*b^2*x^2*arctan(d*tan(b*x + a) + c) + 2*b*x*dilog((2*(I*c*d - d^2 +
d)*tan(b*x + a)^2 - 2*c^2 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*ta
n(b*x + a) + 2*d - 2)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2
*d + 1) + 1) - 2*b*x*dilog((2*(I*c*d - d^2 - d)*tan(b*x + a)^2 - 2*c^2 - 2*
I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 2*d - 2)/((c^2 +
d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) + 2*b*x*dilog((2*
(-I*c*d - d^2 + d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2 + 4*c*d - 2*
I*d^2 + 2*I)*tan(b*x + a) + 2*d - 2)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2
+ c^2 + d^2 - 2*d + 1) + 1) - 2*b*x*dilog((2*(-I*c*d - d^2 - d)*tan(b*x + a
)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2 + 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a) - 2*
d - 2)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) -
2*I*a^2*log(((I*c*d + d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^
2 + 2*I*d + I)*tan(b*x + a) - d - 1)/(tan(b*x + a)^2 + 1)) + 2*I*a^2*log(((
I*c*d + d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)
```

```
*tan(b*x + a) + d - 1)/(tan(b*x + a)^2 + 1)) - 2*I*a^2*log(((I*c*d - d^2 +
d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a)
- d + 1)/(tan(b*x + a)^2 + 1)) + 2*I*a^2*log(((I*c*d - d^2 - d)*tan(b*x + a
)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) + d + 1)/(tan(
b*x + a)^2 + 1)) + (-2*I*b^2*x^2 + 2*I*a^2)*log(-(2*(I*c*d - d^2 + d)*tan(b
*x + a)^2 - 2*c^2 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*tan(b*x +
a) + 2*d - 2)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1))
 + (2*I*b^2*x^2 - 2*I*a^2)*log(-(2*(I*c*d - d^2 - d)*tan(b*x + a)^2 - 2*c^2
 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 2*d - 2)/((c
^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + (2*I*b^2*x^2 -
 2*I*a^2)*log(-(2*(-I*c*d - d^2 + d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*
I*c^2 + 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a) + 2*d - 2)/((c^2 + d^2 - 2*d +
1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) + (-2*I*b^2*x^2 + 2*I*a^2)*log(-(
2*(-I*c*d - d^2 - d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2 + 4*c*d -
2*I*d^2 + 2*I)*tan(b*x + a) - 2*d - 2)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^
2 + c^2 + d^2 + 2*d + 1)) + I*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x
 + a)^2 - c^2 - 2*I*c*d + d^2 + (2*I*c^2 - 4*c*d - 2*I*d^2 + 2*I)*tan(b*x +
 a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - I*
polylog(3, ((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2
+ (-2*I*c^2 - 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d +
1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - I*polylog(3, ((c^2 + 2*I*c*d -
d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 + (2*I*c^2 - 4*c*d - 2*I*d^2
+ 2*I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2
- 2*d + 1)) + I*polylog(3, ((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2
+ 2*I*c*d + d^2 + (-2*I*c^2 - 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 1)/((c^
2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \arctan\left(d \tan\left(bx + a\right) + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(d*tan(b*x + a) + c), x)
```

3.50 $\int \tan^{-1}(c + d\tan(a + bx))\,dx$

Optimal. Leaf size=198

$$\frac{\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b}-\frac{\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b}+\frac{1}{2}ix\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)-\frac{1}{2}ix\log\left(1+\frac{(c+i\ldots}{c\ldots}\right.$$

```
[Out] x*ArcTan[c + d*Tan[a + b*x]] + (I/2)*x*Log[1 + ((1 + I*c + d)*E^((2*I)*a +
(2*I)*b*x))/(1 + I*c - d)] - (I/2)*x*Log[1 + ((c + I*(1 - d))*E^((2*I)*a +
(2*I)*b*x))/(c + I*(1 + d))] + PolyLog[2, -(((1 + I*c + d)*E^((2*I)*a + (2*
I)*b*x))/(1 + I*c - d))]/(4*b) - PolyLog[2, -(((c + I*(1 - d))*E^((2*I)*a +
 (2*I)*b*x))/(c + I*(1 + d)))]/(4*b)
```

Rubi [A] time = 0.23394, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5167, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b}-\frac{\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b}+\frac{1}{2}ix\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)-\frac{1}{2}ix\log\left(1+\frac{(c+i\ldots}{c\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Tan[a + b*x]] + (I/2)*x*Log[1 + ((1 + I*c + d)*E^((2*I)*a +
(2*I)*b*x))/(1 + I*c - d)] - (I/2)*x*Log[1 + ((c + I*(1 - d))*E^((2*I)*a +
(2*I)*b*x))/(c + I*(1 + d))] + PolyLog[2, -(((1 + I*c + d)*E^((2*I)*a + (2*
I)*b*x))/(1 + I*c - d))]/(4*b) - PolyLog[2, -(((c + I*(1 - d))*E^((2*I)*a +
 (2*I)*b*x))/(c + I*(1 + d)))]/(4*b)
```

Rule 5167

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] + (Dist[b*(1 - I*c - d), Int[(x*E^(2*I*a + 2*I*b
*x))/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x)), x], x] - Dist[b*(1
+ I*c + d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 + I*c - d + (1 + I*c + d)*E^(2*I
*a + 2*I*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tan^{-1}(c+d\tan(a+bx))\,dx = x\tan^{-1}(c+d\tan(a+bx)) + (b(1-ic-d))\int \frac{e^{2ia+2ibx}x}{1-ic+d+(1-ic-d)e^{2ia+2ibx}}\,dx - ($$

$$= x\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{2}ix\log\left(1+\frac{(c+i}{c}\right.$$

$$= x\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{2}ix\log\left(1+\frac{(c+i}{c}\right.$$

$$= x\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{2}ix\log\left(1+\frac{(c+i}{c}\right.$$

Mathematica [B] time = 6.76453, size = 555, normalized size = 2.8

$$x\tan^{-1}(d\tan(a+bx)+c) + \frac{x\left(-i\sqrt{-d^2}\left(\text{PolyLog}\left(2,\frac{d^2(1-i\tan(a+bx))}{icd+d^2-i\sqrt{-d^2}}\right)+\log(1-i\tan(a+bx))\log\left(\frac{d^2(-\tan(a+bx))-cd+\sqrt{}}{-cd+id^2+\sqrt{-d^2}}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Tan[a + b*x]] + (x*(-4*a*d*ArcTan[c + d*Tan[a + b*x]] - I*Sq
rt[-d^2]*(Log[1 - I*Tan[a + b*x]]*Log[(-(c*d) + Sqrt[-d^2] - d^2*Tan[a + b*
x])/(-(c*d) + I*d^2 + Sqrt[-d^2])] + PolyLog[2, (d^2*(1 - I*Tan[a + b*x]))/
(I*c*d + d^2 - I*Sqrt[-d^2])]) + I*Sqrt[-d^2]*(Log[1 - I*Tan[a + b*x]]*Log[
(c*d + Sqrt[-d^2] + d^2*Tan[a + b*x])/(c*d - I*d^2 + Sqrt[-d^2])] + PolyLog
[2, (d^2*(1 - I*Tan[a + b*x]))/(I*c*d + d^2 + I*Sqrt[-d^2])]) + I*Sqrt[-d^2
]*(Log[1 + I*Tan[a + b*x]]*Log[(c*d - Sqrt[-d^2] + d^2*Tan[a + b*x])/(c*d +
 I*d^2 - Sqrt[-d^2])] + PolyLog[2, (d^2*(1 + I*Tan[a + b*x]))/((-I)*c*d + d
^2 + I*Sqrt[-d^2])]) - I*Sqrt[-d^2]*(Log[1 + I*Tan[a + b*x]]*Log[(c*d + Sqr
t[-d^2] + d^2*Tan[a + b*x])/(c*d + I*d^2 + Sqrt[-d^2])] + PolyLog[2, (d^2*(
1 + I*Tan[a + b*x]))/(d^2 - I*(c*d + Sqrt[-d^2]))])))/(2*d*(2*a - I*Log[1 -
 I*Tan[a + b*x]] + I*Log[1 + I*Tan[a + b*x]]))
```

Maple [B] time = 0.384, size = 1002, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*tan(b*x+a)),x)
```

```
[Out] 1/b*arctan(tan(b*x+a))*arctan(c+d*tan(b*x+a))+1/2*I/b*arctan((c+d*tan(b*x+a
))/d-c/d)*ln(1-(-I-I*d+c)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b*x+a
))/d-c/d)^2+1)/(-I*d+I-c))+1/2/b*arctan((c+d*tan(b*x+a))/d-c/d)^2+1/4/b*pol
ylog(2,(-I-I*d+c)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d
)^2+1)/(-I*d+I-c))+1/2*d/b/(I+I*d+c)*ln(1-(I-I*d+c)*(1+I*((c+d*tan(b*x+a))/
```

```
d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*arctan((c+d*tan(b*x+a)
)/d-c/d)+1/2/b/(I+I*d+c)*ln(1-(I-I*d+c)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/((
(c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*arctan((c+d*tan(b*x+a))/d-c/d)-1/2
*I/b/(I+I*d+c)*ln(1-(I-I*d+c)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b
*x+a))/d-c/d)^2+1)/(-I*d-I-c))*arctan((c+d*tan(b*x+a))/d-c/d)*c-1/2*I*d/b/(
I+I*d+c)*arctan((c+d*tan(b*x+a))/d-c/d)^2-1/4*I*d/b/(I+I*d+c)*polylog(2,(I-
I*d+c)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*
d-I-c))-1/2*I/b/(I+I*d+c)*arctan((c+d*tan(b*x+a))/d-c/d)^2-1/2/b/(I+I*d+c)*
arctan((c+d*tan(b*x+a))/d-c/d)^2*c-1/4*I/b/(I+I*d+c)*polylog(2,(I-I*d+c)*(1
+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))-1
/4/b/(I+I*d+c)*polylog(2,(I-I*d+c)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*
tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*c
```

Maxima [B] time = 1.88117, size = 585, normalized size = 2.95

$$d\left(\frac{8\,(bx+a)\arctan\left(\frac{d^2\tan(bx+a)+cd}{d}\right)}{d}-\frac{4\,(bx+a)\arctan\left(\frac{cd+\left(d^2+d\right)\tan(bx+a)}{c^2+d^2+2\,d+1},\frac{cd\tan(bx+a)+c^2+d+1}{c^2+d^2+2\,d+1}\right)-4\,(bx+a)\arctan\left(\frac{cd+\left(d^2-d\right)\tan(bx+a)}{c^2+d^2-2\,d+1},\frac{cd\tan(bx+a)+c^2-d+1}{c^2+d^2-2\,d+1}\right)+l\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*(d*(8*(b*x + a)*arctan((d^2*tan(b*x + a) + c*d)/d)/d - (4*(b*x + a)*arc
tan2((c*d + (d^2 + d)*tan(b*x + a))/(c^2 + d^2 + 2*d + 1), (c*d*tan(b*x + a
) + c^2 + d + 1)/(c^2 + d^2 + 2*d + 1)) - 4*(b*x + a)*arctan2((c*d + (d^2 -
 d)*tan(b*x + a))/(c^2 + d^2 - 2*d + 1), (c*d*tan(b*x + a) + c^2 - d + 1)/(
c^2 + d^2 - 2*d + 1)) + log(tan(b*x + a)^2 + 1)*log((d^2*tan(b*x + a)^2 + 2
*c*d*tan(b*x + a) + c^2 + 1)/(c^2 + d^2 + 2*d + 1)) - log(tan(b*x + a)^2 +
1)*log((d^2*tan(b*x + a)^2 + 2*c*d*tan(b*x + a) + c^2 + 1)/(c^2 + d^2 - 2*d
 + 1)) + 2*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + 1)) - 2*dilog(-(I*d*tan
(b*x + a) - d)/(I*c + d - 1)) + 2*dilog((I*d*tan(b*x + a) + d)/(-I*c + d +
1)) - 2*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - 1)))/d) + 8*(b*x + a)*arct
an(d*tan(b*x + a) + c) - 8*(b*x + a)*arctan((d^2*tan(b*x + a) + c*d)/d))/b
```

Fricas [B] time = 2.70716, size = 2894, normalized size = 14.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*arctan(d*tan(b*x + a) + c) + (-2*I*b*x - 2*I*a)*log(-(2*(I*c*d -
 d^2 + d)*tan(b*x + a)^2 - 2*c^2 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 -
2*I)*tan(b*x + a) + 2*d - 2)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 +
d^2 - 2*d + 1)) + (2*I*b*x + 2*I*a)*log(-(2*(I*c*d - d^2 - d)*tan(b*x + a)^
2 - 2*c^2 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 2*d
 - 2)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + (2*I*
b*x + 2*I*a)*log(-(2*(-I*c*d - d^2 + d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d -
(2*I*c^2 + 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a) + 2*d - 2)/((c^2 + d^2 - 2*d
 + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) + (-2*I*b*x - 2*I*a)*log(-(2*(
-I*c*d - d^2 - d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2 + 4*c*d - 2*I
*d^2 + 2*I)*tan(b*x + a) - 2*d - 2)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 +
```

```
 c^2 + d^2 + 2*d + 1)) + 2*I*a*log(((I*c*d + d^2 + d)*tan(b*x + a)^2 - c^2
+ I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d - 1)/(tan(b*x + a)^2
 + 1)) - 2*I*a*log(((I*c*d + d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d + (I*c^2
 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x + a)^2 + 1)) + 2*I*a*l
og(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d
 + I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1)) - 2*I*a*log(((I*c*d - d^2
 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x +
a) + d + 1)/(tan(b*x + a)^2 + 1)) + dilog((2*(I*c*d - d^2 + d)*tan(b*x + a)
^2 - 2*c^2 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) + 2*
d - 2)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1) -
dilog((2*(I*c*d - d^2 - d)*tan(b*x + a)^2 - 2*c^2 - 2*I*c*d - (-2*I*c^2 + 4
*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 2*d - 2)/((c^2 + d^2 + 2*d + 1)*tan(b*
x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) + dilog((2*(-I*c*d - d^2 + d)*tan(b*x
+ a)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2 + 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a) +
 2*d - 2)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1)
 - dilog((2*(-I*c*d - d^2 - d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2
+ 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a) - 2*d - 2)/((c^2 + d^2 + 2*d + 1)*tan
(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arctan\left(d\tan\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(d*tan(b*x + a) + c), x)
```

3.51 $\int \frac{\tan^{-1}(c+d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\tan^{-1}(d\tan(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTan[c + d*Tan[a + b*x]]/x, x]
```

Rubi [A] time = 0.131617, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + d*Tan[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\tan^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Mathematica [A] time = 4.66561, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + d*Tan[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTan[c + d*Tan[a + b*x]]/x, x]
```

Maple [A] time = 0.354, size = 0, normalized size = 0.

$$\int \frac{\arctan\left(c+d\tan\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*tan(b*x+a))/x,x)
```

```
[Out] int(arctan(c+d*tan(b*x+a))/x,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(d\tan\left(bx+a\right)+c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(d*tan(b*x + a) + c)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*tan(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(d\tan\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(d*tan(b*x + a) + c)/x, x)
```

3.52 $\int x^2 \tan^{-1}(c+(1+ic)\tan(a+bx))\,dx$

Optimal. Leaf size=154

$$-\frac{ix\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{3}x^3\tan$$

```
[Out] -(b*x^4)/12 + (x^3*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/3 - (I/6)*x^3*Log[1
 - I*c*E^((2*I)*a + (2*I)*b*x)] - (x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x
)])/(4*b) - ((I/4)*x*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2 + PolyLog
[4, I*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rubi [A] time = 0.237697, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5171, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{3}x^3\tan$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] -(b*x^4)/12 + (x^3*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/3 - (I/6)*x^3*Log[1
 - I*c*E^((2*I)*a + (2*I)*b*x)] - (x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x
)])/(4*b) - ((I/4)*x*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2 + PolyLog
[4, I*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 5171

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tan[a + b*x]])/(f*(m +
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{1}{3}x^3 \tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{3}(ib)\int \frac{x^3}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx$$
$$= -\frac{bx^4}{12} + \frac{1}{3}x^3 \tan^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{3}(bc)\int \frac{e^{2ia+2ibx}x^3}{i(1+ic)+c+ce^{2ia+}$$
$$= -\frac{bx^4}{12} + \frac{1}{3}x^3 \tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{6}ix^3 \log\left(1-ice^{2ia+2ibx}\right) + \frac{1}{2}i$$
$$= -\frac{bx^4}{12} + \frac{1}{3}x^3 \tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{6}ix^3 \log\left(1-ice^{2ia+2ibx}\right) - \frac{x^2}{}$$
$$= -\frac{bx^4}{12} + \frac{1}{3}x^3 \tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{6}ix^3 \log\left(1-ice^{2ia+2ibx}\right) - \frac{x^2}{}$$
$$= -\frac{bx^4}{12} + \frac{1}{3}x^3 \tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{6}ix^3 \log\left(1-ice^{2ia+2ibx}\right) - \frac{x^2}{}$$
$$= -\frac{bx^4}{12} + \frac{1}{3}x^3 \tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{6}ix^3 \log\left(1-ice^{2ia+2ibx}\right) - \frac{x^2}{}$$

Mathematica [A] time = 0.423886, size = 140, normalized size = 0.91

$$\frac{1}{3}x^3 \tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{-6b^2x^2\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right) + 6ibx\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right) + 3\text{PolyLog}\left(4,-\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/3 - ((4*I)*b^3*x^3*Log[1 + I/(c*E^
((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))] +
(6*I)*b*x*PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, (-I)/(c*E
```

```
^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] time = 17.733, size = 1532, normalized size = 10.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x)

[Out] -1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))
*(c-I)/(exp(2*I*(b*x+a))+1))^3+1/6*Pi*x^3-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a
)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2
*I*(b*x+a))+1))+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(c*exp(2*I*
(b*x+a))+I))*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))-1/2*I/b^2*
a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a
))*(-I*c)^(1/2))*x-1/12*b*x^4+1/6*I*x^3*ln(c*exp(2*I*(b*x+a))+I)+1/12*x^3*P
i*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3+1/3*I/b^3*ln(1-I*c*
exp(2*I*(b*x+a)))*a^3+1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x
+a))+1))^3-1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2+
1/12*x^3*Pi*csgn((c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3-1/12*x^3*Pi
*csgn((c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn(I/(ex
p(2*I*(b*x+a))+1))*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))+1/2*I/b
^2*ln(1-I*c*exp(2*I*(b*x+a)))*x*a^2-1/12*x^3*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+
a))+1))^3-1/2/b^3*a^2*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2/b^3*a^2*di
log(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/4*x^2*polylog(2,I*c*exp(2*I*(b*x+a))
)/b-1/6*I*x^3*ln(1-I*c*exp(2*I*(b*x+a)))+1/4/b^3*polylog(2,I*c*exp(2*I*(b*x
+a)))*a^2+1/8*polylog(4,I*c*exp(2*I*(b*x+a)))/b^3-1/12*x^3*Pi*csgn(I*exp(I*
(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1
))*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn(I
*(c*exp(2*I*(b*x+a))+I))*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1)
)^2-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(ex
p(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))-1/3*I*x^3*ln(exp(I*(b*x+a)))-1/6
*I*x^3*ln(c-I)+1/12*x^3*Pi*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))
^2+1/12*x^3*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(
c-I)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp
(2*I*(b*x+a))+1))*csgn((c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))+1/12*x^
3*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+
a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/4*I*x*polylog(3,I*c*exp(2*I*(b*x+a)))/b
^2+1/6*I/b^3*a^3*ln(c*exp(2*I*(b*x+a))+I)-1/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a
))*(-I*c)^(1/2))-1/2*I/b^3*a^3*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/12*x^3
*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a)
)+1))^2-1/12*x^3*Pi*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*csg
n((c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2+1/6*x^3*Pi*csgn(I*exp(I*(b
*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))
*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2
```

Maxima [B] time = 1.14926, size = 417, normalized size = 2.71

$$\frac{\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan((ic+1)\tan(bx+a)+c)}{b^2}-\frac{3\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(-8i\,(bx+a)^3+18i\,(bx+a)^2a-18i\,(bx+a)a^2\right)\mathrm{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*(((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((I*c + 1)*tan
(b*x + a) + c)/b^2 - 3*(-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x +
 a)^2*a^2 + (-8*I*(b*x + a)^3 + 18*I*(b*x + a)^2*a - 18*I*(b*x + a)*a^2)*ar
ctan2(c*cos(2*b*x + 2*a), c*sin(2*b*x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18
*I*(b*x + a)*a - 9*I*a^2)*dilog(I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 -
 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin(2*
b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, I*c*e^(
2*I*b*x + 2*I*a)) + 6*I*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)/(b^2
*(12*c - 12*I)))/b
```

Fricas [C] time = 2.19974, size = 907, normalized size = 5.89

$$-\frac{b^4x^4 - 2i\,b^3x^3\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}e^{(i\,bx+i\,a)}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}e^{(i\,bx+i\,a)}\right) - a^4 - 2i\,a^3 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/12*(b^4*x^4 - 2*I*b^3*x^3*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x -
 2*I*a)/(c - I)) + 6*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6*b^2
*x^2*dilog(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) - a^4 - 2*I*a^3*log(1/2*(2*c*e
^(I*b*x + I*a) + I*sqrt(4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(I*b*x + I*a) -
 I*sqrt(4*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) +
 12*I*b*x*polylog(3, -1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) - (-2*I*b^3*x^3 - 2*
I*a^3)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - (-2*I*b^3*x^3 - 2*I*a^3)*
log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - 12*polylog(4, 1/2*sqrt(4*I*c)*e
^(I*b*x + I*a)) - 12*polylog(4, -1/2*sqrt(4*I*c)*e^(I*b*x + I*a)))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int\frac{x^3}{ice^{2ia}e^{2ibx}-1}\,dx}{3} + \frac{ix^3\log\left(-ic + \frac{ic}{e^{2ia}e^{2ibx}+1} - \frac{ice^{ia}e^{ibx}}{e^{ia}e^{ibx}+e^{-ia}e^{-ibx}} + 1 + \frac{1}{e^{2ia}e^{2ibx}+1} - \frac{e^{ia}e^{ibx}}{e^{ia}e^{ibx}+e^{-ia}e^{-ibx}}\right)}{6} - \frac{ix^3\log\left(ic - \frac{ic}{e^{2ia}e^{2ibx}+1} + \cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] b*Integral(x**3/(I*c*exp(2*I*a)*exp(2*I*b*x) - 1), x)/3 + I*x**3*log(-I*c +
 I*c/(exp(2*I*a)*exp(2*I*b*x) + 1) - I*c*exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(
I*b*x) + exp(-I*a)*exp(-I*b*x)) + 1 + 1/(exp(2*I*a)*exp(2*I*b*x) + 1) - exp
(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) + exp(-I*a)*exp(-I*b*x)))/6 - I*x**3*
log(I*c - I*c/(exp(2*I*a)*exp(2*I*b*x) + 1) + I*c*exp(I*a)*exp(I*b*x)/(exp(
I*a)*exp(I*b*x) + exp(-I*a)*exp(-I*b*x)) + 1 - 1/(exp(2*I*a)*exp(2*I*b*x) +
 1) + exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) + exp(-I*a)*exp(-I*b*x)))/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\arctan\left((i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((I*c + 1)*tan(b*x + a) + c), x)
```

3.53 $\int x \tan^{-1}(c + (1 + ic)\tan(a + bx))\,dx$

Optimal. Leaf size=123

$$-\frac{i\text{PolyLog}\left(3, ice^{2ia+2ibx}\right)}{8b^2} - \frac{x\text{PolyLog}\left(2, ice^{2ia+2ibx}\right)}{4b} - \frac{1}{4}ix^2\log\left(1 - ice^{2ia+2ibx}\right) + \frac{1}{2}x^2\tan^{-1}(c + (1 + ic)\tan(a + bx)$$

```
[Out] -(b*x^3)/6 + (x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/2 - (I/4)*x^2*Log[1 -
 I*c*E^((2*I)*a + (2*I)*b*x)] - (x*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])
/(4*b) - ((I/8)*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rubi [A] time = 0.206246, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5171, 2184, 2190, 2531, 2282, 6589}

$$-\frac{i\text{PolyLog}\left(3, ice^{2ia+2ibx}\right)}{8b^2} - \frac{x\text{PolyLog}\left(2, ice^{2ia+2ibx}\right)}{4b} - \frac{1}{4}ix^2\log\left(1 - ice^{2ia+2ibx}\right) + \frac{1}{2}x^2\tan^{-1}(c + (1 + ic)\tan(a + bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] -(b*x^3)/6 + (x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/2 - (I/4)*x^2*Log[1 -
 I*c*E^((2*I)*a + (2*I)*b*x)] - (x*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])
/(4*b) - ((I/8)*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 5171

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tan[a + b*x]])/(f*(m +
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{2}(ib)\int\frac{x^2}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx$$

$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{2}(bc)\int\frac{e^{2ia+2ibx}x^2}{i(1+ic)+c+ce^{2ia+2ibx}}$$

$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{2}i\int x$$

$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)-\frac{x\mathrm{Li}_2(}{}$$

$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)-\frac{x\mathrm{Li}_2(}{}$$

$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)-\frac{x\mathrm{Li}_2(}{}$$

Mathematica [A] time = 0.311115, size = 110, normalized size = 0.89

$$\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{i\left(2ibx\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)+2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 + I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]
 + PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] time = 9.421, size = 1497, normalized size = 12.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] -1/8*x^2*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^3+1/8*x^2*Pi*csgn(exp(2*I*(b
*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^3+1/4*Pi*x^2-1/8*x^2*Pi*csgn(I*exp(2*I*(
```

```
b*x+a)))^3-1/2*I/b*ln(1-I*c*exp(2*I*(b*x+a)))*x*a+1/2*I/b*a*ln(1-I*exp(I*(b
*x+a))*(-I*c)^(1/2))*x-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b
*x+a))+1))^3-1/4*x*polylog(2,I*c*exp(2*I*(b*x+a)))/b-1/8*I*polylog(3,I*c*ex
p(2*I*(b*x+a)))/b^2+1/4*I*x^2*ln(c*exp(2*I*(b*x+a))+I)+1/8*x^2*Pi*csgn(I*(c
*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3-1/4*I*x^2*ln(1-I*c*exp(2*I*(b*
x+a)))-1/6*b*x^3-1/8*x^2*Pi*csgn((c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1
))^2-1/4/b^2*polylog(2,I*c*exp(2*I*(b*x+a)))*a-1/8*x^2*Pi*csgn(I/(exp(2*I*(
b*x+a))+1))*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))-1/8*x^2*Pi*csg
n(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*
x+a))*(c-I)/(exp(2*I*(b*x+a))+1))+1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*c
sgn(I*(c*exp(2*I*(b*x+a))+I))*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a
))+1))+1/8*x^2*Pi*csgn((c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3-1/8*x
^2*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2+1/2/b^2*a*dilog(1
-I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/2/b^2*a*dilog(1+I*exp(I*(b*x+a))*(-I*c)^(
1/2))+1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/2*I/b^2*a^2*ln(1+
I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/4*I/b^2*ln(1-I*c*exp(2*I*(b*x+a)))*a^2-1/8
*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/4*x^2*Pi*csgn(I
*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/2*I/b*a*ln(1+I*exp(I*(b*x+a))
*(-I*c)^(1/2))*x-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))
+1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))+1/8*x^2*Pi*csgn(I*(c
*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((c*exp(2*I*(b*x+a))+I)/(exp
(2*I*(b*x+a))+1))+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a)
)+1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/4*I/b^2*a^2*ln(
c*exp(2*I*(b*x+a))+I)-1/8*x^2*Pi*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*
x+a))+1))*csgn((c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2+1/8*x^2*Pi*cs
gn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2+1/8*x^2*Pi*
csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*csgn(I/(exp(2
*I*(b*x+a))+1))*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2-1/8*x
^2*Pi*csgn(I*(c*exp(2*I*(b*x+a))+I))*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp(2*I
*(b*x+a))+1))^2-1/2*I*x^2*ln(exp(I*(b*x+a)))-1/4*I*x^2*ln(c-I)+1/8*x^2*Pi*c
sgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))
^2+1/8*x^2*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(c
-I)/(exp(2*I*(b*x+a))+1))^2
```

Maxima [B] time = 1.07632, size = 294, normalized size = 2.39

$$\frac{\frac{\left((bx+a)^2-2\,(bx+a)a\right)\arctan((ic+1)\tan(bx+a)+c)}{b}-\frac{2\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)+\left(-6i\,(bx+a)^2+12i\,(bx+a)a\right)\arctan(c\cos(2\,bx+2}{}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x + a)^2 - 2*(b*x + a)*a)*arctan((I*c + 1)*tan(b*x + a) + c)/b - 2
*(-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(I*c*e^(2*I*b*x + 2*
I*a)) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a), c
*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b*x
+ 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*polylog(3
, I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)/(b*(12*c - 12*I)))/b
```

Fricas [C] time = 2.21656, size = 755, normalized size = 6.14

$$-\frac{2\,b^3x^3-3i\,b^2x^2\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right)+2\,a^3+6\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}e^{(i\,bx+i\,a)}\right)+6\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}e^{(i\,bx+i\,a)}\right)+3i\,a^2\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/12*(2*b^3*x^3 - 3*I*b^2*x^2*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x
 - 2*I*a)/(c - I)) + 2*a^3 + 6*b*x*dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) +
 6*b*x*dilog(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 3*I*a^2*log(1/2*(2*c*e^(I*
b*x + I*a) + I*sqrt(4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(I*b*x + I*a) - I*s
qrt(4*I*c))/c) - (-3*I*b^2*x^2 + 3*I*a^2)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*
a) + 1) - (-3*I*b^2*x^2 + 3*I*a^2)*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1
) + 6*I*polylog(3, 1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6*I*polylog(3, -1/2*s
qrt(4*I*c)*e^(I*b*x + I*a)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int\frac{x^2}{ice^{2ia}e^{2ibx}-1}\,dx}{2}+\frac{ix^2\log\left(-ic+\frac{ic}{e^{2ia}e^{2ibx}+1}-\frac{ice^{ia}e^{ibx}}{e^{ia}e^{ibx}+e^{-ia}e^{-ibx}}+1+\frac{1}{e^{2ia}e^{2ibx}+1}-\frac{e^{ia}e^{ibx}}{e^{ia}e^{ibx}+e^{-ia}e^{-ibx}}\right)}{4}-\frac{ix^2\log\left(ic-\frac{ic}{e^{2ia}e^{2ibx}+1}+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] b*Integral(x**2/(I*c*exp(2*I*a)*exp(2*I*b*x) - 1), x)/2 + I*x**2*log(-I*c +
 I*c/(exp(2*I*a)*exp(2*I*b*x) + 1) - I*c*exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(
I*b*x) + exp(-I*a)*exp(-I*b*x)) + 1 + 1/(exp(2*I*a)*exp(2*I*b*x) + 1) - exp
(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) + exp(-I*a)*exp(-I*b*x)))/4 - I*x**2*
log(I*c - I*c/(exp(2*I*a)*exp(2*I*b*x) + 1) + I*c*exp(I*a)*exp(I*b*x)/(exp(
I*a)*exp(I*b*x) + exp(-I*a)*exp(-I*b*x)) + 1 - 1/(exp(2*I*a)*exp(2*I*b*x) +
 1) + exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) + exp(-I*a)*exp(-I*b*x)))/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\arctan\left((ic+1)\tan\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((I*c + 1)*tan(b*x + a) + c), x)
```

3.54 $\int \tan^{-1}(c+(1+ic)\tan(a+bx))\,dx$

Optimal. Leaf size=85

$$-\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+x\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{bx^2}{2}$$

```
[Out] -(b*x^2)/2 + x*ArcTan[c + (1 + I*c)*Tan[a + b*x]] - (I/2)*x*Log[1 - I*c*E^(
(2*I)*a + (2*I)*b*x)] - PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rubi [A] time = 0.124773, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5163, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+x\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] -(b*x^2)/2 + x*ArcTan[c + (1 + I*c)*Tan[a + b*x]] - (I/2)*x*Log[1 - I*c*E^(
(2*I)*a + (2*I)*b*x)] - PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 5163

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] - Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tan^{-1}(c+(1+ic)\tan(a+bx))\,dx = x\tan^{-1}(c+(1+ic)\tan(a+bx)) - (ib)\int \frac{x}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx$$
$$= -\frac{bx^2}{2} + x\tan^{-1}(c+(1+ic)\tan(a+bx)) + (bc)\int \frac{e^{2ia+2ibx}x}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx$$
$$= -\frac{bx^2}{2} + x\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) + \frac{1}{2}i\int \log\left(1\right.$$
$$= -\frac{bx^2}{2} + x\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) + \frac{\text{Subst}\left(\int \frac{\text{lo}}{}\right.}{}$$
$$= -\frac{bx^2}{2} + x\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) - \frac{\text{Li}_2\left(ice^{2ia+2}\right.}{4b}$$

Mathematica [B] time = 14.2647, size = 967, normalized size = 11.38

$$\frac{ix\left(\right.}{((c+i)\cos(a+bx)+(ic+1)\sin(a+bx))\left(\log(i\tan(bx)+1)\tan(bx)\cos^2(a)+2bx-i\log\left(1-\frac{\sec(bx)((c-i)\cos(a)+i(c+i)\sin(a)}{2c}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (1 + I*c)*Tan[a + b*x]] + (I*x*((2*I)*b*x*Log[2*Cos[b*x]*(Cos[
b*x] - I*Sin[b*x])] - Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((I + c)*Cos[a + b*
x] + (1 + I*c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] + Log[(Sec[b*x]*((
1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*
Log[1 + I*Tan[b*x]] - PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - PolyLog[2, (
Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x
]))/(2*c)] + PolyLog[2, (Sec[b*x]*((1 + I*c)*Cos[a] - (I + c)*Sin[a])*(Cos[
a + b*x] + I*Sin[a + b*x]))/2])*Sec[a + b*x]^2*(Cos[b*x] + I*Sin[b*x])*(I*C
os[b*x] + Sin[b*x])*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(((I
+ c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x])*(2*b*x - I*Log[1 - (Sec[b*x]*((
-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]
- I*Log[1 + (Sec[b*x]*((-1 - I*c)*Cos[a] + (I + c)*Sin[a])*(Cos[a + b*x] +
I*Sin[a + b*x]))/2] - (I*(-I + c)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Log[1
 + I*Tan[b*x]]))/((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]) + ((I + c)
*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((I + c)*Cos[a +
 b*x] + (1 + I*c)*Sin[a + b*x]) - (2*I)*b*x*Tan[b*x] + Log[1 - (Sec[b*x]*((
-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]*
Tan[b*x] - Log[1 + (Sec[b*x]*((-1 - I*c)*Cos[a] + (I + c)*Sin[a])*(Cos[a +
b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] - Log[1 - I*Tan[b*x]]*Tan[b*x] + Cos[a]
^2*Log[1 + I*Tan[b*x]]*Tan[b*x] + Log[1 + I*Tan[b*x]]*Sin[a]^2*Tan[b*x] + (
Log[(Sec[b*x]*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] -
 (2*I)*Sin[a])]*Sec[b*x]^2)/(-I + Tan[b*x]) - (Log[(Sec[b*x]*(Cos[a] - I*Si
n[a])*((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]))/(2*c)]*Sec[b*x]^2)/(
I + Tan[b*x]))*(-I + Tan[a + b*x])*(1 - I*c + (-I + c)*Tan[a + b*x]))
```

Maple [B] time = 0.132, size = 1489, normalized size = 17.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] 1/2/(1+I*c)/b/(I-c)*ln((c+(1+I*c)*tan(b*x+a)-I)/(-2*I+2*c))*ln(-c+(1+I*c)*t
an(b*x+a)+I)*c+1/(1+I*c)/b*arctan(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(-c+(1+
I*c)*tan(b*x+a)+I)*c^2-1/(1+I*c)/b*arctan(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*l
n(c+(1+I*c)*tan(b*x+a)-I)*c^2+1/2/(1+I*c)/b/(I-c)*ln(-1/2*I*(c+(1+I*c)*tan(
b*x+a)+I))*ln(c+(1+I*c)*tan(b*x+a)-I)*c-1/4*I/(1+I*c)/b/(I-c)*ln((c+(1+I*c)
*tan(b*x+a)-I)/(-2*I+2*c))*ln(-c+(1+I*c)*tan(b*x+a)+I)+1/4*I/(1+I*c)/b/(I-c
)*dilog(-1/2*I*(c+(1+I*c)*tan(b*x+a)+I))*c^2-1/8*I/(1+I*c)/b/(I-c)*ln(c+(1+
I*c)*tan(b*x+a)-I)^2*c^2-1/2/(1+I*c)/b/(I-c)*ln(1/2*(c+(1+I*c)*tan(b*x+a)+I
)/c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*c+1/4*I/(1+I*c)/b/(I-c)*dilog((c+(1+I*c)*t
an(b*x+a)-I)/(-2*I+2*c))*c^2-1/4*I/(1+I*c)/b/(I-c)*dilog(1/2*(c+(1+I*c)*tan
(b*x+a)+I)/c)*c^2-1/4*I/(1+I*c)/b/(I-c)*ln(-1/2*I*(c+(1+I*c)*tan(b*x+a)+I))
*ln(c+(1+I*c)*tan(b*x+a)-I)+1/4*I/(1+I*c)/b/(I-c)*ln(1/2*(c+(1+I*c)*tan(b*x
+a)+I)/c)*ln(-c+(1+I*c)*tan(b*x+a)+I)-1/4/(1+I*c)/b/(I-c)*ln(c+(1+I*c)*tan(
b*x+a)-I)^2*c-1/2/(1+I*c)/b/(I-c)*dilog(1/2*(c+(1+I*c)*tan(b*x+a)+I)/c)*c+1
/2/(1+I*c)/b/(I-c)*dilog((c+(1+I*c)*tan(b*x+a)-I)/(-2*I+2*c))*c-1/(1+I*c)/b
*arctan(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(-c+(1+I*c)*tan(b*x+a)+I)+1/(1+I*
c)/b*arctan(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(c+(1+I*c)*tan(b*x+a)-I)+1/8*
I/(1+I*c)/b/(I-c)*ln(c+(1+I*c)*tan(b*x+a)-I)^2+1/4*I/(1+I*c)/b/(I-c)*dilog(
1/2*(c+(1+I*c)*tan(b*x+a)+I)/c)-1/4*I/(1+I*c)/b/(I-c)*dilog((c+(1+I*c)*tan(
b*x+a)-I)/(-2*I+2*c))-1/4*I/(1+I*c)/b/(I-c)*dilog(-1/2*I*(c+(1+I*c)*tan(b*x
+a)+I))+1/2/(1+I*c)/b/(I-c)*dilog(-1/2*I*(c+(1+I*c)*tan(b*x+a)+I))*c+2*I/(1
+I*c)/b*arctan(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(c+(1+I*c)*tan(b*x+a)-I)*c
+1/4*I/(1+I*c)/b/(I-c)*ln(-1/2*I*(c+(1+I*c)*tan(b*x+a)+I))*ln(c+(1+I*c)*tan
(b*x+a)-I)*c^2-1/4*I/(1+I*c)/b/(I-c)*ln(1/2*(c+(1+I*c)*tan(b*x+a)+I)/c)*ln(
-c+(1+I*c)*tan(b*x+a)+I)*c^2+1/4*I/(1+I*c)/b/(I-c)*ln((c+(1+I*c)*tan(b*x+a)
-I)/(-2*I+2*c))*ln(-c+(1+I*c)*tan(b*x+a)+I)*c^2-2*I/(1+I*c)/b*arctan(c+(1+I
*c)*tan(b*x+a))/(2*I-2*c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*c
```

Maxima [B] time = 1.59028, size = 605, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((I*c + 1)*(4*I*(b*x + a)*log((2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x +
 a) + 4*c - 2*I)/(2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x + a) + 2*I))/(I*c +
 1) - I*(4*(b*x + a)*(log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - 2*c + I
) - log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - I)) + I*log(-I*c^2 + (c^2
 - 2*I*c - 1)*tan(b*x + a) - 2*c + I)^2 - 2*I*log(-I*c^2 + (c^2 - 2*I*c - 1
)*tan(b*x + a) - I)*log(-1/2*(c - I)*tan(b*x + a) + 1/2*I*c + 1/2) + 2*I*lo
g(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - I)*log(-1/2*((I*c + 1)*tan(b*x
+ a) + c + I)/c + 1) - 2*I*log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - 2*
c + I)*log(-1/2*I*tan(b*x + a) + 1/2) - 2*I*dilog(1/2*(c - I)*tan(b*x + a)
- 1/2*I*c + 1/2) + 2*I*dilog(1/2*((I*c + 1)*tan(b*x + a) + c + I)/c) - 2*I*
dilog(1/2*I*tan(b*x + a) + 1/2))/(I*c + 1)) - 8*(b*x + a)*arctan((I*c + 1)*
tan(b*x + a) + c) + 4*(-I*b*x - I*a)*log((2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan
(b*x + a) + 4*c - 2*I)/(2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x + a) + 2*I)))
/b
```

Fricas [B] time = 2.30371, size = 547, normalized size = 6.44

$$-\frac{b^2x^2 - i\,bx\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right) - a^2 - (-i\,bx - i\,a)\log\left(\frac{1}{2}\,\sqrt{4i}\,ce^{(i\,bx+i\,a)}+1\right) - (-i\,bx - i\,a)\log\left(-\frac{1}{2}\,\sqrt{4i}\,ce^{(i\,bx+i\,a)}\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - I*b*x*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)
/(c - I)) - a^2 - (-I*b*x - I*a)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) -
 (-I*b*x - I*a)*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - I*a*log(1/2*(2*
c*e^(I*b*x + I*a) + I*sqrt(4*I*c))/c) - I*a*log(1/2*(2*c*e^(I*b*x + I*a) -
I*sqrt(4*I*c))/c) + dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + dilog(-1/2*sqr
t(4*I*c)*e^(I*b*x + I*a)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{x}{ice^{2ia}e^{2ibx}-1}\,dx + \frac{ix\log\left(-ic+\frac{ic}{e^{2ia}e^{2ibx}+1}-\frac{ice^{ia}e^{ibx}}{e^{ia}e^{ibx}+e^{-ia}e^{-ibx}}+1+\frac{1}{e^{2ia}e^{2ibx}+1}-\frac{e^{ia}e^{ibx}}{e^{ia}e^{ibx}+e^{-ia}e^{-ibx}}\right)}{2} - \frac{ix\log\left(ic-\frac{ic}{e^{2ia}e^{2ibx}+1}+\frac{}{e^{ia}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] b*Integral(x/(I*c*exp(2*I*a)*exp(2*I*b*x) - 1), x) + I*x*log(-I*c + I*c/(ex
p(2*I*a)*exp(2*I*b*x) + 1) - I*c*exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) +
 exp(-I*a)*exp(-I*b*x)) + 1 + 1/(exp(2*I*a)*exp(2*I*b*x) + 1) - exp(I*a)*ex
p(I*b*x)/(exp(I*a)*exp(I*b*x) + exp(-I*a)*exp(-I*b*x)))/2 - I*x*log(I*c - I
*c/(exp(2*I*a)*exp(2*I*b*x) + 1) + I*c*exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*
b*x) + exp(-I*a)*exp(-I*b*x)) + 1 - 1/(exp(2*I*a)*exp(2*I*b*x) + 1) + exp(I
*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) + exp(-I*a)*exp(-I*b*x)))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arctan\left((i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((I*c + 1)*tan(b*x + a) + c), x)
```

3.55 $\int \frac{\tan^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=23

$$\text{CannotIntegrate}\left(\frac{\tan^{-1}(c+(1+ic)\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x, x]
```

Rubi [A] time = 0.119529, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx=\int \frac{\tan^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$$

Mathematica [A] time = 0.589706, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x, x]
```

Maple [A] time = 0.408, size = 0, normalized size = 0.

$$\int \frac{\arctan(c+(1+ic)\tan(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(1+I*c)*tan(b*x+a))/x,x)
```

```
[Out] int(arctan(c+(1+I*c)*tan(b*x+a))/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i \log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right)}{2\,x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/(c - I
))/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(1+I*c)*tan(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left((i\,c+1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((I*c + 1)*tan(b*x + a) + c)/x, x)
```

3.56 $\int x^2 \tan^{-1}(c + (-1 + ic)\tan(a + bx))\, dx$

Optimal. Leaf size=155

$$\frac{ix\text{PolyLog}\left(3, -ice^{2ia+2ibx}\right)}{4b^2} - \frac{\text{PolyLog}\left(4, -ice^{2ia+2ibx}\right)}{8b^3} + \frac{x^2\text{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} + \frac{1}{6}ix^3 \log\left(1 + ice^{2ia+2ibx}\right) + \frac{1}{3}$$

```
[Out] (b*x^4)/12 + (x^3*ArcTan[c - (1 - I*c)*Tan[a + b*x]])/3 + (I/6)*x^3*Log[1 +
 I*c*E^((2*I)*a + (2*I)*b*x)] + (x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b
*x)])/(4*b) + ((I/4)*x*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2 - Po
lyLog[4, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rubi [A] time = 0.244238, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5171, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{ix\text{PolyLog}\left(3, -ice^{2ia+2ibx}\right)}{4b^2} - \frac{\text{PolyLog}\left(4, -ice^{2ia+2ibx}\right)}{8b^3} + \frac{x^2\text{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} + \frac{1}{6}ix^3 \log\left(1 + ice^{2ia+2ibx}\right) + \frac{1}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTan[c - (1 - I*c)*Tan[a + b*x]])/3 + (I/6)*x^3*Log[1 +
 I*c*E^((2*I)*a + (2*I)*b*x)] + (x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b
*x)])/(4*b) + ((I/4)*x*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2 - Po
lyLog[4, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 5171

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tan[a + b*x]])/(f*(m +
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+(-1+ic)\tan(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{3}(ib)\int \frac{x^3}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{3}(bc)\int \frac{e^{2ia+2ibx}x^3}{i(-1+ic)+c+ce^{2ia+2ib}}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) - \frac{1}{2}i\int$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) + \frac{x^2\mathrm{Li}_2}{}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) + \frac{x^2\mathrm{Li}_2}{}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) + \frac{x^2\mathrm{Li}_2}{}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) + \frac{x^2\mathrm{Li}_2}{}$$

Mathematica [A] time = 0.439728, size = 137, normalized size = 0.88

$$\frac{1}{24}\left(\frac{6ix\mathrm{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)}{b^2} + \frac{3\mathrm{PolyLog}\left(4,\frac{ie^{-2i(a+bx)}}{c}\right)}{b^3} - \frac{6x^2\mathrm{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)}{b} + 4ix^3\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right) + 8x^3\,\mathrm{t}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTan[c + I*(I + c)*Tan[a + b*x]] + (4*I)*x^3*Log[1 - I/(c*E^((2*I)
*(a + b*x)))] - (6*x^2*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))])/b + ((6*I)*x*
```

```
PolyLog[3, I/(c*E^((2*I)*(a + b*x)))])/b^2 + (3*PolyLog[4, I/(c*E^((2*I)*(a
 + b*x)))])/b^3)/24
```

Maple [C] time = 19.335, size = 1533, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] 1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))*csg
n(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))+1/12*x^3*Pi*csgn((c*exp(2*
I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3-1/12*x^3*Pi*csgn((c*exp(2*I*(b*x+a))-
I)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2
*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2-1/12*x^
3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a
))+1))^2-1/12*x^3*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x
+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1
/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*I
*(b*x+a))+1))+1/6*Pi*x^3+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(c
*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn(I*(c*exp(2*I*
(b*x+a))-I))*csgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3
*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))^2-1/12*
x^3*Pi*csgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((c*exp(2*I*
(b*x+a))-I)/(exp(2*I*(b*x+a))+1))-1/8*polylog(4,-I*c*exp(2*I*(b*x+a)))/b^3-
1/4/b^3*polylog(2,-I*c*exp(2*I*(b*x+a)))*a^2+1/2/b^3*a^2*dilog(1-I*exp(I*(b
*x+a))*(I*c)^(1/2))-1/12*x^3*Pi*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a)
)+1))^2+1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3-1/2
*I/b^2*ln(1+I*c*exp(2*I*(b*x+a)))*x*a^2+1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))
*(I*c)^(1/2))*x+1/12*b*x^4-1/12*x^3*Pi*csgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(2
*I*(b*x+a))+1))^3+1/12*x^3*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))^3+1/2*I/b^
2*a^2*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/2/b^3*a^2*dilog(1+I*exp(I*(b*x
+a))*(I*c)^(1/2))+1/3*I*x^3*ln(exp(I*(b*x+a)))+1/6*I*x^3*ln(I+c)+1/12*x^3*P
i*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3-1/12*x^3*Pi*csgn(I/
(exp(2*I*(b*x+a))+1))*csgn(I*(c*exp(2*I*(b*x+a))-I))*csgn(I*(c*exp(2*I*(b*x
+a))-I)/(exp(2*I*(b*x+a))+1))-1/6*I/b^3*a^3*ln(-c*exp(2*I*(b*x+a))+I)-1/12*
x^3*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn
(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/4*x^2*polylog(2,-I*c*exp(2*
I*(b*x+a)))/b+1/12*x^3*Pi*csgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1
))*csgn((c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/6*I*x^3*ln(c*exp(2
*I*(b*x+a))-I)+1/6*I*x^3*ln(1+I*c*exp(2*I*(b*x+a)))+1/2*I/b^3*a^3*ln(1+I*ex
p(I*(b*x+a))*(I*c)^(1/2))+1/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))-
1/3*I/b^3*ln(1+I*c*exp(2*I*(b*x+a)))*a^3+1/4*I*x*polylog(3,-I*c*exp(2*I*(b*
x+a)))/b^2+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*
csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))-1/6*x^3*Pi*csgn(I*exp(I*(
b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2
```

Maxima [B] time = 1.13526, size = 419, normalized size = 2.7

$$\frac{\frac{\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan((ic-1)\tan(bx+a)+c)}{b^2}+\frac{3\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(8i\,(bx+a)^3-18i\,(bx+a)^2a+18i\,(bx+a)a^2\right)\mathrm{ar}\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*(((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((I*c - 1)*tan
(b*x + a) + c)/b^2 + 3*(-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x +
 a)^2*a^2 + (8*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + 18*I*(b*x + a)*a^2)*arc
tan2(c*cos(2*b*x + 2*a), -c*sin(2*b*x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18
*I*(b*x + a)*a - 9*I*a^2)*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3
- 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin(2
*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, -I*c*e
^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(
b^2*(12*c + 12*I)))/b
```

Fricas [C] time = 1.98512, size = 915, normalized size = 5.9

$$b^4x^4 + 2i\,b^3x^3\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right) - a^4 - 2i\,a^3\log\left(\frac{2\,ce^{(i\,bx}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 + 2*I*b^3*x^3*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x
+ 2*I*a) - I)) + 6*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 6*b^2*
x^2*dilog(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) - a^4 - 2*I*a^3*log(1/2*(2*c*e
^(I*b*x + I*a) + I*sqrt(-4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(I*b*x + I*a)
- I*sqrt(-4*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)
) + 12*I*b*x*polylog(3, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + (2*I*b^3*x^3 +
 2*I*a^3)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) + (2*I*b^3*x^3 + 2*I*a^
3)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - 12*polylog(4, 1/2*sqrt(-4*I
*c)*e^(I*b*x + I*a)) - 12*polylog(4, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)))/b^
3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int\frac{x^3}{ice^{2ia}e^{2ibx}+1}\,dx}{3} + \frac{ix^3\log\left(-ic + \frac{ic}{e^{2ia}e^{2ibx}+1} - \frac{ice^{ia}e^{ibx}}{e^{ia}e^{ibx}+e^{-ia}e^{-ibx}} + 1 - \frac{1}{e^{2ia}e^{2ibx}+1} + \frac{e^{ia}e^{ibx}}{e^{ia}e^{ibx}+e^{-ia}e^{-ibx}}\right)}{6} - \frac{ix^3\log\left(ic - \frac{ic}{e^{2ia}e^{2ibx}+1} + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] b*Integral(x**3/(I*c*exp(2*I*a)*exp(2*I*b*x) + 1), x)/3 + I*x**3*log(-I*c +
 I*c/(exp(2*I*a)*exp(2*I*b*x) + 1) - I*c*exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(
I*b*x) + exp(-I*a)*exp(-I*b*x)) + 1 - 1/(exp(2*I*a)*exp(2*I*b*x) + 1) + exp
(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) + exp(-I*a)*exp(-I*b*x)))/6 - I*x**3*
log(I*c - I*c/(exp(2*I*a)*exp(2*I*b*x) + 1) + I*c*exp(I*a)*exp(I*b*x)/(exp(
I*a)*exp(I*b*x) + exp(-I*a)*exp(-I*b*x)) + 1 + 1/(exp(2*I*a)*exp(2*I*b*x) +
 1) - exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) + exp(-I*a)*exp(-I*b*x)))/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\arctan\left((i\,c-1)\tan\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((I*c - 1)*tan(b*x + a) + c), x)
```

3.57 $\int x \tan^{-1}(c + (-1 + ic)\tan(a + bx))\,dx$

Optimal. Leaf size=124

$$\frac{i\text{PolyLog}\left(3, -ice^{2ia+2ibx}\right)}{8b^2} + \frac{x\text{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} + \frac{1}{4}ix^2\log\left(1 + ice^{2ia+2ibx}\right) + \frac{1}{2}x^2\tan^{-1}(c - (1 - ic)\tan(a + bx))$$

```
[Out] (b*x^3)/6 + (x^2*ArcTan[c - (1 - I*c)*Tan[a + b*x]])/2 + (I/4)*x^2*Log[1 +
I*c*E^((2*I)*a + (2*I)*b*x)] + (x*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) + ((I/8)*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rubi [A] time = 0.207879, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5171, 2184, 2190, 2531, 2282, 6589}

$$\frac{i\text{PolyLog}\left(3, -ice^{2ia+2ibx}\right)}{8b^2} + \frac{x\text{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} + \frac{1}{4}ix^2\log\left(1 + ice^{2ia+2ibx}\right) + \frac{1}{2}x^2\tan^{-1}(c - (1 - ic)\tan(a + bx))$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTan[c - (1 - I*c)*Tan[a + b*x]])/2 + (I/4)*x^2*Log[1 +
I*c*E^((2*I)*a + (2*I)*b*x)] + (x*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) + ((I/8)*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 5171

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tan[a + b*x]])/(f*(m +
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+(-1+ic)\tan(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{2}(ib)\int \frac{x^2}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{2}(bc)\int \frac{e^{2ia+2ibx}x^2}{i(-1+ic)+c+ce^{2ia+}}$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right) - \frac{1}{2}i$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right) + \frac{xL}{}$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right) + \frac{xL}{}$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right) + \frac{xL}{}$$

Mathematica [A] time = 0.306181, size = 111, normalized size = 0.9

$$\frac{i\left(2ibx\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)+2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}+\frac{1}{2}x^2\tan^{-1}(c+i(c+i)\tan(a+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + I*(I + c)*Tan[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 - I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] +
PolyLog[3, I/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] time = 8.505, size = 1498, normalized size = 12.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] 1/8*I*polylog(3,-I*c*exp(2*I*(b*x+a)))/b^2+1/4*x*polylog(2,-I*c*exp(2*I*(b*
x+a)))/b+1/4*Pi*x^2-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(c*exp(2
```

```
*I*(b*x+a))-I))*csgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))+1/8*x^2
*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/8*x^2*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1)
)^3-1/8*x^2*Pi*csgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+1/8*x^
2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(I*exp
(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))-1/4*I*x^2*ln(c*exp(2*I*(b*x+a))-I
)+1/6*b*x^3+1/4*I*x^2*ln(1+I*c*exp(2*I*(b*x+a)))+1/4/b^2*polylog(2,-I*c*exp
(2*I*(b*x+a)))*a-1/2/b^2*a*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/2/b^2*a*
dilog(1-I*exp(I*(b*x+a))*(I*c)^(1/2))-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1
))*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*csgn(I*(I+c))*csgn(I*(I+
c)/(exp(2*I*(b*x+a))+1))^2+1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(
I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))+1/8*x^2*Pi*csgn(I*exp(I*(b*x+a)))^
2*csgn(I*exp(2*I*(b*x+a)))-1/4*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I
*(b*x+a)))^2+1/4*I/b^2*a^2*ln(-c*exp(2*I*(b*x+a))+I)-1/2*I/b^2*a^2*ln(1+I*e
xp(I*(b*x+a))*(I*c)^(1/2))-1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))
+1/2*I/b*ln(1+I*c*exp(2*I*(b*x+a)))*x*a-1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(I*
c)^(1/2))*x-1/2*I/b*a*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/8*x^2*Pi*csgn(
exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3+1/8*x^2*Pi*csgn(I*exp(2*I*(b
*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*
x+a))+1))+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3-
1/8*x^2*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+1/8*x^2*Pi*c
sgn((c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3-1/8*x^2*Pi*csgn((c*exp(2
*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*csgn(I*(c*exp(2*I*(b*x+a)
)-I)/(exp(2*I*(b*x+a))+1))*csgn((c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1)
)+1/8*x^2*Pi*csgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((c*ex
p(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/2*I*x^2*ln(exp(I*(b*x+a)))+1/4*
I*x^2*ln(I+c)-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1)
)*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+1/8*x^2*Pi*csgn(I/(ex
p(2*I*(b*x+a))+1))*csgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/
8*x^2*Pi*csgn(I*(c*exp(2*I*(b*x+a))-I))*csgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(
2*I*(b*x+a))+1))^2+1/4*I/b^2*ln(1+I*c*exp(2*I*(b*x+a)))*a^2-1/8*x^2*Pi*csgn
(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2-
1/8*x^2*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(I+c)
/(exp(2*I*(b*x+a))+1))^2
```

Maxima [B] time = 1.06539, size = 296, normalized size = 2.39

$$\frac{\frac{\left((bx+a)^2-2\,(bx+a)a\right)\arctan((ic-1)\tan(bx+a)+c)}{b}+\frac{2\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)+\left(6i\,(bx+a)^2-12i\,(bx+a)a\right)\arctan(c\cos(2\,bx+2\,a),-\cdots}{\cdots}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x + a)^2 - 2*(b*x + a)*a)*arctan((I*c - 1)*tan(b*x + a) + c)/b + 2
*(-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(-I*c*e^(2*I*b*x + 2
*I*a)) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a), -
c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b*x
 + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*polylog(
3, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b*(12*c + 12*I)))/b
```

Fricas [C] time = 1.94291, size = 760, normalized size = 6.13

$$\frac{2\,b^3x^3+3i\,b^2x^2\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right)+2\,a^3+6\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right)+6\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right)+3i\,a^2\log\left(\frac{2\,ce^{(i\,bx+\cdots}}{\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 3*I*b^2*x^2*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*
x + 2*I*a) - I)) + 2*a^3 + 6*b*x*dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) +
6*b*x*dilog(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 3*I*a^2*log(1/2*(2*c*e^(I*
b*x + I*a) + I*sqrt(-4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(I*b*x + I*a) - I*
sqrt(-4*I*c))/c) + (3*I*b^2*x^2 - 3*I*a^2)*log(1/2*sqrt(-4*I*c)*e^(I*b*x +
I*a) + 1) + (3*I*b^2*x^2 - 3*I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) +
 1) + 6*I*polylog(3, 1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 6*I*polylog(3, -1/
2*sqrt(-4*I*c)*e^(I*b*x + I*a)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int\frac{x^2}{ice^{2ia}e^{2ibx}+1}\,dx}{2}+\frac{ix^2\log\left(-ic+\frac{ic}{e^{2ia}e^{2ibx}+1}-\frac{ice^{ia}e^{ibx}}{e^{ia}e^{ibx}+e^{-ia}e^{-ibx}}+1-\frac{1}{e^{2ia}e^{2ibx}+1}+\frac{e^{ia}e^{ibx}}{e^{ia}e^{ibx}+e^{-ia}e^{-ibx}}\right)}{4}-\frac{ix^2\log\left(ic-\frac{ic}{e^{2ia}e^{2ibx}+1}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] b*Integral(x**2/(I*c*exp(2*I*a)*exp(2*I*b*x) + 1), x)/2 + I*x**2*log(-I*c +
 I*c/(exp(2*I*a)*exp(2*I*b*x) + 1) - I*c*exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(
I*b*x) + exp(-I*a)*exp(-I*b*x)) + 1 - 1/(exp(2*I*a)*exp(2*I*b*x) + 1) + exp
(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) + exp(-I*a)*exp(-I*b*x)))/4 - I*x**2*
log(I*c - I*c/(exp(2*I*a)*exp(2*I*b*x) + 1) + I*c*exp(I*a)*exp(I*b*x)/(exp(
I*a)*exp(I*b*x) + exp(-I*a)*exp(-I*b*x)) + 1 + 1/(exp(2*I*a)*exp(2*I*b*x) +
 1) - exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) + exp(-I*a)*exp(-I*b*x)))/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\arctan\left((i\,c-1)\tan\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((I*c - 1)*tan(b*x + a) + c), x)
```

3.58 $\int \tan^{-1}(c+(-1+ic)\tan(a+bx))\,dx$

Optimal. Leaf size=86

$$\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+x\tan^{-1}(c-(1-ic)\tan(a+bx))+\frac{bx^2}{2}$$

```
[Out] (b*x^2)/2 + x*ArcTan[c - (1 - I*c)*Tan[a + b*x]] + (I/2)*x*Log[1 + I*c*E^((
2*I)*a + (2*I)*b*x)] + PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rubi [A] time = 0.128898, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5163, 2184, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+x\tan^{-1}(c-(1-ic)\tan(a+bx))+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTan[c - (1 - I*c)*Tan[a + b*x]] + (I/2)*x*Log[1 + I*c*E^((
2*I)*a + (2*I)*b*x)] + PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 5163

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] - Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tan^{-1}(c+(-1+ic)\tan(a+bx))\,dx = x\tan^{-1}(c-(1-ic)\tan(a+bx))-(ib)\int \frac{x}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx$$
$$= \frac{bx^2}{2}+x\tan^{-1}(c-(1-ic)\tan(a+bx))-(bc)\int \frac{e^{2ia+2ibx}x}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx$$
$$= \frac{bx^2}{2}+x\tan^{-1}(c-(1-ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)-\frac{1}{2}i\int \log$$
$$= \frac{bx^2}{2}+x\tan^{-1}(c-(1-ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)-\frac{\text{Subst}\Big(}{}$$
$$= \frac{bx^2}{2}+x\tan^{-1}(c-(1-ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+\frac{\text{Li}_2\left(-ic\right.}{4}$$

Mathematica [B] time = 13.6178, size = 847, normalized size = 9.85

$$x\tan^{-1}(c+i(c+i)\tan(a+bx))+\frac{ix\Big(-2ibx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))}{((c-i)\cos(a+bx)+i(c+i)\sin(a+bx))\Big(-\frac{\log\left(\frac{1}{2}\sec(bx)(\cos(a)+i\sin(a))((ic+1)\cos(a+bx)-(c\right.}{\tan(bx)-i}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] x*ArcTan[c + I*(I + c)*Tan[a + b*x]] + (I*x*((-2*I)*b*x*Log[2*Cos[b*x]*(Cos
[b*x] - I*Sin[b*x])] + Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I + c)*Cos[a +
b*x] + I*(I + c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] - Log[(Sec[b*x]*
(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a + b*x]))/2]*Log
[1 + I*Tan[b*x]] + PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] + PolyLog[2, (Sec
[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/
(2*c)] - PolyLog[2, ((Cos[a] + I*Sin[a])*((I + c)*Cos[a] + (1 + I*c)*Sin[a]
)*(-I + Tan[b*x]))/2])*Sec[a + b*x]*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*x] + S
in[b*x]))/(((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x])*(-2*b*x + I*Log
[1 - (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a
+ b*x]))/(2*c)] + (I*(I + c)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Log[1 + I*
Tan[b*x]]))/((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]) + ((1 + I*c)*(
Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((-1 - I*c)*Cos[a
+ b*x] + (I + c)*Sin[a + b*x]) + (2*I)*b*x*Tan[b*x] - Log[1 - (Sec[b*x]*((I
 + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]*Ta
n[b*x] + Log[1 - I*Tan[b*x]]*Tan[b*x] - Log[1 + I*Tan[b*x]]*Tan[b*x] - (Log
[(Sec[b*x]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a + b*
x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + (Log[1 - ((Cos[a] + I*Sin[a])*((I + c
)*Cos[a] + (1 + I*c)*Sin[a])*(-I + Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]
) + (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I + c)*Cos[a + b*x] + I*(I + c)*S
in[a + b*x]))/(2*c)]*Sec[b*x]^2)/(I + Tan[b*x]))*(-I + Tan[a + b*x]))
```

Maple [B] time = 0.143, size = 1681, normalized size = 19.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] 1/4*I/(-1+I*c)/b/(I+c)*ln(-1/2*(-c-(-1+I*c)*tan(b*x+a)+I)/c)*ln(c-(-1+I*c)*
tan(b*x+a)+I)+1/2/(-1+I*c)/b/(I+c)*ln(-1/2*(-c-(-1+I*c)*tan(b*x+a)+I)/c)*ln
(c-(-1+I*c)*tan(b*x+a)+I)*c-1/2/(-1+I*c)/b/(I+c)*ln((-c-(-1+I*c)*tan(b*x+a)
-I)/(-2*I-2*c))*ln(c-(-1+I*c)*tan(b*x+a)+I)*c+1/2/(-1+I*c)/b/(I+c)*ln(-1/2*
I*(c+(-1+I*c)*tan(b*x+a)+I))*ln(-1/2*I*(-c-(-1+I*c)*tan(b*x+a)+I))*c+1/(-1+
I*c)/b*arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c+(-1+I*c)*tan(b*x+a)+I)*
c^2-1/(-1+I*c)/b*arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(
b*x+a)+I)*c^2-1/2/(-1+I*c)/b/(I+c)*ln(-1/2*I*(-c-(-1+I*c)*tan(b*x+a)+I))*ln
(c+(-1+I*c)*tan(b*x+a)+I)*c-1/4*I/(-1+I*c)/b/(I+c)*ln((-c-(-1+I*c)*tan(b*x+
a)-I)/(-2*I-2*c))*ln(c-(-1+I*c)*tan(b*x+a)+I)+1/4*I/(-1+I*c)/b/(I+c)*ln(-1/
2*I*(c+(-1+I*c)*tan(b*x+a)+I))*ln(-1/2*I*(-c-(-1+I*c)*tan(b*x+a)+I))-1/4*I/
(-1+I*c)/b/(I+c)*dilog(-1/2*I*(c+(-1+I*c)*tan(b*x+a)+I))*c^2-1/8*I/(-1+I*c)
/b/(I+c)*ln(c+(-1+I*c)*tan(b*x+a)+I)^2*c^2-1/4*I/(-1+I*c)/b/(I+c)*dilog(-1/
2*(-c-(-1+I*c)*tan(b*x+a)+I)/c)*c^2+1/4*I/(-1+I*c)/b/(I+c)*dilog((-c-(-1+I*
c)*tan(b*x+a)-I)/(-2*I-2*c))*c^2-1/4*I/(-1+I*c)/b/(I+c)*ln(-1/2*I*(-c-(-1+I
*c)*tan(b*x+a)+I))*ln(c+(-1+I*c)*tan(b*x+a)+I)-2*I/(-1+I*c)/b*arctan(c+(-1+
I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)+I)*c-1/4*I/(-1+I*c)/b/(
I+c)*ln(-1/2*I*(c+(-1+I*c)*tan(b*x+a)+I))*ln(-1/2*I*(-c-(-1+I*c)*tan(b*x+a)
+I))*c^2+1/4*I/(-1+I*c)/b/(I+c)*ln(-1/2*I*(-c-(-1+I*c)*tan(b*x+a)+I))*ln(c+
(-1+I*c)*tan(b*x+a)+I)*c^2-1/4*I/(-1+I*c)/b/(I+c)*ln(-1/2*(-c-(-1+I*c)*tan(
b*x+a)+I)/c)*ln(c-(-1+I*c)*tan(b*x+a)+I)*c^2+1/4*I/(-1+I*c)/b/(I+c)*ln((-c-
(-1+I*c)*tan(b*x+a)-I)/(-2*I-2*c))*ln(c-(-1+I*c)*tan(b*x+a)+I)*c^2+2*I/(-1+
I*c)/b*arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c+(-1+I*c)*tan(b*x+a)+I)*
c+1/4*I/(-1+I*c)/b/(I+c)*dilog(-1/2*(-c-(-1+I*c)*tan(b*x+a)+I)/c)-1/4*I/(-1
+I*c)/b/(I+c)*dilog((-c-(-1+I*c)*tan(b*x+a)-I)/(-2*I-2*c))-1/(-1+I*c)/b*arc
tan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c+(-1+I*c)*tan(b*x+a)+I)+1/(-1+I*c)
/b*arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)+I)+1/2/
(-1+I*c)/b/(I+c)*dilog(-1/2*I*(c+(-1+I*c)*tan(b*x+a)+I))*c+1/4/(-1+I*c)/b/(
I+c)*ln(c+(-1+I*c)*tan(b*x+a)+I)^2*c+1/2/(-1+I*c)/b/(I+c)*dilog(-1/2*(-c-(-
1+I*c)*tan(b*x+a)+I)/c)*c-1/2/(-1+I*c)/b/(I+c)*dilog((-c-(-1+I*c)*tan(b*x+a
)-I)/(-2*I-2*c))*c+1/8*I/(-1+I*c)/b/(I+c)*ln(c+(-1+I*c)*tan(b*x+a)+I)^2+1/4
*I/(-1+I*c)/b/(I+c)*dilog(-1/2*I*(c+(-1+I*c)*tan(b*x+a)+I))
```

Maxima [B] time = 1.58137, size = 605, normalized size = 7.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((I*c - 1)*(4*I*(b*x + a)*log((2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x +
 a) + 2*I)/(2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x + a) - 4*c - 2*I))/(I*c -
 1) + I*(4*(b*x + a)*(log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) + 2*c + I
) - log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) - I)) + I*log(-I*c^2 + (c^2
 + 2*I*c - 1)*tan(b*x + a) + 2*c + I)^2 - 2*I*log(-I*c^2 + (c^2 + 2*I*c - 1
)*tan(b*x + a) - I)*log(1/2*(c + I)*tan(b*x + a) - 1/2*I*c + 1/2) + 2*I*log
(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) - I)*log(-1/2*((I*c - 1)*tan(b*x +
 a) + c - I)/c + 1) - 2*I*log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) + 2*c
 + I)*log(-1/2*I*tan(b*x + a) + 1/2) - 2*I*dilog(-1/2*(c + I)*tan(b*x + a)
+ 1/2*I*c + 1/2) + 2*I*dilog(1/2*((I*c - 1)*tan(b*x + a) + c - I)/c) - 2*I*
dilog(1/2*I*tan(b*x + a) + 1/2))/(I*c - 1)) - 8*(b*x + a)*arctan((I*c - 1)*
tan(b*x + a) + c) + 4*(-I*b*x - I*a)*log((2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan
(b*x + a) + 2*I)/(2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x + a) - 4*c - 2*I)))
```

```
/b
```

Fricas [B] time = 1.72881, size = 549, normalized size = 6.38

$$\frac{b^2x^2 + i\,bx\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) - a^2 + (i\,bx + i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)} + 1\right) + (i\,bx + i\,a)\log\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)} + 1\right) - \cdots}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + I*b*x*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a
) - I)) - a^2 + (I*b*x + I*a)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) + (
I*b*x + I*a)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - I*a*log(1/2*(2*c*
e^(I*b*x + I*a) + I*sqrt(-4*I*c))/c) - I*a*log(1/2*(2*c*e^(I*b*x + I*a) - I
*sqrt(-4*I*c))/c) + dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + dilog(-1/2*sq
rt(-4*I*c)*e^(I*b*x + I*a)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{x}{ice^{2ia}e^{2ibx}+1}\,dx + \frac{ix\log\left(-ic + \frac{ic}{e^{2ia}e^{2ibx}+1} - \frac{ice^{ia}e^{ibx}}{e^{ia}e^{ibx}+e^{-ia}e^{-ibx}} + 1 - \frac{1}{e^{2ia}e^{2ibx}+1} + \frac{e^{ia}e^{ibx}}{e^{ia}e^{ibx}+e^{-ia}e^{-ibx}}\right)}{2} - \frac{ix\log\left(ic - \frac{ic}{e^{2ia}e^{2ibx}+1} - \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] b*Integral(x/(I*c*exp(2*I*a)*exp(2*I*b*x) + 1), x) + I*x*log(-I*c + I*c/(ex
p(2*I*a)*exp(2*I*b*x) + 1) - I*c*exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) +
 exp(-I*a)*exp(-I*b*x)) + 1 - 1/(exp(2*I*a)*exp(2*I*b*x) + 1) + exp(I*a)*ex
p(I*b*x)/(exp(I*a)*exp(I*b*x) + exp(-I*a)*exp(-I*b*x)))/2 - I*x*log(I*c - I
*c/(exp(2*I*a)*exp(2*I*b*x) + 1) + I*c*exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*
b*x) + exp(-I*a)*exp(-I*b*x)) + 1 + 1/(exp(2*I*a)*exp(2*I*b*x) + 1) - exp(I
*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) + exp(-I*a)*exp(-I*b*x)))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\arctan\left((i\,c-1)\tan\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((I*c - 1)*tan(b*x + a) + c), x)
```

3.59 $\int \frac{\tan^{-1}(c+(-1+ic)\tan(a+bx))}{x} dx$

Optimal. Leaf size=23

$$\text{CannotIntegrate}\left(\frac{\tan^{-1}(c+(-1+ic)\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x, x]
```

Rubi [A] time = 0.136324, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(c+(-1+ic)\tan(a+bx))}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+(-1+ic)\tan(a+bx))}{x} dx = \int \frac{\tan^{-1}(c+(-1+ic)\tan(a+bx))}{x} dx$$

Mathematica [A] time = 0.931982, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(c+(-1+ic)\tan(a+bx))}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x, x]
```

Maple [A] time = 0.428, size = 0, normalized size = 0.

$$\int \frac{\arctan(c+(-1+ic)\tan(bx+a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

```
[Out] int(arctan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i\,\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right)}{2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a) - I)
)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left((i\,c-1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((I*c - 1)*tan(b*x + a) + c)/x, x)
```

3.60 $\int \tan^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=16

$$-\frac{\tan^{-1}(\cot(a+bx))^2}{2b}$$

```
[Out] -ArcTan[Cot[a + b*x]]^2/(2*b)
```

Rubi [A] time = 0.0030392, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2157, 30}

$$-\frac{\tan^{-1}(\cot(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Cot[a + b*x]],x]
```

```
[Out] -ArcTan[Cot[a + b*x]]^2/(2*b)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(\cot(a+bx))\,dx &= -\frac{\text{Subst}\left(\int x\,dx, x, \tan^{-1}(\cot(a+bx))\right)}{b}\\ &= -\frac{\tan^{-1}(\cot(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.0069552, size = 18, normalized size = 1.12

$$x\tan^{-1}(\cot(a+bx))+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Cot[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTan[Cot[a + b*x]]
```

Maple [B] time = 0.043, size = 51, normalized size = 3.2

$$\frac{\pi x}{2}-\frac{1}{b}\left(-\left(\frac{\pi}{2}-\operatorname{arccot}\left(\cot\left(bx+a\right)\right)\right)\operatorname{arccot}\left(\cot\left(bx+a\right)\right)-\frac{1}{2}\left(\frac{\pi}{2}-\operatorname{arccot}\left(\cot\left(bx+a\right)\right)\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*Pi-arccot(cot(b*x+a)),x)
```

```
[Out] 1/2*Pi*x-1/b*(-(1/2*Pi-arccot(cot(b*x+a)))*arccot(cot(b*x+a))-1/2*(1/2*Pi-a
rccot(cot(b*x+a)))^2)
```

Maxima [A] time = 0.986348, size = 20, normalized size = 1.25

$$-\frac{1}{2}\,bx^{2}+\frac{1}{2}\,\pi x-ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*b*x^2 + 1/2*pi*x - a*x
```

Fricas [A] time = 1.69239, size = 42, normalized size = 2.62

$$-\frac{1}{2}\,bx^{2}+\frac{1}{2}\,(\pi-2\,a)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/2*b*x^2 + 1/2*(pi - 2*a)*x
```

Sympy [A] time = 0.165284, size = 24, normalized size = 1.5

$$\frac{\pi x}{2}-\begin{cases}\frac{\operatorname{acot}^{2}\left(\cot\left(a+bx\right)\right)}{2b} & \text{for}\ b\neq 0\\ x\operatorname{acot}\left(\cot\left(a\right)\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-acot(cot(b*x+a)),x)
```

```
[Out] pi*x/2 - Piecewise((acot(cot(a + b*x))**2/(2*b), Ne(b, 0)), (x*acot(cot(a))
, True))
```

Giac [A] time = 1.0845, size = 20, normalized size = 1.25

$$-\frac{1}{2}\,bx^{2}+\frac{1}{2}\,\pi x-ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/2*b*x^2 + 1/2*pi*x - a*x
```

3.61 $\int x^2 \tan^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=399

$$\frac{ix\text{PolyLog}\left(3,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b^2}-\frac{ix\text{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{8b^3}+\frac{\text{PolyLog}\left(4,\frac{(c+i(d+1}{c+i(}\right.}{8b^3}$$

```
[Out] (x^3*ArcTan[c + d*Cot[a + b*x]])/3 + (I/6)*x^3*Log[1 - ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)] - (I/6)*x^3*Log[1 - ((c + I*(1 + d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 - d))] + (x^2*PolyLog[2, ((1 + I*c - d)*E^(
(2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(4*b) - (x^2*PolyLog[2, ((c + I*(1 +
d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(4*b) + ((I/4)*x*PolyLog[3,
((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/b^2 - ((I/4)*x*Poly
Log[3, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/b^2 - Po
lyLog[4, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)]/(8*b^3) + P
olyLog[4, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))]/(8*b^3
)
```

Rubi [A] time = 0.511481, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5177, 2190, 2531, 6609, 2282, 6589}

$$\frac{ix\text{PolyLog}\left(3,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b^2}-\frac{ix\text{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{8b^3}+\frac{\text{PolyLog}\left(4,\frac{(c+i(d+1}{c+i(}\right.}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Cot[a + b*x]])/3 + (I/6)*x^3*Log[1 - ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)] - (I/6)*x^3*Log[1 - ((c + I*(1 + d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 - d))] + (x^2*PolyLog[2, ((1 + I*c - d)*E^(
(2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(4*b) - (x^2*PolyLog[2, ((c + I*(1 +
d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(4*b) + ((I/4)*x*PolyLog[3,
((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/b^2 - ((I/4)*x*Poly
Log[3, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/b^2 - Po
lyLog[4, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)]/(8*b^3) + P
olyLog[4, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))]/(8*b^3
)
```

Rule 5177

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Cot[a + b*x]])/(f*(m +
 1)), x] + (Dist[(b*(1 + I*c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2
*I*a + 2*I*b*x))/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x)), x], x]
- Dist[(b*(1 - I*c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*a + 2*I
*b*x))/(1 - I*c - d - (1 - I*c + d)*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\tan^{-1}(c+d\cot(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c+d\cot(a+bx))+\frac{1}{3}(b(1+ic-d))\int\frac{e^{2ia+2ibx}x^3}{1+ic+d+(-1-ic+d)e^{2ia+2ibx}}$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-\frac{1}{6}ix^3\log\left(1-\frac{(c}{}\right.$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-\frac{1}{6}ix^3\log\left(1-\frac{(c}{}\right.$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-\frac{1}{6}ix^3\log\left(1-\frac{(c}{}\right.$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-\frac{1}{6}ix^3\log\left(1-\frac{(c}{}\right.$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-\frac{1}{6}ix^3\log\left(1-\frac{(c}{}\right.$$

Mathematica [A] time = 0.856202, size = 359, normalized size = 0.9

$$\frac{1}{3}x^3\tan^{-1}(d\cot(a+bx)+c)+\frac{6b^2x^2\text{PolyLog}\left(2,\frac{(c+i(d-1))e^{2i(a+bx)}}{c-i(d+1)}\right)-6b^2x^2\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2i(a+bx)}}{c-id+i}\right)+6ibx\text{PolyLog}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Cot[a + b*x]])/3 + ((4*I)*b^3*x^3*Log[1 - ((c + I*(-1 + d
))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - (4*I)*b^3*x^3*Log[1 - ((c + I*(1
 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] + 6*b^2*x^2*PolyLog[2, ((c + I*(
-1 + d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - 6*b^2*x^2*PolyLog[2, ((c +
 I*(1 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] + (6*I)*b*x*PolyLog[3, ((c
+ I*(-1 + d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - (6*I)*b*x*PolyLog[3,
((c + I*(1 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] - 3*PolyLog[4, ((c + I
*(-1 + d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] + 3*PolyLog[4, ((c + I*(1
+ d))*E^((2*I)*(a + b*x)))/(I + c - I*d)])/(24*b^3)
```

Maple [C] time = 6.152, size = 7924, normalized size = 19.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+d*cot(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/6*x^3*arctan2(-c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1
)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1) - 1/6*x^3*arctan2(-c*cos(2
*b*x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) - c*s
in(2*b*x + 2*a) - d - 1) + 4*b*d*integrate(1/3*(2*(c^2 + d^2 + 1)*x^3*cos(2
*b*x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^3*sin(2*b*
x + 2*a)^2 - (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x +
2*a) + (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*
cos(2*b*x + 2*a) - (c^2 - d^2 + 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/
(c^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*co
s(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x +
2*a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(
c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^
4 + d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 - 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x +
 2*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*
(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 + c)*d + 2*(
c*d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x +
 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [C] time = 3.21748, size = 4058, normalized size = 10.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(16*b^3*x^3*arctan(d*cot(b*x + a) + c) + 6*b^2*x^2*dilog(-(c^2 + d^2 -
 (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*
sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + 6*b^2*x^2*dilog(-(
c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I
*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - 6*b^2*x^
2*dilog(-(c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2
+ 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1)
 - 6*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a)
 + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d
 + 1) + 1) - 4*I*a^3*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*d +
 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) + 1
/2) + 4*I*a^3*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*d + 1)*cos
(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) + 4
*I*a^3*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x
 + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) - 4*I*a^3
*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a
) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) + 6*I*b*x*polyl
og(3, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2
+ I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) - 6*I*b*x*polylog(3, ((c^2 +
2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x
 + 2*a))/(c^2 + d^2 - 2*d + 1)) - 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^2
+ 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2
 + d^2 + 2*d + 1)) + 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*
x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d
+ 1)) + (4*I*b^3*x^3 + 4*I*a^3)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*
cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)
/(c^2 + d^2 + 2*d + 1)) + (-4*I*b^3*x^3 - 4*I*a^3)*log((c^2 + d^2 - (c^2 -
2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x
 + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1)) + (-4*I*b^3*x^3 - 4*I*a^3)*log((c
^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I
*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) + (4*I*b^3*x^3
 + 4*I*a^3)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (
I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1
)) - 3*polylog(4, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*
c*d - I*d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) + 3*polylog(4, ((
c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin
(2*b*x + 2*a))/(c^2 + d^2 - 2*d + 1)) - 3*polylog(4, ((c^2 - 2*I*c*d - d^2
+ 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2
 + d^2 + 2*d + 1)) + 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*
a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d + 1)))
/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \arctan \left(d\cot \left(bx+a\right)+c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(d*cot(b*x + a) + c), x)
```

3.62 $\int x \tan^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=303

$$\frac{i\text{PolyLog}\left(3,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{8b^2}-\frac{i\text{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^2}+\frac{x\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b}-\frac{x\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2i}}{c+i(1-d}\right.}{4b}$$

```
[Out] (x^2*ArcTan[c + d*Cot[a + b*x]])/2 + (I/4)*x^2*Log[1 - ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)] - (I/4)*x^2*Log[1 - ((c + I*(1 + d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 - d))] + (x*PolyLog[2, ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(4*b) - (x*PolyLog[2, ((c + I*(1 + d))*
E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(4*b) + ((I/8)*PolyLog[3, ((1 +
I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/b^2 - ((I/8)*PolyLog[3, (
(c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/b^2
```

Rubi [A] time = 0.40045, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5177, 2190, 2531, 2282, 6589}

$$\frac{i\text{PolyLog}\left(3,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{8b^2}-\frac{i\text{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^2}+\frac{x\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b}-\frac{x\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2i}}{c+i(1-d}\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Cot[a + b*x]])/2 + (I/4)*x^2*Log[1 - ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)] - (I/4)*x^2*Log[1 - ((c + I*(1 + d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 - d))] + (x*PolyLog[2, ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(4*b) - (x*PolyLog[2, ((c + I*(1 + d))*
E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(4*b) + ((I/8)*PolyLog[3, ((1 +
I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/b^2 - ((I/8)*PolyLog[3, (
(c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/b^2
```

Rule 5177

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Cot[a + b*x]])/(f*(m +
 1)), x] + (Dist[(b*(1 + I*c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2
*I*a + 2*I*b*x))/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x)), x], x]
- Dist[(b*(1 - I*c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*a + 2*I
*b*x))/(1 - I*c - d - (1 - I*c + d)*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+d\cot(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{2}(b(1+ic-d))\int\frac{e^{2ia+2ibx}x^2}{1+ic+d+(-1-ic+d)e^{2ia+2ib}$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{4}ix^2\log\left(1-\right.$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{4}ix^2\log\left(1-\right.$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{4}ix^2\log\left(1-\right.$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{4}ix^2\log\left(1-\right.$$

Mathematica [A] time = 0.538076, size = 270, normalized size = 0.89

$$\frac{1}{2}x^2\tan^{-1}(d\cot(a+bx)+c) + \frac{i\left(-2ibx\text{PolyLog}\left(2,\frac{(c+i(d-1))e^{2i(a+bx)}}{c-i(d+1)}\right)+2ibx\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2i(a+bx)}}{c-id+i}\right)+\text{PolyLog}\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Cot[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 - ((c + I*(-1
+ d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - 2*b^2*x^2*Log[1 - ((c + I*(1
+ d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] - (2*I)*b*x*PolyLog[2, ((c + I*(-
1 + d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] + (2*I)*b*x*PolyLog[2, ((c +
I*(1 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] + PolyLog[3, ((c + I*(-1 + d
))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - PolyLog[3, ((c + I*(1 + d))*E^((
2*I)*(a + b*x)))/(I + c - I*d)]))/b^2
```

Maple [C] time = 24.825, size = 7556, normalized size = 24.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+d*cot(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*x^2*arctan2(-c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1
)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1) - 1/4*x^2*arctan2(-c*cos(2
*b*x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) - c*s
in(2*b*x + 2*a) - d - 1) + 2*b*d*integrate((2*(c^2 + d^2 + 1)*x^2*cos(2*b*x
 + 2*a)^2 + 2*c*d*x^2*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^2*sin(2*b*x +
2*a)^2 - (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a)
 + (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(
2*b*x + 2*a) - (c^2 - d^2 + 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4
 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(4*
b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*a)
^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4
+ d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4 +
d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 - 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a
) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4
 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 + c)*d + 2*(c*d^
3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2*a
))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [C] time = 2.88313, size = 3298, normalized size = 10.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*b^2*x^2*arctan(d*cot(b*x + a) + c) + 2*b*x*dilog(-(c^2 + d^2 - (c^2
 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2
*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + 2*b*x*dilog(-(c^2 + d^2
 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)
*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - 2*b*x*dilog(-(c^2
 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d
^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) - 2*b*x*dilo
g(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d
 - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) + 2*I*
a^2*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2
*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) - 2*I*a^2*log
(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1
/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) - 2*I*a^2*log(-1/2*c
^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*
```

```
c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) + 2*I*a^2*log(-1/2*c^2 + I
*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 +
I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) + (2*I*b^2*x^2 - 2*I*a^2)*log((c
^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I
*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1)) + (-2*I*b^2*x^
2 + 2*I*a^2)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) +
(I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d +
1)) + (-2*I*b^2*x^2 + 2*I*a^2)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*c
os(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/
(c^2 + d^2 - 2*d + 1)) + (2*I*b^2*x^2 - 2*I*a^2)*log((c^2 + d^2 - (c^2 - 2*
I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x +
 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) + I*polylog(3, ((c^2 + 2*I*c*d - d^
2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a))/(c^
2 + d^2 + 2*d + 1)) - I*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2
*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d + 1))
- I*polylog(3, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*
d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) + I*polylog(3, ((c^
2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(
2*b*x + 2*a))/(c^2 + d^2 - 2*d + 1)))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \arctan\left(d \cot\left(bx + a\right) + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(d*cot(b*x + a) + c), x)
```

3.63 $\int \tan^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=198

$$\frac{\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b}-\frac{\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}+\frac{1}{2}ix\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)-\frac{1}{2}ix\log\left(1-\frac{(c+i(d+}{c+i}\right.$$

```
[Out] x*ArcTan[c + d*Cot[a + b*x]] + (I/2)*x*Log[1 - ((1 + I*c - d)*E^((2*I)*a +
(2*I)*b*x))/(1 + I*c + d)] - (I/2)*x*Log[1 - ((c + I*(1 + d))*E^((2*I)*a +
(2*I)*b*x))/(c + I*(1 - d))] + PolyLog[2, ((1 + I*c - d)*E^((2*I)*a + (2*I)
*b*x))/(1 + I*c + d)]/(4*b) - PolyLog[2, ((c + I*(1 + d))*E^((2*I)*a + (2*I
)*b*x))/(c + I*(1 - d))]/(4*b)
```

Rubi [A] time = 0.249962, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5169, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b}-\frac{\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}+\frac{1}{2}ix\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)-\frac{1}{2}ix\log\left(1-\frac{(c+i(d+}{c+i}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + d*Cot[a + b*x]],x]
```


```
[Out] x*ArcTan[c + d*Cot[a + b*x]] + (I/2)*x*Log[1 - ((1 + I*c - d)*E^((2*I)*a +
(2*I)*b*x))/(1 + I*c + d)] - (I/2)*x*Log[1 - ((c + I*(1 + d))*E^((2*I)*a +
(2*I)*b*x))/(c + I*(1 - d))] + PolyLog[2, ((1 + I*c - d)*E^((2*I)*a + (2*I)
*b*x))/(1 + I*c + d)]/(4*b) - PolyLog[2, ((c + I*(1 + d))*E^((2*I)*a + (2*I
)*b*x))/(c + I*(1 - d))]/(4*b)
```


Rule 5169

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcT
an[c + d*Cot[a + b*x]], x] + (Dist[b*(1 + I*c - d), Int[(x*E^(2*I*a + 2*I*b
*x))/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x)), x], x] - Dist[b*(1
- I*c + d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 - I*c - d - (1 - I*c + d)*E^(2*I
*a + 2*I*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tan^{-1}(c+d\cot(a+bx))\,dx = x\tan^{-1}(c+d\cot(a+bx)) + (b(1+ic-d))\int \frac{e^{2ia+2ibx}x}{1+ic+d+(-1-ic+d)e^{2ia+2ibx}}\,dx -$$

$$= x\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{2}ix\log\left(1-\frac{(c+i(}{c}\right.$$

$$= x\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{2}ix\log\left(1-\frac{(c+i(}{c}\right.$$

$$= x\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{2}ix\log\left(1-\frac{(c+i(}{c}\right.$$

Mathematica [B] time = 21.6845, size = 1648, normalized size = 8.32

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Cot[a + b*x]] + (d*(4*a*Sqrt[-d^2]*ArcTan[(c*d + Tan[a + b*x
] + c^2*Tan[a + b*x])/d] + I*d*Log[1 + I*Tan[a + b*x]]*Log[(c*d - Sqrt[-d^2
] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d - Sqrt[-d^2])] + I*d*
Log[1 - I*Tan[a + b*x]]*Log[(c*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a +
b*x])/(-I - I*c^2 + c*d + Sqrt[-d^2])] - I*d*Log[1 + I*Tan[a + b*x]]*Log[(c
*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d + Sqrt[
-d^2])] - I*d*Log[1 - I*Tan[a + b*x]]*Log[(-(c*d) + Sqrt[-d^2] - (1 + c^2)*
Tan[a + b*x])/(I + I*c^2 - c*d + Sqrt[-d^2])] - I*d*PolyLog[2, ((1 + c^2)*(
1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d - I*Sqrt[-d^2])] + I*d*PolyLog[2, ((1
 + c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d + I*Sqrt[-d^2])] - I*d*PolyL
og[2, ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d - I*Sqrt[-d^2])] +
I*d*PolyLog[2, ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d + I*Sqrt[-
d^2])])*((2*a)/(b*(-1 - c^2 - d^2 + Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)]
 - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)])) - (2*(a + b*x))/(b*(-1 -
 c^2 - d^2 + Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)]
 - 2*c*d*Sin[2*(a + b*x)]))))/((d*Log[1 - ((1 + c^2)*(1 - I*Tan[a + b*x]))/
(1 + c^2 + I*c*d - I*Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) - (d
*Log[1 - ((1 + c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d + I*Sqrt[-d^2])]
*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) + (d*Log[(c*d + Sqrt[-d^2] + Tan[a +
b*x] + c^2*Tan[a + b*x])/(-I - I*c^2 + c*d + Sqrt[-d^2])]*Sec[a + b*x]^2)/(
1 - I*Tan[a + b*x]) - (d*Log[(-(c*d) + Sqrt[-d^2] - (1 + c^2)*Tan[a + b*x])
/(I + I*c^2 - c*d + Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) - (d*
Log[1 - ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d - I*Sqrt[-d^2])]*
Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (d*Log[1 - ((1 + c^2)*(1 + I*Tan[a +
 b*x]))/(1 + c^2 - I*c*d + I*Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*
x]) - (d*Log[(c*d - Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^
2 + c*d - Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (d*Log[(c*d +
 Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d + Sqrt[-d^2
])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (I*d*Log[1 + I*Tan[a + b*x]]*(Se
c[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(c*d - Sqrt[-d^2] + Tan[a + b*x] + c^2*
Tan[a + b*x]) + (I*d*Log[1 - I*Tan[a + b*x]]*(Sec[a + b*x]^2 + c^2*Sec[a +
```

```
b*x]^2))/(c*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x]) - (I*d*Log[1
+ I*Tan[a + b*x]]*(Sec[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(c*d + Sqrt[-d^2]
+ Tan[a + b*x] + c^2*Tan[a + b*x]) + (I*(1 + c^2)*d*Log[1 - I*Tan[a + b*x]]
*Sec[a + b*x]^2)/(-(c*d) + Sqrt[-d^2] - (1 + c^2)*Tan[a + b*x]) + (4*a*Sqrt
[-d^2]*(Sec[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(d*(1 + (c*d + Tan[a + b*x] +
 c^2*Tan[a + b*x])^2/d^2)))
```

Maple [B] time = 0.38, size = 1159, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*cot(b*x+a)),x)
```

```
[Out] -1/2/b*arctan(c+d*cot(b*x+a))*Pi+1/b*arctan(c+d*cot(b*x+a))*arccot(cot(b*x+
a))+1/b*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)*arctan((c+d*cot(b*x+a))/d-c/d)
-1/2*I*d/b*ln(1-(I-I*d+c)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c+d*
cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)/
(1+I*c+d)-1/2*I/b*ln(1-(I-I*d+c)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d
*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*cot(b*x+a))/d-c
/d)+c)/(1+I*c+d)-1/2*I/b/(-I-I*d+c)*ln(1-(I-I*d+c)*(1+I*(d*((c+d*cot(b*x+a)
)/d-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c
+d*cot(b*x+a))/d-c/d)+c)*c-1/2*d/b*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)^2/(
1+I*c+d)-1/4*d/b*polylog(2,(I-I*d+c)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2
/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)-1/2/b*arctan(d*(
(c+d*cot(b*x+a))/d-c/d)+c)^2/(1+I*c+d)-1/2/b/(-I-I*d+c)*arctan(d*((c+d*cot(
b*x+a))/d-c/d)+c)^2*c-1/4/b*polylog(2,(I-I*d+c)*(1+I*(d*((c+d*cot(b*x+a))/d
-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)-1/4/b
/(-I-I*d+c)*polylog(2,(I-I*d+c)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*
((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c+1/2*I/b*arctan(d*((c+d*cot(b*
x+a))/d-c/d)+c)*ln(1-(I+I*d+c)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*(
(c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c))+1/2/b*arctan(d*((c+d*cot(b*x+a)
)/d-c/d)+c)^2+1/4/b*polylog(2,(I+I*d+c)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c)
)^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c))
```

Maxima [B] time = 1.91243, size = 710, normalized size = 3.59

$$-\frac{d\left(\frac{8\,(bx+a)\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)}{d}-\frac{8\,(bx+a)\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)-4\arctan\left(cd+\left(c^2+1\right)\tan(bx+a),d\right)\arctan\left(\frac{cd+\left(c^2+d+1\right)\tan(bx+a)}{c^2+d^2+2d+1},-\frac{cd\tan(b}{c^2+}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(d*(8*(b*x + a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d)/d - (8*(b*x +
 a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d) - 4*arctan2(c*d + (c^2 + 1)*ta
n(b*x + a), d)*arctan2((c*d + (c^2 + d + 1)*tan(b*x + a))/(c^2 + d^2 + 2*d
+ 1), -(c*d*tan(b*x + a) - c^2 - d - 1)/(c^2 + d^2 + 2*d + 1)) + 4*arctan2(
c*d + (c^2 + 1)*tan(b*x + a), d)*arctan2(-(c*d + (c^2 - d + 1)*tan(b*x + a)
)/(c^2 + d^2 - 2*d + 1), -(c*d*tan(b*x + a) - c^2 + d - 1)/(c^2 + d^2 - 2*d
 + 1)) - (log(((c^2 + 1)*tan(b*x + a)^2 + c^2 + 1)/(c^2 + d^2 + 2*d + 1)) -
```

```
 log(((c^2 + 1)*tan(b*x + a)^2 + c^2 + 1)/(c^2 + d^2 - 2*d + 1)))*log((c^2
+ 1)*d^2 + 2*(c^3 + c)*d*tan(b*x + a) + (c^4 + 2*c^2 + 1)*tan(b*x + a)^2) -
 2*dilog(((I*c - 1)*tan(b*x + a) + I*d)/(c + I*d + I)) + 2*dilog(((I*c + 1)
*tan(b*x + a) + I*d)/(c + I*d - I)) + 2*dilog(-((I*c - 1)*tan(b*x + a) + I*
d)/(c - I*d + I)) - 2*dilog(-((I*c + 1)*tan(b*x + a) + I*d)/(c - I*d - I)))
/d) - 8*(b*x + a)*arctan(c + d/tan(b*x + a)) - 8*(b*x + a)*arctan((c*d + (c
^2 + 1)*tan(b*x + a))/d))/b
```

Fricas [B] time = 2.96682, size = 2508, normalized size = 12.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*arctan(d*cot(b*x + a) + c) - 2*I*a*log(1/2*c^2 + I*c*d - 1/2*d^2
 - 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d
+ I)*sin(2*b*x + 2*a) + 1/2) + 2*I*a*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c
^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(
2*b*x + 2*a) + 1/2) + 2*I*a*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2
 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x +
2*a) - 1/2) - 2*I*a*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d +
 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1
/2) + (2*I*b*x + 2*I*a)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*
x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 +
d^2 + 2*d + 1)) + (-2*I*b*x - 2*I*a)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2
+ 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) + 2*d
+ 1)/(c^2 + d^2 + 2*d + 1)) + (-2*I*b*x - 2*I*a)*log((c^2 + d^2 - (c^2 + 2*
I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x
+ 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) + (2*I*b*x + 2*I*a)*log((c^2 + d^2
 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)
*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) + dilog(-(c^2 + d^2 - (
c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*si
n(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + dilog(-(c^2 + d^2 -
(c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*si
n(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - dilog(-(c^2 + d^2 -
(c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*s
in(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) - dilog(-(c^2 + d^2 -
 (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*s
in(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \arctan\left(d\cot\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(d*cot(b*x + a) + c), x)
```

3.64 $\int \frac{\tan^{-1}(c+d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\tan^{-1}(d\cot(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTan[c + d*Cot[a + b*x]]/x, x]
```

Rubi [A] time = 0.134324, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + d*Cot[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + d*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+d\cot(a+bx))}{x}\,dx=\int \frac{\tan^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Mathematica [A] time = 4.49295, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + d*Cot[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTan[c + d*Cot[a + b*x]]/x, x]
```

Maple [A] time = 0.405, size = 0, normalized size = 0.

$$\int \frac{\arctan\left(c+d\cot\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*cot(b*x+a))/x,x)
```

```
[Out] int(arctan(c+d*cot(b*x+a))/x,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(d\cot\left(bx+a\right)+c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(d*cot(b*x + a) + c)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*cot(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(d\cot\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(d*cot(b*x + a) + c)/x, x)
```

3.65 $\int x^2 \tan^{-1}(c+(1-ic)\cot(a+bx))\,dx$

Optimal. Leaf size=154

$$\frac{ix\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3}+\frac{x^2\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{3}x^3\text{ta}$$

```
[Out] (b*x^4)/12 + (x^3*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/3 + (I/6)*x^3*Log[1 -
 I*c*E^((2*I)*a + (2*I)*b*x)] + (x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) + ((I/4)*x*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2 - PolyLog[
4, I*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rubi [A] time = 0.255871, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5173, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{ix\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3}+\frac{x^2\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{3}x^3\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```


```
[Out] (b*x^4)/12 + (x^3*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/3 + (I/6)*x^3*Log[1 -
 I*c*E^((2*I)*a + (2*I)*b*x)] + (x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) + ((I/4)*x*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2 - PolyLog[
4, I*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```


Rule 5173

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Cot[a + b*x]])/(f*(m +
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\tan^{-1}(c+(1-ic)\cot(a+bx))\,dx=\frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{3}(ib)\int\frac{x^3}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx$$
$$=\frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{3}(bc)\int\frac{e^{2ia+2ibx}x^3}{-i(1-ic)+c-ce^{2ia+2ibx}}$$
$$=\frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)-\frac{1}{2}i\int x^2$$
$$=\frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{x^2\mathrm{Li}_2(}{}$$
$$=\frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{x^2\mathrm{Li}_2(}{}$$
$$=\frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{x^2\mathrm{Li}_2(}{}$$
$$=\frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{x^2\mathrm{Li}_2(}{}$$

Mathematica [A] time = 0.32611, size = 136, normalized size = 0.88

$$\frac{1}{24}\left(\frac{6ix\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)}{b^2}+\frac{3\text{PolyLog}\left(4,-\frac{ie^{-2i(a+bx)}}{c}\right)}{b^3}-\frac{6x^2\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)}{b}+4ix^3\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)+8\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTan[c + (1 - I*c)*Cot[a + b*x]] + (4*I)*x^3*Log[1 + I/(c*E^((2*I)
*(a + b*x)))] - (6*x^2*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))])/b + ((6*I)
```

```
*x*PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))])/b^2 + (3*PolyLog[4, (-I)/(c*E^
((2*I)*(a + b*x)))])/b^3)/24
```

Maple [C] time = 18.458, size = 1532, normalized size = 10.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] 1/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/12*x^3*Pi*csgn(I*exp(2*
I*(b*x+a)))^3+1/6*Pi*x^3-1/4/b^3*polylog(2,I*c*exp(2*I*(b*x+a)))*a^2+1/2/b^
3*a^2*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/2/b^3*a^2*dilog(1+I*exp(I*(b
*x+a))*(-I*c)^(1/2))+1/4*x^2*polylog(2,I*c*exp(2*I*(b*x+a)))/b+1/6*I*x^3*ln
(1-I*c*exp(2*I*(b*x+a)))-1/8*polylog(4,I*c*exp(2*I*(b*x+a)))/b^3-1/12*x^3*P
i*csgn((c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I*(c
*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3+1/12*x^3*Pi*csgn(I*(I+c)/(exp(
2*I*(b*x+a))-1))^3+1/12*b*x^4-1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(
2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn((c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a)
)-1))^3+1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3+1/1
2*x^3*Pi*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((c*exp(2*
I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(
-I*c)^(1/2))*x-1/2*I/b^2*ln(1-I*c*exp(2*I*(b*x+a)))*x*a^2+1/2*I/b^2*a^2*ln(
1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/6*I*x^3*ln(c*exp(2*I*(b*x+a))+I)+1/12*
x^3*Pi*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I+c)/(exp(2*I*(b*
x+a))-1))+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3
+1/3*I*x^3*ln(exp(I*(b*x+a)))+1/6*I*x^3*ln(I+c)+1/12*x^3*Pi*csgn(I*exp(2*I*
(b*x+a)))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/
(exp(2*I*(b*x+a))-1))-1/12*x^3*Pi*csgn(I*(c*exp(2*I*(b*x+a))+I))*csgn(I/(ex
p(2*I*(b*x+a))-1))*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+1/12
*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b
*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))-1/12*x^3*Pi*csgn(I*(I+c))*csgn(I*(I+c)/(
exp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I+c
)/(exp(2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn(I*(c*exp(2*I*(b*x+a))+I))*csgn(I
*(c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn(I/(exp(2*I
*(b*x+a))-1))*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/12*x^
3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/2*I/b^3*a^3*ln(1+I
*exp(I*(b*x+a))*(-I*c)^(1/2))-1/3*I/b^3*ln(1-I*c*exp(2*I*(b*x+a)))*a^3-1/6*
I/b^3*a^3*ln(c*exp(2*I*(b*x+a))+I)+1/4*I*x*polylog(3,I*c*exp(2*I*(b*x+a)))/
b^2-1/12*x^3*Pi*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((c
*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+
a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a
))-1))^2-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)
/(exp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))*csg
n(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2-1/6*x^3*Pi*csgn(I*exp(I*
(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2
```

Maxima [B] time = 1.13643, size = 417, normalized size = 2.71

$$\frac{\frac{\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan((-ic+1)\cot(bx+a)+c)}{b^2}+\frac{3\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(-8i\,(bx+a)^3+18i\,(bx+a)^2a-18i\,(bx+a)a^2\right)\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*(((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((-I*c + 1)*co
t(b*x + a) + c)/b^2 + 3*(-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 + (-8*I*(b*x + a)^3 + 18*I*(b*x + a)^2*a - 18*I*(b*x + a)*a^2)*a
rctan2(c*cos(2*b*x + 2*a), c*sin(2*b*x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 1
8*I*(b*x + a)*a - 9*I*a^2)*dilog(I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3
- 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin(2
*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, I*c*e^
(2*I*b*x + 2*I*a)) + 6*I*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b^
2*(12*c + 12*I)))/b
```

Fricas [C] time = 1.94354, size = 459, normalized size = 2.98

$$\frac{2\,b^4x^4+4i\,b^3x^3\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right)+6\,b^2x^2\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)-2\,a^4-4i\,a^3\log\left(\frac{ce^{(2i\,bx+2i\,a)}+i}{c}\right)+6i\,bx\mathrm{polylog}\left(3,i\,ce^{(2i\,bx+2i\,a)}\right)\cdots}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(2*b^4*x^4 + 4*I*b^3*x^3*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*
x + 2*I*a) + I)) + 6*b^2*x^2*dilog(I*c*e^(2*I*b*x + 2*I*a)) - 2*a^4 - 4*I*a
^3*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + 6*I*b*x*polylog(3, I*c*e^(2*I*b*x +
 2*I*a)) + (4*I*b^3*x^3 + 4*I*a^3)*log(-I*c*e^(2*I*b*x + 2*I*a) + 1) - 3*po
lylog(4, I*c*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ib\int\frac{x^3}{ce^{2ia}e^{2ibx}+i}\,dx}{3}+\frac{ix^3\log\left(-ic-\frac{ic}{e^{2ia}e^{2ibx}-1}-\frac{ice^{ia}e^{ibx}}{e^{ia}e^{ibx}-e^{-ia}e^{-ibx}}+1+\frac{1}{e^{2ia}e^{2ibx}-1}+\frac{e^{ia}e^{ibx}}{e^{ia}e^{ibx}-e^{-ia}e^{-ibx}}\right)}{6}-\frac{ix^3\log\left(ic+\frac{ic}{e^{2ia}e^{2ibx}-1}+\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] I*b*Integral(x**3/(c*exp(2*I*a)*exp(2*I*b*x) + I), x)/3 + I*x**3*log(-I*c -
 I*c/(exp(2*I*a)*exp(2*I*b*x) - 1) - I*c*exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(
I*b*x) - exp(-I*a)*exp(-I*b*x)) + 1 + 1/(exp(2*I*a)*exp(2*I*b*x) - 1) + exp
(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) - exp(-I*a)*exp(-I*b*x)))/6 - I*x**3*
log(I*c + I*c/(exp(2*I*a)*exp(2*I*b*x) - 1) + I*c*exp(I*a)*exp(I*b*x)/(exp(
I*a)*exp(I*b*x) - exp(-I*a)*exp(-I*b*x)) + 1 - 1/(exp(2*I*a)*exp(2*I*b*x) -
 1) - exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) - exp(-I*a)*exp(-I*b*x)))/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -x^2\arctan\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctan(-(-I*c + 1)*cot(b*x + a) - c), x)
```

3.66 $\int x \tan^{-1}(c + (1 - ic)\cot(a + bx))\,dx$

Optimal. Leaf size=123

$$\frac{i\text{PolyLog}\left(3, ice^{2ia+2ibx}\right)}{8b^2} + \frac{x\text{PolyLog}\left(2, ice^{2ia+2ibx}\right)}{4b} + \frac{1}{4}ix^2\log\left(1 - ice^{2ia+2ibx}\right) + \frac{1}{2}x^2\tan^{-1}(c + (1 - ic)\cot(a + bx)) + \frac{b}{}$$

```
[Out] (b*x^3)/6 + (x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/2 + (I/4)*x^2*Log[1 -
I*c*E^((2*I)*a + (2*I)*b*x)] + (x*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])/
(4*b) + ((I/8)*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rubi [A] time = 0.222363, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5173, 2184, 2190, 2531, 2282, 6589}

$$\frac{i\text{PolyLog}\left(3, ice^{2ia+2ibx}\right)}{8b^2} + \frac{x\text{PolyLog}\left(2, ice^{2ia+2ibx}\right)}{4b} + \frac{1}{4}ix^2\log\left(1 - ice^{2ia+2ibx}\right) + \frac{1}{2}x^2\tan^{-1}(c + (1 - ic)\cot(a + bx)) + \frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/2 + (I/4)*x^2*Log[1 -
I*c*E^((2*I)*a + (2*I)*b*x)] + (x*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])/
(4*b) + ((I/8)*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 5173

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Cot[a + b*x]])/(f*(m +
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+(1-ic)\cot(a+bx))\,dx=\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}(ib)\int\frac{x^2}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{2}(bc)\int\frac{e^{2ia+2ibx}x^2}{-i(1-ic)+c-ce^{2ia+2i}}$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)-\frac{1}{2}i\int$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{x\mathrm{Li}_2}{}$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{x\mathrm{Li}_2}{}$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{x\mathrm{Li}_2}{}$$

Mathematica [A] time = 0.206048, size = 110, normalized size = 0.89

$$\frac{i\left(2ibx\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)+2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]
 + PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] time = 9.3, size = 1497, normalized size = 12.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] -1/8*x^2*Pi*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*c
sgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi
```

```
*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*
(b*x+a))-1))^2-1/8*x^2*Pi*csgn((c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))
^2+1/4*Pi*x^2-1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))*x+1/8*x^2*Pi*cs
gn(I*exp(2*I*(b*x+a)))^3-1/2*I/b*a*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x+1/
8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3-1/8*x^2*Pi*c
sgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I
+c)/(exp(2*I*(b*x+a))-1))^2+1/4*x*polylog(2,I*c*exp(2*I*(b*x+a)))/b+1/2*I/b
*ln(1-I*c*exp(2*I*(b*x+a)))*x*a-1/8*x^2*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp
(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a)
)-1))^3-1/8*x^2*Pi*csgn(I*(c*exp(2*I*(b*x+a))+I))*csgn(I/(exp(2*I*(b*x+a))-
1))*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+1/8*x^2*Pi*csgn(I*(
I+c))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))+1/8*x
^2*Pi*csgn((c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3+1/8*x^2*Pi*csgn(I
*exp(2*I*(b*x+a)))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a
))*(I+c)/(exp(2*I*(b*x+a))-1))+1/6*b*x^3-1/4*I*x^2*ln(c*exp(2*I*(b*x+a))+I)
+1/8*x^2*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^3+1/4/b^2*polylog(2,I*c*exp(
2*I*(b*x+a)))*a+1/4*I*x^2*ln(1-I*c*exp(2*I*(b*x+a)))-1/8*x^2*Pi*csgn(I*(c*e
xp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3-1/2/b^2*a*dilog(1-I*exp(I*(b*x+a
))*(-I*c)^(1/2))-1/2/b^2*a*dilog(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/8*I*pol
ylog(3,I*c*exp(2*I*(b*x+a)))/b^2+1/8*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I
*exp(2*I*(b*x+a)))-1/4*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)
))^2-1/8*x^2*Pi*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((c
*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+1/4*I/b^2*a^2*ln(c*exp(2*I*(b*x+
a))+I)-1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2*I/b^2*a^2*ln(1
+I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(e
xp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))+1/8*x
^2*Pi*csgn(I*(c*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((c*exp(2*I*(
b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/4*I/b^2*ln(1-I*c*exp(2*I*(b*x+a)))*a^2
+1/8*x^2*Pi*csgn(I*(c*exp(2*I*(b*x+a))+I))*csgn(I*(c*exp(2*I*(b*x+a))+I)/(e
xp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c*exp
(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))
)*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2+1/2*I*x^2*ln(exp(I*
(b*x+a)))+1/4*I*x^2*ln(I+c)
```

Maxima [B] time = 1.08603, size = 294, normalized size = 2.39

$$\frac{\frac{\left((bx+a)^2-2\,(bx+a)a\right)\arctan((-i\,c+1)\cot(bx+a)+c)}{b}+\frac{2\,\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)+\left(-6i\,(bx+a)^2+12i\,(bx+a)a\right)\arctan(c\cos(2\,bx+2\,a),\cdots}{\cdots}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x + a)^2 - 2*(b*x + a)*a)*arctan((-I*c + 1)*cot(b*x + a) + c)/b +
2*(-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(I*c*e^(2*I*b*x + 2
*I*a)) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a),
c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b*x
 + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*polylog(
3, I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b*(12*c + 12*I)))/b
```

Fricas [C] time = 1.90519, size = 394, normalized size = 3.2

$$\frac{4\,b^3x^3+6i\,b^2x^2\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right)+4\,a^3+6\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)+6i\,a^2\log\left(\frac{ce^{(2i\,bx+2i\,a)}+i}{c}\right)+\left(6i\,b^2x^2-6i\,a^2\right)\log\left(-i\,ce^{(2\cdots}\right.}{24\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*b^3*x^3 + 6*I*b^2*x^2*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*
x + 2*I*a) + I)) + 4*a^3 + 6*b*x*dilog(I*c*e^(2*I*b*x + 2*I*a)) + 6*I*a^2*l
og((c*e^(2*I*b*x + 2*I*a) + I)/c) + (6*I*b^2*x^2 - 6*I*a^2)*log(-I*c*e^(2*I
*b*x + 2*I*a) + 1) + 3*I*polylog(3, I*c*e^(2*I*b*x + 2*I*a)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ib\int\frac{x^2}{ce^{2ia}e^{2ibx}+i}\,dx}{2}+\frac{ix^2\log\left(-ic-\frac{ic}{e^{2ia}e^{2ibx}-1}-\frac{ice^{ia}e^{ibx}}{e^{ia}e^{ibx}-e^{-ia}e^{-ibx}}+1+\frac{1}{e^{2ia}e^{2ibx}-1}+\frac{e^{ia}e^{ibx}}{e^{ia}e^{ibx}-e^{-ia}e^{-ibx}}\right)}{4}-\frac{ix^2\log\left(ic+\frac{ic}{e^{2ia}e^{2ibx}-1}+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] I*b*Integral(x**2/(c*exp(2*I*a)*exp(2*I*b*x) + I), x)/2 + I*x**2*log(-I*c -
 I*c/(exp(2*I*a)*exp(2*I*b*x) - 1) - I*c*exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(
I*b*x) - exp(-I*a)*exp(-I*b*x)) + 1 + 1/(exp(2*I*a)*exp(2*I*b*x) - 1) + exp
(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) - exp(-I*a)*exp(-I*b*x)))/4 - I*x**2*
log(I*c + I*c/(exp(2*I*a)*exp(2*I*b*x) - 1) + I*c*exp(I*a)*exp(I*b*x)/(exp(
I*a)*exp(I*b*x) - exp(-I*a)*exp(-I*b*x)) + 1 - 1/(exp(2*I*a)*exp(2*I*b*x) -
 1) - exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) - exp(-I*a)*exp(-I*b*x)))/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -x\arctan\left(-(-ic+1)\cot\left(bx+a\right)-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctan(-(-I*c + 1)*cot(b*x + a) - c), x)
```

3.67 $\int \tan^{-1}(c+(1-ic)\cot(a+bx))\,dx$

Optimal. Leaf size=85

$$\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+x\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{bx^2}{2}$$

```
[Out] (b*x^2)/2 + x*ArcTan[c + (1 - I*c)*Cot[a + b*x]] + (I/2)*x*Log[1 - I*c*E^((
2*I)*a + (2*I)*b*x)] + PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rubi [A] time = 0.131453, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5165, 2184, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+x\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTan[c + (1 - I*c)*Cot[a + b*x]] + (I/2)*x*Log[1 - I*c*E^((
2*I)*a + (2*I)*b*x)] + PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 5165

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcT
an[c + d*Cot[a + b*x]], x] - Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tan^{-1}(c+(1-ic)\cot(a+bx))\,dx = x\tan^{-1}(c+(1-ic)\cot(a+bx)) - (ib)\int \frac{x}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx$$
$$= \frac{bx^2}{2} + x\tan^{-1}(c+(1-ic)\cot(a+bx)) + (bc)\int \frac{e^{2ia+2ibx}x}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx$$
$$= \frac{bx^2}{2} + x\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) - \frac{1}{2}i\int \log\Big(1$$
$$= \frac{bx^2}{2} + x\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) - \frac{\mathrm{Subst}\Big(\int \frac{1}{}}{}$$
$$= \frac{bx^2}{2} + x\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) + \frac{\mathrm{Li}_2\left(ice^{2ia+}\right.}{4b}$$

Mathematica [B] time = 15.8397, size = 929, normalized size = 10.93

$$x\tan^{-1}(c+(1-ic)\cot(a+bx)) - \frac{}{(\cot(a+bx)+i)(ic+(c+i)\cot(a+bx)+1)\Big(i\log(i\tan(bx)+1)\tan(bx)\cos^2(a)+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (1 - I*c)*Cot[a + b*x]] - (I*x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos
[b*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((I +
c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] - I*L
og[(Sec[b*x]*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] -
(2*I)*Sin[a])]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x
]] + I*PolyLog[2, (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b
*x] - I*Sin[a + b*x]))/(2*c)] - I*PolyLog[2, (Sec[b*x]*((1 + I*c)*Cos[a] -
(I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2])*(Cos[b*x] - I*Sin[b*x]
)*(Cos[b*x] + I*Sin[b*x]))/((I + Cot[a + b*x])*(1 + I*c + (I + c)*Cot[a + b
*x])*((2*I)*b*x + Log[1 - (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(C
os[a + b*x] - I*Sin[a + b*x]))/(2*c)] + Log[1 + (Sec[b*x]*((-1 - I*c)*Cos[a
] + (I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2] + ((-I + c)*Cos[a +
 b*x]*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]]))/((I + c)*Cos[a + b*x] +
(1 + I*c)*Sin[a + b*x]) + ((I + c)*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x
]])*Sin[a + b*x])/((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]) + 2*b*x*
Tan[b*x] + I*Log[1 - (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a
+ b*x] - I*Sin[a + b*x]))/(2*c)]*Tan[b*x] - I*Log[1 + (Sec[b*x]*((-1 - I*c)
*Cos[a] + (I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] - I*
Log[1 - I*Tan[b*x]]*Tan[b*x] + I*Cos[a]^2*Log[1 + I*Tan[b*x]]*Tan[b*x] + I*
Log[1 + I*Tan[b*x]]*Sin[a]^2*Tan[b*x] + (I*Log[(Sec[b*x]*((1 - I*c)*Cos[a +
 b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*x]^2)/(-I
+ Tan[b*x]) - (I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((I + c)*Cos[a + b*x] +
(1 + I*c)*Sin[a + b*x]))/(2*c)]*Sec[b*x]^2)/(I + Tan[b*x])))
```

Maple [B] time = 0.137, size = 1495, normalized size = 17.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] 1/b/(-1+I*c)*arctan(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(cot(b*x+a)*(-1+I*c)
-c-I)-1/b/(-1+I*c)*arctan(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(c+cot(b*x+a)*
(-1+I*c)+I)-1/2/b/(-1+I*c)/(I+c)*dilog(-1/2*I*(cot(b*x+a)*(-1+I*c)-c+I))*c+
1/4/b/(-1+I*c)/(I+c)*ln(cot(b*x+a)*(-1+I*c)-c-I)^2*c+1/8*I/b/(-1+I*c)/(I+c)
*ln(cot(b*x+a)*(-1+I*c)-c-I)^2+1/4*I/b/(-1+I*c)/(I+c)*dilog(-1/2*(cot(b*x+a
)*(-1+I*c)-c+I)/c)-1/4*I/b/(-1+I*c)/(I+c)*dilog((cot(b*x+a)*(-1+I*c)-c-I)/(
-2*I-2*c))-1/4*I/b/(-1+I*c)/(I+c)*dilog(-1/2*I*(cot(b*x+a)*(-1+I*c)-c+I))+1
/2/b/(-1+I*c)/(I+c)*dilog(-1/2*(cot(b*x+a)*(-1+I*c)-c+I)/c)*c-1/2/b/(-1+I*c
)/(I+c)*dilog((cot(b*x+a)*(-1+I*c)-c-I)/(-2*I-2*c))*c-1/2/b/(-1+I*c)/(I+c)*
ln((cot(b*x+a)*(-1+I*c)-c-I)/(-2*I-2*c))*ln(c+cot(b*x+a)*(-1+I*c)+I)*c-1/b/
(-1+I*c)*arctan(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(cot(b*x+a)*(-1+I*c)-c-I
)*c^2+1/b/(-1+I*c)*arctan(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(c+cot(b*x+a)*
(-1+I*c)+I)*c^2-1/2/b/(-1+I*c)/(I+c)*ln(-1/2*I*(cot(b*x+a)*(-1+I*c)-c+I))*l
n(cot(b*x+a)*(-1+I*c)-c-I)*c-1/4*I/b/(-1+I*c)/(I+c)*ln((cot(b*x+a)*(-1+I*c)
-c-I)/(-2*I-2*c))*ln(c+cot(b*x+a)*(-1+I*c)+I)-1/4*I/b/(-1+I*c)/(I+c)*ln(-1/
2*I*(cot(b*x+a)*(-1+I*c)-c+I))*ln(cot(b*x+a)*(-1+I*c)-c-I)+1/4*I/b/(-1+I*c)
/(I+c)*ln(-1/2*(cot(b*x+a)*(-1+I*c)-c+I)/c)*ln(c+cot(b*x+a)*(-1+I*c)+I)-1/8
*I/b/(-1+I*c)/(I+c)*ln(cot(b*x+a)*(-1+I*c)-c-I)^2*c^2-1/4*I/b/(-1+I*c)/(I+c
)*dilog(-1/2*(cot(b*x+a)*(-1+I*c)-c+I)/c)*c^2+1/2/b/(-1+I*c)/(I+c)*ln(-1/2*
(cot(b*x+a)*(-1+I*c)-c+I)/c)*ln(c+cot(b*x+a)*(-1+I*c)+I)*c+1/4*I/b/(-1+I*c)
/(I+c)*dilog((cot(b*x+a)*(-1+I*c)-c-I)/(-2*I-2*c))*c^2+1/4*I/b/(-1+I*c)/(I+
c)*dilog(-1/2*I*(cot(b*x+a)*(-1+I*c)-c+I))*c^2+1/4*I/b/(-1+I*c)/(I+c)*ln(-1
/2*I*(cot(b*x+a)*(-1+I*c)-c+I))*ln(cot(b*x+a)*(-1+I*c)-c-I)*c^2-1/4*I/b/(-1
+I*c)/(I+c)*ln(-1/2*(cot(b*x+a)*(-1+I*c)-c+I)/c)*ln(c+cot(b*x+a)*(-1+I*c)+I
)*c^2+1/4*I/b/(-1+I*c)/(I+c)*ln((cot(b*x+a)*(-1+I*c)-c-I)/(-2*I-2*c))*ln(c+
cot(b*x+a)*(-1+I*c)+I)*c^2+2*I/b/(-1+I*c)*arctan(cot(b*x+a)*(-1+I*c)-c)/(2*
I+2*c)*ln(c+cot(b*x+a)*(-1+I*c)+I)*c-2*I/b/(-1+I*c)*arctan(cot(b*x+a)*(-1+I
*c)-c)/(2*I+2*c)*ln(cot(b*x+a)*(-1+I*c)-c-I)*c
```

Maxima [B] time = 1.57339, size = 616, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((I*c - 1)*(4*I*(b*x + a)*log((-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) + 4
*c + 2*I)/(-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) - 2*I))/(I*c - 1) - I*(4*(b*
x + a)*(log(-I*c^2 + (c^2 + 1)*tan(b*x + a) + 2*c + I) - log(-I*c^2 + (c^2
+ 1)*tan(b*x + a) - I)) - 2*I*log(-I*c^2 + (c^2 + 1)*tan(b*x + a) + 2*c + I
)*log(-1/2*((I*c + 1)*tan(b*x + a) + c + I)/c + 1) + 2*I*log(-I*c^2 + (c^2
+ 1)*tan(b*x + a) + 2*c + I)*log(tan(b*x + a) - I) - 2*I*log(1/2*(c - I)*ta
n(b*x + a) - 1/2*I*c + 1/2)*log(tan(b*x + a) - I) - I*log(tan(b*x + a) - I)
^2 - 2*I*log(c^2 + 1)*log(I*tan(b*x + a) + 1) + 2*I*log(tan(b*x + a) - I)*l
og(-1/2*I*tan(b*x + a) + 1/2) + 2*I*log(c^2 + 1)*log(-I*tan(b*x + a) + 1) -
 2*I*dilog(-1/2*(c - I)*tan(b*x + a) + 1/2*I*c + 1/2) - 2*I*dilog(1/2*((I*c
 + 1)*tan(b*x + a) + c + I)/c) + 2*I*dilog(1/2*I*tan(b*x + a) + 1/2))/(I*c
- 1)) - 8*(b*x + a)*arctan(c + (-I*c + 1)/tan(b*x + a)) + 4*(-I*b*x - I*a)*
log((-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) + 4*c + 2*I)/(-2*I*c^2 + 2*(c^2 +
1)*tan(b*x + a) - 2*I)))/b
```

Fricas [A] time = 1.96799, size = 309, normalized size = 3.64

$$\frac{2\,b^2x^2 + 2i\,bx\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right) - 2\,a^2 + (2i\,bx + 2i\,a)\log\left(-i\,ce^{(2i\,bx+2i\,a)} + 1\right) - 2i\,a\log\left(\frac{ce^{(2i\,bx+2i\,a)}+i}{c}\right) + \mathrm{Li}_2\left(i\,ce^{(2i\,bx+}\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2 + 2*I*b*x*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2
*I*a) + I)) - 2*a^2 + (2*I*b*x + 2*I*a)*log(-I*c*e^(2*I*b*x + 2*I*a) + 1) -
 2*I*a*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + dilog(I*c*e^(2*I*b*x + 2*I*a)))
/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$ib\int\frac{x}{ce^{2ia}e^{2ibx}+i}\,dx + \frac{ix\log\left(-ic - \frac{ic}{e^{2ia}e^{2ibx}-1} - \frac{ice^{ia}e^{ibx}}{e^{ia}e^{ibx}-e^{-ia}e^{-ibx}} + 1 + \frac{1}{e^{2ia}e^{2ibx}-1} + \frac{e^{ia}e^{ibx}}{e^{ia}e^{ibx}-e^{-ia}e^{-ibx}}\right)}{2} - \frac{ix\log\left(ic + \frac{ic}{e^{2ia}e^{2ibx}-1} + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] I*b*Integral(x/(c*exp(2*I*a)*exp(2*I*b*x) + I), x) + I*x*log(-I*c - I*c/(ex
p(2*I*a)*exp(2*I*b*x) - 1) - I*c*exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) -
 exp(-I*a)*exp(-I*b*x)) + 1 + 1/(exp(2*I*a)*exp(2*I*b*x) - 1) + exp(I*a)*ex
p(I*b*x)/(exp(I*a)*exp(I*b*x) - exp(-I*a)*exp(-I*b*x)))/2 - I*x*log(I*c + I
*c/(exp(2*I*a)*exp(2*I*b*x) - 1) + I*c*exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*
b*x) - exp(-I*a)*exp(-I*b*x)) + 1 - 1/(exp(2*I*a)*exp(2*I*b*x) - 1) - exp(I
*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) - exp(-I*a)*exp(-I*b*x)))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\arctan\left(-(-i\,c + 1)\cot\left(bx + a\right) - c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-arctan(-(-I*c + 1)*cot(b*x + a) - c), x)
```

3.68 $\int \frac{\tan^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=23

$$\text{CannotIntegrate}\left(\frac{\tan^{-1}(c+(1-ic)\cot(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x, x]
```

Rubi [A] time = 0.121414, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\tan^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$$

Mathematica [A] time = 0.628517, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x, x]
```

Maple [A] time = 0.408, size = 0, normalized size = 0.

$$\int -\frac{\arctan(-c-(1-ic)\cot(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

```
[Out] int(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i\,\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right)}{2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a) + I)
)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\arctan\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(-arctan(-(-I*c + 1)*cot(b*x + a) - c)/x, x)
```

3.69 $\int x^2 \tan^{-1}(c+(-1-ic)\cot(a+bx))\,dx$

Optimal. Leaf size=155

$$-\frac{ix\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{3}x^3$$

```
[Out] -(b*x^4)/12 + (x^3*ArcTan[c - (1 + I*c)*Cot[a + b*x]])/3 - (I/6)*x^3*Log[1
+ I*c*E^((2*I)*a + (2*I)*b*x)] - (x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*
b*x)])/(4*b) - ((I/4)*x*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2 + P
olyLog[4, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rubi [A] time = 0.261275, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5173, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{3}x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```


```
[Out] -(b*x^4)/12 + (x^3*ArcTan[c - (1 + I*c)*Cot[a + b*x]])/3 - (I/6)*x^3*Log[1
+ I*c*E^((2*I)*a + (2*I)*b*x)] - (x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*
b*x)])/(4*b) - ((I/4)*x*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2 + P
olyLog[4, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```


Rule 5173

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Cot[a + b*x]])/(f*(m +
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+(-1-ic)\cot(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{3}(ib)\int \frac{x^3}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,d$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{3}(bc)\int \frac{e^{2ia+2ibx}x^3}{-i(-1-ic)+c-ce}$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) + \frac{1}{2}$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) - \frac{x}{}$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) - \frac{x}{}$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) - \frac{x}{}$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) - \frac{x}{}$$

Mathematica [A] time = 0.32077, size = 140, normalized size = 0.9

$$\frac{1}{3}x^3\tan^{-1}(c+(-1-ic)\cot(a+bx)) - \frac{-6b^2x^2\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)+6ibx\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)+3\text{PolyLog}\left(4,\frac{ie^{-}}{}\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (-1 - I*c)*Cot[a + b*x]])/3 - ((4*I)*b^3*x^3*Log[1 - I/(c*E
^((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] + (6
*I)*b*x*PolyLog[3, I/(c*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, I/(c*E^((2*I)*
```

```
(a + b*x)))])/(24*b^3)
```

Maple [C] time = 18.921, size = 1533, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x)
```

```
[Out] -1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/6*Pi*x^3+1/8*polylog(4,-I*c*exp(2
*I*(b*x+a)))/b^3+1/4/b^3*polylog(2,-I*c*exp(2*I*(b*x+a)))*a^2+1/12*x^3*Pi*c
sgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(c
-I)/(exp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-
I)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1)
)+1/12*x^3*Pi*csgn(I*(c*exp(2*I*(b*x+a))-I))*csgn(I/(exp(2*I*(b*x+a))-1))*c
sgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))-1/6*I*x^3*ln(1+I*c*exp(2
*I*(b*x+a)))-1/12*x^3*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))^3-1/2/b^3*a^2*d
ilog(1-I*exp(I*(b*x+a))*(I*c)^(1/2))+1/2*I/b^2*ln(1+I*c*exp(2*I*(b*x+a)))*x
*a^2-1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/2*I/b^2*a^2*ln(1-
I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(
exp(2*I*(b*x+a))-1))^3-1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*
x+a))-1))^2-1/12*b*x^4-1/12*x^3*Pi*csgn(I*(c-I))*csgn(I/(exp(2*I*(b*x+a))-1
))*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))+1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(c-
I)/(exp(2*I*(b*x+a))-1))^3+1/12*x^3*Pi*csgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(2
*I*(b*x+a))-1))^3-1/2/b^3*a^2*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/2))+1/12*x^
3*Pi*csgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*csgn((c*exp(2*I*(b
*x+a))-I)/(exp(2*I*(b*x+a))-1))-1/12*x^3*Pi*csgn(I*(c*exp(2*I*(b*x+a))-I))*
csgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I/(e
xp(2*I*(b*x+a))-1))*csgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2-1
/12*x^3*Pi*csgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*csgn((c*exp(
2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))
)*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2+1/3*I/b^3*ln(1+I*c*
exp(2*I*(b*x+a)))*a^3+1/6*I/b^3*a^3*ln(-c*exp(2*I*(b*x+a))+I)-1/2*I/b^3*a^3
*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))+1/12*x^3*Pi*csgn((c*exp(2*I*(b*x+a))-I)
/(exp(2*I*(b*x+a))-1))^3-1/4*I*x*polylog(3,-I*c*exp(2*I*(b*x+a)))/b^2+1/12*
x^3*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(ex
p(2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a
))-1))^2+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c-I)/(exp(2*I*(b*
x+a))-1))^2-1/12*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1
/3*I*x^3*ln(exp(I*(b*x+a)))-1/6*I*x^3*ln(c-I)-1/12*x^3*Pi*csgn((c*exp(2*I*(
b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2-1/4*x^2*polylog(2,-I*c*exp(2*I*(b*x+a)))
/b+1/6*I*x^3*ln(c*exp(2*I*(b*x+a))-I)-1/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a))*(
I*c)^(1/2))-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))
*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))+1/6*x^3*Pi*csgn(I*exp(I*
(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2
```

Maxima [B] time = 1.12551, size = 419, normalized size = 2.7

$$\frac{\frac{\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan((-i\,c-1)\cot(bx+a)+c)}{b^2}-\frac{3\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(8i\,(bx+a)^3-18i\,(bx+a)^2a+18i\,(bx+a)a^2\right)\mathrm{arcta}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*(((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((-I*c - 1)*co
t(b*x + a) + c)/b^2 - 3*(-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 + (8*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + 18*I*(b*x + a)*a^2)*ar
ctan2(c*cos(2*b*x + 2*a), -c*sin(2*b*x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 1
8*I*(b*x + a)*a - 9*I*a^2)*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3
 - 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin(
2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, -I*c*
e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)/
(b^2*(12*c - 12*I)))/b
```

Fricas [C] time = 1.93349, size = 466, normalized size = 3.01

$$-\frac{2\,b^4x^4 - 4i\,b^3x^3\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}-i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right) - 2\,a^4 - 4i\,a^3\log\left(\frac{ce^{(2i\,bx+2i\,a)}-i}{c}\right) + 6i\,bx\mathrm{polyl}}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/24*(2*b^4*x^4 - 4*I*b^3*x^3*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x
 - 2*I*a)/(c - I)) + 6*b^2*x^2*dilog(-I*c*e^(2*I*b*x + 2*I*a)) - 2*a^4 - 4*
I*a^3*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + 6*I*b*x*polylog(3, -I*c*e^(2*I*b
*x + 2*I*a)) - (-4*I*b^3*x^3 - 4*I*a^3)*log(I*c*e^(2*I*b*x + 2*I*a) + 1) -
3*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ib\int\frac{x^3}{ce^{2ia}e^{2ibx}-i}\,dx}{3} + \frac{ix^3\log\left(-ic - \frac{ic}{e^{2ia}e^{2ibx}-1} - \frac{ice^{ia}e^{ibx}}{e^{ia}e^{ibx}-e^{-ia}e^{-ibx}} + 1 - \frac{1}{e^{2ia}e^{2ibx}-1} - \frac{e^{ia}e^{ibx}}{e^{ia}e^{ibx}-e^{-ia}e^{-ibx}}\right)}{6} - \frac{ix^3\log\left(ic + \frac{ic}{e^{2ia}e^{2ibx}-1} + \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atan(-c-(-1-I*c)*cot(b*x+a)),x)
```

```
[Out] I*b*Integral(x**3/(c*exp(2*I*a)*exp(2*I*b*x) - I), x)/3 + I*x**3*log(-I*c -
 I*c/(exp(2*I*a)*exp(2*I*b*x) - 1) - I*c*exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(
I*b*x) - exp(-I*a)*exp(-I*b*x)) + 1 - 1/(exp(2*I*a)*exp(2*I*b*x) - 1) - exp
(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) - exp(-I*a)*exp(-I*b*x)))/6 - I*x**3*
log(I*c + I*c/(exp(2*I*a)*exp(2*I*b*x) - 1) + I*c*exp(I*a)*exp(I*b*x)/(exp(
I*a)*exp(I*b*x) - exp(-I*a)*exp(-I*b*x)) + 1 + 1/(exp(2*I*a)*exp(2*I*b*x) -
 1) + exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) - exp(-I*a)*exp(-I*b*x)))/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -x^2\arctan\left(-(-i\,c-1)\cot\left(bx+a\right)-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctan(-(-I*c - 1)*cot(b*x + a) - c), x)
```

3.70 $\int x \tan^{-1}(c + (-1 - ic)\cot(a + bx))\,dx$

Optimal. Leaf size=124

$$-\frac{i\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+$$

```
[Out] -(b*x^3)/6 + (x^2*ArcTan[c - (1 + I*c)*Cot[a + b*x]])/2 - (I/4)*x^2*Log[1 +
 I*c*E^((2*I)*a + (2*I)*b*x)] - (x*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x
)])/(4*b) - ((I/8)*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rubi [A] time = 0.223722, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5173, 2184, 2190, 2531, 2282, 6589}

$$-\frac{i\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -(b*x^3)/6 + (x^2*ArcTan[c - (1 + I*c)*Cot[a + b*x]])/2 - (I/4)*x^2*Log[1 +
 I*c*E^((2*I)*a + (2*I)*b*x)] - (x*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x
)])/(4*b) - ((I/8)*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 5173

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Cot[a + b*x]])/(f*(m +
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+(-1-ic)\cot(a+bx))\,dx=\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{2}(ib)\int\frac{x^2}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,dx$$
$$=-\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{2}(bc)\int\frac{e^{2ia+2ibx}x^2}{-i(-1-ic)+c-ce^{2ia+2}}$$
$$=-\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}i\int$$
$$=-\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)-\frac{x\mathrm{Li}_2(}{}$$
$$=-\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)-\frac{x\mathrm{Li}_2(}{}$$
$$=-\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)-\frac{x\mathrm{Li}_2(}{}$$

Mathematica [A] time = 0.196254, size = 110, normalized size = 0.89

$$\frac{1}{2}x^2\tan^{-1}(c+(-1-ic)\cot(a+bx))-\frac{i\left(2ibx\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)+2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (-1 - I*c)*Cot[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 - I/(
c*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] +
 PolyLog[3, I/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] time = 9.099, size = 1498, normalized size = 12.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x)
```

```
[Out] 1/4*Pi*x^2+1/8*x^2*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^3-1
/8*x^2*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*cs
```

```
gn((c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn((c*exp(2*
I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^3+1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(
I*c)^(1/2))+1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))-1/8*x^2*Pi*csg
n(I*(c-I)/(exp(2*I*(b*x+a))-1))^3+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/
(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2-1
/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/4*I/b^2*ln(1+I*c*exp(2*I*(b*x+a)))*a
^2-1/4*I/b^2*a^2*ln(-c*exp(2*I*(b*x+a))+I)+1/2/b^2*a*dilog(1-I*exp(I*(b*x+a
))*(I*c)^(1/2))+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))
*(c-I)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1)
)*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn(I*(
c*exp(2*I*(b*x+a))-I))*csgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^
2-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c*exp(2*I*(b*x+a))-I)/(ex
p(2*I*(b*x+a))-1))^2+1/2*I/b*a*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/6*b*x
^3-1/4*I*x^2*ln(1+I*c*exp(2*I*(b*x+a)))-1/4/b^2*polylog(2,-I*c*exp(2*I*(b*x
+a)))*a+1/2/b^2*a*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/2))+1/4*I*x^2*ln(c*exp(
2*I*(b*x+a))-I)-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-
1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))+1/8*x^2*Pi*csgn(I*(c*
exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^3+1/8*x^2*Pi*csgn(I*(c*exp(2*I*(b
*x+a))-I)/(exp(2*I*(b*x+a))-1))*csgn((c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a
))-1))-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1
))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))+1/8*x^2*Pi*csgn(I*(c
*exp(2*I*(b*x+a))-I))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c*exp(2*I*(b*x+a
))-I)/(exp(2*I*(b*x+a))-1))-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2
*I*(b*x+a))-1))^3-1/8*x^2*Pi*csgn(I*(c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a)
)-1))*csgn((c*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn(I
*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn(I/(exp(2*I*(b*
x+a))-1))*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))^2-1/8*I*polylog(3,-I*c*exp(2*I
*(b*x+a)))/b^2-1/4*x*polylog(2,-I*c*exp(2*I*(b*x+a)))/b-1/2*I/b*ln(1+I*c*ex
p(2*I*(b*x+a)))*x*a+1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/8*x^2*
Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/4*x^2*Pi*csgn(I*exp(
I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/8*x^2*Pi*csgn(I*(c-I))*csgn(I/(exp
(2*I*(b*x+a))-1))*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))-1/2*I*x^2*ln(exp(I*(b*
x+a)))-1/4*I*x^2*ln(c-I)
```

Maxima [B] time = 1.07668, size = 296, normalized size = 2.39

$$\frac{\frac{\left((bx+a)^2-2\,(bx+a)a\right)\arctan((-i\,c-1)\cot(bx+a)+c)}{b} - \frac{2\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)+\left(6i\,(bx+a)^2-12i\,(bx+a)a\right)\arctan(c\cos(2\,bx+\cdots}{\cdots}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x + a)^2 - 2*(b*x + a)*a)*arctan((-I*c - 1)*cot(b*x + a) + c)/b -
2*(-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(-I*c*e^(2*I*b*x +
2*I*a)) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a),
-c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b*
x + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*polylog
(3, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)/(b*(12*c - 12*I)))/b
```

Fricas [C] time = 1.92981, size = 400, normalized size = 3.23

$$-\frac{4\,b^3x^3 - 6i\,b^2x^2\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}-i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right) + 4\,a^3 + 6\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right) + 6i\,a^2\log\left(\frac{ce^{(2i\,bx+2i\,a)}-i}{c}\right) - \left(-6i\,b^2x^2 + \cdots\right.}{24\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/24*(4*b^3*x^3 - 6*I*b^2*x^2*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x
 - 2*I*a)/(c - I)) + 4*a^3 + 6*b*x*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + 6*I*a^
2*log((c*e^(2*I*b*x + 2*I*a) - I)/c) - (-6*I*b^2*x^2 + 6*I*a^2)*log(I*c*e^(
2*I*b*x + 2*I*a) + 1) + 3*I*polylog(3, -I*c*e^(2*I*b*x + 2*I*a)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ib\int \frac{x^2}{ce^{2ia}e^{2ibx}-i}\,dx}{2} + \frac{ix^2\log\left(-ic - \frac{ic}{e^{2ia}e^{2ibx}-1} - \frac{ice^{ia}e^{ibx}}{e^{ia}e^{ibx}-e^{-ia}e^{-ibx}} + 1 - \frac{1}{e^{2ia}e^{2ibx}-1} - \frac{e^{ia}e^{ibx}}{e^{ia}e^{ibx}-e^{-ia}e^{-ibx}}\right)}{4} - \frac{ix^2\log\left(ic + \frac{ic}{e^{2ia}e^{2ibx}-1} + \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atan(-c-(-1-I*c)*cot(b*x+a)),x)
```

```
[Out] I*b*Integral(x**2/(c*exp(2*I*a)*exp(2*I*b*x) - I), x)/2 + I*x**2*log(-I*c -
 I*c/(exp(2*I*a)*exp(2*I*b*x) - 1) - I*c*exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(
I*b*x) - exp(-I*a)*exp(-I*b*x)) + 1 - 1/(exp(2*I*a)*exp(2*I*b*x) - 1) - exp
(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) - exp(-I*a)*exp(-I*b*x)))/4 - I*x**2*
log(I*c + I*c/(exp(2*I*a)*exp(2*I*b*x) - 1) + I*c*exp(I*a)*exp(I*b*x)/(exp(
I*a)*exp(I*b*x) - exp(-I*a)*exp(-I*b*x)) + 1 + 1/(exp(2*I*a)*exp(2*I*b*x) -
 1) + exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) - exp(-I*a)*exp(-I*b*x)))/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -x\arctan\left(-(-ic-1)\cot(bx+a)-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctan(-(-I*c - 1)*cot(b*x + a) - c), x)
```

3.71 $\int \tan^{-1}(c + (-1 - ic)\cot(a + bx))\,dx$

Optimal. Leaf size=86

$$-\frac{\text{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} - \frac{1}{2}ix\log\left(1 + ice^{2ia+2ibx}\right) + x\tan^{-1}(c - (1 + ic)\cot(a + bx)) - \frac{bx^2}{2}$$

```
[Out] -(b*x^2)/2 + x*ArcTan[c - (1 + I*c)*Cot[a + b*x]] - (I/2)*x*Log[1 + I*c*E^(
(2*I)*a + (2*I)*b*x)] - PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rubi [A] time = 0.133581, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5165, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} - \frac{1}{2}ix\log\left(1 + ice^{2ia+2ibx}\right) + x\tan^{-1}(c - (1 + ic)\cot(a + bx)) - \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -(b*x^2)/2 + x*ArcTan[c - (1 + I*c)*Cot[a + b*x]] - (I/2)*x*Log[1 + I*c*E^(
(2*I)*a + (2*I)*b*x)] - PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 5165

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcT
an[c + d*Cot[a + b*x]], x] - Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tan^{-1}(c+(-1-ic)\cot(a+bx))\,dx = x\tan^{-1}(c-(1+ic)\cot(a+bx))-(ib)\int \frac{x}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,dx$$

$$= -\frac{bx^2}{2}+x\tan^{-1}(c-(1+ic)\cot(a+bx))-(bc)\int \frac{e^{2ia+2ibx}x}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,dx$$

$$= -\frac{bx^2}{2}+x\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}i\int \log\Big(1$$

$$= -\frac{bx^2}{2}+x\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+\frac{\text{Subst}\Big(\int}{}$$

$$= -\frac{bx^2}{2}+x\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)-\frac{\text{Li}_2\left(-ice^{2ia}\right.}{4b}$$

Mathematica [B] time = 12.7368, size = 872, normalized size = 10.14

$$x\tan^{-1}(c+(-ic-1)\cot(a+bx))+\frac{ix\csc(a+bx)\Big(2bx\log(2\cos(bx)(\cos(bx)-i\sin}{(\cot(a+bx)+i)((c-i)\cos(a+bx)+i(c+i)\sin(a+bx))\Bigg(\frac{\log\left(\frac{1}{2}\sec(bx)(\cos(a)+i\sin(a))((i}{i}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (-1 - I*c)*Cot[a + b*x]] + (I*x*Csc[a + b*x]*(2*b*x*Log[2*Cos[
b*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I +
c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] - I*L
og[(Sec[b*x]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a +
b*x]))/2]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] +
I*PolyLog[2, (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] -
I*Sin[a + b*x]))/(2*c)] - I*PolyLog[2, ((Cos[a] + I*Sin[a])*((I + c)*Cos[a]
 + (1 + I*c)*Sin[a])*(-I + Tan[b*x]))/2])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*x]
 + I*Sin[b*x]))/((I + Cot[a + b*x])*((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[
a + b*x])*((-2*I)*b*x - Log[1 - (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a
])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)] - (Log[1 - I*Tan[b*x]]*((I + c)*
Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]))/((-I + c)*Cos[a + b*x] + I*(I + c)*
Sin[a + b*x]) + (Log[1 + I*Tan[b*x]]*((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[
a + b*x]))/((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]) + (Log[(Sec[b*x
]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a + b*x]))/2]*S
ec[b*x]^2)/(1 + I*Tan[b*x]) - 2*b*x*Tan[b*x] - I*Log[1 - (Sec[b*x]*((I + c)
*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]*Tan[b*x
] + I*Log[1 - I*Tan[b*x]]*Tan[b*x] - I*Log[1 + I*Tan[b*x]]*Tan[b*x] + (I*Lo
g[1 - ((Cos[a] + I*Sin[a])*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(-I + Tan[b*
x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + (I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*
((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]))/(2*c)]*Sec[b*x]^2)/(I + T
an[b*x])))
```

Maple [B] time = 0.136, size = 1753, normalized size = 20.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(-c-(-1-I*c)*cot(b*x+a)),x)
```

```
[Out] 1/2/b/(1+I*c)/(I-c)*dilog((-(1+I*c)*cot(b*x+a)+c-I)/(-2*I+2*c))*c-1/2/b/(1+
I*c)/(I-c)*dilog(1/2*(-(1+I*c)*cot(b*x+a)+c+I)/c)*c+1/8*I/b/(1+I*c)/(I-c)*l
n((1+I*c)*cot(b*x+a)-c+I)^2-1/4*I/b/(1+I*c)/(I-c)*dilog((-(1+I*c)*cot(b*x+a
)+c-I)/(-2*I+2*c))+1/4*I/b/(1+I*c)/(I-c)*dilog(1/2*(-(1+I*c)*cot(b*x+a)+c+I
)/c)+1/4*I/b/(1+I*c)/(I-c)*dilog(-1/2*I*((1+I*c)*cot(b*x+a)-c+I))+1/b/(1+I*
c)*arctan(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(-(1+I*c)*cot(b*x+a)-c+I)-1/2/
b/(1+I*c)/(I-c)*dilog(-1/2*I*((1+I*c)*cot(b*x+a)-c+I))*c-1/4/b/(1+I*c)/(I-c
)*ln((1+I*c)*cot(b*x+a)-c+I)^2*c+1/4*I/b/(1+I*c)/(I-c)*dilog((-(1+I*c)*cot(
b*x+a)+c-I)/(-2*I+2*c))*c^2-1/b/(1+I*c)*arctan(-c+(1+I*c)*cot(b*x+a))/(2*I-
2*c)*ln((1+I*c)*cot(b*x+a)-c+I)+1/4*I/b/(1+I*c)/(I-c)*ln(-1/2*I*((1+I*c)*co
t(b*x+a)-c+I))*ln(-1/2*I*(-(1+I*c)*cot(b*x+a)+c+I))-1/4*I/b/(1+I*c)/(I-c)*l
n(-1/2*I*(-(1+I*c)*cot(b*x+a)+c+I))*ln((1+I*c)*cot(b*x+a)-c+I)-1/4*I/b/(1+I
*c)/(I-c)*dilog(1/2*(-(1+I*c)*cot(b*x+a)+c+I)/c)*c^2-1/4*I/b/(1+I*c)/(I-c)*
ln((-(1+I*c)*cot(b*x+a)+c-I)/(-2*I+2*c))*ln(-(1+I*c)*cot(b*x+a)-c+I)+1/4*I/
b/(1+I*c)/(I-c)*ln(1/2*(-(1+I*c)*cot(b*x+a)+c+I)/c)*ln(-(1+I*c)*cot(b*x+a)-
c+I)-1/b/(1+I*c)*arctan(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(-(1+I*c)*cot(b*
x+a)-c+I)*c^2+1/b/(1+I*c)*arctan(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln((1+I*c
)*cot(b*x+a)-c+I)*c^2+1/2/b/(1+I*c)/(I-c)*ln(-1/2*I*(-(1+I*c)*cot(b*x+a)+c+
I))*ln((1+I*c)*cot(b*x+a)-c+I)*c+1/2/b/(1+I*c)/(I-c)*ln((-(1+I*c)*cot(b*x+a
)+c-I)/(-2*I+2*c))*ln(-(1+I*c)*cot(b*x+a)-c+I)*c-1/2/b/(1+I*c)/(I-c)*ln(1/2
*(-(1+I*c)*cot(b*x+a)+c+I)/c)*ln(-(1+I*c)*cot(b*x+a)-c+I)*c-1/4*I/b/(1+I*c)
/(I-c)*dilog(-1/2*I*((1+I*c)*cot(b*x+a)-c+I))*c^2-1/8*I/b/(1+I*c)/(I-c)*ln(
(1+I*c)*cot(b*x+a)-c+I)^2*c^2-1/2/b/(1+I*c)/(I-c)*ln(-1/2*I*((1+I*c)*cot(b*
x+a)-c+I))*ln(-1/2*I*(-(1+I*c)*cot(b*x+a)+c+I))*c-1/4*I/b/(1+I*c)/(I-c)*ln(
-1/2*I*((1+I*c)*cot(b*x+a)-c+I))*ln(-1/2*I*(-(1+I*c)*cot(b*x+a)+c+I))*c^2+1
/4*I/b/(1+I*c)/(I-c)*ln(-1/2*I*(-(1+I*c)*cot(b*x+a)+c+I))*ln((1+I*c)*cot(b*
x+a)-c+I)*c^2+1/4*I/b/(1+I*c)/(I-c)*ln((-(1+I*c)*cot(b*x+a)+c-I)/(-2*I+2*c)
)*ln(-(1+I*c)*cot(b*x+a)-c+I)*c^2-1/4*I/b/(1+I*c)/(I-c)*ln(1/2*(-(1+I*c)*co
t(b*x+a)+c+I)/c)*ln(-(1+I*c)*cot(b*x+a)-c+I)*c^2+2*I/b/(1+I*c)*arctan(-c+(1
+I*c)*cot(b*x+a))/(2*I-2*c)*ln(-(1+I*c)*cot(b*x+a)-c+I)*c-2*I/b/(1+I*c)*arc
tan(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln((1+I*c)*cot(b*x+a)-c+I)*c
```

Maxima [B] time = 1.59481, size = 616, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((I*c + 1)*(4*I*(b*x + a)*log((-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) - 2
*I)/(-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) - 4*c + 2*I))/(I*c + 1) + I*(4*(b*
x + a)*(log(-I*c^2 + (c^2 + 1)*tan(b*x + a) - 2*c + I) - log(-I*c^2 + (c^2
+ 1)*tan(b*x + a) - I)) - 2*I*log(-I*c^2 + (c^2 + 1)*tan(b*x + a) - 2*c + I
)*log(-1/2*((I*c - 1)*tan(b*x + a) + c - I)/c + 1) + 2*I*log(-I*c^2 + (c^2
+ 1)*tan(b*x + a) - 2*c + I)*log(tan(b*x + a) - I) - 2*I*log(-1/2*(c + I)*t
an(b*x + a) + 1/2*I*c + 1/2)*log(tan(b*x + a) - I) - I*log(tan(b*x + a) - I
)^2 - 2*I*log(c^2 + 1)*log(I*tan(b*x + a) + 1) + 2*I*log(tan(b*x + a) - I)*
log(-1/2*I*tan(b*x + a) + 1/2) + 2*I*log(c^2 + 1)*log(-I*tan(b*x + a) + 1)
- 2*I*dilog(1/2*(c + I)*tan(b*x + a) - 1/2*I*c + 1/2) - 2*I*dilog(1/2*((I*c
 - 1)*tan(b*x + a) + c - I)/c) + 2*I*dilog(1/2*I*tan(b*x + a) + 1/2))/(I*c
+ 1)) - 8*(b*x + a)*arctan(c + (-I*c - 1)/tan(b*x + a)) + 4*(-I*b*x - I*a)*
log((-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) - 2*I)/(-2*I*c^2 + 2*(c^2 + 1)*tan
(b*x + a) - 4*c + 2*I)))/b
```

Fricas [A] time = 1.98425, size = 313, normalized size = 3.64

$$-\frac{2\,b^2x^2-2i\,bx\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}-i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right)-2\,a^2-(-2i\,bx-2i\,a)\log\left(i\,ce^{(2i\,bx+2i\,a)}+1\right)-2i\,a\log\left(\frac{ce^{(2i\,bx+2i\,a)}-i}{c}\right)+\mathrm{Li}_2\left(-\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*x^2 - 2*I*b*x*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x - 2*
I*a)/(c - I)) - 2*a^2 - (-2*I*b*x - 2*I*a)*log(I*c*e^(2*I*b*x + 2*I*a) + 1)
 - 2*I*a*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + dilog(-I*c*e^(2*I*b*x + 2*I*a
)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$ib\int\frac{x}{ce^{2ia}e^{2ibx}-i}\,dx+\frac{ix\log\left(-ic-\frac{ic}{e^{2ia}e^{2ibx}-1}-\frac{ice^{ia}e^{ibx}}{e^{ia}e^{ibx}-e^{-ia}e^{-ibx}}+1-\frac{1}{e^{2ia}e^{2ibx}-1}-\frac{e^{ia}e^{ibx}}{e^{ia}e^{ibx}-e^{-ia}e^{-ibx}}\right)}{2}-\frac{ix\log\left(ic+\frac{ic}{e^{2ia}e^{2ibx}-1}+\frac{}{e^{ia}e}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(-c-(-1-I*c)*cot(b*x+a)),x)
```

```
[Out] I*b*Integral(x/(c*exp(2*I*a)*exp(2*I*b*x) - I), x) + I*x*log(-I*c - I*c/(ex
p(2*I*a)*exp(2*I*b*x) - 1) - I*c*exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) -
 exp(-I*a)*exp(-I*b*x)) + 1 - 1/(exp(2*I*a)*exp(2*I*b*x) - 1) - exp(I*a)*ex
p(I*b*x)/(exp(I*a)*exp(I*b*x) - exp(-I*a)*exp(-I*b*x)))/2 - I*x*log(I*c + I
*c/(exp(2*I*a)*exp(2*I*b*x) - 1) + I*c*exp(I*a)*exp(I*b*x)/(exp(I*a)*exp(I*
b*x) - exp(-I*a)*exp(-I*b*x)) + 1 + 1/(exp(2*I*a)*exp(2*I*b*x) - 1) + exp(I
*a)*exp(I*b*x)/(exp(I*a)*exp(I*b*x) - exp(-I*a)*exp(-I*b*x)))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\arctan\left(-(-i\,c-1)\cot\left(bx+a\right)-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-arctan(-(-I*c - 1)*cot(b*x + a) - c), x)
```

3.72 $\int \frac{\tan^{-1}(c+(-1-ic)\cot(a+bx))}{x} dx$

Optimal. Leaf size=23

$$\text{CannotIntegrate}\left(\frac{\tan^{-1}(c+(-1-ic)\cot(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x, x]
```

Rubi [A] time = 0.118817, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(c+(-1-ic)\cot(a+bx))}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+(-1-ic)\cot(a+bx))}{x} dx = \int \frac{\tan^{-1}(c+(-1-ic)\cot(a+bx))}{x} dx$$

Mathematica [A] time = 0.644584, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(c+(-1-ic)\cot(a+bx))}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x, x]
```

Maple [A] time = 0.431, size = 0, normalized size = 0.

$$\int -\frac{\arctan(-c-(-1-ic)\cot(bx+a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x)
```

```
[Out] int(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i \log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}-i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right)}{2\,x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x - 2*I*a)/(c - I
))/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(-c-(-1-I*c)*cot(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\arctan\left(-(-i\,c-1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(-arctan(-(-I*c - 1)*cot(b*x + a) - c)/x, x)
```

3.73 $\int \tan^{-1}(\sinh(x))\,dx$

Optimal. Leaf size=39

$$i\text{PolyLog}(2,-ie^x)-i\text{PolyLog}(2,ie^x)-2x\tan^{-1}(e^x)+x\tan^{-1}(\sinh(x))$$

```
[Out] -2*x*ArcTan[E^x] + x*ArcTan[Sinh[x]] + I*PolyLog[2, (-I)*E^x] - I*PolyLog[2
, I*E^x]
```

Rubi [A] time = 0.0317576, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {5203, 4180, 2279, 2391}

$$i\text{PolyLog}(2,-ie^x)-i\text{PolyLog}(2,ie^x)-2x\tan^{-1}(e^x)+x\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sinh[x]],x]
```

```
[Out] -2*x*ArcTan[E^x] + x*ArcTan[Sinh[x]] + I*PolyLog[2, (-I)*E^x] - I*PolyLog[2
, I*E^x]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \tan^{-1}(\sinh(x))\,dx &= x\tan^{-1}(\sinh(x))-\int x\text{sech}(x)\,dx\\
&= -2x\tan^{-1}(e^x)+x\tan^{-1}(\sinh(x))+i\int \log(1-ie^x)\,dx-i\int \log(1+ie^x)\,dx\\
&= -2x\tan^{-1}(e^x)+x\tan^{-1}(\sinh(x))+i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^x\right)-i\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\right.\\
&= -2x\tan^{-1}(e^x)+x\tan^{-1}(\sinh(x))+i\text{Li}_2(-ie^x)-i\text{Li}_2(ie^x)
\end{aligned}$$

Mathematica [A] time = 0.0308815, size = 64, normalized size = 1.64

$$x\tan^{-1}(\sinh(x)) + i\left(\text{PolyLog}(2, -ie^{-x}) - \text{PolyLog}(2, ie^{-x}) + x(\log(1 - ie^{-x}) - \log(1 + ie^{-x}))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sinh[x]],x]
```

```
[Out] x*ArcTan[Sinh[x]] + I*(x*(Log[1 - I/E^x] - Log[1 + I/E^x]) + PolyLog[2, (-I
)/E^x] - PolyLog[2, I/E^x])
```

Maple [B] time = 0.051, size = 142, normalized size = 3.6

$$x\arctan(\sinh(x)) - i\text{dilog}(-i\cosh(x) - i\sinh(x)) - i(\ln(-i\cosh(x) - i\sinh(x)) - x)\ln\left((1-i)\cosh\left(\frac{x}{2}\right) + (1+i)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(sinh(x)),x)
```

```
[Out] x*arctan(sinh(x))-I*dilog(-I*cosh(x)-I*sinh(x))-I*(ln(-I*cosh(x)-I*sinh(x))
-x)*ln((1-I)*cosh(1/2*x)+(1+I)*sinh(1/2*x))+I*dilog(I*cosh(x)+I*sinh(x))+I*
(ln(I*cosh(x)+I*sinh(x))-x)*ln((1+I)*cosh(1/2*x)+(1-I)*sinh(1/2*x))+1/2*I*(
-ln(-I*cosh(x)-I*sinh(x))+ln(I*cosh(x)+I*sinh(x)))*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right) - 2\int \frac{xe^x}{e^{(2x)} + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(sinh(x)),x, algorithm="maxima")
```

```
[Out] x*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 2*integrate(x*e^x/(e^(2*x) + 1), x)
```

Fricas [B] time = 1.96049, size = 217, normalized size = 5.56

$$x\arctan(\sinh(x)) + ix\log(i\cosh(x) + i\sinh(x) + 1) - ix\log(-i\cosh(x) - i\sinh(x) + 1) - i\,\text{Li}_2(i\cosh(x) + i\sinh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(sinh(x)),x, algorithm="fricas")
```

```
[Out] x*arctan(sinh(x)) + I*x*log(I*cosh(x) + I*sinh(x) + 1) - I*x*log(-I*cosh(x)
 - I*sinh(x) + 1) - I*dilog(I*cosh(x) + I*sinh(x)) + I*dilog(-I*cosh(x) - I
*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{atan}(\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(sinh(x)),x)
```

```
[Out] Integral(atan(sinh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arctan(\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(arctan(sinh(x)), x)
```

3.74 $\int x \tan^{-1}(\sinh(x))\,dx$

Optimal. Leaf size=74

$$ix\text{PolyLog}(2,-ie^x)-ix\text{PolyLog}(2,ie^x)-i\text{PolyLog}(3,-ie^x)+i\text{PolyLog}(3,ie^x)+x^2\left(-\tan^{-1}(e^x)\right)+\frac{1}{2}x^2\tan^{-1}(\sinh($$

```
[Out] -(x^2*ArcTan[E^x]) + (x^2*ArcTan[Sinh[x]])/2 + I*x*PolyLog[2, (-I)*E^x] - I
*x*PolyLog[2, I*E^x] - I*PolyLog[3, (-I)*E^x] + I*PolyLog[3, I*E^x]
```

Rubi [A] time = 0.061271, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5205, 4180, 2531, 2282, 6589}

$$ix\text{PolyLog}(2,-ie^x)-ix\text{PolyLog}(2,ie^x)-i\text{PolyLog}(3,-ie^x)+i\text{PolyLog}(3,ie^x)+x^2\left(-\tan^{-1}(e^x)\right)+\frac{1}{2}x^2\tan^{-1}(\sinh($$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[Sinh[x]],x]
```


```
[Out] -(x^2*ArcTan[E^x]) + (x^2*ArcTan[Sinh[x]])/2 + I*x*PolyLog[2, (-I)*E^x] - I
*x*PolyLog[2, I*E^x] - I*PolyLog[3, (-I)*E^x] + I*PolyLog[3, I*E^x]
```


Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(\sinh(x))\,dx = \frac{1}{2}x^2\tan^{-1}(\sinh(x)) - \frac{1}{2}\int x^2\text{sech}(x)\,dx$$
$$= -x^2\tan^{-1}\left(e^x\right) + \frac{1}{2}x^2\tan^{-1}(\sinh(x)) + i\int x\log\left(1-ie^x\right)\,dx - i\int x\log\left(1+ie^x\right)\,dx$$
$$= -x^2\tan^{-1}\left(e^x\right) + \frac{1}{2}x^2\tan^{-1}(\sinh(x)) + ix\text{Li}_2\left(-ie^x\right) - ix\text{Li}_2\left(ie^x\right) - i\int \text{Li}_2\left(-ie^x\right)\,dx + i\int \text{Li}$$
$$= -x^2\tan^{-1}\left(e^x\right) + \frac{1}{2}x^2\tan^{-1}(\sinh(x)) + ix\text{Li}_2\left(-ie^x\right) - ix\text{Li}_2\left(ie^x\right) - i\,\text{Subst}\left(\int \frac{\text{Li}_2(-ix)}{x}\,dx, x$$
$$= -x^2\tan^{-1}\left(e^x\right) + \frac{1}{2}x^2\tan^{-1}(\sinh(x)) + ix\text{Li}_2\left(-ie^x\right) - ix\text{Li}_2\left(ie^x\right) - i\text{Li}_3\left(-ie^x\right) + i\text{Li}_3\left(ie^x\right)$$

Mathematica [A] time = 0.0253573, size = 105, normalized size = 1.42

$$\frac{1}{2}x^2\tan^{-1}(\sinh(x)) - \frac{1}{2}i\left(-2x\left(\text{PolyLog}\left(2,-ie^{-x}\right) - \text{PolyLog}\left(2,ie^{-x}\right)\right) - 2\left(\text{PolyLog}\left(3,-ie^{-x}\right) - \text{PolyLog}\left(3,ie^{-x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[Sinh[x]],x]
```

```
[Out] (x^2*ArcTan[Sinh[x]])/2 - (I/2)*(-(x^2*(Log[1 - I/E^x] - Log[1 + I/E^x])) -
 2*x*(PolyLog[2, (-I)/E^x] - PolyLog[2, I/E^x]) - 2*(PolyLog[3, (-I)/E^x] -
 PolyLog[3, I/E^x]))
```

Maple [C] time = 0.388, size = 732, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(sinh(x)),x)
```

```
[Out] 1/4*Pi*x^2+1/8*Pi*x^2*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-I)^2)^2-1/8*Pi
*x^2*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+I)^2)^2-1/4*Pi*x^2*csgn(I*(exp(
x)+I))*csgn(I*(exp(x)+I)^2)^2-1/8*Pi*x^2*csgn(I*(exp(x)+I)^2)*csgn(I*exp(-x
)*(exp(x)+I)^2)^2-1/8*Pi*x^2*csgn(exp(-x)*(exp(x)+I)^2)^2-1/8*Pi*x^2*csgn(I
*exp(-x)*(exp(x)+I)^2)*csgn(exp(-x)*(exp(x)+I)^2)^2-1/8*Pi*x^2*csgn(I*(exp(
x)-I))^2*csgn(I*(exp(x)-I)^2)+1/4*Pi*x^2*csgn(I*(exp(x)-I))*csgn(I*(exp(x)-
I)^2)^2+1/8*Pi*x^2*csgn(I*exp(-x)*(exp(x)+I)^2)*csgn(exp(-x)*(exp(x)+I)^2)+
1/8*Pi*x^2*csgn(I*exp(-x)*(exp(x)-I)^2)*csgn(exp(-x)*(exp(x)-I)^2)^2+1/8*Pi
*x^2*csgn(I*(exp(x)-I)^2)*csgn(I*exp(-x)*(exp(x)-I)^2)^2+1/8*Pi*x^2*csgn(I*
(exp(x)+I))^2*csgn(I*(exp(x)+I)^2)+1/8*Pi*x^2*csgn(I*exp(-x)*(exp(x)+I)^2)^
3-I*x*polylog(2,I*exp(x))-1/8*Pi*x^2*csgn(I*(exp(x)-I)^2)^3+1/8*Pi*x^2*csgn
(exp(-x)*(exp(x)+I)^2)^3-1/8*Pi*x^2*csgn(I*(exp(x)-I)^2)*csgn(I*exp(-x))*cs
gn(I*exp(-x)*(exp(x)-I)^2)+1/8*Pi*x^2*csgn(I*(exp(x)+I)^2)*csgn(I*exp(-x))*
csgn(I*exp(-x)*(exp(x)+I)^2)+1/8*Pi*x^2*csgn(exp(-x)*(exp(x)-I)^2)^3-1/8*Pi
```

```
*x^2*csgn(exp(-x)*(exp(x)-I)^2)^2-1/8*Pi*x^2*csgn(I*exp(-x)*(exp(x)-I)^2)^3
-1/2*I*x^2*ln(1-I*exp(x))-1/2*I*x^2*ln(exp(x)-I)+1/2*I*x^2*ln(exp(x)+I)+1/2
*I*x^2*ln(1+I*exp(x))-1/8*Pi*x^2*csgn(I*exp(-x)*(exp(x)-I)^2)*csgn(exp(-x)*
(exp(x)-I)^2)+1/8*Pi*x^2*csgn(I*(exp(x)+I)^2)^3+I*x*polylog(2,-I*exp(x))-I*
polylog(3,-I*exp(x))+I*polylog(3,I*exp(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)-\int\frac{x^2e^x}{e^{(2x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - integrate(x^2*e^x/(e^(2*x) + 1),
 x)
```

Fricas [C] time = 1.97886, size = 351, normalized size = 4.74

$$\frac{1}{2}x^2\arctan\left(\sinh\left(x\right)\right)+\frac{1}{2}ix^2\log\left(i\cosh\left(x\right)+i\sinh\left(x\right)+1\right)-\frac{1}{2}ix^2\log\left(-i\cosh\left(x\right)-i\sinh\left(x\right)+1\right)-ix\mathrm{Li}_2\left(i\cosh\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*arctan(sinh(x)) + 1/2*I*x^2*log(I*cosh(x) + I*sinh(x) + 1) - 1/2*I*
x^2*log(-I*cosh(x) - I*sinh(x) + 1) - I*x*dilog(I*cosh(x) + I*sinh(x)) + I*
x*dilog(-I*cosh(x) - I*sinh(x)) + I*polylog(3, I*cosh(x) + I*sinh(x)) - I*p
olylog(3, -I*cosh(x) - I*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{atan}\left(\sinh\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(sinh(x)),x)
```

```
[Out] Integral(x*atan(sinh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\arctan\left(\sinh\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(sinh(x)), x)
```

3.75 $\int x^2 \tan^{-1}(\sinh(x))\,dx$

Optimal. Leaf size=108

$$ix^2\text{PolyLog}\left(2,-ie^x\right)-ix^2\text{PolyLog}\left(2,ie^x\right)-2ix\text{PolyLog}\left(3,-ie^x\right)+2ix\text{PolyLog}\left(3,ie^x\right)+2i\text{PolyLog}\left(4,-ie^x\right)-2i\text{Po}$$

```
[Out] (-2*x^3*ArcTan[E^x])/3 + (x^3*ArcTan[Sinh[x]])/3 + I*x^2*PolyLog[2, (-I)*E^
x] - I*x^2*PolyLog[2, I*E^x] - (2*I)*x*PolyLog[3, (-I)*E^x] + (2*I)*x*PolyL
og[3, I*E^x] + (2*I)*PolyLog[4, (-I)*E^x] - (2*I)*PolyLog[4, I*E^x]
```

Rubi [A] time = 0.0878868, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {5205, 4180, 2531, 6609, 2282, 6589}

$$ix^2\text{PolyLog}\left(2,-ie^x\right)-ix^2\text{PolyLog}\left(2,ie^x\right)-2ix\text{PolyLog}\left(3,-ie^x\right)+2ix\text{PolyLog}\left(3,ie^x\right)+2i\text{PolyLog}\left(4,-ie^x\right)-2i\text{Po}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[Sinh[x]],x]
```


```
[Out] (-2*x^3*ArcTan[E^x])/3 + (x^3*ArcTan[Sinh[x]])/3 + I*x^2*PolyLog[2, (-I)*E^
x] - I*x^2*PolyLog[2, I*E^x] - (2*I)*x*PolyLog[3, (-I)*E^x] + (2*I)*x*PolyL
og[3, I*E^x] + (2*I)*PolyLog[4, (-I)*E^x] - (2*I)*PolyLog[4, I*E^x]
```


Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
```

```
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}(\sinh(x))\,dx &= \frac{1}{3}x^3\tan^{-1}(\sinh(x))-\frac{1}{3}\int x^3\mathbf{sech}(x)\,dx\\
&= -\frac{2}{3}x^3\tan^{-1}\left(e^x\right)+\frac{1}{3}x^3\tan^{-1}(\sinh(x))+i\int x^2\log\left(1-ie^x\right)\,dx-i\int x^2\log\left(1+ie^x\right)\,dx\\
&= -\frac{2}{3}x^3\tan^{-1}\left(e^x\right)+\frac{1}{3}x^3\tan^{-1}(\sinh(x))+ix^2\mathrm{Li}_2\left(-ie^x\right)-ix^2\mathrm{Li}_2\left(ie^x\right)-2i\int x\mathrm{Li}_2\left(-ie^x\right)\,dx+2i\\
&= -\frac{2}{3}x^3\tan^{-1}\left(e^x\right)+\frac{1}{3}x^3\tan^{-1}(\sinh(x))+ix^2\mathrm{Li}_2\left(-ie^x\right)-ix^2\mathrm{Li}_2\left(ie^x\right)-2ix\mathrm{Li}_3\left(-ie^x\right)+2ix\mathrm{Li}_3\left(i\right.\\
&= -\frac{2}{3}x^3\tan^{-1}\left(e^x\right)+\frac{1}{3}x^3\tan^{-1}(\sinh(x))+ix^2\mathrm{Li}_2\left(-ie^x\right)-ix^2\mathrm{Li}_2\left(ie^x\right)-2ix\mathrm{Li}_3\left(-ie^x\right)+2ix\mathrm{Li}_3\left(i\right.\\
&= -\frac{2}{3}x^3\tan^{-1}\left(e^x\right)+\frac{1}{3}x^3\tan^{-1}(\sinh(x))+ix^2\mathrm{Li}_2\left(-ie^x\right)-ix^2\mathrm{Li}_2\left(ie^x\right)-2ix\mathrm{Li}_3\left(-ie^x\right)+2ix\mathrm{Li}_3\left(i\right.\end{aligned}$$

Mathematica [B] time = 0.107332, size = 356, normalized size = 3.3

$$\frac{1}{192}i\Big(192x^2\mathrm{PolyLog}\left(2,-ie^x\right)+192i\pi x\mathrm{PolyLog}\left(2,ie^x\right)+384x\mathrm{PolyLog}\left(3,-ie^{-x}\right)-384x\mathrm{PolyLog}\left(3,-ie^x\right)-48(\pi-2i$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[Sinh[x]],x]
```

```
[Out] (I/192)*(7*Pi^4 + (8*I)*Pi^3*x + 24*Pi^2*x^2 - (32*I)*Pi*x^3 - 16*x^4 - (64
*I)*x^3*ArcTan[Sinh[x]] + (8*I)*Pi^3*Log[1 + I/E^x] + 48*Pi^2*x*Log[1 + I/E
^x] - (96*I)*Pi*x^2*Log[1 + I/E^x] - 64*x^3*Log[1 + I/E^x] - 48*Pi^2*x*Log[
1 - I*E^x] + (96*I)*Pi*x^2*Log[1 - I*E^x] - (8*I)*Pi^3*Log[1 + I*E^x] + 64*
x^3*Log[1 + I*E^x] + (8*I)*Pi^3*Log[Tan[(Pi + (2*I)*x)/4]] - 48*(Pi - (2*I)
*x)^2*PolyLog[2, (-I)/E^x] + 192*x^2*PolyLog[2, (-I)*E^x] - 48*Pi^2*PolyLog
[2, I*E^x] + (192*I)*Pi*x*PolyLog[2, I*E^x] + (192*I)*Pi*PolyLog[3, (-I)/E^
x] + 384*x*PolyLog[3, (-I)/E^x] - 384*x*PolyLog[3, (-I)*E^x] - (192*I)*Pi*P
olyLog[3, I*E^x] + 384*PolyLog[4, (-I)/E^x] + 384*PolyLog[4, (-I)*E^x])
```

Maple [C] time = 0.227, size = 758, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(sinh(x)),x)
```

```
[Out] 1/6*Pi*x^3+1/12*Pi*x^3*csgn(I*exp(-x)*(exp(x)+I)^2)*csgn(exp(-x)*(exp(x)+I)
^2)+1/12*Pi*x^3*csgn(I*exp(-x)*(exp(x)-I)^2)*csgn(exp(-x)*(exp(x)-I)^2)^2+1
/12*Pi*x^3*csgn(I*(exp(x)+I))^2*csgn(I*(exp(x)+I)^2)-1/6*Pi*x^3*csgn(I*(exp
(x)+I))*csgn(I*(exp(x)+I)^2)^2-1/12*Pi*x^3*csgn(I*(exp(x)+I)^2)*csgn(I*exp(
-x)*(exp(x)+I)^2)^2-1/12*Pi*x^3*csgn(I*exp(-x)*(exp(x)+I)^2)*csgn(exp(-x)*(
exp(x)+I)^2)^2-1/12*Pi*x^3*csgn(I*(exp(x)-I))^2*csgn(I*(exp(x)-I)^2)+1/6*Pi
*x^3*csgn(I*(exp(x)-I))*csgn(I*(exp(x)-I)^2)^2+1/12*Pi*x^3*csgn(I*(exp(x)-I
)^2)*csgn(I*exp(-x)*(exp(x)-I)^2)^2-1/12*Pi*x^3*csgn(I*(exp(x)-I)^2)*csgn(I
*exp(-x))*csgn(I*exp(-x)*(exp(x)-I)^2)+1/12*Pi*x^3*csgn(I*(exp(x)+I)^2)*csg
n(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+I)^2)+1/12*Pi*x^3*csgn(I*exp(-x))*csgn(
I*exp(-x)*(exp(x)-I)^2)^2-1/12*Pi*x^3*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x
)+I)^2)^2-1/12*Pi*x^3*csgn(I*exp(-x)*(exp(x)-I)^2)*csgn(exp(-x)*(exp(x)-I)^
2)+1/12*Pi*x^3*csgn(exp(-x)*(exp(x)+I)^2)^3+1/12*Pi*x^3*csgn(exp(-x)*(exp(x
)-I)^2)^3-1/12*Pi*x^3*csgn(exp(-x)*(exp(x)+I)^2)^2+1/3*I*x^3*ln(1+I*exp(x))
-1/3*I*x^3*ln(exp(x)-I)+1/3*I*x^3*ln(exp(x)+I)-1/3*I*x^3*ln(1-I*exp(x))-I*x
^2*polylog(2,I*exp(x))-2*I*x*polylog(3,-I*exp(x))-1/12*Pi*x^3*csgn(I*exp(-x
)*(exp(x)-I)^2)^3-1/12*Pi*x^3*csgn(I*(exp(x)-I)^2)^3+1/12*Pi*x^3*csgn(I*(ex
p(x)+I)^2)^3+1/12*Pi*x^3*csgn(I*exp(-x)*(exp(x)+I)^2)^3-1/12*Pi*x^3*csgn(ex
p(-x)*(exp(x)-I)^2)^2-2*I*polylog(4,I*exp(x))+I*x^2*polylog(2,-I*exp(x))+2*
I*x*polylog(3,I*exp(x))+2*I*polylog(4,-I*exp(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}x^3\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)-2\int\frac{x^3e^x}{3\left(e^{(2x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 2*integrate(1/3*x^3*e^x/(e^(2*x)
 + 1), x)
```

Fricas [C] time = 1.88555, size = 477, normalized size = 4.42

$$\frac{1}{3}x^3\arctan\left(\sinh\left(x\right)\right)+\frac{1}{3}ix^3\log\left(i\cosh\left(x\right)+i\sinh\left(x\right)+1\right)-\frac{1}{3}ix^3\log\left(-i\cosh\left(x\right)-i\sinh\left(x\right)+1\right)-ix^2\mathrm{Li}_2\left(i\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arctan(sinh(x)) + 1/3*I*x^3*log(I*cosh(x) + I*sinh(x) + 1) - 1/3*I*
x^3*log(-I*cosh(x) - I*sinh(x) + 1) - I*x^2*dilog(I*cosh(x) + I*sinh(x)) +
I*x^2*dilog(-I*cosh(x) - I*sinh(x)) + 2*I*x*polylog(3, I*cosh(x) + I*sinh(x
)) - 2*I*x*polylog(3, -I*cosh(x) - I*sinh(x)) - 2*I*polylog(4, I*cosh(x) +
I*sinh(x)) + 2*I*polylog(4, -I*cosh(x) - I*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{atan}(\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(sinh(x)),x)
```

```
[Out] Integral(x**2*atan(sinh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \arctan(\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(sinh(x)), x)
```

3.76 $\int (e+fx)^3 \tan^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=299

$$\frac{3if^2(e+fx)\text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}-\frac{3if(e+fx)^2\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2}+\frac{3if}{}$$

```
[Out] -((e + f*x)^4*ArcTan[E^(2*a + 2*b*x)])/(4*f) + ((e + f*x)^4*ArcTan[Tanh[a +
 b*x]])/(4*f) + ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((
I/4)*(e + f*x)^3*PolyLog[2, I*E^(2*a + 2*b*x)])/b - (((3*I)/8)*f*(e + f*x)^
2*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/b^2 + (((3*I)/8)*f*(e + f*x)^2*PolyLog[
3, I*E^(2*a + 2*b*x)])/b^2 + (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, (-I)*E^(2*
a + 2*b*x)])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/
b^3 - (((3*I)/16)*f^3*PolyLog[5, (-I)*E^(2*a + 2*b*x)])/b^4 + (((3*I)/16)*f
^3*PolyLog[5, I*E^(2*a + 2*b*x)])/b^4
```

Rubi [A] time = 0.209102, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5183, 4180, 2531, 6609, 2282, 6589}

$$\frac{3if^2(e+fx)\text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}-\frac{3if(e+fx)^2\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2}+\frac{3if}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcTan[Tanh[a + b*x]],x]
```


```
[Out] -((e + f*x)^4*ArcTan[E^(2*a + 2*b*x)])/(4*f) + ((e + f*x)^4*ArcTan[Tanh[a +
 b*x]])/(4*f) + ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((
I/4)*(e + f*x)^3*PolyLog[2, I*E^(2*a + 2*b*x)])/b - (((3*I)/8)*f*(e + f*x)^
2*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/b^2 + (((3*I)/8)*f*(e + f*x)^2*PolyLog[
3, I*E^(2*a + 2*b*x)])/b^2 + (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, (-I)*E^(2*
a + 2*b*x)])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/
b^3 - (((3*I)/16)*f^3*PolyLog[5, (-I)*E^(2*a + 2*b*x)])/b^4 + (((3*I)/16)*f
^3*PolyLog[5, I*E^(2*a + 2*b*x)])/b^4
```


Rule 5183

```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTan[Tanh[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)^3 \tan^{-1}(\tanh(a+bx))\,dx = \frac{(e+fx)^4\tan^{-1}(\tanh(a+bx))}{4f} - \frac{b\int (e+fx)^4 \text{sech}(2a+2bx)\,dx}{4f}$$
$$= -\frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4\tan^{-1}(\tanh(a+bx))}{4f} + \frac{1}{2}i\int (e+fx)^3 \log\left(\right.$$
$$= -\frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4\tan^{-1}(\tanh(a+bx))}{4f} + \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2\cdots}\right.}{4b}$$
$$= -\frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4\tan^{-1}(\tanh(a+bx))}{4f} + \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2\cdots}\right.}{4b}$$
$$= -\frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4\tan^{-1}(\tanh(a+bx))}{4f} + \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2\cdots}\right.}{4b}$$
$$= -\frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4\tan^{-1}(\tanh(a+bx))}{4f} + \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2\cdots}\right.}{4b}$$
$$= -\frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4\tan^{-1}(\tanh(a+bx))}{4f} + \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2\cdots}\right.}{4b}$$

Mathematica [B] time = 4.93567, size = 600, normalized size = 2.01

$$\frac{1}{4}x\left(6e^2fx+4e^3+4ef^2x^2+f^3x^3\right)\tan^{-1}(\tanh(a+bx)) - \frac{i\left(6b^2e^2f\text{PolyLog}\left(3,-ie^{2(a+bx)}\right)-6b^2e^2f\text{PolyLog}\left(3,ie^{2(a+bx)}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTan[Tanh[a + b*x]])/4 - (
(I/16)*(8*b^4*e^3*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 - I
*E^(2*(a + b*x))] + 8*b^4*e*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] + 2*b^4*f^3*
x^4*Log[1 - I*E^(2*(a + b*x))] - 8*b^4*e^3*x*Log[1 + I*E^(2*(a + b*x))] - 1
2*b^4*e^2*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 + I*E^(2
*(a + b*x))] - 2*b^4*f^3*x^4*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*(e + f*x)^3
*PolyLog[2, (-I)*E^(2*(a + b*x))] + 4*b^3*(e + f*x)^3*PolyLog[2, I*E^(2*(a
+ b*x))] + 6*b^2*e^2*f*PolyLog[3, (-I)*E^(2*(a + b*x))] + 12*b^2*e*f^2*x*Po
lyLog[3, (-I)*E^(2*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^(2*(a + b*
x))] - 6*b^2*e^2*f*PolyLog[3, I*E^(2*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3
, I*E^(2*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^(2*(a + b*x))] - 6*b*e*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] - 6*b*f^3*x*PolyLog[4, (-I)*E^(2*(a +
b*x))] + 6*b*e*f^2*PolyLog[4, I*E^(2*(a + b*x))] + 6*b*f^3*x*PolyLog[4, I*E
^(2*(a + b*x))] + 3*f^3*PolyLog[5, (-I)*E^(2*(a + b*x))] - 3*f^3*PolyLog[5,
 I*E^(2*(a + b*x))]))/b^4
```

Maple [C] time = 6.493, size = 7275, normalized size = 24.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arctan(tanh(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\left(f^3x^4 + 4\,ef^2x^3 + 6\,e^2fx^2 + 4\,e^3x\right)\arctan\left(\frac{e^{(2\,bx+2\,a)}-1}{e^{(2\,bx+2\,a)}+1}\right) - \int \frac{\left(bf^3x^4e^{(2\,a)} + 4\,bef^2x^3e^{(2\,a)} + 6\,be^2fx^2e^{(2\,a)} + 4\,be^3x\right.}{2\left(e^{(4\,bx+4\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*arctan((e^(2*b*x + 2*a)
 - 1)/(e^(2*b*x + 2*a) + 1)) - integrate(1/2*(b*f^3*x^4*e^(2*a) + 4*b*e*f^2
*x^3*e^(2*a) + 6*b*e^2*f*x^2*e^(2*a) + 4*b*e^3*x*e^(2*a))*e^(2*b*x)/(e^(4*b
*x + 4*a) + 1), x)
```

Fricas [C] time = 3.09964, size = 4070, normalized size = 13.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(24*I*f^3*polylog(5, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2
4*I*f^3*polylog(5, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I*f
^3*polylog(5, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I*f^3*po
```

```
lylog(5, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^4*f^3*x^4
+ 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*arctan(sinh(b*x + a)/cos
h(b*x + a)) + (-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4
*I*b^3*e^3)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-4*I*b^
3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(-1/2
*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (4*I*b^3*f^3*x^3 + 12*I*b^3*e
*f^2*x^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(1/2*sqrt(-4*I)*(cosh(b*x +
 a) + sinh(b*x + a))) + (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^
2*f*x + 4*I*b^3*e^3)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
 + (-I*b^4*f^3*x^4 - 4*I*b^4*e*f^2*x^3 - 6*I*b^4*e^2*f*x^2 - 4*I*b^4*e^3*x
- 4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(1/2*
sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 - 4*I*b^4*
e*f^2*x^3 - 6*I*b^4*e^2*f*x^2 - 4*I*b^4*e^3*x - 4*I*a*b^3*e^3 + 6*I*a^2*b^2
*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + s
inh(b*x + a)) + 1) + (I*b^4*f^3*x^4 + 4*I*b^4*e*f^2*x^3 + 6*I*b^4*e^2*f*x^2
 + 4*I*b^4*e^3*x + 4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*
a^4*f^3)*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^4*f
^3*x^4 + 4*I*b^4*e*f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x + 4*I*a*b^3*
e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(-1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + (4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f +
 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*
x + a)) + (4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)
*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-4*I*a*b^3*e^3 +
6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(I*sqrt(-4*I) + 2*cosh(
b*x + a) + 2*sinh(b*x + a)) + (-4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3
*b*e*f^2 + I*a^4*f^3)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)
) + (-24*I*b*f^3*x - 24*I*b*e*f^2)*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a)
+ sinh(b*x + a))) + (-24*I*b*f^3*x - 24*I*b*e*f^2)*polylog(4, -1/2*sqrt(4*I
)*(cosh(b*x + a) + sinh(b*x + a))) + (24*I*b*f^3*x + 24*I*b*e*f^2)*polylog(
4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (24*I*b*f^3*x + 24*I*b
*e*f^2)*polylog(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (12*I
*b^2*f^3*x^2 + 24*I*b^2*e*f^2*x + 12*I*b^2*e^2*f)*polylog(3, 1/2*sqrt(4*I)*
(cosh(b*x + a) + sinh(b*x + a))) + (12*I*b^2*f^3*x^2 + 24*I*b^2*e*f^2*x + 1
2*I*b^2*e^2*f)*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) +
 (-12*I*b^2*f^3*x^2 - 24*I*b^2*e*f^2*x - 12*I*b^2*e^2*f)*polylog(3, 1/2*sqr
t(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-12*I*b^2*f^3*x^2 - 24*I*b^2*e*
f^2*x - 12*I*b^2*e^2*f)*polylog(3, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*
x + a))))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e + fx)^3 \operatorname{atan}(\tanh(a + bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*atan(tanh(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**3*atan(tanh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx + e)^3 \arctan(\tanh(bx + a))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(tanh(b*x+a)),x, algorithm="giac")

[Out] integrate((f*x + e)^3*arctan(tanh(b*x + a)), x)
```

3.77 $\int (e+fx)^2 \tan^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=229

$$-\frac{if(e+fx)\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2}+\frac{if(e+fx)\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2}+\frac{if^2\text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3}-\frac{if^2\text{PolyLog}\left(4,ie^{2\ldots}\right.}{8b^3}$$

```
[Out] -((e + f*x)^3*ArcTan[E^(2*a + 2*b*x)])/(3*f) + ((e + f*x)^3*ArcTan[Tanh[a +
 b*x]])/(3*f) + ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((
I/4)*(e + f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b - ((I/4)*f*(e + f*x)*Poly
Log[3, (-I)*E^(2*a + 2*b*x)])/b^2 + ((I/4)*f*(e + f*x)*PolyLog[3, I*E^(2*a
+ 2*b*x)])/b^2 + ((I/8)*f^2*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/b^3 - ((I/8)*
f^2*PolyLog[4, I*E^(2*a + 2*b*x)])/b^3
```

Rubi [A] time = 0.15181, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5183, 4180, 2531, 6609, 2282, 6589}

$$-\frac{if(e+fx)\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2}+\frac{if(e+fx)\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2}+\frac{if^2\text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3}-\frac{if^2\text{PolyLog}\left(4,ie^{2\ldots}\right.}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcTan[Tanh[a + b*x]],x]
```


```
[Out] -((e + f*x)^3*ArcTan[E^(2*a + 2*b*x)])/(3*f) + ((e + f*x)^3*ArcTan[Tanh[a +
 b*x]])/(3*f) + ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((
I/4)*(e + f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b - ((I/4)*f*(e + f*x)*Poly
Log[3, (-I)*E^(2*a + 2*b*x)])/b^2 + ((I/4)*f*(e + f*x)*PolyLog[3, I*E^(2*a
+ 2*b*x)])/b^2 + ((I/8)*f^2*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/b^3 - ((I/8)*
f^2*PolyLog[4, I*E^(2*a + 2*b*x)])/b^3
```


Rule 5183

```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTan[Tanh[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)^2\tan^{-1}(\tanh(a+bx))\,dx = \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} - \frac{b\int (e+fx)^3\text{sech}(2a+2bx)\,dx}{3f}$$

$$= -\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} + \frac{1}{2}i\int (e+fx)^2\,\text{l}$$

$$= -\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} + \frac{i(e+fx)^2\text{Li}_2\left(-\right.}{4b}$$

$$= -\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} + \frac{i(e+fx)^2\text{Li}_2\left(-\right.}{4b}$$

$$= -\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} + \frac{i(e+fx)^2\text{Li}_2\left(-\right.}{4b}$$

$$= -\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} + \frac{i(e+fx)^2\text{Li}_2\left(-\right.}{4b}$$

Mathematica [A] time = 2.62151, size = 375, normalized size = 1.64

$$\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\tan^{-1}(\tanh(a+bx)) - \frac{i\left(-6b^2(e+fx)^2\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+6b^2(e+fx)^2\text{PolyLog}\left(2,ie^{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTan[Tanh[a + b*x]])/3 - ((I/24)*(12*b^3*e
^2*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 - I*E^(2*(a + b*x))]
 + 4*b^3*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] - 12*b^3*e^2*x*Log[1 + I*E^(2*(
a + b*x))] - 12*b^3*e*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*f^2*x^3*Log[
1 + I*E^(2*(a + b*x))] - 6*b^2*(e + f*x)^2*PolyLog[2, (-I)*E^(2*(a + b*x))]
 + 6*b^2*(e + f*x)^2*PolyLog[2, I*E^(2*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I
)*E^(2*(a + b*x))] + 6*b*f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] - 6*b*e*f*P
olyLog[3, I*E^(2*(a + b*x))] - 6*b*f^2*x*PolyLog[3, I*E^(2*(a + b*x))] - 3*
```

```
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] + 3*f^2*PolyLog[4, I*E^(2*(a + b*x))])
)/b^3
```

Maple [C] time = 8.241, size = 5425, normalized size = 23.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arctan(tanh(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\left(f^2x^3+3\,efx^2+3\,e^2x\right)\arctan\left(\frac{e^{(2\,bx+2\,a)}-1}{e^{(2\,bx+2\,a)}+1}\right)-\int\frac{2\left(bf^2x^3e^{(2\,a)}+3\,befx^2e^{(2\,a)}+3\,be^2xe^{(2\,a)}\right)e^{(2\,bx)}}{3\left(e^{(4\,bx+4\,a)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x
+ 2*a) + 1)) - integrate(2/3*(b*f^2*x^3*e^(2*a) + 3*b*e*f*x^2*e^(2*a) + 3*b
*e^2*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1), x)
```

Fricas [C] time = 2.67548, size = 2903, normalized size = 12.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(-6*I*f^2*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6
*I*f^2*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*f^2
*polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*f^2*polyl
og(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^3*f^2*x^3 + 3
*b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(sinh(b*x + a)/cosh(b*x + a)) + (-3*I*b^2
*f^2*x^2 - 6*I*b^2*e*f*x - 3*I*b^2*e^2)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a)
+ sinh(b*x + a))) + (-3*I*b^2*f^2*x^2 - 6*I*b^2*e*f*x - 3*I*b^2*e^2)*dilog(
-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (3*I*b^2*f^2*x^2 + 6*I*b^
2*e*f*x + 3*I*b^2*e^2)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))
) + (3*I*b^2*f^2*x^2 + 6*I*b^2*e*f*x + 3*I*b^2*e^2)*dilog(-1/2*sqrt(-4*I)*(
cosh(b*x + a) + sinh(b*x + a))) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 - 3*I*b
^3*e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(1/2*sqrt(4*I)*(co
sh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 - 3*I
*b^3*e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(-1/2*sqrt(4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3
*I*b^3*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(1/2*sqrt(-4*I
)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 +
```

```
 3*I*b^3*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(-1/2*sqrt(-
4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (3*I*a*b^2*e^2 - 3*I*a^2*b*e*f
+ I*a^3*f^2)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (3*I*a*
b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2
*sinh(b*x + a)) + (-3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(I*sqrt(-
4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-3*I*a*b^2*e^2 + 3*I*a^2*b*e*f
 - I*a^3*f^2)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (6*I
*b*f^2*x + 6*I*b*e*f)*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x +
a))) + (6*I*b*f^2*x + 6*I*b*e*f)*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) + (-6*I*b*f^2*x - 6*I*b*e*f)*polylog(3, 1/2*sqrt(-4*I)*(co
sh(b*x + a) + sinh(b*x + a))) + (-6*I*b*f^2*x - 6*I*b*e*f)*polylog(3, -1/2*
sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e+fx\right)^2 \operatorname{atan}\left(\tanh\left(a+bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*atan(tanh(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**2*atan(tanh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)^2 \arctan\left(\tanh\left(bx+a\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*arctan(tanh(b*x + a)), x)
```

3.78 $\int (e+fx)\tan^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=159

$$-\frac{ifPolyLog\left(3,-ie^{2a+2bx}\right)}{8b^2}+\frac{ifPolyLog\left(3,ie^{2a+2bx}\right)}{8b^2}+\frac{i(e+fx)PolyLog\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i(e+fx)PolyLog\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[Out] -((e + f*x)^2*ArcTan[E^(2*a + 2*b*x)])/(2*f) + ((e + f*x)^2*ArcTan[Tanh[a +
 b*x]])/(2*f) + ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((I/
4)*(e + f*x)*PolyLog[2, I*E^(2*a + 2*b*x)])/b - ((I/8)*f*PolyLog[3, (-I)*E^
(2*a + 2*b*x)])/b^2 + ((I/8)*f*PolyLog[3, I*E^(2*a + 2*b*x)])/b^2
```

Rubi [A] time = 0.0973901, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5183, 4180, 2531, 2282, 6589}

$$-\frac{ifPolyLog\left(3,-ie^{2a+2bx}\right)}{8b^2}+\frac{ifPolyLog\left(3,ie^{2a+2bx}\right)}{8b^2}+\frac{i(e+fx)PolyLog\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i(e+fx)PolyLog\left(2,ie^{2a+2bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] -((e + f*x)^2*ArcTan[E^(2*a + 2*b*x)])/(2*f) + ((e + f*x)^2*ArcTan[Tanh[a +
 b*x]])/(2*f) + ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((I/
4)*(e + f*x)*PolyLog[2, I*E^(2*a + 2*b*x)])/b - ((I/8)*f*PolyLog[3, (-I)*E^
(2*a + 2*b*x)])/b^2 + ((I/8)*f*PolyLog[3, I*E^(2*a + 2*b*x)])/b^2
```

Rule 5183

```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTan[Tanh[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int (e+fx)\tan^{-1}(\tanh(a+bx))\,dx &= \frac{(e+fx)^2\tan^{-1}(\tanh(a+bx))}{2f} - \frac{b\int (e+fx)^2\operatorname{sech}(2a+2bx)\,dx}{2f} \\
&= -\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\tanh(a+bx))}{2f} + \frac{1}{2}i\int (e+fx)\log \\
&= -\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\tanh(a+bx))}{2f} + \frac{i(e+fx)\text{Li}_2\left(-ie\right.}{4b} \\
&= -\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\tanh(a+bx))}{2f} + \frac{i(e+fx)\text{Li}_2\left(-ie\right.}{4b} \\
&= -\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\tanh(a+bx))}{2f} + \frac{i(e+fx)\text{Li}_2\left(-ie\right.}{4b}
\end{aligned}
$$

Mathematica [A] time = 1.70609, size = 278, normalized size = 1.75

$$
-\frac{if\left(-2bx\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+2bx\text{PolyLog}\left(2,ie^{2(a+bx)}\right)+\text{PolyLog}\left(3,-ie^{2(a+bx)}\right)-\text{PolyLog}\left(3,ie^{2(a+bx)}\right)+2b\right.}{8b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] e*x*ArcTan[Tanh[a + b*x]] + (f*x^2*ArcTan[Tanh[a + b*x]])/2 - (e*(-(((-4*I)
*a + Pi - (4*I)*b*x)*(Log[1 - I*E^(2*(a + b*x))] - Log[1 + I*E^(2*(a + b*x)
)])) + ((-4*I)*a + Pi)*Log[Cot[((4*I)*a + Pi + (4*I)*b*x)/4]] - (2*I)*(Poly
Log[2, (-I)*E^(2*(a + b*x))] - PolyLog[2, I*E^(2*(a + b*x))])))/(8*b) - ((I
/8)*f*(2*b^2*x^2*Log[1 - I*E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + I*E^(2*(a +
 b*x))] - 2*b*x*PolyLog[2, (-I)*E^(2*(a + b*x))] + 2*b*x*PolyLog[2, I*E^(2*
(a + b*x))] + PolyLog[3, (-I)*E^(2*(a + b*x))] - PolyLog[3, I*E^(2*(a + b*x
))]))/b^2
```

Maple [C] time = 7.638, size = 2414, normalized size = 15.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arctan(tanh(b*x+a)),x)
```

```
[Out] 1/8*Pi*x^2*f*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3+1/8*Pi*x^2
*f*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3-1/4*Pi*x*e*csgn(I*(e
```

```
xp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I)
/(exp(2*b*x+2*a)+1))^3+1/4*Pi*x*e*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+
2*a)+1))^3-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1
+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*f*csgn((1+I)*(exp(2
*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*f*csgn(I/(exp(2*
b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*f*c
sgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+1
/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b
*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I)/(ex
p(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))+1/8*Pi*x
^2*f*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*
a)+I)/(exp(2*b*x+2*a)+1))-1/4*I/b^2*f*a^2*ln(exp(2*b*x+2*a)+I)-1/8*Pi*x^2*f
*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2
+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp
(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+1/4*Pi*x*e*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-1/4*Pi*x*e*csgn((1+I)*(exp(
2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-1/4*I*ln(exp(2*b*x+2*a)-I)*f*x^2-1/2*I*
ln(exp(2*b*x+2*a)-I)*e*x-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+
2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2+1/4*I*f/b^2*a^
2*ln(-exp(2*b*x+2*a)+I)+1/2*I*(1/2*f*x^2+e*x)*ln(exp(2*b*x+2*a)+I)+1/8*I*f*
polylog(3,I*exp(2*b*x+2*a))/b^2-1/2*I*e/b*(b*x+a)*ln(((-I)^(1/2)+exp(b*x+a)
)/(-I)^(1/2))-1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a
)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))+1/8*Pi*f*x^2+1/4*Pi*e*x
-1/2*I/b*e*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I/b*e*dilog(((-I)^
(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*e/b*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*
I*e/b*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/4*Pi*x*e*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2+1/4*Pi*x*e*csgn(I*(exp(2*b*
x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-1/4*Pi*x*e*csgn(
I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-1/4*P
i*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2
*a)-I)/(exp(2*b*x+2*a)+1))+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+
2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))+1/2*I/b^2*f*a*di
log(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I/b^2*f*a*dilog(((-I)^(1/2)+exp
(b*x+a))/(-I)^(1/2))+1/2*I*e/b*(b*x+a)*ln(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*e/
b*(b*x+a)*ln(1-exp(b*x+a)*(-1)^(3/4))+1/4*I*f/b^2*(b*x+a)^2*ln(1+I*exp(2*b*
x+2*a))+1/4*I*f/b^2*(b*x+a)*polylog(2,-I*exp(2*b*x+2*a))-1/2*I*f/b^2*a*dilo
g(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*f/b^2*a*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/4
*I*f/b^2*(b*x+a)^2*ln(1-I*exp(2*b*x+2*a))-1/4*I*f/b^2*(b*x+a)*polylog(2,I*e
xp(2*b*x+2*a))-1/2*I*e/b*(b*x+a)*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/8
*Pi*x^2*f*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-1/4*Pi*x*e*cs
gn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+1/4*Pi*x*e*csgn((1+I)*(ex
p(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I
)/(exp(2*b*x+2*a)+1))^3+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2
*a)+1))^3+1/2*I/b*a*e*ln(exp(2*b*x+2*a)+I)-1/2*I*e/b*a*ln(-exp(2*b*x+2*a)+I
)+1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I
*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))-1/4*Pi*x*e*csgn(I/(exp(2*b*x+2*a)+1
))*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))
+1/4*Pi*x*e*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(e
xp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))-1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b
^2+1/2*I*f/b^2*a*(b*x+a)*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*f/b^2
*a*(b*x+a)*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/2*I*f/b^2*a*(b*x+a)*ln(
1+exp(b*x+a)*(-1)^(3/4))-1/2*I*f/b^2*a*(b*x+a)*ln(1-exp(b*x+a)*(-1)^(3/4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(fx^2 + 2\,ex\right)\arctan\left(\frac{e^{(2\,bx+2\,a)} - 1}{e^{(2\,bx+2\,a)} + 1}\right) - \int \frac{\left(bfx^2e^{(2\,a)} + 2\,bexe^{(2\,a)}\right)e^{(2\,bx)}}{e^{(4\,bx+4\,a)} + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctan(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*e*x)*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)) - i
ntegrate((b*f*x^2*e^(2*a) + 2*b*e*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1
), x)
```

Fricas [C] time = 2.32183, size = 1894, normalized size = 11.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctan(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*f*x^2 + 2*b^2*e*x)*arctan(sinh(b*x + a)/cosh(b*x + a)) + (-2*I*
b*f*x - 2*I*b*e)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-2
*I*b*f*x - 2*I*b*e)*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) +
 (2*I*b*f*x + 2*I*b*e)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))
) + (2*I*b*f*x + 2*I*b*e)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x +
 a))) + (-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e + I*a^2*f)*log(1/2*sqrt(4*I
)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*
a*b*e + I*a^2*f)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) +
(I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I*a^2*f)*log(1/2*sqrt(-4*I)*(cosh(
b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I
*a^2*f)*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (2*I*a*b
*e - I*a^2*f)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (2*I*a
*b*e - I*a^2*f)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-2
*I*a*b*e + I*a^2*f)*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) +
 (-2*I*a*b*e + I*a^2*f)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x +
a)) + 2*I*f*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I
*f*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*f*polyl
og(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*f*polylog(3, -1
/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e + fx\right) \operatorname{atan}{\left(\tanh{\left(a + bx \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*atan(tanh(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*atan(tanh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx + e\right) \arctan\left(\tanh\left(bx + a\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctan(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*arctan(tanh(b*x + a)), x)
```

3.79 $\int \tan^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=74

$$\frac{i\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}-x\tan^{-1}\left(e^{2a+2bx}\right)+x\tan^{-1}(\tanh(a+bx))$$

```
[Out] -(x*ArcTan[E^(2*a + 2*b*x)]) + x*ArcTan[Tanh[a + b*x]] + ((I/4)*PolyLog[2,
(-I)*E^(2*a + 2*b*x)])/b - ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b
```

Rubi [A] time = 0.0424545, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5179, 4180, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}-x\tan^{-1}\left(e^{2a+2bx}\right)+x\tan^{-1}(\tanh(a+bx))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Tanh[a + b*x]],x]
```

```
[Out] -(x*ArcTan[E^(2*a + 2*b*x)]) + x*ArcTan[Tanh[a + b*x]] + ((I/4)*PolyLog[2,
(-I)*E^(2*a + 2*b*x)])/b - ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b
```

Rule 5179

```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTan[Tanh[a + b
*x]], x] - Dist[b, Int[x*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tan^{-1}(\tanh(a+bx))\,dx = x\tan^{-1}(\tanh(a+bx)) - b\int x\text{sech}(2a+2bx)\,dx$$
$$= -x\tan^{-1}\left(e^{2a+2bx}\right) + x\tan^{-1}(\tanh(a+bx)) + \frac{1}{2}i\int \log\left(1-ie^{2a+2bx}\right)dx - \frac{1}{2}i\int \log\left(1+ie^{2a+2bx}\right)dx$$
$$= -x\tan^{-1}\left(e^{2a+2bx}\right) + x\tan^{-1}(\tanh(a+bx)) + \frac{i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}dx, x, e^{2a+2bx}\right)}{4b} - \frac{i\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}dx, x, e^{2a+2bx}\right)}{4b}$$
$$= -x\tan^{-1}\left(e^{2a+2bx}\right) + x\tan^{-1}(\tanh(a+bx)) + \frac{i\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b} - \frac{i\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}$$

Mathematica [A] time = 0.0618684, size = 132, normalized size = 1.78

$$x\tan^{-1}(\tanh(a+bx)) - \frac{-2i\left(\text{PolyLog}\left(2,-ie^{2(a+bx)}\right) - \text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right) - (-4ia-4ibx+\pi)\left(\log\left(1-ie^{2(a+bx)}\right) - \log\left(1+ie^{2(a+bx)}\right)\right) + \ldots}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[Tanh[a + b*x]] - (-(((-4*I)*a + Pi - (4*I)*b*x)*(Log[1 - I*E^(2*(a
 + b*x))] - Log[1 + I*E^(2*(a + b*x))])) + ((-4*I)*a + Pi)*Log[Cot[((4*I)*a
 + Pi + (4*I)*b*x)/4]] - (2*I)*(PolyLog[2, (-I)*E^(2*(a + b*x))] - PolyLog[
2, I*E^(2*(a + b*x))]))/(8*b)
```

Maple [B] time = 0.121, size = 440, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(tanh(b*x+a)),x)
```

```
[Out] 1/b*arctanh(tanh(b*x+a))*arctan(tanh(b*x+a))-1/4*I/b*dilog(-I*cosh(2*arctan
h(tanh(b*x+a)))-I*sinh(2*arctanh(tanh(b*x+a))))+1/2*I/b*ln((1-I)/(1-tanh(b*
x+a)^2)^(1/2)+(1+I)*tanh(b*x+a)/(1-tanh(b*x+a)^2)^(1/2))*arctanh(tanh(b*x+a
))-1/4*I/b*ln((1-I)/(1-tanh(b*x+a)^2)^(1/2)+(1+I)*tanh(b*x+a)/(1-tanh(b*x+a
)^2)^(1/2))*ln(-I*cosh(2*arctanh(tanh(b*x+a)))-I*sinh(2*arctanh(tanh(b*x+a)
)))+1/4*I/b*dilog(I*cosh(2*arctanh(tanh(b*x+a)))+I*sinh(2*arctanh(tanh(b*x+
a))))-1/2*I/b*ln((1+I)/(1-tanh(b*x+a)^2)^(1/2)+(1-I)*tanh(b*x+a)/(1-tanh(b*
x+a)^2)^(1/2))*arctanh(tanh(b*x+a))+1/4*I/b*ln((1+I)/(1-tanh(b*x+a)^2)^(1/2
)+(1-I)*tanh(b*x+a)/(1-tanh(b*x+a)^2)^(1/2))*ln(I*cosh(2*arctanh(tanh(b*x+a
)))+I*sinh(2*arctanh(tanh(b*x+a))))-1/4*I/b*arctanh(tanh(b*x+a))*ln(-I*cosh
(2*arctanh(tanh(b*x+a)))-I*sinh(2*arctanh(tanh(b*x+a))))+1/4*I/b*arctanh(ta
nh(b*x+a))*ln(I*cosh(2*arctanh(tanh(b*x+a)))+I*sinh(2*arctanh(tanh(b*x+a)))
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x\arctan\left(\frac{e^{(2\,bx+2\,a)}-1}{e^{(2\,bx+2\,a)}+1}\right) - 2\,b\int \frac{xe^{(2\,bx+2\,a)}}{e^{(4\,bx+4\,a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] x*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)) - 2*b*integrate(x*e^(
2*b*x + 2*a)/(e^(4*b*x + 4*a) + 1), x)
```

Fricas [B] time = 2.10203, size = 1098, normalized size = 14.84

$$\frac{2\,bx\arctan\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)+(-i\,bx-i\,a)\log\left(\frac{1}{2}\sqrt{4i}(\cosh(bx+a)+\sinh(bx+a))+1\right)+(-i\,bx-i\,a)\log\left(-\frac{1}{2}\sqrt{4i}(\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(sinh(b*x + a)/cosh(b*x + a)) + (-I*b*x - I*a)*log(1/2*sqr
t(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt(
4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(1/2*sqrt(-4*I
)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(-1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + I*a*log(I*sqrt(4*I) + 2*cosh(b*x + a
) + 2*sinh(b*x + a)) + I*a*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x
+ a)) - I*a*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - I*a*log
(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - I*dilog(1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) + I*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
+ I*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{atan}(\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(tanh(b*x+a)),x)
```

```
[Out] Integral(atan(tanh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arctan(\tanh(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(tanh(b*x + a)), x)
```

3.80 $\int \frac{\tan^{-1}(\tanh(a+bx))}{e+fx}\,dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\tan^{-1}(\tanh(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate[ArcTan[Tanh[a + b*x]]/(e + f*x), x]
```

Rubi [A] time = 0.0378732, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(\tanh(a+bx))}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[Tanh[a + b*x]]/(e + f*x),x]
```

```
[Out] Defer[Int][ArcTan[Tanh[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\tan^{-1}(\tanh(a+bx))}{e+fx}\,dx$$

Mathematica [A] time = 6.44085, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(\tanh(a+bx))}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[Tanh[a + b*x]]/(e + f*x),x]
```

```
[Out] Integrate[ArcTan[Tanh[a + b*x]]/(e + f*x), x]
```

Maple [A] time = 0.606, size = 0, normalized size = 0.

$$\int \frac{\arctan(\tanh(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(tanh(b*x+a))/(f*x+e),x)
```

```
[Out] int(arctan(tanh(b*x+a))/(f*x+e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(\tanh(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(arctan(tanh(b*x + a))/(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(\tanh(bx+a))}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(arctan(tanh(b*x + a))/(f*x + e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}(\tanh(a+bx))}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(tanh(b*x+a))/(f*x+e),x)
```

```
[Out] Integral(atan(tanh(a + b*x))/(e + f*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(\tanh(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(arctan(tanh(b*x + a))/(f*x + e), x)
```

3.81 $\int x^2 \tan^{-1}(c + d\tanh(a + bx))\,dx$

Optimal. Leaf size=355

$$-\frac{ix\text{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}+\frac{ix\text{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^3}-\frac{i\text{PolyLog}\left(4,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^3}$$

```
[Out] (x^3*ArcTan[c + d*Tanh[a + b*x]])/3 + (I/6)*x^3*Log[1 + ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/6)*x^3*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x^2*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c
+ d))])/b - ((I/4)*x^2*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c -
d))])/b - ((I/4)*x*PolyLog[3, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))]
)/b^2 + ((I/4)*x*PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/
b^2 + ((I/8)*PolyLog[4, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b^3
- ((I/8)*PolyLog[4, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b^3
```

Rubi [A] time = 0.461361, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5199, 2190, 2531, 6609, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}+\frac{ix\text{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^3}-\frac{i\text{PolyLog}\left(4,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Tanh[a + b*x]])/3 + (I/6)*x^3*Log[1 + ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/6)*x^3*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x^2*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c
+ d))])/b - ((I/4)*x^2*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c -
d))])/b - ((I/4)*x*PolyLog[3, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))]
)/b^2 + ((I/4)*x*PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/
b^2 + ((I/8)*PolyLog[4, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b^3
- ((I/8)*PolyLog[4, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b^3
```

Rule 5199

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tanh[a + b*x]])/(f*(m
 + 1)), x] + (Dist[(I*b*(I - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^
(2*a + 2*b*x))/(I - c + d + (I - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[(I*
b*(I + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(I + c
- d + (I + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\tan^{-1}(c+d\tanh(a+bx))\,dx=\frac{1}{3}x^3\tan^{-1}(c+d\tanh(a+bx))+\frac{1}{3}(b(1-i(c+d)))\int\frac{e^{2a+2bx}x^3}{i+c-d+(i+c+d)e^{2a+}}$$

$$=\frac{1}{3}x^3\tan^{-1}(c+d\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{6}ix^3\log\left(1\right.$$

$$=\frac{1}{3}x^3\tan^{-1}(c+d\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{6}ix^3\log\left(1\right.$$

$$=\frac{1}{3}x^3\tan^{-1}(c+d\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{6}ix^3\log\left(1\right.$$

$$=\frac{1}{3}x^3\tan^{-1}(c+d\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{6}ix^3\log\left(1\right.$$

$$=\frac{1}{3}x^3\tan^{-1}(c+d\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{6}ix^3\log\left(1\right.$$

Mathematica [A] time = 5.225, size = 305, normalized size = 0.86

$$\frac{1}{3}x^3\tan^{-1}(d\tanh(a+bx)+c)+\frac{i\left(6b^2x^2\text{PolyLog}\left(2,-\frac{(c+d-i)e^{2(a+bx)}}{c-d-i}\right)-6b^2x^2\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2(a+bx)}}{c-d+i}\right)-6bx\text{PolyL}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Tanh[a + b*x]])/3 + ((I/24)*(4*b^3*x^3*Log[1 + ((-I + c +
 d)*E^(2*(a + b*x)))/(-I + c - d)] - 4*b^3*x^3*Log[1 + ((I + c + d)*E^(2*(a
 + b*x)))/(I + c - d)] + 6*b^2*x^2*PolyLog[2, -(((-I + c + d)*E^(2*(a + b*x
)))/(-I + c - d))] - 6*b^2*x^2*PolyLog[2, -(((I + c + d)*E^(2*(a + b*x)))/(
I + c - d))] - 6*b*x*PolyLog[3, -(((-I + c + d)*E^(2*(a + b*x)))/(-I + c -
d))] + 6*b*x*PolyLog[3, -(((I + c + d)*E^(2*(a + b*x)))/(I + c - d))] + 3*P
olyLog[4, -(((-I + c + d)*E^(2*(a + b*x)))/(-I + c - d))] - 3*PolyLog[4, -(
((I + c + d)*E^(2*(a + b*x)))/(I + c - d))]))/b^3
```

Maple [C] time = 6.736, size = 6981, normalized size = 19.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+d*tanh(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}x^3\arctan\left(\frac{\left(ce^{(2a)}+de^{(2a)}\right)e^{(2bx)}+c-d}{e^{(2bx+2a)}+1}\right)-4bd\int\frac{x^3e^{(2bx+2a)}}{3\left(c^2-2cd+d^2+\left(c^2e^{(4a)}+2cde^{(4a)}+d^2e^{(4a)}+e^{(4a)}\right)e^{(4bx)}+2\left(c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) + c - d)/(e^(2*b*x + 2*a)
 + 1)) - 4*b*d*integrate(1/3*x^3*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*
e^(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a)
 - d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Fricas [C] time = 2.92378, size = 3641, normalized size = 10.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arctan((c*cosh(b*x + a) + d*sinh(b*x + a))/cosh(b*x + a)) +
3*I*b^2*x^2*dilog(1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2
+ 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*I*b^2*x^2*dilog(-1/2*sqrt(-(4*c^
2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x +
 a))) - 3*I*b^2*x^2*dilog(1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*
d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 3*I*b^2*x^2*dilog(-1/2*sqr
t(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + si
nh(b*x + a))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2
+ 2*c*d + d^2 + 1)*sinh(b*x + a) + (c^2 - d^2 - 2*I*d + 1)*sqrt(-(4*c^2 - 4
*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) - I*a^3*log(2*(c^2 + 2*c*d + d^
```

```
2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2
 - 2*I*d + 1)*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) +
 I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 +
 1)*sinh(b*x + a) + (c^2 - d^2 + 2*I*d + 1)*sqrt(-(4*c^2 - 4*d^2 - 8*I*d +
4)/(c^2 - 2*c*d + d^2 + 1))) + I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x
 + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 + 2*I*d + 1)*s
qrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) - 6*I*b*x*polylo
g(3, 1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b
*x + a) + sinh(b*x + a))) - 6*I*b*x*polylog(3, -1/2*sqrt(-(4*c^2 - 4*d^2 +
8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*
b*x*polylog(3, 1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1
))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*b*x*polylog(3, -1/2*sqrt(-(4*c^2
- 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a
))) + (I*b^3*x^3 + I*a^3)*log(1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 -
2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*x^3 + I*a^3
)*log(-1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh
(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*x^3 - I*a^3)*log(1/2*sqrt(-(4*c^2
 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x +
a)) + 1) + (-I*b^3*x^3 - I*a^3)*log(-1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/
(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*I*polylog
(4, 1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*
x + a) + sinh(b*x + a))) + 6*I*polylog(4, -1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d
 + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*polyl
og(4, 1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(
b*x + a) + sinh(b*x + a))) - 6*I*polylog(4, -1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I
*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \arctan\left(d \tanh\left(bx + a\right) + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(d*tanh(b*x + a) + c), x)
```

3.82 $\int x\tan^{-1}(c+d\tanh(a+bx))\,dx$

Optimal. Leaf size=267

$$-\frac{i\text{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^2}+\frac{i\text{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{ix\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}$$

```
[Out] (x^2*ArcTan[c + d*Tanh[a + b*x]])/2 + (I/4)*x^2*Log[1 + ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/4)*x^2*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c +
d))])/b - ((I/4)*x*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))]
)/b - ((I/8)*PolyLog[3, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b^2
+ ((I/8)*PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b^2
```

Rubi [A] time = 0.373405, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5199, 2190, 2531, 2282, 6589}

$$-\frac{i\text{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^2}+\frac{i\text{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{ix\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + d*Tanh[a + b*x]],x]
```


```
[Out] (x^2*ArcTan[c + d*Tanh[a + b*x]])/2 + (I/4)*x^2*Log[1 + ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/4)*x^2*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c +
d))])/b - ((I/4)*x*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))]
)/b - ((I/8)*PolyLog[3, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b^2
+ ((I/8)*PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b^2
```


Rule 5199

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tanh[a + b*x]])/(f*(m
 + 1)), x] + (Dist[(I*b*(I - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^
(2*a + 2*b*x))/(I - c + d + (I - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[(I*
b*(I + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(I + c
- d + (I + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+d\tanh(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}(b(1-i(c+d)))\int \frac{e^{2a+2bx}x^2}{i+c-d+(i+c+d)e^{2a+2}}$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{4}ix^2\log\left(1+\right.$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{4}ix^2\log\left(1+\right.$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{4}ix^2\log\left(1+\right.$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{4}ix^2\log\left(1+\right.$$

Mathematica [A] time = 3.9925, size = 229, normalized size = 0.86

$$\frac{1}{2}x^2\tan^{-1}(d\tanh(a+bx)+c) + \frac{i\left(2bx\text{PolyLog}\left(2,-\frac{(c+d-i)e^{2(a+bx)}}{c-d-i}\right) - 2bx\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2(a+bx)}}{c-d+i}\right) - \text{PolyLog}\left(3,-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Tanh[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + ((-I + c +
d)*E^(2*(a + b*x)))/(-I + c - d)] - 2*b^2*x^2*Log[1 + ((I + c + d)*E^(2*(a
+ b*x)))/(I + c - d)] + 2*b*x*PolyLog[2, -(((-I + c + d)*E^(2*(a + b*x)))/(
-I + c - d))] - 2*b*x*PolyLog[2, -(((I + c + d)*E^(2*(a + b*x)))/(I + c - d
))] - PolyLog[3, -(((-I + c + d)*E^(2*(a + b*x)))/(-I + c - d))] + PolyLog[
3, -(((I + c + d)*E^(2*(a + b*x)))/(I + c - d))]))/b^2
```

Maple [C] time = 16.382, size = 6640, normalized size = 24.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+d*tanh(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,x^2\arctan\left(\frac{\left(ce^{(2\,a)}+de^{(2\,a)}\right)e^{(2\,bx)}+c-d}{e^{(2\,bx+2\,a)}+1}\right)-2\,bd\int\frac{x^2e^{(2\,bx+2\,a)}}{c^2-2\,cd+d^2+\left(c^2e^{(4\,a)}+2\,cde^{(4\,a)}+d^2e^{(4\,a)}+e^{(4\,a)}\right)e^{(4\,bx)}+2\left(c^2e^{(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) + c - d)/(e^(2*b*x + 2*a)
 + 1)) - 2*b*d*integrate(x^2*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4
*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) - d
^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Fricas [C] time = 2.63717, size = 2985, normalized size = 11.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arctan((c*cosh(b*x + a) + d*sinh(b*x + a))/cosh(b*x + a)) +
2*I*b*x*dilog(1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1)
)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*b*x*dilog(-1/2*sqrt(-(4*c^2 - 4*d^
2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) -
2*I*b*x*dilog(1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1)
)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*b*x*dilog(-1/2*sqrt(-(4*c^2 - 4*d^
2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) +
I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 +
1)*sinh(b*x + a) + (c^2 - d^2 - 2*I*d + 1)*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4
)/(c^2 - 2*c*d + d^2 + 1))) + I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x
+ a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 - 2*I*d + 1)*sq
rt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) - I*a^2*log(2*(c^
2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a
) + (c^2 - d^2 + 2*I*d + 1)*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d
+ d^2 + 1))) - I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 +
 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 + 2*I*d + 1)*sqrt(-(4*c^2 - 4*
d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) + (I*b^2*x^2 - I*a^2)*log(1/2*sq
rt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + s
inh(b*x + a)) + 1) + (I*b^2*x^2 - I*a^2)*log(-1/2*sqrt(-(4*c^2 - 4*d^2 + 8*
I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-
I*b^2*x^2 + I*a^2)*log(1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d +
 d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*x^2 + I*a^2)*log(
-1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x +
 a) + sinh(b*x + a)) + 1) - 2*I*polylog(3, 1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d
 + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*polyl
og(3, -1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh
```

```
(b*x + a) + sinh(b*x + a))) + 2*I*polylog(3, 1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I
*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*pol
ylog(3, -1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(co
sh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \arctan\left(d \tanh\left(bx + a\right) + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(d*tanh(b*x + a) + c), x)
```

3.83 $\int \tan^{-1}(c + d\tanh(a + bx))\,dx$

Optimal. Leaf size=174

$$\frac{i\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b} - \frac{i\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b} + \frac{1}{2}ix\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right) - \frac{1}{2}ix\log\left(1+\frac{(c+d+}{c-d}\right.$$

```
[Out] x*ArcTan[c + d*Tanh[a + b*x]] + (I/2)*x*Log[1 + ((I - c - d)*E^(2*a + 2*b*x
))/(I - c + d)] - (I/2)*x*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)
] + ((I/4)*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b - ((
I/4)*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b
```

Rubi [A] time = 0.230635, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5191, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b} - \frac{i\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b} + \frac{1}{2}ix\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right) - \frac{1}{2}ix\log\left(1+\frac{(c+d+}{c-d}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Tanh[a + b*x]] + (I/2)*x*Log[1 + ((I - c - d)*E^(2*a + 2*b*x
))/(I - c + d)] - (I/2)*x*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)
] + ((I/4)*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b - ((
I/4)*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b
```

Rule 5191

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tan[c + d*Tanh[a + b*x]], x] + (Dist[I*b*(I - c - d), Int[(x*E^(2*a + 2*b*x
))/(I - c + d + (I - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[I*b*(I + c + d)
, Int[(x*E^(2*a + 2*b*x))/(I + c - d + (I + c + d)*E^(2*a + 2*b*x)), x], x]
) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tan^{-1}(c+d\tanh(a+bx))\,dx = x\tan^{-1}(c+d\tanh(a+bx)) + (b(1-i(c+d)))\int \frac{e^{2a+2bx}x}{i+c-d+(i+c+d)e^{2a+2bx}}\,dx -$$
$$= x\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{2}ix\log\left(1+\frac{(i+c}{i}\right.$$
$$= x\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{2}ix\log\left(1+\frac{(i+c}{i}\right.$$
$$= x\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{2}ix\log\left(1+\frac{(i+c}{i}\right.$$

Mathematica [A] time = 3.90743, size = 288, normalized size = 1.66

$$\frac{d\text{PolyLog}\left(2,-\frac{(c^2+2cd+d^2+1)e^{2(a+bx)}}{c^2-d^2+2\sqrt{-d^2}+1}\right) - d\text{PolyLog}\left(2,\frac{(c^2+2cd+d^2+1)e^{2(a+bx)}}{-c^2+d^2+2\sqrt{-d^2}-1}\right) - 2d(a+bx)\log\left(\frac{2((c+d)^2+1)e^{2(a+bx)}}{2c^2-2d^2-4\sqrt{-d^2}+2}+1\right) + 2d(a}{4b\sqrt{-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Tanh[a + b*x]] + (4*a*Sqrt[-d^2]*ArcTan[(1 + c^2 - d^2 + (1
+ c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(2*d)] - 2*d*(a + b*x)*Log[1 + (2*(1
+ (c + d)^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] + 2*d*(a
+ b*x)*Log[1 + ((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d
^2])] + d*PolyLog[2, -(((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 -
 d^2 + 2*Sqrt[-d^2]))] - d*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*
x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])])/(4*b*Sqrt[-d^2])
```

Maple [B] time = 0.085, size = 350, normalized size = 2.

$$\frac{\arctan(c+d\tanh(bx+a))\ln(d\tanh(bx+a)+d)}{2b} - \frac{\arctan(c+d\tanh(bx+a))\ln(d\tanh(bx+a)-d)}{2b} - \frac{\frac{i}{4}\ln(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*tanh(b*x+a)),x)
```

```
[Out] 1/2/b*arctan(c+d*tanh(b*x+a))*ln(d*tanh(b*x+a)+d)-1/2/b*arctan(c+d*tanh(b*x
+a))*ln(d*tanh(b*x+a)-d)-1/4*I/b*ln(d*tanh(b*x+a)-d)*ln((-d*tanh(b*x+a)+I-c
)/(I-c-d))+1/4*I/b*ln(d*tanh(b*x+a)-d)*ln((d*tanh(b*x+a)+c+I)/(I+c+d))-1/4*
I/b*dilog((-d*tanh(b*x+a)+I-c)/(I-c-d))+1/4*I/b*dilog((d*tanh(b*x+a)+c+I)/(
I+c+d))+1/4*I/b*ln(d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)+I-c)/(I-c+d))-1/4*I/
b*ln(d*tanh(b*x+a)+d)*ln((d*tanh(b*x+a)+c+I)/(I+c-d))+1/4*I/b*dilog((-d*tan
h(b*x+a)+I-c)/(I-c+d))-1/4*I/b*dilog((d*tanh(b*x+a)+c+I)/(I+c-d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-4\,bd\int \frac{xe^{(2\,bx+2\,a)}}{c^2-2\,cd+d^2+\left(c^2e^{(4\,a)}+2\,cde^{(4\,a)}+d^2e^{(4\,a)}+e^{(4\,a)}\right)e^{(4\,bx)}+2\left(c^2e^{(2\,a)}-d^2e^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+1}\,dx + x\,\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -4*b*d*integrate(x*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a) + 2*c*
d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) - d^2*e^(2*a)
 + e^(2*a))*e^(2*b*x) + 1), x) + x*arctan(((c*e^(2*a) + d*e^(2*a))*e^(2*b*x
) + c - d)/(e^(2*b*x + 2*a) + 1))
```

Fricas [B] time = 5.62637, size = 2288, normalized size = 13.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan((c*cosh(b*x + a) + d*sinh(b*x + a))/cosh(b*x + a)) - I*a*
log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sin
h(b*x + a) + (c^2 - d^2 - 2*I*d + 1)*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2
 - 2*c*d + d^2 + 1))) - I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2
*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 - 2*I*d + 1)*sqrt(-(4*c
^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) + I*a*log(2*(c^2 + 2*c*d
+ d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + (c^2 -
 d^2 + 2*I*d + 1)*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1)
)) + I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2
 + 1)*sinh(b*x + a) - (c^2 - d^2 + 2*I*d + 1)*sqrt(-(4*c^2 - 4*d^2 - 8*I*d
+ 4)/(c^2 - 2*c*d + d^2 + 1))) + (I*b*x + I*a)*log(1/2*sqrt(-(4*c^2 - 4*d^2
 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1)
 + (I*b*x + I*a)*log(-1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d +
d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(1/2*sqr
t(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + si
nh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d +
4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + I*dilog(
1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x +
a) + sinh(b*x + a))) + I*dilog(-1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2
- 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(1/2*sqrt(-(4
*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*
x + a))) - I*dilog(-1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^
2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arctan\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(d*tanh(b*x + a) + c), x)
```

3.84 $\int \frac{\tan^{-1}(c+d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\tan^{-1}(d\tanh(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTan[c + d*Tanh[a + b*x]]/x, x]
```

Rubi [A] time = 0.126024, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + d*Tanh[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\tan^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Mathematica [A] time = 8.14036, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + d*Tanh[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTan[c + d*Tanh[a + b*x]]/x, x]
```

Maple [A] time = 0.358, size = 0, normalized size = 0.

$$\int \frac{\arctan(c+d\tanh(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*tanh(b*x+a))/x,x)
```

```
[Out] int(arctan(c+d*tanh(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(d\tanh(bx+a)+c)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctan(d*tanh(b*x + a) + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(d\tanh(bx+a)+c)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(d*tanh(b*x + a) + c)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*tanh(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(d\tanh(bx+a)+c)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(d*tanh(b*x + a) + c)/x, x)
```

3.85 $\int x^2 \tan^{-1}(c + (i + c)\tanh(a + bx))\,dx$

Optimal. Leaf size=142

$$-\frac{ix\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{1}{3}x^3\ \text{ta}$$

```
[Out] (-I/12)*b*x^4 + (x^3*ArcTan[c + (I + c)*Tanh[a + b*x]])/3 + (I/6)*x^3*Log[1
 + I*c*E^(2*a + 2*b*x)] + ((I/4)*x^2*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
- ((I/4)*x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2 + ((I/8)*PolyLog[4, (-I)
*c*E^(2*a + 2*b*x)])/b^3
```

Rubi [A] time = 0.228086, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5195, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{1}{3}x^3\ \text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (-I/12)*b*x^4 + (x^3*ArcTan[c + (I + c)*Tanh[a + b*x]])/3 + (I/6)*x^3*Log[1
 + I*c*E^(2*a + 2*b*x)] + ((I/4)*x^2*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
- ((I/4)*x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2 + ((I/8)*PolyLog[4, (-I)
*c*E^(2*a + 2*b*x)])/b^3
```

Rule 5195

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tanh[a + b*x]])/(f*(m
 + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+(i+c)\tanh(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx)) - \frac{1}{3}b\int \frac{x^3}{-i+ce^{2a+2bx}}\,dx$$

$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{3}(ibc)\int \frac{e^{2a+2bx}x^3}{-i+ce^{2a+2bx}}\,dx$$

$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) -$$

$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) +$$

$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) +$$

$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) +$$

$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) +$$

Mathematica [A] time = 5.12782, size = 128, normalized size = 0.9

$$\frac{i\left(-6b^2x^2\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)-6bx\text{PolyLog}\left(3,\frac{ie^{-2(a+bx)}}{c}\right)-3\text{PolyLog}\left(4,\frac{ie^{-2(a+bx)}}{c}\right)+4b^3x^3\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{24b^3}+\frac{1}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (I + c)*Tanh[a + b*x]])/3 + ((I/24)*(4*b^3*x^3*Log[1 - I/(c
*E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, I/(c*E^(2*(a + b*x)))] - 6*b*x*Po
lyLog[3, I/(c*E^(2*(a + b*x)))] - 3*PolyLog[4, I/(c*E^(2*(a + b*x)))]))/b^3
```

Maple [C] time = 11.773, size = 1555, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] 1/6*Pi*x^3+1/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^3*a^3*ln(1-
I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(I*c)^(1/2))+1
/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))-1/12*Pi*x^3*csgn(I*(2*I*exp(
2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)
+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*
x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*e
xp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c
-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))
^2-1/12*Pi*x^3*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)
+1))^2-1/6*I/b^3*a^3*ln(-exp(2*b*x+2*a)*c+I)+1/12*Pi*x^3*csgn(I*(2*I*exp(2*
b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+1/12*Pi*x^3*csgn((2*I*ex
p(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3-1/6*I*x^3*ln(2*exp(2
*b*x+2*a)*c-2*I)-1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a
)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))-1/3/b^3*a^3/(I+c)*l
n(exp(b*x+a))+1/3/b^2/(I+c)*x*a^3+1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/
b+1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3+1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*
a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*
x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+1/12*Pi*x^3*csgn(I*(2*exp(2*
b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+1/12
*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3+1/12*Pi*x^3*csg
n(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))
^2-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(
2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-
2*I)/(exp(2*b*x+2*a)+1))^3+1/6*I*x^3*ln(1+I*c*exp(2*b*x+2*a))-1/3*I/b^3*ln(
1+I*c*exp(2*b*x+2*a))*a^3-1/4*I/b^3*polylog(2,-I*c*exp(2*b*x+2*a))*a^2+1/6*
I*x^3*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)-1/12*Pi*x^3*csgn(I*(2*I*exp
(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a
)*c)/(exp(2*b*x+2*a)+1))^2-1/2*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*x*a^2+1/2*I/b
^2*a^2*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*x+1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(I*
c)^(1/2))*x-1/4*I/b^3*c/(I+c)*a^4-1/12*I*c*b/(I+c)*x^4+1/4/b^3/(I+c)*a^4+1/
12*b/(I+c)*x^4-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2
*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))-1/12*Pi*x^3
*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+1/3*I/b^3*c*a^3/(I+c)*
ln(exp(b*x+a))-1/3*I/b^2*c/(I+c)*x*a^3-1/4*I*x*polylog(3,-I*c*exp(2*b*x+2*a
))/b^2
```

Maxima [A] time = 5.93211, size = 174, normalized size = 1.23

$$\frac{1}{3}x^3\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)+\frac{4}{9}\left(\frac{3\,x^4}{4i\,c-4}-\frac{4\,b^3x^3\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3(-}{-2\,b^4(-i\,c+1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan((c + I)*tanh(b*x + a) + c) + 4/9*(3*x^4/(4*I*c - 4) - (4*b^3
*x^3*log(I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-I*c*e^(2*b*x + 2*a)) -
```

```
6*b*x*polylog(3, -I*c*e^(2*b*x + 2*a)) + 3*polylog(4, -I*c*e^(2*b*x + 2*a)
))/(b^4*(2*I*c - 2)))*b*(c + I)
```

Fricas [C] time = 1.98988, size = 868, normalized size = 6.11

$$-i b^4x^4 + 2i b^3x^3 \log\left(-\frac{(c+i)e^{(2bx+2a)}}{ce^{(2bx+2a)}-i}\right) + 6i b^2x^2 \mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i c}e^{(bx+a)}\right) + 6i b^2x^2 \mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i c}e^{(bx+a)}\right) + i a^4 - 2i a^3 \log\left(\frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-I*b^4*x^4 + 2*I*b^3*x^3*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2
*a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*b^2*x^2*d
ilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) + I*a^4 - 2*I*a^3*log(1/2*(2*c*e^(b*x +
 a) + I*sqrt(-4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c
))/c) - 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 12*I*b*x*polylo
g(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)) + (2*I*b^3*x^3 + 2*I*a^3)*log(1/2*sqrt(
-4*I*c)*e^(b*x + a) + 1) + (2*I*b^3*x^3 + 2*I*a^3)*log(-1/2*sqrt(-4*I*c)*e^
(b*x + a) + 1) + 12*I*polylog(4, 1/2*sqrt(-4*I*c)*e^(b*x + a)) + 12*I*polyl
og(4, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ib\int\frac{x^3}{ice^{2a}e^{2bx}+1}\,dx}{3} + \frac{ix^3\log\left(-ic + \frac{ic}{e^{2a}e^{2bx}+1} - \frac{ice^ae^{bx}}{e^ae^{bx}+e^{-a}e^{-bx}} + 1 - \frac{1}{e^{2a}e^{2bx}+1} + \frac{e^ae^{bx}}{e^ae^{bx}+e^{-a}e^{-bx}}\right)}{6} - \frac{ix^3\log\left(ic - \frac{ic}{e^{2a}e^{2bx}+1} + \frac{ice}{e^ae^{bx}+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] -I*b*Integral(x**3/(I*c*exp(2*a)*exp(2*b*x) + 1), x)/3 + I*x**3*log(-I*c +
I*c/(exp(2*a)*exp(2*b*x) + 1) - I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(
-a)*exp(-b*x)) + 1 - 1/(exp(2*a)*exp(2*b*x) + 1) + exp(a)*exp(b*x)/(exp(a)*
exp(b*x) + exp(-a)*exp(-b*x)))/6 - I*x**3*log(I*c - I*c/(exp(2*a)*exp(2*b*x
) + 1) + I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(-a)*exp(-b*x)) + 1 + 1/
(exp(2*a)*exp(2*b*x) + 1) - exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(-a)*exp(
-b*x)))/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \arctan\left((c+i)\tanh(bx+a)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((c + I)*tanh(b*x + a) + c), x)
```

3.86 $\int x \tan^{-1}(c + (i + c) \tanh(a + bx)) \, dx$

Optimal. Leaf size=113

$$-\frac{i\text{PolyLog}\left(3, -ice^{2a+2bx}\right)}{8b^2} + \frac{ix\text{PolyLog}\left(2, -ice^{2a+2bx}\right)}{4b} + \frac{1}{4}ix^2 \log\left(1 + ice^{2a+2bx}\right) + \frac{1}{2}x^2 \tan^{-1}(c + (c + i) \tanh(a + bx))$$

```
[Out] (-I/6)*b*x^3 + (x^2*ArcTan[c + (I + c)*Tanh[a + b*x]])/2 + (I/4)*x^2*Log[1
+ I*c*E^(2*a + 2*b*x)] + ((I/4)*x*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b - (
(I/8)*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2
```

Rubi [A] time = 0.194972, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5195, 2184, 2190, 2531, 2282, 6589}

$$-\frac{i\text{PolyLog}\left(3, -ice^{2a+2bx}\right)}{8b^2} + \frac{ix\text{PolyLog}\left(2, -ice^{2a+2bx}\right)}{4b} + \frac{1}{4}ix^2 \log\left(1 + ice^{2a+2bx}\right) + \frac{1}{2}x^2 \tan^{-1}(c + (c + i) \tanh(a + bx))$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (-I/6)*b*x^3 + (x^2*ArcTan[c + (I + c)*Tanh[a + b*x]])/2 + (I/4)*x^2*Log[1
+ I*c*E^(2*a + 2*b*x)] + ((I/4)*x*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b - (
(I/8)*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2
```

Rule 5195

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tanh[a + b*x]])/(f*(m
 + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+(i+c)\tanh(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx)) - \frac{1}{2}b\int\frac{x^2}{-i+ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}x^2}{-i+ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)-\frac{1}{2}$$
$$= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+$$
$$= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+$$
$$= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+$$

Mathematica [A] time = 5.23494, size = 102, normalized size = 0.9

$$\frac{i\left(-2bx\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(3,\frac{ie^{-2(a+bx)}}{c}\right)+2b^2x^2\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{8b^2}+\frac{1}{2}x^2\tan^{-1}(c+(c+i)\tanh(a+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (I + c)*Tanh[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 - I/(c*
E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, I/(c*E^(2*(a + b*x)))] - PolyLog[3, I/
(c*E^(2*(a + b*x)))]))/b^2
```

Maple [C] time = 6.903, size = 1519, normalized size = 13.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] 1/4*Pi*x^2-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)
+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*csgn((2*I*exp(2*b*x+2
```

```
*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+1/8*Pi*x^2*csgn((2*exp(2*b*x+
2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3+1/4*I/b^2*a^2*ln(-exp(2*b*x+2*a)*c+I)-1/8
*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b
*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*
I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*csgn(I
*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(
2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I
*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+1/2*I/b*ln(1+I*c*exp(2*b*x+
2*a))*x*a-1/2*I/b*a*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*x-1/2*I/b*a*ln(1-I*exp(b
*x+a)*(I*c)^(1/2))*x+1/3*I/b^2*c/(I+c)*a^3-1/6*I*c*b/(I+c)*x^3-1/8*Pi*x^2*c
sgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*
b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*c
sgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+1/2/b^2*a^2/(I+c)*ln(exp(b*x+a))-1/2
/b/(I+c)*x*a^2+1/4*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*a^2+1/4*I/b^2*polylog(2,-
I*c*exp(2*b*x+2*a))*a-1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^
2*a^2*ln(1-I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^2*a*dilog(1+I*exp(b*x+a)*(I*c)
^(1/2))-1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))-1/8*I*polylog(3,-I*c*
exp(2*b*x+2*a))/b^2+1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)-1/4
*I*x^2*ln(2*exp(2*b*x+2*a)*c-2*I)+1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*e
xp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+1/2*I/b*c/(I+c)*x*a^2-1/2*I/b^2*c*a^
2/(I+c)*ln(exp(b*x+a))+1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/8*Pi*x^2*
csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3-1/8*Pi*x^2*csgn((2*I*
exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/4*I*x^2*ln(1+I*c
*exp(2*b*x+2*a))-1/3/b^2/(I+c)*a^3+1/6*b/(I+c)*x^3+1/8*Pi*x^2*csgn(I*(2*I*e
xp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2
*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+
2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*
a)+1))^2-1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2
```

Maxima [A] time = 5.79743, size = 143, normalized size = 1.27

$$\left(\frac{2\,x^3}{3i\,c-3}-\frac{2\,b^2x^2\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-i\,ce^{(2\,bx+2\,a)})}{-2\,b^3(-i\,c+1)}\right)b(c+i)+\frac{1}{2}\,x^2\arctan\left((c+i)\tanh\right($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] (2*x^3/(3*I*c - 3) - (2*b^2*x^2*log(I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(
-I*c*e^(2*b*x + 2*a)) - polylog(3, -I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c - 2))
)*b*(c + I) + 1/2*x^2*arctan((c + I)*tanh(b*x + a) + c)
```

Fricas [C] time = 1.88472, size = 718, normalized size = 6.35

$$-2i\,b^3x^3+3i\,b^2x^2\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right)-2i\,a^3+6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}e^{(bx+a)}\right)+6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}e^{(bx+a)}\right)+3i\,a^2\log\left(\frac{2\,ce^{(bx+}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x +
 2*a) - I)) - 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*b
*x*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a)
+ I*sqrt(-4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c
) + (3*I*b^2*x^2 - 3*I*a^2)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (3*I*b^
2*x^2 - 3*I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 6*I*polylog(3, 1/
2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*polylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a))
)/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ib\int\frac{x^2}{ice^{2a}e^{2bx}+1}\,dx}{2}+\frac{ix^2\log\left(-ic+\frac{ic}{e^{2a}e^{2bx}+1}-\frac{ice^ae^{bx}}{e^ae^{bx}+e^{-a}e^{-bx}}+1-\frac{1}{e^{2a}e^{2bx}+1}+\frac{e^ae^{bx}}{e^ae^{bx}+e^{-a}e^{-bx}}\right)}{4}-\frac{ix^2\log\left(ic-\frac{ic}{e^{2a}e^{2bx}+1}+\frac{ice}{e^ae^{bx}+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] -I*b*Integral(x**2/(I*c*exp(2*a)*exp(2*b*x) + 1), x)/2 + I*x**2*log(-I*c +
I*c/(exp(2*a)*exp(2*b*x) + 1) - I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(
-a)*exp(-b*x)) + 1 - 1/(exp(2*a)*exp(2*b*x) + 1) + exp(a)*exp(b*x)/(exp(a)*
exp(b*x) + exp(-a)*exp(-b*x)))/4 - I*x**2*log(I*c - I*c/(exp(2*a)*exp(2*b*x
) + 1) + I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(-a)*exp(-b*x)) + 1 + 1/
(exp(2*a)*exp(2*b*x) + 1) - exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(-a)*exp(
-b*x)))/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((c + I)*tanh(b*x + a) + c), x)
```

3.87 $\int \tan^{-1}(c+(i+c)\tanh(a+bx))\,dx$

Optimal. Leaf size=79

$$\frac{i\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+x\tan^{-1}(c+(c+i)\tanh(a+bx))-\frac{1}{2}ibx^2$$

```
[Out] (-I/2)*b*x^2 + x*ArcTan[c + (I + c)*Tanh[a + b*x]] + (I/2)*x*Log[1 + I*c*E^
(2*a + 2*b*x)] + ((I/4)*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
```

Rubi [A] time = 0.117691, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5187, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+x\tan^{-1}(c+(c+i)\tanh(a+bx))-\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (-I/2)*b*x^2 + x*ArcTan[c + (I + c)*Tanh[a + b*x]] + (I/2)*x*Log[1 + I*c*E^
(2*a + 2*b*x)] + ((I/4)*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
```

Rule 5187

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tan[c + d*Tanh[a + b*x]], x] - Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tan^{-1}(c+(i+c)\tanh(a+bx))\,dx = x\tan^{-1}(c+(i+c)\tanh(a+bx)) - b\int \frac{x}{-i+ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\tanh(a+bx)) + (ibc)\int \frac{e^{2a+2bx}x}{-i+ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) - \frac{1}{2}i\int \text{lo}$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) - \frac{i\,\text{Subs}}{}$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) + \frac{i\text{Li}_2(-}{}$$

Mathematica [A] time = 1.70071, size = 71, normalized size = 0.9

$$\frac{i\left(2bx\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}+x\tan^{-1}(c+(c+i)\tanh(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (I + c)*Tanh[a + b*x]] + ((I/4)*(2*b*x*Log[1 - I/(c*E^(2*(a +
b*x)))] - PolyLog[2, I/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] time = 0.145, size = 1381, normalized size = 17.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] -1/4*I/(I+c)^2/b*ln(c-(I+c)*tanh(b*x+a)+I)*ln(-1/2*(-c-(I+c)*tanh(b*x+a)+I)
/c)*c^2+1/4*I/(I+c)^2/b*ln(c-(I+c)*tanh(b*x+a)+I)*ln((-c-(I+c)*tanh(b*x+a)-
I)/(-2*I-2*c))*c^2-1/4*I/(I+c)^2/b*ln(-1/2*I*(c+(I+c)*tanh(b*x+a)+I))*ln(-1
/2*I*(-c-(I+c)*tanh(b*x+a)+I))*c^2+1/(I+c)/b*arctan(c+(I+c)*tanh(b*x+a))/(2
*I+2*c)*ln(c+(I+c)*tanh(b*x+a)+I)*c^2-1/(I+c)/b*arctan(c+(I+c)*tanh(b*x+a))
/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)*c^2+1/4*I/(I+c)^2/b*ln(-1/2*I*(-c-(I+c
)*tanh(b*x+a)+I))*ln(c+(I+c)*tanh(b*x+a)+I)*c^2-1/8*I/(I+c)^2/b*ln(c+(I+c)*
tanh(b*x+a)+I)^2*c^2-1/4*I/(I+c)^2/b*dilog(-1/2*(-c-(I+c)*tanh(b*x+a)+I)/c)
*c^2+1/4*I/(I+c)^2/b*dilog((-c-(I+c)*tanh(b*x+a)-I)/(-2*I-2*c))*c^2+1/4*I/(
I+c)^2/b*ln(-1/2*I*(c+(I+c)*tanh(b*x+a)+I))*ln(-1/2*I*(-c-(I+c)*tanh(b*x+a)
+I))-1/4*I/(I+c)^2/b*ln(-1/2*I*(-c-(I+c)*tanh(b*x+a)+I))*ln(c+(I+c)*tanh(b*
x+a)+I)+1/4*I/(I+c)^2/b*ln(c-(I+c)*tanh(b*x+a)+I)*ln(-1/2*(-c-(I+c)*tanh(b*
x+a)+I)/c)-1/4*I/(I+c)^2/b*ln(c-(I+c)*tanh(b*x+a)+I)*ln((-c-(I+c)*tanh(b*x+
a)-I)/(-2*I-2*c))-1/(I+c)/b*arctan(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(c+(I+c
)*tanh(b*x+a)+I)+1/(I+c)/b*arctan(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(c-(I+c)
*tanh(b*x+a)+I)-1/2/(I+c)^2/b*ln(c-(I+c)*tanh(b*x+a)+I)*ln((-c-(I+c)*tanh(b
*x+a)-I)/(-2*I-2*c))*c+1/2/(I+c)^2/b*ln(-1/2*I*(c+(I+c)*tanh(b*x+a)+I))*ln(
-1/2*I*(-c-(I+c)*tanh(b*x+a)+I))*c+1/4/(I+c)^2/b*ln(c+(I+c)*tanh(b*x+a)+I)^
2*c+1/2/(I+c)^2/b*dilog(-1/2*(-c-(I+c)*tanh(b*x+a)+I)/c)*c-1/2/(I+c)^2/b*di
log((-c-(I+c)*tanh(b*x+a)-I)/(-2*I-2*c))*c+1/2/(I+c)^2/b*dilog(-1/2*I*(c+(I
```

```
+c)*tanh(b*x+a)+I))*c+1/8*I/(I+c)^2/b*ln(c+(I+c)*tanh(b*x+a)+I)^2+1/4*I/(I+
c)^2/b*dilog(-1/2*(-c-(I+c)*tanh(b*x+a)+I)/c)-1/4*I/(I+c)^2/b*dilog((-c-(I+
c)*tanh(b*x+a)-I)/(-2*I-2*c))+1/4*I/(I+c)^2/b*dilog(-1/2*I*(c+(I+c)*tanh(b*
x+a)+I))-1/4*I/(I+c)^2/b*dilog(-1/2*I*(c+(I+c)*tanh(b*x+a)+I))*c^2+2*I/(I+c
)/b*arctan(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(c+(I+c)*tanh(b*x+a)+I)*c-1/2/(
I+c)^2/b*ln(-1/2*I*(-c-(I+c)*tanh(b*x+a)+I))*ln(c+(I+c)*tanh(b*x+a)+I)*c+1/
2/(I+c)^2/b*ln(c-(I+c)*tanh(b*x+a)+I)*ln(-1/2*(-c-(I+c)*tanh(b*x+a)+I)/c)*c
-2*I/(I+c)/b*arctan(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I
)*c
```

Maxima [A] time = 5.86429, size = 108, normalized size = 1.37

$$2\,b(c+i)\left(\frac{2\,x^2}{2i\,c-2}-\frac{2\,bx\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c+1)}\right)+x\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*b*(c + I)*(2*x^2/(2*I*c - 2) - (2*b*x*log(I*c*e^(2*b*x + 2*a) + 1) + dilo
g(-I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c - 2))) + x*arctan((c + I)*tanh(b*x + a
) + c)
```

Fricas [B] time = 1.9047, size = 518, normalized size = 6.56

$$\frac{-i\,b^2x^2+i\,bx\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right)+i\,a^2+(i\,bx+i\,a)\log\left(\frac{1}{2}\,\sqrt{-4i\,c}e^{(bx+a)}+1\right)+(i\,bx+i\,a)\log\left(-\frac{1}{2}\,\sqrt{-4i\,c}e^{(bx+a)}+1\right)-i\,a}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b^2*x^2 + I*b*x*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) - I
)) + I*a^2 + (I*b*x + I*a)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (I*b*x +
 I*a)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - I*a*log(1/2*(2*c*e^(b*x + a)
 + I*sqrt(-4*I*c))/c) - I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c) +
 I*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + I*dilog(-1/2*sqrt(-4*I*c)*e^(b*x +
 a)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-ib\int\frac{x}{ice^{2a}e^{2bx}+1}\,dx+\frac{ix\log\left(-ic+\frac{ic}{e^{2a}e^{2bx}+1}-\frac{ice^{a}e^{bx}}{e^{a}e^{bx}+e^{-a}e^{-bx}}+1-\frac{1}{e^{2a}e^{2bx}+1}+\frac{e^{a}e^{bx}}{e^{a}e^{bx}+e^{-a}e^{-bx}}\right)}{2}-\frac{ix\log\left(ic-\frac{ic}{e^{2a}e^{2bx}+1}+\frac{ice^{a}e}{e^{a}e^{bx}+e^{-}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] -I*b*Integral(x/(I*c*exp(2*a)*exp(2*b*x) + 1), x) + I*x*log(-I*c + I*c/(exp
(2*a)*exp(2*b*x) + 1) - I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(-a)*exp(
```

```
-b*x)) + 1 - 1/(exp(2*a)*exp(2*b*x) + 1) + exp(a)*exp(b*x)/(exp(a)*exp(b*x)
 + exp(-a)*exp(-b*x)))/2 - I*x*log(I*c - I*c/(exp(2*a)*exp(2*b*x) + 1) + I*
c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(-a)*exp(-b*x)) + 1 + 1/(exp(2*a)*e
xp(2*b*x) + 1) - exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(-a)*exp(-b*x)))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arctan\left((c+i)\tanh\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((c + I)*tanh(b*x + a) + c), x)
```

3.88 $\int \frac{\tan^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=21

$$\text{CannotIntegrate}\left(\frac{\tan^{-1}(c+(c+i)\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTan[c + (I + c)*Tanh[a + b*x]]/x, x]
```

Rubi [A] time = 0.11195, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + (I + c)*Tanh[a + b*x]]/x,x]

[Out] Defer[Int][ArcTan[c + (I + c)*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx=\int \frac{\tan^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$$

Mathematica [A] time = 3.59822, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + (I + c)*Tanh[a + b*x]]/x,x]

[Out] Integrate[ArcTan[c + (I + c)*Tanh[a + b*x]]/x, x]
```

Maple [A] time = 0.428, size = 0, normalized size = 0.

$$\int \frac{\arctan\left(c+(i+c)\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(I+c)*tanh(b*x+a))/x,x)

[Out] int(arctan(c+(I+c)*tanh(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$ibx - \frac{1}{4}\left(2\pi - 4ia + 2\arctan(c, -1) - i\log\left(c^2 + 1\right)\right)\log(x) + \frac{1}{2}\int \frac{\arctan\left(ce^{(2bx+2a)}\right)}{x}\,dx - \frac{1}{4}i\int \frac{\log\left(c^2e^{(4bx+4a)}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] I*b*x - 1/4*(2*pi - 4*I*a + 2*arctan2(c, -1) - I*log(c^2 + 1))*log(x) + 1/2
*integrate(arctan(c*e^(2*b*x + 2*a))/x, x) - 1/4*I*integrate(log(c^2*e^(4*b
*x + 4*a) + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i\log\left(-\frac{(c+i)e^{(2bx+2a)}}{ce^{(2bx+2a)}-i}\right)}{2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) - I))/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(I+c)*tanh(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left((c+i)\tanh(bx+a)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((c + I)*tanh(b*x + a) + c)/x, x)
```

3.89 $\int x^2 \tan^{-1}(c-(i-c)\tanh(a+bx))\,dx$

Optimal. Leaf size=145

$$\frac{ix\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}-\frac{ix^2\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)+\frac{1}{3}x^3\tan^{-1}(c$$

```
[Out] (I/12)*b*x^4 + (x^3*ArcTan[c - (I - c)*Tanh[a + b*x]])/3 - (I/6)*x^3*Log[1
- I*c*E^(2*a + 2*b*x)] - ((I/4)*x^2*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b + ((
I/4)*x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2 - ((I/8)*PolyLog[4, I*c*E^(2*a
+ 2*b*x)])/b^3
```

Rubi [A] time = 0.220334, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5195, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{ix\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}-\frac{ix^2\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)+\frac{1}{3}x^3\tan^{-1}(c$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTan[c - (I - c)*Tanh[a + b*x]])/3 - (I/6)*x^3*Log[1
- I*c*E^(2*a + 2*b*x)] - ((I/4)*x^2*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b + ((
I/4)*x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2 - ((I/8)*PolyLog[4, I*c*E^(2*a
+ 2*b*x)])/b^3
```

Rule 5195

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tanh[a + b*x]])/(f*(m
 + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^2 \tan^{-1}(c-(i-c)\tanh(a+bx))\,dx &= \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{3}b\int \frac{x^3}{i+ce^{2a+2bx}}\,dx \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{3}(ibc)\int \frac{e^{2a+2bx}x^3}{i+ce^{2a+2bx}}\,dx \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) + \frac{1}{2} \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) - \frac{ix}{} \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) - \frac{ix}{} \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) - \frac{ix}{} \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) - \frac{ix}{}
\end{aligned}$$

Mathematica [A] time = 5.10254, size = 128, normalized size = 0.88

$$\frac{1}{3}x^3\tan^{-1}(c+(c-i)\tanh(a+bx)) - \frac{i\left(-6b^2x^2\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right) - 6bx\text{PolyLog}\left(3,-\frac{ie^{-2(a+bx)}}{c}\right) - 3\text{PolyLog}\left(4,\right.\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (-I + c)*Tanh[a + b*x]])/3 - ((I/24)*(4*b^3*x^3*Log[1 + I/(
c*E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - 6*b*
x*PolyLog[3, (-I)/(c*E^(2*(a + b*x)))] - 3*PolyLog[4, (-I)/(c*E^(2*(a + b*x
)))]))/b^3
```

Maple [C] time = 10.954, size = 1570, normalized size = 10.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] -1/6*Pi*x^3+1/12*Pi*x^3*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(
2*b*x+2*a)+1))^2+1/2*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*x*a^2-1/2*I/b^2*a^2*ln(
1-I*exp(b*x+a)*(-I*c)^(1/2))*x-1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3+1/4*
I*x*polylog(3,I*c*exp(2*b*x+2*a))/b^2-1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*
c+2*I)/(exp(2*b*x+2*a)+1))^3+1/6*I*x^3*ln(-2*exp(2*b*x+2*a)*c-2*I)-1/6*I*x^
3*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)+1/12*Pi*x^3*csgn(I*(-2*I*exp(2*
b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+1/12*Pi*x^3*csgn((-2*I*e
xp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3-1/4/b^3/(I-c)*a^4-1
/12*b/(I-c)*x^4-1/6*I*x^3*ln(1-I*c*exp(2*b*x+2*a))+1/3*I/b^3*ln(1-I*c*exp(2
*b*x+2*a))*a^3+1/4*I/b^3*polylog(2,I*c*exp(2*b*x+2*a))*a^2-1/2*I/b^3*a^3*ln
(1-I*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))
-1/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b^3*a^2*dilog(1+I*e
xp(b*x+a)*(-I*c)^(1/2))+1/12*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*
x+2*a)+1))^3+1/12*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^
2-1/4*I/b^3*c/(I-c)*a^4+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*
b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))+1/6*I/b^3*a^
3*ln(exp(2*b*x+2*a)*c+I)-1/12*I*c*b/(I-c)*x^4+1/12*Pi*x^3*csgn(I/(exp(2*b*x
+2*a)+1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1
))^2+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*
c+2*I)/(exp(2*b*x+2*a)+1))^2-1/12*Pi*x^3*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(
2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2
*a)+1))^2+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))*c
sgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-1/12*Pi*x^3*csgn(I*(-2*I
*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*b*
x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+1/3/b^3*a^3/(I-c)*ln(exp(b*x
+a))-1/3/b^2/(I-c)*x*a^3+1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*e
xp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))-1
/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b
*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)
+1))-1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b-1/12*Pi*x^3*csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+1/3*I/b^3*
c*a^3/(I-c)*ln(exp(b*x+a))-1/12*Pi*x^3*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*
b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*
c)/(exp(2*b*x+2*a)+1))^2-1/3*I/b^2*c/(I-c)*x*a^3-1/2*I/b^2*a^2*ln(1+I*exp(b
*x+a)*(-I*c)^(1/2))*x
```

Maxima [A] time = 5.8979, size = 174, normalized size = 1.2

$$\frac{1}{3}x^3\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)-\frac{4}{9}\left(\frac{3x^4}{4ic+4}-\frac{4b^3x^3\log\left(-ice^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(ice^{(2bx+2a)}\right)-6bx\mathrm{Li}_3(i\,c$$
$$\overline{-2b^4(-ic-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan((c - I)*tanh(b*x + a) + c) - 4/9*(3*x^4/(4*I*c + 4) - (4*b^3
*x^3*log(-I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(I*c*e^(2*b*x + 2*a)) -
```

```
 6*b*x*polylog(3, I*c*e^(2*b*x + 2*a)) + 3*polylog(4, I*c*e^(2*b*x + 2*a)))
/(b^4*(2*I*c + 2)))*b*(c - I)
```

Fricas [C] time = 2.01688, size = 857, normalized size = 5.91

$$\frac{i\,b^4x^4+2i\,b^3x^3\log\left(-\frac{\left(ce^{(2\,bx+2\,a)}+i\right)e^{(-2\,bx-2\,a)}}{c-i}\right)-6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}e^{(bx+a)}\right)-6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}e^{(bx+a)}\right)-i\,a^4+2i\,a^3\log\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*I*b^3*x^3*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)
/(c - I)) - 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - 6*I*b^2*x^2*di
log(-1/2*sqrt(4*I*c)*e^(b*x + a)) - I*a^4 + 2*I*a^3*log(1/2*(2*c*e^(b*x + a
) + I*sqrt(4*I*c))/c) + 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c
) + 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(b*x + a)) + 12*I*b*x*polylog(3,
-1/2*sqrt(4*I*c)*e^(b*x + a)) + (-2*I*b^3*x^3 - 2*I*a^3)*log(1/2*sqrt(4*I*c
)*e^(b*x + a) + 1) + (-2*I*b^3*x^3 - 2*I*a^3)*log(-1/2*sqrt(4*I*c)*e^(b*x +
 a) + 1) - 12*I*polylog(4, 1/2*sqrt(4*I*c)*e^(b*x + a)) - 12*I*polylog(4, -
1/2*sqrt(4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ib\int\frac{x^3}{ice^{2a}e^{2bx}-1}\,dx}{3}+\frac{ix^3\log\left(-ic+\frac{ic}{e^{2a}e^{2bx}+1}-\frac{ice^{a}e^{bx}}{e^{a}e^{bx}+e^{-a}e^{-bx}}+1+\frac{1}{e^{2a}e^{2bx}+1}-\frac{e^{a}e^{bx}}{e^{a}e^{bx}+e^{-a}e^{-bx}}\right)}{6}-\frac{ix^3\log\left(ic-\frac{ic}{e^{2a}e^{2bx}+1}+\frac{ice^{a}e^{bx}+\cdots}{e^{a}e^{bx}+\cdots}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] -I*b*Integral(x**3/(I*c*exp(2*a)*exp(2*b*x) - 1), x)/3 + I*x**3*log(-I*c +
I*c/(exp(2*a)*exp(2*b*x) + 1) - I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(
-a)*exp(-b*x)) + 1 + 1/(exp(2*a)*exp(2*b*x) + 1) - exp(a)*exp(b*x)/(exp(a)*
exp(b*x) + exp(-a)*exp(-b*x)))/6 - I*x**3*log(I*c - I*c/(exp(2*a)*exp(2*b*x
) + 1) + I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(-a)*exp(-b*x)) + 1 - 1/
(exp(2*a)*exp(2*b*x) + 1) + exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(-a)*exp(
-b*x)))/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((c - I)*tanh(b*x + a) + c), x)
```

3.90 $\int x\tan^{-1}(c-(i-c)\tanh(a+bx))\,dx$

Optimal. Leaf size=116

$$\frac{i\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}-\frac{ix\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{1}{2}x^2\tan^{-1}(c-(-c+i)\tanh(a+bx))+\frac{1}{6}$$

```
[Out] (I/6)*b*x^3 + (x^2*ArcTan[c - (I - c)*Tanh[a + b*x]])/2 - (I/4)*x^2*Log[1 -
 I*c*E^(2*a + 2*b*x)] - ((I/4)*x*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b + ((I/8
)*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2
```

Rubi [A] time = 0.196986, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5195, 2184, 2190, 2531, 2282, 6589}

$$\frac{i\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}-\frac{ix\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{1}{2}x^2\tan^{-1}(c-(-c+i)\tanh(a+bx))+\frac{1}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcTan[c - (I - c)*Tanh[a + b*x]])/2 - (I/4)*x^2*Log[1 -
 I*c*E^(2*a + 2*b*x)] - ((I/4)*x*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b + ((I/8
)*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2
```

Rule 5195

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tanh[a + b*x]])/(f*(m
 + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c-(i-c)\tanh(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx))-\frac{1}{2}b\int\frac{x^2}{i+ce^{2a+2bx}}\,dx$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx))-\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}x^2}{i+ce^{2a+2bx}}\,dx$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{1}{2}i$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)-\frac{ixL}{}$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)-\frac{ixL}{}$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)-\frac{ixL}{}$$

Mathematica [A] time = 4.98064, size = 102, normalized size = 0.88

$$\frac{1}{2}x^2\tan^{-1}(c+(c-i)\tanh(a+bx))-\frac{i\left(-2bx\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(3,-\frac{ie^{-2(a+bx)}}{c}\right)+2b^2x^2\log\left(1+\frac{ie^{-2(a+}}{c}\right.\right.}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (-I + c)*Tanh[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 + I/(c
*E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - PolyLog[3
, (-I)/(c*E^(2*(a + b*x)))]))/b^2
```

Maple [C] time = 3.744, size = 1534, normalized size = 13.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] -1/4*Pi*x^2+1/8*Pi*x^2*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2
*b*x+2*a)+1))^3+1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1)
```

```
)^3-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(
exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp
(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/3/b^2/(I-c)*a^3-1/6
*b*x^3/(I-c)+1/2/b/(I-c)*x*a^2+1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+
2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))-
1/4*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*a^2-1/4*I/b^2*polylog(2,I*c*exp(2*b*x+2*
a))*a+1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b^2*a^2*ln(1+I*ex
p(b*x+a)*(-I*c)^(1/2))+1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I
/b^2*a*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*
a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2
*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*
a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x
+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2
*a)+1))^2-1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2
*b*x+2*a)+1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)
+1))^2-1/4*I/b^2*a^2*ln(exp(2*b*x+2*a)*c+I)+1/8*I*polylog(3,I*c*exp(2*b*x+2
*a))/b^2+1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-1/8
*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^3-1/8*Pi*x^2*cs
gn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*c
sgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+1/8*Pi*x
^2*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/4*
I*x^2*ln(-2*exp(2*b*x+2*a)*c-2*I)-1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b
*x+2*a)*c)-1/4*I*x^2*ln(1-I*c*exp(2*b*x+2*a))+1/8*Pi*x^2*csgn(I/(exp(2*b*x+
2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(
exp(2*b*x+2*a)+1))-1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*
c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2
*b*x+2*a)+1))+1/2*I/b*c/(I-c)*x*a^2-1/2*I/b^2*c*a^2/(I-c)*ln(exp(b*x+a))-1/
2/b^2*a^2/(I-c)*ln(exp(b*x+a))-1/2*I/b*ln(1-I*c*exp(2*b*x+2*a))*x*a+1/2*I/b
*a*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x+1/2*I/b*a*ln(1+I*exp(b*x+a)*(-I*c)^(1/
2))*x+1/3*I/b^2*c/(I-c)*a^3-1/6*I*c*b*x^3/(I-c)-1/4*I*x*polylog(2,I*c*exp(2
*b*x+2*a))/b
```

Maxima [A] time = 5.92242, size = 144, normalized size = 1.24

$$-\left(\frac{2\,x^3}{3i\,c+3}-\frac{2\,b^2x^2\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(i\,ce^{(2\,bx+2\,a)})}{-2\,b^3(-i\,c-1)}\right)b(c-i)+\frac{1}{2}\,x^2\arctan\left((c-i)\tanh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -(2*x^3/(3*I*c + 3) - (2*b^2*x^2*log(-I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g(I*c*e^(2*b*x + 2*a)) - polylog(3, I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c + 2))
)*b*(c - I) + 1/2*x^2*arctan((c - I)*tanh(b*x + a) + c)
```

Fricas [C] time = 1.95827, size = 710, normalized size = 6.12

$$2i\,b^3x^3+3i\,b^2x^2\log\left(-\frac{\left(ce^{(2\,bx+2\,a)}+i\right)e^{(-2\,bx-2\,a)}}{c-i}\right)+2i\,a^3-6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}e^{(bx+a)}\right)-6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}e^{(bx+a)}\right)-3i\,a^2\log\left(\frac{2\,ce}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*
a)/(c - I)) + 2*I*a^3 - 6*I*b*x*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - 6*I*b*
x*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) +
I*sqrt(4*I*c))/c) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) +
(-3*I*b^2*x^2 + 3*I*a^2)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (-3*I*b^2*x
^2 + 3*I*a^2)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + 6*I*polylog(3, 1/2*sq
rt(4*I*c)*e^(b*x + a)) + 6*I*polylog(3, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ib\int\frac{x^2}{ice^{2a}e^{2bx}-1}\,dx}{2}+\frac{ix^2\log\left(-ic+\frac{ic}{e^{2a}e^{2bx}+1}-\frac{ice^ae^{bx}}{e^ae^{bx}+e^{-a}e^{-bx}}+1+\frac{1}{e^{2a}e^{2bx}+1}-\frac{e^ae^{bx}}{e^ae^{bx}+e^{-a}e^{-bx}}\right)}{4}-\frac{ix^2\log\left(ic-\frac{ic}{e^{2a}e^{2bx}+1}+\frac{ice}{e^ae^{bx}+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] -I*b*Integral(x**2/(I*c*exp(2*a)*exp(2*b*x) - 1), x)/2 + I*x**2*log(-I*c +
I*c/(exp(2*a)*exp(2*b*x) + 1) - I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(
-a)*exp(-b*x)) + 1 + 1/(exp(2*a)*exp(2*b*x) + 1) - exp(a)*exp(b*x)/(exp(a)*
exp(b*x) + exp(-a)*exp(-b*x)))/4 - I*x**2*log(I*c - I*c/(exp(2*a)*exp(2*b*x
) + 1) + I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(-a)*exp(-b*x)) + 1 - 1/
(exp(2*a)*exp(2*b*x) + 1) + exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(-a)*exp(
-b*x)))/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((c - I)*tanh(b*x + a) + c), x)
```

3.91 $\int \tan^{-1}(c-(i-c)\tanh(a+bx))\,dx$

Optimal. Leaf size=82

$$-\frac{i\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+x\tan^{-1}(c-(-c+i)\tanh(a+bx))+\frac{1}{2}ibx^2$$

```
[Out] (I/2)*b*x^2 + x*ArcTan[c - (I - c)*Tanh[a + b*x]] - (I/2)*x*Log[1 - I*c*E^(
2*a + 2*b*x)] - ((I/4)*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b
```

Rubi [A] time = 0.118922, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5187, 2184, 2190, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+x\tan^{-1}(c-(-c+i)\tanh(a+bx))+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTan[c - (I - c)*Tanh[a + b*x]] - (I/2)*x*Log[1 - I*c*E^(
2*a + 2*b*x)] - ((I/4)*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b
```

Rule 5187

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tan[c + d*Tanh[a + b*x]], x] - Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \tan^{-1}(c-(i-c)\tanh(a+bx))\,dx &= x\tan^{-1}(c-(i-c)\tanh(a+bx))-b\int \frac{x}{i+ce^{2a+2bx}}\,dx \\
&= \frac{1}{2}ibx^2+x\tan^{-1}(c-(i-c)\tanh(a+bx))-(ibc)\int \frac{e^{2a+2bx}x}{i+ce^{2a+2bx}}\,dx \\
&= \frac{1}{2}ibx^2+x\tan^{-1}(c-(i-c)\tanh(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+\frac{1}{2}i\int \log \\
&= \frac{1}{2}ibx^2+x\tan^{-1}(c-(i-c)\tanh(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+\frac{i\,\mathrm{Subst}\left(\right.}{} \\
&= \frac{1}{2}ibx^2+x\tan^{-1}(c-(i-c)\tanh(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)-\frac{i\mathrm{Li}_2\left(ice^2\right.}{4b}
\end{aligned}
$$

Mathematica [A] time = 1.66507, size = 71, normalized size = 0.87

$$
x\tan^{-1}(c+(c-i)\tanh(a+bx))-\frac{i\left(2bx\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-\mathrm{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (-I + c)*Tanh[a + b*x]] - ((I/4)*(2*b*x*Log[1 + I/(c*E^(2*(a +
 b*x)))] - PolyLog[2, (-I)/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] time = 0.143, size = 1351, normalized size = 16.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] 1/4*I/b/(c-I)/(I-c)*dilog(-1/2*I*((c-I)*tanh(b*x+a)+c+I))*c^2-1/8*I/b/(c-I)
/(I-c)*ln((c-I)*tanh(b*x+a)+c-I)^2*c^2+1/4*I/b/(c-I)/(I-c)*dilog(((c-I)*tan
h(b*x+a)+c-I)/(-2*I+2*c))*c^2-1/4*I/b/(c-I)/(I-c)*dilog(1/2*((c-I)*tanh(b*x
+a)+c+I)/c)*c^2-1/4*I/b/(c-I)/(I-c)*ln(-1/2*I*((c-I)*tanh(b*x+a)+c+I))*ln((
c-I)*tanh(b*x+a)+c-I)+1/2/b/(c-I)/(I-c)*ln((c-I)*tanh(b*x+a)-c+I)*ln(((c-I)
*tanh(b*x+a)+c-I)/(-2*I+2*c))*c-1/2/b/(c-I)/(I-c)*ln((c-I)*tanh(b*x+a)-c+I)
*ln(1/2*((c-I)*tanh(b*x+a)+c+I)/c)*c+1/2/b/(c-I)/(I-c)*ln(-1/2*I*((c-I)*tan
h(b*x+a)+c+I))*ln((c-I)*tanh(b*x+a)+c-I)*c-1/b/(c-I)*arctan((c-I)*tanh(b*x+
a)+c)/(2*I-2*c)*ln((c-I)*tanh(b*x+a)+c-I)*c^2-1/4*I/b/(c-I)/(I-c)*ln((c-I)*
tanh(b*x+a)-c+I)*ln(((c-I)*tanh(b*x+a)+c-I)/(-2*I+2*c))+1/b/(c-I)*arctan((c
-I)*tanh(b*x+a)+c)/(2*I-2*c)*ln((c-I)*tanh(b*x+a)-c+I)*c^2+1/4*I/b/(c-I)/(I
-c)*ln((c-I)*tanh(b*x+a)-c+I)*ln(1/2*((c-I)*tanh(b*x+a)+c+I)/c)-1/4*I/b/(c-
I)/(I-c)*dilog(((c-I)*tanh(b*x+a)+c-I)/(-2*I+2*c))+1/4*I/b/(c-I)/(I-c)*dilo
g(1/2*((c-I)*tanh(b*x+a)+c+I)/c)+1/b/(c-I)*arctan((c-I)*tanh(b*x+a)+c)/(2*I
-2*c)*ln((c-I)*tanh(b*x+a)+c-I)-1/b/(c-I)*arctan((c-I)*tanh(b*x+a)+c)/(2*I-
2*c)*ln((c-I)*tanh(b*x+a)-c+I)+1/2/b/(c-I)/(I-c)*dilog(-1/2*I*((c-I)*tanh(b
*x+a)+c+I))*c-1/4/b/(c-I)/(I-c)*ln((c-I)*tanh(b*x+a)+c-I)^2*c+1/2/b/(c-I)/(
I-c)*dilog(((c-I)*tanh(b*x+a)+c-I)/(-2*I+2*c))*c-1/2/b/(c-I)/(I-c)*dilog(1/
2*((c-I)*tanh(b*x+a)+c+I)/c)*c-1/4*I/b/(c-I)/(I-c)*dilog(-1/2*I*((c-I)*tanh
(b*x+a)+c+I))+1/8*I/b/(c-I)/(I-c)*ln((c-I)*tanh(b*x+a)+c-I)^2+1/4*I/b/(c-I)
```

```
/(I-c)*ln((c-I)*tanh(b*x+a)-c+I)*ln(((c-I)*tanh(b*x+a)+c-I)/(-2*I+2*c))*c^2
-1/4*I/b/(c-I)/(I-c)*ln((c-I)*tanh(b*x+a)-c+I)*ln(1/2*((c-I)*tanh(b*x+a)+c+
I)/c)*c^2-2*I/b/(c-I)*arctan((c-I)*tanh(b*x+a)+c)/(2*I-2*c)*ln((c-I)*tanh(b
*x+a)-c+I)*c+2*I/b/(c-I)*arctan((c-I)*tanh(b*x+a)+c)/(2*I-2*c)*ln((c-I)*tan
h(b*x+a)+c-I)*c+1/4*I/b/(c-I)/(I-c)*ln(-1/2*I*((c-I)*tanh(b*x+a)+c+I))*ln((
c-I)*tanh(b*x+a)+c-I)*c^2
```

Maxima [A] time = 5.94616, size = 108, normalized size = 1.32

$$-2\,b(c-i)\left(\frac{2\,x^2}{2i\,c+2}-\frac{2\,bx\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c-1)}\right)+x\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*(c - I)*(2*x^2/(2*I*c + 2) - (2*b*x*log(-I*c*e^(2*b*x + 2*a) + 1) + di
log(I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c + 2))) + x*arctan((c - I)*tanh(b*x +
a) + c)
```

Fricas [B] time = 1.98497, size = 513, normalized size = 6.26

$$\frac{i\,b^2x^2+i\,bx\log\left(-\frac{\left(ce^{(2\,bx+2\,a)}+i\right)e^{(-2\,bx-2\,a)}}{c-i}\right)-i\,a^2+(-i\,bx-i\,a)\log\left(\frac{1}{2}\,\sqrt{4i\,c}e^{(bx+a)}+1\right)+(-i\,bx-i\,a)\log\left(-\frac{1}{2}\,\sqrt{4i\,c}e^{(bx+a)}+1\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 + I*b*x*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/(c - I
)) - I*a^2 + (-I*b*x - I*a)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (-I*b*x
- I*a)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + I*a*log(1/2*(2*c*e^(b*x + a)
 + I*sqrt(4*I*c))/c) + I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) - I
*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - I*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a))
)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-ib\int\frac{x}{ice^{2a}e^{2bx}-1}\,dx+\frac{ix\log\left(-ic+\frac{ic}{e^{2a}e^{2bx}+1}-\frac{ice^ae^{bx}}{e^ae^{bx}+e^{-a}e^{-bx}}+1+\frac{1}{e^{2a}e^{2bx}+1}-\frac{e^ae^{bx}}{e^ae^{bx}+e^{-a}e^{-bx}}\right)}{2}-\frac{ix\log\left(ic-\frac{ic}{e^{2a}e^{2bx}+1}+\frac{ice^ae}{e^ae^{bx}+e^-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] -I*b*Integral(x/(I*c*exp(2*a)*exp(2*b*x) - 1), x) + I*x*log(-I*c + I*c/(exp
(2*a)*exp(2*b*x) + 1) - I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(-a)*exp(
-b*x)) + 1 + 1/(exp(2*a)*exp(2*b*x) + 1) - exp(a)*exp(b*x)/(exp(a)*exp(b*x)
 + exp(-a)*exp(-b*x)))/2 - I*x*log(I*c - I*c/(exp(2*a)*exp(2*b*x) + 1) + I*
c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(-a)*exp(-b*x)) + 1 - 1/(exp(2*a)*e
```

```
xp(2*b*x) + 1) + exp(a)*exp(b*x)/(exp(a)*exp(b*x) + exp(-a)*exp(-b*x)))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arctan\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((c - I)*tanh(b*x + a) + c), x)
```

3.92 $\int \frac{\tan^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\tan^{-1}(c-(-c+i)\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTan[c - (I - c)*Tanh[a + b*x]]/x, x]
```

Rubi [A] time = 0.115669, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c - (I - c)*Tanh[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c - (I - c)*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\tan^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$$

Mathematica [A] time = 3.60212, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c - (I - c)*Tanh[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTan[c - (I - c)*Tanh[a + b*x]]/x, x]
```

Maple [A] time = 0.439, size = 0, normalized size = 0.

$$\int \frac{\arctan\left(c-(i-c)\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c-(I-c)*tanh(b*x+a))/x,x)
```

```
[Out] int(arctan(c-(I-c)*tanh(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-i\,bx - \frac{1}{4}\left(2\,\pi + 4i\,a - 2\,\arctan(c) + i\,\log\left(c^2+1\right)\right)\log(x) - \frac{1}{2}\int \frac{\arctan\left(ce^{(2\,bx+2\,a)}, -1\right)}{x}\,dx + \frac{1}{4}i\int \frac{\log\left(c^2e^{(4\,bx+4\,a}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x - 1/4*(2*pi + 4*I*a - 2*arctan(c) + I*log(c^2 + 1))*log(x) - 1/2*int
egrate(arctan2(c*e^(2*b*x + 2*a), -1)/x, x) + 1/4*I*integrate(log(c^2*e^(4*
b*x + 4*a) + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i\,\log\left(-\frac{\left(ce^{(2\,bx+2\,a)}+i\right)e^{(-2\,bx-2\,a)}}{c-i}\right)}{2\,x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/(c - I))/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c-(I-c)*tanh(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((c - I)*tanh(b*x + a) + c)/x, x)
```

3.93 $\int (e + fx)^3 \tan^{-1}(\coth(a + bx))\,dx$

Optimal. Leaf size=299

$$-\frac{3if^2(e+fx)\text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3}+\frac{3if^2(e+fx)\text{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}+\frac{3if(e+fx)^2\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2}-\frac{3if(e}{}$$

```
[Out] ((e + f*x)^4*ArcTan[E^(2*a + 2*b*x)])/(4*f) + ((e + f*x)^4*ArcTan[Coth[a +
b*x]])/(4*f) - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I
/4)*(e + f*x)^3*PolyLog[2, I*E^(2*a + 2*b*x)])/b + (((3*I)/8)*f*(e + f*x)^2
*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/b^2 - (((3*I)/8)*f*(e + f*x)^2*PolyLog[3
, I*E^(2*a + 2*b*x)])/b^2 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, (-I)*E^(2*a
 + 2*b*x)])/b^3 + (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/b
^3 + (((3*I)/16)*f^3*PolyLog[5, (-I)*E^(2*a + 2*b*x)])/b^4 - (((3*I)/16)*f^
3*PolyLog[5, I*E^(2*a + 2*b*x)])/b^4
```

Rubi [A] time = 0.208555, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5185, 4180, 2531, 6609, 2282, 6589}

$$-\frac{3if^2(e+fx)\text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3}+\frac{3if^2(e+fx)\text{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}+\frac{3if(e+fx)^2\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2}-\frac{3if(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcTan[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^4*ArcTan[E^(2*a + 2*b*x)])/(4*f) + ((e + f*x)^4*ArcTan[Coth[a +
b*x]])/(4*f) - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I
/4)*(e + f*x)^3*PolyLog[2, I*E^(2*a + 2*b*x)])/b + (((3*I)/8)*f*(e + f*x)^2
*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/b^2 - (((3*I)/8)*f*(e + f*x)^2*PolyLog[3
, I*E^(2*a + 2*b*x)])/b^2 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, (-I)*E^(2*a
 + 2*b*x)])/b^3 + (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/b
^3 + (((3*I)/16)*f^3*PolyLog[5, (-I)*E^(2*a + 2*b*x)])/b^4 - (((3*I)/16)*f^
3*PolyLog[5, I*E^(2*a + 2*b*x)])/b^4
```

Rule 5185

```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTan[Coth[a + b*x]])/(f*(m + 1)), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)^3\tan^{-1}(\coth(a+bx))\,dx = \frac{(e+fx)^4\tan^{-1}(\coth(a+bx))}{4f}+\frac{b\int (e+fx)^4\text{sech}(2a+2bx)\,dx}{4f}$$
$$= \frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f}+\frac{(e+fx)^4\tan^{-1}(\coth(a+bx))}{4f}-\frac{1}{2}i\int (e+fx)^3\log$$
$$= \frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f}+\frac{(e+fx)^4\tan^{-1}(\coth(a+bx))}{4f}-\frac{i(e+fx)^3\text{Li}_2\left(-ie\right.}{4b}$$
$$= \frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f}+\frac{(e+fx)^4\tan^{-1}(\coth(a+bx))}{4f}-\frac{i(e+fx)^3\text{Li}_2\left(-ie\right.}{4b}$$
$$= \frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f}+\frac{(e+fx)^4\tan^{-1}(\coth(a+bx))}{4f}-\frac{i(e+fx)^3\text{Li}_2\left(-ie\right.}{4b}$$
$$= \frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f}+\frac{(e+fx)^4\tan^{-1}(\coth(a+bx))}{4f}-\frac{i(e+fx)^3\text{Li}_2\left(-ie\right.}{4b}$$
$$= \frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f}+\frac{(e+fx)^4\tan^{-1}(\coth(a+bx))}{4f}-\frac{i(e+fx)^3\text{Li}_2\left(-ie\right.}{4b}$$

Mathematica [B] time = 4.81517, size = 600, normalized size = 2.01

$$\frac{1}{4}x\left(6e^2fx+4e^3+4ef^2x^2+f^3x^3\right)\tan^{-1}(\coth(a+bx))+\frac{i\left(6b^2e^2f\text{PolyLog}\left(3,-ie^{2(a+bx)}\right)-6b^2e^2f\text{PolyLog}\left(3,ie^{2(a\right.\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcTan[Coth[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTan[Coth[a + b*x]])/4 + (
(I/16)*(8*b^4*e^3*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 - I
*E^(2*(a + b*x))] + 8*b^4*e*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] + 2*b^4*f^3*
x^4*Log[1 - I*E^(2*(a + b*x))] - 8*b^4*e^3*x*Log[1 + I*E^(2*(a + b*x))] - 1
2*b^4*e^2*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 + I*E^(2
*(a + b*x))] - 2*b^4*f^3*x^4*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*(e + f*x)^3
*PolyLog[2, (-I)*E^(2*(a + b*x))] + 4*b^3*(e + f*x)^3*PolyLog[2, I*E^(2*(a
+ b*x))] + 6*b^2*e^2*f*PolyLog[3, (-I)*E^(2*(a + b*x))] + 12*b^2*e*f^2*x*Po
lyLog[3, (-I)*E^(2*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^(2*(a + b*
x))] - 6*b^2*e^2*f*PolyLog[3, I*E^(2*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3
, I*E^(2*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^(2*(a + b*x))] - 6*b*e*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] - 6*b*f^3*x*PolyLog[4, (-I)*E^(2*(a +
b*x))] + 6*b*e*f^2*PolyLog[4, I*E^(2*(a + b*x))] + 6*b*f^3*x*PolyLog[4, I*E
^(2*(a + b*x))] + 3*f^3*PolyLog[5, (-I)*E^(2*(a + b*x))] - 3*f^3*PolyLog[5,
 I*E^(2*(a + b*x))]))/b^4
```

Maple [C] time = 4.787, size = 7275, normalized size = 24.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arctan(coth(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\left(f^3x^4 + 4\,ef^2x^3 + 6\,e^2fx^2 + 4\,e^3x\right)\arctan\left(e^{(2\,bx+2\,a)} + 1, e^{(2\,bx+2\,a)} - 1\right) + \int \frac{\left(bf^3x^4e^{(2\,a)} + 4\,bef^2x^3e^{(2\,a)} + 6\,be^2fx^2e^{(2\,a)}\right.}{2\left(e^{(4\,bx+4\,a)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*arctan2(e^(2*b*x + 2*a)
 + 1, e^(2*b*x + 2*a) - 1) + integrate(1/2*(b*f^3*x^4*e^(2*a) + 4*b*e*f^2*x
^3*e^(2*a) + 6*b*e^2*f*x^2*e^(2*a) + 4*b*e^3*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x
 + 4*a) + 1), x)
```

Fricas [C] time = 2.99838, size = 4072, normalized size = 13.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(-24*I*f^3*polylog(5, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
24*I*f^3*polylog(5, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 24*I*
f^3*polylog(5, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 24*I*f^3*p
```

```
olylog(5, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^4*f^3*x^4
 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*arctan(cosh(b*x + a)/si
nh(b*x + a)) + (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x + 4
*I*b^3*e^3)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (4*I*b^3
*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(-1/2*
sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-4*I*b^3*f^3*x^3 - 12*I*b^3*e
*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(1/2*sqrt(-4*I)*(cosh(b*x +
 a) + sinh(b*x + a))) + (-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e
^2*f*x - 4*I*b^3*e^3)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))
) + (I*b^4*f^3*x^4 + 4*I*b^4*e*f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x
+ 4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(1/2*
sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^4*f^3*x^4 + 4*I*b^4*e
*f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x + 4*I*a*b^3*e^3 - 6*I*a^2*b^2*
e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + si
nh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 - 4*I*b^4*e*f^2*x^3 - 6*I*b^4*e^2*f*x^2
 - 4*I*b^4*e^3*x - 4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*
a^4*f^3)*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^4*
f^3*x^4 - 4*I*b^4*e*f^2*x^3 - 6*I*b^4*e^2*f*x^2 - 4*I*b^4*e^3*x - 4*I*a*b^3
*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(-1/2*sqrt(-4*I)
*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f
 - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(
b*x + a)) + (-4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f
^3)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (4*I*a*b^3*e^3
- 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(I*sqrt(-4*I) + 2*cos
h(b*x + a) + 2*sinh(b*x + a)) + (4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^
3*b*e*f^2 - I*a^4*f^3)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a
)) + (24*I*b*f^3*x + 24*I*b*e*f^2)*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a)
+ sinh(b*x + a))) + (24*I*b*f^3*x + 24*I*b*e*f^2)*polylog(4, -1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a))) + (-24*I*b*f^3*x - 24*I*b*e*f^2)*polylog(
4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-24*I*b*f^3*x - 24*I*
b*e*f^2)*polylog(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-12
*I*b^2*f^3*x^2 - 24*I*b^2*e*f^2*x - 12*I*b^2*e^2*f)*polylog(3, 1/2*sqrt(4*I
)*(cosh(b*x + a) + sinh(b*x + a))) + (-12*I*b^2*f^3*x^2 - 24*I*b^2*e*f^2*x
- 12*I*b^2*e^2*f)*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))
) + (12*I*b^2*f^3*x^2 + 24*I*b^2*e*f^2*x + 12*I*b^2*e^2*f)*polylog(3, 1/2*s
qrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (12*I*b^2*f^3*x^2 + 24*I*b^2*e
*f^2*x + 12*I*b^2*e^2*f)*polylog(3, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b
*x + a))))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*atan(coth(b*x+a)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^3 \arctan(\coth(bx+a))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*arctan(coth(b*x + a)), x)
```

3.94 $\int (e+fx)^2 \tan^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=229

$$\frac{if(e+fx)\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2}-\frac{if(e+fx)\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2}-\frac{if^2\text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3}+\frac{if^2\text{PolyLog}\left(4,i\right.}{8b^3}$$

```
[Out] ((e + f*x)^3*ArcTan[E^(2*a + 2*b*x)])/(3*f) + ((e + f*x)^3*ArcTan[Coth[a +
b*x]])/(3*f) - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I
/4)*(e + f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b + ((I/4)*f*(e + f*x)*PolyL
og[3, (-I)*E^(2*a + 2*b*x)])/b^2 - ((I/4)*f*(e + f*x)*PolyLog[3, I*E^(2*a +
 2*b*x)])/b^2 - ((I/8)*f^2*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/b^3 + ((I/8)*f
^2*PolyLog[4, I*E^(2*a + 2*b*x)])/b^3
```

Rubi [A] time = 0.15292, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5185, 4180, 2531, 6609, 2282, 6589}

$$\frac{if(e+fx)\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2}-\frac{if(e+fx)\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2}-\frac{if^2\text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3}+\frac{if^2\text{PolyLog}\left(4,i\right.}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcTan[Coth[a + b*x]],x]
```


```
[Out] ((e + f*x)^3*ArcTan[E^(2*a + 2*b*x)])/(3*f) + ((e + f*x)^3*ArcTan[Coth[a +
b*x]])/(3*f) - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I
/4)*(e + f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b + ((I/4)*f*(e + f*x)*PolyL
og[3, (-I)*E^(2*a + 2*b*x)])/b^2 - ((I/4)*f*(e + f*x)*PolyLog[3, I*E^(2*a +
 2*b*x)])/b^2 - ((I/8)*f^2*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/b^3 + ((I/8)*f
^2*PolyLog[4, I*E^(2*a + 2*b*x)])/b^3
```


Rule 5185

```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTan[Coth[a + b*x]])/(f*(m + 1)), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)^2\tan^{-1}(\coth(a+bx))\,dx = \frac{(e+fx)^3\tan^{-1}(\coth(a+bx))}{3f}+\frac{b\int (e+fx)^3\operatorname{sech}(2a+2bx)\,dx}{3f}$$
$$=\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f}+\frac{(e+fx)^3\tan^{-1}(\coth(a+bx))}{3f}-\frac{1}{2}i\int (e+fx)^2\log\left(1\right.$$
$$=\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f}+\frac{(e+fx)^3\tan^{-1}(\coth(a+bx))}{3f}-\frac{i(e+fx)^2\text{Li}_2\left(-ie^{2a+\cdots}\right.}{4b}$$
$$=\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f}+\frac{(e+fx)^3\tan^{-1}(\coth(a+bx))}{3f}-\frac{i(e+fx)^2\text{Li}_2\left(-ie^{2a+\cdots}\right.}{4b}$$
$$=\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f}+\frac{(e+fx)^3\tan^{-1}(\coth(a+bx))}{3f}-\frac{i(e+fx)^2\text{Li}_2\left(-ie^{2a+\cdots}\right.}{4b}$$
$$=\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f}+\frac{(e+fx)^3\tan^{-1}(\coth(a+bx))}{3f}-\frac{i(e+fx)^2\text{Li}_2\left(-ie^{2a+\cdots}\right.}{4b}$$

Mathematica [A] time = 2.6565, size = 375, normalized size = 1.64

$$\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\tan^{-1}(\coth(a+bx))+\frac{i\left(-6b^2(e+fx)^2\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+6b^2(e+fx)^2\text{PolyLog}\left(2,ie^{2(a+\cdots}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcTan[Coth[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTan[Coth[a + b*x]])/3 + ((I/24)*(12*b^3*e
^2*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 - I*E^(2*(a + b*x))]
 + 4*b^3*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] - 12*b^3*e^2*x*Log[1 + I*E^(2*(
a + b*x))] - 12*b^3*e*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*f^2*x^3*Log[
1 + I*E^(2*(a + b*x))] - 6*b^2*(e + f*x)^2*PolyLog[2, (-I)*E^(2*(a + b*x))]
 + 6*b^2*(e + f*x)^2*PolyLog[2, I*E^(2*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I
)*E^(2*(a + b*x))] + 6*b*f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] - 6*b*e*f*P
olyLog[3, I*E^(2*(a + b*x))] - 6*b*f^2*x*PolyLog[3, I*E^(2*(a + b*x))] - 3*
```

```
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] + 3*f^2*PolyLog[4, I*E^(2*(a + b*x))])
)/b^3
```

Maple [C] time = 7.741, size = 5425, normalized size = 23.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arctan(coth(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\left(f^2x^3+3\,efx^2+3\,e^2x\right)\arctan\left(e^{(2\,bx+2\,a)}+1,e^{(2\,bx+2\,a)}-1\right)+\int\frac{2\left(bf^2x^3e^{(2\,a)}+3\,befx^2e^{(2\,a)}+3\,be^2xe^{(2\,a)}\right)e^{(2\,bx)}}{3\left(e^{(4\,bx+4\,a)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x +
 2*a) - 1) + integrate(2/3*(b*f^2*x^3*e^(2*a) + 3*b*e*f*x^2*e^(2*a) + 3*b*e
^2*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1), x)
```

Fricas [C] time = 2.62113, size = 2901, normalized size = 12.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*I*f^2*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*
I*f^2*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*f^2*
polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*f^2*polylo
g(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^3*f^2*x^3 + 3*
b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(cosh(b*x + a)/sinh(b*x + a)) + (3*I*b^2*f
^2*x^2 + 6*I*b^2*e*f*x + 3*I*b^2*e^2)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) +
sinh(b*x + a))) + (3*I*b^2*f^2*x^2 + 6*I*b^2*e*f*x + 3*I*b^2*e^2)*dilog(-1/
2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-3*I*b^2*f^2*x^2 - 6*I*b^2*
e*f*x - 3*I*b^2*e^2)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
+ (-3*I*b^2*f^2*x^2 - 6*I*b^2*e*f*x - 3*I*b^2*e^2)*dilog(-1/2*sqrt(-4*I)*(c
osh(b*x + a) + sinh(b*x + a))) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3*I*b^3
*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(1/2*sqrt(4*I)*(cosh
(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3*I*b^
3*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(-1/2*sqrt(4*I)*(co
sh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 - 3*I
*b^3*e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 -
```

```
3*I*b^3*e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(-1/2*sqrt(-4
*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-3*I*a*b^2*e^2 + 3*I*a^2*b*e*f
- I*a^3*f^2)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-3*I*a
*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) +
2*sinh(b*x + a)) + (3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(I*sqrt(-
4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (3*I*a*b^2*e^2 - 3*I*a^2*b*e*f
+ I*a^3*f^2)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-6*I
*b*f^2*x - 6*I*b*e*f)*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x +
a))) + (-6*I*b*f^2*x - 6*I*b*e*f)*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a)
+ sinh(b*x + a))) + (6*I*b*f^2*x + 6*I*b*e*f)*polylog(3, 1/2*sqrt(-4*I)*(co
sh(b*x + a) + sinh(b*x + a))) + (6*I*b*f^2*x + 6*I*b*e*f)*polylog(3, -1/2*s
qrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*atan(coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^2 \arctan(\coth(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*arctan(coth(b*x + a)), x)
```

3.95 $\int (e+fx)\tan^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=159

$$\frac{ifPolyLog\left(3,-ie^{2a+2bx}\right)}{8b^2}-\frac{ifPolyLog\left(3,ie^{2a+2bx}\right)}{8b^2}-\frac{i(e+fx)PolyLog\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)PolyLog\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[Out] ((e + f*x)^2*ArcTan[E^(2*a + 2*b*x)])/(2*f) + ((e + f*x)^2*ArcTan[Coth[a +
b*x]])/(2*f) - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I/4
)*(e + f*x)*PolyLog[2, I*E^(2*a + 2*b*x)])/b + ((I/8)*f*PolyLog[3, (-I)*E^(
2*a + 2*b*x)])/b^2 - ((I/8)*f*PolyLog[3, I*E^(2*a + 2*b*x)])/b^2
```

Rubi [A] time = 0.100106, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5185, 4180, 2531, 2282, 6589}

$$\frac{ifPolyLog\left(3,-ie^{2a+2bx}\right)}{8b^2}-\frac{ifPolyLog\left(3,ie^{2a+2bx}\right)}{8b^2}-\frac{i(e+fx)PolyLog\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)PolyLog\left(2,ie^{2a+2bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcTan[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^2*ArcTan[E^(2*a + 2*b*x)])/(2*f) + ((e + f*x)^2*ArcTan[Coth[a +
b*x]])/(2*f) - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I/4
)*(e + f*x)*PolyLog[2, I*E^(2*a + 2*b*x)])/b + ((I/8)*f*PolyLog[3, (-I)*E^(
2*a + 2*b*x)])/b^2 - ((I/8)*f*PolyLog[3, I*E^(2*a + 2*b*x)])/b^2
```

Rule 5185

```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTan[Coth[a + b*x]])/(f*(m + 1)), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)\tan^{-1}(\coth(a+bx))\,dx = \frac{(e+fx)^2\tan^{-1}(\coth(a+bx))}{2f} + \frac{b\int (e+fx)^2\operatorname{sech}(2a+2bx)\,dx}{2f}$$
$$= \frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\coth(a+bx))}{2f} - \frac{1}{2}i\int (e+fx)\log\left(1-\right.$$
$$= \frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\coth(a+bx))}{2f} - \frac{i(e+fx)\text{Li}_2\left(-ie^{2a+2bx}\right.}{4b}$$
$$= \frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\coth(a+bx))}{2f} - \frac{i(e+fx)\text{Li}_2\left(-ie^{2a+2bx}\right.}{4b}$$
$$= \frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\coth(a+bx))}{2f} - \frac{i(e+fx)\text{Li}_2\left(-ie^{2a+2bx}\right.}{4b}$$

Mathematica [A] time = 1.77281, size = 278, normalized size = 1.75

$$\frac{if\left(-2bx\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+2bx\text{PolyLog}\left(2,ie^{2(a+bx)}\right)+\text{PolyLog}\left(3,-ie^{2(a+bx)}\right)-\text{PolyLog}\left(3,ie^{2(a+bx)}\right)+2b^2x^2\,\text{lo}\right.}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*ArcTan[Coth[a + b*x]],x]
```

```
[Out] e*x*ArcTan[Coth[a + b*x]] + (f*x^2*ArcTan[Coth[a + b*x]])/2 + (e*(-(((-4*I)
*a + Pi - (4*I)*b*x)*(Log[1 - I*E^(2*(a + b*x))] - Log[1 + I*E^(2*(a + b*x)
)])) + ((-4*I)*a + Pi)*Log[Cot[((4*I)*a + Pi + (4*I)*b*x)/4]] - (2*I)*(Poly
Log[2, (-I)*E^(2*(a + b*x))] - PolyLog[2, I*E^(2*(a + b*x))])))/(8*b) + ((I
/8)*f*(2*b^2*x^2*Log[1 - I*E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + I*E^(2*(a +
 b*x))] - 2*b*x*PolyLog[2, (-I)*E^(2*(a + b*x))] + 2*b*x*PolyLog[2, I*E^(2*
(a + b*x))] + PolyLog[3, (-I)*E^(2*(a + b*x))] - PolyLog[3, I*E^(2*(a + b*x
))]))/b^2
```

Maple [C] time = 7.496, size = 2415, normalized size = 15.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arctan(coth(b*x+a)),x)
```

```
[Out] -1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2
*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+1/2*I/b*e*dilog(((-I)^(1/2)-exp(b*x+a))/
```

(-I)^(1/2))+1/2*I/b*e*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/2*I*e/b*d
ilog(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*e/b*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/4*
Pi*x*e*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*f*csg
n((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*f*csgn(I*(exp(2
*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3+1/2*I*(-1/2*f*x^2-e*x)*ln(exp(2*b*x+2*a)
+I)+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(
exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I
))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*f*csgn((1+I)*
(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a
)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^
2+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(
2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I))
csgn(I(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+1/4*I*ln(exp(2*b*x+2*a)-I)
*f*x^2+1/2*I*ln(exp(2*b*x+2*a)-I)*e*x+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I))
csgn(I(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-1/4*Pi*x*e*csgn(I/(exp(2*b
*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+1/4*Pi*x*e*csgn
(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-1/2*
I/b*a*e*ln(exp(2*b*x+2*a)+I)+1/4*Pi*x*e*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(
2*b*x+2*a)-1))^3+1/4*I/b^2*f*a^2*ln(exp(2*b*x+2*a)+I)+1/8*I*f*polylog(3,-I*
exp(2*b*x+2*a))/b^2+1/8*Pi*f*x^2+1/4*Pi*e*x-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+
2*a)+I)/(exp(2*b*x+2*a)-1))^3+1/8*Pi*x^2*f*csgn((1-I)*(exp(2*b*x+2*a)+I)/(e
xp(2*b*x+2*a)-1))^3+1/8*Pi*x^2*f*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2
*a)-1))^3+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3-1/4*Pi
*x*e*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^3+1/4*Pi*x*e*csgn((1-I)*
(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^3+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a
)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-
1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*
b*x+2*a)+I)/(exp(2*b*x+2*a)-1))+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2
*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-1/4*Pi*x*e*
csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-1/8*I*f*polylog(3,I*exp
(2*b*x+2*a))/b^2+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+
2*a)+I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2*a)-1))*csgn(
I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2
*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+1/4*Pi*x*e*csgn(I*(
exp(2*b*x+2*a)-I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(ex
p(2*b*x+2*a)-1))-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I/(exp(2*b*x+2*
a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))-1/8*Pi*x^2*f*csgn(I*(e
xp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b
*x+2*a)-1))-1/4*I*f/b^2*a^2*ln(-exp(2*b*x+2*a)+I)+1/2*I*e/b*a*ln(-exp(2*b*x
+2*a)+I)-1/4*I*f/b^2*(b*x+a)*polylog(2,-I*exp(2*b*x+2*a))-1/2*I*e/b*(b*x+a)
*ln(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*e/b*(b*x+a)*ln(1-exp(b*x+a)*(-1)^(3/4))+
1/2*I*f/b^2*a*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*f/b^2*a*dilog(1-exp(b*x+
a)*(-1)^(3/4))-1/2*I/b^2*f*a*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*
I/b^2*f*a*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*e/b*(b*x+a)*ln(((
-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*e/b*(b*x+a)*ln(((-I)^(1/2)+exp(b*x+
a))/(-I)^(1/2))+1/4*I*f/b^2*(b*x+a)^2*ln(1-I*exp(2*b*x+2*a))+1/4*I*f/b^2*(b
*x+a)*polylog(2,I*exp(2*b*x+2*a))+1/2*I*f/b^2*a*(b*x+a)*ln(1+exp(b*x+a)*(-1
)^(3/4))+1/2*I*f/b^2*a*(b*x+a)*ln(1-exp(b*x+a)*(-1)^(3/4))-1/2*I*f/b^2*a*(b
*x+a)*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I*f/b^2*a*(b*x+a)*ln(((-I)
^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/4*I*f/b^2*(b*x+a)^2*ln(1+I*exp(2*b*x+2*a))
+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*
(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+
I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1)
)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(fx^2+2\,ex\right)\arctan\left(e^{(2\,bx+2\,a)}+1,e^{(2\,bx+2\,a)}-1\right)+\int\frac{\left(bfx^2e^{(2\,a)}+2\,bexe^{(2\,a)}\right)e^{(2\,bx)}}{e^{(4\,bx+4\,a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctan(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*e*x)*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x + 2*a) - 1) + int
egrate((b*f*x^2*e^(2*a) + 2*b*e*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1),
 x)
```

Fricas [C] time = 2.61824, size = 1894, normalized size = 11.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctan(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*f*x^2 + 2*b^2*e*x)*arctan(cosh(b*x + a)/sinh(b*x + a)) + (2*I*b
*f*x + 2*I*b*e)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (2*I
*b*f*x + 2*I*b*e)*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (
-2*I*b*f*x - 2*I*b*e)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
 + (-2*I*b*f*x - 2*I*b*e)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x +
 a))) + (I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I*a^2*f)*log(1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*
b*e - I*a^2*f)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-
I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e + I*a^2*f)*log(1/2*sqrt(-4*I)*(cosh(b
*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e + I
*a^2*f)*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-2*I*a*
b*e + I*a^2*f)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-2*I
*a*b*e + I*a^2*f)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (
2*I*a*b*e - I*a^2*f)*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a))
+ (2*I*a*b*e - I*a^2*f)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x +
a)) - 2*I*f*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I
*f*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*f*polyl
og(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*f*polylog(3, -1
/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*atan(coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)\arctan\left(\coth\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctan(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*arctan(coth(b*x + a)), x)
```

3.96 $\int \tan^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=73

$$-\frac{i\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}+x\tan^{-1}\left(e^{2a+2bx}\right)+x\tan^{-1}(\coth(a+bx))$$

```
[Out] x*ArcTan[E^(2*a + 2*b*x)] + x*ArcTan[Coth[a + b*x]] - ((I/4)*PolyLog[2, (-I
)*E^(2*a + 2*b*x)])/b + ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b
```

Rubi [A] time = 0.043107, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5181, 4180, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}+x\tan^{-1}\left(e^{2a+2bx}\right)+x\tan^{-1}(\coth(a+bx))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[E^(2*a + 2*b*x)] + x*ArcTan[Coth[a + b*x]] - ((I/4)*PolyLog[2, (-I
)*E^(2*a + 2*b*x)])/b + ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b
```

Rule 5181

```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTan[Coth[a + b
*x]], x] + Dist[b, Int[x*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tan^{-1}(\coth(a+bx))\,dx = x\tan^{-1}(\coth(a+bx)) + b\int x\text{sech}(2a+2bx)\,dx$$
$$= x\tan^{-1}\left(e^{2a+2bx}\right) + x\tan^{-1}(\coth(a+bx)) - \frac{1}{2}i\int \log\left(1-ie^{2a+2bx}\right)dx + \frac{1}{2}i\int \log\left(1+i\right.$$
$$= x\tan^{-1}\left(e^{2a+2bx}\right) + x\tan^{-1}(\coth(a+bx)) - \frac{i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}dx, x, e^{2a+2bx}\right)}{4b} + \frac{i\,\text{Subst}}{}$$
$$= x\tan^{-1}\left(e^{2a+2bx}\right) + x\tan^{-1}(\coth(a+bx)) - \frac{i\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b} + \frac{i\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}$$

Mathematica [A] time = 0.0546063, size = 132, normalized size = 1.81

$$x\tan^{-1}(\coth(a+bx)) + \frac{-2i\left(\text{PolyLog}\left(2,-ie^{2(a+bx)}\right) - \text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right) - (-4ia-4ibx+\pi)\left(\log\left(1-ie^{2(a+bx)}\right)\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[Coth[a + b*x]] + (-(((-4*I)*a + Pi - (4*I)*b*x)*(Log[1 - I*E^(2*(a
 + b*x))] - Log[1 + I*E^(2*(a + b*x))])) + ((-4*I)*a + Pi)*Log[Cot[((4*I)*a
 + Pi + (4*I)*b*x)/4]] - (2*I)*(PolyLog[2, (-I)*E^(2*(a + b*x))] - PolyLog[
2, I*E^(2*(a + b*x))]))/(8*b)
```

Maple [B] time = 0.118, size = 440, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(coth(b*x+a)),x)
```

```
[Out] 1/b*arctanh(coth(b*x+a))*arctan(coth(b*x+a))-1/4*I/b*dilog(-I*cosh(2*arctan
h(coth(b*x+a)))-I*sinh(2*arctanh(coth(b*x+a))))+1/2*I/b*ln((1-I)/(1-coth(b*
x+a)^2)^(1/2)+(1+I)*coth(b*x+a)/(1-coth(b*x+a)^2)^(1/2))*arctanh(coth(b*x+a
))-1/4*I/b*ln((1-I)/(1-coth(b*x+a)^2)^(1/2)+(1+I)*coth(b*x+a)/(1-coth(b*x+a
)^2)^(1/2))*ln(-I*cosh(2*arctanh(coth(b*x+a)))-I*sinh(2*arctanh(coth(b*x+a)
)))+1/4*I/b*dilog(I*cosh(2*arctanh(coth(b*x+a)))+I*sinh(2*arctanh(coth(b*x+
a))))-1/2*I/b*ln((1+I)/(1-coth(b*x+a)^2)^(1/2)+(1-I)*coth(b*x+a)/(1-coth(b*
x+a)^2)^(1/2))*arctanh(coth(b*x+a))+1/4*I/b*ln((1+I)/(1-coth(b*x+a)^2)^(1/2
)+(1-I)*coth(b*x+a)/(1-coth(b*x+a)^2)^(1/2))*ln(I*cosh(2*arctanh(coth(b*x+a
)))+I*sinh(2*arctanh(coth(b*x+a))))-1/4*I/b*arctanh(coth(b*x+a))*ln(-I*cosh
(2*arctanh(coth(b*x+a)))-I*sinh(2*arctanh(coth(b*x+a))))+1/4*I/b*arctanh(co
th(b*x+a))*ln(I*cosh(2*arctanh(coth(b*x+a)))+I*sinh(2*arctanh(coth(b*x+a)))
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x\arctan\left(e^{(2bx+2a)}+1, e^{(2bx+2a)}-1\right) + 2b\int \frac{xe^{(2bx+2a)}}{e^{(4bx+4a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] x*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x + 2*a) - 1) + 2*b*integrate(x*e^(2*
b*x + 2*a)/(e^(4*b*x + 4*a) + 1), x)
```

Fricas [B] time = 2.44648, size = 1098, normalized size = 15.04

$$\frac{2\,bx\arctan\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right) + (i\,bx + i\,a)\log\left(\frac{1}{2}\,\sqrt{4i}(\cosh\left(bx+a\right) + \sinh\left(bx+a\right)) + 1\right) + (i\,bx + i\,a)\log\left(-\frac{1}{2}\,\sqrt{4i}(\cosh\left(bx + \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(cosh(b*x + a)/sinh(b*x + a)) + (I*b*x + I*a)*log(1/2*sqrt
(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(-1/2*sqrt(4*
I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(1/2*sqrt(-4*I)
*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) - I*a*log(I*sqrt(4*I) + 2*cosh(b*x + a
) + 2*sinh(b*x + a)) - I*a*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x
+ a)) + I*a*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + I*a*log
(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + I*dilog(1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a))) + I*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) - I*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
- I*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{atan}{\left(\coth{\left(a + bx \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(coth(b*x+a)),x)
```

```
[Out] Integral(atan(coth(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arctan\left(\coth\left(bx + a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(coth(b*x + a)), x)
```

3.97 $\int \frac{\tan^{-1}(\coth(a+bx))}{e+fx} dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\tan^{-1}(\coth(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate[ArcTan[Coth[a + b*x]]/(e + f*x), x]
```

Rubi [A] time = 0.0432478, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(\coth(a+bx))}{e+fx} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[Coth[a + b*x]]/(e + f*x),x]
```

```
[Out] Defer[Int][ArcTan[Coth[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(\coth(a+bx))}{e+fx} dx = \int \frac{\tan^{-1}(\coth(a+bx))}{e+fx} dx$$

Mathematica [A] time = 4.32902, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(\coth(a+bx))}{e+fx} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[Coth[a + b*x]]/(e + f*x),x]
```

```
[Out] Integrate[ArcTan[Coth[a + b*x]]/(e + f*x), x]
```

Maple [A] time = 0.872, size = 0, normalized size = 0.

$$\int \frac{\arctan(\coth(bx+a))}{fx+e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(coth(b*x+a))/(f*x+e),x)
```

```
[Out] int(arctan(coth(b*x+a))/(f*x+e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(\coth(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(arctan(coth(b*x + a))/(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(\coth(bx+a))}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(arctan(coth(b*x + a))/(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(coth(b*x+a))/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(\coth(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(arctan(coth(b*x + a))/(f*x + e), x)
```

3.98 $\int x^2 \tan^{-1}(c + d \coth(a + bx))\, dx$

Optimal. Leaf size=351

$$-\frac{ix\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}+\frac{ix\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^3}-\frac{i\text{PolyLog}\left(4,\frac{(c+d+i)e^{2a}}{c-d+i}\right)}{8b^3}$$

```
[Out] (x^3*ArcTan[c + d*Coth[a + b*x]])/3 + (I/6)*x^3*Log[1 - ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/6)*x^3*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x^2*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c +
d)])/b - ((I/4)*x^2*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/
b - ((I/4)*x*PolyLog[3, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^2 + (
(I/4)*x*PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^2 + ((I/8)
*PolyLog[4, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^3 - ((I/8)*PolyLo
g[4, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^3
```

Rubi [A] time = 0.464925, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5201, 2190, 2531, 6609, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}+\frac{ix\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^3}-\frac{i\text{PolyLog}\left(4,\frac{(c+d+i)e^{2a}}{c-d+i}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Coth[a + b*x]])/3 + (I/6)*x^3*Log[1 - ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/6)*x^3*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x^2*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c +
d)])/b - ((I/4)*x^2*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/
b - ((I/4)*x*PolyLog[3, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^2 + (
(I/4)*x*PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^2 + ((I/8)
*PolyLog[4, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^3 - ((I/8)*PolyLo
g[4, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^3
```

Rule 5201

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Coth[a + b*x]])/(f*(m
 + 1)), x] + (-Dist[(I*b*(I - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E
^(2*a + 2*b*x))/(I - c + d - (I - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[(I
*b*(I + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(I + c
 - d - (I + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) - \frac{1}{3}(b(1-i(c+d)))\int \frac{e^{2a+2bx}x^3}{i+c-d+(-i-c-d)e^{2a+2bx}}$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3\log\left(1-\frac{(i\right.$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3\log\left(1-\frac{(i\right.$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3\log\left(1-\frac{(i\right.$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3\log\left(1-\frac{(i\right.$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3\log\left(1-\frac{(i\right.$$

Mathematica [A] time = 5.47724, size = 299, normalized size = 0.85

$$\frac{1}{3}x^3\tan^{-1}(d\coth(a+bx)+c) + \frac{i\left(6b^2x^2\text{PolyLog}\left(2,\frac{(c+d-i)e^{2(a+bx)}}{c-d-i}\right) - 6b^2x^2\text{PolyLog}\left(2,\frac{(c+d+i)e^{2(a+bx)}}{c-d+i}\right) - 6bx\text{PolyLog}\left(3,\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Coth[a + b*x]])/3 + ((I/24)*(4*b^3*x^3*Log[1 + ((-I + c +
 d)*E^(2*(a + b*x)))/(I - c + d)] - 4*b^3*x^3*Log[1 + ((I + c + d)*E^(2*(a
+ b*x)))/(-I - c + d)] + 6*b^2*x^2*PolyLog[2, ((-I + c + d)*E^(2*(a + b*x))
)/(-I + c - d)] - 6*b^2*x^2*PolyLog[2, ((I + c + d)*E^(2*(a + b*x)))/(I + c
 - d)] - 6*b*x*PolyLog[3, ((-I + c + d)*E^(2*(a + b*x)))/(-I + c - d)] + 6*
b*x*PolyLog[3, ((I + c + d)*E^(2*(a + b*x)))/(I + c - d)] + 3*PolyLog[4, ((
-I + c + d)*E^(2*(a + b*x)))/(-I + c - d)] - 3*PolyLog[4, ((I + c + d)*E^(2
*(a + b*x)))/(I + c - d)]))/b^3
```

Maple [C] time = 5.885, size = 6909, normalized size = 19.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+d*coth(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}x^3\arctan\left(\left(ce^{(2a)}+de^{(2a)}\right)e^{(2bx)}-c+d,e^{(2bx+2a)}-1\right)+4bd\int\frac{x^3e^{(2bx}}{3\left(c^2-2cd+d^2+\left(c^2e^{(4a)}+2cde^{(4a)}+d^2e^{(4a)}+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan2((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) - c + d, e^(2*b*x + 2*a)
- 1) + 4*b*d*integrate(1/3*x^3*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^
(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) -
 d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Fricas [C] time = 3.26848, size = 3614, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arctan((d*cosh(b*x + a) + c*sinh(b*x + a))/sinh(b*x + a)) +
3*I*b^2*x^2*dilog(1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 +
 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*I*b^2*x^2*dilog(-1/2*sqrt((4*c^2
- 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a
))) - 3*I*b^2*x^2*dilog(1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d +
 d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 3*I*b^2*x^2*dilog(-1/2*sqrt((
4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b
*x + a))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*
c*d + d^2 + 1)*sinh(b*x + a) + (c^2 - d^2 - 2*I*d + 1)*sqrt((4*c^2 - 4*d^2
+ 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1
)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 - 2*
```

```
I*d + 1)*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) + I*a^3
*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*si
nh(b*x + a) + (c^2 - d^2 + 2*I*d + 1)*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2
 - 2*c*d + d^2 + 1))) + I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) +
 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 + 2*I*d + 1)*sqrt((4*
c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) - 6*I*b*x*polylog(3, 1/2
*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) +
 sinh(b*x + a))) - 6*I*b*x*polylog(3, -1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)
/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*b*x*polylo
g(3, 1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*
x + a) + sinh(b*x + a))) + 6*I*b*x*polylog(3, -1/2*sqrt((4*c^2 - 4*d^2 - 8*
I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + (I*b^3
*x^3 + I*a^3)*log(1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 +
 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*x^3 + I*a^3)*log(-1/2*sq
rt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + si
nh(b*x + a)) + 1) + (-I*b^3*x^3 - I*a^3)*log(1/2*sqrt((4*c^2 - 4*d^2 - 8*I*
d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*
b^3*x^3 - I*a^3)*log(-1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d
^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*I*polylog(4, 1/2*sqrt((4*
c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x
 + a))) + 6*I*polylog(4, -1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d
 + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*polylog(4, 1/2*sqrt((4*
c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x
 + a))) - 6*I*polylog(4, -1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d
 + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \arctan\left(d\coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(d*coth(b*x + a) + c), x)
```

3.99 $\int x \tan^{-1}(c + d \coth(a + bx)) \, dx$

Optimal. Leaf size=265

$$-\frac{i\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^2}+\frac{i\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{ix\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a}}{c-d+i}\right)}{4b}$$

```
[Out] (x^2*ArcTan[c + d*Coth[a + b*x]])/2 + (I/4)*x^2*Log[1 - ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/4)*x^2*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)
])/b - ((I/4)*x*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b -
((I/8)*PolyLog[3, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^2 + ((I/8)*
PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^2
```

Rubi [A] time = 0.370437, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5201, 2190, 2531, 2282, 6589}

$$-\frac{i\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^2}+\frac{i\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{ix\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a}}{c-d+i}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Coth[a + b*x]])/2 + (I/4)*x^2*Log[1 - ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/4)*x^2*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)
])/b - ((I/4)*x*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b -
((I/8)*PolyLog[3, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^2 + ((I/8)*
PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^2
```

Rule 5201

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Coth[a + b*x]])/(f*(m
 + 1)), x] + (-Dist[(I*b*(I - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E
^(2*a + 2*b*x))/(I - c + d - (I - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[(I
*b*(I + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(I + c
 - d - (I + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+d\coth(a+bx))\,dx=\frac{1}{2}x^2\tan^{-1}(c+d\coth(a+bx))-\frac{1}{2}(b(1-i(c+d)))\int\frac{e^{2a+2bx}x^2}{i+c-d+(-i-c-d)e^{2a+2bx}}$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{4}ix^2\log\left(1-\frac{(i-}{}\right.$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{4}ix^2\log\left(1-\frac{(i-}{}\right.$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{4}ix^2\log\left(1-\frac{(i-}{}\right.$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{4}ix^2\log\left(1-\frac{(i-}{}\right.$$

Mathematica [A] time = 4.07315, size = 225, normalized size = 0.85

$$\frac{1}{2}x^2\tan^{-1}(d\coth(a+bx)+c)+\frac{i\left(2bx\text{PolyLog}\left(2,\frac{(c+d-i)e^{2(a+bx)}}{c-d-i}\right)-2bx\text{PolyLog}\left(2,\frac{(c+d+i)e^{2(a+bx)}}{c-d+i}\right)-\text{PolyLog}\left(3,\frac{(c+d-i)e^{2}}{c-d-}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Coth[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + ((-I + c +
d)*E^(2*(a + b*x)))/(I - c + d)] - 2*b^2*x^2*Log[1 + ((I + c + d)*E^(2*(a +
 b*x)))/(-I - c + d)] + 2*b*x*PolyLog[2, ((-I + c + d)*E^(2*(a + b*x)))/(-I
 + c - d)] - 2*b*x*PolyLog[2, ((I + c + d)*E^(2*(a + b*x)))/(I + c - d)] -
PolyLog[3, ((-I + c + d)*E^(2*(a + b*x)))/(-I + c - d)] + PolyLog[3, ((I +
c + d)*E^(2*(a + b*x)))/(I + c - d)]))/b^2
```

Maple [C] time = 16.575, size = 6529, normalized size = 24.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+d*coth(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2\arctan\left(\left(ce^{(2a)}+de^{(2a)}\right)e^{(2bx)}-c+d,e^{(2bx+2a)}-1\right)+2bd\int\frac{x^2e^{(2bx+}}{c^2-2cd+d^2+\left(c^2e^{(4a)}+2cde^{(4a)}+d^2e^{(4a)}+e^{(4a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) - c + d, e^(2*b*x + 2*a)
- 1) + 2*b*d*integrate(x^2*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a
) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) - d^2
*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Fricas [C] time = 3.02052, size = 2963, normalized size = 11.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arctan((d*cosh(b*x + a) + c*sinh(b*x + a))/sinh(b*x + a)) +
2*I*b*x*dilog(1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))
*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*b*x*dilog(-1/2*sqrt((4*c^2 - 4*d^2
+ 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*
I*b*x*dilog(1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(
cosh(b*x + a) + sinh(b*x + a))) - 2*I*b*x*dilog(-1/2*sqrt((4*c^2 - 4*d^2 -
8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*a^
2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*s
inh(b*x + a) + (c^2 - d^2 - 2*I*d + 1)*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^
2 - 2*c*d + d^2 + 1))) + I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a)
+ 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 - 2*I*d + 1)*sqrt((4
*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) - I*a^2*log(2*(c^2 + 2*
c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + (c
^2 - d^2 + 2*I*d + 1)*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 +
 1))) - I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d
+ d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 + 2*I*d + 1)*sqrt((4*c^2 - 4*d^2 - 8*
I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) + (I*b^2*x^2 - I*a^2)*log(1/2*sqrt((4*c^
2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x +
 a)) + 1) + (I*b^2*x^2 - I*a^2)*log(-1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(
c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*x^2
+ I*a^2)*log(1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*x^2 + I*a^2)*log(-1/2*sqrt((
4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b
*x + a)) + 1) - 2*I*polylog(3, 1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 -
2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*polylog(3, -1/2*sq
rt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + si
```

```
nh(b*x + a))) + 2*I*polylog(3, 1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 -
2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*polylog(3, -1/2*sq
rt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + si
nh(b*x + a))))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \arctan\left(d \coth\left(bx + a\right) + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(d*coth(b*x + a) + c), x)
```

3.100 $\int \tan^{-1}(c + d \coth(a + bx))\, dx$

Optimal. Leaf size=174

$$\frac{i\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{i\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}+\frac{1}{2}ix\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)-\frac{1}{2}ix\log\left(1-\frac{(c+d+\right.}{c-d}$$

```
[Out] x*ArcTan[c + d*Coth[a + b*x]] + (I/2)*x*Log[1 - ((I - c - d)*E^(2*a + 2*b*x
))/(I - c + d)] - (I/2)*x*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)
] + ((I/4)*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b - ((I/4
)*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b
```

Rubi [A] time = 0.227623, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5193, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{i\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}+\frac{1}{2}ix\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)-\frac{1}{2}ix\log\left(1-\frac{(c+d+\right.}{c-d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Coth[a + b*x]] + (I/2)*x*Log[1 - ((I - c - d)*E^(2*a + 2*b*x
))/(I - c + d)] - (I/2)*x*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)
] + ((I/4)*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b - ((I/4
)*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b
```

Rule 5193

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tan[c + d*Coth[a + b*x]], x] + (-Dist[I*b*(I - c - d), Int[(x*E^(2*a + 2*b*
x))/(I - c + d - (I - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[I*b*(I + c + d
), Int[(x*E^(2*a + 2*b*x))/(I + c - d - (I + c + d)*E^(2*a + 2*b*x)), x], x
]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tan^{-1}(c+d\coth(a+bx))\,dx = x\tan^{-1}(c+d\coth(a+bx)) - (b(1-i(c+d)))\int \frac{e^{2a+2bx}x}{i+c-d+(-i-c-d)e^{2a+2bx}}\,dx + (b$$

$$= x\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{2}ix\log\left(1-\frac{(i+c+d}{i+c}\right.$$

$$= x\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{2}ix\log\left(1-\frac{(i+c+d}{i+c}\right.$$

$$= x\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{2}ix\log\left(1-\frac{(i+c+d}{i+c}\right.$$

Mathematica [A] time = 3.70343, size = 287, normalized size = 1.65

$$\frac{d\text{PolyLog}\left(2,\frac{(c^2+2cd+d^2+1)e^{2(a+bx)}}{c^2-d^2+2\sqrt{-d^2}+1}\right) - d\text{PolyLog}\left(2,-\frac{(c^2+2cd+d^2+1)e^{2(a+bx)}}{-c^2+d^2+2\sqrt{-d^2}-1}\right) + 2d(a+bx)\log\left(1-\frac{((c+d)^2+1)e^{2(a+bx)}}{c^2-d^2+2\sqrt{-d^2}+1}\right) - 2d(a+b}{4b\sqrt{-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Coth[a + b*x]] + (4*a*Sqrt[-d^2]*ArcTan[(1 + c^2 - d^2 - (1
+ c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(2*d)] + 2*d*(a + b*x)*Log[1 - ((1 +
(c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] - 2*d*(a + b*x)
*Log[1 + ((1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])]
 + d*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 +
2*Sqrt[-d^2])] - d*PolyLog[2, -(((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(
-1 - c^2 + d^2 + 2*Sqrt[-d^2]))])/(4*b*Sqrt[-d^2])
```

Maple [B] time = 0.091, size = 350, normalized size = 2.

$$\frac{\arctan(c+d\coth(bx+a))\ln(d\coth(bx+a)+d)}{2b} - \frac{\arctan(c+d\coth(bx+a))\ln(d\coth(bx+a)-d)}{2b} - \frac{\frac{i}{4}\ln(d\coth(}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*coth(b*x+a)),x)
```

```
[Out] 1/2/b*arctan(c+d*coth(b*x+a))*ln(d*coth(b*x+a)+d)-1/2/b*arctan(c+d*coth(b*x
+a))*ln(d*coth(b*x+a)-d)-1/4*I/b*ln(d*coth(b*x+a)-d)*ln((-d*coth(b*x+a)+I-c
)/(I-c-d))+1/4*I/b*ln(d*coth(b*x+a)-d)*ln((d*coth(b*x+a)+c+I)/(I+c+d))-1/4*
I/b*dilog((-d*coth(b*x+a)+I-c)/(I-c-d))+1/4*I/b*dilog((d*coth(b*x+a)+c+I)/(
I+c+d))+1/4*I/b*ln(d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)+I-c)/(I-c+d))-1/4*I/
b*ln(d*coth(b*x+a)+d)*ln((d*coth(b*x+a)+c+I)/(I+c-d))+1/4*I/b*dilog((-d*cot
h(b*x+a)+I-c)/(I-c+d))-1/4*I/b*dilog((d*coth(b*x+a)+c+I)/(I+c-d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$4bd\int \frac{xe^{(2bx+2a)}}{c^2-2cd+d^2+\left(c^2e^{(4a)}+2cde^{(4a)}+d^2e^{(4a)}+e^{(4a)}\right)e^{(4bx)}-2\left(c^2e^{(2a)}-d^2e^{(2a)}+e^{(2a)}\right)e^{(2bx)}+1}\,dx + x\arctan\left((\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 4*b*d*integrate(x*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a) + 2*c*d
*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) - d^2*e^(2*a)
+ e^(2*a))*e^(2*b*x) + 1), x) + x*arctan2((c*e^(2*a) + d*e^(2*a))*e^(2*b*x)
 - c + d, e^(2*b*x + 2*a) - 1)
```

Fricas [B] time = 6.50466, size = 2272, normalized size = 13.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan((d*cosh(b*x + a) + c*sinh(b*x + a))/sinh(b*x + a)) - I*a*
log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sin
h(b*x + a) + (c^2 - d^2 - 2*I*d + 1)*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2
- 2*c*d + d^2 + 1))) - I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*
(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 - 2*I*d + 1)*sqrt((4*c^2
 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) + I*a*log(2*(c^2 + 2*c*d +
d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + (c^2 - d
^2 + 2*I*d + 1)*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1)))
+ I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 +
1)*sinh(b*x + a) - (c^2 - d^2 + 2*I*d + 1)*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)
/(c^2 - 2*c*d + d^2 + 1))) + (I*b*x + I*a)*log(1/2*sqrt((4*c^2 - 4*d^2 + 8*
I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I
*b*x + I*a)*log(-1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 +
1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(1/2*sqrt((4*c
^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x
+ a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2
- 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + I*dilog(1/2*sqrt
((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh
(b*x + a))) + I*dilog(-1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d +
d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(1/2*sqrt((4*c^2 - 4*d^
2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) -
I*dilog(-1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cos
h(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arctan\left(d\coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(d*coth(b*x + a) + c), x)
```

3.101 $\int \frac{\tan^{-1}(c+d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\tan^{-1}(d\coth(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTan[c + d*Coth[a + b*x]]/x, x]
```

Rubi [A] time = 0.121101, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + d*Coth[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\tan^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Mathematica [A] time = 8.3992, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + d*Coth[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTan[c + d*Coth[a + b*x]]/x, x]
```

Maple [A] time = 0.387, size = 0, normalized size = 0.

$$\int \frac{\arctan(c+d\coth(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*coth(b*x+a))/x,x)
```

```
[Out] int(arctan(c+d*coth(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctan(d*coth(b*x + a) + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(d\coth\left(bx+a\right)+c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(d*coth(b*x + a) + c)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*coth(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(d*coth(b*x + a) + c)/x, x)
```

3.102 $\int x^2 \tan^{-1}(c + (i + c)\coth(a + bx))\,dx$

Optimal. Leaf size=142

$$-\frac{ix\text{PolyLog}\left(3, ice^{2a+2bx}\right)}{4b^2} + \frac{i\text{PolyLog}\left(4, ice^{2a+2bx}\right)}{8b^3} + \frac{ix^2\text{PolyLog}\left(2, ice^{2a+2bx}\right)}{4b} + \frac{1}{6}ix^3\log\left(1 - ice^{2a+2bx}\right) + \frac{1}{3}x^3\tan$$

```
[Out] (-I/12)*b*x^4 + (x^3*ArcTan[c + (I + c)*Coth[a + b*x]])/3 + (I/6)*x^3*Log[1
 - I*c*E^(2*a + 2*b*x)] + ((I/4)*x^2*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b - (
(I/4)*x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2 + ((I/8)*PolyLog[4, I*c*E^(2*a
 + 2*b*x)])/b^3
```

Rubi [A] time = 0.235911, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5197, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(3, ice^{2a+2bx}\right)}{4b^2} + \frac{i\text{PolyLog}\left(4, ice^{2a+2bx}\right)}{8b^3} + \frac{ix^2\text{PolyLog}\left(2, ice^{2a+2bx}\right)}{4b} + \frac{1}{6}ix^3\log\left(1 - ice^{2a+2bx}\right) + \frac{1}{3}x^3\tan$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (-I/12)*b*x^4 + (x^3*ArcTan[c + (I + c)*Coth[a + b*x]])/3 + (I/6)*x^3*Log[1
 - I*c*E^(2*a + 2*b*x)] + ((I/4)*x^2*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b - (
(I/4)*x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2 + ((I/8)*PolyLog[4, I*c*E^(2*a
 + 2*b*x)])/b^3
```

Rule 5197

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Coth[a + b*x]])/(f*(m
 + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+(i+c)\coth(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx)) - \frac{1}{3}b\int \frac{x^3}{-i-ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx)) - \frac{1}{3}(ibc)\int \frac{e^{2a+2bx}x^3}{-i-ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) - \frac{1}{2}i$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) + \frac{ix^2}{}$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) + \frac{ix^2}{}$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) + \frac{ix^2}{}$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) + \frac{ix^2}{}$$

Mathematica [A] time = 1.57162, size = 128, normalized size = 0.9

$$\frac{i\left(-6b^2x^2\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)-6bx\text{PolyLog}\left(3,-\frac{ie^{-2(a+bx)}}{c}\right)-3\text{PolyLog}\left(4,-\frac{ie^{-2(a+bx)}}{c}\right)+4b^3x^3\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)\right)}{24b^3}+\frac{1}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (I + c)*Coth[a + b*x]])/3 + ((I/24)*(4*b^3*x^3*Log[1 + I/(c
*E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - 6*b*x
*PolyLog[3, (-I)/(c*E^(2*(a + b*x)))] - 3*PolyLog[4, (-I)/(c*E^(2*(a + b*x)
))]))/b^3
```

Maple [C] time = 12.993, size = 1554, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] 1/6*Pi*x^3-1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn
(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-1/12*Pi*x^
3*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((
2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/12*Pi*x^3*cs
gn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1)
)^2-1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3+1/8*I
*polylog(4,I*c*exp(2*b*x+2*a))/b^3+1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/
b-1/12*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2-1/12*Pi*x
^3*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/12*
Pi*x^3*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3-1/3/b^3*a^3/(I+c
)*ln(exp(b*x+a))+1/3/b^2/(I+c)*x*a^3-1/6*I/b^3*a^3*ln(exp(2*b*x+2*a)*c+I)+1
/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1
))^3+1/6*I*x^3*ln(1-I*c*exp(2*b*x+2*a))-1/2*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*
x*a^2+1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x+1/2*I/b^2*a^2*ln(1+I*
exp(b*x+a)*(-I*c)^(1/2))*x+1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b
*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)
/(exp(2*b*x+2*a)-1))+1/12*Pi*x^3*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*
c)/(exp(2*b*x+2*a)-1))^3-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I
*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-1/12*Pi*x^3*csgn(
I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c+2*I
)/(exp(2*b*x+2*a)-1))+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*
x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+1/12*Pi*x^3*
csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x
+2*a)-1))^2-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)
*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))+1/12*Pi*x^3*cs
gn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*cs
gn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))-1/4*I*x*po
lylog(3,I*c*exp(2*b*x+2*a))/b^2-1/3*I/b^3*ln(1-I*c*exp(2*b*x+2*a))*a^3-1/4*
I/b^3*polylog(2,I*c*exp(2*b*x+2*a))*a^2+1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(-I
*c)^(1/2))+1/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b^3*a^2*dilo
g(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(-I*c)^(1
/2))-1/4*I/b^3*c/(I+c)*a^4-1/12*I*c*b/(I+c)*x^4+1/4/b^3/(I+c)*a^4+1/12*b/(I
+c)*x^4-1/6*I*x^3*ln(2*exp(2*b*x+2*a)*c+2*I)+1/6*I*x^3*ln(2*I*exp(2*b*x+2*a
)+2*exp(2*b*x+2*a)*c)+1/3*I/b^3*c*a^3/(I+c)*ln(exp(b*x+a))-1/3*I/b^2*c/(I+c
)*x*a^3
```

Maxima [A] time = 5.84651, size = 174, normalized size = 1.23

$$\frac{1}{3}x^3\arctan\left((c+i)\coth\left(bx+a\right)+c\right)+\frac{4}{9}\left(\frac{3\,x^4}{4i\,c-4}-\frac{4\,b^3x^3\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}\cdots}{-2\,b^4(-i\,c+1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan((c + I)*coth(b*x + a) + c) + 4/9*(3*x^4/(4*I*c - 4) - (4*b^3
*x^3*log(-I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(I*c*e^(2*b*x + 2*a)) -
```

```
6*b*x*polylog(3, I*c*e^(2*b*x + 2*a)) + 3*polylog(4, I*c*e^(2*b*x + 2*a)))
/(b^4*(2*I*c - 2)))*b*(c + I)
```

Fricas [C] time = 2.29184, size = 855, normalized size = 6.02

$$\frac{-i\,b^4x^4 + 2i\,b^3x^3\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}e^{(bx+a)}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}e^{(bx+a)}\right) + i\,a^4 - 2i\,a^3\log\left(\frac{2\,ce^{(bx+a)}\ldots}{2\,c}\right.\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-I*b^4*x^4 + 2*I*b^3*x^3*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2
*a) + I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + 6*I*b^2*x^2*di
log(-1/2*sqrt(4*I*c)*e^(b*x + a)) + I*a^4 - 2*I*a^3*log(1/2*(2*c*e^(b*x + a
) + I*sqrt(4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c
) - 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(b*x + a)) - 12*I*b*x*polylog(3,
-1/2*sqrt(4*I*c)*e^(b*x + a)) + (2*I*b^3*x^3 + 2*I*a^3)*log(1/2*sqrt(4*I*c)
*e^(b*x + a) + 1) + (2*I*b^3*x^3 + 2*I*a^3)*log(-1/2*sqrt(4*I*c)*e^(b*x + a
) + 1) + 12*I*polylog(4, 1/2*sqrt(4*I*c)*e^(b*x + a)) + 12*I*polylog(4, -1/
2*sqrt(4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int\frac{x^3}{ce^{2a}e^{2bx}+i}\,dx}{3} + \frac{ix^3\log\left(-ic - \frac{ic}{e^{2a}e^{2bx}-1} - \frac{ice^ae^{bx}}{e^ae^{bx}-e^{-a}e^{-bx}} + 1 + \frac{1}{e^{2a}e^{2bx}-1} + \frac{e^ae^{bx}}{e^ae^{bx}-e^{-a}e^{-bx}}\right)}{6} - \frac{ix^3\log\left(ic + \frac{ic}{e^{2a}e^{2bx}-1} + \frac{ice^ae^{bx}}{e^ae^{bx}-e^{-a}e^{-bx}}\ldots\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] b*Integral(x**3/(c*exp(2*a)*exp(2*b*x) + I), x)/3 + I*x**3*log(-I*c - I*c/(
exp(2*a)*exp(2*b*x) - 1) - I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*e
xp(-b*x)) + 1 + 1/(exp(2*a)*exp(2*b*x) - 1) + exp(a)*exp(b*x)/(exp(a)*exp(b
*x) - exp(-a)*exp(-b*x)))/6 - I*x**3*log(I*c + I*c/(exp(2*a)*exp(2*b*x) - 1
) + I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*exp(-b*x)) + 1 - 1/(exp(
2*a)*exp(2*b*x) - 1) - exp(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*exp(-b*x)
))/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\arctan\left((c+i)\coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((c + I)*coth(b*x + a) + c), x)
```

3.103 $\int x \tan^{-1}(c + (i + c) \coth(a + bx))\, dx$

Optimal. Leaf size=113

$$-\frac{i\text{PolyLog}\left(3, ice^{2a+2bx}\right)}{8b^2} + \frac{ix\text{PolyLog}\left(2, ice^{2a+2bx}\right)}{4b} + \frac{1}{4}ix^2 \log\left(1 - ice^{2a+2bx}\right) + \frac{1}{2}x^2 \tan^{-1}(c + (c + i)\coth(a + bx))$$

```
[Out] (-I/6)*b*x^3 + (x^2*ArcTan[c + (I + c)*Coth[a + b*x]])/2 + (I/4)*x^2*Log[1
- I*c*E^(2*a + 2*b*x)] + ((I/4)*x*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b - ((I/
8)*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2
```

Rubi [A] time = 0.205965, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5197, 2184, 2190, 2531, 2282, 6589}

$$-\frac{i\text{PolyLog}\left(3, ice^{2a+2bx}\right)}{8b^2} + \frac{ix\text{PolyLog}\left(2, ice^{2a+2bx}\right)}{4b} + \frac{1}{4}ix^2 \log\left(1 - ice^{2a+2bx}\right) + \frac{1}{2}x^2 \tan^{-1}(c + (c + i)\coth(a + bx))$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (-I/6)*b*x^3 + (x^2*ArcTan[c + (I + c)*Coth[a + b*x]])/2 + (I/4)*x^2*Log[1
- I*c*E^(2*a + 2*b*x)] + ((I/4)*x*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b - ((I/
8)*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2
```

Rule 5197

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Coth[a + b*x]])/(f*(m
 + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+(i+c)\coth(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx)) - \frac{1}{2}b\int \frac{x^2}{-i-ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{6}ibx^3 + \frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx)) - \frac{1}{2}(ibc)\int \frac{e^{2a+2bx}x^2}{-i-ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{6}ibx^3 + \frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right) - \frac{1}{2}i\int$$
$$= -\frac{1}{6}ibx^3 + \frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right) + \frac{ix\mathrm{Li}_2}{}$$
$$= -\frac{1}{6}ibx^3 + \frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right) + \frac{ix\mathrm{Li}_2}{}$$
$$= -\frac{1}{6}ibx^3 + \frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right) + \frac{ix\mathrm{Li}_2}{}$$

Mathematica [A] time = 1.47469, size = 102, normalized size = 0.9

$$\frac{i\left(-2bx\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(3,-\frac{ie^{-2(a+bx)}}{c}\right)+2b^2x^2\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)\right)}{8b^2}+\frac{1}{2}x^2\tan^{-1}(c+(c+i)\coth(a+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (I + c)*Coth[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + I/(c*
E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - PolyLog[3,
 (-I)/(c*E^(2*(a + b*x)))]))/b^2
```

Maple [C] time = 4.582, size = 1518, normalized size = 13.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] 1/4*Pi*x^2+1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3+1
/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b+1/3*I/b^2*c/(I+c)*a^3-1/6*I*c*b/(I+c
```

```
)*x^3-1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+1/2/b^
2*a^2/(I+c)*ln(exp(b*x+a))-1/2/b/(I+c)*x*a^2-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2
*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(e
xp(2*b*x+2*a)-1))+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x
+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*csgn((2*I*exp(2*b*x+2*a)+2*ex
p(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/2*I/b*c/(I+c)*x*a^2-1/2*I/b^2*c*a^2
/(I+c)*ln(exp(b*x+a))-1/8*I*polylog(3,I*c*exp(2*b*x+2*a))/b^2-1/4*I*x^2*ln(
2*exp(2*b*x+2*a)*c+2*I)+1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)
+1/4*I/b^2*a^2*ln(exp(2*b*x+2*a)*c+I)-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c
+2*I)/(exp(2*b*x+2*a)-1))^3+1/8*Pi*x^2*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)-1))^3-1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(
2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)
*c)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)-1))^3-1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(-I*c)^(1/2
))+1/4*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*a^2+1/4*I/b^2*polylog(2,I*c*exp(2*b*x
+2*a))*a-1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b^2*a^2*ln(1+I
*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b^2*a*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))+1/
8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*
b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+1/4*I*x^2*ln(1-I*c*exp(2*b*x+2*a))-1/
8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*
b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))+1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)-1))+1/2*I/b*ln(1-I*c*exp(2*b*x+2*a))*x*a-1/2*I/b*a
*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x-1/2*I/b*a*ln(1+I*exp(b*x+a)*(-I*c)^(1/2)
)*x-1/3/b^2/(I+c)*a^3+1/6*b/(I+c)*x^3-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))
*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/8*P
i*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp
(2*b*x+2*a)-1))^2-1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)
)*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/8*
Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*
a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))
```

Maxima [A] time = 5.89234, size = 143, normalized size = 1.27

$$\left(\frac{2\,x^{3}}{3i\,c-3}-\frac{2\,b^{2}x^{2}\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_{2}\left(i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_{3}(i\,ce^{(2\,bx+2\,a)})}{-2\,b^{3}(-i\,c+1)}\right)b(c+i)+\frac{1}{2}\,x^{2}\arctan\left((c+i)\coth\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] (2*x^3/(3*I*c - 3) - (2*b^2*x^2*log(-I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(I*c*e^(2*b*x + 2*a)) - polylog(3, I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c - 2)))
*b*(c + I) + 1/2*x^2*arctan((c + I)*coth(b*x + a) + c)
```

Fricas [C] time = 2.26382, size = 707, normalized size = 6.26

$$-2i\,b^{3}x^{3}+3i\,b^{2}x^{2}\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right)-2i\,a^{3}+6i\,bx\mathrm{Li}_{2}\left(\frac{1}{2}\,\sqrt{4i\,c}e^{(bx+a)}\right)+6i\,bx\mathrm{Li}_{2}\left(-\frac{1}{2}\,\sqrt{4i\,c}e^{(bx+a)}\right)+3i\,a^{2}\log\left(\frac{2\,ce^{(bx+}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x +
 2*a) + I)) - 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + 6*I*b*
x*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) +
I*sqrt(4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) +
(3*I*b^2*x^2 - 3*I*a^2)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (3*I*b^2*x^2
 - 3*I*a^2)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 6*I*polylog(3, 1/2*sqrt
(4*I*c)*e^(b*x + a)) - 6*I*polylog(3, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int \frac{x^2}{ce^{2a}e^{2bx}+i}\,dx}{2}+\frac{ix^2\log\left(-ic-\frac{ic}{e^{2a}e^{2bx}-1}-\frac{ice^ae^{bx}}{e^ae^{bx}-e^{-a}e^{-bx}}+1+\frac{1}{e^{2a}e^{2bx}-1}+\frac{e^ae^{bx}}{e^ae^{bx}-e^{-a}e^{-bx}}\right)}{4}-\frac{ix^2\log\left(ic+\frac{ic}{e^{2a}e^{2bx}-1}+\frac{ice^ae^{bx}}{e^ae^{bx}-e^{-a}e^{-bx}}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] b*Integral(x**2/(c*exp(2*a)*exp(2*b*x) + I), x)/2 + I*x**2*log(-I*c - I*c/(
exp(2*a)*exp(2*b*x) - 1) - I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*e
xp(-b*x)) + 1 + 1/(exp(2*a)*exp(2*b*x) - 1) + exp(a)*exp(b*x)/(exp(a)*exp(b
*x) - exp(-a)*exp(-b*x)))/4 - I*x**2*log(I*c + I*c/(exp(2*a)*exp(2*b*x) - 1
) + I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*exp(-b*x)) + 1 - 1/(exp(
2*a)*exp(2*b*x) - 1) - exp(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*exp(-b*x)
))/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\arctan\left((c+i)\coth(bx+a)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((c + I)*coth(b*x + a) + c), x)
```

3.104 $\int \tan^{-1}(c+(i+c)\coth(a+bx))\,dx$

Optimal. Leaf size=79

$$\frac{i\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+x\tan^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{2}ibx^2$$

```
[Out] (-I/2)*b*x^2 + x*ArcTan[c + (I + c)*Coth[a + b*x]] + (I/2)*x*Log[1 - I*c*E^
(2*a + 2*b*x)] + ((I/4)*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b
```

Rubi [A] time = 0.122423, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5189, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+x\tan^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (-I/2)*b*x^2 + x*ArcTan[c + (I + c)*Coth[a + b*x]] + (I/2)*x*Log[1 - I*c*E^
(2*a + 2*b*x)] + ((I/4)*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b
```

Rule 5189

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tan[c + d*Coth[a + b*x]], x] - Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tan^{-1}(c+(i+c)\coth(a+bx))\,dx = x\tan^{-1}(c+(i+c)\coth(a+bx)) - b\int \frac{x}{-i-ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\coth(a+bx)) - (ibc)\int \frac{e^{2a+2bx}x}{-i-ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) - \frac{1}{2}i\int \log\left(\right.$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) - \frac{i\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) + \frac{i\text{Li}_2\left(ice^{2a}\right.}{4b}$$

Mathematica [A] time = 0.660659, size = 71, normalized size = 0.9

$$\frac{i\left(2bx\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}+x\tan^{-1}(c+(c+i)\coth(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (I + c)*Coth[a + b*x]] + ((I/4)*(2*b*x*Log[1 + I/(c*E^(2*(a +
b*x)))] - PolyLog[2, (-I)/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] time = 0.152, size = 1381, normalized size = 17.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] 1/(I+c)/b*arctan(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c+(I+c)*coth(b*x+a)+I)*c
^2-1/(I+c)/b*arctan(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I
)*c^2+1/4*I/(I+c)^2/b*ln(-1/2*I*(-c-(I+c)*coth(b*x+a)+I))*ln(c+(I+c)*coth(b
*x+a)+I)*c^2-1/4*I/(I+c)^2/b*ln(c-(I+c)*coth(b*x+a)+I)*ln(-1/2*(-c-(I+c)*co
th(b*x+a)+I)/c)*c^2+1/4*I/(I+c)^2/b*ln(c-(I+c)*coth(b*x+a)+I)*ln((-c-(I+c)*
coth(b*x+a)-I)/(-2*I-2*c))*c^2-1/4*I/(I+c)^2/b*ln(-1/2*I*(c+(I+c)*coth(b*x+
a)+I))*ln(-1/2*I*(-c-(I+c)*coth(b*x+a)+I))*c^2+1/2/(I+c)^2/b*ln(c-(I+c)*cot
h(b*x+a)+I)*ln(-1/2*(-c-(I+c)*coth(b*x+a)+I)/c)*c-1/4*I/(I+c)^2/b*dilog((-c
-(I+c)*coth(b*x+a)-I)/(-2*I-2*c))+1/8*I/(I+c)^2/b*ln(c+(I+c)*coth(b*x+a)+I)
^2+1/4*I/(I+c)^2/b*dilog(-1/2*I*(c+(I+c)*coth(b*x+a)+I))+1/4/(I+c)^2/b*ln(c
+(I+c)*coth(b*x+a)+I)^2*c+1/2/(I+c)^2/b*dilog(-1/2*I*(c+(I+c)*coth(b*x+a)+I
))*c+2*I/(I+c)/b*arctan(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c+(I+c)*coth(b*x+
a)+I)*c-2*I/(I+c)/b*arctan(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b
*x+a)+I)*c-1/2/(I+c)^2/b*ln(c-(I+c)*coth(b*x+a)+I)*ln((-c-(I+c)*coth(b*x+a)
-I)/(-2*I-2*c))*c+1/(I+c)/b*arctan(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c
)*coth(b*x+a)+I)-1/4*I/(I+c)^2/b*ln(-1/2*I*(-c-(I+c)*coth(b*x+a)+I))*ln(c+(
I+c)*coth(b*x+a)+I)+1/4*I/(I+c)^2/b*ln(c-(I+c)*coth(b*x+a)+I)*ln(-1/2*(-c-(
I+c)*coth(b*x+a)+I)/c)-1/4*I/(I+c)^2/b*ln(c-(I+c)*coth(b*x+a)+I)*ln((-c-(I+
c)*coth(b*x+a)-I)/(-2*I-2*c))-1/8*I/(I+c)^2/b*ln(c+(I+c)*coth(b*x+a)+I)^2*c
^2-1/4*I/(I+c)^2/b*dilog(-1/2*I*(c+(I+c)*coth(b*x+a)+I))*c^2-1/4*I/(I+c)^2/
```

```
b*dilog(-1/2*(-c-(I+c)*coth(b*x+a)+I)/c)*c^2+1/4*I/(I+c)^2/b*dilog((-c-(I+c
)*coth(b*x+a)-I)/(-2*I-2*c))*c^2+1/4*I/(I+c)^2/b*ln(-1/2*I*(c+(I+c)*coth(b*
x+a)+I))*ln(-1/2*I*(-c-(I+c)*coth(b*x+a)+I))-1/(I+c)/b*arctan(c+(I+c)*coth(
b*x+a))/(2*I+2*c)*ln(c+(I+c)*coth(b*x+a)+I)+1/2/(I+c)^2/b*ln(-1/2*I*(c+(I+c
)*coth(b*x+a)+I))*ln(-1/2*I*(-c-(I+c)*coth(b*x+a)+I))*c-1/2/(I+c)^2/b*ln(-1
/2*I*(-c-(I+c)*coth(b*x+a)+I))*ln(c+(I+c)*coth(b*x+a)+I)*c+1/2/(I+c)^2/b*di
log(-1/2*(-c-(I+c)*coth(b*x+a)+I)/c)*c-1/2/(I+c)^2/b*dilog((-c-(I+c)*coth(b
*x+a)-I)/(-2*I-2*c))*c+1/4*I/(I+c)^2/b*dilog(-1/2*(-c-(I+c)*coth(b*x+a)+I)/
c)
```

Maxima [A] time = 5.92393, size = 108, normalized size = 1.37

$$2\,b(c+i)\left(\frac{2\,x^2}{2i\,c-2}-\frac{2\,bx\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c+1)}\right)+x\arctan\left((c+i)\coth\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*b*(c + I)*(2*x^2/(2*I*c - 2) - (2*b*x*log(-I*c*e^(2*b*x + 2*a) + 1) + dil
og(I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c - 2))) + x*arctan((c + I)*coth(b*x + a
) + c)
```

Fricas [B] time = 2.24685, size = 510, normalized size = 6.46

$$\frac{-i\,b^2x^2+i\,bx\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right)+i\,a^2+(i\,bx+i\,a)\log\left(\frac{1}{2}\,\sqrt{4i\,c}e^{(bx+a)}+1\right)+(i\,bx+i\,a)\log\left(-\frac{1}{2}\,\sqrt{4i\,c}e^{(bx+a)}+1\right)-i\,a}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b^2*x^2 + I*b*x*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) + I
)) + I*a^2 + (I*b*x + I*a)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (I*b*x +
I*a)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - I*a*log(1/2*(2*c*e^(b*x + a) +
 I*sqrt(4*I*c))/c) - I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) + I*d
ilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + I*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)))/
b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{x}{ce^{2a}e^{2bx}+i}\,dx+\frac{ix\log\left(-ic-\frac{ic}{e^{2a}e^{2bx}-1}-\frac{ice^{a}e^{bx}}{e^{a}e^{bx}-e^{-a}e^{-bx}}+1+\frac{1}{e^{2a}e^{2bx}-1}+\frac{e^{a}e^{bx}}{e^{a}e^{bx}-e^{-a}e^{-bx}}\right)}{2}-\frac{ix\log\left(ic+\frac{ic}{e^{2a}e^{2bx}-1}+\frac{ice^{a}e^{\cdots}}{e^{a}e^{bx}-e^{-\cdots}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] b*Integral(x/(c*exp(2*a)*exp(2*b*x) + I), x) + I*x*log(-I*c - I*c/(exp(2*a)
*exp(2*b*x) - 1) - I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*exp(-b*x)
```

```
) + 1 + 1/(exp(2*a)*exp(2*b*x) - 1) + exp(a)*exp(b*x)/(exp(a)*exp(b*x) - ex
p(-a)*exp(-b*x)))/2 - I*x*log(I*c + I*c/(exp(2*a)*exp(2*b*x) - 1) + I*c*exp
(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*exp(-b*x)) + 1 - 1/(exp(2*a)*exp(2*
b*x) - 1) - exp(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*exp(-b*x)))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arctan\left((c+i)\coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((c + I)*coth(b*x + a) + c), x)
```

3.105 $\int \frac{\tan^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=21

$$\text{CannotIntegrate}\left(\frac{\tan^{-1}(c+(c+i)\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTan[c + (I + c)*Coth[a + b*x]]/x, x]
```

Rubi [A] time = 0.109887, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + (I + c)*Coth[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + (I + c)*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx=\int \frac{\tan^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$$

Mathematica [A] time = 3.59387, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + (I + c)*Coth[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTan[c + (I + c)*Coth[a + b*x]]/x, x]
```

Maple [A] time = 0.438, size = 0, normalized size = 0.

$$\int \frac{\arctan\left(c+(i+c)\coth\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(I+c)*coth(b*x+a))/x,x)
```

```
[Out] int(arctan(c+(I+c)*coth(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$ibx - \frac{1}{4}\left(2\pi - 4ia + 2\arctan(c, -1) - i\log\left(c^2 + 1\right)\right)\log(x) + \frac{1}{2}\int \frac{\arctan\left(ce^{(2bx+2a)}, -1\right)}{x}\,dx - \frac{1}{4}i\int \frac{\log\left(c^2e^{(4bx+4a)}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] I*b*x - 1/4*(2*pi - 4*I*a + 2*arctan2(c, -1) - I*log(c^2 + 1))*log(x) + 1/2
*integrate(arctan2(c*e^(2*b*x + 2*a), -1)/x, x) - 1/4*I*integrate(log(c^2*e
^(4*b*x + 4*a) + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i\log\left(-\frac{(c+i)e^{(2bx+2a)}}{ce^{(2bx+2a)}+i}\right)}{2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) + I))/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(I+c)*coth(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left((c+i)\coth(bx+a)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((c + I)*coth(b*x + a) + c)/x, x)
```

3.106 $\int x^2 \tan^{-1}(c-(i-c)\coth(a+bx))\,dx$

Optimal. Leaf size=145

$$\frac{ix\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}-\frac{ix^2\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{1}{3}x^3$$

```
[Out] (I/12)*b*x^4 + (x^3*ArcTan[c - (I - c)*Coth[a + b*x]])/3 - (I/6)*x^3*Log[1
+ I*c*E^(2*a + 2*b*x)] - ((I/4)*x^2*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b +
 ((I/4)*x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2 - ((I/8)*PolyLog[4, (-I)*
c*E^(2*a + 2*b*x)])/b^3
```

Rubi [A] time = 0.231205, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5197, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{ix\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}-\frac{ix^2\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{1}{3}x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTan[c - (I - c)*Coth[a + b*x]])/3 - (I/6)*x^3*Log[1
+ I*c*E^(2*a + 2*b*x)] - ((I/4)*x^2*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b +
 ((I/4)*x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2 - ((I/8)*PolyLog[4, (-I)*
c*E^(2*a + 2*b*x)])/b^3
```

Rule 5197

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Coth[a + b*x]])/(f*(m
 + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\tan^{-1}(c-(i-c)\coth(a+bx))\,dx=\frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{3}b\int\frac{x^3}{i-ce^{2a+2bx}}\,dx$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{3}(ibc)\int\frac{e^{2a+2bx}x^3}{i-ce^{2a+2bx}}\,dx$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{1}{2}i\int$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)-\frac{ix^2\mathrm{Li}}{}$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)-\frac{ix^2\mathrm{Li}}{}$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)-\frac{ix^2\mathrm{Li}}{}$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)-\frac{ix^2\mathrm{Li}}{}$$

Mathematica [A] time = 1.57459, size = 128, normalized size = 0.88

$$\frac{1}{3}x^3\tan^{-1}(c+(c-i)\coth(a+bx))-\frac{i\left(-6b^2x^2\mathrm{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)-6bx\mathrm{PolyLog}\left(3,\frac{ie^{-2(a+bx)}}{c}\right)-3\mathrm{PolyLog}\left(4,\frac{ie^{-2(a+bx}}{c}\right.\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (-I + c)*Coth[a + b*x]])/3 - ((I/24)*(4*b^3*x^3*Log[1 - I/(
c*E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, I/(c*E^(2*(a + b*x)))] - 6*b*x*P
olyLog[3, I/(c*E^(2*(a + b*x)))] - 3*PolyLog[4, I/(c*E^(2*(a + b*x)))]))/b^
3
```

Maple [C] time = 12.849, size = 1571, normalized size = 10.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] 1/12*Pi*x^3*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1
))^2-1/6*Pi*x^3+1/12*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1
))^2+1/2*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*x*a^2-1/2*I/b^2*a^2*ln(1+I*exp(b*x+
a)*(I*c)^(1/2))*x-1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*x+1/12*Pi*x^
3*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a
)*c-2*I)/(exp(2*b*x+2*a)-1))-1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3-1/6*I
*x^3*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)+1/4*I*x*polylog(3,-I*c*exp(2
*b*x+2*a))/b^2-1/6*I*x^3*ln(1+I*c*exp(2*b*x+2*a))+1/6*I*x^3*ln(-2*exp(2*b*x
+2*a)*c+2*I)-1/4/b^3/(I-c)*a^4-1/12*b/(I-c)*x^4+1/12*Pi*x^3*csgn(I/(exp(2*b
*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I
)/(exp(2*b*x+2*a)-1))-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(
2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2+1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-
1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1
/12*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3+1/12*Pi*x^3*
csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3-1/12*
Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3-1/4*I/b^3*c/(I
-c)*a^4-1/12*I*c*b/(I-c)*x^4-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*
(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))+1/12*Pi*x^3*csgn((-2*I*exp(2*b*x+2*a)+2*
exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3-1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+
2*a))/b+1/3/b^3*a^3/(I-c)*ln(exp(b*x+a))-1/3/b^2/(I-c)*x*a^3+1/3*I/b^3*ln(1
+I*c*exp(2*b*x+2*a))*a^3+1/4*I/b^3*polylog(2,-I*c*exp(2*b*x+2*a))*a^2-1/2*I
/b^3*a^3*ln(1+I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(I*
c)^(1/2))-1/2*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^3*a^2*dil
og(1-I*exp(b*x+a)*(I*c)^(1/2))+1/6*I/b^3*a^3*ln(-exp(2*b*x+2*a)*c+I)-1/12*P
i*x^3*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b
*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/12*Pi*x^3*csgn(I*(2*exp
(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-1
/12*Pi*x^3*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-
1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))+1/3*I
/b^3*c*a^3/(I-c)*ln(exp(b*x+a))-1/12*Pi*x^3*csgn(I*(-2*I*exp(2*b*x+2*a)+2*e
xp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+
2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(
exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-1/3*
I/b^2*c/(I-c)*x*a^3
```

Maxima [A] time = 5.85591, size = 174, normalized size = 1.2

$$\frac{1}{3}\,x^{3}\arctan\left((c-i)\coth\left(bx+a\right)+c\right)-\frac{4}{9}\left(\frac{3\,x^{4}}{4i\,c+4}-\frac{4\,b^{3}x^{3}\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^{2}x^{2}\mathrm{Li}_{2}\left(-i\,ce^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_{3}}{-2\,b^{4}(-i\,c-1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan((c - I)*coth(b*x + a) + c) - 4/9*(3*x^4/(4*I*c + 4) - (4*b^3
*x^3*log(I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-I*c*e^(2*b*x + 2*a)) -
```

```
 6*b*x*polylog(3, -I*c*e^(2*b*x + 2*a)) + 3*polylog(4, -I*c*e^(2*b*x + 2*a)
))/(b^4*(2*I*c + 2)))*b*(c - I)
```

Fricas [C] time = 2.26299, size = 871, normalized size = 6.01

$$\frac{i b^4 x^4 + 2i b^3 x^3 \log\left(-\frac{\left(ce^{(2bx+2a)}-i\right)e^{(-2bx-2a)}}{c-i}\right) - 6i b^2 x^2 \text{Li}_2\left(\frac{1}{2}\sqrt{-4ic}e^{(bx+a)}\right) - 6i b^2 x^2 \text{Li}_2\left(-\frac{1}{2}\sqrt{-4ic}e^{(bx+a)}\right) - i a^4 + 2i a^3 \log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*I*b^3*x^3*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)
/(c - I)) - 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*b^2*x^2*d
ilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) - I*a^4 + 2*I*a^3*log(1/2*(2*c*e^(b*x +
 a) + I*sqrt(-4*I*c))/c) + 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c
))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(b*x + a)) + 12*I*b*x*polylo
g(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)) + (-2*I*b^3*x^3 - 2*I*a^3)*log(1/2*sqrt
(-4*I*c)*e^(b*x + a) + 1) + (-2*I*b^3*x^3 - 2*I*a^3)*log(-1/2*sqrt(-4*I*c)*
e^(b*x + a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 12*I*pol
ylog(4, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int \frac{x^3}{ce^{2a}e^{2bx}-i}\,dx}{3} + \frac{ix^3\log\left(-ic - \frac{ic}{e^{2a}e^{2bx}-1} - \frac{ice^a e^{bx}}{e^a e^{bx}-e^{-a}e^{-bx}} + 1 - \frac{1}{e^{2a}e^{2bx}-1} - \frac{e^a e^{bx}}{e^a e^{bx}-e^{-a}e^{-bx}}\right)}{6} - \frac{ix^3\log\left(ic + \frac{ic}{e^{2a}e^{2bx}-1} + \frac{ice^a e^{bx}}{e^a e^{bx}-e^{-a}e^{-bx}}\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] b*Integral(x**3/(c*exp(2*a)*exp(2*b*x) - I), x)/3 + I*x**3*log(-I*c - I*c/(
exp(2*a)*exp(2*b*x) - 1) - I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*e
xp(-b*x)) + 1 - 1/(exp(2*a)*exp(2*b*x) - 1) - exp(a)*exp(b*x)/(exp(a)*exp(b
*x) - exp(-a)*exp(-b*x)))/6 - I*x**3*log(I*c + I*c/(exp(2*a)*exp(2*b*x) - 1
) + I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*exp(-b*x)) + 1 + 1/(exp(
2*a)*exp(2*b*x) - 1) + exp(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*exp(-b*x)
))/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \arctan\left((c-i)\coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((c - I)*coth(b*x + a) + c), x)
```

3.107 $\int x \tan^{-1}(c-(i-c)\coth(a+bx))\,dx$

Optimal. Leaf size=116

$$\frac{i\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}-\frac{ix\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{1}{2}x^2\tan^{-1}(c-(-c+i)\coth(a+bx$$

```
[Out] (I/6)*b*x^3 + (x^2*ArcTan[c - (I - c)*Coth[a + b*x]])/2 - (I/4)*x^2*Log[1 +
 I*c*E^(2*a + 2*b*x)] - ((I/4)*x*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b + ((
I/8)*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2
```

Rubi [A] time = 0.201336, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5197, 2184, 2190, 2531, 2282, 6589}

$$\frac{i\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}-\frac{ix\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{1}{2}x^2\tan^{-1}(c-(-c+i)\coth(a+bx$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```



```
[Out] (I/6)*b*x^3 + (x^2*ArcTan[c - (I - c)*Coth[a + b*x]])/2 - (I/4)*x^2*Log[1 +
 I*c*E^(2*a + 2*b*x)] - ((I/4)*x*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b + ((
I/8)*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2
```


Rule 5197

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Coth[a + b*x]])/(f*(m
 + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c-(i-c)\coth(a+bx))\,dx=\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{2}b\int\frac{x^2}{i-ce^{2a+2bx}}\,dx$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}x^2}{i-ce^{2a+2bx}}\,dx$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{1}{2}i\int x$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)-\frac{ix\text{Li}_2\left(-\right.}{}$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)-\frac{ix\text{Li}_2\left(-\right.}{}$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)-\frac{ix\text{Li}_2\left(-\right.}{}$$

Mathematica [A] time = 1.49472, size = 102, normalized size = 0.88

$$\frac{1}{2}x^2\tan^{-1}(c+(c-i)\coth(a+bx))-\frac{i\left(-2bx\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(3,\frac{ie^{-2(a+bx)}}{c}\right)+2b^2x^2\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (-I + c)*Coth[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 - I/(c
*E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, I/(c*E^(2*(a + b*x)))] - PolyLog[3, I
/(c*E^(2*(a + b*x)))]))/b^2
```

Maple [C] time = 7.276, size = 1535, normalized size = 13.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] 1/8*Pi*x^2*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1)
)^2+1/4*I*x^2*ln(-2*exp(2*b*x+2*a)*c+2*I)-1/4*Pi*x^2+1/8*Pi*x^2*csgn((2*exp
```

```
(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a
)*c-2*I)/(exp(2*b*x+2*a)-1))^3+1/3/b^2/(I-c)*a^3-1/6*b*x^3/(I-c)+1/2/b/(I-c
)*x*a^2-1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*
(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*c
sgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c
-2*I)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*
b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3-1/4*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*a^2-1/
4*I/b^2*polylog(2,-I*c*exp(2*b*x+2*a))*a+1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(I
*c)^(1/2))+1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^2*a*dilog(1
+I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))+1/
8*I*polylog(3,-I*c*exp(2*b*x+2*a))/b^2+1/2*I/b*a*ln(1+I*exp(b*x+a)*(I*c)^(1
/2))*x-1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)-1/4*I*x^2*ln(1+I
*c*exp(2*b*x+2*a))+1/8*Pi*x^2*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)
/(exp(2*b*x+2*a)-1))^3+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(
2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))-1/8*
Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2
*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))
+1/2*I/b*a*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*x+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+
2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2+1/2*I/b*
c/(I-c)*x*a^2-1/2*I/b^2*c*a^2/(I-c)*ln(exp(b*x+a))-1/2/b^2*a^2/(I-c)*ln(exp
(b*x+a))+1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3-1/4
*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a
)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2
*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)
/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))-1/4*
I/b^2*a^2*ln(-exp(2*b*x+2*a)*c+I)-1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*
exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I
*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn(I/(exp(2*b*
x+2*a)-1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-
1))^2+1/3*I/b^2*c/(I-c)*a^3-1/6*I*c*b*x^3/(I-c)-1/2*I/b*ln(1+I*c*exp(2*b*x+
2*a))*x*a
```

Maxima [A] time = 5.75899, size = 144, normalized size = 1.24

$$-\left(\frac{2\,x^3}{3i\,c+3}-\frac{2\,b^2x^2\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-i\,ce^{(2\,bx+2\,a)})}{-2\,b^3(-i\,c-1)}\right)b(c-i)+\frac{1}{2}\,x^2\arctan\left((c-i)\,\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -(2*x^3/(3*I*c + 3) - (2*b^2*x^2*log(I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-I*c*e^(2*b*x + 2*a)) - polylog(3, -I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c + 2)
))*b*(c - I) + 1/2*x^2*arctan((c - I)*coth(b*x + a) + c)
```

Fricas [C] time = 2.23493, size = 721, normalized size = 6.22

$$2i\,b^3x^3+3i\,b^2x^2\log\left(-\frac{\left(ce^{(2\,bx+2\,a)}-i\right)e^{(-2\,bx-2\,a)}}{c-i}\right)+2i\,a^3-6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}e^{(bx+a)}\right)-6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}e^{(bx+a)}\right)-3i\,a^2\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*
a)/(c - I)) + 2*I*a^3 - 6*I*b*x*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*b
*x*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a)
+ I*sqrt(-4*I*c))/c) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c
) + (-3*I*b^2*x^2 + 3*I*a^2)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (-3*I*
b^2*x^2 + 3*I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + 6*I*polylog(3,
1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*polylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a
)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int \frac{x^2}{ce^{2a}e^{2bx}-i}\,dx}{2} + \frac{ix^2\log\left(-ic - \frac{ic}{e^{2a}e^{2bx}-1} - \frac{ice^{a}e^{bx}}{e^{a}e^{bx}-e^{-a}e^{-bx}} + 1 - \frac{1}{e^{2a}e^{2bx}-1} - \frac{e^{a}e^{bx}}{e^{a}e^{bx}-e^{-a}e^{-bx}}\right)}{4} - \frac{ix^2\log\left(ic + \frac{ic}{e^{2a}e^{2bx}-1} + \frac{ice^{a}e^{bx}}{e^{a}e^{bx}-e^{-a}e^{-bx}}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] b*Integral(x**2/(c*exp(2*a)*exp(2*b*x) - I), x)/2 + I*x**2*log(-I*c - I*c/(
exp(2*a)*exp(2*b*x) - 1) - I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*e
xp(-b*x)) + 1 - 1/(exp(2*a)*exp(2*b*x) - 1) - exp(a)*exp(b*x)/(exp(a)*exp(b
*x) - exp(-a)*exp(-b*x)))/4 - I*x**2*log(I*c + I*c/(exp(2*a)*exp(2*b*x) - 1
) + I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*exp(-b*x)) + 1 + 1/(exp(
2*a)*exp(2*b*x) - 1) + exp(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*exp(-b*x)
))/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\arctan\left((c-i)\coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((c - I)*coth(b*x + a) + c), x)
```

3.108 $\int \tan^{-1}(c-(i-c)\coth(a+bx))\,dx$

Optimal. Leaf size=82

$$-\frac{i\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+x\tan^{-1}(c-(-c+i)\coth(a+bx))+\frac{1}{2}ibx^2$$

```
[Out] (I/2)*b*x^2 + x*ArcTan[c - (I - c)*Coth[a + b*x]] - (I/2)*x*Log[1 + I*c*E^(
2*a + 2*b*x)] - ((I/4)*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
```

Rubi [A] time = 0.120685, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5189, 2184, 2190, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+x\tan^{-1}(c-(-c+i)\coth(a+bx))+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTan[c - (I - c)*Coth[a + b*x]] - (I/2)*x*Log[1 + I*c*E^(
2*a + 2*b*x)] - ((I/4)*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
```

Rule 5189

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tan[c + d*Coth[a + b*x]], x] - Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tan^{-1}(c-(i-c)\coth(a+bx))\,dx = x\tan^{-1}(c-(i-c)\coth(a+bx)) - b\int \frac{x}{i-ce^{2a+2bx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\coth(a+bx)) + (ibc)\int \frac{e^{2a+2bx}x}{i-ce^{2a+2bx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) + \frac{1}{2}i\int \log\left(1 -\right.$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) + \frac{i\,\mathrm{Subst}\left(\int \frac{\mathrm{lo}}{}\right.}{}$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) - \frac{i\mathrm{Li}_2\left(-ice^{2a+}\right.}{4b}$$

Mathematica [A] time = 0.662361, size = 71, normalized size = 0.87

$$x\tan^{-1}(c+(c-i)\coth(a+bx)) - \frac{i\left(2bx\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right) - \mathrm{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (-I + c)*Coth[a + b*x]] - ((I/4)*(2*b*x*Log[1 - I/(c*E^(2*(a +
 b*x)))] - PolyLog[2, I/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] time = 0.144, size = 1351, normalized size = 16.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] 1/4*I/b/(c-I)/(I-c)*dilog(1/2*((c-I)*coth(b*x+a)+c+I)/c)-1/4*I/b/(c-I)/(I-c
)*dilog(((c-I)*coth(b*x+a)+c-I)/(-2*I+2*c))-1/4*I/b/(c-I)/(I-c)*dilog(-1/2*
I*((c-I)*coth(b*x+a)+c+I))+1/8*I/b/(c-I)/(I-c)*ln((c-I)*coth(b*x+a)+c-I)^2+
1/b/(c-I)*arctan((c-I)*coth(b*x+a)+c)/(2*I-2*c)*ln((c-I)*coth(b*x+a)+c-I)+1
/2/b/(c-I)/(I-c)*dilog(-1/2*I*((c-I)*coth(b*x+a)+c+I))*c-1/4/b/(c-I)/(I-c)*
ln((c-I)*coth(b*x+a)+c-I)^2*c-1/2/b/(c-I)/(I-c)*dilog(1/2*((c-I)*coth(b*x+a
)+c+I)/c)*c+1/2/b/(c-I)/(I-c)*dilog(((c-I)*coth(b*x+a)+c-I)/(-2*I+2*c))*c-1
/b/(c-I)*arctan((c-I)*coth(b*x+a)+c)/(2*I-2*c)*ln((c-I)*coth(b*x+a)-c+I)+1/
4*I/b/(c-I)/(I-c)*ln((c-I)*coth(b*x+a)-c+I)*ln(1/2*((c-I)*coth(b*x+a)+c+I)/
c)+1/4*I/b/(c-I)/(I-c)*dilog(((c-I)*coth(b*x+a)+c-I)/(-2*I+2*c))*c^2+1/b/(c
-I)*arctan((c-I)*coth(b*x+a)+c)/(2*I-2*c)*ln((c-I)*coth(b*x+a)-c+I)*c^2-1/2
/b/(c-I)/(I-c)*ln((c-I)*coth(b*x+a)-c+I)*ln(1/2*((c-I)*coth(b*x+a)+c+I)/c)*
c+1/2/b/(c-I)/(I-c)*ln((c-I)*coth(b*x+a)-c+I)*ln(((c-I)*coth(b*x+a)+c-I)/(-
2*I+2*c))*c+1/2/b/(c-I)/(I-c)*ln(-1/2*I*((c-I)*coth(b*x+a)+c+I))*ln((c-I)*c
oth(b*x+a)+c-I)*c-1/4*I/b/(c-I)/(I-c)*ln((c-I)*coth(b*x+a)-c+I)*ln(((c-I)*c
oth(b*x+a)+c-I)/(-2*I+2*c))-1/b/(c-I)*arctan((c-I)*coth(b*x+a)+c)/(2*I-2*c)
*ln((c-I)*coth(b*x+a)+c-I)*c^2-1/4*I/b/(c-I)/(I-c)*ln(-1/2*I*((c-I)*coth(b*
x+a)+c+I))*ln((c-I)*coth(b*x+a)+c-I)+1/4*I/b/(c-I)/(I-c)*dilog(-1/2*I*((c-I
)*coth(b*x+a)+c+I))*c^2-1/8*I/b/(c-I)/(I-c)*ln((c-I)*coth(b*x+a)+c-I)^2*c^2
-1/4*I/b/(c-I)/(I-c)*dilog(1/2*((c-I)*coth(b*x+a)+c+I)/c)*c^2+1/4*I/b/(c-I)
```

```
/(I-c)*ln((c-I)*coth(b*x+a)-c+I)*ln(((c-I)*coth(b*x+a)+c-I)/(-2*I+2*c))*c^2
+2*I/b/(c-I)*arctan((c-I)*coth(b*x+a)+c)/(2*I-2*c)*ln((c-I)*coth(b*x+a)+c-I
)*c-2*I/b/(c-I)*arctan((c-I)*coth(b*x+a)+c)/(2*I-2*c)*ln((c-I)*coth(b*x+a)-
c+I)*c+1/4*I/b/(c-I)/(I-c)*ln(-1/2*I*((c-I)*coth(b*x+a)+c+I))*ln((c-I)*coth
(b*x+a)+c-I)*c^2-1/4*I/b/(c-I)/(I-c)*ln((c-I)*coth(b*x+a)-c+I)*ln(1/2*((c-I
)*coth(b*x+a)+c+I)/c)*c^2
```

Maxima [A] time = 5.82855, size = 108, normalized size = 1.32

$$-2\,b(c-i)\left(\frac{2\,x^2}{2i\,c+2}-\frac{2\,bx\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c-1)}\right)+x\arctan\left((c-i)\coth\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*(c - I)*(2*x^2/(2*I*c + 2) - (2*b*x*log(I*c*e^(2*b*x + 2*a) + 1) + dil
og(-I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c + 2))) + x*arctan((c - I)*coth(b*x +
a) + c)
```

Fricas [B] time = 2.2388, size = 521, normalized size = 6.35

$$\frac{i\,b^2x^2+i\,bx\log\left(-\frac{\left(ce^{(2\,bx+2\,a)}-i\right)e^{(-2\,bx-2\,a)}}{c-i}\right)-i\,a^2+(-i\,bx-i\,a)\log\left(\frac{1}{2}\,\sqrt{-4i\,c}e^{(bx+a)}+1\right)+(-i\,bx-i\,a)\log\left(-\frac{1}{2}\,\sqrt{-4i\,c}e^{(bx}\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 + I*b*x*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/(c - I
)) - I*a^2 + (-I*b*x - I*a)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (-I*b*x
 - I*a)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + I*a*log(1/2*(2*c*e^(b*x +
a) + I*sqrt(-4*I*c))/c) + I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c)
 - I*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - I*dilog(-1/2*sqrt(-4*I*c)*e^(b*x
 + a)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{x}{ce^{2a}e^{2bx}-i}\,dx+\frac{ix\log\left(-ic-\frac{ic}{e^{2a}e^{2bx}-1}-\frac{ice^ae^{bx}}{e^ae^{bx}-e^{-a}e^{-bx}}+1-\frac{1}{e^{2a}e^{2bx}-1}-\frac{e^ae^{bx}}{e^ae^{bx}-e^{-a}e^{-bx}}\right)}{2}-\frac{ix\log\left(ic+\frac{ic}{e^{2a}e^{2bx}-1}+\frac{ice^ae^{b}}{e^ae^{bx}-e^{-a}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] b*Integral(x/(c*exp(2*a)*exp(2*b*x) - I), x) + I*x*log(-I*c - I*c/(exp(2*a)
*exp(2*b*x) - 1) - I*c*exp(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*exp(-b*x)
) + 1 - 1/(exp(2*a)*exp(2*b*x) - 1) - exp(a)*exp(b*x)/(exp(a)*exp(b*x) - ex
p(-a)*exp(-b*x)))/2 - I*x*log(I*c + I*c/(exp(2*a)*exp(2*b*x) - 1) + I*c*exp
(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*exp(-b*x)) + 1 + 1/(exp(2*a)*exp(2*
```

```
b*x) - 1) + exp(a)*exp(b*x)/(exp(a)*exp(b*x) - exp(-a)*exp(-b*x)))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arctan\left((c-i)\coth(bx+a)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a)),x, algorithm="giac")

[Out] integrate(arctan((c - I)*coth(b*x + a) + c), x)
```

3.109 $\int \frac{\tan^{-1}(c-(i-c)\coth(a+bx))}{x} dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\tan^{-1}(c-(-c+i)\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTan[c - (I - c)*Coth[a + b*x]]/x, x]
```

Rubi [A] time = 0.11197, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(c-(i-c)\coth(a+bx))}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c - (I - c)*Coth[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c - (I - c)*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c-(i-c)\coth(a+bx))}{x} dx = \int \frac{\tan^{-1}(c-(i-c)\coth(a+bx))}{x} dx$$

Mathematica [A] time = 3.62889, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(c-(i-c)\coth(a+bx))}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c - (I - c)*Coth[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTan[c - (I - c)*Coth[a + b*x]]/x, x]
```

Maple [A] time = 0.447, size = 0, normalized size = 0.

$$\int \frac{\arctan\left(c-(i-c)\coth\left(bx+a\right)\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c-(I-c)*coth(b*x+a))/x,x)
```

```
[Out] int(arctan(c-(I-c)*coth(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-ibx - \frac{1}{4}\left(2\pi + 4ia - 2\arctan(c) + i\log\left(c^2+1\right)\right)\log(x) - \frac{1}{2}\int \frac{\arctan\left(ce^{(2bx+2a)}\right)}{x}\,dx + \frac{1}{4}i\int \frac{\log\left(c^2e^{(4bx+4a)}+1\right)}{x}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x - 1/4*(2*pi + 4*I*a - 2*arctan(c) + I*log(c^2 + 1))*log(x) - 1/2*int
egrate(arctan(c*e^(2*b*x + 2*a))/x, x) + 1/4*I*integrate(log(c^2*e^(4*b*x +
 4*a) + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i\log\left(-\frac{\left(ce^{(2bx+2a)}-i\right)e^{(-2bx-2a)}}{c-i}\right)}{2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/(c - I))/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c-(I-c)*coth(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left((c-i)\coth(bx+a)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((c - I)*coth(b*x + a) + c)/x, x)
```

3.110 $\int \tan^{-1}(e^x)\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}i\text{PolyLog}(2,-ie^x)-\frac{1}{2}i\text{PolyLog}(2,ie^x)$$

```
[Out] (I/2)*PolyLog[2, (-I)*E^x] - (I/2)*PolyLog[2, I*E^x]
```

Rubi [A] time = 0.0255403, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {2282, 4848, 2391}

$$\frac{1}{2}i\text{PolyLog}(2,-ie^x)-\frac{1}{2}i\text{PolyLog}(2,ie^x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[E^x],x]

[Out] (I/2)*PolyLog[2, (-I)*E^x] - (I/2)*PolyLog[2, I*E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(e^x)\,dx &= \text{Subst}\left(\int \frac{\tan^{-1}(x)}{x}\,dx, x, e^x\right)\\ &= \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^x\right)-\frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^x\right)\\ &= \frac{1}{2}i\text{Li}_2(-ie^x)-\frac{1}{2}i\text{Li}_2(ie^x)\end{aligned}$$

Mathematica [A] time = 0.043881, size = 59, normalized size = 1.9

$$x\tan^{-1}(e^x)-\frac{1}{2}i\left(-\text{PolyLog}(2,-ie^x)+\text{PolyLog}(2,ie^x)+x\left(\log(1-ie^x)-\log(1+ie^x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[E^x],x]
```

```
[Out] x*ArcTan[E^x] - (I/2)*(x*(Log[1 - I*E^x] - Log[1 + I*E^x]) - PolyLog[2, (-I
)*E^x] + PolyLog[2, I*E^x])
```

Maple [B] time = 0.049, size = 59, normalized size = 1.9

$$\ln(\mathrm{e}^{x})\arctan(\mathrm{e}^{x})+\frac{i}{2}\ln(\mathrm{e}^{x})\ln(1+i\mathrm{e}^{x})-\frac{i}{2}\ln(\mathrm{e}^{x})\ln(1-i\mathrm{e}^{x})+\frac{i}{2}\mathrm{dilog}(1+i\mathrm{e}^{x})-\frac{i}{2}\mathrm{dilog}(1-i\mathrm{e}^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(exp(x)),x)
```

```
[Out] ln(exp(x))*arctan(exp(x))+1/2*I*ln(exp(x))*ln(1+I*exp(x))-1/2*I*ln(exp(x))*
ln(1-I*exp(x))+1/2*I*dilog(1+I*exp(x))-1/2*I*dilog(1-I*exp(x))
```

Maxima [B] time = 1.50996, size = 46, normalized size = 1.48

$$x\arctan(e^{x})-\frac{1}{4}\pi\log\left(e^{(2x)}+1\right)-\frac{1}{2}i\,\mathrm{Li}_2(ie^{x}+1)+\frac{1}{2}i\,\mathrm{Li}_2(-ie^{x}+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x)),x, algorithm="maxima")
```

```
[Out] x*arctan(e^x) - 1/4*pi*log(e^(2*x) + 1) - 1/2*I*dilog(I*e^x + 1) + 1/2*I*di
log(-I*e^x + 1)
```

Fricas [B] time = 2.16789, size = 147, normalized size = 4.74

$$x\arctan(e^{x})+\frac{1}{2}ix\log(ie^{x}+1)-\frac{1}{2}ix\log(-ie^{x}+1)-\frac{1}{2}i\,\mathrm{Li}_2(ie^{x})+\frac{1}{2}i\,\mathrm{Li}_2(-ie^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x)),x, algorithm="fricas")
```

```
[Out] x*arctan(e^x) + 1/2*I*x*log(I*e^x + 1) - 1/2*I*x*log(-I*e^x + 1) - 1/2*I*di
log(I*e^x) + 1/2*I*dilog(-I*e^x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{atan}(e^{x})\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(exp(x)),x)
```

```
[Out] Integral(atan(exp(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arctan\left(e^x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(arctan(e^x), x)
```

3.111 $\int x \tan^{-1}(e^x) \, dx$

Optimal. Leaf size=63

$$\frac{1}{2}ix\text{PolyLog}(2, -ie^x) - \frac{1}{2}ix\text{PolyLog}(2, ie^x) - \frac{1}{2}i\text{PolyLog}(3, -ie^x) + \frac{1}{2}i\text{PolyLog}(3, ie^x)$$

```
[Out] (I/2)*x*PolyLog[2, (-I)*E^x] - (I/2)*x*PolyLog[2, I*E^x] - (I/2)*PolyLog[3,
 (-I)*E^x] + (I/2)*PolyLog[3, I*E^x]
```

Rubi [A] time = 0.0427798, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5143, 2531, 2282, 6589}

$$\frac{1}{2}ix\text{PolyLog}(2, -ie^x) - \frac{1}{2}ix\text{PolyLog}(2, ie^x) - \frac{1}{2}i\text{PolyLog}(3, -ie^x) + \frac{1}{2}i\text{PolyLog}(3, ie^x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[E^x],x]
```

```
[Out] (I/2)*x*PolyLog[2, (-I)*E^x] - (I/2)*x*PolyLog[2, I*E^x] - (I/2)*PolyLog[3,
 (-I)*E^x] + (I/2)*PolyLog[3, I*E^x]
```

Rule 5143

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}(e^x)\,dx &= \frac{1}{2}i\int x\log(1-ie^x)\,dx-\frac{1}{2}i\int x\log(1+ie^x)\,dx\\ &= \frac{1}{2}ix\mathrm{Li}_2(-ie^x)-\frac{1}{2}ix\mathrm{Li}_2(ie^x)-\frac{1}{2}i\int \mathrm{Li}_2(-ie^x)\,dx+\frac{1}{2}i\int \mathrm{Li}_2(ie^x)\,dx\\ &= \frac{1}{2}ix\mathrm{Li}_2(-ie^x)-\frac{1}{2}ix\mathrm{Li}_2(ie^x)-\frac{1}{2}i\,\mathrm{Subst}\left(\int\frac{\mathrm{Li}_2(-ix)}{x}\,dx,x,e^x\right)+\frac{1}{2}i\,\mathrm{Subst}\left(\int\frac{\mathrm{Li}_2(ix)}{x}\,dx,x,e^x\right)\\ &= \frac{1}{2}ix\mathrm{Li}_2(-ie^x)-\frac{1}{2}ix\mathrm{Li}_2(ie^x)-\frac{1}{2}i\mathrm{Li}_3(-ie^x)+\frac{1}{2}i\mathrm{Li}_3(ie^x)\end{aligned}$$

Mathematica [A] time = 0.0088854, size = 50, normalized size = 0.79

$$\frac{1}{2}i\left(x\mathrm{PolyLog}(2,-ie^x)-x\mathrm{PolyLog}(2,ie^x)-\mathrm{PolyLog}(3,-ie^x)+\mathrm{PolyLog}(3,ie^x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[E^x],x]
```

```
[Out] (I/2)*(x*PolyLog[2, (-I)*E^x] - x*PolyLog[2, I*E^x] - PolyLog[3, (-I)*E^x]
+ PolyLog[3, I*E^x])
```

Maple [A] time = 0.089, size = 44, normalized size = 0.7

$$\frac{i}{2}x\mathrm{polylog}(2,-ie^x)-\frac{i}{2}x\mathrm{polylog}(2,ie^x)-\frac{i}{2}\mathrm{polylog}(3,-ie^x)+\frac{i}{2}\mathrm{polylog}(3,ie^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(exp(x)),x)
```

```
[Out] 1/2*I*x*polylog(2,-I*exp(x))-1/2*I*x*polylog(2,I*exp(x))-1/2*I*polylog(3,-I
*exp(x))+1/2*I*polylog(3,I*exp(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2\arctan(e^x)-\int\frac{x^2e^x}{2\left(e^{(2x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(e^x) - integrate(1/2*x^2*e^x/(e^(2*x) + 1), x)
```

Fricas [C] time = 2.16289, size = 238, normalized size = 3.78

$$\frac{1}{2}x^2\arctan(e^x)+\frac{1}{4}ix^2\log(ie^x+1)-\frac{1}{4}ix^2\log(-ie^x+1)-\frac{1}{2}ix\mathrm{Li}_2(ie^x)+\frac{1}{2}ix\mathrm{Li}_2(-ie^x)+\frac{1}{2}i\,\mathrm{polylog}(3,ie^x)-\frac{1}{2}i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*arctan(e^x) + 1/4*I*x^2*log(I*e^x + 1) - 1/4*I*x^2*log(-I*e^x + 1)
- 1/2*I*x*dilog(I*e^x) + 1/2*I*x*dilog(-I*e^x) + 1/2*I*polylog(3, I*e^x) -
1/2*I*polylog(3, -I*e^x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{atan}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(exp(x)),x)
```

```
[Out] Integral(x*atan(exp(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \arctan(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(e^x), x)
```

3.112 $\int x^2 \tan^{-1}(e^x) \, dx$

Optimal. Leaf size=91

$$\frac{1}{2}ix^2\text{PolyLog}(2,-ie^x)-\frac{1}{2}ix^2\text{PolyLog}(2,ie^x)-ix\text{PolyLog}(3,-ie^x)+ix\text{PolyLog}(3,ie^x)+i\text{PolyLog}(4,-ie^x)-i\text{Pol}$$

```
[Out] (I/2)*x^2*PolyLog[2, (-I)*E^x] - (I/2)*x^2*PolyLog[2, I*E^x] - I*x*PolyLog[
3, (-I)*E^x] + I*x*PolyLog[3, I*E^x] + I*PolyLog[4, (-I)*E^x] - I*PolyLog[4
, I*E^x]
```

Rubi [A] time = 0.0661729, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5143, 2531, 6609, 2282, 6589}

$$\frac{1}{2}ix^2\text{PolyLog}(2,-ie^x)-\frac{1}{2}ix^2\text{PolyLog}(2,ie^x)-ix\text{PolyLog}(3,-ie^x)+ix\text{PolyLog}(3,ie^x)+i\text{PolyLog}(4,-ie^x)-i\text{Pol}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[E^x],x]
```

```
[Out] (I/2)*x^2*PolyLog[2, (-I)*E^x] - (I/2)*x^2*PolyLog[2, I*E^x] - I*x*PolyLog[
3, (-I)*E^x] + I*x*PolyLog[3, I*E^x] + I*PolyLog[4, (-I)*E^x] - I*PolyLog[4
, I*E^x]
```

Rule 5143

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}(e^x)\,dx &= \frac{1}{2}i\int x^2\log(1-ie^x)\,dx-\frac{1}{2}i\int x^2\log(1+ie^x)\,dx\\
&= \frac{1}{2}ix^2\mathrm{Li}_2(-ie^x)-\frac{1}{2}ix^2\mathrm{Li}_2(ie^x)-i\int x\mathrm{Li}_2(-ie^x)\,dx+i\int x\mathrm{Li}_2(ie^x)\,dx\\
&= \frac{1}{2}ix^2\mathrm{Li}_2(-ie^x)-\frac{1}{2}ix^2\mathrm{Li}_2(ie^x)-ix\mathrm{Li}_3(-ie^x)+ix\mathrm{Li}_3(ie^x)+i\int \mathrm{Li}_3(-ie^x)\,dx-i\int \mathrm{Li}_3(ie^x)\,dx\\
&= \frac{1}{2}ix^2\mathrm{Li}_2(-ie^x)-\frac{1}{2}ix^2\mathrm{Li}_2(ie^x)-ix\mathrm{Li}_3(-ie^x)+ix\mathrm{Li}_3(ie^x)+i\,\mathrm{Subst}\left(\int\frac{\mathrm{Li}_3(-ix)}{x}\,dx,x,e^x\right)-i\,\mathrm{Subs}\\
&= \frac{1}{2}ix^2\mathrm{Li}_2(-ie^x)-\frac{1}{2}ix^2\mathrm{Li}_2(ie^x)-ix\mathrm{Li}_3(-ie^x)+ix\mathrm{Li}_3(ie^x)+i\mathrm{Li}_4(-ie^x)-i\mathrm{Li}_4(ie^x)\end{aligned}$$

Mathematica [A] time = 0.0080973, size = 91, normalized size = 1.

$$\frac{1}{2}ix^2\mathrm{PolyLog}(2,-ie^x)-\frac{1}{2}ix^2\mathrm{PolyLog}(2,ie^x)-ix\mathrm{PolyLog}(3,-ie^x)+ix\mathrm{PolyLog}(3,ie^x)+i\mathrm{PolyLog}(4,-ie^x)-i\mathrm{PolyL}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[E^x],x]
```

```
[Out] (I/2)*x^2*PolyLog[2, (-I)*E^x] - (I/2)*x^2*PolyLog[2, I*E^x] - I*x*PolyLog[
3, (-I)*E^x] + I*x*PolyLog[3, I*E^x] + I*PolyLog[4, (-I)*E^x] - I*PolyLog[4
, I*E^x]
```

Maple [A] time = 0.079, size = 70, normalized size = 0.8

$$\frac{i}{2}x^2\mathrm{polylog}(2,-ie^x)-\frac{i}{2}x^2\mathrm{polylog}(2,ie^x)-ix\mathrm{polylog}(3,-ie^x)+ix\mathrm{polylog}(3,ie^x)+i\mathrm{polylog}(4,-ie^x)-i\mathrm{polylog}(4,i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(exp(x)),x)
```

```
[Out] 1/2*I*x^2*polylog(2,-I*exp(x))-1/2*I*x^2*polylog(2,I*exp(x))-I*x*polylog(3,
-I*exp(x))+I*x*polylog(3,I*exp(x))+I*polylog(4,-I*exp(x))-I*polylog(4,I*exp
(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}x^3\arctan(e^x)-\int\frac{x^3e^x}{3\left(e^{(2x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(e^x) - integrate(1/3*x^3*e^x/(e^(2*x) + 1), x)
```

Fricas [C] time = 2.28235, size = 298, normalized size = 3.27

$$\frac{1}{3}x^3\arctan(e^x) + \frac{1}{6}ix^3\log(ie^x+1) - \frac{1}{6}ix^3\log(-ie^x+1) - \frac{1}{2}ix^2\mathrm{Li}_2(ie^x) + \frac{1}{2}ix^2\mathrm{Li}_2(-ie^x) + ix\mathrm{polylog}(3, ie^x) - i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arctan(e^x) + 1/6*I*x^3*log(I*e^x + 1) - 1/6*I*x^3*log(-I*e^x + 1)
- 1/2*I*x^2*dilog(I*e^x) + 1/2*I*x^2*dilog(-I*e^x) + I*x*polylog(3, I*e^x)
- I*x*polylog(3, -I*e^x) - I*polylog(4, I*e^x) + I*polylog(4, -I*e^x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\operatorname{atan}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(exp(x)),x)
```

```
[Out] Integral(x**2*atan(exp(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\arctan(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(e^x), x)
```

3.113 $\int \tan^{-1}\left(e^{a+bx}\right)dx$

Optimal. Leaf size=45

$$\frac{i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b}-\frac{i\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b}$$

```
[Out] ((I/2)*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*PolyLog[2, I*E^(a + b*x)])/
b
```

Rubi [A] time = 0.027565, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 4848, 2391}

$$\frac{i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b}-\frac{i\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*PolyLog[2, I*E^(a + b*x)])/
b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}\left(e^{a+bx}\right)dx&=\frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{x}\,dx,x,e^{a+bx}\right)}{b}\\&=\frac{i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{a+bx}\right)}{2b}-\frac{i\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,e^{a+bx}\right)}{2b}\\&=\frac{i\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{i\text{Li}_2\left(ie^{a+bx}\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.100697, size = 83, normalized size = 1.84

$$x\tan^{-1}\left(e^{a+bx}\right)-\frac{i\left(-\text{PolyLog}\left(2,-ie^{a+bx}\right)+\text{PolyLog}\left(2,ie^{a+bx}\right)+bx\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[E^(a + b*x)],x]
```

```
[Out] x*ArcTan[E^(a + b*x)] - ((I/2)*(b*x*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^(
a + b*x)]) - PolyLog[2, (-I)*E^(a + b*x)] + PolyLog[2, I*E^(a + b*x)]))/b
```

Maple [B] time = 0.049, size = 106, normalized size = 2.4

$$\frac{\ln\left(e^{bx+a}\right)\arctan\left(e^{bx+a}\right)}{b}+\frac{\frac{i}{2}\ln\left(e^{bx+a}\right)\ln\left(1+ie^{bx+a}\right)}{b}-\frac{\frac{i}{2}\ln\left(e^{bx+a}\right)\ln\left(1-ie^{bx+a}\right)}{b}+\frac{\frac{i}{2}\text{dilog}\left(1+ie^{bx+a}\right)}{b}-\frac{\frac{i}{2}\text{dilog}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(exp(b*x+a)),x)
```

```
[Out] 1/b*ln(exp(b*x+a))*arctan(exp(b*x+a))+1/2*I/b*ln(exp(b*x+a))*ln(1+I*exp(b*x
+a))-1/2*I/b*ln(exp(b*x+a))*ln(1-I*exp(b*x+a))+1/2*I/b*dilog(1+I*exp(b*x+a)
)-1/2*I/b*dilog(1-I*exp(b*x+a))
```

Maxima [B] time = 1.49925, size = 85, normalized size = 1.89

$$\frac{(bx+a)\arctan\left(e^{(bx+a)}\right)}{b}-\frac{\pi\log\left(e^{(2\,bx+2\,a)}+1\right)+2i\,\mathrm{Li}_2\left(i\,e^{(bx+a)}+1\right)-2i\,\mathrm{Li}_2\left(-i\,e^{(bx+a)}+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] (b*x + a)*arctan(e^(b*x + a))/b - 1/4*(pi*log(e^(2*b*x + 2*a) + 1) + 2*I*di
log(I*e^(b*x + a) + 1) - 2*I*dilog(-I*e^(b*x + a) + 1))/b
```

Fricas [B] time = 2.23242, size = 297, normalized size = 6.6

$$\frac{2\,bx\arctan\left(e^{(bx+a)}\right)+i\,a\log\left(e^{(bx+a)}+i\right)-i\,a\log\left(e^{(bx+a)}-i\right)+(i\,bx+i\,a)\log\left(i\,e^{(bx+a)}+1\right)+(-i\,bx-i\,a)\log\left(-i\,e^{(bx}\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(e^(b*x + a)) + I*a*log(e^(b*x + a) + I) - I*a*log(e^(b*x
+ a) - I) + (I*b*x + I*a)*log(I*e^(b*x + a) + 1) + (-I*b*x - I*a)*log(-I*e^
(b*x + a) + 1) - I*dilog(I*e^(b*x + a)) + I*dilog(-I*e^(b*x + a)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{atan}\left(e^{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(exp(b*x+a)),x)
```

```
[Out] Integral(atan(exp(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arctan\left(e^{(bx+a)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(e^(b*x + a)), x)
```

3.114 $\int x \tan^{-1}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=91

$$-\frac{i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{2b^2}+\frac{i\text{PolyLog}\left(3,ie^{a+bx}\right)}{2b^2}+\frac{ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b}-\frac{ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b}$$

```
[Out] ((I/2)*x*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*x*PolyLog[2, I*E^(a + b*x
)])/b - ((I/2)*PolyLog[3, (-I)*E^(a + b*x)])/b^2 + ((I/2)*PolyLog[3, I*E^(a
 + b*x)])/b^2
```

Rubi [A] time = 0.0579984, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5143, 2531, 2282, 6589}

$$-\frac{i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{2b^2}+\frac{i\text{PolyLog}\left(3,ie^{a+bx}\right)}{2b^2}+\frac{ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b}-\frac{ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*x*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*x*PolyLog[2, I*E^(a + b*x
)])/b - ((I/2)*PolyLog[3, (-I)*E^(a + b*x)])/b^2 + ((I/2)*PolyLog[3, I*E^(a
 + b*x)])/b^2
```

Rule 5143

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x\tan^{-1}\left(e^{a+bx}\right)dx &= \frac{1}{2}i\int x\log\left(1-ie^{a+bx}\right)dx-\frac{1}{2}i\int x\log\left(1+ie^{a+bx}\right)dx \\
&= \frac{ix\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix\text{Li}_2\left(ie^{a+bx}\right)}{2b}-\frac{i\int\text{Li}_2\left(-ie^{a+bx}\right)dx}{2b}+\frac{i\int\text{Li}_2\left(ie^{a+bx}\right)dx}{2b} \\
&= \frac{ix\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix\text{Li}_2\left(ie^{a+bx}\right)}{2b}-\frac{i\,\text{Subst}\left(\int\frac{\text{Li}_2(-ix)}{x}dx,x,e^{a+bx}\right)}{2b^2}+\frac{i\,\text{Subst}\left(\int\frac{\text{Li}_2(ix)}{x}dx,x,e^{a+bx}\right)}{2b^2} \\
&= \frac{ix\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix\text{Li}_2\left(ie^{a+bx}\right)}{2b}-\frac{i\text{Li}_3\left(-ie^{a+bx}\right)}{2b^2}+\frac{i\text{Li}_3\left(ie^{a+bx}\right)}{2b^2}
\end{aligned}
$$

Mathematica [A] time = 0.0130628, size = 71, normalized size = 0.78

$$
\frac{i\left(bx\text{PolyLog}\left(2,-ie^{a+bx}\right)-bx\text{PolyLog}\left(2,ie^{a+bx}\right)-\text{PolyLog}\left(3,-ie^{a+bx}\right)+\text{PolyLog}\left(3,ie^{a+bx}\right)\right)}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*(b*x*PolyLog[2, (-I)*E^(a + b*x)] - b*x*PolyLog[2, I*E^(a + b*x)] -
PolyLog[3, (-I)*E^(a + b*x)] + PolyLog[3, I*E^(a + b*x)]))/b^2
```

Maple [B] time = 0.171, size = 349, normalized size = 3.8

$$
\frac{\frac{i}{2}\ln\left(1+ie^{bx+a}\right)xa}{b}-\frac{\frac{i}{2}\text{polylog}\left(2,ie^{bx+a}\right)a}{b^2}-\frac{\frac{i}{2}\ln\left(-i\left(-e^{bx+a}+i\right)\right)xa}{b}+\frac{\frac{i}{2}\text{polylog}\left(2,-ie^{bx+a}\right)a}{b^2}+\frac{\frac{i}{2}\text{dilog}\left(-i\left(e^{bx+a}+\right.\right.}{b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(exp(b*x+a)),x)
```

```
[Out] 1/2*I/b*ln(1+I*exp(b*x+a))*x*a-1/2*I/b^2*polylog(2,I*exp(b*x+a))*a-1/2*I/b*
ln(-I*(-exp(b*x+a)+I))*x*a+1/2*I/b^2*polylog(2,-I*exp(b*x+a))*a+1/2*I/b^2*d
ilog(-I*(exp(b*x+a)+I))*a+1/2*I/b^2*dilog(-I*exp(b*x+a))*a+1/2*I*polylog(3,
I*exp(b*x+a))/b^2+1/2*I/b*ln(-I*(exp(b*x+a)+I))*x*a-1/2*I*x*polylog(2,I*exp
(b*x+a))/b-1/2*I/b^2*ln(-I*(-exp(b*x+a)+I))*a^2-1/2*I/b^2*a^2*ln(1-I*exp(b*
x+a))+1/2*I/b^2*ln(-I*exp(b*x+a))*ln(-I*(-exp(b*x+a)+I))*a+1/2*I*x*polylog(
2,-I*exp(b*x+a))/b-1/2*I*polylog(3,-I*exp(b*x+a))/b^2+1/2*I/b^2*a^2*ln(1+I*
exp(b*x+a))-1/2*I/b*ln(1-I*exp(b*x+a))*x*a+1/2*I/b^2*ln(-I*(exp(b*x+a)+I))*
a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\frac{1}{2}x^2\arctan\left(e^{(bx+a)}\right)-b\int\frac{x^2e^{(bx+a)}}{2\left(e^{(2bx+2a)}+1\right)}dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(e^(b*x + a)) - b*integrate(1/2*x^2*e^(b*x + a)/(e^(2*b*x + 2
*a) + 1), x)
```

Fricas [C] time = 2.33572, size = 431, normalized size = 4.74

$$\frac{2\,b^2x^2\arctan\left(e^{(bx+a)}\right)-2i\,bx\mathrm{Li}_2\left(i\,e^{(bx+a)}\right)+2i\,bx\mathrm{Li}_2\left(-i\,e^{(bx+a)}\right)-i\,a^2\log\left(e^{(bx+a)}+i\right)+i\,a^2\log\left(e^{(bx+a)}-i\right)+\left(i\,b^2x^2\right.}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arctan(e^(b*x + a)) - 2*I*b*x*dilog(I*e^(b*x + a)) + 2*I*b*x
*dilog(-I*e^(b*x + a)) - I*a^2*log(e^(b*x + a) + I) + I*a^2*log(e^(b*x + a)
 - I) + (I*b^2*x^2 - I*a^2)*log(I*e^(b*x + a) + 1) + (-I*b^2*x^2 + I*a^2)*l
og(-I*e^(b*x + a) + 1) + 2*I*polylog(3, I*e^(b*x + a)) - 2*I*polylog(3, -I*
e^(b*x + a)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{atan}\left(e^{a}e^{bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(exp(b*x+a)),x)
```

```
[Out] Integral(x*atan(exp(a)*exp(b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\arctan\left(e^{(bx+a)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(e^(b*x + a)), x)
```

3.115 $\int x^2 \tan^{-1}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=133

$$-\frac{ix\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^2}+\frac{ix\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^2}+\frac{i\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^3}-\frac{i\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^3}+\frac{ix^2\text{PolyLog}\left(2,-ie\right.}{2b}$$

```
[Out] ((I/2)*x^2*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*x^2*PolyLog[2, I*E^(a +
 b*x)])/b - (I*x*PolyLog[3, (-I)*E^(a + b*x)])/b^2 + (I*x*PolyLog[3, I*E^(a
 + b*x)])/b^2 + (I*PolyLog[4, (-I)*E^(a + b*x)])/b^3 - (I*PolyLog[4, I*E^(a
 + b*x)])/b^3
```

Rubi [A] time = 0.0899561, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5143, 2531, 6609, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^2}+\frac{ix\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^2}+\frac{i\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^3}-\frac{i\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^3}+\frac{ix^2\text{PolyLog}\left(2,-ie\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*x^2*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*x^2*PolyLog[2, I*E^(a +
 b*x)])/b - (I*x*PolyLog[3, (-I)*E^(a + b*x)])/b^2 + (I*x*PolyLog[3, I*E^(a
 + b*x)])/b^2 + (I*PolyLog[4, (-I)*E^(a + b*x)])/b^3 - (I*PolyLog[4, I*E^(a
 + b*x)])/b^3
```

Rule 5143

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}\left(e^{a+bx}\right)dx &= \frac{1}{2}i\int x^2\log\left(1-ie^{a+bx}\right)dx-\frac{1}{2}i\int x^2\log\left(1+ie^{a+bx}\right)dx\\ &= \frac{ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix^2\text{Li}_2\left(ie^{a+bx}\right)}{2b}-\frac{i\int x\text{Li}_2\left(-ie^{a+bx}\right)dx}{b}+\frac{i\int x\text{Li}_2\left(ie^{a+bx}\right)dx}{b}\\ &= \frac{ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix^2\text{Li}_2\left(ie^{a+bx}\right)}{2b}-\frac{ix\text{Li}_3\left(-ie^{a+bx}\right)}{b^2}+\frac{ix\text{Li}_3\left(ie^{a+bx}\right)}{b^2}+\frac{i\int\text{Li}_3\left(-ie^{a+bx}\right)dx}{b^2}\\ &= \frac{ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix^2\text{Li}_2\left(ie^{a+bx}\right)}{2b}-\frac{ix\text{Li}_3\left(-ie^{a+bx}\right)}{b^2}+\frac{ix\text{Li}_3\left(ie^{a+bx}\right)}{b^2}+\frac{i\,\text{Subst}\left(\int\frac{\text{Li}_3(-ix)}{x}dx\right.}{b^3}\\ &= \frac{ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix^2\text{Li}_2\left(ie^{a+bx}\right)}{2b}-\frac{ix\text{Li}_3\left(-ie^{a+bx}\right)}{b^2}+\frac{ix\text{Li}_3\left(ie^{a+bx}\right)}{b^2}+\frac{i\text{Li}_4\left(-ie^{a+bx}\right)}{b^3}-\frac{i\text{Li}_4}{}\end{aligned}$$

Mathematica [A] time = 0.0103102, size = 133, normalized size = 1.

$$-\frac{ix\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^2}+\frac{ix\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^2}+\frac{i\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^3}-\frac{i\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^3}+\frac{ix^2\text{PolyLog}\left(2,\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*x^2*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*x^2*PolyLog[2, I*E^(a +
 b*x)])/b - (I*x*PolyLog[3, (-I)*E^(a + b*x)])/b^2 + (I*x*PolyLog[3, I*E^(a
 + b*x)])/b^2 + (I*PolyLog[4, (-I)*E^(a + b*x)])/b^3 - (I*PolyLog[4, I*E^(a
 + b*x)])/b^3
```

Maple [B] time = 0.182, size = 407, normalized size = 3.1

$$\frac{-\frac{i}{2}x^2\text{polylog}\left(2,i\mathrm{e}^{bx+a}\right)}{b}-\frac{i\text{polylog}\left(3,-i\mathrm{e}^{bx+a}\right)x}{b^2}-\frac{\frac{i}{2}\ln\left(-i\left(\mathrm{e}^{bx+a}+i\right)\right)a^3}{b^3}-\frac{\frac{i}{2}\text{polylog}\left(2,-i\mathrm{e}^{bx+a}\right)a^2}{b^3}+\frac{i\text{polylog}\left(4\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(exp(b*x+a)),x)
```

```
[Out] -1/2*I*x^2*polylog(2,I*exp(b*x+a))/b-I*x*polylog(3,-I*exp(b*x+a))/b^2-1/2*I
/b^3*ln(-I*(exp(b*x+a)+I))*a^3-1/2*I/b^3*polylog(2,-I*exp(b*x+a))*a^2+I*pol
ylog(4,-I*exp(b*x+a))/b^3+I*x*polylog(3,I*exp(b*x+a))/b^2-1/2*I/b^3*a^3*ln(
1+I*exp(b*x+a))+1/2*I*x^2*polylog(2,-I*exp(b*x+a))/b-I*polylog(4,I*exp(b*x+
a))/b^3+1/2*I/b^3*polylog(2,I*exp(b*x+a))*a^2+1/2*I/b^3*ln(-I*(-exp(b*x+a)+
I))*a^3+1/2*I/b^3*a^3*ln(1-I*exp(b*x+a))-1/2*I/b^3*dilog(-I*exp(b*x+a))*a^2
-1/2*I/b^3*ln(-I*exp(b*x+a))*ln(-I*(-exp(b*x+a)+I))*a^2-1/2*I/b^2*ln(1+I*ex
```

```
p(b*x+a))*x*a^2-1/2*I/b^3*dilog(-I*(exp(b*x+a)+I))*a^2+1/2*I/b^2*ln(1-I*exp
(b*x+a))*x*a^2-1/2*I/b^2*ln(-I*(exp(b*x+a)+I))*x*a^2+1/2*I/b^2*ln(-I*(-exp(
b*x+a)+I))*x*a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\,x^3\arctan\left(e^{(bx+a)}\right)-b\int\frac{x^3e^{(bx+a)}}{3\left(e^{(2\,bx+2\,a)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(e^(b*x + a)) - b*integrate(1/3*x^3*e^(b*x + a)/(e^(2*b*x + 2
*a) + 1), x)
```

Fricas [C] time = 2.35137, size = 540, normalized size = 4.06

$$\frac{2\,b^3x^3\arctan\left(e^{(bx+a)}\right)-3i\,b^2x^2\mathrm{Li}_2\left(i\,e^{(bx+a)}\right)+3i\,b^2x^2\mathrm{Li}_2\left(-i\,e^{(bx+a)}\right)+i\,a^3\log\left(e^{(bx+a)}+i\right)-i\,a^3\log\left(e^{(bx+a)}-i\right)+6i\,bx\mathrm{p}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arctan(e^(b*x + a)) - 3*I*b^2*x^2*dilog(I*e^(b*x + a)) + 3*I
*b^2*x^2*dilog(-I*e^(b*x + a)) + I*a^3*log(e^(b*x + a) + I) - I*a^3*log(e^(
b*x + a) - I) + 6*I*b*x*polylog(3, I*e^(b*x + a)) - 6*I*b*x*polylog(3, -I*e
^(b*x + a)) + (I*b^3*x^3 + I*a^3)*log(I*e^(b*x + a) + 1) + (-I*b^3*x^3 - I*
a^3)*log(-I*e^(b*x + a) + 1) - 6*I*polylog(4, I*e^(b*x + a)) + 6*I*polylog(
4, -I*e^(b*x + a)))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,\mathrm{atan}\left(e^ae^{bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(exp(b*x+a)),x)
```

```
[Out] Integral(x**2*atan(exp(a)*exp(b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\arctan\left(e^{(bx+a)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(e^(b*x + a)), x)
```

3.116 $\int \tan^{-1}\left(a+bf^{c+dx}\right)dx$

Optimal. Leaf size=196

$$\frac{i\text{PolyLog}\left(2,1-\frac{2}{1-i\left(a+bf^{c+dx}\right)}\right)}{2d\log(f)}-\frac{i\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(-a+i)\left(1-i\left(a+bf^{c+dx}\right)\right)}\right)}{2d\log(f)}-\frac{\log\left(\frac{2}{1-i\left(a+bf^{c+dx}\right)}\right)\tan^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}+\frac{\log\left(\frac{\cdots}{(-\cdots}\right.\cdots}{\cdots}$$

```
[Out] -((ArcTan[a + b*f^(c + d*x)]*Log[2/(1 - I*(a + b*f^(c + d*x)))])/(d*Log[f])
) + (ArcTan[a + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((I - a)*(1 - I*(a + b
*f^(c + d*x))))])/(d*Log[f]) + ((I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*f^(c +
 d*x)))])/(d*Log[f]) - ((I/2)*PolyLog[2, 1 - (2*b*f^(c + d*x))/((I - a)*(1
- I*(a + b*f^(c + d*x))))])/(d*Log[f])
```

Rubi [A] time = 0.14815, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2282, 5047, 4856, 2402, 2315, 2447}

$$\frac{i\text{PolyLog}\left(2,1-\frac{2}{1-i\left(a+bf^{c+dx}\right)}\right)}{2d\log(f)}-\frac{i\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(-a+i)\left(1-i\left(a+bf^{c+dx}\right)\right)}\right)}{2d\log(f)}-\frac{\log\left(\frac{2}{1-i\left(a+bf^{c+dx}\right)}\right)\tan^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}+\frac{\log\left(\frac{\cdots}{(-\cdots}\right.\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] -((ArcTan[a + b*f^(c + d*x)]*Log[2/(1 - I*(a + b*f^(c + d*x)))])/(d*Log[f])
) + (ArcTan[a + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((I - a)*(1 - I*(a + b
*f^(c + d*x))))])/(d*Log[f]) + ((I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*f^(c +
 d*x)))])/(d*Log[f]) - ((I/2)*PolyLog[2, 1 - (2*b*f^(c + d*x))/((I - a)*(1
- I*(a + b*f^(c + d*x))))])/(d*Log[f])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \tan^{-1}\left(a+bf^{c+dx}\right)dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(a+bx)}{x}\,dx, x, f^{c+dx}\right)}{d\log(f)}$$

$$= \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bf^{c+dx}\right)}{bd\log(f)}$$

$$= -\frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i\left(a+bf^{c+dx}\right)}\right)}{d\log(f)} + \frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)\left(1-i\left(a+bf^{c+dx}\right)\right)}\right)}{d\log(f)} + \frac{\text{Su}}{}$$

$$= -\frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i\left(a+bf^{c+dx}\right)}\right)}{d\log(f)} + \frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)\left(1-i\left(a+bf^{c+dx}\right)\right)}\right)}{d\log(f)} - \frac{i\text{Li}}{}$$

$$= -\frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i\left(a+bf^{c+dx}\right)}\right)}{d\log(f)} + \frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)\left(1-i\left(a+bf^{c+dx}\right)\right)}\right)}{d\log(f)} + \frac{i\text{L}}{}$$

Mathematica [A] time = 0.180523, size = 167, normalized size = 0.85

$$x\tan^{-1}\left(a+bf^{c+dx}\right) - \frac{b\left(\text{PolyLog}\left(2,-\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right) - \text{PolyLog}\left(2,-\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right) + dx\log(f)\left(\log\left(\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}+1\right) - \log\left(\frac{b^2f^{c}}{ab+\sqrt{}}\right.\right.\right.}{2\sqrt{-b^2}d\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] x*ArcTan[a + b*f^(c + d*x)] - (b*(d*x*Log[f]*(Log[1 + (b^2*f^(c + d*x))/(a*
b - Sqrt[-b^2])] - Log[1 + (b^2*f^(c + d*x))/(a*b + Sqrt[-b^2])]) + PolyLog
[2, -((b^2*f^(c + d*x))/(a*b - Sqrt[-b^2]))] - PolyLog[2, -((b^2*f^(c + d*x
))/(a*b + Sqrt[-b^2]))]))/(2*Sqrt[-b^2]*d*Log[f])
```

Maple [A] time = 0.053, size = 186, normalized size = 1.

$$\frac{\ln\left(bf^{dx+c}\right)\arctan\left(a+bf^{dx+c}\right)}{d\ln\left(f\right)}+\frac{\frac{i}{2}\ln\left(bf^{dx+c}\right)}{d\ln\left(f\right)}\ln\left(\frac{-bf^{dx+c}-a+i}{i-a}\right)-\frac{\frac{i}{2}\ln\left(bf^{dx+c}\right)}{d\ln\left(f\right)}\ln\left(\frac{bf^{dx+c}+a+i}{i+a}\right)+\frac{\frac{i}{2}}{d\ln\left(f\right)}\mathrm{dilog}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a+b*f^(d*x+c)),x)
```

```
[Out] 1/d/ln(f)*ln(b*f^(d*x+c))*arctan(a+b*f^(d*x+c))+1/2*I/d/ln(f)*ln(b*f^(d*x+c
))*ln((-b*f^(d*x+c)-a+I)/(I-a))-1/2*I/d/ln(f)*ln(b*f^(d*x+c))*ln((b*f^(d*x+
c)+a+I)/(I+a))+1/2*I/d/ln(f)*dilog((-b*f^(d*x+c)-a+I)/(I-a))-1/2*I/d/ln(f)*
dilog((b*f^(d*x+c)+a+I)/(I+a))
```

Maxima [A] time = 1.69977, size = 302, normalized size = 1.54

$$\frac{\arctan\left(bf^{dx+c}+a\right)\log\left(f^{dx+c}\right)}{d\log\left(f\right)}-\frac{\arctan\left(\frac{bf^{dx+c}}{a^2+1},-\frac{abf^{dx+c}}{a^2+1}\right)\log\left(b^2f^{2dx+2c}+2abf^{dx+c}+a^2+1\right)-\arctan\left(bf^{dx+c}+a\right)\mathrm{lo}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] arctan(b*f^(d*x + c) + a)*log(f^(d*x + c))/(d*log(f)) - 1/2*(arctan2(b*f^(d
*x + c)/(a^2 + 1), -a*b*f^(d*x + c)/(a^2 + 1))*log(b^2*f^(2*d*x + 2*c) + 2*
a*b*f^(d*x + c) + a^2 + 1) - arctan(b*f^(d*x + c) + a)*log(b^2*f^(2*d*x + 2
*c)/(a^2 + 1)) + 2*arctan((b^2*f^(d*x + c) + a*b)/b)*log(f^(d*x + c)) + I*d
ilog((I*b*f^(d*x + c) + I*a + 1)/(I*a + 1)) - I*dilog((I*b*f^(d*x + c) + I*
a - 1)/(I*a - 1)))/(d*log(f))
```

Fricas [A] time = 2.41286, size = 554, normalized size = 2.83

$$\frac{2\,dx\arctan\left(bf^{dx+c}+a\right)\log\left(f\right)+ic\log\left(bf^{dx+c}+a+i\right)\log\left(f\right)-ic\log\left(bf^{dx+c}+a-i\right)\log\left(f\right)+\left(idx+ic\right)\log\left(f\right)\mathrm{log}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*x*arctan(b*f^(d*x + c) + a)*log(f) + I*c*log(b*f^(d*x + c) + a + I
)*log(f) - I*c*log(b*f^(d*x + c) + a - I)*log(f) + (I*d*x + I*c)*log(f)*log
((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + (-I*d*x - I*c)*log(f)*log
((a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + I*dilog(-(a^2 + (a*b + I*
b)*f^(d*x + c) + 1)/(a^2 + 1) + 1) - I*dilog(-(a^2 + (a*b - I*b)*f^(d*x + c
) + 1)/(a^2 + 1) + 1))/(d*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a+b*f**(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arctan\left(bf^{dx+c}+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*f^(d*x + c) + a), x)
```

3.117 $\int x \tan^{-1}\left(a+bf^{c+dx}\right) dx$

Optimal. Leaf size=232

$$\frac{i\text{PolyLog}\left(3,\frac{ibf^{c+dx}}{1-ia}\right)}{2d^2\log^2(f)}-\frac{i\text{PolyLog}\left(3,-\frac{ibf^{c+dx}}{1+ia}\right)}{2d^2\log^2(f)}-\frac{ix\text{PolyLog}\left(2,\frac{ibf^{c+dx}}{1-ia}\right)}{2d\log(f)}+\frac{ix\text{PolyLog}\left(2,-\frac{ibf^{c+dx}}{1+ia}\right)}{2d\log(f)}-\frac{1}{4}ix^2\log\left(1-\frac{ibf^{c+}}{1-}\right.$$

```
[Out] (x^2*ArcTan[a + b*f^(c + d*x)])/2 - (I/4)*x^2*Log[1 - (I*b*f^(c + d*x))/(1
- I*a)] + (I/4)*x^2*Log[1 + (I*b*f^(c + d*x))/(1 + I*a)] - ((I/2)*x*PolyLog
[2, (I*b*f^(c + d*x))/(1 - I*a)])/(d*Log[f]) + ((I/2)*x*PolyLog[2, ((-I)*b*
f^(c + d*x))/(1 + I*a)])/(d*Log[f]) + ((I/2)*PolyLog[3, (I*b*f^(c + d*x))/(
1 - I*a)])/(d^2*Log[f]^2) - ((I/2)*PolyLog[3, ((-I)*b*f^(c + d*x))/(1 + I*a
)])/(d^2*Log[f]^2)
```

Rubi [A] time = 0.150929, antiderivative size = 250, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5143, 2532, 2531, 2282, 6589}

$$-\frac{i\text{PolyLog}\left(3,\frac{bf^{c+dx}}{-a+i}\right)}{2d^2\log^2(f)}+\frac{i\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+i}\right)}{2d^2\log^2(f)}+\frac{ix\text{PolyLog}\left(2,\frac{bf^{c+dx}}{-a+i}\right)}{2d\log(f)}-\frac{ix\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+i}\right)}{2d\log(f)}+\frac{1}{4}ix^2\log\left(-ia-ibf\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x*ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] (I/4)*x^2*Log[1 - I*a - I*b*f^(c + d*x)] - (I/4)*x^2*Log[1 + I*a + I*b*f^(c
 + d*x)] + (I/4)*x^2*Log[1 - (b*f^(c + d*x))/(I - a)] - (I/4)*x^2*Log[1 + (
b*f^(c + d*x))/(I + a)] + ((I/2)*x*PolyLog[2, (b*f^(c + d*x))/(I - a)])/(d*
Log[f]) - ((I/2)*x*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]) - ((I
/2)*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d^2*Log[f]^2) + ((I/2)*PolyLog[3,
 -((b*f^(c + d*x))/(I + a))])/(d^2*Log[f]^2)
```

Rule 5143

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}\left(a+bf^{c+dx}\right)dx=\frac{1}{2}i\int x\log\left(1-ia-ibf^{c+dx}\right)dx-\frac{1}{2}i\int x\log\left(1+ia+ibf^{c+dx}\right)dx$$
$$=\frac{1}{4}ix^2\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)-\frac{1}{4}ix^2$$
$$=\frac{1}{4}ix^2\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)-\frac{1}{4}ix^2$$
$$=\frac{1}{4}ix^2\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)-\frac{1}{4}ix^2$$
$$=\frac{1}{4}ix^2\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)-\frac{1}{4}ix^2$$

Mathematica [A] time = 0.0872073, size = 236, normalized size = 1.02

$$i\left(-2\text{PolyLog}\left(3,\frac{bf^{c+dx}}{-a+i}\right)+2\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+i}\right)+2dx\log(f)\text{PolyLog}\left(2,\frac{bf^{c+dx}}{-a+i}\right)-2dx\log(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+i}\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] ((I/4)*(d^2*x^2*Log[f]^2*Log[1 - I*a - I*b*f^(c + d*x)] - d^2*x^2*Log[f]^2*
Log[1 + I*a + I*b*f^(c + d*x)] - d^2*x^2*Log[f]^2*Log[(I + a + b*f^(c + d*x
))/(I + a)] + d^2*x^2*Log[f]^2*Log[1 + (b*f^(c + d*x))/(-I + a)] + 2*d*x*Lo
g[f]*PolyLog[2, (b*f^(c + d*x))/(I - a)] - 2*d*x*Log[f]*PolyLog[2, -((b*f^(
c + d*x))/(I + a))] - 2*PolyLog[3, (b*f^(c + d*x))/(I - a)] + 2*PolyLog[3,
-((b*f^(c + d*x))/(I + a))]))/(d^2*Log[f]^2)
```

Maple [B] time = 0.242, size = 672, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a+b*f^(d*x+c)),x)
```

```
[Out] -1/4*I*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x^2-1/4*I*x^2*ln(1+I*(a+b*f^(d*x+c)))+
1/4*I*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*x^2+1/4*I/d^2*ln(1-I*b/(-I*a-1)*f^(d*x
)*f^c)*c^2+1/2*I/d^2/ln(f)*c*dilog((b*f^(d*x)*f^c+I+a)/(I+a))-1/2*I/d^2*c^2
*ln((b*f^(d*x)*f^c+a-I)/(-I+a))-1/4*I/d^2*c^2*ln(1-I*a-I*f^(d*x)*f^c*b)-1/2
*I/d^2/ln(f)*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*c-1/2*I/d*ln(1-I*b/(1-I*a)*
f^(d*x)*f^c)*x*c+1/2*I/d*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*x*c+1/4*I/d^2*c^2*l
n(I*f^(d*x)*f^c*b+I*a+1)+1/2*I/d^2/ln(f)^2*polylog(3,I*b/(1-I*a)*f^(d*x)*f^
c)+1/2*I/d*c*ln((b*f^(d*x)*f^c+I+a)/(I+a))*x+1/2*I/d^2/ln(f)*polylog(2,I*b/
(-I*a-1)*f^(d*x)*f^c)*c+1/4*I*x^2*ln(1-I*(a+b*f^(d*x+c)))+1/2*I/d^2*c^2*ln(
(b*f^(d*x)*f^c+I+a)/(I+a))-1/4*I/d^2*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*c^2-1/2*
I/d^2/ln(f)*c*dilog((b*f^(d*x)*f^c+a-I)/(-I+a))-1/2*I/d/ln(f)*polylog(2,I*b
/(1-I*a)*f^(d*x)*f^c)*x-1/2*I/d^2/ln(f)^2*polylog(3,I*b/(-I*a-1)*f^(d*x)*f^
c)+1/2*I/d/ln(f)*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*x-1/2*I/d*c*ln((b*f^(d
*x)*f^c+a-I)/(-I+a))*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.47553, size = 778, normalized size = 3.35

$$\frac{2\,d^2x^2\arctan\left(bf^{dx+c}+a\right)\log\left(f\right)^2-i\,c^2\log\left(bf^{dx+c}+a+i\right)\log\left(f\right)^2+i\,c^2\log\left(bf^{dx+c}+a-i\right)\log\left(f\right)^2+2i\,dx\mathrm{Li}_2\left(-\frac{a^2+\cdots}{\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d^2*x^2*arctan(b*f^(d*x + c) + a)*log(f)^2 - I*c^2*log(b*f^(d*x + c)
 + a + I)*log(f)^2 + I*c^2*log(b*f^(d*x + c) + a - I)*log(f)^2 + 2*I*d*x*di
log(-(a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f) - 2*I*d*x*di
log(-(a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f) + (I*d^2*x^2
 - I*c^2)*log(f)^2*log((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + (-I
*d^2*x^2 + I*c^2)*log(f)^2*log((a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1
)) - 2*I*polylog(3, -(a*b + I*b)*f^(d*x + c)/(a^2 + 1)) + 2*I*polylog(3, -(
a*b - I*b)*f^(d*x + c)/(a^2 + 1)))/(d^2*log(f)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a+b*f**(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \arctan\left(bf^{dx+c} + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(b*f^(d*x + c) + a), x)
```

3.118 $\int x^2 \tan^{-1}\left(a + bf^{c+dx}\right) dx$

Optimal. Leaf size=302

$$\frac{ix\text{PolyLog}\left(3,\frac{ibf^{c+dx}}{1-ia}\right)}{d^2\log^2(f)} - \frac{ix\text{PolyLog}\left(3,-\frac{ibf^{c+dx}}{1+ia}\right)}{d^2\log^2(f)} - \frac{i\text{PolyLog}\left(4,\frac{ibf^{c+dx}}{1-ia}\right)}{d^3\log^3(f)} + \frac{i\text{PolyLog}\left(4,-\frac{ibf^{c+dx}}{1+ia}\right)}{d^3\log^3(f)} - \frac{ix^2\text{PolyLog}\left(2,\frac{ibf}{1\cdots}\right.}{2d\log(f)}$$

```
[Out] (x^3*ArcTan[a + b*f^(c + d*x)])/3 - (I/6)*x^3*Log[1 - (I*b*f^(c + d*x))/(1
- I*a)] + (I/6)*x^3*Log[1 + (I*b*f^(c + d*x))/(1 + I*a)] - ((I/2)*x^2*PolyL
og[2, (I*b*f^(c + d*x))/(1 - I*a)])/(d*Log[f]) + ((I/2)*x^2*PolyLog[2, ((-I
)*b*f^(c + d*x))/(1 + I*a)])/(d*Log[f]) + (I*x*PolyLog[3, (I*b*f^(c + d*x))
/(1 - I*a)])/(d^2*Log[f]^2) - (I*x*PolyLog[3, ((-I)*b*f^(c + d*x))/(1 + I*a
)])/(d^2*Log[f]^2) - (I*PolyLog[4, (I*b*f^(c + d*x))/(1 - I*a)])/(d^3*Log[f
]^3) + (I*PolyLog[4, ((-I)*b*f^(c + d*x))/(1 + I*a)])/(d^3*Log[f]^3)
```

Rubi [A] time = 0.201092, antiderivative size = 313, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5143, 2532, 2531, 6609, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(3,\frac{bf^{c+dx}}{-a+i}\right)}{d^2\log^2(f)} + \frac{ix\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+i}\right)}{d^2\log^2(f)} + \frac{i\text{PolyLog}\left(4,\frac{bf^{c+dx}}{-a+i}\right)}{d^3\log^3(f)} - \frac{i\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a+i}\right)}{d^3\log^3(f)} + \frac{ix^2\text{PolyLog}\left(2,\frac{bf^{c}}{-a\cdots}\right.}{2d\log(f)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2*ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] (I/6)*x^3*Log[1 - I*a - I*b*f^(c + d*x)] - (I/6)*x^3*Log[1 + I*a + I*b*f^(c
 + d*x)] + (I/6)*x^3*Log[1 - (b*f^(c + d*x))/(I - a)] - (I/6)*x^3*Log[1 + (
b*f^(c + d*x))/(I + a)] + ((I/2)*x^2*PolyLog[2, (b*f^(c + d*x))/(I - a)])/(
d*Log[f]) - ((I/2)*x^2*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]) -
 (I*x*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d^2*Log[f]^2) + (I*x*PolyLog[3,
 -((b*f^(c + d*x))/(I + a))])/(d^2*Log[f]^2) + (I*PolyLog[4, (b*f^(c + d*x)
)/(I - a)])/(d^3*Log[f]^3) - (I*PolyLog[4, -((b*f^(c + d*x))/(I + a))])/(d^
3*Log[f]^3)
```

Rule 5143

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tan^{-1}\left(a+bf^{c+dx}\right) dx = \frac{1}{2}i\int x^2 \log\left(1-ia-ibf^{c+dx}\right) dx - \frac{1}{2}i\int x^2\log\left(1+ia+ibf^{c+dx}\right) dx$$

$$= \frac{1}{6}ix^3\log\left(1-ia-ibf^{c+dx}\right) - \frac{1}{6}ix^3\log\left(1+ia+ibf^{c+dx}\right) + \frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right) - \frac{1}{6}ix^3$$

$$= \frac{1}{6}ix^3\log\left(1-ia-ibf^{c+dx}\right) - \frac{1}{6}ix^3\log\left(1+ia+ibf^{c+dx}\right) + \frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right) - \frac{1}{6}ix^3$$

$$= \frac{1}{6}ix^3\log\left(1-ia-ibf^{c+dx}\right) - \frac{1}{6}ix^3\log\left(1+ia+ibf^{c+dx}\right) + \frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right) - \frac{1}{6}ix^3$$

$$= \frac{1}{6}ix^3\log\left(1-ia-ibf^{c+dx}\right) - \frac{1}{6}ix^3\log\left(1+ia+ibf^{c+dx}\right) + \frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right) - \frac{1}{6}ix^3$$

$$= \frac{1}{6}ix^3\log\left(1-ia-ibf^{c+dx}\right) - \frac{1}{6}ix^3\log\left(1+ia+ibf^{c+dx}\right) + \frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right) - \frac{1}{6}ix^3$$

Mathematica [A] time = 0.0137312, size = 334, normalized size = 1.11

$$\frac{ix\text{PolyLog}\left(3,\frac{ibf^{c+dx}}{1-ia}\right)}{d^2\log^2(f)} - \frac{ix\text{PolyLog}\left(3,-\frac{ibf^{c+dx}}{1+ia}\right)}{d^2\log^2(f)} + \frac{i\text{PolyLog}\left(4,\frac{bf^{c+dx}}{-a+i}\right)}{d^3\log^3(f)} - \frac{i\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a+i}\right)}{d^3\log^3(f)} - \frac{ix^2\text{PolyLog}\left(2,\right.}{2d\log(f)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] (I/6)*x^3*Log[1 - I*a - I*b*f^(c + d*x)] - (I/6)*x^3*Log[1 + I*a + I*b*f^(c
 + d*x)] - (I/6)*x^3*Log[1 - (I*b*f^(c + d*x))/(1 - I*a)] + (I/6)*x^3*Log[1
 + (I*b*f^(c + d*x))/(1 + I*a)] - ((I/2)*x^2*PolyLog[2, (I*b*f^(c + d*x))/(
1 - I*a)])/(d*Log[f]) + ((I/2)*x^2*PolyLog[2, ((-I)*b*f^(c + d*x))/(1 + I*a
)])/(d*Log[f]) + (I*x*PolyLog[3, (I*b*f^(c + d*x))/(1 - I*a)])/(d^2*Log[f]^
2) - (I*x*PolyLog[3, ((-I)*b*f^(c + d*x))/(1 + I*a)])/(d^2*Log[f]^2) + (I*P
olyLog[4, (b*f^(c + d*x))/(I - a)])/(d^3*Log[f]^3) - (I*PolyLog[4, -((b*f^(
c + d*x))/(I + a))])/(d^3*Log[f]^3)
```

Maple [B] time = 0.217, size = 758, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a+b*f^(d*x+c)),x)
```

```
[Out] -1/2*I/d^2*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*x*c^2-1/6*I/d^3*c^3*ln(I*f^(d*x)*
f^c*b+I*a+1)-1/2*I/d/ln(f)*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*x^2+1/2*I/d/l
n(f)*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*x^2-I/d^3/ln(f)^3*polylog(4,I*b/(1
-I*a)*f^(d*x)*f^c)-1/2*I/d^3/ln(f)*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*c^2+
1/2*I/d^3/ln(f)*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*c^2-1/6*I*x^3*ln(1+I*(a+
b*f^(d*x+c)))+I/d^2/ln(f)^2*polylog(3,I*b/(1-I*a)*f^(d*x)*f^c)*x-1/2*I/d^2*
c^2*ln((b*f^(d*x)*f^c+I+a)/(I+a))*x+1/2*I/d^3/ln(f)*c^2*dilog((b*f^(d*x)*f^
c+a-I)/(-I+a))+1/2*I/d^2*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x*c^2+1/6*I*ln(1-I*b
/(-I*a-1)*f^(d*x)*f^c)*x^3+1/3*I/d^3*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*c^3+1/2*
I/d^3*c^3*ln((b*f^(d*x)*f^c+a-I)/(-I+a))-1/2*I/d^3*c^3*ln((b*f^(d*x)*f^c+I+
a)/(I+a))-1/2*I/d^3/ln(f)*c^2*dilog((b*f^(d*x)*f^c+I+a)/(I+a))+1/6*I*x^3*ln
(1-I*(a+b*f^(d*x+c)))-1/6*I*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x^3+1/6*I/d^3*c^3
*ln(1-I*a-I*f^(d*x)*f^c*b)+1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+a-I)/(-I+a))*x+I
/d^3/ln(f)^3*polylog(4,I*b/(-I*a-1)*f^(d*x)*f^c)-1/3*I/d^3*ln(1-I*b/(-I*a-1
)*f^(d*x)*f^c)*c^3-I/d^2/ln(f)^2*polylog(3,I*b/(-I*a-1)*f^(d*x)*f^c)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.47284, size = 967, normalized size = 3.2

$$\frac{2\,d^3x^3\arctan\left(bf^{dx+c}+a\right)\log\left(f\right)^3+3i\,d^2x^2\mathrm{Li}_2\left(-\frac{a^2+(ab+i\,b)f^{dx+c}+1}{a^2+1}+1\right)\log\left(f\right)^2-3i\,d^2x^2\mathrm{Li}_2\left(-\frac{a^2+(ab-i\,b)f^{dx+c}+1}{a^2+1}+1\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*d^3*x^3*arctan(b*f^(d*x + c) + a)*log(f)^3 + 3*I*d^2*x^2*dilog(-(a^2
 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f)^2 - 3*I*d^2*x^2*dilog
(-(a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f)^2 + I*c^3*log(b
*f^(d*x + c) + a + I)*log(f)^3 - I*c^3*log(b*f^(d*x + c) + a - I)*log(f)^3
+ (I*d^3*x^3 + I*c^3)*log(f)^3*log((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2
 + 1)) + (-I*d^3*x^3 - I*c^3)*log(f)^3*log((a^2 + (a*b - I*b)*f^(d*x + c) +
 1)/(a^2 + 1)) - 6*I*d*x*log(f)*polylog(3, -(a*b + I*b)*f^(d*x + c)/(a^2 +
1)) + 6*I*d*x*log(f)*polylog(3, -(a*b - I*b)*f^(d*x + c)/(a^2 + 1)) + 6*I*p
olylog(4, -(a*b + I*b)*f^(d*x + c)/(a^2 + 1)) - 6*I*polylog(4, -(a*b - I*b)
*f^(d*x + c)/(a^2 + 1)))/(d^3*log(f)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a+b*f**(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \arctan\left(bf^{dx+c} + a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(b*f^(d*x + c) + a), x)
```

3.119 $\int e^{-x} \tan^{-1}(e^x)\, dx$

Optimal. Leaf size=25

$$x-\frac{1}{2}\log\left(e^{2x}+1\right)-e^{-x}\tan^{-1}\left(e^x\right)$$

```
[Out] x - ArcTan[E^x]/E^x - Log[1 + E^(2*x)]/2
```

Rubi [A] time = 0.0211932, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {2194, 5207, 2282, 36, 29, 31}

$$x-\frac{1}{2}\log\left(e^{2x}+1\right)-e^{-x}\tan^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[E^x]/E^x,x]
```

```
[Out] x - ArcTan[E^x]/E^x - Log[1 + E^(2*x)]/2
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5207

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] && !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-x}\tan^{-1}(e^x)\,dx &= -e^{-x}\tan^{-1}(e^x)+\int \frac{1}{1+e^{2x}}\,dx\\ &= -e^{-x}\tan^{-1}(e^x)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,e^{2x}\right)\\ &= -e^{-x}\tan^{-1}(e^x)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,e^{2x}\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,e^{2x}\right)\\ &= x-e^{-x}\tan^{-1}(e^x)-\frac{1}{2}\log\left(1+e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.0172051, size = 25, normalized size = 1.

$$x-\frac{1}{2}\log\left(e^{2x}+1\right)-e^{-x}\tan^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[E^x]/E^x,x]
```

```
[Out] x - ArcTan[E^x]/E^x - Log[1 + E^(2*x)]/2
```

Maple [A] time = 0.04, size = 23, normalized size = 0.9

$$-\frac{\arctan(e^x)}{e^x}-\frac{\ln\left((e^x)^2+1\right)}{2}+\ln(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(exp(x))/exp(x),x)
```

```
[Out] -arctan(exp(x))/exp(x)-1/2*ln(exp(x)^2+1)+ln(exp(x))
```

Maxima [A] time = 0.946223, size = 26, normalized size = 1.04

$$-\arctan(e^x)\,e^{(-x)}-\frac{1}{2}\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x))/exp(x),x, algorithm="maxima")
```

```
[Out] -arctan(e^x)*e^(-x) - 1/2*log(e^(-2*x) + 1)
```

Fricas [A] time = 2.14754, size = 82, normalized size = 3.28

$$\frac{1}{2}\left(2\,xe^x-e^x\log\left(e^{(2x)}+1\right)-2\arctan(e^x)\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x))/exp(x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x*e^x - e^x*log(e^(2*x) + 1) - 2*arctan(e^x))*e^(-x)
```

Sympy [A] time = 11.4519, size = 19, normalized size = 0.76

$$x - \frac{\log\left(e^{2x} + 1\right)}{2} - e^{-x}\operatorname{atan}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(exp(x))/exp(x),x)
```

```
[Out] x - log(exp(2*x) + 1)/2 - exp(-x)*atan(exp(x))
```

Giac [A] time = 1.09933, size = 27, normalized size = 1.08

$$-\arctan\left(e^{x}\right)e^{(-x)} + x - \frac{1}{2}\log\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x))/exp(x),x, algorithm="giac")
```

```
[Out] -arctan(e^x)*e^(-x) + x - 1/2*log(e^(2*x) + 1)
```

3.120 $\int \frac{\tan^{-1}(x)}{(-1+x)^3} \, dx$

Optimal. Leaf size=45

$$\frac{1}{8}\log\left(x^2+1\right)+\frac{1}{4(1-x)}-\frac{1}{4}\log(1-x)-\frac{\tan^{-1}(x)}{2(1-x)^2}$$

```
[Out] 1/(4*(1 - x)) - ArcTan[x]/(2*(1 - x)^2) - Log[1 - x]/4 + Log[1 + x^2]/8
```

Rubi [A] time = 0.0317409, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4862, 710, 801, 260}

$$\frac{1}{8}\log\left(x^2+1\right)+\frac{1}{4(1-x)}-\frac{1}{4}\log(1-x)-\frac{\tan^{-1}(x)}{2(1-x)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x]/(-1 + x)^3,x]
```

```
[Out] 1/(4*(1 - x)) - ArcTan[x]/(2*(1 - x)^2) - Log[1 - x]/4 + Log[1 + x^2]/8
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tan^{-1}(x)}{(-1+x)^3}\,dx = -\frac{\tan^{-1}(x)}{2(1-x)^2} + \frac{1}{2}\int \frac{1}{(-1+x)^2\left(1+x^2\right)}\,dx$$
$$= \frac{1}{4(1-x)} - \frac{\tan^{-1}(x)}{2(1-x)^2} + \frac{1}{4}\int \frac{-1-x}{(-1+x)\left(1+x^2\right)}\,dx$$
$$= \frac{1}{4(1-x)} - \frac{\tan^{-1}(x)}{2(1-x)^2} + \frac{1}{4}\int \left(\frac{1}{1-x} + \frac{x}{1+x^2}\right)dx$$
$$= \frac{1}{4(1-x)} - \frac{\tan^{-1}(x)}{2(1-x)^2} - \frac{1}{4}\log(1-x) + \frac{1}{4}\int \frac{x}{1+x^2}\,dx$$
$$= \frac{1}{4(1-x)} - \frac{\tan^{-1}(x)}{2(1-x)^2} - \frac{1}{4}\log(1-x) + \frac{1}{8}\log\left(1+x^2\right)$$

Mathematica [A] time = 0.0328314, size = 35, normalized size = 0.78

$$\frac{1}{8}\left(\log\left(x^2+1\right) - \frac{2}{x-1} - 2\log(1-x) - \frac{4\tan^{-1}(x)}{(x-1)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]/(-1 + x)^3,x]
```

```
[Out] (-2/(-1 + x) - (4*ArcTan[x])/(-1 + x)^2 - 2*Log[1 - x] + Log[1 + x^2])/8
```

Maple [A] time = 0.043, size = 32, normalized size = 0.7

$$-\frac{\arctan(x)}{2\,(x-1)^2} - \frac{1}{4x-4} - \frac{\ln(x-1)}{4} + \frac{\ln\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)/(x-1)^3,x)
```

```
[Out] -1/2/(x-1)^2*arctan(x)-1/4/(x-1)-1/4*ln(x-1)+1/8*ln(x^2+1)
```

Maxima [A] time = 1.44204, size = 42, normalized size = 0.93

$$-\frac{1}{4\,(x-1)} - \frac{\arctan(x)}{2\,(x-1)^2} + \frac{1}{8}\log\left(x^2+1\right) - \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/(-1+x)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(x - 1) - 1/2*arctan(x)/(x - 1)^2 + 1/8*log(x^2 + 1) - 1/4*log(x - 1)
```

Fricas [A] time = 2.39151, size = 144, normalized size = 3.2

$$\frac{\left(x^2-2x+1\right)\log\left(x^2+1\right) - 2\left(x^2-2x+1\right)\log(x-1) - 2x - 4\arctan(x) + 2}{8\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/(-1+x)^3,x, algorithm="fricas")
```

```
[Out] 1/8*((x^2 - 2*x + 1)*log(x^2 + 1) - 2*(x^2 - 2*x + 1)*log(x - 1) - 2*x - 4*
arctan(x) + 2)/(x^2 - 2*x + 1)
```

Sympy [B] time = 0.652303, size = 153, normalized size = 3.4

$$-\frac{2x^2\log(x-1)}{8x^2-16x+8}+\frac{x^2\log(x^2+1)}{8x^2-16x+8}-\frac{x^2}{8x^2-16x+8}+\frac{4x\log(x-1)}{8x^2-16x+8}-\frac{2x\log(x^2+1)}{8x^2-16x+8}-\frac{2\log(x-1)}{8x^2-16x+8}+\frac{\log(x^2+1}{8x^2-16x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)/(-1+x)**3,x)
```

```
[Out] -2*x**2*log(x - 1)/(8*x**2 - 16*x + 8) + x**2*log(x**2 + 1)/(8*x**2 - 16*x
+ 8) - x**2/(8*x**2 - 16*x + 8) + 4*x*log(x - 1)/(8*x**2 - 16*x + 8) - 2*x*
log(x**2 + 1)/(8*x**2 - 16*x + 8) - 2*log(x - 1)/(8*x**2 - 16*x + 8) + log(
x**2 + 1)/(8*x**2 - 16*x + 8) - 4*atan(x)/(8*x**2 - 16*x + 8) + 1/(8*x**2 -
 16*x + 8)
```

Giac [A] time = 1.10366, size = 43, normalized size = 0.96

$$-\frac{1}{4(x-1)}-\frac{\arctan(x)}{2(x-1)^2}+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/(-1+x)^3,x, algorithm="giac")
```

```
[Out] -1/4/(x - 1) - 1/2*arctan(x)/(x - 1)^2 + 1/8*log(x^2 + 1) - 1/4*log(abs(x -
 1))
```

3.121 $\int \frac{\tan^{-1}(1+2x)}{(4+3x)^3} \, dx$

Optimal. Leaf size=64

$$-\frac{5}{578}\log\left(2x^2+2x+1\right)-\frac{1}{34(3x+4)}+\frac{5}{289}\log(3x+4)-\frac{\tan^{-1}(2x+1)}{6(3x+4)^2}+\frac{8}{867}\tan^{-1}(2x+1)$$

```
[Out] -1/(34*(4 + 3*x)) + (8*ArcTan[1 + 2*x])/867 - ArcTan[1 + 2*x]/(6*(4 + 3*x)^
2) + (5*Log[4 + 3*x])/289 - (5*Log[1 + 2*x + 2*x^2])/578
```

Rubi [A] time = 0.067086, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5045, 1982, 709, 800, 634, 617, 204, 628}

$$-\frac{5}{578}\log\left(2x^2+2x+1\right)-\frac{1}{34(3x+4)}+\frac{5}{289}\log(3x+4)-\frac{\tan^{-1}(2x+1)}{6(3x+4)^2}+\frac{8}{867}\tan^{-1}(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[1 + 2*x]/(4 + 3*x)^3,x]
```

```
[Out] -1/(34*(4 + 3*x)) + (8*ArcTan[1 + 2*x])/867 - ArcTan[1 + 2*x]/(6*(4 + 3*x)^
2) + (5*Log[4 + 3*x])/289 - (5*Log[1 + 2*x + 2*x^2])/578
```

Rule 5045

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
 + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 1982

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(1+2x)}{(4+3x)^3}\,dx &= -\frac{\tan^{-1}(1+2x)}{6(4+3x)^2}+\frac{1}{3}\int \frac{1}{(4+3x)^2\left(1+(1+2x)^2\right)}\,dx\\
&= -\frac{\tan^{-1}(1+2x)}{6(4+3x)^2}+\frac{1}{3}\int \frac{1}{(4+3x)^2\left(2+4x+4x^2\right)}\,dx\\
&= -\frac{1}{34(4+3x)}-\frac{\tan^{-1}(1+2x)}{6(4+3x)^2}+\frac{1}{102}\int \frac{4-12x}{(4+3x)\left(2+4x+4x^2\right)}\,dx\\
&= -\frac{1}{34(4+3x)}-\frac{\tan^{-1}(1+2x)}{6(4+3x)^2}+\frac{1}{102}\int \left(\frac{90}{17(4+3x)}-\frac{2(7+30x)}{17\left(1+2x+2x^2\right)}\right)dx\\
&= -\frac{1}{34(4+3x)}-\frac{\tan^{-1}(1+2x)}{6(4+3x)^2}+\frac{5}{289}\log(4+3x)-\frac{1}{867}\int \frac{7+30x}{1+2x+2x^2}\,dx\\
&= -\frac{1}{34(4+3x)}-\frac{\tan^{-1}(1+2x)}{6(4+3x)^2}+\frac{5}{289}\log(4+3x)-\frac{5}{578}\int \frac{2+4x}{1+2x+2x^2}\,dx+\frac{8}{867}\int \frac{1}{1+2x+\cdots}\\
&= -\frac{1}{34(4+3x)}-\frac{\tan^{-1}(1+2x)}{6(4+3x)^2}+\frac{5}{289}\log(4+3x)-\frac{5}{578}\log\left(1+2x+2x^2\right)-\frac{8}{867}\operatorname{Subst}\left(\int \cdots\right.\\
&= -\frac{1}{34(4+3x)}+\frac{8}{867}\tan^{-1}(1+2x)-\frac{\tan^{-1}(1+2x)}{6(4+3x)^2}+\frac{5}{289}\log(4+3x)-\frac{5}{578}\log\left(1+2x+2x^{\cdots}\right.
\end{aligned}$$

Mathematica [C] time = 0.0439864, size = 81, normalized size = 1.27

$$\frac{-289\tan^{-1}(2x+1)+(3x+4)((-15+8i)(3x+4)\log((1+i)x+i)-(15+8i)(3x+4)\log(1+(1+i)x)+90x\log(3x \cdots}{1734(3x+4)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[1 + 2*x]/(4 + 3*x)^3,x]
```

```
[Out] (-289*ArcTan[1 + 2*x] + (4 + 3*x)*(-51 - (15 - 8*I)*(4 + 3*x)*Log[I + (1 +
I)*x] - (15 + 8*I)*(4 + 3*x)*Log[1 + (1 + I)*x] + 120*Log[4 + 3*x] + 90*x*L
og[4 + 3*x]))/(1734*(4 + 3*x)^2)
```

Maple [A] time = 0.044, size = 54, normalized size = 0.8

$$-\frac{2\arctan(1+2x)}{3(8+6x)^2}-\frac{5\ln\left((1+2x)^2+1\right)}{578}+\frac{8\arctan(1+2x)}{867}-\frac{1}{136+102x}+\frac{5\ln(8+6x)}{289}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(1+2*x)/(4+3*x)^3,x)
```

```
[Out] -2/3/(8+6*x)^2*arctan(1+2*x)-5/578*ln((1+2*x)^2+1)+8/867*arctan(1+2*x)-1/17
/(8+6*x)+5/289*ln(8+6*x)
```

Maxima [A] time = 1.44811, size = 73, normalized size = 1.14

$$-\frac{1}{34(3x+4)}-\frac{\arctan(2x+1)}{6(3x+4)^2}+\frac{8}{867}\arctan(2x+1)-\frac{5}{578}\log\left(2x^2+2x+1\right)+\frac{5}{289}\log(3x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+2*x)/(4+3*x)^3,x, algorithm="maxima")
```

```
[Out] -1/34/(3*x + 4) - 1/6*arctan(2*x + 1)/(3*x + 4)^2 + 8/867*arctan(2*x + 1) -
 5/578*log(2*x^2 + 2*x + 1) + 5/289*log(3*x + 4)
```

Fricas [A] time = 2.42057, size = 219, normalized size = 3.42

$$\frac{\left(48x^2+128x-11\right)\arctan(2x+1)-5\left(9x^2+24x+16\right)\log\left(2x^2+2x+1\right)+10\left(9x^2+24x+16\right)\log(3x+4)-51}{578\left(9x^2+24x+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+2*x)/(4+3*x)^3,x, algorithm="fricas")
```

```
[Out] 1/578*((48*x^2 + 128*x - 11)*arctan(2*x + 1) - 5*(9*x^2 + 24*x + 16)*log(2*
x^2 + 2*x + 1) + 10*(9*x^2 + 24*x + 16)*log(3*x + 4) - 51*x - 68)/(9*x^2 +
24*x + 16)
```

Sympy [B] time = 0.779718, size = 223, normalized size = 3.48

$$\frac{90x^2\log\left(x+\frac{4}{3}\right)}{5202x^2+13872x+9248}-\frac{45x^2\log\left(2x^2+2x+1\right)}{5202x^2+13872x+9248}+\frac{48x^2\operatorname{atan}(2x+1)}{5202x^2+13872x+9248}+\frac{240x\log\left(x+\frac{4}{3}\right)}{5202x^2+13872x+9248}-\frac{120x\,\mathrm{l}}{5202x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(1+2*x)/(4+3*x)**3,x)
```

```
[Out] 90*x**2*log(x + 4/3)/(5202*x**2 + 13872*x + 9248) - 45*x**2*log(2*x**2 + 2*
x + 1)/(5202*x**2 + 13872*x + 9248) + 48*x**2*atan(2*x + 1)/(5202*x**2 + 13
872*x + 9248) + 240*x*log(x + 4/3)/(5202*x**2 + 13872*x + 9248) - 120*x*log
(2*x**2 + 2*x + 1)/(5202*x**2 + 13872*x + 9248) + 128*x*atan(2*x + 1)/(5202
*x**2 + 13872*x + 9248) - 51*x/(5202*x**2 + 13872*x + 9248) + 160*log(x + 4
/3)/(5202*x**2 + 13872*x + 9248) - 80*log(2*x**2 + 2*x + 1)/(5202*x**2 + 13
872*x + 9248) - 11*atan(2*x + 1)/(5202*x**2 + 13872*x + 9248) - 68/(5202*x*
*2 + 13872*x + 9248)
```

Giac [A] time = 1.10174, size = 74, normalized size = 1.16

$$-\frac{1}{34(3x+4)}-\frac{\arctan(2x+1)}{6(3x+4)^2}+\frac{8}{867}\arctan(2x+1)-\frac{5}{578}\log\left(2x^2+2x+1\right)+\frac{5}{289}\log(|3x+4|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+2*x)/(4+3*x)^3,x, algorithm="giac")
```

```
[Out] -1/34/(3*x + 4) - 1/6*arctan(2*x + 1)/(3*x + 4)^2 + 8/867*arctan(2*x + 1) -
 5/578*log(2*x^2 + 2*x + 1) + 5/289*log(abs(3*x + 4))
```

3.122 $\int \tan^{-1}\left(\sqrt{1+x}\right) dx$

Optimal. Leaf size=30

$$-\sqrt{x+1}+x\tan^{-1}\left(\sqrt{x+1}\right)+2\tan^{-1}\left(\sqrt{x+1}\right)$$

```
[Out] -Sqrt[1 + x] + 2*ArcTan[Sqrt[1 + x]] + x*ArcTan[Sqrt[1 + x]]
```

Rubi [A] time = 0.0110627, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5203, 80, 63, 203}

$$-\sqrt{x+1}+x\tan^{-1}\left(\sqrt{x+1}\right)+2\tan^{-1}\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[1 + x]],x]

[Out] -Sqrt[1 + x] + 2*ArcTan[Sqrt[1 + x]] + x*ArcTan[Sqrt[1 + x]]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \tan^{-1}\left(\sqrt{1+x}\right) dx &= x\tan^{-1}\left(\sqrt{1+x}\right) - \int \frac{x}{\sqrt{1+x}(4+2x)} dx \\
&= -\sqrt{1+x} + x\tan^{-1}\left(\sqrt{1+x}\right) + 2\int \frac{1}{\sqrt{1+x}(4+2x)} dx \\
&= -\sqrt{1+x} + x\tan^{-1}\left(\sqrt{1+x}\right) + 4\,\mathrm{Subst}\left(\int \frac{1}{2+2x^2} dx, x, \sqrt{1+x}\right) \\
&= -\sqrt{1+x} + 2\tan^{-1}\left(\sqrt{1+x}\right) + x\tan^{-1}\left(\sqrt{1+x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.009583, size = 22, normalized size = 0.73

$$(x+2)\tan^{-1}\left(\sqrt{x+1}\right) - \sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[1 + x]],x]

[Out] -Sqrt[1 + x] + (2 + x)*ArcTan[Sqrt[1 + x]]
```

Maple [A] time = 0.038, size = 25, normalized size = 0.8

$$(x+1)\arctan\left(\sqrt{x+1}\right) - \sqrt{x+1} + \arctan\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan((x+1)^(1/2)),x)

[Out] (x+1)*arctan((x+1)^(1/2))-(x+1)^(1/2)+arctan((x+1)^(1/2))
```

Maxima [A] time = 1.43828, size = 32, normalized size = 1.07

$$(x+1)\arctan\left(\sqrt{x+1}\right) - \sqrt{x+1} + \arctan\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan((1+x)^(1/2)),x, algorithm="maxima")

[Out] (x + 1)*arctan(sqrt(x + 1)) - sqrt(x + 1) + arctan(sqrt(x + 1))
```

Fricas [A] time = 2.39461, size = 58, normalized size = 1.93

$$(x+2)\arctan\left(\sqrt{x+1}\right) - \sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan((1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] (x + 2)*arctan(sqrt(x + 1)) - sqrt(x + 1)
```

Sympy [A] time = 0.255563, size = 26, normalized size = 0.87

$$x \operatorname{atan}\left(\sqrt{x+1}\right) - \sqrt{x+1} + 2 \operatorname{atan}\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan((1+x)**(1/2)),x)
```

```
[Out] x*atan(sqrt(x + 1)) - sqrt(x + 1) + 2*atan(sqrt(x + 1))
```

Giac [A] time = 1.1122, size = 32, normalized size = 1.07

$$(x+1)\arctan\left(\sqrt{x+1}\right) - \sqrt{x+1} + \arctan\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan((1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] (x + 1)*arctan(sqrt(x + 1)) - sqrt(x + 1) + arctan(sqrt(x + 1))
```

3.123 $\int \frac{1}{(1+x^2)(2+\tan^{-1}(x))} \, dx$

Optimal. Leaf size=5

$$\log\left(\tan^{-1}(x)+2\right)$$

```
[Out] Log[2 + ArcTan[x]]
```

Rubi [A] time = 0.0245949, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {4882}

$$\log\left(\tan^{-1}(x)+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(2 + ArcTan[x])),x]
```

```
[Out] Log[2 + ArcTan[x]]
```

Rule 4882

```
Int[1/(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[Log[RemoveContent[a + b*ArcTan[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)\left(2+\tan^{-1}(x)\right)} \, dx = \log\left(2+\tan^{-1}(x)\right)$$

Mathematica [A] time = 0.0293372, size = 5, normalized size = 1.

$$\log\left(\tan^{-1}(x)+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(2 + ArcTan[x])),x]
```

```
[Out] Log[2 + ArcTan[x]]
```

Maple [A] time = 0.036, size = 6, normalized size = 1.2

$$\ln\left(2+\arctan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(2+arctan(x)),x)
```

```
[Out] ln(2+arctan(x))
```

Maxima [A] time = 0.953525, size = 7, normalized size = 1.4

$$\log\left(\arctan\left(x\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(2+arctan(x)),x, algorithm="maxima")
```

```
[Out] log(arctan(x) + 2)
```

Fricas [A] time = 1.9855, size = 27, normalized size = 5.4

$$\log\left(\arctan\left(x\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(2+arctan(x)),x, algorithm="fricas")
```

```
[Out] log(arctan(x) + 2)
```

Sympy [A] time = 0.349717, size = 5, normalized size = 1.

$$\log\left(\operatorname{atan}\left(x\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(2+atan(x)),x)
```

```
[Out] log(atan(x) + 2)
```

Giac [A] time = 1.10889, size = 7, normalized size = 1.4

$$\log\left(\arctan\left(x\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(2+arctan(x)),x, algorithm="giac")
```

```
[Out] log(arctan(x) + 2)
```

3.124 $\int \frac{1}{(a+ax^2)(b-2b\tan^{-1}(x))}\,dx$

Optimal. Leaf size=17

$$-\frac{\log\left(1-2\tan^{-1}(x)\right)}{2ab}$$

```
[Out] -Log[1 - 2*ArcTan[x]]/(2*a*b)
```

Rubi [A] time = 0.0393312, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {4882}

$$-\frac{\log\left(1-2\tan^{-1}(x)\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x^2)*(b - 2*b*ArcTan[x])),x]

[Out] -Log[1 - 2*ArcTan[x]]/(2*a*b)
```

Rule 4882

```
Int[1/(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[Log[RemoveContent[a + b*ArcTan[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{1}{\left(a+ax^2\right)\left(b-2b\tan^{-1}(x)\right)}\,dx=-\frac{\log\left(1-2\tan^{-1}(x)\right)}{2ab}$$

Mathematica [A] time = 0.0445816, size = 17, normalized size = 1.

$$-\frac{\log\left(2\tan^{-1}(x)-1\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x^2)*(b - 2*b*ArcTan[x])),x]

[Out] -Log[-1 + 2*ArcTan[x]]/(2*a*b)
```

Maple [A] time = 0.073, size = 19, normalized size = 1.1

$$-\frac{\ln\left(2\,b\arctan(x)-b\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+a)/(b-2*b*arctan(x)),x)
```

```
[Out] -1/2/a*ln(2*b*arctan(x)-b)/b
```

Maxima [A] time = 0.969344, size = 22, normalized size = 1.29

$$-\frac{\log\left(|2\arctan\left(x\right)-1|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arctan(x)),x, algorithm="maxima")
```

```
[Out] -1/2*log(abs(2*arctan(x) - 1))/(a*b)
```

Fricas [A] time = 1.79571, size = 45, normalized size = 2.65

$$-\frac{\log\left(2\arctan\left(x\right)-1\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arctan(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*arctan(x) - 1)/(a*b)
```

Sympy [A] time = 0.70524, size = 14, normalized size = 0.82

$$-\frac{\log\left(\operatorname{atan}\left(x\right)-\frac{1}{2}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+a)/(b-2*b*atan(x)),x)
```

```
[Out] -log(atan(x) - 1/2)/(2*a*b)
```

Giac [A] time = 1.09499, size = 22, normalized size = 1.29

$$-\frac{\log\left(|2\arctan\left(x\right)-1|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arctan(x)),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*arctan(x) - 1))/(a*b)
```

3.125 $\int \frac{x+x^3+(1+x)^2\tan^{-1}(x)}{(1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=18

$$\frac{1}{x+1}+\log(x+1)+\frac{1}{2}\tan^{-1}(x)^2$$

```
[Out] (1 + x)^(-1) + ArcTan[x]^2/2 + Log[1 + x]
```

Rubi [A] time = 0.150804, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6725, 43, 4884}

$$\frac{1}{x+1}+\log(x+1)+\frac{1}{2}\tan^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^3 + (1 + x)^2*ArcTan[x])/((1 + x)^2*(1 + x^2)),x]
```

```
[Out] (1 + x)^(-1) + ArcTan[x]^2/2 + Log[1 + x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x+x^3+(1+x)^2\tan^{-1}(x)}{(1+x)^2\left(1+x^2\right)}\,dx &= \int \left(\frac{x}{(1+x)^2}+\frac{\tan^{-1}(x)}{1+x^2}\right)dx\\
&= \int \frac{x}{(1+x)^2}\,dx+\int \frac{\tan^{-1}(x)}{1+x^2}\,dx\\
&= \frac{1}{2}\tan^{-1}(x)^2+\int \left(-\frac{1}{(1+x)^2}+\frac{1}{1+x}\right)dx\\
&= \frac{1}{1+x}+\frac{1}{2}\tan^{-1}(x)^2+\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.030539, size = 18, normalized size = 1.

$$\frac{1}{x+1}+\log(x+1)+\frac{1}{2}\tan^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^3 + (1 + x)^2*ArcTan[x])/((1 + x)^2*(1 + x^2)),x]
```

```
[Out] (1 + x)^(-1) + ArcTan[x]^2/2 + Log[1 + x]
```

Maple [A] time = 0.045, size = 17, normalized size = 0.9

$$(x+1)^{-1}+\frac{(\arctan(x))^2}{2}+\ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+x^3+(x+1)^2*arctan(x))/(x+1)^2/(x^2+1),x)
```

```
[Out] 1/(x+1)+1/2*arctan(x)^2+ln(x+1)
```

Maxima [A] time = 1.91213, size = 22, normalized size = 1.22

$$\frac{1}{2}\arctan(x)^2+\frac{1}{x+1}+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x)^2 + 1/(x + 1) + log(x + 1)
```

Fricas [A] time = 1.80419, size = 84, normalized size = 4.67

$$\frac{(x+1)\arctan(x)^2+2(x+1)\log(x+1)+2}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*((x + 1)*arctan(x)^2 + 2*(x + 1)*log(x + 1) + 2)/(x + 1)
```

Sympy [B] time = 0.856435, size = 53, normalized size = 2.94

$$\frac{2x\log(x+1)}{2x+2}+\frac{x\operatorname{atan}^2(x)}{2x+2}+\frac{2\log(x+1)}{2x+2}+\frac{\operatorname{atan}^2(x)}{2x+2}+\frac{2}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x**3+(1+x)**2*atan(x))/(1+x)**2/(x**2+1),x)
```

```
[Out] 2*x*log(x + 1)/(2*x + 2) + x*atan(x)**2/(2*x + 2) + 2*log(x + 1)/(2*x + 2)
+ atan(x)**2/(2*x + 2) + 2/(2*x + 2)
```

Giac [B] time = 1.13125, size = 45, normalized size = 2.5

$$\frac{1}{2}\arctan\left(-(x+1)\left(\frac{1}{x+1}-1\right)\right)^2+\frac{1}{x+1}-\log\left(-\frac{1}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(-(x + 1)*(1/(x + 1) - 1))^2 + 1/(x + 1) - log(-1/(x + 1))
```

3.126 $\int -x^3 \tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=68

$$\frac{\pi x^4}{16}+\frac{x^{7/2}}{56}-\frac{x^{5/2}}{40}+\frac{x^{3/2}}{24}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}}{8}+\frac{1}{8}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -Sqrt[x]/8 + x^(3/2)/24 - x^(5/2)/40 + x^(7/2)/56 + (Pi*x^4)/16 + ArcTan[Sq
rt[x]]/8 - (x^4*ArcTan[Sqrt[x]])/8
```

Rubi [A] time = 0.0279441, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5159, 30, 5033, 50, 63, 203}

$$\frac{\pi x^4}{16}+\frac{x^{7/2}}{56}-\frac{x^{5/2}}{40}+\frac{x^{3/2}}{24}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}}{8}+\frac{1}{8}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-(x^3*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] -Sqrt[x]/8 + x^(3/2)/24 - x^(5/2)/40 + x^(7/2)/56 + (Pi*x^4)/16 + ArcTan[Sq
rt[x]]/8 - (x^4*ArcTan[Sqrt[x]])/8
```

Rule 5159

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int -x^3 \tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right) dx &= -\left(\frac{1}{2}\int x^3 \tan^{-1}\left(\sqrt{x}\right) dx\right)+\frac{1}{4}\pi\int x^3\, dx\\
&= \frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{16}\int \frac{x^{7/2}}{1+x}\, dx\\
&= \frac{x^{7/2}}{56}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{16}\int \frac{x^{5/2}}{1+x}\, dx\\
&= -\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{16}\int \frac{x^{3/2}}{1+x}\, dx\\
&= \frac{x^{3/2}}{24}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{16}\int \frac{\sqrt{x}}{1+x}\, dx\\
&= -\frac{\sqrt{x}}{8}+\frac{x^{3/2}}{24}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{16}\int \frac{1}{\sqrt{x}(1+x)}\, dx\\
&= -\frac{\sqrt{x}}{8}+\frac{x^{3/2}}{24}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\, dx, x, \sqrt{x}\right)\\
&= -\frac{\sqrt{x}}{8}+\frac{x^{3/2}}{24}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}+\frac{1}{8}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0593455, size = 58, normalized size = 0.85

$$\frac{1}{8}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{840}\sqrt{x}\left(-15x^3+21x^2+210x^{7/2}\tan^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)-35x+105\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(x^3*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] ArcTan[Sqrt[x]]/8 - (Sqrt[x]*(105 - 35*x + 21*x^2 - 15*x^3 + 210*x^(7/2)*Ar
cTan[Sqrt[x] - Sqrt[1 + x]]))/840
```

Maple [A] time = 0.056, size = 45, normalized size = 0.7

$$-\frac{x^4}{4}\arctan\left(\sqrt{x}-\sqrt{x+1}\right)+\frac{1}{56}x^{\frac{7}{2}}-\frac{1}{40}x^{\frac{5}{2}}+\frac{1}{24}x^{\frac{3}{2}}-\frac{1}{8}\sqrt{x}+\frac{1}{8}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*arctan(x^(1/2)-(x+1)^(1/2)),x)
```

```
[Out] -1/4*x^4*arctan(x^(1/2)-(x+1)^(1/2))+1/56*x^(7/2)-1/40*x^(5/2)+1/24*x^(3/2)
-1/8*x^(1/2)+1/8*arctan(x^(1/2))
```

Maxima [A] time = 1.58877, size = 59, normalized size = 0.87

$$\frac{1}{4}x^4\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{56}x^{\frac{7}{2}}-\frac{1}{40}x^{\frac{5}{2}}+\frac{1}{24}x^{\frac{3}{2}}-\frac{1}{8}\sqrt{x}+\frac{1}{8}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan(sqrt(x + 1) - sqrt(x)) + 1/56*x^(7/2) - 1/40*x^(5/2) + 1/24*
x^(3/2) - 1/8*sqrt(x) + 1/8*arctan(sqrt(x))
```

Fricas [A] time = 1.96809, size = 124, normalized size = 1.82

$$\frac{1}{4}\left(x^4-1\right)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{840}\left(15x^3-21x^2+35x-105\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(x^4 - 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/840*(15*x^3 - 21*x^2 + 35*x
 - 105)*sqrt(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**3*atan(x**(1/2)-(1+x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12872, size = 59, normalized size = 0.87

$$-\frac{1}{4}x^4\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{56}x^{\frac{7}{2}}-\frac{1}{40}x^{\frac{5}{2}}+\frac{1}{24}x^{\frac{3}{2}}-\frac{1}{8}\sqrt{x}+\frac{1}{8}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*x^4*arctan(-sqrt(x + 1) + sqrt(x)) + 1/56*x^(7/2) - 1/40*x^(5/2) + 1/2
4*x^(3/2) - 1/8*sqrt(x) + 1/8*arctan(sqrt(x))
```

3.127 $\int -x^2 \tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right) dx$

Optimal. Leaf size=59

$$\frac{\pi x^3}{12}+\frac{x^{5/2}}{30}-\frac{x^{3/2}}{18}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x}}{6}-\frac{1}{6}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] Sqrt[x]/6 - x^(3/2)/18 + x^(5/2)/30 + (Pi*x^3)/12 - ArcTan[Sqrt[x]]/6 - (x^
3*ArcTan[Sqrt[x]])/6
```

Rubi [A] time = 0.0259393, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5159, 30, 5033, 50, 63, 203}

$$\frac{\pi x^3}{12}+\frac{x^{5/2}}{30}-\frac{x^{3/2}}{18}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x}}{6}-\frac{1}{6}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-(x^2*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] Sqrt[x]/6 - x^(3/2)/18 + x^(5/2)/30 + (Pi*x^3)/12 - ArcTan[Sqrt[x]]/6 - (x^
3*ArcTan[Sqrt[x]])/6
```

Rule 5159

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int -x^2 \tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right) dx &= -\left(\frac{1}{2}\int x^2 \tan^{-1}\left(\sqrt{x}\right) dx\right)+\frac{1}{4}\pi\int x^2 dx\\
&= \frac{\pi x^3}{12}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{12}\int \frac{x^{5/2}}{1+x} dx\\
&= \frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{12}\int \frac{x^{3/2}}{1+x} dx\\
&= -\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{12}\int \frac{\sqrt{x}}{1+x} dx\\
&= \frac{\sqrt{x}}{6}-\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{12}\int \frac{1}{\sqrt{x}(1+x)} dx\\
&= \frac{\sqrt{x}}{6}-\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{x}\right)\\
&= \frac{\sqrt{x}}{6}-\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{1}{6}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0321391, size = 53, normalized size = 0.9

$$\frac{1}{90}\left(-\sqrt{x}\left(-3x^2+30x^{5/2}\tan^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)+5x-15\right)-15\tan^{-1}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(x^2*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] (-15*ArcTan[Sqrt[x]] - Sqrt[x]*(-15 + 5*x - 3*x^2 + 30*x^(5/2)*ArcTan[Sqrt[
x] - Sqrt[1 + x]]))/90
```

Maple [A] time = 0.051, size = 40, normalized size = 0.7

$$-\frac{x^3}{3}\arctan\left(\sqrt{x}-\sqrt{x+1}\right)+\frac{1}{30}x^{\frac{5}{2}}-\frac{1}{18}x^{\frac{3}{2}}+\frac{1}{6}\sqrt{x}-\frac{1}{6}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctan(x^(1/2)-(x+1)^(1/2)),x)
```

```
[Out] -1/3*x^3*arctan(x^(1/2)-(x+1)^(1/2))+1/30*x^(5/2)-1/18*x^(3/2)+1/6*x^(1/2)-
1/6*arctan(x^(1/2))
```

Maxima [A] time = 1.59477, size = 53, normalized size = 0.9

$$\frac{1}{3}x^3\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{30}x^{\frac{5}{2}}-\frac{1}{18}x^{\frac{3}{2}}+\frac{1}{6}\sqrt{x}-\frac{1}{6}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(sqrt(x + 1) - sqrt(x)) + 1/30*x^(5/2) - 1/18*x^(3/2) + 1/6*s
qrt(x) - 1/6*arctan(sqrt(x))
```

Fricas [A] time = 1.96665, size = 107, normalized size = 1.81

$$\frac{1}{3}\left(x^3+1\right)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{90}\left(3x^2-5x+15\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/90*(3*x^2 - 5*x + 15)*sqrt(
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atan(x**(1/2)-(1+x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13207, size = 53, normalized size = 0.9

$$-\frac{1}{3}x^3\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{30}x^{\frac{5}{2}}-\frac{1}{18}x^{\frac{3}{2}}+\frac{1}{6}\sqrt{x}-\frac{1}{6}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/3*x^3*arctan(-sqrt(x + 1) + sqrt(x)) + 1/30*x^(5/2) - 1/18*x^(3/2) + 1/6
*sqrt(x) - 1/6*arctan(sqrt(x))
```

3.128 $\int -x \tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right) dx$

Optimal. Leaf size=50

$$\frac{\pi x^2}{8}+\frac{x^{3/2}}{12}-\frac{1}{4}x^2\tan^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}}{4}+\frac{1}{4}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -Sqrt[x]/4 + x^(3/2)/12 + (Pi*x^2)/8 + ArcTan[Sqrt[x]]/4 - (x^2*ArcTan[Sqrt
[x]])/4
```

Rubi [A] time = 0.0176926, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5159, 30, 5033, 50, 63, 203}

$$\frac{\pi x^2}{8}+\frac{x^{3/2}}{12}-\frac{1}{4}x^2\tan^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}}{4}+\frac{1}{4}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-(x*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] -Sqrt[x]/4 + x^(3/2)/12 + (Pi*x^2)/8 + ArcTan[Sqrt[x]]/4 - (x^2*ArcTan[Sqrt
[x]])/4
```

Rule 5159

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int -x \tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right) dx &= -\left(\frac{1}{2}\int x \tan^{-1}\left(\sqrt{x}\right) dx\right)+\frac{1}{4}\pi \int x\, dx \\
&= \frac{\pi x^2}{8}-\frac{1}{4}x^2 \tan^{-1}\left(\sqrt{x}\right)+\frac{1}{8}\int \frac{x^{3/2}}{1+x}\, dx \\
&= \frac{x^{3/2}}{12}+\frac{\pi x^2}{8}-\frac{1}{4}x^2 \tan^{-1}\left(\sqrt{x}\right)-\frac{1}{8}\int \frac{\sqrt{x}}{1+x}\, dx \\
&= -\frac{\sqrt{x}}{4}+\frac{x^{3/2}}{12}+\frac{\pi x^2}{8}-\frac{1}{4}x^2 \tan^{-1}\left(\sqrt{x}\right)+\frac{1}{8}\int \frac{1}{\sqrt{x}(1+x)}\, dx \\
&= -\frac{\sqrt{x}}{4}+\frac{x^{3/2}}{12}+\frac{\pi x^2}{8}-\frac{1}{4}x^2 \tan^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\, dx,x,\sqrt{x}\right) \\
&= -\frac{\sqrt{x}}{4}+\frac{x^{3/2}}{12}+\frac{\pi x^2}{8}+\frac{1}{4}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{4}x^2 \tan^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0293783, size = 48, normalized size = 0.96

$$\frac{1}{12}\left(3\tan^{-1}\left(\sqrt{x}\right)-\sqrt{x}\left(6x^{3/2}\tan^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)-x+3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(x*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] (3*ArcTan[Sqrt[x]] - Sqrt[x]*(3 - x + 6*x^(3/2)*ArcTan[Sqrt[x] - Sqrt[1 + x
]]))/12
```

Maple [A] time = 0.054, size = 35, normalized size = 0.7

$$-\frac{x^2}{2}\arctan\left(\sqrt{x}-\sqrt{x+1}\right)+\frac{1}{12}x^{\frac{3}{2}}-\frac{1}{4}\sqrt{x}+\frac{1}{4}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctan(x^(1/2)-(x+1)^(1/2)),x)
```

```
[Out] -1/2*x^2*arctan(x^(1/2)-(x+1)^(1/2))+1/12*x^(3/2)-1/4*x^(1/2)+1/4*arctan(x^
(1/2))
```

Maxima [A] time = 1.56974, size = 46, normalized size = 0.92

$$\frac{1}{2}x^2\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{12}x^{\frac{3}{2}}-\frac{1}{4}\sqrt{x}+\frac{1}{4}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(sqrt(x + 1) - sqrt(x)) + 1/12*x^(3/2) - 1/4*sqrt(x) + 1/4*ar
ctan(sqrt(x))
```

Fricas [A] time = 1.87696, size = 92, normalized size = 1.84

$$\frac{1}{2}\left(x^2-1\right)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{12}\left(x-3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 - 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/12*(x - 3)*sqrt(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atan(x**(1/2)-(1+x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11343, size = 46, normalized size = 0.92

$$-\frac{1}{2}x^2\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{12}x^{\frac{3}{2}}-\frac{1}{4}\sqrt{x}+\frac{1}{4}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*x^2*arctan(-sqrt(x + 1) + sqrt(x)) + 1/12*x^(3/2) - 1/4*sqrt(x) + 1/4*
arctan(sqrt(x))
```

3.129 $\int -\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=37

$$\frac{\pi x}{4}+\frac{\sqrt{x}}{2}-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] Sqrt[x]/2 + (Pi*x)/4 - ArcTan[Sqrt[x]]/2 - (x*ArcTan[Sqrt[x]])/2
```

Rubi [A] time = 0.0100762, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5159, 8, 5027, 50, 63, 203}

$$\frac{\pi x}{4}+\frac{\sqrt{x}}{2}-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-ArcTan[Sqrt[x] - Sqrt[1 + x]],x]
```

```
[Out] Sqrt[x]/2 + (Pi*x)/4 - ArcTan[Sqrt[x]]/2 - (x*ArcTan[Sqrt[x]])/2
```

Rule 5159

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5027

```
Int[ArcTan[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTan[c*x^n], x] - Dist[c
*n, Int[x^n/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int -\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)\,dx &= -\left(\frac{1}{2}\int \tan^{-1}\left(\sqrt{x}\right)\,dx\right)+\frac{1}{4}\pi\int 1\,dx\\ &= \frac{\pi x}{4}-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int \frac{\sqrt{x}}{1+x}\,dx\\ &= \frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int \frac{1}{\sqrt{x}(1+x)}\,dx\\ &= \frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\ &= \frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.39669, size = 39, normalized size = 1.05

$$\frac{\sqrt{x}}{2}-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)-x\tan^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-ArcTan[Sqrt[x] - Sqrt[1 + x]],x]

[Out] Sqrt[x]/2 - ArcTan[Sqrt[x]]/2 - x*ArcTan[Sqrt[x] - Sqrt[1 + x]]
```

Maple [A] time = 0.054, size = 28, normalized size = 0.8

$$-x\arctan\left(\sqrt{x}-\sqrt{x+1}\right)+\frac{1}{2}\sqrt{x}-\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(x^(1/2)-(x+1)^(1/2)),x)

[Out] -x*arctan(x^(1/2)-(x+1)^(1/2))+1/2*x^(1/2)-1/2*arctan(x^(1/2))
```

Maxima [A] time = 1.59613, size = 35, normalized size = 0.95

$$x\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{2}\sqrt{x}-\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")

[Out] x*arctan(sqrt(x + 1) - sqrt(x)) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

Fricas [A] time = 1.91506, size = 72, normalized size = 1.95

$$(x+1)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{2}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] (x + 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/2*sqrt(x)
```

Sympy [A] time = 77.464, size = 29, normalized size = 0.78

$$\frac{\sqrt{x}}{2}-x\,\mathrm{atan}\left(\sqrt{x}-\sqrt{x+1}\right)-\frac{\mathrm{atan}\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2)),x)
```

```
[Out] sqrt(x)/2 - x*atan(sqrt(x) - sqrt(x + 1)) - atan(sqrt(x))/2
```

Giac [A] time = 1.11131, size = 36, normalized size = 0.97

$$-x\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{2}\sqrt{x}-\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -x*arctan(-sqrt(x + 1) + sqrt(x)) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

3.130 $\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x} \, dx$

Optimal. Leaf size=42

$$-\frac{1}{2}i\text{PolyLog}\left(2,-i\sqrt{x}\right)+\frac{1}{2}i\text{PolyLog}\left(2,i\sqrt{x}\right)+\frac{1}{4}\pi\log(x)$$

```
[Out] (Pi*Log[x])/4 - (I/2)*PolyLog[2, (-I)*Sqrt[x]] + (I/2)*PolyLog[2, I*Sqrt[x]
]
```

Rubi [A] time = 0.0422181, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5159, 29, 5031, 4848, 2391}

$$-\frac{1}{2}i\text{PolyLog}\left(2,-i\sqrt{x}\right)+\frac{1}{2}i\text{PolyLog}\left(2,i\sqrt{x}\right)+\frac{1}{4}\pi\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x),x]
```

```
[Out] (Pi*Log[x])/4 - (I/2)*PolyLog[2, (-I)*Sqrt[x]] + (I/2)*PolyLog[2, I*Sqrt[x]
]
```

Rule 5159

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx = -\left(\frac{1}{2}\int\frac{\tan^{-1}\left(\sqrt{x}\right)}{x}\,dx\right)+\frac{1}{4}\pi\int\frac{1}{x}\,dx$$
$$= \frac{1}{4}\pi\log(x)-\text{Subst}\left(\int\frac{\tan^{-1}(x)}{x}\,dx,x,\sqrt{x}\right)$$
$$= \frac{1}{4}\pi\log(x)-\frac{1}{2}i\,\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,\sqrt{x}\right)+\frac{1}{2}i\,\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,\sqrt{x}\right)$$
$$= \frac{1}{4}\pi\log(x)-\frac{1}{2}i\text{Li}_2\left(-i\sqrt{x}\right)+\frac{1}{2}i\text{Li}_2\left(i\sqrt{x}\right)$$

Mathematica [A] time = 0.184779, size = 84, normalized size = 2.

$$-\log(x)\tan^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)+\frac{1}{4}i\left(-2\text{PolyLog}\left(2,-i\sqrt{x}\right)+2\text{PolyLog}\left(2,i\sqrt{x}\right)+\left(\log\left(1-i\sqrt{x}\right)-\log\left(1+i\sqrt{x}\right)\right)\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x),x]
```

```
[Out] -(ArcTan[Sqrt[x] - Sqrt[1 + x]]*Log[x]) + (I/4)*((Log[1 - I*Sqrt[x]] - Log[
1 + I*Sqrt[x]])*Log[x] - 2*PolyLog[2, (-I)*Sqrt[x]] + 2*PolyLog[2, I*Sqrt[x
]])
```

Maple [B] time = 1.115, size = 374, normalized size = 8.9

$$2\arctan\left(\sqrt{x}-\sqrt{x+1}\right)\ln\left(1-\frac{1+i\left(\sqrt{x}-\sqrt{x+1}\right)}{\sqrt{\left(\sqrt{x}-\sqrt{x+1}\right)^2+1}}\right)-2i\text{polylog}\left(2,\left(1+i\left(\sqrt{x}-\sqrt{x+1}\right)\right)\frac{1}{\sqrt{\left(\sqrt{x}-\sqrt{x+1}\right)^2+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(x^(1/2)-(x+1)^(1/2))/x,x)
```

```
[Out] 2*arctan(x^(1/2)-(x+1)^(1/2))*ln(1-(1+I*(x^(1/2)-(x+1)^(1/2)))/((x^(1/2)-(x
+1)^(1/2))^2+1)^(1/2))-2*I*polylog(2,(1+I*(x^(1/2)-(x+1)^(1/2)))/((x^(1/2)-
(x+1)^(1/2))^2+1)^(1/2))+2*arctan(x^(1/2)-(x+1)^(1/2))*ln(1+(1+I*(x^(1/2)-(
x+1)^(1/2)))/((x^(1/2)-(x+1)^(1/2))^2+1)^(1/2))-2*I*polylog(2,-(1+I*(x^(1/2
)-(x+1)^(1/2)))/((x^(1/2)-(x+1)^(1/2))^2+1)^(1/2))-2*arctan(x^(1/2)-(x+1)^(
1/2))*ln(1+(1+I*(x^(1/2)-(x+1)^(1/2)))^4/((x^(1/2)-(x+1)^(1/2))^2+1)^2)+1/2
*I*polylog(2,-(1+I*(x^(1/2)-(x+1)^(1/2)))^4/((x^(1/2)-(x+1)^(1/2))^2+1)^2)+
2*arctan(x^(1/2)-(x+1)^(1/2))*ln((1+I*(x^(1/2)-(x+1)^(1/2)))^2/((x^(1/2)-(x
+1)^(1/2))^2+1)+1)-I*polylog(2,-(1+I*(x^(1/2)-(x+1)^(1/2)))^2/((x^(1/2)-(x+
1)^(1/2))^2+1))
```

Maxima [A] time = 1.54996, size = 58, normalized size = 1.38

$$\frac{1}{4}\pi\log(x+1)+\arctan\left(\sqrt{x+1}-\sqrt{x}\right)\log(x)+\frac{1}{2}i\,\text{Li}_2\left(i\sqrt{x}+1\right)-\frac{1}{2}i\,\text{Li}_2\left(-i\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] 1/4*pi*log(x + 1) + arctan(sqrt(x + 1) - sqrt(x))*log(x) + 1/2*I*dilog(I*sq
rt(x) + 1) - 1/2*I*dilog(-I*sqrt(x) + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(sqrt(x + 1) - sqrt(x))/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(-arctan(-sqrt(x + 1) + sqrt(x))/x, x)
```

3.131 $\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx$

Optimal. Leaf size=41

$$\frac{1}{2\sqrt{x}}-\frac{\pi}{4x}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x}+\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -Pi/(4*x) + 1/(2*Sqrt[x]) + ArcTan[Sqrt[x]]/2 + ArcTan[Sqrt[x]]/(2*x)
```

Rubi [A] time = 0.0221297, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5159, 30, 5033, 51, 63, 203}

$$\frac{1}{2\sqrt{x}}-\frac{\pi}{4x}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x}+\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^2),x]
```

```
[Out] -Pi/(4*x) + 1/(2*Sqrt[x]) + ArcTan[Sqrt[x]]/2 + ArcTan[Sqrt[x]]/(2*x)
```

Rule 5159

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx &= -\left(\frac{1}{2}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^2}\,dx\right)+\frac{1}{4}\pi\int \frac{1}{x^2}\,dx\\ &= -\frac{\pi}{4x}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x}-\frac{1}{4}\int \frac{1}{x^{3/2}(1+x)}\,dx\\ &= -\frac{\pi}{4x}+\frac{1}{2\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x}+\frac{1}{4}\int \frac{1}{\sqrt{x}(1+x)}\,dx\\ &= -\frac{\pi}{4x}+\frac{1}{2\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x}+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\ &= -\frac{\pi}{4x}+\frac{1}{2\sqrt{x}}+\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)+\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x}\end{aligned}$$

Mathematica [A] time = 0.0322639, size = 40, normalized size = 0.98

$$\frac{1}{2\sqrt{x}}+\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)+\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^2),x]
```

```
[Out] 1/(2*Sqrt[x]) + ArcTan[Sqrt[x]]/2 + ArcTan[Sqrt[x] - Sqrt[1 + x]]/x
```

Maple [B] time = 0.055, size = 57, normalized size = 1.4

$$\frac{1}{x}\arctan\left(\sqrt{x}-\sqrt{x+1}\right)+\frac{1}{2}\frac{1}{\sqrt{x}}+\frac{1}{2}\mathrm{Artanh}\left(\sqrt{x+1}\right)+\frac{1}{2}\arctan\left(\sqrt{x}\right)+\frac{1}{4}\ln\left(\sqrt{x+1}-1\right)-\frac{1}{4}\ln\left(\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(x^(1/2)-(x+1)^(1/2))/x^2,x)
```

```
[Out] arctan(x^(1/2)-(x+1)^(1/2))/x+1/2/x^(1/2)+1/2*arctanh((x+1)^(1/2))+1/2*arct
an(x^(1/2))+1/4*ln((x+1)^(1/2)-1)-1/4*ln((x+1)^(1/2)+1)
```

Maxima [A] time = 1.62095, size = 39, normalized size = 0.95

$$-\frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{x}+\frac{1}{2\sqrt{x}}+\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] -arctan(sqrt(x + 1) - sqrt(x))/x + 1/2/sqrt(x) + 1/2*arctan(sqrt(x))
```

Fricas [A] time = 2.02482, size = 81, normalized size = 1.98

$$-\frac{2\,(x+1)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)-\sqrt{x}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(x + 1)*arctan(sqrt(x + 1) - sqrt(x)) - sqrt(x))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2))/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11505, size = 38, normalized size = 0.93

$$\frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{x}+\frac{1}{2\,\sqrt{x}}+\frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] arctan(-sqrt(x + 1) + sqrt(x))/x + 1/2/sqrt(x) + 1/2*arctan(sqrt(x))
```

3.132 $\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx$

Optimal. Leaf size=50

$$\frac{1}{12x^{3/2}}-\frac{\pi}{8x^2}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}-\frac{1}{4\sqrt{x}}-\frac{1}{4}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -Pi/(8*x^2) + 1/(12*x^(3/2)) - 1/(4*Sqrt[x]) - ArcTan[Sqrt[x]]/4 + ArcTan[S
qrt[x]]/(4*x^2)
```

Rubi [A] time = 0.0242087, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5159, 30, 5033, 51, 63, 203}

$$\frac{1}{12x^{3/2}}-\frac{\pi}{8x^2}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}-\frac{1}{4\sqrt{x}}-\frac{1}{4}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^3),x]
```

```
[Out] -Pi/(8*x^2) + 1/(12*x^(3/2)) - 1/(4*Sqrt[x]) - ArcTan[Sqrt[x]]/4 + ArcTan[S
qrt[x]]/(4*x^2)
```

Rule 5159

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx &= -\left(\frac{1}{2}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^3}\,dx\right)+\frac{1}{4}\pi\int \frac{1}{x^3}\,dx\\
&= -\frac{\pi}{8x^2}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}-\frac{1}{8}\int \frac{1}{x^{5/2}(1+x)}\,dx\\
&= -\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}+\frac{1}{8}\int \frac{1}{x^{3/2}(1+x)}\,dx\\
&= -\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}-\frac{1}{4\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}-\frac{1}{8}\int \frac{1}{\sqrt{x}(1+x)}\,dx\\
&= -\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}-\frac{1}{4\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}-\frac{1}{4\sqrt{x}}-\frac{1}{4}\tan^{-1}\left(\sqrt{x}\right)+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}
\end{aligned}$$

Mathematica [A] time = 0.0288469, size = 48, normalized size = 0.96

$$-\frac{3x^2\tan^{-1}\left(\sqrt{x}\right)+(3x-1)\sqrt{x}-6\tan^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^3),x]
```

```
[Out] -(Sqrt[x]*(-1 + 3*x) + 3*x^2*ArcTan[Sqrt[x]] - 6*ArcTan[Sqrt[x] - Sqrt[1 +
x]])/(12*x^2)
```

Maple [A] time = 0.07, size = 35, normalized size = 0.7

$$\frac{1}{2x^2}\arctan\left(\sqrt{x}-\sqrt{x+1}\right)+\frac{1}{12}x^{-\frac{3}{2}}-\frac{1}{4}\arctan\left(\sqrt{x}\right)-\frac{1}{4}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(x^(1/2)-(x+1)^(1/2))/x^3,x)
```

```
[Out] 1/2*arctan(x^(1/2)-(x+1)^(1/2))/x^2+1/12/x^(3/2)-1/4*arctan(x^(1/2))-1/4/x^
(1/2)
```

Maxima [A] time = 1.60696, size = 46, normalized size = 0.92

$$-\frac{1}{4\sqrt{x}}-\frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{2\,x^2}+\frac{1}{12\,x^{\frac{3}{2}}}-\frac{1}{4}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4/sqrt(x) - 1/2*arctan(sqrt(x + 1) - sqrt(x))/x^2 + 1/12/x^(3/2) - 1/4*a
rctan(sqrt(x))
```

Fricas [A] time = 1.96096, size = 100, normalized size = 2.

$$\frac{6\left(x^2-1\right)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)-(3\,x-1)\sqrt{x}}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(x^2 - 1)*arctan(sqrt(x + 1) - sqrt(x)) - (3*x - 1)*sqrt(x))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2))/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10773, size = 46, normalized size = 0.92

$$-\frac{3\,x-1}{12\,x^{\frac{3}{2}}}+\frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{2\,x^2}-\frac{1}{4}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*x - 1)/x^(3/2) + 1/2*arctan(-sqrt(x + 1) + sqrt(x))/x^2 - 1/4*arct
an(sqrt(x))
```

3.133 $\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx$

Optimal. Leaf size=59

$$-\frac{1}{18x^{3/2}}+\frac{1}{30x^{5/2}}-\frac{\pi}{12x^3}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}+\frac{1}{6\sqrt{x}}+\frac{1}{6}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -Pi/(12*x^3) + 1/(30*x^(5/2)) - 1/(18*x^(3/2)) + 1/(6*Sqrt[x]) + ArcTan[Sqr
t[x]]/6 + ArcTan[Sqrt[x]]/(6*x^3)
```

Rubi [A] time = 0.0245805, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5159, 30, 5033, 51, 63, 203}

$$-\frac{1}{18x^{3/2}}+\frac{1}{30x^{5/2}}-\frac{\pi}{12x^3}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}+\frac{1}{6\sqrt{x}}+\frac{1}{6}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^4),x]
```

```
[Out] -Pi/(12*x^3) + 1/(30*x^(5/2)) - 1/(18*x^(3/2)) + 1/(6*Sqrt[x]) + ArcTan[Sqr
t[x]]/6 + ArcTan[Sqrt[x]]/(6*x^3)
```

Rule 5159

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx &= -\left(\frac{1}{2}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^4}\,dx\right)+\frac{1}{4}\pi\int \frac{1}{x^4}\,dx\\
&= -\frac{\pi}{12x^3}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}-\frac{1}{12}\int \frac{1}{x^{7/2}(1+x)}\,dx\\
&= -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}+\frac{1}{12}\int \frac{1}{x^{5/2}(1+x)}\,dx\\
&= -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}-\frac{1}{18x^{3/2}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}-\frac{1}{12}\int \frac{1}{x^{3/2}(1+x)}\,dx\\
&= -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}-\frac{1}{18x^{3/2}}+\frac{1}{6\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}+\frac{1}{12}\int \frac{1}{\sqrt{x}(1+x)}\,dx\\
&= -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}-\frac{1}{18x^{3/2}}+\frac{1}{6\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}-\frac{1}{18x^{3/2}}+\frac{1}{6\sqrt{x}}+\frac{1}{6}\tan^{-1}\left(\sqrt{x}\right)+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}
\end{aligned}
$$

Mathematica [A] time = 0.040509, size = 51, normalized size = 0.86

$$
\frac{1}{90}\left(-\frac{-15x^2+5x-3}{x^{5/2}}+\frac{30\tan^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)}{x^3}+15\tan^{-1}\left(\sqrt{x}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^4),x]
```

```
[Out] (-((-3 + 5*x - 15*x^2)/x^(5/2)) + 15*ArcTan[Sqrt[x]] + (30*ArcTan[Sqrt[x] -
 Sqrt[1 + x]])/x^3)/90
```

Maple [A] time = 0.048, size = 40, normalized size = 0.7

$$
\frac{1}{3\,x^3}\arctan\left(\sqrt{x}-\sqrt{x+1}\right)+\frac{1}{6}\arctan\left(\sqrt{x}\right)+\frac{1}{30}x^{-\frac{5}{2}}+\frac{1}{6}\frac{1}{\sqrt{x}}-\frac{1}{18}x^{-\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(x^(1/2)-(x+1)^(1/2))/x^4,x)
```

```
[Out] 1/3*arctan(x^(1/2)-(x+1)^(1/2))/x^3+1/6*arctan(x^(1/2))+1/30/x^(5/2)+1/6/x^
(1/2)-1/18/x^(3/2)
```

Maxima [A] time = 1.62479, size = 53, normalized size = 0.9

$$\frac{1}{6\sqrt{x}} - \frac{1}{18\,x^{\frac{3}{2}}} - \frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{3\,x^3} + \frac{1}{30\,x^{\frac{5}{2}}} + \frac{1}{6}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6/sqrt(x) - 1/18/x^(3/2) - 1/3*arctan(sqrt(x + 1) - sqrt(x))/x^3 + 1/30/x
^(5/2) + 1/6*arctan(sqrt(x))
```

Fricas [A] time = 2.06878, size = 115, normalized size = 1.95

$$-\frac{30\left(x^3+1\right)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)-\left(15\,x^2-5\,x+3\right)\sqrt{x}}{90\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] -1/90*(30*(x^3 + 1)*arctan(sqrt(x + 1) - sqrt(x)) - (15*x^2 - 5*x + 3)*sqrt
(x))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2))/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13073, size = 53, normalized size = 0.9

$$\frac{15\,x^2-5\,x+3}{90\,x^{\frac{5}{2}}} + \frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{3\,x^3} + \frac{1}{6}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] 1/90*(15*x^2 - 5*x + 3)/x^(5/2) + 1/3*arctan(-sqrt(x + 1) + sqrt(x))/x^3 +
1/6*arctan(sqrt(x))
```

3.134 $\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{m+1}}{c(m+1)\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^(1 + m))/(c*(1 + m)*Sqrt
[d - (c^2*d*x^2)/a])
```

Rubi [A] time = 0.106523, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5157, 5155}

$$\frac{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{m+1}}{c(m+1)\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^m/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^(1 + m))/(c*(1 + m)*Sqrt
[d - (c^2*d*x^2)/a])
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx &= \frac{\sqrt{a-c^2x^2}\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{a-c^2x^2}}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}\\ &= \frac{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{1+m}}{c(1+m)\sqrt{d-\frac{c^2dx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0604835, size = 63, normalized size = 1.

$$\frac{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{m+1}}{c(m+1)\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^m/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^(1 + m))/(c*(1 + m)*Sqrt
[d - (c^2*d*x^2)/a])
```

Maple [A] time = 0.756, size = 73, normalized size = 1.2

$$-\frac{c^2x^2-a}{(1+m)c}\left(\arctan\left(cx\frac{1}{\sqrt{-c^2x^2+a}}\right)\right)^{1+m}\frac{1}{\sqrt{-\frac{d(c^2x^2-a)}{a}}}\frac{1}{\sqrt{-c^2x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x)
```

```
[Out] -arctan(c*x/(-c^2*x^2+a)^(1/2))^(1+m)/(1+m)*(c^2*x^2-a)/(-d*(c^2*x^2-a)/a)^
(1/2)/(-c^2*x^2+a)^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.43247, size = 252, normalized size = 4.

$$-\frac{\sqrt{-c^2x^2+a}a\left(-\arctan\left(\frac{\sqrt{-c^2x^2+a}cx}{c^2x^2-a}\right)\right)^m\sqrt{-\frac{c^2dx^2-ad}{a}}\arctan\left(\frac{\sqrt{-c^2x^2+a}cx}{c^2x^2-a}\right)}{acdm+acd-\left(c^3dm+c^3d\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -sqrt(-c^2*x^2 + a)*a*(-arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a)))^m*sqr
t(-(c^2*d*x^2 - a*d)/a)*arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a))/(a*c*d
```

```
*m + a*c*d - (c^3*d*m + c^3*d)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^{m}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c*x/(-c**2*x**2+a)**(1/2))**m/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(atan(c*x/sqrt(a - c**2*x**2))**m/sqrt(-d*(-1 + c**2*x**2/a)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^{m}}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))^m/sqrt(-c^2*d*x^2/a + d), x)
```

3.135 $\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3)/(3*c*Sqrt[d - (c^2*d*
x^2)/a])
```

Rubi [A] time = 0.0981454, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5157, 5155}

$$\frac{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3)/(3*c*Sqrt[d - (c^2*d*
x^2)/a])
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx &= \frac{\sqrt{a-c^2x^2}\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{a-c^2x^2}}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}\\ &= \frac{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0302251, size = 59, normalized size = 1.

$$\frac{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3)/(3*c*Sqrt[d - (c^2*d*
x^2)/a])
```

Maple [A] time = 0.68, size = 72, normalized size = 1.2

$$-\frac{a}{3d\left(c^2x^2-a\right)c}\sqrt{-\frac{d\left(c^2x^2-a\right)}{a}}\sqrt{-c^2x^2+a}\left(\arctan\left(cx\frac{1}{\sqrt{-c^2x^2+a}}\right)\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x)
```

```
[Out] -1/3*(-d*(c^2*x^2-a)/a)^(1/2)*(-c^2*x^2+a)^(1/2)/d/(c^2*x^2-a)/c*arctan(c*x
/(-c^2*x^2+a)^(1/2))^3*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{a\sqrt{-\frac{c^2dx^2-ad}{a}}\arctan\left(\frac{\sqrt{-c^2x^2+a}cx}{c^2x^2-a}\right)^2}{c^2dx^2-ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-a*sqrt(-(c^2*d*x^2 - a*d)/a)*arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x
^2 - a))^2/(c^2*d*x^2 - a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^{2}\left(\frac{cx}{\sqrt{a-c^{2}x^{2}}}\right)}{\sqrt{-d\left(-1+\frac{c^{2}x^{2}}{a}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c*x/(-c**2*x**2+a)**(1/2))**2/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(atan(c*x/sqrt(a - c**2*x**2))**2/sqrt(-d*(-1 + c**2*x**2/a)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{-c^{2}x^{2}+a}}\right)^{2}}{\sqrt{-\frac{c^{2}dx^{2}}{a}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))^2/sqrt(-c^2*d*x^2/a + d), x)
```

3.136 $\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2)/(2*c*Sqrt[d - (c^2*d*
x^2)/a])
```

Rubi [A] time = 0.0621355, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {5157, 5155}

$$\frac{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2)/(2*c*Sqrt[d - (c^2*d*
x^2)/a])
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx &= \frac{\sqrt{a-c^2x^2}\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{a-c^2x^2}}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}\\ &= \frac{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0228845, size = 59, normalized size = 1.

$$\frac{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2)/(2*c*Sqrt[d - (c^2*d*
x^2)/a])
```

Maple [A] time = 0.675, size = 72, normalized size = 1.2

$$-\frac{a}{2\,d\left(c^2x^2-a\right)c}\sqrt{-\frac{d\left(c^2x^2-a\right)}{a}}\sqrt{-c^2x^2+a}\left(\arctan\left(cx\frac{1}{\sqrt{-c^2x^2+a}}\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x)
```

```
[Out] -1/2*(-d*(c^2*x^2-a)/a)^(1/2)*(-c^2*x^2+a)^(1/2)/d/(c^2*x^2-a)/c*arctan(c*x
/(-c^2*x^2+a)^(1/2))^2*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a\sqrt{-\frac{c^2dx^2-ad}{a}}\arctan\left(\frac{\sqrt{-c^2x^2+a}cx}{c^2x^2-a}\right)}{c^2dx^2-ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(a*sqrt(-(c^2*d*x^2 - a*d)/a)*arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^
2 - a))/(c^2*d*x^2 - a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c*x/(-c**2*x**2+a)**(1/2))/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(atan(c*x/sqrt(a - c**2*x**2))/sqrt(-d*(-1 + c**2*x**2/a)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))/sqrt(-c^2*d*x^2/a + d), x)
```

3.137 $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$

Optimal. Leaf size=55

$$\frac{\sqrt{a-c^2x^2}\log\left(\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[Out] (Sqrt[a - c^2*x^2]*Log[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]])/(c*Sqrt[d - (c^2*d
*x^2)/a])
```

Rubi [A] time = 0.107999, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5157, 5153}

$$\frac{\sqrt{a-c^2x^2}\log\left(\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]),x]
```

```
[Out] (Sqrt[a - c^2*x^2]*Log[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]])/(c*Sqrt[d - (c^2*d
*x^2)/a])
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rule 5153

```
Int[1/(ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*Sqrt[(a_.) + (b_.)*(
x_)^2]), x_Symbol] :> Simp[(1*Log[ArcTan[(c*x)/Sqrt[a + b*x^2]]])/c, x] /;
FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx &= \frac{\sqrt{a-c^2x^2}\int \frac{1}{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}\\ &= \frac{\sqrt{a-c^2x^2}\log\left(\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0547002, size = 55, normalized size = 1.

$$\frac{\sqrt{a-c^2x^2}\log\left(\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]),x]
```

```
[Out] (Sqrt[a - c^2*x^2]*Log[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]])/(c*Sqrt[d - (c^2*d
*x^2)/a])
```

Maple [A] time = 1.108, size = 71, normalized size = 1.3

$$-\frac{a}{d\left(c^2x^2-a\right)c}\sqrt{-\frac{d\left(c^2x^2-a\right)}{a}}\sqrt{-c^2x^2+a}\ln\left(\arctan\left(cx\frac{1}{\sqrt{-c^2x^2+a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x)
```

```
[Out] -(-d*(c^2*x^2-a)/a)^(1/2)*(-c^2*x^2+a)^(1/2)/d/(c^2*x^2-a)/c*ln(arctan(c*x/
(-c^2*x^2+a)^(1/2)))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-\frac{c^2dx^2}{a}+d}\,\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))), x)
```

Fricas [A] time = 1.80791, size = 165, normalized size = 3.

$$-\frac{\sqrt{-c^2x^2+a}\,a\sqrt{-\frac{c^2dx^2-ad}{a}}\log\left(2\arctan\left(\frac{\sqrt{-c^2x^2+a}cx}{c^2x^2-a}\right)\right)}{c^3dx^2-acd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -sqrt(-c^2*x^2 + a)*a*sqrt(-(c^2*d*x^2 - a*d)/a)*log(2*arctan(sqrt(-c^2*x^2
 + a)*c*x/(c^2*x^2 - a)))/(c^3*d*x^2 - a*c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}\operatorname{atan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(c*x/(-c**2*x**2+a)**(1/2))/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*(-1 + c**2*x**2/a))*atan(c*x/sqrt(a - c**2*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\frac{c^2dx^2}{a}+d}\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))), x)
```

3.138 $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{a-c^2x^2}}{c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}$$

```
[Out] -(Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^
2]]))
```

Rubi [A] time = 0.0985449, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5157, 5155}

$$-\frac{\sqrt{a-c^2x^2}}{c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2),x]
```

```
[Out] -(Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^
2]]))
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx &= \frac{\sqrt{a-c^2x^2}\int \frac{1}{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}\\
&= -\frac{\sqrt{a-c^2x^2}}{c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}
\end{aligned}$$

Mathematica [A] time = 0.0259737, size = 57, normalized size = 1.

$$-\frac{\sqrt{a-c^2x^2}}{c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2),x]
```

```
[Out] -(Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^
2]]))
```

Maple [A] time = 0.672, size = 71, normalized size = 1.3

$$\frac{a}{d\left(c^2x^2-a\right)c}\sqrt{-\frac{d\left(c^2x^2-a\right)}{a}}\sqrt{-c^2x^2+a}\left(\arctan\left(cx\frac{1}{\sqrt{-c^2x^2+a}}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x)
```

```
[Out] (-d*(c^2*x^2-a)/a)^(1/2)*(-c^2*x^2+a)^(1/2)/d/(c^2*x^2-a)/c*a/arctan(c*x/(-
c^2*x^2+a)^(1/2))
```

Maxima [A] time = 1.53785, size = 39, normalized size = 0.68

$$-\frac{\sqrt{a}}{c\sqrt{d}\arctan\left(cx, \sqrt{-c^2x^2+a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -sqrt(a)/(c*sqrt(d)*arctan2(c*x, sqrt(-c^2*x^2 + a)))
```

Fricas [A] time = 1.85097, size = 158, normalized size = 2.77

$$-\frac{\sqrt{-c^2x^2+a}a\sqrt{-\frac{c^2dx^2-ad}{a}}}{\left(c^3dx^2-acd\right)\arctan\left(\frac{\sqrt{-c^2x^2+a}cx}{c^2x^2-a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -sqrt(-c^2*x^2 + a)*a*sqrt(-(c^2*d*x^2 - a*d)/a)/((c^3*d*x^2 - a*c*d)*arcta
n(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}\operatorname{atan}^2\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(c*x/(-c**2*x**2+a)**(1/2))**2/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*(-1 + c**2*x**2/a))*atan(c*x/sqrt(a - c**2*x**2))**2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\frac{c^2dx^2}{a}+d}\,\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))^2), x)
```

3.139 $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx$

Optimal. Leaf size=59

$$-\frac{\sqrt{a-c^2x^2}}{2c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}$$

```
[Out] -Sqrt[a - c^2*x^2]/(2*c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x
^2]]^2)
```

Rubi [A] time = 0.10101, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5157, 5155}

$$-\frac{\sqrt{a-c^2x^2}}{2c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3),x]
```

```
[Out] -Sqrt[a - c^2*x^2]/(2*c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x
^2]]^2)
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx &= \frac{\sqrt{a-c^2x^2}\int \frac{1}{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}\\ &= -\frac{\sqrt{a-c^2x^2}}{2c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\end{aligned}$$

Mathematica [A] time = 0.0261237, size = 59, normalized size = 1.

$$-\frac{\sqrt{a-c^2x^2}}{2c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3),x]
```

```
[Out] -Sqrt[a - c^2*x^2]/(2*c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x
^2]]^2)
```

Maple [A] time = 0.675, size = 72, normalized size = 1.2

$$\frac{a}{2d\left(c^2x^2-a\right)c}\sqrt{-\frac{d\left(c^2x^2-a\right)}{a}}\sqrt{-c^2x^2+a}\left(\arctan\left(cx\frac{1}{\sqrt{-c^2x^2+a}}\right)\right)^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x)
```

```
[Out] 1/2*(-d*(c^2*x^2-a)/a)^(1/2)*(-c^2*x^2+a)^(1/2)/d/(c^2*x^2-a)/c*a/arctan(c*
x/(-c^2*x^2+a)^(1/2))^2
```

Maxima [A] time = 1.69293, size = 39, normalized size = 0.66

$$-\frac{\sqrt{a}}{2\,c\sqrt{d}\arctan\left(cx,\sqrt{-c^2x^2+a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*sqrt(a)/(c*sqrt(d)*arctan2(c*x, sqrt(-c^2*x^2 + a))^2)
```

Fricas [A] time = 1.88121, size = 165, normalized size = 2.8

$$\frac{\sqrt{-c^2x^2+a}\,a\sqrt{-\frac{c^2dx^2-ad}{a}}}{2\left(c^3dx^2-acd\right)\arctan\left(\frac{\sqrt{-c^2x^2+a}cx}{c^2x^2-a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*sqrt(-c^2*x^2 + a)*a*sqrt(-(c^2*d*x^2 - a*d)/a)/((c^3*d*x^2 - a*c*d)*ar
ctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a))^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}\operatorname{atan}^3\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(c*x/(-c**2*x**2+a)**(1/2))**3/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*(-1 + c**2*x**2/a))*atan(c*x/sqrt(a - c**2*x**2))**3),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\frac{c^2dx^2}{a}+d}\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))^3), x)
```

3.140 $\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)^{m+1}}{e(m+1)\sqrt{a+bx^2}}$$

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^(1
 + m))/(e*(1 + m)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.107659, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {5157, 5155}

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)^{m+1}}{e(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^m/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^(1
 + m))/(e*(1 + m)*Sqrt[a + b*x^2])
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^{1+m}}{e(1+m)\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.221558, size = 66, normalized size = 0.92

$$\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)^{m+1}}{e(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^m/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]]^(1
 + m))/(e*(1 + m)*Sqrt[a + b*x^2])
```

Maple [F] time = 0.9, size = 0, normalized size = 0.

$$\int \left(\arctan\left(ex\frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)\right)^m \frac{1}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x)
```

```
[Out] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.13816, size = 270, normalized size = 3.75

$$-\frac{\sqrt{bx^2+a}\left(-\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)\right)^m\sqrt{-\frac{be^2x^2+ae^2}{b}}\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)}{aem+(bem+be)x^2+ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -sqrt(b*x^2 + a)*(-arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e))
)^m*sqrt(-(b*e^2*x^2 + a*e^2)/b)*arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b
*e*x^2 + a*e))/(a*e*m + (b*e*m + b*e)*x^2 + a*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^m\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**m/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**m/sqrt(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^m}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^m/sqrt(b*x^2 + a), x)
```

3.141 $\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)^3}{3e\sqrt{a+bx^2}}$$

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^3)
/(3*e*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0979628, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {5157, 5155}

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)^3}{3e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^3)
/(3*e*Sqrt[a + b*x^2])
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}{3e\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.125058, size = 62, normalized size = 0.91

$$\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)^3}{3e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]]^3)
/(3*e*Sqrt[a + b*x^2])
```

Maple [F] time = 0.722, size = 0, normalized size = 0.

$$\int \left(\arctan\left(ex\frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)\right)^2 \frac{1}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
```

```
[Out] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)^2}{\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e))^2/sqrt(b*
x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^2\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**2/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**2/sqrt(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^2}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^2/sqrt(b*x^2 + a), x)
```

3.142 $\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)^2}{2e\sqrt{a+bx^2}}$$

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2)
/(2*e*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0618708, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5157, 5155}

$$\frac{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)^2}{2e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2)
/(2*e*Sqrt[a + b*x^2])
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\,dx}{\sqrt{a+bx^2}}$$
$$= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{2e\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0595942, size = 62, normalized size = 0.91

$$\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)^2}{2e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]]^2)
/(2*e*Sqrt[a + b*x^2])
```

Maple [F] time = 0.717, size = 0, normalized size = 0.

$$\int \arctan\left(ex\frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)\frac{1}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
```

```
[Out] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e))/sqrt(b*x
^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(atan(e*x/sqrt(-a*e**2/b - e**2*x**2))/sqrt(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))/sqrt(b*x^2 + a), x)
```

3.143 $\int \frac{1}{\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\log\left(\tan^{-1}\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)\right)}{e\sqrt{a+bx^2}}$$

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*Log[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]
]])/(e*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.107628, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {5157, 5153}

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\log\left(\tan^{-1}\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)\right)}{e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]),x]
```

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*Log[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]
]])/(e*Sqrt[a + b*x^2])
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rule 5153

```
Int[1/(ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*Sqrt[(a_.) + (b_.)*(
x_)^2]), x_Symbol] :> Simp[(1*Log[ArcTan[(c*x)/Sqrt[a + b*x^2]]])/c, x] /;
FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx}{\sqrt{a+bx^2}}$$

$$= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\log\left(\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)\right)}{e\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.104103, size = 58, normalized size = 0.91

$$\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\log\left(\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)\right)}{e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]),x]
```

```
[Out] (Sqrt[-((e^2*(a + b*x^2))/b)]*Log[ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]
]])/(e*Sqrt[a + b*x^2])
```

Maple [F] time = 0.715, size = 0, normalized size = 0.

$$\int \left(\arctan\left(ex\frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)\right)^{-1}\frac{1}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
```

```
[Out] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))), x)
```

Fricas [A] time = 1.76519, size = 170, normalized size = 2.66

$$\frac{\sqrt{bx^2+a}\sqrt{-\frac{be^2x^2+ae^2}{b}}\log\left(2\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)\right)}{bex^2+ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*sqrt(-(b*e^2*x^2 + a*e^2)/b)*log(2*arctan(b*x*sqrt(-(b*e^2*
x^2 + a*e^2)/b)/(b*e*x^2 + a*e)))/(b*e*x^2 + a*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^2}\,\operatorname{atan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*atan(e*x/sqrt(-a*e**2/b - e**2*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))), x)
```

3.144 $\int \frac{1}{\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}{e\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)}$$

```
[Out] -(Sqrt[-((a*e^2)/b) - e^2*x^2]/(e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^
2)/b) - e^2*x^2]]))
```

Rubi [A] time = 0.101419, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {5157, 5155}

$$-\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}{e\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2),x]
```

```
[Out] -(Sqrt[-((a*e^2)/b) - e^2*x^2]/(e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^
2)/b) - e^2*x^2]]))
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}}{e\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}$$

Mathematica [A] time = 0.0914722, size = 60, normalized size = 0.91

$$\frac{e\sqrt{a+bx^2}}{b\sqrt{-\frac{e^2(a+bx^2)}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2),
x]
```

```
[Out] (e*Sqrt[a + b*x^2])/(b*Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^
2*(a + b*x^2))/b)]])
```

Maple [F] time = 0.728, size = 0, normalized size = 0.

$$\int \left(\arctan\left(ex\frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)\right)^{-2}\frac{1}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
```

```
[Out] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
```

Maxima [A] time = 1.51578, size = 62, normalized size = 0.94

$$-\frac{\sqrt{-bx^2-a}}{\sqrt{bx^2+a}\sqrt{b}\arctan\left(\sqrt{b}x,\sqrt{-bx^2-a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -sqrt(-b*x^2 - a)/(sqrt(b*x^2 + a)*sqrt(b)*arctan2(sqrt(b)*x, sqrt(-b*x^2 -
 a)))
```

Fricas [A] time = 1.8452, size = 163, normalized size = 2.47

$$\frac{\sqrt{bx^2+a}\sqrt{-\frac{be^2x^2+ae^2}{b}}}{(bex^2+ae)\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*sqrt(-(b*e^2*x^2 + a*e^2)/b)/((b*e*x^2 + a*e)*arctan(b*x*sq
rt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^2}\operatorname{atan}^2\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**2/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^2), x)
```

3.145 $\int \frac{1}{\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}{2e\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)^2}$$

```
[Out] -Sqrt[-((a*e^2)/b) - e^2*x^2]/(2*e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e
^2)/b) - e^2*x^2]]^2)
```

Rubi [A] time = 0.100236, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {5157, 5155}

$$-\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}{2e\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^3),x]
```

```
[Out] -Sqrt[-((a*e^2)/b) - e^2*x^2]/(2*e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e
^2)/b) - e^2*x^2]]^2)
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}}{2e\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}$$

Mathematica [A] time = 0.088125, size = 62, normalized size = 0.91

$$-\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}}{2e\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^3),
x]
```

```
[Out] -Sqrt[-((e^2*(a + b*x^2))/b)]/(2*e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((e^2
*(a + b*x^2))/b)]]^2)
```

Maple [F] time = 0.711, size = 0, normalized size = 0.

$$\int \left(\arctan\left(ex\frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)\right)^{-3}\frac{1}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x)
```

```
[Out] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x)
```

Maxima [A] time = 1.71051, size = 80, normalized size = 1.18

$$-\frac{\sqrt{bx^2+a}\sqrt{-bx^2-a}\sqrt{b}}{2\left(b^2x^2+ab\right)\arctan\left(\sqrt{b}x,\sqrt{-bx^2-a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*sqrt(b*x^2 + a)*sqrt(-b*x^2 - a)*sqrt(b)/((b^2*x^2 + a*b)*arctan2(sqrt
(b)*x, sqrt(-b*x^2 - a))^2)
```

Fricas [A] time = 1.91877, size = 173, normalized size = 2.54

$$-\frac{\sqrt{bx^2+a}\sqrt{-\frac{be^2x^2+ae^2}{b}}}{2\left(bex^2+ae\right)\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/2*sqrt(b*x^2 + a)*sqrt(-(b*e^2*x^2 + a*e^2)/b)/((b*e*x^2 + a*e)*arctan(b
*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e))^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^2}\,\operatorname{atan}^3\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**3/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^3), x)
```

3.146 $\int \frac{\tan^{-1}(c(a+bx))\log(d(a+bx))}{a+bx}\,dx$

Optimal. Leaf size=101

$$\frac{i\log(d(a+bx))\text{PolyLog}(2,-ic(a+bx))}{2b}-\frac{i\log(d(a+bx))\text{PolyLog}(2,ic(a+bx))}{2b}-\frac{i\text{PolyLog}(3,-ic(a+bx))}{2b}+\frac{i\text{PolyL}}{}$$

```
[Out] ((I/2)*Log[d*(a + b*x)]*PolyLog[2, (-I)*c*(a + b*x)])/b - ((I/2)*Log[d*(a +
 b*x)]*PolyLog[2, I*c*(a + b*x)])/b - ((I/2)*PolyLog[3, (-I)*c*(a + b*x)])/
b + ((I/2)*PolyLog[3, I*c*(a + b*x)])/b
```

Rubi [A] time = 0.266421, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4848, 2391, 5209, 2444, 2433, 2374, 6589}

$$\frac{i\log(d(a+bx))\text{PolyLog}(2,-ic(a+bx))}{2b}-\frac{i\log(d(a+bx))\text{PolyLog}(2,ic(a+bx))}{2b}-\frac{i\text{PolyLog}(3,-ic(a+bx))}{2b}+\frac{i\text{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTan[c*(a + b*x)]*Log[d*(a + b*x)])/(a + b*x),x]
```


```
[Out] ((I/2)*Log[d*(a + b*x)]*PolyLog[2, (-I)*c*(a + b*x)])/b - ((I/2)*Log[d*(a +
 b*x)]*PolyLog[2, I*c*(a + b*x)])/b - ((I/2)*PolyLog[3, (-I)*c*(a + b*x)])/
b + ((I/2)*PolyLog[3, I*c*(a + b*x)])/b
```


Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5209

```
Int[(ArcTan[v_]*Log[w_])/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[I/2, Int[(
Log[1 - I*v]*Log[w])/(a + b*x), x], x] - Dist[I/2, Int[(Log[1 + I*v]*Log[w]
)/(a + b*x), x], x] /; FreeQ[{a, b}, x] && LinearQ[v, x] && LinearQ[w, x] &
& EqQ[Simplify[D[v/(a + b*x), x]], 0] && EqQ[Simplify[D[w/(a + b*x), x]], 0
]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
```

```
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(c(a+bx))\log(d(a+bx))}{a+bx}\,dx = \frac{1}{2}i\int \frac{\log(d(a+bx))\log(1-ic(a+bx))}{a+bx}\,dx - \frac{1}{2}i\int \frac{\log(d(a+bx))\log(1+ic(a+bx))}{a+bx}\,dx$$
$$= \frac{1}{2}i\int \frac{\log(d(a+bx))\log(1-iac-ibcx)}{a+bx}\,dx - \frac{1}{2}i\int \frac{\log(d(a+bx))\log(1+iac+ibcx)}{a+bx}\,dx$$
$$= \frac{i\,\text{Subst}\left(\int \frac{\log(dx)\log\left(\frac{iabc+b(1-iac)}{b}-icx\right)}{x}\,dx, x, a+bx\right)}{2b} - \frac{i\,\text{Subst}\left(\int \frac{\log(dx)\log\left(\frac{-iabc+b\ldots}{b}\ldots\right)}{x}\ldots\right)}{2b}$$
$$= \frac{i\log(d(a+bx))\text{Li}_2(-ic(a+bx))}{2b} - \frac{i\log(d(a+bx))\text{Li}_2(ic(a+bx))}{2b} - \frac{i\,\text{Subst}(\ldots}{\ldots}$$
$$= \frac{i\log(d(a+bx))\text{Li}_2(-ic(a+bx))}{2b} - \frac{i\log(d(a+bx))\text{Li}_2(ic(a+bx))}{2b} - \frac{i\text{Li}_3(-ic\ldots}{2\ldots}$$

Mathematica [A] time = 0.137435, size = 79, normalized size = 0.78

$$\frac{i(\log(d(a+bx))\text{PolyLog}(2,-ic(a+bx)) - \log(d(a+bx))\text{PolyLog}(2,ic(a+bx)) - \text{PolyLog}(3,-ic(a+bx)) + \text{PolyLo}\ldots}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTan[c*(a + b*x)]*Log[d*(a + b*x)])/(a + b*x),x]
```

```
[Out] ((I/2)*(Log[d*(a + b*x)]*PolyLog[2, (-I)*c*(a + b*x)] - Log[d*(a + b*x)]*Po
lyLog[2, I*c*(a + b*x)] - PolyLog[3, (-I)*c*(a + b*x)] + PolyLog[3, I*c*(a
+ b*x)]))/b
```

Maple [F] time = 1.936, size = 0, normalized size = 0.

$$\int \frac{\arctan(c(bx+a))\ln(d(bx+a))}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c*(b*x+a))*ln(d*(b*x+a))/(b*x+a),x)
```

```
[Out] int(arctan(c*(b*x+a))*ln(d*(b*x+a))/(b*x+a),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*(b*x+a))*log(d*(b*x+a))/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(bcx+ac\right)\log\left(bdx+ad\right)}{bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*(b*x+a))*log(d*(b*x+a))/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*c*x + a*c)*log(b*d*x + a*d)/(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c*(b*x+a))*ln(d*(b*x+a))/(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\arctan\left((bx+a)c\right)\log\left((bx+a)d\right)}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*(b*x+a))*log(d*(b*x+a))/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arctan((b*x + a)*c)*log((b*x + a)*d)/(b*x + a), x)
```

3.147 $\int e^{c(a+bx)}\tan^{-1}(\sinh(ac+bcx))\,dx$

Optimal. Leaf size=48

$$\frac{e^{ac+bcx}\tan^{-1}(\sinh(c(a+bx)))}{bc}-\frac{\log\left(e^{2c(a+bx)}+1\right)}{bc}$$

```
[Out] (E^(a*c + b*c*x)*ArcTan[Sinh[c*(a + b*x)]])/(b*c) - Log[1 + E^(2*c*(a + b*x
))]/(b*c)
```

Rubi [A] time = 0.0753947, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2194, 5207, 2282, 12, 260}

$$\frac{e^{ac+bcx}\tan^{-1}(\sinh(c(a+bx)))}{bc}-\frac{\log\left(e^{2c(a+bx)}+1\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Sinh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTan[Sinh[c*(a + b*x)]])/(b*c) - Log[1 + E^(2*c*(a + b*x
))]/(b*c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5207

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] && !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int e^{c(a+bx)}\tan^{-1}(\sinh(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x\tan^{-1}(\sinh(x))\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int e^x\text{sech}(x)\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2x}{1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\sinh(c(a+bx)))}{bc} - \frac{2\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

Mathematica [A] time = 0.10271, size = 61, normalized size = 1.27

$$-\frac{\log\left(e^{2c(a+bx)}+1\right)+e^{c(a+bx)}\tan^{-1}\left(\frac{1}{2}e^{-c(a+bx)}-\frac{1}{2}e^{c(a+bx)}\right)}{bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Sinh[a*c + b*c*x]],x]
```

```
[Out] -((E^(c*(a + b*x))*ArcTan[1/(2*E^(c*(a + b*x))) - E^(c*(a + b*x))/2] + Log[
1 + E^(2*c*(a + b*x))])/(b*c))
```

Maple [C] time = 0.671, size = 1299, normalized size = 27.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x)
```

```
[Out] 2*a/b+1/2/c/b*exp(c*(b*x+a))*Pi-I/c/b*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-I)+I
/c/b*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+I)-ln(1+exp(2*c*(b*x+a)))/b/c+1/4/c/b
*Pi*csgn(I*(exp(c*(b*x+a))+I))^2*csgn(I*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x+a)
)-1/4/c/b*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))
+I)^2)^2*exp(c*(b*x+a))+1/2/c/b*Pi*csgn(I*(exp(c*(b*x+a))-I))*csgn(I*(exp(c
*(b*x+a))-I)^2)^2*exp(c*(b*x+a))+1/4/c/b*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*cs
gn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))-1/4/c/b*Pi*csgn
(I*(exp(c*(b*x+a))-I))^2*csgn(I*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))-1/4/c/
b*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*
x+a))*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))+1/4/c/b*Pi*csgn(I*(exp(c*(b*x+a)
)+I)^2)*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2
)*exp(c*(b*x+a))+1/4/c/b*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^3*ex
p(c*(b*x+a))+1/4/c/b*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))+1/4/c
/b*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))-1/4/c/b
*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I*exp(-c*
(b*x+a))*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))+1/4/c/b*Pi*csgn(exp(-c*(b*x
+a))*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(exp(-c*(b*x+a))
*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(exp(-c*(b*x+a))*(ex
p(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))+1/4/c/b*Pi*csgn(I*exp(-c*(b*x+a)))*csgn
(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))+1/4/c/b*Pi*csgn(I
```

```
*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))
-I)^2)^2*exp(c*(b*x+a))+1/4/c/b*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I
)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x+a))-1/4/c/b*Pi*c
sgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*
x+a))-I)^2)*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a)
)+I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))-1/2/c/b
*Pi*csgn(I*(exp(c*(b*x+a))+I))*csgn(I*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a)
)-1/4/c/b*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*
x+a))+I)^2)^2*exp(c*(b*x+a))
```

Maxima [A] time = 1.51609, size = 65, normalized size = 1.35

$$\frac{\arctan\left(\sinh\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}-\frac{\log\left(e^{\left(2\,bcx+2\,ac\right)}+1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(sinh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - log(e^(2*b*c*x + 2*a*c) +
 1)/(b*c)
```

Fricas [A] time = 1.9274, size = 192, normalized size = 4.

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\sinh\left(bcx+ac\right)\right)-\log\left(\frac{2\,\cosh\left(bcx+ac\right)}{\cosh\left(bcx+ac\right)-\sinh\left(bcx+ac\right)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] ((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(sinh(b*c*x + a*c)) - log(2*
cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c))))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(sinh(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11608, size = 88, normalized size = 1.83

$$\frac{\left(\arctan\left(\frac{1}{2}\,e^{\left(bcx+ac\right)}-\frac{1}{2}\,e^{\left(-bcx-ac\right)}\right)e^{\left(bcx\right)}-e^{\left(-ac\right)}\log\left(e^{\left(2\,bcx+2\,ac\right)}+1\right)\right)e^{\left(ac\right)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] (arctan(1/2*e^(b*c*x + a*c) - 1/2*e^(-b*c*x - a*c))*e^(b*c*x) - e^(-a*c)*lo
g(e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

3.148 $\int e^{c(a+bx)} \tan^{-1}(\cosh(ac+bcx))\,dx$

Optimal. Leaf size=103

$$-\frac{\left(1-\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}-\frac{\left(1+\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc}$$

```
[Out] (E^(a*c + b*c*x)*ArcTan[Cosh[c*(a + b*x)]])/(b*c) - ((1 - Sqrt[2])*Log[3 -
2*Sqrt[2] + E^(2*c*(a + b*x))])/(2*b*c) - ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
+ E^(2*c*(a + b*x))])/(2*b*c)
```

Rubi [A] time = 0.154934, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2194, 5207, 2282, 12, 1247, 632, 31}

$$-\frac{\left(1-\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}-\frac{\left(1+\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Cosh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTan[Cosh[c*(a + b*x)]])/(b*c) - ((1 - Sqrt[2])*Log[3 -
2*Sqrt[2] + E^(2*c*(a + b*x))])/(2*b*c) - ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
+ E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5207

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int e^{c(a+bx)}\tan^{-1}(\cosh(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x\tan^{-1}(\cosh(x))\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{e^x\sinh(x)}{1+\cosh^2(x)}\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2x(-1+x^2)}{1+6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc} - \frac{2\,\text{Subst}\left(\int \frac{x(-1+x^2)}{1+6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{-1+x}{1+6x+x^2}\,dx, x, e^{2ac+2bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc} - \frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{3-2\sqrt{2}+x}\,dx, x, e^{2ac+2bcx}\right)}{2bc} - \frac{(1 \ldots}{}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc} - \frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2ac+2bcx}\right)}{2bc} - \frac{\left(1+\sqrt{2}\right)\text{lo}\ldots}{}$$

Mathematica [C] time = 0.144337, size = 146, normalized size = 1.42

$$\frac{\text{RootSum}\left[\#1^4+6\#1^2+1\&, \frac{-7\#1^2\log\left(e^{c(a+bx)}-\#1\right)+7\#1^2ac+7\#1^2bcx-\log\left(e^{c(a+bx)}-\#1\right)+ac+bcx}{3\#1^2+1}\&\right]-4c(a+bx)+2e^{c(a+bx)}\tan^{-1}\left(\frac{1}{2}e^{-c\ldots}\right.}{2bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Cosh[a*c + b*c*x]],x]
```

```
[Out] (-4*c*(a + b*x) + 2*E^(c*(a + b*x))*ArcTan[(1 + E^(2*c*(a + b*x)))/(2*E^(c*
(a + b*x)))] + RootSum[1 + 6*#1^2 + #1^4 & , (a*c + b*c*x - Log[E^(c*(a + b
*x)) - #1] + 7*a*c*#1^2 + 7*b*c*x*#1^2 - 7*Log[E^(c*(a + b*x)) - #1]*#1^2)/
(1 + 3*#1^2) & ])/(2*b*c)
```

Maple [C] time = 0.801, size = 1375, normalized size = 13.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x)
```

```
[Out] -1/2*I/c/b*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1-2*I*exp(c*(b*x+a)))+1/4/c/b
*Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^3*exp(
c*(b*x+a))+1/4/c/b*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(
2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I*(-ex
p(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x
+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I*exp(-c*(b*x+
a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x+a))*(-exp(2*
c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I*exp(-c
*(b*x+a)))*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(I*exp(-c*(
b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a))-1/4/c/b*Pi
*csgn(exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^3*exp(c*(b*
x+a))+1/4/c/b*Pi*csgn(exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)
)))^3*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+
2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*
x+a))))^2*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(
b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/c/b*P
i*csgn(I*exp(-c*(b*x+a)))*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*c
sgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a
))+1/4/c/b*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))
))^3*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)
)))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c
*(b*x+a))-1/4/c/b*Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*
(b*x+a))))*csgn(exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*e
xp(c*(b*x+a))-1/4/c/b*Pi*csgn(exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(
c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(exp(-c*(b*x+a))*(exp(2*c*(b*x
+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/c/b*Pi*csgn(I*exp(-c*(b*x+
a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x+a))*(exp(2*c*
(b*x+a))+1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a))-1/2/c/b*ln(exp(2*c*(b*x+a))+
(1+2^(1/2))^2)*2^(1/2)+1/2/c/b*exp(c*(b*x+a))*Pi+1/2/c/b*ln(exp(2*c*(b*x+a)
)+(2^(1/2)-1)^2)*2^(1/2)+2*a/b-1/2/c/b*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)+1
/2*I/c/b*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))-1/2/c/b*l
n(exp(2*c*(b*x+a))+(2^(1/2)-1)^2)
```

Maxima [A] time = 1.53232, size = 177, normalized size = 1.72

$$\frac{\arctan\left(\cosh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}-\frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2\,bcx-2\,ac)}-3}{2\sqrt{2}+e^{(-2\,bcx-2\,ac)}+3}\right)}{2\,bc}-\frac{2\left(bcx+ac\right)}{bc}-\frac{\log\left(6\,e^{(-2\,bcx-2\,ac)}+e^{(-4\,bcx-4\,ac)}+1\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(cosh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 1/2*sqrt(2)*log(-(2*sqrt(
2) - e^(-2*b*c*x - 2*a*c) - 3)/(2*sqrt(2) + e^(-2*b*c*x - 2*a*c) + 3))/(b*c
) - 2*(b*c*x + a*c)/(b*c) - 1/2*log(6*e^(-2*b*c*x - 2*a*c) + e^(-4*b*c*x -
4*a*c) + 1)/(b*c)
```

Fricas [B] time = 1.99728, size = 591, normalized size = 5.74

$$\frac{2\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\cosh\left(bcx+ac\right)\right)+\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh\left(bcx+ac\right)^{2}-4\left(3\sqrt{2}-4\right)\cosh\left(bcx+ac\right)\sinh\left(\right.}{\cosh\left(bcx+ac\right)^{2}+\sinh\left(\right.}\right.}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(cosh(b*c*x + a*c)) +
sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(b*c*x + a*c)^2 - 4*(3*sqrt(2) - 4)*cos
h(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) - 3)*sinh(b*c*x + a*c)^2 +
2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 + 3)) - log(2*(co
sh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 + 3)/(cosh(b*c*x + a*c)^2 - 2*cosh(
b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)^2)))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(cosh(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12482, size = 208, normalized size = 2.02

$$\frac{\left(\sqrt{2}e^{(-ac)}\log\left(-\frac{2\sqrt{2}e^{(2ac)}-e^{(2bcx+4ac)}-3e^{(2ac)}}{2\sqrt{2}e^{(2ac)}+e^{(2bcx+4ac)}+3e^{(2ac)}}\right)+2\arctan\left(\frac{1}{2}e^{(bcx+ac)}+\frac{1}{2}e^{(-bcx-ac)}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(4bcx+4ac)}+6e^{(2bcx+2ac)}\right.}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*e^(-a*c)*log(-(2*sqrt(2)*e^(2*a*c) - e^(2*b*c*x + 4*a*c) - 3*e
^(2*a*c))/(2*sqrt(2)*e^(2*a*c) + e^(2*b*c*x + 4*a*c) + 3*e^(2*a*c))) + 2*ar
ctan(1/2*e^(b*c*x + a*c) + 1/2*e^(-b*c*x - a*c))*e^(b*c*x) - e^(-a*c)*log(e
^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

3.149 $\int e^{c(a+bx)} \tan^{-1}(\tanh(ac+bcx))\,dx$

Optimal. Leaf size=180

$$-\frac{\log\left(e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc}+\frac{\log\left(e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc}+\frac{\tan^{-1}\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc}-\frac{\tan^{-1}\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}bc}+$$

```
[Out] ArcTan[1 - Sqrt[2]*E^(a*c + b*c*x)]/(Sqrt[2]*b*c) - ArcTan[1 + Sqrt[2]*E^(a
*c + b*c*x)]/(Sqrt[2]*b*c) + (E^(a*c + b*c*x)*ArcTan[Tanh[c*(a + b*x)]])/(b
*c) - Log[1 + E^(2*c*(a + b*x)) - Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
+ Log[1 + E^(2*c*(a + b*x)) + Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
```

Rubi [A] time = 0.181831, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2194, 5207, 12, 2249, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc}+\frac{\log\left(e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc}+\frac{\tan^{-1}\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc}-\frac{\tan^{-1}\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}bc}+$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Tanh[a*c + b*c*x]],x]
```


```
[Out] ArcTan[1 - Sqrt[2]*E^(a*c + b*c*x)]/(Sqrt[2]*b*c) - ArcTan[1 + Sqrt[2]*E^(a
*c + b*c*x)]/(Sqrt[2]*b*c) + (E^(a*c + b*c*x)*ArcTan[Tanh[c*(a + b*x)]])/(b
*c) - Log[1 + E^(2*c*(a + b*x)) - Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
+ Log[1 + E^(2*c*(a + b*x)) + Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
```


Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5207

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{c(a+bx)}\tan^{-1}(\tanh(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x\tan^{-1}(\tanh(x))\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2e^{3x}}{1+e^{4x}}\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc} - \frac{2\,\text{Subst}\left(\int \frac{e^{3x}}{1+e^{4x}}\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^{ac+bcx}\right)}{bc} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\right.}{b}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^{ac+bcx}\right)}{2bc} - \frac{\text{Subst}\left(\int\right.}{}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc} - \frac{\log\left(1-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} + \frac{\log\left(1+\sqrt{2}\right.}{}$$
$$= \frac{\tan^{-1}\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} - \frac{\tan^{-1}\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)}{bc}$$

Mathematica [C] time = 0.117015, size = 89, normalized size = 0.49

$$\frac{\text{RootSum}\left[\#1^4+1\&, \frac{-\log\left(e^{c(a+bx)}-\#1\right)+ac+bcx}{\#1}\&\right] + 2e^{c(a+bx)}\tan^{-1}\left(\frac{e^{2c(a+bx)}-1}{e^{2c(a+bx)}+1}\right)}{2bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Tanh[a*c + b*c*x]],x]
```

```
[Out] (2*E^(c*(a + b*x))*ArcTan[(-1 + E^(2*c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))]
 + RootSum[1 + #1^4 & , (a*c + b*c*x - Log[E^(c*(a + b*x)) - #1])/#1 & ])/(
2*b*c)
```

Maple [C] time = 0.73, size = 1355, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x)
```

```
[Out] -1/4*I/c/b*ln(exp(c*(b*x+a))-1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)-1/4/c/b*ln(
exp(c*(b*x+a))-1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)-1/4/c/b*ln(exp(c*(b*x+a))
-1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)+1/4/c/b*ln(exp(c*(b*x+a))+1/2*2^(1/2)+1
/2*I*2^(1/2))*2^(1/2)+1/4/c/b*ln(exp(c*(b*x+a))+1/2*2^(1/2)-1/2*I*2^(1/2))*
2^(1/2)+1/4/c/b*exp(c*(b*x+a))*Pi-1/4/c/b*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*c
sgn(I*(exp(2*c*(b*x+a))-I))*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))
))*exp(c*(b*x+a))+1/4/c/b*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(
b*x+a))+I))*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a)
)+1/4/c/b*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))*csgn((1-I)*(
```

```
exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/c/b*Pi*csgn(
I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^2
*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*
x+a))+I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I*(exp(2*c*
(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*
c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp
(2*c*(b*x+a))))*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))*exp(c
*(b*x+a))+1/4/c/b*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*csgn
((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))-1/4/c/b*Pi
*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a
))))^2*exp(c*(b*x+a))+1/4/c/b*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I*(exp(2
*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/2*I/c/b*exp(c*(b*x+
a))*ln(exp(2*c*(b*x+a))+I)+1/4*I/c/b*ln(exp(c*(b*x+a))-1/2*2^(1/2)-1/2*I*2^
(1/2))*2^(1/2)-1/4/c/b*Pi*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a
))))^2*exp(c*(b*x+a))-1/2*I/c/b*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-I)-1/4/c
/b*Pi*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a)
)-1/4*I/c/b*ln(exp(c*(b*x+a))+1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)+1/4/c/b*Pi
*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^3*exp(c*(b*x+a))+1/4/c/b
*Pi*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^3*exp(c*(b*x+a))+
1/4/c/b*Pi*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^3*exp(c*(b
*x+a))-1/4/c/b*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^3*exp(c
*(b*x+a))+1/4*I/c/b*ln(exp(c*(b*x+a))+1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.52341, size = 225, normalized size = 1.25

$$\frac{\arctan\left(\tanh\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}-\frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{\left(bcx+ac\right)}\right)\right)}{2\,bc}-\frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{\left(bcx+ac\right)}\right)\right)}{2\,bc}+\frac{\sqrt{2}\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2) + 2*e^(b*c*x + a*c)))/(b*c) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2
)*(sqrt(2) - 2*e^(b*c*x + a*c)))/(b*c) + 1/4*sqrt(2)*log(sqrt(2)*e^(b*c*x +
 a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c) - 1/4*sqrt(2)*log(-sqrt(2)*e^(b*c*x
+ a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Fricas [B] time = 2.23642, size = 1152, normalized size = 6.4

$$4\sqrt{2}bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}e^{\left(bcx+ac\right)}+\sqrt{2}\sqrt{\sqrt{2}b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac{3}{4}}e^{\left(bcx+ac\right)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{\left(2\,bcx+2\,ac\right)}}bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}-1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*arctan(-sqrt(2)*b*c*(1/(b^4*c^4))^(1
/4)*e^(b*c*x + a*c) + sqrt(2)*sqrt(sqrt(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b
*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c))*b*c*(1/(b^4*
c^4))^(1/4) - 1) + 4*sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*arctan(-sqrt(2)*b*c*(1
/(b^4*c^4))^(1/4)*e^(b*c*x + a*c) + sqrt(2)*sqrt(-sqrt(2)*b^3*c^3*(1/(b^4*c
```

```
^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c
))*b*c*(1/(b^4*c^4))^(1/4) + 1) + sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*log(sqrt(
2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4))
+ e^(2*b*c*x + 2*a*c)) - sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*log(-sqrt(2)*b^3*c
^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b
*c*x + 2*a*c)) + 4*arctan((e^(2*b*c*x + 2*a*c) - 1)/(e^(2*b*c*x + 2*a*c) +
1))*e^(b*c*x + a*c))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(tanh(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41832, size = 344, normalized size = 1.91

$$-\frac{1}{4}\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{(-ac)}+2e^{(bcx)}\right)e^{(ac)}\right)e^{(-11ac)}}{bc}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{(-ac)}-2e^{(bcx)}\right)e^{(ac)}\right)e^{(-11ac)}}{bc}-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(-a*c) + 2*e^(b*c*x))*e^(a*c)
)*e^(-11*a*c)/(b*c) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(-a*c) - 2*e
^(b*c*x))*e^(a*c))*e^(-11*a*c)/(b*c) - sqrt(2)*e^(-11*a*c)*log(sqrt(2)*e^(b
*c*x - a*c) + e^(2*b*c*x) + e^(-2*a*c))/(b*c) + sqrt(2)*e^(-11*a*c)*log(-sq
rt(2)*e^(b*c*x - a*c) + e^(2*b*c*x) + e^(-2*a*c))/(b*c))*e^(11*a*c) - 1/4*(
4*pi*e^(b*c*x + a*c)*floor(1/4*(3*pi - 4*arctan(e^(-2*a*c)))/pi) - pi*e^(b*
c*x + a*c) + 4*arctan(e^(-2*b*c*x - 2*a*c))*e^(b*c*x + a*c))/(b*c)
```

3.150 $\int e^{c(a+bx)} \tan^{-1}(\coth(ac+bcx))\,dx$

Optimal. Leaf size=180

$$\frac{\log\left(e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc}-\frac{\log\left(e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc}-\frac{\tan^{-1}\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc}+\frac{\tan^{-1}\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}bc}+\frac{e^{ac+bc}}{}$$

```
[Out] -(ArcTan[1 - Sqrt[2]*E^(a*c + b*c*x)]/(Sqrt[2]*b*c)) + ArcTan[1 + Sqrt[2]*E
^(a*c + b*c*x)]/(Sqrt[2]*b*c) + (E^(a*c + b*c*x)*ArcTan[Coth[c*(a + b*x)]])
/(b*c) + Log[1 + E^(2*c*(a + b*x)) - Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*
c) - Log[1 + E^(2*c*(a + b*x)) + Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
```

Rubi [A] time = 0.179136, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2194, 5207, 12, 2249, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc}-\frac{\log\left(e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc}-\frac{\tan^{-1}\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc}+\frac{\tan^{-1}\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}bc}+\frac{e^{ac+bc}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Coth[a*c + b*c*x]],x]
```


```
[Out] -(ArcTan[1 - Sqrt[2]*E^(a*c + b*c*x)]/(Sqrt[2]*b*c)) + ArcTan[1 + Sqrt[2]*E
^(a*c + b*c*x)]/(Sqrt[2]*b*c) + (E^(a*c + b*c*x)*ArcTan[Coth[c*(a + b*x)]])
/(b*c) + Log[1 + E^(2*c*(a + b*x)) - Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*
c) - Log[1 + E^(2*c*(a + b*x)) + Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
```


Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5207

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{c(a+bx)}\tan^{-1}(\coth(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x\tan^{-1}(\coth(x))\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2e^{3x}}{-1-e^{4x}}\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} - \frac{2\,\text{Subst}\left(\int \frac{e^{3x}}{-1-e^{4x}}\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1-x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{1-x^2}{-1-x^4}\,dx, x, e^{ac+bcx}\right)}{bc} - \frac{\text{Subst}\left(\int \frac{1+x^2}{-1-x^4}\right.}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^{ac+bcx}\right)}{2bc} + \frac{\text{Subst}\left(\int \frac{}{1+}\right.}{}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} + \frac{\log\left(1-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} - \frac{\log\left(1+\sqrt{2}e^{ac+}\right.}{2\sqrt{}}$$
$$= -\frac{\tan^{-1}\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{\tan^{-1}\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc}$$

Mathematica [C] time = 0.111277, size = 89, normalized size = 0.49

$$\frac{\text{RootSum}\left[\#1^4+1\&, \frac{\log\left(e^{c(a+bx)}-\#1\right)-ac-bcx}{\#1}\&\right]+2e^{c(a+bx)}\tan^{-1}\left(\frac{e^{2c(a+bx)}+1}{e^{2c(a+bx)}-1}\right)}{2bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Coth[a*c + b*c*x]],x]
```

```
[Out] (2*E^(c*(a + b*x))*ArcTan[(1 + E^(2*c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))]
 + RootSum[1 + #1^4 & , (-(a*c) - b*c*x + Log[E^(c*(a + b*x)) - #1])/#1 & ]
)/(2*b*c)
```

Maple [C] time = 0.591, size = 1355, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x)
```

```
[Out] -1/4*I/c/b*ln(exp(c*(b*x+a))+1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)-1/4/c/b*Pi*
csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4/c/b*
Pi*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))+1
/4/c/b*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a)
)+1/4/c/b*Pi*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^3*exp(c*
(b*x+a))+1/4/c/b*ln(exp(c*(b*x+a))-1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)+1/4/c
/b*ln(exp(c*(b*x+a))-1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)-1/4/c/b*ln(exp(c*(b
*x+a))+1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)-1/4/c/b*ln(exp(c*(b*x+a))+1/2*2^(
1/2)-1/2*I*2^(1/2))*2^(1/2)+1/4/c/b*exp(c*(b*x+a))*Pi+1/4*I/c/b*ln(exp(c*(b
```

```
*x+a))-1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)+1/4/c/b*Pi*csgn(I*(exp(2*c*(b*x+a
))+I)/(exp(2*c*(b*x+a))-1))*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a
))-1))^2*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*
x+a))-1))*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*
x+a))-1/4/c/b*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*csgn((1-
I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/c/b*Pi*csg
n(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*csgn((1+I)*(exp(2*c*(b*x+a))
-I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I*(exp(2*c*(b*x+a)
)-I))*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/
4/c/b*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*
(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(
I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/2*I/c/b*exp
(c*(b*x+a))*ln(exp(2*c*(b*x+a))+I)+1/4/c/b*Pi*csgn(I*(exp(2*c*(b*x+a))+I))*
csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/c/b*
Pi*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1
/4/c/b*Pi*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*
x+a))+1/4/c/b*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I/(exp(2*c*(b*x+a))-1))*
csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))-1/4/c/b*Pi
*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(2*c*
(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/2*I/c/b*exp(c*(b*x+a))*l
n(exp(2*c*(b*x+a))-I)-1/4*I/c/b*ln(exp(c*(b*x+a))-1/2*2^(1/2)-1/2*I*2^(1/2)
)*2^(1/2)+1/4*I/c/b*ln(exp(c*(b*x+a))+1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.54574, size = 225, normalized size = 1.25

$$\frac{\arctan\left(\coth\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}+\frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{(bcx+ac)}\right)\right)}{2\,bc}+\frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{(bcx+ac)}\right)\right)}{2\,bc}-\frac{\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(coth(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2) + 2*e^(b*c*x + a*c)))/(b*c) + 1/2*sqrt(2)*arctan(-1/2*sqrt(2
)*(sqrt(2) - 2*e^(b*c*x + a*c)))/(b*c) - 1/4*sqrt(2)*log(sqrt(2)*e^(b*c*x +
 a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c) + 1/4*sqrt(2)*log(-sqrt(2)*e^(b*c*x
+ a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Fricas [B] time = 2.2317, size = 1153, normalized size = 6.41

$$-\frac{4\sqrt{2}bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}e^{(bcx+ac)}+\sqrt{2}\sqrt{\sqrt{2}b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac{3}{4}}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2\,bcx+2\,ac)}}bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*arctan(-sqrt(2)*b*c*(1/(b^4*c^4))^(
1/4)*e^(b*c*x + a*c) + sqrt(2)*sqrt(sqrt(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(
b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c))*b*c*(1/(b^4
*c^4))^(1/4) - 1) + 4*sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*arctan(-sqrt(2)*b*c*(
1/(b^4*c^4))^(1/4)*e^(b*c*x + a*c) + sqrt(2)*sqrt(-sqrt(2)*b^3*c^3*(1/(b^4*
```

```
c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*
c))*b*c*(1/(b^4*c^4))^(1/4) + 1) + sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*log(sqrt
(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4))
 + e^(2*b*c*x + 2*a*c)) - sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*log(-sqrt(2)*b^3*
c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*
b*c*x + 2*a*c)) - 4*arctan((e^(2*b*c*x + 2*a*c) + 1)/(e^(2*b*c*x + 2*a*c) -
 1))*e^(b*c*x + a*c))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(coth(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36402, size = 344, normalized size = 1.91

$$\frac{1}{4}\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{(-ac)}+2e^{(bcx)}\right)e^{(ac)}\right)e^{(-11\,ac)}}{bc}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{(-ac)}-2e^{(bcx)}\right)e^{(ac)}\right)e^{(-11\,ac)}}{bc}-\frac{\sqrt{2}e^{(-11}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(-a*c) + 2*e^(b*c*x))*e^(a*c))
*e^(-11*a*c)/(b*c) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(-a*c) - 2*e^
(b*c*x))*e^(a*c))*e^(-11*a*c)/(b*c) - sqrt(2)*e^(-11*a*c)*log(sqrt(2)*e^(b*
c*x - a*c) + e^(2*b*c*x) + e^(-2*a*c))/(b*c) + sqrt(2)*e^(-11*a*c)*log(-sqr
t(2)*e^(b*c*x - a*c) + e^(2*b*c*x) + e^(-2*a*c))/(b*c))*e^(11*a*c) + 1/4*(4
*pi*e^(b*c*x + a*c)*floor(1/4*(5*pi - 4*arctan(e^(-2*a*c)))/pi) - 3*pi*e^(b
*c*x + a*c) + 4*arctan(e^(-2*b*c*x - 2*a*c))*e^(b*c*x + a*c))/(b*c)
```

3.151 $\int e^{c(a+bx)} \tan^{-1}(\mathbf{sech}(ac+bcx))\,dx$

Optimal. Leaf size=103

$$\frac{\left(1-\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tan^{-1}(\text{sech}(c(a+bx)))}{bc}$$

```
[Out] (E^(a*c + b*c*x)*ArcTan[Sech[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
2*Sqrt[2] + E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
+ E^(2*c*(a + b*x))])/(2*b*c)
```

Rubi [A] time = 0.145389, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2194, 5207, 2282, 12, 1247, 632, 31}

$$\frac{\left(1-\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tan^{-1}(\text{sech}(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Sech[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTan[Sech[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
2*Sqrt[2] + E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
+ E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5207

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\tan^{-1}(\text{sech}(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\tan^{-1}(\text{sech}(x))\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\text{sech}(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int \frac{e^x\text{sech}(x)\tanh(x)}{1+\text{sech}^2(x)}\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\text{sech}(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int \frac{2x(-1+x^2)}{1+6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\text{sech}(c(a+bx)))}{bc}+\frac{2\,\text{Subst}\left(\int \frac{x(-1+x^2)}{1+6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\text{sech}(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int \frac{-1+x}{1+6x+x^2}\,dx, x, e^{2ac+2bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\text{sech}(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{3-2\sqrt{2}+x}\,dx, x, e^{2ac+2bcx}\right)}{2bc}+\frac{(1}{}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\text{sech}(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2ac+2bcx}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\text{lo}}{}\end{aligned}$$

Mathematica [C] time = 0.147943, size = 145, normalized size = 1.41

$$\frac{\text{RootSum}\left[\#1^4+6\#1^2+1\&,\frac{7\#1^2\log\left(e^{c(a+bx)}-\#1\right)-7\#1^2ac-7\#1^2bcx+\log\left(e^{c(a+bx)}-\#1\right)-ac-bcx}{3\#1^2+1}\&\right]+4c(a+bx)+2e^{c(a+bx)}\tan^{-1}\left(\frac{2e^{c(a+}}{e^{2c(a+b}}\right.}{2bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Sech[a*c + b*c*x]],x]
```

```
[Out] (4*c*(a + b*x) + 2*E^(c*(a + b*x))*ArcTan[(2*E^(c*(a + b*x)))/(1 + E^(2*c*(
a + b*x)))] + RootSum[1 + 6*#1^2 + #1^4 & , (-(a*c) - b*c*x + Log[E^(c*(a +
 b*x)) - #1] - 7*a*c*#1^2 - 7*b*c*x*#1^2 + 7*Log[E^(c*(a + b*x)) - #1]*#1^2
)/(1 + 3*#1^2) & ])/(2*b*c)
```

Maple [C] time = 0.609, size = 842, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x)
```

```
[Out] -1/2*I/c/b*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))-1/4/c/b
*Pi*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^3*
exp(c*(b*x+a))+1/4/c/b*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x
+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/c/b*P
i*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))+
1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/c/b*Pi*csg
n(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(I/(1+exp(2*c*(b*x+a))))*c
sgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))*exp(c*(
b*x+a))+1/4/c/b*Pi*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(I/
(1+exp(2*c*(b*x+a))))*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+ex
p(2*c*(b*x+a))))*exp(c*(b*x+a))+1/4/c/b*Pi*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*
exp(c*(b*x+a))))*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c
*(b*x+a))))^2*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I
*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*
x+a))-1/4/c/b*Pi*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c
*(b*x+a))))^3*exp(c*(b*x+a))-1/2/c/b*ln(exp(2*c*(b*x+a))+(2^(1/2)-1)^2)*2^(
1/2)+1/2/c/b*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)*2^(1/2)-2*a/b+1/2/c/b*ln(ex
p(2*c*(b*x+a))+(2^(1/2)-1)^2)+1/2/c/b*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)+1/
2*I/c/b*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1-2*I*exp(c*(b*x+a)))
```

Maxima [A] time = 1.57521, size = 228, normalized size = 2.21

$$\frac{\arctan\left(\operatorname{sech}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} - \frac{3\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2bcx+2ac)}-3}{2\sqrt{2}+e^{(2bcx+2ac)}+3}\right)}{8bc} + \frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2bcx-2ac)}-3}{2\sqrt{2}+e^{(-2bcx-2ac)}+3}\right)}{8bc} + \frac{\log\left(e^{(4bcx+4ac)}+6\right.}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(sech(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 3/8*sqrt(2)*log(-(2*sqrt(
2) - e^(2*b*c*x + 2*a*c) - 3)/(2*sqrt(2) + e^(2*b*c*x + 2*a*c) + 3))/(b*c)
+ 1/8*sqrt(2)*log(-(2*sqrt(2) - e^(-2*b*c*x - 2*a*c) - 3)/(2*sqrt(2) + e^(-
2*b*c*x - 2*a*c) + 3))/(b*c) + 1/2*log(e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x +
 2*a*c) + 1)/(b*c)
```

Fricas [B] time = 2.06773, size = 741, normalized size = 7.19

$$2\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\frac{2\left(\cosh(bcx+ac)+\sinh(bcx+ac)\right)}{\cosh(bcx+ac)^2+2\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^2+1}\right)+\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)c}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(2*(cosh(b*c*x + a*c)
+ sinh(b*c*x + a*c))/(cosh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)*sinh(b*c*x
+ a*c) + sinh(b*c*x + a*c)^2 + 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*
c*x + a*c)^2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2
*sqrt(2) + 3)*sinh(b*c*x + a*c)^2 + 2*sqrt(2) + 3)/(cosh(b*c*x + a*c)^2 + s
```

```
inh(b*c*x + a*c)^2 + 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2
 + 3)/(cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b
*c*x + a*c)^2)))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(sech(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13004, size = 208, normalized size = 2.02

$$-\frac{\left(\sqrt{2}e^{(-ac)}\log\left(-\frac{2\sqrt{2}e^{(2\,ac)}-e^{(2\,bcx+4\,ac)}-3\,e^{(2\,ac)}}{2\sqrt{2}e^{(2\,ac)}+e^{(2\,bcx+4\,ac)}+3\,e^{(2\,ac)}}\right)-2\,\arctan\left(\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)\right)e}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*e^(-a*c)*log(-(2*sqrt(2)*e^(2*a*c) - e^(2*b*c*x + 4*a*c) - 3*
e^(2*a*c))/(2*sqrt(2)*e^(2*a*c) + e^(2*b*c*x + 4*a*c) + 3*e^(2*a*c))) - 2*a
rctan(2/(e^(b*c*x + a*c) + e^(-b*c*x - a*c)))*e^(b*c*x) - e^(-a*c)*log(e^(4
*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

3.152 $\int e^{c(a+bx)} \tan^{-1}(\mathbf{csch}(ac + bcx))\,dx$

Optimal. Leaf size=47

$$\frac{\log\left(e^{2c(a+bx)}+1\right)}{bc}+\frac{e^{ac+bcx}\tan^{-1}(\operatorname{csch}(c(a+bx)))}{bc}$$

```
[Out] (E^(a*c + b*c*x)*ArcTan[Csch[c*(a + b*x)]])/(b*c) + Log[1 + E^(2*c*(a + b*x
))]/(b*c)
```

Rubi [A] time = 0.0793913, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2194, 5207, 2282, 12, 260}

$$\frac{\log\left(e^{2c(a+bx)}+1\right)}{bc}+\frac{e^{ac+bcx}\tan^{-1}(\operatorname{csch}(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Csch[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTan[Csch[c*(a + b*x)]])/(b*c) + Log[1 + E^(2*c*(a + b*x
))]/(b*c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5207

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] && !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int e^{c(a+bx)}\tan^{-1}(\text{csch}(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\tan^{-1}(\text{csch}(x))\,dx,x,ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx}\tan^{-1}(\text{csch}(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int e^x\text{sech}(x)\,dx,x,ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx}\tan^{-1}(\text{csch}(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int \frac{2x}{1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx}\tan^{-1}(\text{csch}(c(a+bx)))}{bc}+\frac{2\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx}\tan^{-1}(\text{csch}(c(a+bx)))}{bc}+\frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}
\end{aligned}
$$

Mathematica [A] time = 0.106656, size = 57, normalized size = 1.21

$$
\frac{\log\left(e^{2c(a+bx)}+1\right)+e^{c(a+bx)}\tan^{-1}\left(\frac{2e^{c(a+bx)}}{e^{2c(a+bx)}-1}\right)}{bc}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Csch[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*ArcTan[(2*E^(c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))] + Log
[1 + E^(2*c*(a + b*x))])/(b*c)
```

Maple [C] time = 0.513, size = 885, normalized size = 18.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x)
```

```
[Out] I/c/b*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-I)+1/4/c/b*Pi*csgn(I*(exp(c*(b*x+a))
-I)^2/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I/(exp(2*c*(b*
x+a))-1))*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a)
)-1/4/c/b*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(
2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/c/b*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*c
sgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1
))*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I*(exp(c*(b*x+a))+I))^2*csgn(I*(exp(c*(b*
x+a))+I)^2)*exp(c*(b*x+a))+1/2/c/b*Pi*csgn(I*(exp(c*(b*x+a))+I))*csgn(I*(ex
p(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)
*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))
-1))*exp(c*(b*x+a))+1/4/c/b*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(c*(
b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/c/b*Pi*csgn(I*(exp(
c*(b*x+a))-I))^2*csgn(I*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))-1/2/c/b*Pi*csg
n(I*(exp(c*(b*x+a))-I))*csgn(I*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))+1/4/c
/b*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))-1/4/c/b*Pi*csgn(I*(exp(
c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))+1/4/c/b*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*c
sgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/c/b*P
i*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))-2*a/b+
ln(1+exp(2*c*(b*x+a)))/b/c-I/c/b*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+I)
```

Maxima [A] time = 1.51905, size = 63, normalized size = 1.34

$$\frac{\arctan\left(\operatorname{csch}\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}+\frac{\log\left(e^{\left(2\,bcx+2\,ac\right)}+1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(csch(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(2*b*c*x + 2*a*c) +
 1)/(b*c)
```

Fricas [B] time = 1.90537, size = 343, normalized size = 7.3

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\frac{2\left(\cosh(bcx+ac)+\sinh(bcx+ac)\right)}{\cosh(bcx+ac)^{2}+2\,\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^{2}-1}\right)+\log\left(\frac{2\,\cosh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] ((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(2*(cosh(b*c*x + a*c) + sinh
(b*c*x + a*c))/(cosh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)
 + sinh(b*c*x + a*c)^2 - 1)) + log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) -
 sinh(b*c*x + a*c))))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int e^{bcx}\operatorname{atan}\left(\operatorname{csch}\left(ac+bcx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(csch(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*atan(csch(a*c + b*c*x)), x)
```

Giac [A] time = 1.15124, size = 89, normalized size = 1.89

$$\frac{\left(\arctan\left(\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}\right)e^{(bcx)}+e^{(-ac)}\log\left(e^{(2\,bcx+2\,ac)}+1\right)\right)e^{(ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] (arctan(2/(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))*e^(b*c*x) + e^(-a*c)*log(e^
(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

3.153 $\int \frac{\left(a+b\tan^{-1}(cx^n)\right)\left(d+e\log\left(fx^m\right)\right)}{x}\,dx$

Optimal. Leaf size=163

$$\frac{ibd\text{PolyLog}\left(2,-icx^n\right)}{2n}-\frac{ibd\text{PolyLog}\left(2,icx^n\right)}{2n}+\frac{ibe\log\left(fx^m\right)\text{PolyLog}\left(2,-icx^n\right)}{2n}-\frac{ibe\log\left(fx^m\right)\text{PolyLog}\left(2,icx^n\right)}{2n}-$$

```
[Out] a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) + ((I/2)*b*d*PolyLog[2, (-I)*c*x^n])/
n + ((I/2)*b*e*Log[f*x^m]*PolyLog[2, (-I)*c*x^n])/n - ((I/2)*b*d*PolyLog[2,
 I*c*x^n])/n - ((I/2)*b*e*Log[f*x^m]*PolyLog[2, I*c*x^n])/n - ((I/2)*b*e*m*
PolyLog[3, (-I)*c*x^n])/n^2 + ((I/2)*b*e*m*PolyLog[3, I*c*x^n])/n^2
```

Rubi [A] time = 0.571665, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2301, 6742, 5031, 4848, 2391, 5007, 5005, 2374, 6589}

$$\frac{ibd\text{PolyLog}\left(2,-icx^n\right)}{2n}-\frac{ibd\text{PolyLog}\left(2,icx^n\right)}{2n}+\frac{ibe\log\left(fx^m\right)\text{PolyLog}\left(2,-icx^n\right)}{2n}-\frac{ibe\log\left(fx^m\right)\text{PolyLog}\left(2,icx^n\right)}{2n}-$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```


```
[Out] a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) + ((I/2)*b*d*PolyLog[2, (-I)*c*x^n])/
n + ((I/2)*b*e*Log[f*x^m]*PolyLog[2, (-I)*c*x^n])/n - ((I/2)*b*d*PolyLog[2,
 I*c*x^n])/n - ((I/2)*b*e*Log[f*x^m]*PolyLog[2, I*c*x^n])/n - ((I/2)*b*e*m*
PolyLog[3, (-I)*c*x^n])/n^2 + ((I/2)*b*e*m*PolyLog[3, I*c*x^n])/n^2
```


Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5007

```
Int[(Log[(d_.)*(x_)^(m_.)]*(ArcTan[(c_.)*(x_)^(n_.)]*(b_.) + (a_)))/(x_), x
_Symbol] :> Dist[a, Int[Log[d*x^m]/x, x], x] + Dist[b, Int[(Log[d*x^m]*ArcT
an[c*x^n])/x, x], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

Rule 5005

```
Int[(ArcTan[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Dis
t[I/2, Int[(Log[d*x^m]*Log[1 - I*c*x^n])/x, x], x] - Dist[I/2, Int[(Log[d*x
^m]*Log[1 + I*c*x^n])/x, x], x] /; FreeQ[{c, d, m, n}, x]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\tan^{-1}\left(cx^n\right)\right)\left(d+e\log\left(fx^m\right)\right)}{x}\,dx &= \int \left(\frac{d\left(a+b\tan^{-1}\left(cx^n\right)\right)}{x}+\frac{e\left(a+b\tan^{-1}\left(cx^n\right)\right)\log\left(fx^m\right)}{x}\right)dx\\
&= d\int \frac{a+b\tan^{-1}\left(cx^n\right)}{x}\,dx+e\int \frac{\left(a+b\tan^{-1}\left(cx^n\right)\right)\log\left(fx^m\right)}{x}\,dx\\
&= (ae)\int \frac{\log\left(fx^m\right)}{x}\,dx+(be)\int \frac{\tan^{-1}\left(cx^n\right)\log\left(fx^m\right)}{x}\,dx+\frac{d\,\mathrm{Subst}\left(\int \frac{a}{\ldots}\right.}{\ldots}\\
&= ad\log(x)+\frac{ae\log^2\left(fx^m\right)}{2m}+\frac{1}{2}(ibe)\int \frac{\log\left(fx^m\right)\log\left(1-icx^n\right)}{x}\,dx-\frac{1}{2}(i\ldots\\
&= ad\log(x)+\frac{ae\log^2\left(fx^m\right)}{2m}+\frac{ibd\mathrm{Li}_2\left(-icx^n\right)}{2n}+\frac{ibe\log\left(fx^m\right)\mathrm{Li}_2\left(-icx^n\right)}{2n}\ldots\\
&= ad\log(x)+\frac{ae\log^2\left(fx^m\right)}{2m}+\frac{ibd\mathrm{Li}_2\left(-icx^n\right)}{2n}+\frac{ibe\log\left(fx^m\right)\mathrm{Li}_2\left(-icx^n\right)}{2n}\ldots
\end{aligned}$$

Mathematica [C] time = 0.323576, size = 116, normalized size = 0.71

$$\frac{bcx^n\left(d+e\log\left(fx^m\right)\right)\mathrm{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},-c^2x^{2n}\right)}{n}-\frac{bcemx^n\mathrm{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2},\frac{1}{2},\ldots\right.\right.}{n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTan[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] -((b*c*e*m*x^n*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, -(c^2
*x^(2*n))])/n^2) + (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, -(
c^2*x^(2*n))]*(d + e*Log[f*x^m]))/n + (a*Log[x]*(2*d - e*m*Log[x] + 2*e*Log
```

```
[f*x^m]))/2
```

Maple [C] time = 0.422, size = 896, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^n))*(d+e*ln(f*x^m))/x,x)
```

```
[Out] -1/4/n*Pi*dilog(1-I*c*x^n)*b*e*csgn(I*f*x^m)^3+1/n*ln(f)*ln(x^n)*a*e-1/2*I*
b*e*m*polylog(3,-I*c*x^n)/n^2+1/4/n*Pi*dilog(1+I*c*x^n)*b*e*csgn(I*x^m)*csg
n(I*f*x^m)*csgn(I*f)-1/4/n*Pi*dilog(1-I*c*x^n)*b*e*csgn(I*x^m)*csgn(I*f*x^m
)*csgn(I*f)+1/2*I*e*b/n*ln(-I*(-c*x^n+I))*ln(-I*c*x^n)*m*ln(x)+1/2*I/n*Pi*l
n(x^n)*a*e*csgn(I*x^m)*csgn(I*f*x^m)^2+1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*f*x^m)
^2*csgn(I*f)-1/2*I/n*dilog(1-I*c*x^n)*b*d-1/4/n*Pi*dilog(1+I*c*x^n)*b*e*csg
n(I*x^m)*csgn(I*f*x^m)^2+1/4/n*Pi*dilog(1-I*c*x^n)*b*e*csgn(I*x^m)*csgn(I*f
*x^m)^2+1/4/n*Pi*dilog(1-I*c*x^n)*b*e*csgn(I*f*x^m)^2*csgn(I*f)-1/2*I*e*b/n
*dilog(-I*c*x^n)*ln(x^m)-1/2*I/n*ln(f)*dilog(1-I*c*x^n)*b*e+1/2*I/n*ln(f)*d
ilog(1+I*c*x^n)*b*e-1/2*I*e*b/n*dilog(-I*(c*x^n+I))*ln(x^m)+1/2*I*e*b*ln(-I
*(c*x^n+I))*ln(x)^2*m+1/2*I*e*b/n*m*ln(x)*polylog(2,-I*c*x^n)-1/2*I*e*b*ln(
1-I*c*x^n)*ln(x)^2*m+1/2*I*e*b*ln(1-I*c*x^n)*ln(x^m)*ln(x)+1/4/n*Pi*dilog(1
+I*c*x^n)*b*e*csgn(I*f*x^m)^3-1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*f*x^m)^3-1/2*I*
e*b/n*ln(-I*(-c*x^n+I))*ln(-I*c*x^n)*ln(x^m)+1/2*e*a/m*ln(x^m)^2+1/n*ln(x^n
)*a*d-1/4/n*Pi*dilog(1+I*c*x^n)*b*e*csgn(I*f*x^m)^2*csgn(I*f)-1/2*I*e*b*ln(
1+I*c*x^n)*ln(x^m)*ln(x)-1/2*I*e*b*ln(-I*(-c*x^n+I))*ln(x)^2*m+1/2*I*e*b*ln
(-I*(-c*x^n+I))*ln(x^m)*ln(x)-1/2*I*e*b*ln(x^m)*ln(-I*(c*x^n+I))*ln(x)+1/2*
I*e*b*ln(1+I*c*x^n)*ln(x)^2*m+1/2*I/n*dilog(1+I*c*x^n)*b*d+1/2*I*e*b/n*dilo
g(-I*(c*x^n+I))*m*ln(x)-1/2*I*e*b/n*m*ln(x)*polylog(2,I*c*x^n)+1/2*I*e*b/n*
dilog(-I*c*x^n)*m*ln(x)-1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*x^m)*csgn(I*f*x^m)*cs
gn(I*f)+1/2*I*b*e*m*polylog(3,I*c*x^n)/n^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ae\log\left(fx^m\right)^2}{2\,m} + ad\log(x) - \frac{1}{2}\left(bem\log(x)^2 - 2\,be\log(x)\log(x^m) - 2\left(be\log\left(f\right) + bd\right)\log(x)\right)\arctan(cx^n) - \int -\frac{bcem}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/2*(b*e*m*log(x)^2 - 2*b*e*log(x)*lo
g(x^m) - 2*(b*e*log(f) + b*d)*log(x))*arctan(c*x^n) - integrate(-1/2*(b*c*e
*m*n*x^n*log(x)^2 - 2*b*c*e*n*x^n*log(x)*log(x^m) - 2*(b*c*e*log(f) + b*c*d
)*n*x^n*log(x))/(c^2*x*x^(2*n) + x), x)
```

Fricas [C] time = 2.17444, size = 711, normalized size = 4.36

$$\frac{2\,aemn^2\log(x)^2 + 2i\,bem\text{polylog}(3, i\,cx^n) - 2i\,bem\text{polylog}(3, -i\,cx^n) + 2\left(bemn^2\log(x)^2 + 2\left(ben^2\log\left(f\right) + bdn^2\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*e*m*n^2*log(x)^2 + 2*I*b*e*m*polylog(3, I*c*x^n) - 2*I*b*e*m*polyl
og(3, -I*c*x^n) + 2*(b*e*m*n^2*log(x)^2 + 2*(b*e*n^2*log(f) + b*d*n^2)*log(
x))*arctan(c*x^n) + (-2*I*b*e*m*n*log(x) - 2*I*b*e*n*log(f) - 2*I*b*d*n)*di
log(I*c*x^n) + (2*I*b*e*m*n*log(x) + 2*I*b*e*n*log(f) + 2*I*b*d*n)*dilog(-I
*c*x^n) + (I*b*e*m*n^2*log(x)^2 + (2*I*b*e*n^2*log(f) + 2*I*b*d*n^2)*log(x)
)*log(I*c*x^n + 1) + (-I*b*e*m*n^2*log(x)^2 + (-2*I*b*e*n^2*log(f) - 2*I*b*
d*n^2)*log(x))*log(-I*c*x^n + 1) + 4*(a*e*n^2*log(f) + a*d*n^2)*log(x))/n^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**n))*(d+e*ln(f*x**m))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx^n) + a)\left(e \log\left(fx^m\right) + d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
             fi;
             #both result and optimal complex
             if leaf_count_result<=2*leaf_count_optimal then
                return "A";
             else
                return "B";
             end if
          else #result contains complex but optimal is not
             if debug then
                print("result contains complex but optimal is not");
             fi;
             return "C";
          end if
       else # result do not contain complex
            # this assumes optimal do not as well
            if debug then
                print("result do not contain complex, this assumes optimal do
    not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
               if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
               fi;
               return "A";
            else
               if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
               fi;
               return "B";
            end if
       end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
    (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```