Computer algebra independent integration tests

5-Inverse-trig-functions/5.3-Inverse-tangent/5.3.6-Exponentials-of-inverse-tangent

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [385]. This is test number [152].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (385)	% 0. (0)
Mathematica	% 95.58 (368)	% 4.42 (17)
Maple	% 52.73 (203)	% 47.27 (182)
Maxima	% 28.83 (111)	% 71.17 (274)
Fricas	% 74.29 (286)	% 25.71 (99)
Sympy	% 17.14 (66)	% 82.86 (319)
Giac	% 29.35 (113)	% 70.65 (272)

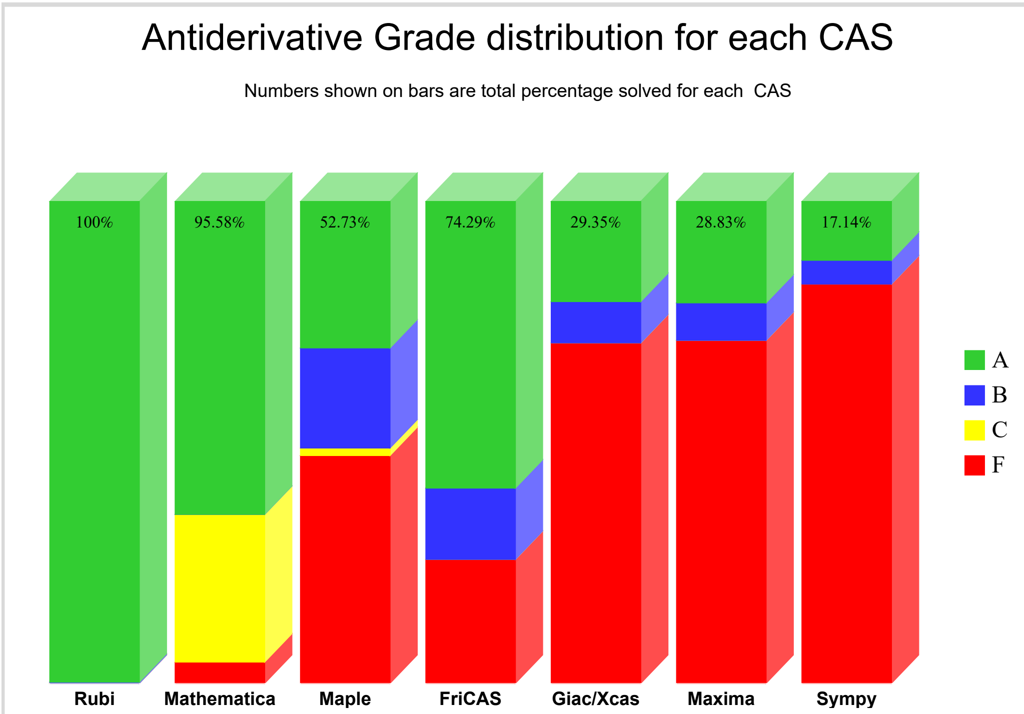
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

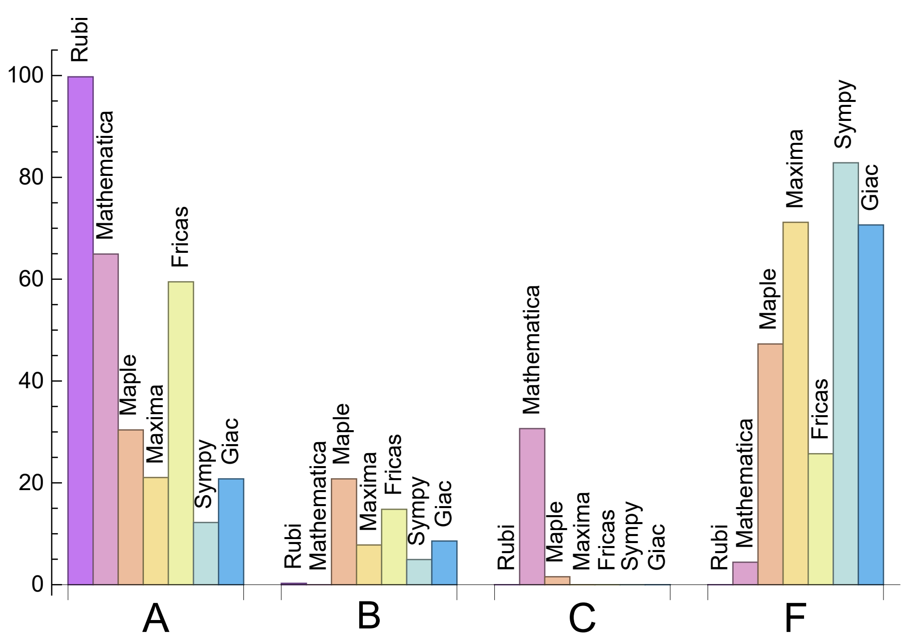
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.74	0.26	0.	0.
Mathematica	64.94	0.	30.65	4.42
Maple	30.39	20.78	1.56	47.27
Maxima	21.04	7.79	0.	71.17
Fricas	59.48	14.81	0.	25.71
Sympy	12.21	4.94	0.	82.86
Giac	20.78	8.57	0.	70.65

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	142.58	1.01	95.	1.
Mathematica	0.07	85.12	0.8	76.5	0.81
Maple	0.09	284.74	2.48	87.	1.42
Maxima	1.29	142.37	1.89	81.	1.61
Fricas	2.01	457.57	3.28	310.	2.48
Sympy	5.88	331.44	5.88	42.5	0.98
Giac	1.13	131.27	1.95	92.	1.6

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {340, 344, 345, 346}

Mathematica {6, 7, 8, 9, 10, 14, 23, 24, 25, 26, 30, 39, 40, 41, 42, 43, 47, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 140, 141, 142, 143, 154, 157, 162, 163, 164, 165, 167, 168, 169, 170, 175, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 202, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 240, 247, 248, 249, 250, 251, 252, 262, 263, 264, 265, 266, 276, 277, 278, 279, 280, 290, 291, 292, 293, 294, 302, 304, 307, 309, 311, 313, 316, 318, 329, 331, 334, 336, 339, 340, 343, 344, 345, 346, 347, 365}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
```

```
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

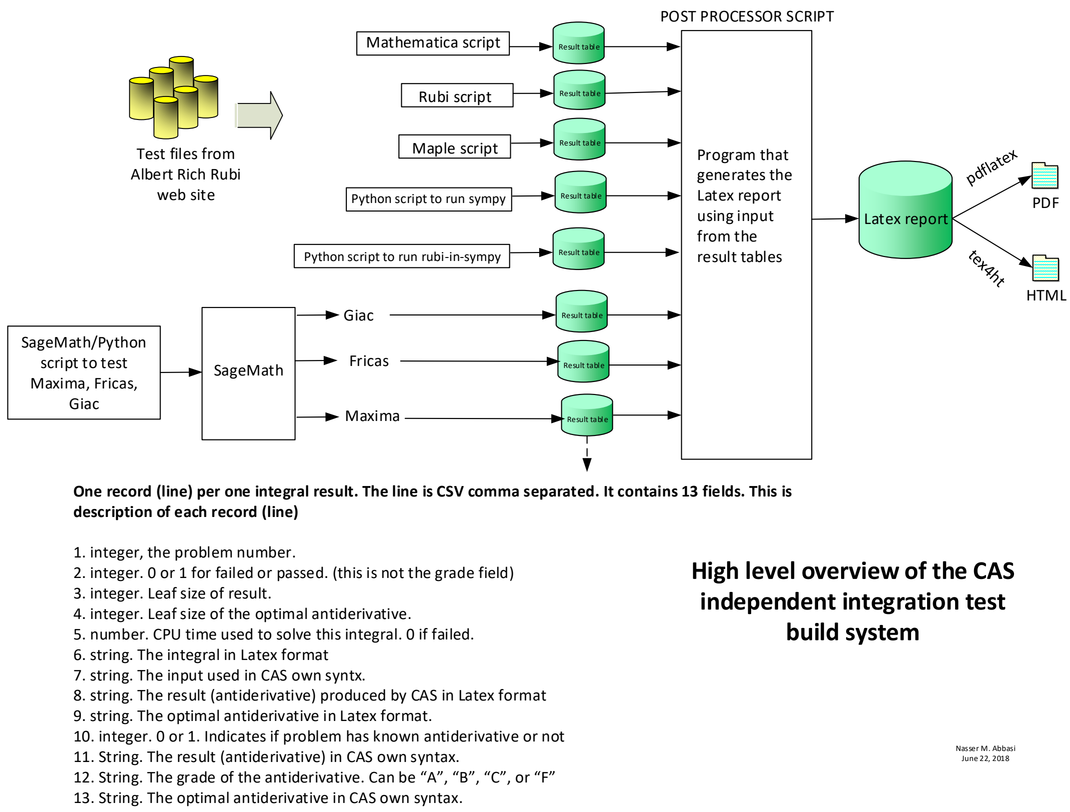
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

B grade: { 344 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 134, 135, 136, 137, 138, 139, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 365, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

B grade: { }

C grade: { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 140, 141, 142, 143, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 248, 249, 250, 251, 252, 263, 264, 265, 266, 277, 278, 279, 280, 291, 292, 293, 294, 302, 311, 343, 347 }

F grade: { 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 236, 364, 366, 367, 368, 369, 370}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 44, 45, 46, 47, 48, 49, 50, 51, 140, 141, 165, 166, 202, 203, 248, 249, 250, 251, 252, 256, 257, 258, 263, 264, 265, 266, 270, 271, 272, 277, 278, 279, 280, 284, 285, 286, 291, 292, 293, 294, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 310, 312, 314, 315, 316, 317, 318, 319, 321, 323, 324, 325, 326, 327, 328, 330, 332, 333, 335, 343, 347, 372, 373, 374, 375, 376, 377, 378, 380, 381, 382, 383, 384, 385 }

B grade: { 6, 35, 36, 37, 38, 39, 40, 41, 42, 43, 52, 53, 54, 55, 56, 57, 58, 59, 60, 162, 163, 164, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 307, 309, 311, 313, 320, 322, 329, 331, 334, 336, 379 }

C grade: { 134, 135, 136, 137, 138, 139 }

F grade: { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 253, 254, 255, 259, 260, 261, 262, 267, 268, 269, 273, 274, 275, 276, 281, 282, 283, 287, 288, 289, 290, 295, 296, 297, 337, 338, 339, 340, 341, 342, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 191, 192, 193, 198, 199, 200, 201, 202, 203, 211, 248, 263, 277, 291, 301, 303, 304, 306, 307, 308, 315, 317, 322, 323, 325, 326, 333, 335, 343, 372, 383, 384

B grade: { 53, 171, 172, 173, 174, 175, 176, 177, 178, 179, 189, 190, 204, 205, 206, 207, 208, 209, 210, 302, 305, 309, 319, 320, 321, 327, 375, 376, 377, 385 }

C grade: { }

F grade: { 39, 40, 41, 42, 43, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 180, 181, 182, 183, 184, 185, 186, 187, 188, 194, 195, 196, 197, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 310, 311, 312, 313, 314, 316, 318, 324, 328, 329, 330, 331, 332, 334, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374, 378, 379, 380, 381, 382 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 162, 163, 164, 165, 166, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 189, 190, 191, 192, 193, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 216, 217, 218, 219, 221, 222, 223, 226, 227, 228, 229, 231, 232, 233, 248, 249, 250, 251, 252, 256, 257, 258, 263, 264, 265, 266, 270, 271, 272, 277, 278, 279, 280, 284, 285, 286, 291, 292, 293, 294, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 319, 320, 321, 322, 325, 326, 327, 328, 329, 330, 331, 334, 335, 336, 343, 347, 372, 373, 374, 377, 378 }

B grade: { 6, 7, 23, 24, 32, 39, 40, 56, 57, 65, 75, 93, 102, 111, 112, 167, 168, 169, 170, 185, 186, 187, 188, 194, 195, 196, 197, 212, 213, 214, 215, 220, 224, 225, 230, 234, 235, 310, 311, 312, 313, 314, 315, 316, 317, 318, 323, 324, 332, 333, 375, 376, 379, 380, 381, 384, 385 }

C grade: { }

F grade: { 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 253, 254, 255, 259, 260, 261, 262, 267, 268, 269, 273, 274, 275, 276, 281, 282, 283, 287, 288, 289, 290, 295, 296, 297, 337, 338, 339, 340, 341, 342, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 382, 383 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 27, 28, 29, 30, 31, 32, 33, 34, 44, 45, 46, 47, 48, 49, 50, 51, 141, 166, 175, 202, 301, 303, 305, 306, 308, 319, 321, 377, 378 }

B grade: { 136, 137, 171, 172, 173, 174, 176, 177, 198, 199, 200, 201, 203, 204, 323, 324, 326, 376, 379 }

C grade: { }

F grade: { 19, 20, 21, 22, 23, 24, 25, 26, 35, 36, 37, 38, 39, 40, 41, 42, 43, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 302, 304, 307, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 322, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 380, 381, 382, 383, 384, 385 }

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A grade: { 1, 2, 3, 4, 5, 11, 12, 13, 14, 15, 16, 17, 18, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 49, 51, 162, 163, 164, 165, 166, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 189, 190, 191, 192, 193, 194, 202, 207, 208, 209, 248, 263, 277, 291, 301, 302, 303, 305, 306, 308, 309, 313, 316, 319, 321, 322, 323, 324, 325, 326, 331, 334, 343, 372, 373, 377 }

B grade: { 6, 7, 8, 9, 10, 39, 44, 45, 46, 47, 48, 50, 183, 184, 198, 199, 200, 201, 203, 204, 205, 206, 210, 211, 320, 327, 329, 336, 374, 375, 376, 378, 379 }

C grade: { }

F grade: { 19, 20, 21, 22, 23, 24, 25, 26, 40, 41, 42, 43, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 167, 168, 169, 170, 185, 186, 187, 188, 195, 196, 197, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 304, 307, 310, 311, 312, 314, 315, 317, 318, 328, 330, 332, 333, 335, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 380, 381, 382, 383, 384, 385 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	64	128	151	169	138	111
normalized size	1	1.	0.57	1.13	1.34	1.5	1.22	0.98
time (sec)	N/A	0.088	0.053	0.071	0.984	1.813	5.702	1.128

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	56	109	126	144	119	99
normalized size	1	1.	0.62	1.21	1.4	1.6	1.32	1.1
time (sec)	N/A	0.064	0.039	0.072	0.991	1.718	5.414	1.115

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	46	89	100	123	75	85
normalized size	1	1.	0.61	1.19	1.33	1.64	1.	1.13
time (sec)	N/A	0.047	0.031	0.074	1.016	1.682	3.825	1.13

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	38	69	73	101	51	73
normalized size	1	1.	0.9	1.64	1.74	2.4	1.21	1.74
time (sec)	N/A	0.019	0.027	0.067	0.982	1.684	4.434	1.114

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	48	46	77	68	55
normalized size	1	1.	0.9	1.66	1.59	2.66	2.34	1.9
time (sec)	N/A	0.009	0.015	0.053	1.024	1.616	1.336	1.114

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	25	25	29	48	45	143	53	93
normalized size	1	1.	1.16	1.92	1.8	5.72	2.12	3.72
time (sec)	N/A	0.037	0.014	0.063	1.016	1.753	2.895	1.145

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	38	38	47	34	42	154	26	103
normalized size	1	1.	1.24	0.89	1.11	4.05	0.68	2.71
time (sec)	N/A	0.038	0.03	0.063	1.	1.753	2.525	1.177

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	57	53	68	197	48	209
normalized size	1	1.	0.9	0.84	1.08	3.13	0.76	3.32
time (sec)	N/A	0.05	0.04	0.071	0.977	1.68	3.54	1.136

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	70	75	93	220	75	221
normalized size	1	1.	0.78	0.83	1.03	2.44	0.83	2.46
time (sec)	N/A	0.069	0.051	0.067	0.991	1.699	3.864	1.144

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	76	97	119	242	122	324
normalized size	1	1.	0.67	0.86	1.05	2.14	1.08	2.87
time (sec)	N/A	0.09	0.057	0.07	1.049	1.722	5.449	1.155

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	55	76	112	41	65
normalized size	1	1.	1.	1.15	1.58	2.33	0.85	1.35
time (sec)	N/A	0.036	0.02	0.038	1.477	1.498	0.348	1.103

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	47	63	88	31	53
normalized size	1	1.	1.	1.21	1.62	2.26	0.79	1.36
time (sec)	N/A	0.028	0.013	0.036	1.526	1.593	0.359	1.091

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	38	51	69	22	41
normalized size	1	1.	1.	1.31	1.76	2.38	0.76	1.41
time (sec)	N/A	0.019	0.01	0.036	1.476	1.598	0.165	1.087

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	19	19	30	30	38	45	12	22
normalized size	1	1.	1.58	1.58	2.	2.37	0.63	1.16
time (sec)	N/A	0.009	0.009	0.036	1.508	1.546	0.311	1.104

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	23	28	39	10	20
normalized size	1	1.	1.	1.77	2.15	3.	0.77	1.54
time (sec)	N/A	0.019	0.006	0.041	1.506	1.685	0.408	1.132

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	42	69	20	31
normalized size	1	1.	1.	1.31	1.62	2.65	0.77	1.19
time (sec)	N/A	0.024	0.009	0.043	1.487	1.8	0.428	1.097

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	45	57	97	29	43
normalized size	1	1.	1.	1.25	1.58	2.69	0.81	1.19
time (sec)	N/A	0.027	0.01	0.041	1.514	1.599	0.46	1.117

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	55	69	119	39	57
normalized size	1	1.	1.	1.15	1.44	2.48	0.81	1.19
time (sec)	N/A	0.03	0.012	0.043	1.503	1.713	0.508	1.09

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	80	143	170	220	0	0
normalized size	1	1.	0.58	1.04	1.24	1.61	0.	0.
time (sec)	N/A	0.621	0.06	0.079	1.011	1.726	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	63	123	144	201	0	0
normalized size	1	1.	0.62	1.21	1.41	1.97	0.	0.
time (sec)	N/A	0.568	0.051	0.073	1.02	1.674	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	54	104	119	176	0	0
normalized size	1	1.	0.59	1.13	1.29	1.91	0.	0.
time (sec)	N/A	0.326	0.059	0.071	1.037	1.715	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	42	81	89	146	0	0
normalized size	1	1.	0.7	1.35	1.48	2.43	0.	0.
time (sec)	N/A	0.045	0.032	0.054	0.996	1.623	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	51	51	55	77	82	250	0	0
normalized size	1	1.	1.08	1.51	1.61	4.9	0.	0.
time (sec)	N/A	0.663	0.039	0.064	1.	1.721	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	61	80	84	247	0	0
normalized size	1	1.	0.97	1.27	1.33	3.92	0.	0.
time (sec)	N/A	0.56	0.044	0.069	1.051	1.792	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	79	105	112	294	0	0
normalized size	1	1.	0.86	1.14	1.22	3.2	0.	0.
time (sec)	N/A	0.601	0.077	0.072	1.05	1.712	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	89	141	138	316	0	0
normalized size	1	1.	0.76	1.21	1.18	2.7	0.	0.
time (sec)	N/A	0.616	0.075	0.07	1.053	1.734	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	70	104	177	58	85
normalized size	1	1.	1.	1.08	1.6	2.72	0.89	1.31
time (sec)	N/A	0.045	0.043	0.046	1.538	1.498	0.5	1.096

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	60	92	147	46	72
normalized size	1	1.	1.	1.13	1.74	2.77	0.87	1.36
time (sec)	N/A	0.04	0.031	0.046	1.492	1.621	0.491	1.117

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	53	81	127	37	61
normalized size	1	1.	1.	1.18	1.8	2.82	0.82	1.36
time (sec)	N/A	0.026	0.024	0.045	1.507	1.577	0.484	1.118

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	31	31	42	41	61	95	26	35
normalized size	1	1.	1.35	1.32	1.97	3.06	0.84	1.13
time (sec)	N/A	0.014	0.02	0.043	1.521	1.539	0.422	1.125

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	30	49	15	19
normalized size	1	1.	1.	0.94	1.88	3.06	0.94	1.19
time (sec)	N/A	0.022	0.01	0.047	1.526	1.601	0.475	1.108

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	45	72	131	37	50
normalized size	1	1.	1.	1.18	1.89	3.45	0.97	1.32
time (sec)	N/A	0.031	0.024	0.049	1.534	1.66	0.609	1.107

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	60	93	171	51	63
normalized size	1	1.	1.	1.15	1.79	3.29	0.98	1.21
time (sec)	N/A	0.035	0.036	0.048	1.534	1.712	0.685	1.095

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	68	104	189	63	80
normalized size	1	1.	1.	1.1	1.68	3.05	1.02	1.29
time (sec)	N/A	0.039	0.035	0.052	1.512	1.784	0.758	1.103

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	56	187	103	146	0	99
normalized size	1	1.	0.62	2.08	1.14	1.62	0.	1.1
time (sec)	N/A	0.066	0.043	0.156	1.496	1.804	0.	1.119

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	46	168	80	124	0	85
normalized size	1	1.	0.61	2.24	1.07	1.65	0.	1.13
time (sec)	N/A	0.047	0.031	0.077	1.549	1.671	0.	1.108

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	38	152	57	103	0	73
normalized size	1	1.	0.9	3.62	1.36	2.45	0.	1.74
time (sec)	N/A	0.02	0.026	0.069	1.563	1.763	0.	1.134

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	97	34	78	0	57
normalized size	1	1.	0.9	3.34	1.17	2.69	0.	1.97
time (sec)	N/A	0.009	0.013	0.058	1.56	1.665	0.	1.106

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	25	25	29	121	0	143	0	92
normalized size	1	1.	1.16	4.84	0.	5.72	0.	3.68
time (sec)	N/A	0.037	0.014	0.078	0.	1.783	0.	1.134

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	38	38	47	194	0	153	0	0
normalized size	1	1.	1.24	5.11	0.	4.03	0.	0.
time (sec)	N/A	0.037	0.026	0.095	0.	1.662	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	57	219	0	196	0	0
normalized size	1	1.	0.9	3.48	0.	3.11	0.	0.
time (sec)	N/A	0.051	0.041	0.079	0.	1.814	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	70	237	0	221	0	0
normalized size	1	1.	0.78	2.63	0.	2.46	0.	0.
time (sec)	N/A	0.07	0.048	0.081	0.	1.726	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	76	259	0	243	0	0
normalized size	1	1.	0.67	2.29	0.	2.15	0.	0.
time (sec)	N/A	0.089	0.057	0.082	0.	1.783	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	55	59	112	41	108
normalized size	1	1.	1.	1.12	1.2	2.29	0.84	2.2
time (sec)	N/A	0.035	0.018	0.043	1.003	1.632	0.36	1.135

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	47	47	88	31	92
normalized size	1	1.	1.	1.18	1.18	2.2	0.78	2.3
time (sec)	N/A	0.03	0.013	0.046	1.037	1.537	0.36	1.096

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	38	38	69	22	78
normalized size	1	1.	1.	1.27	1.27	2.3	0.73	2.6
time (sec)	N/A	0.021	0.01	0.041	1.088	1.518	0.175	1.111

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	20	20	30	30	22	45	14	90
normalized size	1	1.	1.5	1.5	1.1	2.25	0.7	4.5
time (sec)	N/A	0.01	0.009	0.041	1.019	1.523	0.33	1.109

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	23	16	39	10	63
normalized size	1	1.	1.	1.64	1.14	2.79	0.71	4.5
time (sec)	N/A	0.02	0.007	0.044	1.026	1.624	0.418	1.096

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	34	46	70	20	50
normalized size	1	1.	1.	1.26	1.7	2.59	0.74	1.85
time (sec)	N/A	0.026	0.009	0.044	1.055	1.745	0.421	1.099

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	45	68	97	27	85
normalized size	1	1.	1.	1.22	1.84	2.62	0.73	2.3
time (sec)	N/A	0.027	0.011	0.046	1.009	1.66	0.474	1.096

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	55	77	117	39	104
normalized size	1	1.	1.	1.12	1.57	2.39	0.8	2.12
time (sec)	N/A	0.032	0.015	0.048	1.047	1.643	0.52	1.102

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	80	296	292	219	0	0
normalized size	1	1.	0.58	2.16	2.13	1.6	0.	0.
time (sec)	N/A	0.624	0.056	0.087	1.551	1.75	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	63	224	244	200	0	0
normalized size	1	1.	0.62	2.2	2.39	1.96	0.	0.
time (sec)	N/A	0.574	0.049	0.079	1.541	1.667	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	60	226	151	174	0	0
normalized size	1	1.	0.65	2.46	1.64	1.89	0.	0.
time (sec)	N/A	0.324	0.04	0.078	1.558	1.657	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	42	219	88	144	0	0
normalized size	1	1.	0.7	3.65	1.47	2.4	0.	0.
time (sec)	N/A	0.044	0.03	0.055	1.506	1.647	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	52	52	55	257	0	252	0	0
normalized size	1	1.	1.06	4.94	0.	4.85	0.	0.
time (sec)	N/A	0.572	0.04	0.084	0.	1.736	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	61	305	0	247	0	0
normalized size	1	1.	0.95	4.77	0.	3.86	0.	0.
time (sec)	N/A	0.552	0.048	0.086	0.	1.643	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	79	376	0	292	0	0
normalized size	1	1.	0.85	4.04	0.	3.14	0.	0.
time (sec)	N/A	0.593	0.074	0.095	0.	1.679	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	89	392	0	316	0	0
normalized size	1	1.	0.75	3.32	0.	2.68	0.	0.
time (sec)	N/A	0.604	0.07	0.09	0.	1.756	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	95	416	0	346	0	0
normalized size	1	1.	0.68	2.99	0.	2.49	0.	0.
time (sec)	N/A	0.662	0.077	0.089	0.	1.772	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	339	339	82	0	0	645	0	0
normalized size	1	1.	0.24	0.	0.	1.9	0.	0.
time (sec)	N/A	0.228	0.036	0.18	0.	1.845	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	295	295	63	0	0	597	0	0
normalized size	1	1.	0.21	0.	0.	2.02	0.	0.
time (sec)	N/A	0.179	0.021	0.13	0.	1.688	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	268	268	41	0	0	493	0	0
normalized size	1	1.	0.15	0.	0.	1.84	0.	0.
time (sec)	N/A	0.147	0.023	0.102	0.	1.699	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	97	0	0	660	0	0
normalized size	1	1.	0.36	0.	0.	2.47	0.	0.
time (sec)	N/A	0.174	0.034	0.12	0.	1.706	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	71	0	0	363	0	0
normalized size	1	1.	0.77	0.	0.	3.95	0.	0.
time (sec)	N/A	0.034	0.015	0.12	0.	1.634	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	81	0	0	401	0	0
normalized size	1	1.	0.61	0.	0.	3.04	0.	0.
time (sec)	N/A	0.043	0.018	0.141	0.	1.81	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	93	0	0	436	0	0
normalized size	1	1.	0.55	0.	0.	2.56	0.	0.
time (sec)	N/A	0.063	0.025	0.14	0.	1.741	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	202	202	99	0	0	463	0	0
normalized size	1	1.	0.49	0.	0.	2.29	0.	0.
time (sec)	N/A	0.079	0.028	0.141	0.	1.797	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	240	240	111	0	0	497	0	0
normalized size	1	1.	0.46	0.	0.	2.07	0.	0.
time (sec)	N/A	0.102	0.032	0.144	0.	1.745	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	337	337	148	0	0	756	0	0
normalized size	1	1.	0.44	0.	0.	2.24	0.	0.
time (sec)	N/A	0.215	0.105	0.175	0.	1.705	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	339	339	82	0	0	674	0	0
normalized size	1	1.	0.24	0.	0.	1.99	0.	0.
time (sec)	N/A	0.21	0.038	0.148	0.	1.721	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	295	295	61	0	0	640	0	0
normalized size	1	1.	0.21	0.	0.	2.17	0.	0.
time (sec)	N/A	0.177	0.016	0.141	0.	1.68	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	268	268	41	0	0	536	0	0
normalized size	1	1.	0.15	0.	0.	2.	0.	0.
time (sec)	N/A	0.143	0.027	0.127	0.	1.545	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	96	0	0	671	0	0
normalized size	1	1.	0.36	0.	0.	2.51	0.	0.
time (sec)	N/A	0.166	0.03	0.139	0.	1.567	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	68	0	0	381	0	0
normalized size	1	1.	0.74	0.	0.	4.14	0.	0.
time (sec)	N/A	0.032	0.014	0.138	0.	1.65	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	81	0	0	423	0	0
normalized size	1	1.	0.61	0.	0.	3.2	0.	0.
time (sec)	N/A	0.042	0.017	0.146	0.	1.72	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	93	0	0	447	0	0
normalized size	1	1.	0.55	0.	0.	2.63	0.	0.
time (sec)	N/A	0.063	0.023	0.139	0.	1.739	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	202	202	99	0	0	477	0	0
normalized size	1	1.	0.49	0.	0.	2.36	0.	0.
time (sec)	N/A	0.081	0.028	0.141	0.	1.844	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	373	373	96	0	0	759	0	0
normalized size	1	1.	0.26	0.	0.	2.03	0.	0.
time (sec)	N/A	0.255	0.043	0.189	0.	1.726	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	371	371	86	0	0	689	0	0
normalized size	1	1.	0.23	0.	0.	1.86	0.	0.
time (sec)	N/A	0.244	0.035	0.153	0.	1.785	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	324	324	72	0	0	637	0	0
normalized size	1	1.	0.22	0.	0.	1.97	0.	0.
time (sec)	N/A	0.21	0.034	0.151	0.	1.755	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	41	0	0	549	0	0
normalized size	1	1.	0.14	0.	0.	1.84	0.	0.
time (sec)	N/A	0.178	0.038	0.13	0.	1.643	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	293	293	112	0	0	716	0	0
normalized size	1	1.	0.38	0.	0.	2.44	0.	0.
time (sec)	N/A	0.219	0.037	0.144	0.	1.693	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	87	0	0	377	0	0
normalized size	1	1.	0.72	0.	0.	3.12	0.	0.
time (sec)	N/A	0.038	0.021	0.15	0.	1.639	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	99	0	0	420	0	0
normalized size	1	1.	0.61	0.	0.	2.58	0.	0.
time (sec)	N/A	0.051	0.023	0.146	0.	1.738	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	203	203	106	0	0	452	0	0
normalized size	1	1.	0.52	0.	0.	2.23	0.	0.
time (sec)	N/A	0.082	0.03	0.155	0.	1.702	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	233	233	118	0	0	482	0	0
normalized size	1	1.	0.51	0.	0.	2.07	0.	0.
time (sec)	N/A	0.1	0.034	0.158	0.	2.079	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	337	337	127	0	0	733	0	0
normalized size	1	1.	0.38	0.	0.	2.18	0.	0.
time (sec)	N/A	0.213	0.101	0.146	0.	2.163	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	339	339	73	0	0	648	0	0
normalized size	1	1.	0.22	0.	0.	1.91	0.	0.
time (sec)	N/A	0.216	0.026	0.144	0.	2.214	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	295	295	63	0	0	618	0	0
normalized size	1	1.	0.21	0.	0.	2.09	0.	0.
time (sec)	N/A	0.173	0.021	0.141	0.	2.155	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	268	268	41	0	0	493	0	0
normalized size	1	1.	0.15	0.	0.	1.84	0.	0.
time (sec)	N/A	0.146	0.028	0.128	0.	2.028	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	96	0	0	672	0	0
normalized size	1	1.	0.36	0.	0.	2.52	0.	0.
time (sec)	N/A	0.161	0.028	0.139	0.	2.013	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	69	0	0	369	0	0
normalized size	1	1.	0.75	0.	0.	4.01	0.	0.
time (sec)	N/A	0.035	0.015	0.136	0.	2.104	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	81	0	0	413	0	0
normalized size	1	1.	0.61	0.	0.	3.13	0.	0.
time (sec)	N/A	0.043	0.021	0.142	0.	2.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	92	0	0	443	0	0
normalized size	1	1.	0.54	0.	0.	2.61	0.	0.
time (sec)	N/A	0.069	0.024	0.154	0.	2.099	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	202	202	99	0	0	475	0	0
normalized size	1	1.	0.49	0.	0.	2.35	0.	0.
time (sec)	N/A	0.081	0.031	0.146	0.	2.095	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	337	337	127	0	0	740	0	0
normalized size	1	1.	0.38	0.	0.	2.2	0.	0.
time (sec)	N/A	0.217	0.102	0.161	0.	2.041	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	339	339	73	0	0	678	0	0
normalized size	1	1.	0.22	0.	0.	2.	0.	0.
time (sec)	N/A	0.219	0.029	0.158	0.	2.053	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	295	295	63	0	0	620	0	0
normalized size	1	1.	0.21	0.	0.	2.1	0.	0.
time (sec)	N/A	0.178	0.014	0.158	0.	2.134	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	268	268	39	0	0	536	0	0
normalized size	1	1.	0.15	0.	0.	2.	0.	0.
time (sec)	N/A	0.15	0.039	0.135	0.	2.052	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	97	0	0	662	0	0
normalized size	1	1.	0.36	0.	0.	2.48	0.	0.
time (sec)	N/A	0.168	0.028	0.146	0.	2.112	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	69	0	0	373	0	0
normalized size	1	1.	0.75	0.	0.	4.05	0.	0.
time (sec)	N/A	0.032	0.016	0.155	0.	2.039	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	81	0	0	414	0	0
normalized size	1	1.	0.61	0.	0.	3.14	0.	0.
time (sec)	N/A	0.043	0.019	0.158	0.	2.107	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	93	0	0	446	0	0
normalized size	1	1.	0.55	0.	0.	2.62	0.	0.
time (sec)	N/A	0.065	0.023	0.158	0.	2.094	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	202	202	99	0	0	473	0	0
normalized size	1	1.	0.49	0.	0.	2.34	0.	0.
time (sec)	N/A	0.081	0.032	0.158	0.	2.109	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	373	373	100	0	0	876	0	0
normalized size	1	1.	0.27	0.	0.	2.35	0.	0.
time (sec)	N/A	0.254	0.041	0.179	0.	2.067	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	371	371	91	0	0	783	0	0
normalized size	1	1.	0.25	0.	0.	2.11	0.	0.
time (sec)	N/A	0.241	0.03	0.173	0.	1.865	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	324	324	63	0	0	757	0	0
normalized size	1	1.	0.19	0.	0.	2.34	0.	0.
time (sec)	N/A	0.206	0.028	0.173	0.	1.785	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	39	0	0	644	0	0
normalized size	1	1.	0.13	0.	0.	2.15	0.	0.
time (sec)	N/A	0.172	0.047	0.128	0.	1.71	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	293	293	106	0	0	879	0	0
normalized size	1	1.	0.36	0.	0.	3.	0.	0.
time (sec)	N/A	0.219	0.058	0.177	0.	1.863	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	69	0	0	491	0	0
normalized size	1	1.	0.57	0.	0.	4.06	0.	0.
time (sec)	N/A	0.039	0.017	0.175	0.	1.817	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	81	0	0	543	0	0
normalized size	1	1.	0.5	0.	0.	3.33	0.	0.
time (sec)	N/A	0.05	0.021	0.172	0.	1.79	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	203	203	93	0	0	574	0	0
normalized size	1	1.	0.46	0.	0.	2.83	0.	0.
time (sec)	N/A	0.079	0.025	0.174	0.	1.794	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	233	233	99	0	0	610	0	0
normalized size	1	1.	0.42	0.	0.	2.62	0.	0.
time (sec)	N/A	0.099	0.033	0.179	0.	1.755	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	319	319	73	0	0	695	0	0
normalized size	1	1.	0.23	0.	0.	2.18	0.	0.
time (sec)	N/A	0.384	0.034	0.055	0.	1.762	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	278	278	57	0	0	645	0	0
normalized size	1	1.	0.21	0.	0.	2.32	0.	0.
time (sec)	N/A	0.338	0.023	0.05	0.	1.787	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	262	262	34	0	0	636	0	0
normalized size	1	1.	0.13	0.	0.	2.43	0.	0.
time (sec)	N/A	0.31	0.017	0.044	0.	1.691	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	430	430	90	0	0	1108	0	0
normalized size	1	1.	0.21	0.	0.	2.58	0.	0.
time (sec)	N/A	0.458	0.031	0.054	0.	1.79	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	253	253	64	0	0	635	0	0
normalized size	1	1.	0.25	0.	0.	2.51	0.	0.
time (sec)	N/A	0.156	0.013	0.049	0.	1.735	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	280	280	72	0	0	672	0	0
normalized size	1	1.	0.26	0.	0.	2.4	0.	0.
time (sec)	N/A	0.176	0.017	0.049	0.	1.733	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	319	319	81	0	0	733	0	0
normalized size	1	1.	0.25	0.	0.	2.3	0.	0.
time (sec)	N/A	0.2	0.022	0.056	0.	1.736	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	73	0	0	397	0	0
normalized size	1	1.	0.41	0.	0.	2.24	0.	0.
time (sec)	N/A	0.055	0.032	0.049	0.	1.689	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	54	0	0	362	0	0
normalized size	1	1.	0.39	0.	0.	2.59	0.	0.
time (sec)	N/A	0.036	0.02	0.05	0.	1.697	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	34	0	0	346	0	0
normalized size	1	1.	0.29	0.	0.	2.98	0.	0.
time (sec)	N/A	0.021	0.017	0.043	0.	1.703	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	90	0	0	423	0	0
normalized size	1	1.	0.55	0.	0.	2.6	0.	0.
time (sec)	N/A	0.04	0.03	0.047	0.	1.656	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	59	0	0	352	0	0
normalized size	1	1.	0.53	0.	0.	3.17	0.	0.
time (sec)	N/A	0.029	0.012	0.047	0.	1.7	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	69	0	0	387	0	0
normalized size	1	1.	0.49	0.	0.	2.73	0.	0.
time (sec)	N/A	0.04	0.014	0.05	0.	1.683	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	741	741	83	0	0	1531	0	0
normalized size	1	1.	0.11	0.	0.	2.07	0.	0.
time (sec)	N/A	0.733	0.038	0.074	0.	1.901	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	689	689	63	0	0	1304	0	0
normalized size	1	1.	0.09	0.	0.	1.89	0.	0.
time (sec)	N/A	0.495	0.022	0.068	0.	1.897	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	674	674	41	0	0	1115	0	0
normalized size	1	1.	0.06	0.	0.	1.65	0.	0.
time (sec)	N/A	0.434	0.022	0.05	0.	1.798	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	859	859	97	0	0	1423	0	0
normalized size	1	1.	0.11	0.	0.	1.66	0.	0.
time (sec)	N/A	0.546	0.034	0.064	0.	1.887	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	328	328	71	0	0	819	0	0
normalized size	1	1.	0.22	0.	0.	2.5	0.	0.
time (sec)	N/A	0.132	0.016	0.063	0.	1.822	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	364	364	84	0	0	883	0	0
normalized size	1	1.	0.23	0.	0.	2.43	0.	0.
time (sec)	N/A	0.161	0.022	0.065	0.	1.83	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	94	748	0	0	0	0
normalized size	1	1.	0.82	6.56	0.	0.	0.	0.
time (sec)	N/A	0.095	0.045	0.625	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	58	417	0	0	0	0
normalized size	1	1.	1.16	8.34	0.	0.	0.	0.
time (sec)	N/A	0.042	0.021	0.516	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	175	0	0	269	0
normalized size	1	1.	0.74	4.49	0.	0.	6.9	0.
time (sec)	N/A	0.023	0.007	0.463	0.	0.	3.935	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	158	0	0	578	0
normalized size	1	1.	0.74	4.05	0.	0.	14.82	0.
time (sec)	N/A	0.023	0.008	0.382	0.	0.	31.517	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	58	428	0	0	0	0
normalized size	1	1.	1.16	8.56	0.	0.	0.	0.
time (sec)	N/A	0.039	0.021	0.536	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	94	1196	0	0	0	0
normalized size	1	1.	0.82	10.4	0.	0.	0.	0.
time (sec)	N/A	0.09	0.034	0.657	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	113	146	0	0	0	0
normalized size	1	1.	0.71	0.92	0.	0.	0.	0.
time (sec)	N/A	0.774	0.077	0.438	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	85	71	0	0	95	0
normalized size	1	1.	1.08	0.9	0.	0.	1.2	0.
time (sec)	N/A	0.041	0.037	0.293	0.	0.	3.366	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	85	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.041	0.414	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	113	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.705	0.071	0.41	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.207	0.132	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.182	0.059	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.174	0.126	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.186	0.134	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.199	0.128	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.232	0.164	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.195	0.181	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.198	0.167	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.171	0.06	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.22	0.119	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	210	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.18	0.083	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	116	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.049	0.082	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	105	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.025	0.074	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	53	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.029	0.067	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	106	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.026	0.166	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	82	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.015	0.174	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	114	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.03	0.179	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	119	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.055	0.19	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	276	276	217	656	0	520	0	289
normalized size	1	1.	0.79	2.38	0.	1.88	0.	1.05
time (sec)	N/A	0.201	0.492	0.138	0.	1.784	0.	1.157

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	176	465	0	386	0	220
normalized size	1	1.	0.88	2.31	0.	1.92	0.	1.09
time (sec)	N/A	0.192	0.273	0.117	0.	1.719	0.	1.173

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	135	302	0	278	0	158
normalized size	1	1.	0.79	1.77	0.	1.63	0.	0.92
time (sec)	N/A	0.125	0.134	0.117	0.	1.693	0.	1.151

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	108	171	0	200	0	103
normalized size	1	1.	0.98	1.55	0.	1.82	0.	0.94
time (sec)	N/A	0.075	0.113	0.109	0.	1.65	0.	1.155

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	28	69	0	144	36	69
normalized size	1	1.	0.54	1.33	0.	2.77	0.69	1.33
time (sec)	N/A	0.034	0.018	0.088	0.	1.676	2.802	1.162

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	132	157	0	402	0	0
normalized size	1	1.	1.48	1.76	0.	4.52	0.	0.
time (sec)	N/A	0.075	0.135	0.105	0.	1.693	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	115	236	0	545	0	0
normalized size	1	1.	0.88	1.82	0.	4.19	0.	0.
time (sec)	N/A	0.064	0.082	0.109	0.	1.705	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	149	405	0	1103	0	0
normalized size	1	1.	0.74	2.01	0.	5.49	0.	0.
time (sec)	N/A	0.138	0.118	0.113	0.	1.83	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	283	283	242	611	0	1767	0	0
normalized size	1	1.	0.86	2.16	0.	6.24	0.	0.
time (sec)	N/A	0.183	0.263	0.118	0.	1.824	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	347	203	284	2315	176
normalized size	1	1.	1.	3.77	2.21	3.09	25.16	1.91
time (sec)	N/A	0.083	0.067	0.039	1.499	1.576	13.475	1.113

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	255	159	201	1212	119
normalized size	1	1.	1.	3.54	2.21	2.79	16.83	1.65
time (sec)	N/A	0.059	0.058	0.038	1.46	1.552	6.433	1.096

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	176	117	132	513	77
normalized size	1	1.	1.	3.26	2.17	2.44	9.5	1.43
time (sec)	N/A	0.048	0.032	0.037	1.503	1.62	2.694	1.091

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	107	89	88	148	49
normalized size	1	1.	1.	2.89	2.41	2.38	4.	1.32
time (sec)	N/A	0.03	0.019	0.038	1.462	1.878	1.034	1.09

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	20	20	32	51	62	50	14	23
normalized size	1	1.	1.6	2.55	3.1	2.5	0.7	1.15
time (sec)	N/A	0.012	0.011	0.039	1.457	1.732	0.373	1.112

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	31	149	108	73	1538	49
normalized size	1	1.	0.82	3.92	2.84	1.92	40.47	1.29
time (sec)	N/A	0.034	0.018	0.043	1.479	1.812	2.541	1.116

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	39	260	169	111	3550	92
normalized size	1	1.	0.71	4.73	3.07	2.02	64.55	1.67
time (sec)	N/A	0.041	0.025	0.044	1.512	1.937	8.194	1.092

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	63	406	254	181	0	132
normalized size	1	1.	0.83	5.34	3.34	2.38	0.	1.74
time (sec)	N/A	0.049	0.032	0.048	1.492	1.939	0.	1.136

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	88	560	355	247	0	184
normalized size	1	1.	0.95	6.02	3.82	2.66	0.	1.98
time (sec)	N/A	0.056	0.046	0.052	1.532	1.911	0.	1.096

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	324	324	249	933	0	822	0	525
normalized size	1	1.	0.77	2.88	0.	2.54	0.	1.62
time (sec)	N/A	0.267	0.447	0.131	0.	2.056	0.	1.169

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	201	711	0	633	0	446
normalized size	1	1.	0.81	2.86	0.	2.54	0.	1.79
time (sec)	N/A	0.242	0.261	0.129	0.	2.077	0.	1.185

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	160	519	0	489	0	375
normalized size	1	1.	0.7	2.29	0.	2.15	0.	1.65
time (sec)	N/A	0.169	0.236	0.12	0.	2.018	0.	1.162

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	132	358	0	366	0	319
normalized size	1	1.	0.81	2.2	0.	2.25	0.	1.96
time (sec)	N/A	0.12	0.163	0.116	0.	2.035	0.	1.157

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	45	362	0	265	0	262
normalized size	1	1.	0.48	3.85	0.	2.82	0.	2.79
time (sec)	N/A	0.042	0.039	0.092	0.	2.16	0.	1.169

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	229	818	0	983	0	0
normalized size	1	1.	1.71	6.1	0.	7.34	0.	0.
time (sec)	N/A	0.095	0.714	0.11	0.	2.501	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	143	1358	0	1030	0	0
normalized size	1	1.	0.81	7.72	0.	5.85	0.	0.
time (sec)	N/A	0.083	0.151	0.117	0.	2.405	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	264	264	180	1955	0	1462	0	0
normalized size	1	1.	0.68	7.41	0.	5.54	0.	0.
time (sec)	N/A	0.158	0.206	0.116	0.	2.399	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	338	338	277	2624	0	2215	0	0
normalized size	1	1.	0.82	7.76	0.	6.55	0.	0.
time (sec)	N/A	0.27	0.421	0.119	0.	2.627	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	276	276	248	1208	616	524	0	289
normalized size	1	1.	0.9	4.38	2.23	1.9	0.	1.05
time (sec)	N/A	0.225	0.758	0.223	1.632	2.36	0.	1.127

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	202	894	416	389	0	220
normalized size	1	1.	1.	4.45	2.07	1.94	0.	1.09
time (sec)	N/A	0.194	0.472	0.122	1.563	2.239	0.	1.132

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	162	605	217	282	0	158
normalized size	1	1.	0.95	3.54	1.27	1.65	0.	0.92
time (sec)	N/A	0.126	0.281	0.124	1.511	2.284	0.	1.127

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	131	350	131	200	0	105
normalized size	1	1.	1.19	3.18	1.19	1.82	0.	0.95
time (sec)	N/A	0.076	0.129	0.08	1.513	2.291	0.	1.13

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	28	122	47	146	0	70
normalized size	1	1.	0.54	2.35	0.9	2.81	0.	1.35
time (sec)	N/A	0.034	0.019	0.054	1.499	2.17	0.	1.123

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	132	283	0	404	0	104
normalized size	1	1.	1.48	3.18	0.	4.54	0.	1.17
time (sec)	N/A	0.065	0.079	0.116	0.	2.32	0.	1.169

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	114	602	0	545	0	0
normalized size	1	1.	0.88	4.63	0.	4.19	0.	0.
time (sec)	N/A	0.064	0.053	0.145	0.	2.248	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	149	1146	0	1106	0	0
normalized size	1	1.	0.74	5.7	0.	5.5	0.	0.
time (sec)	N/A	0.117	0.111	0.124	0.	2.365	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	283	282	251	1738	0	1766	0	0
normalized size	1	1.	0.89	6.14	0.	6.24	0.	0.
time (sec)	N/A	0.183	0.232	0.129	0.	2.452	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	95	292	138	286	2315	317
normalized size	1	1.	0.96	2.95	1.39	2.89	23.38	3.2
time (sec)	N/A	0.086	0.07	0.043	1.022	2.026	13.427	1.126

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	211	100	203	1212	234
normalized size	1	1.	1.	2.74	1.3	2.64	15.74	3.04
time (sec)	N/A	0.059	0.058	0.043	1.05	2.126	6.412	1.094

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	55	143	70	135	513	162
normalized size	1	1.	0.93	2.42	1.19	2.29	8.69	2.75
time (sec)	N/A	0.048	0.034	0.043	1.03	2.121	2.692	1.125

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	85	49	89	148	108
normalized size	1	1.	1.	2.12	1.22	2.22	3.7	2.7
time (sec)	N/A	0.032	0.02	0.043	1.024	2.194	1.029	1.116

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	23	23	32	40	26	50	15	51
normalized size	1	1.	1.39	1.74	1.13	2.17	0.65	2.22
time (sec)	N/A	0.013	0.011	0.042	0.987	2.094	0.378	1.104

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	34	74	63	73	1538	104
normalized size	1	1.	0.83	1.8	1.54	1.78	37.51	2.54
time (sec)	N/A	0.036	0.019	0.063	1.021	2.22	2.543	1.099

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	42	152	153	109	3550	143
normalized size	1	1.	0.68	2.45	2.47	1.76	57.26	2.31
time (sec)	N/A	0.042	0.023	0.052	1.016	2.142	8.056	1.111

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	66	246	220	181	0	212
normalized size	1	1.	0.81	3.04	2.72	2.23	0.	2.62
time (sec)	N/A	0.052	0.036	0.053	1.031	2.183	0.	1.108

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	91	349	300	248	0	273
normalized size	1	1.	0.89	3.42	2.94	2.43	0.	2.68
time (sec)	N/A	0.061	0.046	0.054	1.058	2.121	0.	1.132

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	324	324	299	2058	1847	818	0	536
normalized size	1	1.	0.92	6.35	5.7	2.52	0.	1.65
time (sec)	N/A	0.281	0.478	0.138	1.66	2.402	0.	1.141

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	244	1529	1322	635	0	459
normalized size	1	1.	0.98	6.14	5.31	2.55	0.	1.84
time (sec)	N/A	0.246	0.354	0.102	1.658	2.363	0.	1.162

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	198	1026	842	485	0	386
normalized size	1	1.	0.86	4.48	3.68	2.12	0.	1.69
time (sec)	N/A	0.168	0.295	0.094	1.567	2.439	0.	1.176

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	157	676	396	367	0	325
normalized size	1	1.	0.96	4.15	2.43	2.25	0.	1.99
time (sec)	N/A	0.119	0.3	0.086	1.55	2.344	0.	1.154

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	45	329	139	262	0	265
normalized size	1	1.	0.48	3.5	1.48	2.79	0.	2.82
time (sec)	N/A	0.044	0.035	0.062	1.489	2.226	0.	1.142

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	172	1278	0	980	0	0
normalized size	1	1.	1.28	9.54	0.	7.31	0.	0.
time (sec)	N/A	0.094	0.545	0.13	0.	2.501	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	178	178	135	1917	0	1029	0	0
normalized size	1	1.	0.76	10.77	0.	5.78	0.	0.
time (sec)	N/A	0.085	0.171	0.132	0.	2.376	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	264	264	184	3042	0	1458	0	0
normalized size	1	1.	0.7	11.52	0.	5.52	0.	0.
time (sec)	N/A	0.16	0.222	0.135	0.	2.488	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	339	339	268	4390	0	2219	0	0
normalized size	1	1.	0.79	12.95	0.	6.55	0.	0.
time (sec)	N/A	0.271	0.387	0.142	0.	2.679	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	494	494	121	0	0	1432	0	0
normalized size	1	1.	0.24	0.	0.	2.9	0.	0.
time (sec)	N/A	0.405	0.085	0.312	0.	2.461	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	410	410	81	0	0	1015	0	0
normalized size	1	1.	0.2	0.	0.	2.48	0.	0.
time (sec)	N/A	0.307	0.046	0.224	0.	2.334	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	338	338	45	0	0	636	0	0
normalized size	1	1.	0.13	0.	0.	1.88	0.	0.
time (sec)	N/A	0.195	0.016	0.194	0.	2.355	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	395	395	124	0	0	1141	0	0
normalized size	1	1.	0.31	0.	0.	2.89	0.	0.
time (sec)	N/A	0.246	0.087	0.215	0.	2.442	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	110	0	0	1575	0	0
normalized size	1	1.	0.54	0.	0.	7.68	0.	0.
time (sec)	N/A	0.103	0.027	0.227	0.	2.376	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	494	494	121	0	0	1571	0	0
normalized size	1	1.	0.24	0.	0.	3.18	0.	0.
time (sec)	N/A	0.383	0.088	0.22	0.	2.519	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	410	410	79	0	0	1123	0	0
normalized size	1	1.	0.19	0.	0.	2.74	0.	0.
time (sec)	N/A	0.292	0.042	0.22	0.	2.316	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	338	338	45	0	0	693	0	0
normalized size	1	1.	0.13	0.	0.	2.05	0.	0.
time (sec)	N/A	0.197	0.017	0.232	0.	2.405	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	427	427	122	0	0	1840	0	0
normalized size	1	1.	0.29	0.	0.	4.31	0.	0.
time (sec)	N/A	0.251	0.096	0.218	0.	2.698	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	106	0	0	1886	0	0
normalized size	1	1.	0.5	0.	0.	8.94	0.	0.
time (sec)	N/A	0.108	0.022	0.24	0.	2.722	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	494	494	99	0	0	1445	0	0
normalized size	1	1.	0.2	0.	0.	2.93	0.	0.
time (sec)	N/A	0.401	0.088	0.23	0.	2.635	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	410	410	84	0	0	1027	0	0
normalized size	1	1.	0.2	0.	0.	2.5	0.	0.
time (sec)	N/A	0.307	0.04	0.233	0.	2.516	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	338	338	45	0	0	649	0	0
normalized size	1	1.	0.13	0.	0.	1.92	0.	0.
time (sec)	N/A	0.197	0.017	0.203	0.	2.549	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	395	395	126	0	0	1296	0	0
normalized size	1	1.	0.32	0.	0.	3.28	0.	0.
time (sec)	N/A	0.206	0.033	0.22	0.	2.825	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	210	210	107	0	0	1875	0	0
normalized size	1	1.	0.51	0.	0.	8.93	0.	0.
time (sec)	N/A	0.101	0.026	0.225	0.	2.691	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	494	494	98	0	0	1566	0	0
normalized size	1	1.	0.2	0.	0.	3.17	0.	0.
time (sec)	N/A	0.39	0.085	0.245	0.	2.739	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	410	410	84	0	0	1115	0	0
normalized size	1	1.	0.2	0.	0.	2.72	0.	0.
time (sec)	N/A	0.297	0.038	0.233	0.	2.641	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	338	338	43	0	0	679	0	0
normalized size	1	1.	0.13	0.	0.	2.01	0.	0.
time (sec)	N/A	0.197	0.017	0.211	0.	2.449	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	427	427	128	0	0	1673	0	0
normalized size	1	1.	0.3	0.	0.	3.92	0.	0.
time (sec)	N/A	0.24	0.036	0.234	0.	2.85	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	107	0	0	1667	0	0
normalized size	1	1.	0.51	0.	0.	7.9	0.	0.
time (sec)	N/A	0.118	0.026	0.232	0.	2.622	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.807	0.131	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	260	260	272	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.186	0.3	0.197	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	160	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	0.135	0.063	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	128	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.114	0.059	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	60	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.028	0.055	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	170	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.034	0.198	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	125	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.022	0.193	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	173	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.064	0.205	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	102	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.028	0.322	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.014	0.184	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.011	0.056	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	45	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.015	0.051	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	13	13	35	13	16	31	0	16
normalized size	1	1.	2.69	1.	1.23	2.38	0.	1.23
time (sec)	N/A	0.026	0.007	0.036	1.491	2.044	0.	1.11

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	50	50	60	39	0	90	0	0
normalized size	1	1.	1.2	0.78	0.	1.8	0.	0.
time (sec)	N/A	0.05	0.024	0.036	0.	2.09	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	114	55	0	153	0	0
normalized size	1	1.	1.37	0.66	0.	1.84	0.	0.
time (sec)	N/A	0.078	0.249	0.036	0.	2.023	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	123	71	0	220	0	0
normalized size	1	1.	1.06	0.61	0.	1.9	0.	0.
time (sec)	N/A	0.11	0.264	0.038	0.	1.969	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	153	87	0	304	0	0
normalized size	1	1.	1.03	0.58	0.	2.04	0.	0.
time (sec)	N/A	0.14	0.258	0.037	0.	1.972	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.023	0.282	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.018	0.28	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.02	0.29	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	37	0	99	0	0
normalized size	1	1.	1.	1.06	0.	2.83	0.	0.
time (sec)	N/A	0.036	0.015	0.038	0.	1.875	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	54	0	157	0	0
normalized size	1	1.	0.83	0.75	0.	2.18	0.	0.
time (sec)	N/A	0.073	0.034	0.036	0.	2.081	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	79	70	0	215	0	0
normalized size	1	1.	0.73	0.65	0.	1.99	0.	0.
time (sec)	N/A	0.113	0.035	0.036	0.	2.019	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.024	0.279	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.013	0.191	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.012	0.057	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	46	46	37	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.019	0.054	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	18	18	34	16	20	39	0	20
normalized size	1	1.	1.89	0.89	1.11	2.17	0.	1.11
time (sec)	N/A	0.028	0.006	0.034	1.526	2.008	0.	1.083

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	53	55	40	0	90	0	0
normalized size	1	1.	1.04	0.75	0.	1.7	0.	0.
time (sec)	N/A	0.057	0.018	0.037	0.	1.929	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	86	57	0	153	0	0
normalized size	1	1.	0.98	0.65	0.	1.74	0.	0.
time (sec)	N/A	0.089	0.117	0.037	0.	2.124	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	122	73	0	212	0	0
normalized size	1	1.	0.99	0.59	0.	1.72	0.	0.
time (sec)	N/A	0.12	0.245	0.039	0.	2.029	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.021	0.279	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.018	0.276	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.018	0.28	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	39	0	101	0	0
normalized size	1	1.	1.	1.05	0.	2.73	0.	0.
time (sec)	N/A	0.04	0.016	0.039	0.	2.029	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	62	56	0	163	0	0
normalized size	1	1.	0.82	0.74	0.	2.14	0.	0.
time (sec)	N/A	0.081	0.034	0.04	0.	2.018	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	81	72	0	228	0	0
normalized size	1	1.	0.71	0.63	0.	2.	0.	0.
time (sec)	N/A	0.125	0.038	0.037	0.	1.867	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.026	0.323	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.014	0.19	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.013	0.058	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	45	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.021	0.052	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	16	16	36	16	31	34	0	20
normalized size	1	1.	2.25	1.	1.94	2.12	0.	1.25
time (sec)	N/A	0.027	0.008	0.038	1.047	1.969	0.	1.093

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	60	41	0	93	0	0
normalized size	1	1.	1.11	0.76	0.	1.72	0.	0.
time (sec)	N/A	0.054	0.022	0.036	0.	1.79	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	91	57	0	155	0	0
normalized size	1	1.	1.02	0.64	0.	1.74	0.	0.
time (sec)	N/A	0.083	0.148	0.036	0.	2.036	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	127	73	0	223	0	0
normalized size	1	1.	1.02	0.59	0.	1.8	0.	0.
time (sec)	N/A	0.119	0.347	0.039	0.	1.986	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.022	0.301	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.018	0.288	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.021	0.286	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	39	0	100	0	0
normalized size	1	1.	0.97	1.03	0.	2.63	0.	0.
time (sec)	N/A	0.039	0.02	0.035	0.	2.035	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	62	56	0	158	0	0
normalized size	1	1.	0.81	0.73	0.	2.05	0.	0.
time (sec)	N/A	0.08	0.042	0.036	0.	2.022	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	81	72	0	216	0	0
normalized size	1	1.	0.7	0.63	0.	1.88	0.	0.
time (sec)	N/A	0.124	0.046	0.039	0.	1.951	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.022	0.326	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.014	0.192	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.012	0.059	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	46	46	37	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.021	0.051	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	18	18	34	18	31	42	0	20
normalized size	1	1.	1.89	1.	1.72	2.33	0.	1.11
time (sec)	N/A	0.028	0.008	0.036	1.064	1.953	0.	1.119

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	55	42	0	93	0	0
normalized size	1	1.	1.02	0.78	0.	1.72	0.	0.
time (sec)	N/A	0.058	0.021	0.038	0.	2.039	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	85	59	0	155	0	0
normalized size	1	1.	0.96	0.66	0.	1.74	0.	0.
time (sec)	N/A	0.088	0.145	0.038	0.	2.083	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	121	75	0	215	0	0
normalized size	1	1.	0.98	0.6	0.	1.73	0.	0.
time (sec)	N/A	0.121	0.322	0.039	0.	1.998	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.024	0.3	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.018	0.283	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.021	0.285	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	41	0	103	0	0
normalized size	1	1.	0.97	1.08	0.	2.71	0.	0.
time (sec)	N/A	0.039	0.021	0.036	0.	1.947	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	62	58	0	165	0	0
normalized size	1	1.	0.81	0.75	0.	2.14	0.	0.
time (sec)	N/A	0.08	0.046	0.037	0.	1.86	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	81	74	0	230	0	0
normalized size	1	1.	0.7	0.64	0.	2.	0.	0.
time (sec)	N/A	0.125	0.049	0.038	0.	1.974	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	45	85	115	41	41
normalized size	1	1.	0.84	0.9	1.7	2.3	0.82	0.82
time (sec)	N/A	0.043	0.022	0.049	1.489	1.851	0.589	1.118

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	48	113	163	204	0	32
normalized size	1	1.	0.66	1.55	2.23	2.79	0.	0.44
time (sec)	N/A	0.039	0.012	0.067	0.995	1.886	0.	1.14

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	40	59	69	22	34
normalized size	1	1.	1.	1.33	1.97	2.3	0.73	1.13
time (sec)	N/A	0.038	0.014	0.044	1.477	1.775	0.458	1.116

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	41	41	52	63	66	126	0	0
normalized size	1	1.	1.27	1.54	1.61	3.07	0.	0.
time (sec)	N/A	0.035	0.03	0.064	0.988	1.767	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	26	32	30	12	18
normalized size	1	1.	1.	1.73	2.13	2.	0.8	1.2
time (sec)	N/A	0.029	0.005	0.033	1.497	1.782	0.09	1.099

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	26	16	31	12	18
normalized size	1	1.	1.	1.62	1.	1.94	0.75	1.12
time (sec)	N/A	0.032	0.005	0.033	0.973	1.701	0.07	1.115

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	41	41	56	143	45	126	0	0
normalized size	1	1.	1.37	3.49	1.1	3.07	0.	0.
time (sec)	N/A	0.035	0.058	0.068	1.467	1.956	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	41	55	68	24	38
normalized size	1	1.	1.	1.28	1.72	2.12	0.75	1.19
time (sec)	N/A	0.038	0.015	0.043	0.979	1.726	0.388	1.091

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	82	262	146	204	0	32
normalized size	1	1.	1.12	3.59	2.	2.79	0.	0.44
time (sec)	N/A	0.038	0.072	0.069	1.549	1.95	0.	1.152

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	69	84	0	815	0	0
normalized size	1	1.	0.53	0.64	0.	6.22	0.	0.
time (sec)	N/A	0.083	0.033	0.151	0.	2.217	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	71	800	0	417	0	0
normalized size	1	1.	0.74	8.33	0.	4.34	0.	0.
time (sec)	N/A	0.085	0.022	0.229	0.	2.225	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	55	61	0	798	0	0
normalized size	1	1.	0.65	0.73	0.	9.5	0.	0.
time (sec)	N/A	0.077	0.026	0.143	0.	2.262	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	91	204	0	332	0	97
normalized size	1	1.	1.44	3.24	0.	5.27	0.	1.54
time (sec)	N/A	0.06	0.027	0.172	0.	2.009	0.	1.141

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	53	0	575	0	0
normalized size	1	1.	1.	1.26	0.	13.69	0.	0.
time (sec)	N/A	0.065	0.013	0.125	0.	2.1	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	42	20	578	0	0
normalized size	1	1.	1.	0.98	0.47	13.44	0.	0.
time (sec)	N/A	0.066	0.016	0.109	0.998	2.255	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	117	87	0	332	0	101
normalized size	1	1.	1.86	1.38	0.	5.27	0.	1.6
time (sec)	N/A	0.061	0.058	0.201	0.	1.993	0.	1.323

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	60	66	47	801	0	0
normalized size	1	1.	0.7	0.77	0.55	9.31	0.	0.
time (sec)	N/A	0.076	0.028	0.142	0.999	2.255	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	132	136	0	417	0	0
normalized size	1	1.	1.38	1.42	0.	4.34	0.	0.
time (sec)	N/A	0.078	0.08	0.172	0.	2.164	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	24	20	80	84	44	26
normalized size	1	1.	0.69	0.57	2.29	2.4	1.26	0.74
time (sec)	N/A	0.038	0.016	0.049	1.511	1.914	0.647	1.117

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	47	269	128	176	0	150
normalized size	1	1.	0.7	4.01	1.91	2.63	0.	2.24
time (sec)	N/A	0.039	0.015	0.067	0.996	1.869	0.	1.162

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	15	47	45	26	18
normalized size	1	1.	0.95	0.79	2.47	2.37	1.37	0.95
time (sec)	N/A	0.032	0.018	0.047	1.48	1.789	0.59	1.098

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	48	104	61	117	0	89
normalized size	1	1.	0.72	1.55	0.91	1.75	0.	1.33
time (sec)	N/A	0.039	0.013	0.062	0.967	1.946	0.	1.148

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	21	38	38	116	39	55
normalized size	1	1.	0.75	1.36	1.36	4.14	1.39	1.96
time (sec)	N/A	0.042	0.016	0.037	1.456	1.939	0.474	1.11

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	21	25	0	116	36	59
normalized size	1	1.	0.72	0.86	0.	4.	1.24	2.03
time (sec)	N/A	0.041	0.016	0.053	0.	1.833	0.404	1.119

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	48	93	80	117	0	93
normalized size	1	1.	0.72	1.39	1.19	1.75	0.	1.39
time (sec)	N/A	0.04	0.014	0.067	1.475	1.896	0.	1.217

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	16	18	46	27	19
normalized size	1	1.	0.95	0.84	0.95	2.42	1.42	1.
time (sec)	N/A	0.032	0.018	0.033	1.008	1.671	0.395	1.096

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	47	92	135	176	0	150
normalized size	1	1.	0.7	1.37	2.01	2.63	0.	2.24
time (sec)	N/A	0.039	0.014	0.062	0.979	1.924	0.	1.187

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	56	48	0	220	0	0
normalized size	1	1.	0.59	0.51	0.	2.32	0.	0.
time (sec)	N/A	0.08	0.029	0.15	0.	2.05	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	77	940	0	149	0	182
normalized size	1	1.	1.12	13.62	0.	2.16	0.	2.64
time (sec)	N/A	0.065	0.034	0.082	0.	2.836	0.	1.13

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	57	44	0	150	0	0
normalized size	1	1.	1.16	0.9	0.	3.06	0.	0.
time (sec)	N/A	0.069	0.036	0.056	0.	2.16	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	78	398	0	101	0	103
normalized size	1	1.	1.44	7.37	0.	1.87	0.	1.91
time (sec)	N/A	0.055	0.027	0.074	0.	2.38	0.	1.145

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	51	58	0	698	0	0
normalized size	1	1.	0.58	0.66	0.	7.93	0.	0.
time (sec)	N/A	0.081	0.031	0.138	0.	2.442	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	60	86	70	698	0	0
normalized size	1	1.	0.67	0.97	0.79	7.84	0.	0.
time (sec)	N/A	0.081	0.034	0.154	1.023	2.472	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	78	137	0	101	0	105
normalized size	1	1.	1.44	2.54	0.	1.87	0.	1.94
time (sec)	N/A	0.055	0.033	0.07	0.	2.381	0.	1.223

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	57	45	39	151	0	0
normalized size	1	1.	1.16	0.92	0.8	3.08	0.	0.
time (sec)	N/A	0.072	0.041	0.041	1.023	2.237	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	77	307	0	149	0	181
normalized size	1	1.	1.12	4.45	0.	2.16	0.	2.62
time (sec)	N/A	0.066	0.039	0.072	0.	2.755	0.	1.155

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	90	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.023	0.191	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	88	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.016	0.064	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	56	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.018	0.055	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	131	206	141	0	0	0	0	0
normalized size	1	1.57	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.227	0.113	0.266	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	121	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	0.12	0.254	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	109	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	0.06	0.318	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	18	18	42	18	23	36	0	23
normalized size	1	1.	2.33	1.	1.28	2.	0.	1.28
time (sec)	N/A	0.03	0.007	0.04	1.524	2.239	0.	1.085

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	65	132	120	0	0	0	0	0
normalized size	1	2.03	1.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.036	0.323	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	90	180	142	0	0	0	0	0
normalized size	1	2.	1.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	0.046	0.326	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	126	242	174	0	0	0	0	0
normalized size	1	1.92	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	0.068	0.346	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	165	166	0	666	0	0
normalized size	1	1.	0.91	0.92	0.	3.68	0.	0.
time (sec)	N/A	0.175	0.44	0.04	0.	2.004	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	118	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.105	0.285	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	117	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.052	0.284	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	117	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.035	0.286	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	217	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.311	0.247	0.286	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	214	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.289	0.192	0.302	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	248	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.357	0.292	0.29	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	206	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.342	0.167	0.294	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	187	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.187	0.137	0.289	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	117	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.025	0.03	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	120	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.203	0.043	0.287	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	142	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	0.065	0.289	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	159	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.253	0.077	0.293	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.063	0.255	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.046	0.271	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	0.043	0.341	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	123	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.053	0.266	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	49	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.459	0.112	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	51	51	96	0	0	0	0	0
normalized size	1	1.	1.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.152	0.553	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.444	0.48	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.678	180.	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.194	0.33	0.309	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.211	0.433	0.309	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	0.57	0.317	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	115	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.032	0.31	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	39	41	103	89	0	74
normalized size	1	1.	0.74	0.77	1.94	1.68	0.	1.4
time (sec)	N/A	0.065	0.028	0.046	1.035	2.283	0.	1.131

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	39	41	0	92	0	78
normalized size	1	1.	0.74	0.77	0.	1.74	0.	1.47
time (sec)	N/A	0.061	0.025	0.046	0.	2.212	0.	1.144

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	55	62	0	162	0	487
normalized size	1	1.	0.92	1.03	0.	2.7	0.	8.12
time (sec)	N/A	0.112	0.024	0.06	0.	2.243	0.	1.142

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	36	34	394	1139	0	429
normalized size	1	1.	0.95	0.89	10.37	29.97	0.	11.29
time (sec)	N/A	0.079	1.055	0.342	2.013	10.849	0.	1.137

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	36	35	209	429	202	204
normalized size	1	1.	0.95	0.92	5.5	11.29	5.32	5.37
time (sec)	N/A	0.078	0.193	0.13	1.587	2.491	111.484	1.144

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	36	34	84	104	60	65
normalized size	1	1.	0.95	0.89	2.21	2.74	1.58	1.71
time (sec)	N/A	0.077	0.031	0.051	1.504	2.023	0.859	1.159

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	36	62	0	104	56	119
normalized size	1	1.	0.95	1.63	0.	2.74	1.47	3.13
time (sec)	N/A	0.077	0.033	0.053	0.	2.133	0.894	1.123

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	36	218	0	429	199	204
normalized size	1	1.	0.95	5.74	0.	11.29	5.24	5.37
time (sec)	N/A	0.078	0.194	0.086	0.	2.565	110.205	1.119

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	63	58	0	1369	0	0
normalized size	1	1.	0.97	0.89	0.	21.06	0.	0.
time (sec)	N/A	0.201	0.482	0.098	0.	4.18	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	58	0	436	0	0
normalized size	1	1.	1.	0.89	0.	6.71	0.	0.
time (sec)	N/A	0.204	0.092	0.07	0.	2.396	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	74	87	0	0	0	0
normalized size	1	1.	0.52	0.61	0.	0.	0.	0.
time (sec)	N/A	0.219	0.051	0.161	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	75	86	74	0	0	0
normalized size	1	1.	0.52	0.6	0.52	0.	0.	0.
time (sec)	N/A	0.224	0.057	0.152	1.007	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	58	126	437	0	0
normalized size	1	1.	1.	0.89	1.94	6.72	0.	0.
time (sec)	N/A	0.208	0.101	0.074	1.087	2.417	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	63	58	370	1370	0	0
normalized size	1	1.	0.97	0.89	5.69	21.08	0.	0.
time (sec)	N/A	0.209	0.493	0.089	1.46	4.287	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [131] had the largest ratio of [1.25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.	14	0.286
2	A	5	4	1.	14	0.286
3	A	7	5	1.	14	0.357
4	A	3	3	1.	12	0.25
5	A	3	3	1.	10	0.3
6	A	6	6	1.	14	0.429
7	A	5	5	1.	14	0.357

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	6	6	1.	14	0.429
9	A	7	6	1.	14	0.429
10	A	8	6	1.	14	0.429
11	A	3	2	1.	14	0.143
12	A	3	2	1.	14	0.143
13	A	3	2	1.	12	0.167
14	A	3	2	1.	10	0.2
15	A	3	2	1.	14	0.143
16	A	3	2	1.	14	0.143
17	A	3	2	1.	14	0.143
18	A	3	2	1.	14	0.143
19	A	14	11	1.	14	0.786
20	A	10	9	1.	14	0.643
21	A	9	7	1.	12	0.583
22	A	5	5	1.	10	0.5
23	A	8	7	1.	14	0.5
24	A	8	7	1.	14	0.5
25	A	12	8	1.	14	0.571
26	A	14	9	1.	14	0.643
27	A	3	2	1.	14	0.143
28	A	3	2	1.	14	0.143
29	A	3	2	1.	12	0.167
30	A	3	2	1.	10	0.2
31	A	3	2	1.	14	0.143
32	A	3	2	1.	14	0.143
33	A	3	2	1.	14	0.143
34	A	3	2	1.	14	0.143
35	A	5	4	1.	14	0.286
36	A	7	5	1.	14	0.357
37	A	3	3	1.	12	0.25
38	A	3	3	1.	10	0.3
39	A	6	6	1.	14	0.429
40	A	5	5	1.	14	0.357
41	A	6	6	1.	14	0.429
42	A	7	6	1.	14	0.429
43	A	8	6	1.	14	0.429
44	A	3	2	1.	14	0.143
45	A	3	2	1.	14	0.143
46	A	3	2	1.	12	0.167
47	A	3	2	1.	10	0.2
48	A	3	2	1.	14	0.143
49	A	3	2	1.	14	0.143
50	A	3	2	1.	14	0.143
51	A	3	2	1.	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	14	11	1.	14	0.786
53	A	10	9	1.	14	0.643
54	A	9	7	1.	12	0.583
55	A	5	5	1.	10	0.5
56	A	8	7	1.	14	0.5
57	A	8	7	1.	14	0.5
58	A	12	8	1.	14	0.571
59	A	14	9	1.	14	0.643
60	A	19	9	1.	14	0.643
61	A	15	12	1.	16	0.75
62	A	14	11	1.	14	0.786
63	A	13	10	1.	12	0.833
64	A	17	14	1.	16	0.875
65	A	6	6	1.	16	0.375
66	A	7	7	1.	16	0.438
67	A	9	8	1.	16	0.5
68	A	10	8	1.	16	0.5
69	A	11	8	1.	16	0.5
70	A	15	12	1.	16	0.75
71	A	15	12	1.	16	0.75
72	A	14	11	1.	14	0.786
73	A	13	10	1.	12	0.833
74	A	17	14	1.	16	0.875
75	A	6	6	1.	16	0.375
76	A	7	7	1.	16	0.438
77	A	9	8	1.	16	0.5
78	A	10	8	1.	16	0.5
79	A	16	13	1.	16	0.812
80	A	16	12	1.	16	0.75
81	A	15	11	1.	14	0.786
82	A	14	11	1.	12	0.917
83	A	19	16	1.	16	1.
84	A	7	6	1.	16	0.375
85	A	8	7	1.	16	0.438
86	A	10	9	1.	16	0.562
87	A	11	9	1.	16	0.562
88	A	15	12	1.	16	0.75
89	A	15	12	1.	16	0.75
90	A	14	11	1.	14	0.786
91	A	13	10	1.	12	0.833
92	A	17	14	1.	16	0.875
93	A	6	6	1.	16	0.375
94	A	7	7	1.	16	0.438
95	A	9	8	1.	16	0.5

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	10	8	1.	16	0.5
97	A	15	12	1.	16	0.75
98	A	15	12	1.	16	0.75
99	A	14	11	1.	14	0.786
100	A	13	10	1.	12	0.833
101	A	17	14	1.	16	0.875
102	A	6	6	1.	16	0.375
103	A	7	7	1.	16	0.438
104	A	9	8	1.	16	0.5
105	A	10	8	1.	16	0.5
106	A	16	13	1.	16	0.812
107	A	16	12	1.	16	0.75
108	A	15	11	1.	14	0.786
109	A	14	11	1.	12	0.917
110	A	19	16	1.	16	1.
111	A	7	6	1.	16	0.375
112	A	8	7	1.	16	0.438
113	A	10	9	1.	16	0.562
114	A	11	9	1.	16	0.562
115	A	16	12	1.	14	0.857
116	A	15	11	1.	12	0.917
117	A	14	10	1.	10	1.
118	A	25	13	1.	14	0.929
119	A	13	9	1.	14	0.643
120	A	14	10	1.	14	0.714
121	A	16	11	1.	14	0.786
122	A	5	5	1.	14	0.357
123	A	4	4	1.	12	0.333
124	A	3	3	1.	10	0.3
125	A	4	4	1.	14	0.286
126	A	3	3	1.	14	0.214
127	A	4	4	1.	14	0.286
128	A	27	13	1.	16	0.812
129	A	26	12	1.	14	0.857
130	A	25	11	1.	12	0.917
131	A	39	20	1.	16	1.25
132	A	16	13	1.	16	0.812
133	A	17	14	1.	16	0.875
134	A	4	4	1.	14	0.286
135	A	4	4	1.	14	0.286
136	A	3	3	1.	14	0.214
137	A	3	3	1.	14	0.214
138	A	4	4	1.	14	0.286
139	A	4	4	1.	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
140	A	9	5	1.	14	0.357
141	A	4	3	1.	14	0.214
142	A	4	3	1.	14	0.214
143	A	9	5	1.	14	0.357
144	A	2	2	1.	16	0.125
145	A	2	2	1.	16	0.125
146	A	2	2	1.	16	0.125
147	A	2	2	1.	16	0.125
148	A	2	2	1.	16	0.125
149	A	2	2	1.	16	0.125
150	A	2	2	1.	12	0.167
151	A	2	2	1.	12	0.167
152	A	2	2	1.	16	0.125
153	A	2	2	1.	15	0.133
154	A	4	4	1.	15	0.267
155	A	4	4	1.	15	0.267
156	A	3	3	1.	13	0.231
157	A	2	2	1.	11	0.182
158	A	4	4	1.	15	0.267
159	A	2	2	1.	15	0.133
160	A	3	3	1.	15	0.2
161	A	5	5	1.	15	0.333
162	A	8	8	1.	16	0.5
163	A	7	7	1.	16	0.438
164	A	7	7	1.	16	0.438
165	A	6	6	1.	14	0.429
166	A	5	5	1.	12	0.417
167	A	7	7	1.	16	0.438
168	A	4	4	1.	16	0.25
169	A	5	5	1.	16	0.312
170	A	7	6	1.	16	0.375
171	A	3	2	1.	16	0.125
172	A	3	2	1.	16	0.125
173	A	3	2	1.	16	0.125
174	A	3	2	1.	14	0.143
175	A	3	2	1.	12	0.167
176	A	3	2	1.	16	0.125
177	A	3	2	1.	16	0.125
178	A	3	2	1.	16	0.125
179	A	3	2	1.	16	0.125
180	A	9	8	1.	16	0.5
181	A	8	8	1.	16	0.5
182	A	8	7	1.	16	0.438
183	A	7	6	1.	14	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
184	A	6	6	1.	12	0.5
185	A	8	8	1.	16	0.5
186	A	5	4	1.	16	0.25
187	A	6	5	1.	16	0.312
188	A	8	7	1.	16	0.438
189	A	8	8	1.	16	0.5
190	A	7	7	1.	16	0.438
191	A	7	7	1.	16	0.438
192	A	6	6	1.	14	0.429
193	A	5	5	1.	12	0.417
194	A	7	7	1.	16	0.438
195	A	4	4	1.	16	0.25
196	A	5	5	1.	16	0.312
197	A	7	6	1.	16	0.375
198	A	3	2	1.	16	0.125
199	A	3	2	1.	16	0.125
200	A	3	2	1.	16	0.125
201	A	3	2	1.	14	0.143
202	A	3	2	1.	12	0.167
203	A	3	2	1.	16	0.125
204	A	3	2	1.	16	0.125
205	A	3	2	1.	16	0.125
206	A	3	2	1.	16	0.125
207	A	9	8	1.	16	0.5
208	A	8	8	1.	16	0.5
209	A	8	7	1.	16	0.438
210	A	7	6	1.	14	0.429
211	A	6	6	1.	12	0.5
212	A	8	8	1.	16	0.5
213	A	5	4	1.	16	0.25
214	A	6	5	1.	16	0.312
215	A	8	7	1.	16	0.438
216	A	15	12	1.	18	0.667
217	A	14	11	1.	16	0.688
218	A	13	10	1.	14	0.714
219	A	15	12	1.	18	0.667
220	A	6	6	1.	18	0.333
221	A	15	12	1.	18	0.667
222	A	14	11	1.	16	0.688
223	A	13	10	1.	14	0.714
224	A	17	14	1.	18	0.778
225	A	6	6	1.	18	0.333
226	A	15	12	1.	18	0.667
227	A	14	11	1.	16	0.688

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
228	A	13	10	1.	14	0.714
229	A	14	11	1.	18	0.611
230	A	5	5	1.	18	0.278
231	A	15	12	1.	18	0.667
232	A	14	11	1.	16	0.688
233	A	13	10	1.	14	0.714
234	A	17	14	1.	18	0.778
235	A	6	6	1.	18	0.333
236	A	4	3	1.	14	0.214
237	A	4	4	1.	14	0.286
238	A	4	4	1.	14	0.286
239	A	3	3	1.	12	0.25
240	A	2	2	1.	10	0.2
241	A	4	4	1.	14	0.286
242	A	2	2	1.	14	0.143
243	A	3	3	1.	14	0.214
244	A	3	3	1.	19	0.158
245	A	2	2	1.	19	0.105
246	A	2	2	1.	17	0.118
247	A	2	2	1.	6	0.333
248	A	1	1	1.	19	0.053
249	A	2	2	1.	19	0.105
250	A	3	2	1.	19	0.105
251	A	4	2	1.	19	0.105
252	A	5	2	1.	19	0.105
253	A	3	3	1.	21	0.143
254	A	3	3	1.	21	0.143
255	A	3	3	1.	21	0.143
256	A	1	1	1.	21	0.048
257	A	2	2	1.	21	0.095
258	A	3	2	1.	21	0.095
259	A	3	3	1.	21	0.143
260	A	2	2	1.	21	0.095
261	A	2	2	1.	19	0.105
262	A	2	2	1.	8	0.25
263	A	1	1	1.	21	0.048
264	A	2	2	1.	21	0.095
265	A	3	2	1.	21	0.095
266	A	4	2	1.	21	0.095
267	A	3	3	1.	23	0.13
268	A	3	3	1.	23	0.13
269	A	3	3	1.	23	0.13
270	A	1	1	1.	23	0.043
271	A	2	2	1.	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
272	A	3	2	1.	23	0.087
273	A	3	3	1.	21	0.143
274	A	2	2	1.	21	0.095
275	A	2	2	1.	19	0.105
276	A	2	2	1.	8	0.25
277	A	1	1	1.	21	0.048
278	A	2	2	1.	21	0.095
279	A	3	2	1.	21	0.095
280	A	4	2	1.	21	0.095
281	A	3	3	1.	23	0.13
282	A	3	3	1.	23	0.13
283	A	3	3	1.	23	0.13
284	A	1	1	1.	23	0.043
285	A	2	2	1.	23	0.087
286	A	3	2	1.	23	0.087
287	A	3	3	1.	21	0.143
288	A	2	2	1.	21	0.095
289	A	2	2	1.	19	0.105
290	A	2	2	1.	8	0.25
291	A	1	1	1.	21	0.048
292	A	2	2	1.	21	0.095
293	A	3	2	1.	21	0.095
294	A	4	2	1.	21	0.095
295	A	3	3	1.	23	0.13
296	A	3	3	1.	23	0.13
297	A	3	3	1.	23	0.13
298	A	1	1	1.	23	0.043
299	A	2	2	1.	23	0.087
300	A	3	2	1.	23	0.087
301	A	3	2	1.	24	0.083
302	A	5	4	1.	24	0.167
303	A	3	2	1.	24	0.083
304	A	4	4	1.	24	0.167
305	A	2	2	1.	24	0.083
306	A	2	2	1.	24	0.083
307	A	4	4	1.	24	0.167
308	A	3	2	1.	24	0.083
309	A	5	4	1.	24	0.167
310	A	4	3	1.	25	0.12
311	A	5	5	1.	25	0.2
312	A	4	3	1.	25	0.12
313	A	4	4	1.	25	0.16
314	A	3	3	1.	25	0.12
315	A	3	3	1.	25	0.12

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
316	A	4	4	1.	25	0.16
317	A	4	3	1.	25	0.12
318	A	5	5	1.	25	0.2
319	A	3	2	1.	24	0.083
320	A	3	3	1.	24	0.125
321	A	2	2	1.	24	0.083
322	A	3	3	1.	24	0.125
323	A	4	3	1.	24	0.125
324	A	4	3	1.	24	0.125
325	A	3	3	1.	24	0.125
326	A	2	2	1.	24	0.083
327	A	3	3	1.	24	0.125
328	A	4	3	1.	25	0.12
329	A	3	3	1.	25	0.12
330	A	3	3	1.	25	0.12
331	A	3	3	1.	25	0.12
332	A	5	4	1.	25	0.16
333	A	5	4	1.	25	0.16
334	A	3	3	1.	25	0.12
335	A	3	3	1.	25	0.12
336	A	3	3	1.	25	0.12
337	A	2	2	1.	21	0.095
338	A	2	2	1.	19	0.105
339	A	2	2	1.	8	0.25
340	A	4	4	1.57	24	0.167
341	A	4	4	1.	24	0.167
342	A	3	3	1.	22	0.136
343	A	1	1	1.	21	0.048
344	B	3	3	2.03	24	0.125
345	A	5	5	2.	24	0.208
346	A	6	6	1.92	24	0.25
347	A	4	2	1.	21	0.095
348	A	3	3	1.	23	0.13
349	A	3	3	1.	23	0.13
350	A	3	3	1.	23	0.13
351	A	5	5	1.	26	0.192
352	A	5	5	1.	26	0.192
353	A	5	5	1.	26	0.192
354	A	5	5	1.	26	0.192
355	A	4	4	1.	24	0.167
356	A	3	3	1.	23	0.13
357	A	3	3	1.	26	0.115
358	A	4	4	1.	26	0.154
359	A	6	6	1.	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
360	A	3	3	1.	23	0.13
361	A	3	3	1.	23	0.13
362	A	3	3	1.	23	0.13
363	A	3	3	1.	23	0.13
364	A	2	2	1.	22	0.091
365	A	2	2	1.	24	0.083
366	A	2	2	1.	24	0.083
367	A	2	2	1.	24	0.083
368	A	3	3	1.	26	0.115
369	A	3	3	1.	26	0.115
370	A	3	3	1.	26	0.115
371	A	3	3	1.	21	0.143
372	A	3	3	1.	24	0.125
373	A	3	3	1.	24	0.125
374	A	1	1	1.	35	0.029
375	A	2	2	1.	26	0.077
376	A	2	2	1.	26	0.077
377	A	2	2	1.	26	0.077
378	A	2	2	1.	26	0.077
379	A	2	2	1.	26	0.077
380	A	3	3	1.	28	0.107
381	A	3	3	1.	28	0.107
382	A	4	3	1.	28	0.107
383	A	4	3	1.	28	0.107
384	A	3	3	1.	28	0.107
385	A	3	3	1.	28	0.107

Chapter 3

Listing of integrals

3.1 $\int e^{i\tan^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=113

$$\frac{ix^4\sqrt{a^2x^2+1}}{5a}+\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{4ix^2\sqrt{a^2x^2+1}}{15a^3}+\frac{(-45ax+64i)\sqrt{a^2x^2+1}}{120a^5}+\frac{3\sinh^{-1}(ax)}{8a^5}$$

```
[Out] (((-4*I)/15)*x^2*Sqrt[1 + a^2*x^2])/a^3 + (x^3*Sqrt[1 + a^2*x^2])/(4*a^2) +
 ((I/5)*x^4*Sqrt[1 + a^2*x^2])/a + ((64*I - 45*a*x)*Sqrt[1 + a^2*x^2])/(120
*a^5) + (3*ArcSinh[a*x])/(8*a^5)
```

Rubi [A] time = 0.0878502, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5060, 833, 780, 215}

$$\frac{ix^4\sqrt{a^2x^2+1}}{5a}+\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{4ix^2\sqrt{a^2x^2+1}}{15a^3}+\frac{(-45ax+64i)\sqrt{a^2x^2+1}}{120a^5}+\frac{3\sinh^{-1}(ax)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])*x^4,x]
```

```
[Out] (((-4*I)/15)*x^2*Sqrt[1 + a^2*x^2])/a^3 + (x^3*Sqrt[1 + a^2*x^2])/(4*a^2) +
 ((I/5)*x^4*Sqrt[1 + a^2*x^2])/a + ((64*I - 45*a*x)*Sqrt[1 + a^2*x^2])/(120
*a^5) + (3*ArcSinh[a*x])/(8*a^5)
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{i\tan^{-1}(ax)}x^4\,dx &= \int \frac{x^4(1+iax)}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{ix^4\sqrt{1+a^2x^2}}{5a}+\frac{\int \frac{x^3\left(-4ia+5a^2x\right)}{\sqrt{1+a^2x^2}}\,dx}{5a^2}\\
&= \frac{x^3\sqrt{1+a^2x^2}}{4a^2}+\frac{ix^4\sqrt{1+a^2x^2}}{5a}+\frac{\int \frac{x^2\left(-15a^2-16ia^3x\right)}{\sqrt{1+a^2x^2}}\,dx}{20a^4}\\
&= -\frac{4ix^2\sqrt{1+a^2x^2}}{15a^3}+\frac{x^3\sqrt{1+a^2x^2}}{4a^2}+\frac{ix^4\sqrt{1+a^2x^2}}{5a}+\frac{\int \frac{x\left(32ia^3-45a^4x\right)}{\sqrt{1+a^2x^2}}\,dx}{60a^6}\\
&= -\frac{4ix^2\sqrt{1+a^2x^2}}{15a^3}+\frac{x^3\sqrt{1+a^2x^2}}{4a^2}+\frac{ix^4\sqrt{1+a^2x^2}}{5a}+\frac{(64i-45ax)\sqrt{1+a^2x^2}}{120a^5}+\frac{3\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{8a^4}\\
&= -\frac{4ix^2\sqrt{1+a^2x^2}}{15a^3}+\frac{x^3\sqrt{1+a^2x^2}}{4a^2}+\frac{ix^4\sqrt{1+a^2x^2}}{5a}+\frac{(64i-45ax)\sqrt{1+a^2x^2}}{120a^5}+\frac{3\sinh^{-1}(ax)}{8a^5}
\end{aligned}
$$

Mathematica [A] time = 0.0529357, size = 64, normalized size = 0.57

$$\frac{45\sinh^{-1}(ax)+\sqrt{a^2x^2+1}\left(24ia^4x^4+30a^3x^3-32ia^2x^2-45ax+64i\right)}{120a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x])*x^4,x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(64*I - 45*a*x - (32*I)*a^2*x^2 + 30*a^3*x^3 + (24*I)*a^
4*x^4) + 45*ArcSinh[a*x])/(120*a^5)
```

Maple [A] time = 0.071, size = 128, normalized size = 1.1

$$\frac{\frac{i}{5}x^4}{a}\sqrt{a^2x^2+1}-\frac{\frac{4i}{15}x^2}{a^3}\sqrt{a^2x^2+1}+\frac{\frac{8i}{15}}{a^5}\sqrt{a^2x^2+1}+\frac{x^3}{4a^2}\sqrt{a^2x^2+1}-\frac{3x}{8a^4}\sqrt{a^2x^2+1}+\frac{3}{8a^4}\ln\left(a^2x\frac{1}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x)
```

```
[Out] 1/5*I*x^4*(a^2*x^2+1)^(1/2)/a-4/15*I*x^2*(a^2*x^2+1)^(1/2)/a^3+8/15*I/a^5*(
a^2*x^2+1)^(1/2)+1/4*x^3*(a^2*x^2+1)^(1/2)/a^2-3/8/a^4*x*(a^2*x^2+1)^(1/2)+
3/8/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Maxima [A] time = 0.984417, size = 151, normalized size = 1.34

$$\frac{i\sqrt{a^2x^2+1}x^4}{5a}+\frac{\sqrt{a^2x^2+1}x^3}{4a^2}-\frac{4i\sqrt{a^2x^2+1}x^2}{15a^3}-\frac{3\sqrt{a^2x^2+1}x}{8a^4}+\frac{3\operatorname{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\sqrt{a^2}a^4}+\frac{8i\sqrt{a^2x^2+1}}{15a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/5*I*sqrt(a^2*x^2 + 1)*x^4/a + 1/4*sqrt(a^2*x^2 + 1)*x^3/a^2 - 4/15*I*sqrt
(a^2*x^2 + 1)*x^2/a^3 - 3/8*sqrt(a^2*x^2 + 1)*x/a^4 + 3/8*arcsinh(a^2*x/sqr
t(a^2))/(sqrt(a^2)*a^4) + 8/15*I*sqrt(a^2*x^2 + 1)/a^5
```

Fricas [A] time = 1.81326, size = 169, normalized size = 1.5

$$\frac{\left(24i\,a^4x^4+30\,a^3x^3-32i\,a^2x^2-45\,ax+64i\right)\sqrt{a^2x^2+1}-45\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{120\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x, algorithm="fricas")
```

```
[Out] 1/120*((24*I*a^4*x^4 + 30*a^3*x^3 - 32*I*a^2*x^2 - 45*a*x + 64*I)*sqrt(a^2*
x^2 + 1) - 45*log(-a*x + sqrt(a^2*x^2 + 1)))/a^5
```

Sympy [A] time = 5.70249, size = 138, normalized size = 1.22

$$ia\left(\begin{cases}\frac{x^4\sqrt{a^2x^2+1}}{5a^2}-\frac{4x^2\sqrt{a^2x^2+1}}{15a^4}+\frac{8\sqrt{a^2x^2+1}}{15a^6} & \text{for } a\neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)+\frac{x^5}{4\sqrt{a^2x^2+1}}-\frac{x^3}{8a^2\sqrt{a^2x^2+1}}-\frac{3x}{8a^4\sqrt{a^2x^2+1}}+\frac{3\operatorname{asinh}(ax)}{8a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**4,x)
```

```
[Out] I*a*Piecewise((x**4*sqrt(a**2*x**2 + 1)/(5*a**2) - 4*x**2*sqrt(a**2*x**2 +
1)/(15*a**4) + 8*sqrt(a**2*x**2 + 1)/(15*a**6), Ne(a, 0)), (x**6/6, True))
+ x**5/(4*sqrt(a**2*x**2 + 1)) - x**3/(8*a**2*sqrt(a**2*x**2 + 1)) - 3*x/(8
*a**4*sqrt(a**2*x**2 + 1)) + 3*asinh(a*x)/(8*a**5)
```

Giac [A] time = 1.12757, size = 111, normalized size = 0.98

$$\frac{1}{120}\sqrt{a^2x^2+1}\left(\left(2\left(3\left(\frac{4\,ix}{a}+\frac{5}{a^2}\right)x-\frac{16\,i}{a^3}\right)x-\frac{45}{a^4}\right)x+\frac{64\,i}{a^5}\right)-\frac{3\,\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{8\,a^4|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x, algorithm="giac")
```

```
[Out] 1/120*sqrt(a^2*x^2 + 1)*((2*(3*(4*i*x/a + 5/a^2)*x - 16*i/a^3)*x - 45/a^4)*
x + 64*i/a^5) - 3/8*log(-x*abs(a) + sqrt(a^2*x^2 + 1))/(a^4*abs(a))
```

3.2 $\int e^{i \tan^{-1}(ax)} x^3 \, dx$

Optimal. Leaf size=90

$$\frac{ix^3\sqrt{a^2x^2+1}}{4a}+\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{(16+9iax)\sqrt{a^2x^2+1}}{24a^4}+\frac{3i\sinh^{-1}(ax)}{8a^4}$$

```
[Out] (x^2*Sqrt[1 + a^2*x^2])/(3*a^2) + ((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - ((16 +
(9*I)*a*x)*Sqrt[1 + a^2*x^2])/(24*a^4) + (((3*I)/8)*ArcSinh[a*x])/a^4
```

Rubi [A] time = 0.0642709, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5060, 833, 780, 215}

$$\frac{ix^3\sqrt{a^2x^2+1}}{4a}+\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{(16+9iax)\sqrt{a^2x^2+1}}{24a^4}+\frac{3i\sinh^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])*x^3,x]
```

```
[Out] (x^2*Sqrt[1 + a^2*x^2])/(3*a^2) + ((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - ((16 +
(9*I)*a*x)*Sqrt[1 + a^2*x^2])/(24*a^4) + (((3*I)/8)*ArcSinh[a*x])/a^4
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{i\tan^{-1}(ax)}x^3\,dx &= \int \frac{x^3(1+iax)}{\sqrt{1+a^2x^2}}\,dx\\ &= \frac{ix^3\sqrt{1+a^2x^2}}{4a} + \frac{\int \frac{x^2\left(-3ia+4a^2x\right)}{\sqrt{1+a^2x^2}}\,dx}{4a^2}\\ &= \frac{x^2\sqrt{1+a^2x^2}}{3a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} + \frac{\int \frac{x\left(-8a^2-9ia^3x\right)}{\sqrt{1+a^2x^2}}\,dx}{12a^4}\\ &= \frac{x^2\sqrt{1+a^2x^2}}{3a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{(16+9iax)\sqrt{1+a^2x^2}}{24a^4} + \frac{(3i)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{8a^3}\\ &= \frac{x^2\sqrt{1+a^2x^2}}{3a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{(16+9iax)\sqrt{1+a^2x^2}}{24a^4} + \frac{3i\sinh^{-1}(ax)}{8a^4}\end{aligned}$$

Mathematica [A] time = 0.0385748, size = 56, normalized size = 0.62

$$\frac{\sqrt{a^2x^2+1}\left(6ia^3x^3+8a^2x^2-9iax-16\right)+9i\sinh^{-1}(ax)}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x])*x^3,x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(-16 - (9*I)*a*x + 8*a^2*x^2 + (6*I)*a^3*x^3) + (9*I)*Ar
cSinh[a*x])/(24*a^4)
```

Maple [A] time = 0.072, size = 109, normalized size = 1.2

$$\frac{\frac{i}{4}x^3}{a}\sqrt{a^2x^2+1}-\frac{\frac{3i}{8}x}{a^3}\sqrt{a^2x^2+1}+\frac{\frac{3i}{8}}{a^3}\ln\left(a^2x\frac{1}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2}}+\frac{x^2}{3a^2}\sqrt{a^2x^2+1}-\frac{2}{3a^4}\sqrt{a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x)
```

```
[Out] 1/4*I*x^3*(a^2*x^2+1)^(1/2)/a-3/8*I/a^3*x*(a^2*x^2+1)^(1/2)+3/8*I/a^3*ln(a^
2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)+1/3*x^2*(a^2*x^2+1)^(1/2)/a^
2-2/3*(a^2*x^2+1)^(1/2)/a^4
```

Maxima [A] time = 0.990504, size = 126, normalized size = 1.4

$$\frac{i\sqrt{a^2x^2+1}x^3}{4a}+\frac{\sqrt{a^2x^2+1}x^2}{3a^2}-\frac{3i\sqrt{a^2x^2+1}x}{8a^3}+\frac{3i\,\operatorname{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\sqrt{a^2}a^3}-\frac{2\sqrt{a^2x^2+1}}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/4*I*sqrt(a^2*x^2 + 1)*x^3/a + 1/3*sqrt(a^2*x^2 + 1)*x^2/a^2 - 3/8*I*sqrt(
a^2*x^2 + 1)*x/a^3 + 3/8*I*arcsinh(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^3) - 2/3*s
```

```
qrt(a^2*x^2 + 1)/a^4
```

Fricas [A] time = 1.71779, size = 144, normalized size = 1.6

$$\frac{\left(6i\,a^3x^3+8\,a^2x^2-9i\,ax-16\right)\sqrt{a^2x^2+1}-9i\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/24*((6*I*a^3*x^3 + 8*a^2*x^2 - 9*I*a*x - 16)*sqrt(a^2*x^2 + 1) - 9*I*log(
-a*x + sqrt(a^2*x^2 + 1)))/a^4
```

Sympy [A] time = 5.4138, size = 119, normalized size = 1.32

$$\frac{iax^5}{4\sqrt{a^2x^2+1}}+\begin{cases}\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{2\sqrt{a^2x^2+1}}{3a^4} & \text{for}\ a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}-\frac{ix^3}{8a\sqrt{a^2x^2+1}}-\frac{3ix}{8a^3\sqrt{a^2x^2+1}}+\frac{3i\operatorname{asinh}(ax)}{8a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**3,x)
```

```
[Out] I*a*x**5/(4*sqrt(a**2*x**2 + 1)) + Piecewise((x**2*sqrt(a**2*x**2 + 1)/(3*a
**2) - 2*sqrt(a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) - I*x**3/
(8*a*sqrt(a**2*x**2 + 1)) - 3*I*x/(8*a**3*sqrt(a**2*x**2 + 1)) + 3*I*asinh(
a*x)/(8*a**4)
```

Giac [A] time = 1.11541, size = 99, normalized size = 1.1

$$\frac{1}{24}\sqrt{a^2x^2+1}\left(\left(2\left(\frac{3\,ix}{a}+\frac{4}{a^2}\right)x-\frac{9\,i}{a^3}\right)x-\frac{16}{a^4}\right)-\frac{3\,i\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{8\,a^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/24*sqrt(a^2*x^2 + 1)*((2*(3*i*x/a + 4/a^2)*x - 9*i/a^3)*x - 16/a^4) - 3/8
*i*log(-x*abs(a) + sqrt(a^2*x^2 + 1))/(a^3*abs(a))
```

3.3 $\int e^{i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=75

$$\frac{i\left(a^2x^2+1\right)^{3/2}}{3a^3}+\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{i\sqrt{a^2x^2+1}}{a^3}-\frac{\sinh^{-1}(ax)}{2a^3}$$

```
[Out] ((-I)*Sqrt[1 + a^2*x^2])/a^3 + (x*Sqrt[1 + a^2*x^2])/(2*a^2) + ((I/3)*(1 +
a^2*x^2)^(3/2))/a^3 - ArcSinh[a*x]/(2*a^3)
```

Rubi [A] time = 0.0471978, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5060, 797, 641, 195, 215}

$$\frac{i\left(a^2x^2+1\right)^{3/2}}{3a^3}+\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{i\sqrt{a^2x^2+1}}{a^3}-\frac{\sinh^{-1}(ax)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])*x^2,x]
```

```
[Out] ((-I)*Sqrt[1 + a^2*x^2])/a^3 + (x*Sqrt[1 + a^2*x^2])/(2*a^2) + ((I/3)*(1 +
a^2*x^2)^(3/2))/a^3 - ArcSinh[a*x]/(2*a^3)
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{i\tan^{-1}(ax)}x^2\,dx = \int \frac{x^2(1+iax)}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{\int \frac{1+iax}{\sqrt{1+a^2x^2}}\,dx}{a^2} + \frac{\int (1+iax)\sqrt{1+a^2x^2}\,dx}{a^2}$$
$$= -\frac{i\sqrt{1+a^2x^2}}{a^3} + \frac{i\left(1+a^2x^2\right)^{3/2}}{3a^3} - \frac{\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{a^2} + \frac{\int \sqrt{1+a^2x^2}\,dx}{a^2}$$
$$= -\frac{i\sqrt{1+a^2x^2}}{a^3} + \frac{x\sqrt{1+a^2x^2}}{2a^2} + \frac{i\left(1+a^2x^2\right)^{3/2}}{3a^3} - \frac{\sinh^{-1}(ax)}{a^3} + \frac{\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a^2}$$
$$= -\frac{i\sqrt{1+a^2x^2}}{a^3} + \frac{x\sqrt{1+a^2x^2}}{2a^2} + \frac{i\left(1+a^2x^2\right)^{3/2}}{3a^3} - \frac{\sinh^{-1}(ax)}{2a^3}$$

Mathematica [A] time = 0.0312297, size = 46, normalized size = 0.61

$$\frac{-3\sinh^{-1}(ax) + \left(2ia^2x^2 + 3ax - 4i\right)\sqrt{a^2x^2+1}}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x])*x^2,x]
```

```
[Out] ((-4*I + 3*a*x + (2*I)*a^2*x^2)*Sqrt[1 + a^2*x^2] - 3*ArcSinh[a*x])/(6*a^3)
```

Maple [A] time = 0.074, size = 89, normalized size = 1.2

$$\frac{\frac{i}{3}x^2}{a}\sqrt{a^2x^2+1} - \frac{\frac{2i}{3}}{a^3}\sqrt{a^2x^2+1} + \frac{x}{2a^2}\sqrt{a^2x^2+1} - \frac{1}{2a^2}\ln\left(a^2x\frac{1}{\sqrt{a^2}} + \sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x)
```

```
[Out] 1/3*I/a*x^2*(a^2*x^2+1)^(1/2)-2/3*I/a^3*(a^2*x^2+1)^(1/2)+1/2*x*(a^2*x^2+1)
^(1/2)/a^2-1/2/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Maxima [A] time = 1.01625, size = 100, normalized size = 1.33

$$\frac{i\sqrt{a^2x^2+1}x^2}{3a} + \frac{\sqrt{a^2x^2+1}x}{2a^2} - \frac{\operatorname{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}a^2} - \frac{2i\sqrt{a^2x^2+1}}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*I*sqrt(a^2*x^2 + 1)*x^2/a + 1/2*sqrt(a^2*x^2 + 1)*x/a^2 - 1/2*arcsinh(a
^2*x/sqrt(a^2))/(sqrt(a^2)*a^2) - 2/3*I*sqrt(a^2*x^2 + 1)/a^3
```

Fricas [A] time = 1.68213, size = 123, normalized size = 1.64

$$\frac{\sqrt{a^2x^2+1}\left(2i\,a^2x^2+3\,ax-4i\right)+3\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(a^2*x^2 + 1)*(2*I*a^2*x^2 + 3*a*x - 4*I) + 3*log(-a*x + sqrt(a^2*
x^2 + 1)))/a^3
```

Sympy [A] time = 3.82505, size = 75, normalized size = 1.

$$ia\left(\begin{cases}\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{2\sqrt{a^2x^2+1}}{3a^4} & \text{for}\ a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)+\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\operatorname{asinh}(ax)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**2,x)
```

```
[Out] I*a*Piecewise((x**2*sqrt(a**2*x**2 + 1)/(3*a**2) - 2*sqrt(a**2*x**2 + 1)/(3
*a**4), Ne(a, 0)), (x**4/4, True)) + x*sqrt(a**2*x**2 + 1)/(2*a**2) - asinh
(a*x)/(2*a**3)
```

Giac [A] time = 1.13049, size = 85, normalized size = 1.13

$$\frac{1}{6}\sqrt{a^2x^2+1}\left(\left(\frac{2\,ix}{a}+\frac{3}{a^2}\right)x-\frac{4\,i}{a^3}\right)+\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{2\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(a^2*x^2 + 1)*((2*i*x/a + 3/a^2)*x - 4*i/a^3) + 1/2*log(-x*abs(a) +
 sqrt(a^2*x^2 + 1))/(a^2*abs(a))
```

3.4 $\int e^{i \tan^{-1}(ax)} x \, dx$

Optimal. Leaf size=42

$$\frac{(2+iax)\sqrt{a^2x^2+1}}{2a^2} - \frac{i\sinh^{-1}(ax)}{2a^2}$$

```
[Out] ((2 + I*a*x)*Sqrt[1 + a^2*x^2])/(2*a^2) - ((I/2)*ArcSinh[a*x])/a^2
```

Rubi [A] time = 0.019259, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5060, 780, 215}

$$\frac{(2+iax)\sqrt{a^2x^2+1}}{2a^2} - \frac{i\sinh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])*x,x]
```

```
[Out] ((2 + I*a*x)*Sqrt[1 + a^2*x^2])/(2*a^2) - ((I/2)*ArcSinh[a*x])/a^2
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{i \tan^{-1}(ax)} x \, dx &= \int \frac{x(1+iax)}{\sqrt{1+a^2x^2}} \, dx \\
&= \frac{(2+iax)\sqrt{1+a^2x^2}}{2a^2} - \frac{i\int \frac{1}{\sqrt{1+a^2x^2}} \, dx}{2a} \\
&= \frac{(2+iax)\sqrt{1+a^2x^2}}{2a^2} - \frac{i\sinh^{-1}(ax)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0272107, size = 38, normalized size = 0.9

$$\frac{(2+iax)\sqrt{a^2x^2+1} - i\sinh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x])*x,x]
```

```
[Out] ((2 + I*a*x)*Sqrt[1 + a^2*x^2] - I*ArcSinh[a*x])/(2*a^2)
```

Maple [A] time = 0.067, size = 69, normalized size = 1.6

$$\frac{\frac{i}{2}x}{a}\sqrt{a^2x^2+1}-\frac{\frac{i}{2}}{a}\ln\left(a^2x\frac{1}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2}}+\frac{1}{a^2}\sqrt{a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x)
```

```
[Out] 1/2*I/a*x*(a^2*x^2+1)^(1/2)-1/2*I/a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))
/(a^2)^(1/2)+(a^2*x^2+1)^(1/2)/a^2
```

Maxima [A] time = 0.982332, size = 73, normalized size = 1.74

$$\frac{i\sqrt{a^2x^2+1}x}{2a}-\frac{i\,\text{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}a}+\frac{\sqrt{a^2x^2+1}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*I*sqrt(a^2*x^2 + 1)*x/a - 1/2*I*arcsinh(a^2*x/sqrt(a^2))/(sqrt(a^2)*a)
+ sqrt(a^2*x^2 + 1)/a^2
```

Fricas [A] time = 1.68409, size = 101, normalized size = 2.4

$$\frac{\sqrt{a^2x^2+1}(i\,ax+2)+i\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2*x^2 + 1)*(I*a*x + 2) + I*log(-a*x + sqrt(a^2*x^2 + 1)))/a^2
```

Sympy [A] time = 4.43354, size = 51, normalized size = 1.21

$$\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ \frac{\sqrt{a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}+\frac{ix\sqrt{a^2x^2+1}}{2a}-\frac{i\,\text{asinh}\,(ax)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x,x)
```

```
[Out] Piecewise((x**2/2, Eq(a**2, 0)), (sqrt(a**2*x**2 + 1)/a**2, True)) + I*x*sq
rt(a**2*x**2 + 1)/(2*a) - I*asinh(a*x)/(2*a**2)
```

Giac [A] time = 1.11395, size = 73, normalized size = 1.74

$$\frac{1}{2}\sqrt{a^2x^2+1}\left(\frac{ix}{a}+\frac{2}{a^2}\right)+\frac{i\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{2\,a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a^2*x^2 + 1)*(i*x/a + 2/a^2) + 1/2*i*log(-x*abs(a) + sqrt(a^2*x^2
+ 1))/(a*abs(a))
```

3.5 $\int e^{i \tan^{-1}(ax)} dx$

Optimal. Leaf size=29

$$\frac{\sinh^{-1}(ax)}{a}+\frac{i\sqrt{a^2x^2+1}}{a}$$

```
[Out] (I*Sqrt[1 + a^2*x^2])/a + ArcSinh[a*x]/a
```

Rubi [A] time = 0.0090885, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5059, 641, 215}

$$\frac{\sinh^{-1}(ax)}{a}+\frac{i\sqrt{a^2x^2+1}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x]),x]
```

```
[Out] (I*Sqrt[1 + a^2*x^2])/a + ArcSinh[a*x]/a
```

Rule 5059

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 - I*a*x)^((I*n + 1)/2)
/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2]), x] /; FreeQ[a, x] && Intege
rQ[(I*n - 1)/2]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{i \tan^{-1}(ax)} dx &= \int \frac{1+iax}{\sqrt{1+a^2x^2}} dx \\
&= \frac{i\sqrt{1+a^2x^2}}{a}+\int \frac{1}{\sqrt{1+a^2x^2}} dx \\
&= \frac{i\sqrt{1+a^2x^2}}{a}+\frac{\sinh^{-1}(ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0147939, size = 26, normalized size = 0.9

$$\frac{\sinh^{-1}(ax)+i\sqrt{a^2x^2+1}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x]),x]
```

```
[Out] (I*Sqrt[1 + a^2*x^2] + ArcSinh[a*x])/a
```

Maple [A] time = 0.053, size = 48, normalized size = 1.7

$$\ln\left(a^2x\frac{1}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2}}+\frac{i}{a}\sqrt{a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)+I*(a^2*x^2+1)^(1/2)/a
```

Maxima [A] time = 1.02399, size = 46, normalized size = 1.59

$$\frac{\operatorname{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}+\frac{i\sqrt{a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(a^2*x/sqrt(a^2))/sqrt(a^2) + I*sqrt(a^2*x^2 + 1)/a
```

Fricas [A] time = 1.61595, size = 77, normalized size = 2.66

$$\frac{i\sqrt{a^2x^2+1}-\log\left(-ax+\sqrt{a^2x^2+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (I*sqrt(a^2*x^2 + 1) - log(-a*x + sqrt(a^2*x^2 + 1)))/a
```

Sympy [A] time = 1.3362, size = 68, normalized size = 2.34

$$ia\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ \frac{\sqrt{a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)+\begin{cases}\sqrt{-\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\\ \sqrt{\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] I*a*Piecewise((x**2/2, Eq(a**2, 0)), (sqrt(a**2*x**2 + 1)/a**2, True)) + Pi
ecewise((sqrt(-1/a**2)*asin(x*sqrt(-a**2)), a**2 < 0), (sqrt(a**(-2))*asinh
(x*sqrt(a**2)), a**2 > 0))
```

Giac [A] time = 1.11413, size = 55, normalized size = 1.9

$$\frac{\sqrt{a^2x^2+1}i}{a} - \frac{\log\left(-x|a| + \sqrt{a^2x^2+1}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(a^2*x^2 + 1)*i/a - log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a)
```

3.6 $\int \frac{e^{i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=25

$$-\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)+i\sinh^{-1}(ax)$$

```
[Out] I*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rubi [A] time = 0.0365401, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5060, 844, 215, 266, 63, 208}

$$-\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)+i\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/x,x]

[Out] I*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{i\tan^{-1}(ax)}}{x}\,dx &= \int \frac{1+iax}{x\sqrt{1+a^2x^2}}\,dx\\
&= (ia)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx + \int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\
&= i\sinh^{-1}(ax) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right)\\
&= i\sinh^{-1}(ax) + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a^2}\\
&= i\sinh^{-1}(ax) - \tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0139497, size = 29, normalized size = 1.16

$$-\log\left(\sqrt{a^2x^2+1}+1\right) + i\sinh^{-1}(ax) + \log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a*x])/x,x]
```

```
[Out] I*ArcSinh[a*x] + Log[x] - Log[1 + Sqrt[1 + a^2*x^2]]
```

Maple [B] time = 0.063, size = 48, normalized size = 1.9

$$ia\ln\left(a^2x\frac{1}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2}} - \text{Artanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] I*a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)-arctanh(1/(a^2*x^2+
1)^(1/2))
```

Maxima [A] time = 1.01602, size = 45, normalized size = 1.8

$$\frac{i\,a\,\text{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}} - \text{arsinh}\left(\frac{1}{\sqrt{a^2}|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] I*a*arcsinh(a^2*x/sqrt(a^2))/sqrt(a^2) - arcsinh(1/(sqrt(a^2)*abs(x)))
```

Fricas [B] time = 1.75347, size = 143, normalized size = 5.72

$$-\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-i\log\left(-ax+\sqrt{a^2x^2+1}\right)+\log\left(-ax+\sqrt{a^2x^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -log(-a*x + sqrt(a^2*x^2 + 1) + 1) - I*log(-a*x + sqrt(a^2*x^2 + 1)) + log(
-a*x + sqrt(a^2*x^2 + 1) - 1)
```

Sympy [A] time = 2.89457, size = 53, normalized size = 2.12

$$ia\left(\begin{cases}\sqrt{-\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\\ \sqrt{\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\end{cases}\right)-\operatorname{asinh}\left(\frac{1}{ax}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] I*a*Piecewise((sqrt(-1/a**2)*asin(x*sqrt(-a**2)), a**2 < 0), (sqrt(a**(-2))
*asinh(x*sqrt(a**2)), a**2 > 0)) - asinh(1/(a*x))
```

Giac [B] time = 1.14457, size = 93, normalized size = 3.72

$$-\frac{ai\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|}-\log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right)+\log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -a*i*log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a) - log(abs(-x*abs(a) + sqrt(a
^2*x^2 + 1) + 1)) + log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) - 1))
```

3.7 $\int \frac{e^{i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=38

$$-\frac{\sqrt{a^2x^2+1}}{x}-ia\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -(Sqrt[1 + a^2*x^2]/x) - I*a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rubi [A] time = 0.0382376, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5060, 807, 266, 63, 208}

$$-\frac{\sqrt{a^2x^2+1}}{x}-ia\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/x^2,x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/x) - I*a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{1+iax}{x^2\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{x}+(ia)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{x}+\frac{1}{2}(ia)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1+a^2x^2}}{x}+\frac{i\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)}{a}\\
&= -\frac{\sqrt{1+a^2x^2}}{x}-ia\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0295662, size = 47, normalized size = 1.24

$$
-\frac{\sqrt{a^2x^2+1}}{x}-ia\log\left(\sqrt{a^2x^2+1}+1\right)+ia\log(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a*x])/x^2,x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/x) + I*a*Log[x] - I*a*Log[1 + Sqrt[1 + a^2*x^2]]
```

Maple [A] time = 0.063, size = 34, normalized size = 0.9

$$
-\frac{1}{x}\sqrt{a^2x^2+1}-ia\text{Artanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] -(a^2*x^2+1)^(1/2)/x-I*a*arctanh(1/(a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.00049, size = 42, normalized size = 1.11

$$
-i\,a\,\text{arsinh}\left(\frac{1}{\sqrt{a^2}|x|}\right)-\frac{\sqrt{a^2x^2+1}}{x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -I*a*arcsinh(1/(sqrt(a^2)*abs(x))) - sqrt(a^2*x^2 + 1)/x
```

Fricas [B] time = 1.75318, size = 154, normalized size = 4.05

$$\frac{-i\,ax\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+i\,ax\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-ax-\sqrt{a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (-I*a*x*log(-a*x + sqrt(a^2*x^2 + 1) + 1) + I*a*x*log(-a*x + sqrt(a^2*x^2 +
 1) - 1) - a*x - sqrt(a^2*x^2 + 1))/x
```

Sympy [A] time = 2.52476, size = 26, normalized size = 0.68

$$-a\sqrt{1+\frac{1}{a^2x^2}}-ia\operatorname{asinh}\left(\frac{1}{ax}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] -a*sqrt(1 + 1/(a**2*x**2)) - I*a*asinh(1/(a*x))
```

Giac [B] time = 1.17661, size = 103, normalized size = 2.71

$$-ai\log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right)+ai\log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right)+\frac{2\,|a|}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -a*i*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) + 1)) + a*i*log(abs(-x*abs(a) +
sqrt(a^2*x^2 + 1) - 1)) + 2*abs(a)/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)
```

3.8 $\int \frac{e^{i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=63

$$-\frac{ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -Sqrt[1 + a^2*x^2]/(2*x^2) - (I*a*Sqrt[1 + a^2*x^2])/x + (a^2*ArcTanh[Sqrt[
1 + a^2*x^2]])/2
```

Rubi [A] time = 0.0502899, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5060, 835, 807, 266, 63, 208}

$$-\frac{ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/x^3,x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(2*x^2) - (I*a*Sqrt[1 + a^2*x^2])/x + (a^2*ArcTanh[Sqrt[
1 + a^2*x^2]])/2
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{1+iax}{x^3\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{1}{2}\int \frac{-2ia+a^2x}{x^2\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{ia\sqrt{1+a^2x^2}}{x}-\frac{1}{2}a^2\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{ia\sqrt{1+a^2x^2}}{x}-\frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{ia\sqrt{1+a^2x^2}}{x}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{ia\sqrt{1+a^2x^2}}{x}+\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0396952, size = 57, normalized size = 0.9

$$\frac{1}{2}\left(\frac{(-1-2iax)\sqrt{a^2x^2+1}}{x^2}+a^2\log\left(\sqrt{a^2x^2+1}+1\right)+a^2(-\log(x))\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a*x])/x^3,x]
```

```
[Out] (((-1 - (2*I)*a*x)*Sqrt[1 + a^2*x^2])/x^2 - a^2*Log[x] + a^2*Log[1 + Sqrt[1
 + a^2*x^2]])/2
```

Maple [A] time = 0.071, size = 53, normalized size = 0.8

$$-\frac{1}{2\,x^2}\sqrt{a^2x^2+1}+\frac{a^2}{2}\text{Artanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)-\frac{ia}{x}\sqrt{a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x)
```

```
[Out] -1/2*(a^2*x^2+1)^(1/2)/x^2+1/2*a^2*arctanh(1/(a^2*x^2+1)^(1/2))-I*a*(a^2*x^
2+1)^(1/2)/x
```

Maxima [A] time = 0.976611, size = 68, normalized size = 1.08

$$\frac{1}{2}a^2\operatorname{arsinh}\left(\frac{1}{\sqrt{a^2}|x|}\right)-\frac{i\sqrt{a^2x^2+1}a}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*arcsinh(1/(sqrt(a^2)*abs(x))) - I*sqrt(a^2*x^2 + 1)*a/x - 1/2*sqrt(
a^2*x^2 + 1)/x^2
```

Fricas [A] time = 1.68001, size = 197, normalized size = 3.13

$$\frac{a^2x^2\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-a^2x^2\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-2i\,a^2x^2+\sqrt{a^2x^2+1}(-2i\,ax-1)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - a^2*x^2*log(-a*x + sqrt(a^
2*x^2 + 1) - 1) - 2*I*a^2*x^2 + sqrt(a^2*x^2 + 1)*(-2*I*a*x - 1))/x^2
```

Sympy [A] time = 3.53996, size = 48, normalized size = 0.76

$$-ia^2\sqrt{1+\frac{1}{a^2x^2}}+\frac{a^2\operatorname{asinh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{1+\frac{1}{a^2x^2}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] -I*a**2*sqrt(1 + 1/(a**2*x**2)) + a**2*asinh(1/(a*x))/2 - a*sqrt(1 + 1/(a**
2*x**2))/(2*x)
```

Giac [B] time = 1.13557, size = 209, normalized size = 3.32

$$\frac{1}{2}a^2\log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right)-\frac{1}{2}a^2\log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right)+\frac{\left(x|a|-\sqrt{a^2x^2+1}\right)^3a^2+2\left(x|a|-\sqrt{a^2x^2+1}\right)}{\left(\left(x|a|-\sqrt{a^2x^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) + 1)) - 1/2*a^2*log(abs(-x*ab
s(a) + sqrt(a^2*x^2 + 1) - 1)) + ((x*abs(a) - sqrt(a^2*x^2 + 1))^3*a^2 + 2*
(x*abs(a) - sqrt(a^2*x^2 + 1))^2*a*i*abs(a) + (x*abs(a) - sqrt(a^2*x^2 + 1)
)*a^2 - 2*a*i*abs(a))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)^2
```

3.9 $\int \frac{e^{i \tan^{-1}(ax)}}{x^4} \, dx$

Optimal. Leaf size=90

$$\frac{2a^2\sqrt{a^2x^2+1}}{3x}-\frac{ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{1}{2}ia^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -Sqrt[1 + a^2*x^2]/(3*x^3) - ((I/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (2*a^2*Sqrt[
1 + a^2*x^2])/(3*x) + (I/2)*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rubi [A] time = 0.0694304, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5060, 835, 807, 266, 63, 208}

$$\frac{2a^2\sqrt{a^2x^2+1}}{3x}-\frac{ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{1}{2}ia^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/x^4,x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(3*x^3) - ((I/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (2*a^2*Sqrt[
1 + a^2*x^2])/(3*x) + (I/2)*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{i\tan^{-1}(ax)}}{x^4}\,dx &= \int \frac{1+iax}{x^4\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{1}{3}\int \frac{-3ia+2a^2x}{x^3\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{ia\sqrt{1+a^2x^2}}{2x^2}+\frac{1}{6}\int \frac{-4a^2-3ia^3x}{x^2\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{ia\sqrt{1+a^2x^2}}{2x^2}+\frac{2a^2\sqrt{1+a^2x^2}}{3x}-\frac{1}{2}\left(ia^3\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{ia\sqrt{1+a^2x^2}}{2x^2}+\frac{2a^2\sqrt{1+a^2x^2}}{3x}-\frac{1}{4}\left(ia^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{ia\sqrt{1+a^2x^2}}{2x^2}+\frac{2a^2\sqrt{1+a^2x^2}}{3x}-\frac{1}{2}(ia)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)\\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{ia\sqrt{1+a^2x^2}}{2x^2}+\frac{2a^2\sqrt{1+a^2x^2}}{3x}+\frac{1}{2}ia^3\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0512706, size = 70, normalized size = 0.78

$$\frac{1}{6}\left(\frac{\sqrt{a^2x^2+1}\left(4a^2x^2-3iax-2\right)}{x^3}+3ia^3\log\left(\sqrt{a^2x^2+1}+1\right)-3ia^3\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a*x])/x^4,x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-2 - (3*I)*a*x + 4*a^2*x^2))/x^3 - (3*I)*a^3*Log[x] +
(3*I)*a^3*Log[1 + Sqrt[1 + a^2*x^2]])/6
```

Maple [A] time = 0.067, size = 75, normalized size = 0.8

$$ia\left(-\frac{1}{2x^2}\sqrt{a^2x^2+1}+\frac{a^2}{2}\text{Artanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)-\frac{1}{3x^3}\sqrt{a^2x^2+1}+\frac{2a^2}{3x}\sqrt{a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x)
```

```
[Out] I*a*(-1/2*(a^2*x^2+1)^(1/2)/x^2+1/2*a^2*arctanh(1/(a^2*x^2+1)^(1/2)))-1/3*(
a^2*x^2+1)^(1/2)/x^3+2/3*a^2*(a^2*x^2+1)^(1/2)/x
```

Maxima [A] time = 0.990776, size = 93, normalized size = 1.03

$$\frac{1}{2} i a^3 \operatorname{arsinh}\left(\frac{1}{\sqrt{a^2}|x|}\right) + \frac{2\sqrt{a^2x^2+1}a^2}{3x} - \frac{i\sqrt{a^2x^2+1}a}{2x^2} - \frac{\sqrt{a^2x^2+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*I*a^3*arcsinh(1/(sqrt(a^2)*abs(x))) + 2/3*sqrt(a^2*x^2 + 1)*a^2/x - 1/2
*I*sqrt(a^2*x^2 + 1)*a/x^2 - 1/3*sqrt(a^2*x^2 + 1)/x^3
```

Fricas [A] time = 1.69883, size = 220, normalized size = 2.44

$$\frac{3i a^3x^3 \log\left(-ax + \sqrt{a^2x^2+1}+1\right) - 3i a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}-1\right) + 4a^3x^3 + \left(4a^2x^2 - 3i ax - 2\right)\sqrt{a^2x^2+1}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*I*a^3*x^3*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 3*I*a^3*x^3*log(-a*x +
 sqrt(a^2*x^2 + 1) - 1) + 4*a^3*x^3 + (4*a^2*x^2 - 3*I*a*x - 2)*sqrt(a^2*x^
2 + 1))/x^3
```

Sympy [A] time = 3.86385, size = 75, normalized size = 0.83

$$\frac{2a^3\sqrt{1+\frac{1}{a^2x^2}}}{3} + \frac{ia^3\operatorname{asinh}\left(\frac{1}{ax}\right)}{2} - \frac{ia^2\sqrt{1+\frac{1}{a^2x^2}}}{2x} - \frac{a\sqrt{1+\frac{1}{a^2x^2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] 2*a**3*sqrt(1 + 1/(a**2*x**2))/3 + I*a**3*asinh(1/(a*x))/2 - I*a**2*sqrt(1
+ 1/(a**2*x**2))/(2*x) - a*sqrt(1 + 1/(a**2*x**2))/(3*x**2)
```

Giac [B] time = 1.14421, size = 221, normalized size = 2.46

$$\frac{1}{2}a^3 i \log\left(\left|-x|a| + \sqrt{a^2x^2+1}+1\right|\right) - \frac{1}{2}a^3 i\log\left(\left|-x|a| + \sqrt{a^2x^2+1}-1\right|\right) + \frac{3\left(x|a| - \sqrt{a^2x^2+1}\right)^5 a^3 i - 3\left(x|a| - \sqrt{a^2x^2+1}\right.}{3\left(\left(x|a| - \sqrt{a^2x^2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*a^3*i*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) + 1)) - 1/2*a^3*i*log(abs(-
x*abs(a) + sqrt(a^2*x^2 + 1) - 1)) + 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 + 1))^
5*a^3*i - 3*(x*abs(a) - sqrt(a^2*x^2 + 1))*a^3*i + 12*(x*abs(a) - sqrt(a^2*
x^2 + 1))^2*a^2*abs(a) - 4*a^2*abs(a))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 -
1)^3
```

3.10 $\int \frac{e^{i\tan^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=113

$$\frac{2ia^3\sqrt{a^2x^2+1}}{3x}+\frac{3a^2\sqrt{a^2x^2+1}}{8x^2}-\frac{ia\sqrt{a^2x^2+1}}{3x^3}-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{3}{8}a^4\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -Sqrt[1 + a^2*x^2]/(4*x^4) - ((I/3)*a*Sqrt[1 + a^2*x^2])/x^3 + (3*a^2*Sqrt[
1 + a^2*x^2])/(8*x^2) + (((2*I)/3)*a^3*Sqrt[1 + a^2*x^2])/x - (3*a^4*ArcTan
h[Sqrt[1 + a^2*x^2]])/8
```

Rubi [A] time = 0.0904184, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5060, 835, 807, 266, 63, 208}

$$\frac{2ia^3\sqrt{a^2x^2+1}}{3x}+\frac{3a^2\sqrt{a^2x^2+1}}{8x^2}-\frac{ia\sqrt{a^2x^2+1}}{3x^3}-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{3}{8}a^4\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/x^5,x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(4*x^4) - ((I/3)*a*Sqrt[1 + a^2*x^2])/x^3 + (3*a^2*Sqrt[
1 + a^2*x^2])/(8*x^2) + (((2*I)/3)*a^3*Sqrt[1 + a^2*x^2])/x - (3*a^4*ArcTan
h[Sqrt[1 + a^2*x^2]])/8
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{i\tan^{-1}(ax)}}{x^5}\,dx &= \int \frac{1+iax}{x^5\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{1}{4}\int \frac{-4ia+3a^2x}{x^4\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{1}{12}\int \frac{-9a^2-8ia^3x}{x^3\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{1}{24}\int \frac{16ia^3-9a^4x}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} + \frac{2ia^3\sqrt{1+a^2x^2}}{3x} + \frac{1}{8}\left(3a^4\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} + \frac{2ia^3\sqrt{1+a^2x^2}}{3x} + \frac{1}{16}\left(3a^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx \right. \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} + \frac{2ia^3\sqrt{1+a^2x^2}}{3x} + \frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \right. \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} + \frac{2ia^3\sqrt{1+a^2x^2}}{3x} - \frac{3}{8}a^4\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0574363, size = 76, normalized size = 0.67

$$
\frac{1}{24}\left(\frac{\sqrt{a^2x^2+1}\left(16ia^3x^3+9a^2x^2-8iax-6\right)}{x^4} - 9a^4\log\left(\sqrt{a^2x^2+1}+1\right) + 9a^4\log(x)\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a*x])/x^5,x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-6 - (8*I)*a*x + 9*a^2*x^2 + (16*I)*a^3*x^3))/x^4 + 9*
a^4*Log[x] - 9*a^4*Log[1 + Sqrt[1 + a^2*x^2]])/24
```

Maple [A] time = 0.07, size = 97, normalized size = 0.9

$$
-\frac{1}{4x^4}\sqrt{a^2x^2+1} - \frac{3a^2}{4}\left(-\frac{1}{2x^2}\sqrt{a^2x^2+1} + \frac{a^2}{2}\text{Artanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right) + ia\left(-\frac{1}{3x^3}\sqrt{a^2x^2+1} + \frac{2a^2}{3x}\sqrt{a^2x^2+1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x)
```

```
[Out] -1/4*(a^2*x^2+1)^(1/2)/x^4-3/4*a^2*(-1/2*(a^2*x^2+1)^(1/2)/x^2+1/2*a^2*arct
anh(1/(a^2*x^2+1)^(1/2)))+I*a*(-1/3*(a^2*x^2+1)^(1/2)/x^3+2/3*a^2*(a^2*x^2+
1)^(1/2)/x)
```

Maxima [A] time = 1.04937, size = 119, normalized size = 1.05

$$-\frac{3}{8}\,a^{4}\,\operatorname{arsinh}\left(\frac{1}{\sqrt{a^{2}}|x|}\right)+\frac{2i\,\sqrt{a^{2}x^{2}+1}a^{3}}{3\,x}+\frac{3\,\sqrt{a^{2}x^{2}+1}a^{2}}{8\,x^{2}}-\frac{i\,\sqrt{a^{2}x^{2}+1}a}{3\,x^{3}}-\frac{\sqrt{a^{2}x^{2}+1}}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -3/8*a^4*arcsinh(1/(sqrt(a^2)*abs(x))) + 2/3*I*sqrt(a^2*x^2 + 1)*a^3/x + 3/
8*sqrt(a^2*x^2 + 1)*a^2/x^2 - 1/3*I*sqrt(a^2*x^2 + 1)*a/x^3 - 1/4*sqrt(a^2*
x^2 + 1)/x^4
```

Fricas [A] time = 1.72206, size = 242, normalized size = 2.14

$$-\frac{9\,a^{4}x^{4}\log\left(-ax+\sqrt{a^{2}x^{2}+1}+1\right)-9\,a^{4}x^{4}\log\left(-ax+\sqrt{a^{2}x^{2}+1}-1\right)-16i\,a^{4}x^{4}-\left(16i\,a^{3}x^{3}+9\,a^{2}x^{2}-8i\,ax-6\right)\sqrt{a^{2}x^{2}}}{24\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(9*a^4*x^4*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 9*a^4*x^4*log(-a*x + s
qrt(a^2*x^2 + 1) - 1) - 16*I*a^4*x^4 - (16*I*a^3*x^3 + 9*a^2*x^2 - 8*I*a*x
- 6)*sqrt(a^2*x^2 + 1))/x^4
```

Sympy [A] time = 5.44941, size = 122, normalized size = 1.08

$$\frac{2ia^{4}\sqrt{1+\frac{1}{a^{2}x^{2}}}}{3}-\frac{3a^{4}\operatorname{asinh}\left(\frac{1}{ax}\right)}{8}+\frac{3a^{3}}{8x\sqrt{1+\frac{1}{a^{2}x^{2}}}}-\frac{ia^{2}\sqrt{1+\frac{1}{a^{2}x^{2}}}}{3x^{2}}+\frac{a}{8x^{3}\sqrt{1+\frac{1}{a^{2}x^{2}}}}-\frac{1}{4ax^{5}\sqrt{1+\frac{1}{a^{2}x^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x**5,x)
```

```
[Out] 2*I*a**4*sqrt(1 + 1/(a**2*x**2))/3 - 3*a**4*asinh(1/(a*x))/8 + 3*a**3/(8*x*
sqrt(1 + 1/(a**2*x**2))) - I*a**2*sqrt(1 + 1/(a**2*x**2))/(3*x**2) + a/(8*x
**3*sqrt(1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(1 + 1/(a**2*x**2)))
```

Giac [B] time = 1.15453, size = 324, normalized size = 2.87

$$-\frac{3}{8}\,a^4\log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right)+\frac{3}{8}\,a^4\log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right)-\frac{9\left(x|a|-\sqrt{a^2x^2+1}\right)^7a^4-33\left(x|a|-\sqrt{a^2x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -3/8*a^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) + 1)) + 3/8*a^4*log(abs(-x*a
bs(a) + sqrt(a^2*x^2 + 1) - 1)) - 1/12*(9*(x*abs(a) - sqrt(a^2*x^2 + 1))^7*
a^4 - 33*(x*abs(a) - sqrt(a^2*x^2 + 1))^5*a^4 - 48*(x*abs(a) - sqrt(a^2*x^2
 + 1))^4*a^3*i*abs(a) - 33*(x*abs(a) - sqrt(a^2*x^2 + 1))^3*a^4 + 64*(x*abs
(a) - sqrt(a^2*x^2 + 1))^2*a^3*i*abs(a) + 9*(x*abs(a) - sqrt(a^2*x^2 + 1))*
a^4 - 16*a^3*i*abs(a))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)^4
```

3.11 $\int e^{2i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=48

$$\frac{x^2}{a^2}-\frac{2ix}{a^3}-\frac{2\log(ax+i)}{a^4}+\frac{2ix^3}{3a}-\frac{x^4}{4}$$

```
[Out] ((-2*I)*x)/a^3 + x^2/a^2 + (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I + a*x])/a^4
```

Rubi [A] time = 0.0361756, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$\frac{x^2}{a^2}-\frac{2ix}{a^3}-\frac{2\log(ax+i)}{a^4}+\frac{2ix^3}{3a}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])*x^3,x]
```

```
[Out] ((-2*I)*x)/a^3 + x^2/a^2 + (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I + a*x])/a^4
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int e^{2i\tan^{-1}(ax)}x^3\,dx &= \int \frac{x^3(1+iax)}{1-iax}\,dx\\
&= \int \left(-\frac{2i}{a^3}+\frac{2x}{a^2}+\frac{2ix^2}{a}-x^3-\frac{2}{a^3(i+ax)}\right)dx\\
&= -\frac{2ix}{a^3}+\frac{x^2}{a^2}+\frac{2ix^3}{3a}-\frac{x^4}{4}-\frac{2\log(i+ax)}{a^4}
\end{aligned}$$

Mathematica [A] time = 0.0196513, size = 48, normalized size = 1.

$$\frac{x^2}{a^2}-\frac{2ix}{a^3}-\frac{2\log(ax+i)}{a^4}+\frac{2ix^3}{3a}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])*x^3,x]
```

```
[Out] ((-2*I)*x)/a^3 + x^2/a^2 + (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I + a*x])/a^4
```

Maple [A] time = 0.038, size = 55, normalized size = 1.2

$$-\frac{x^4}{4}+\frac{\frac{2i}{3}x^3}{a}+\frac{x^2}{a^2}-\frac{2\,ix}{a^3}-\frac{\ln\left(a^2x^2+1\right)}{a^4}+\frac{2\,i\arctan\left(ax\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)*x^3,x)
```

```
[Out] -1/4*x^4+2/3*I*x^3/a+x^2/a^2-2*I*x/a^3-1/a^4*ln(a^2*x^2+1)+2*I/a^4*arctan(a
*x)
```

Maxima [A] time = 1.47701, size = 76, normalized size = 1.58

$$-\frac{3\,a^3x^4-8i\,a^2x^3-12\,ax^2+24i\,x}{12\,a^3}+\frac{2i\,\arctan\left(ax\right)}{a^4}-\frac{\log\left(a^2x^2+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^3*x^4 - 8*I*a^2*x^3 - 12*a*x^2 + 24*I*x)/a^3 + 2*I*arctan(a*x)/a
^4 - log(a^2*x^2 + 1)/a^4
```

Fricas [A] time = 1.49833, size = 112, normalized size = 2.33

$$-\frac{3\,a^4x^4-8i\,a^3x^3-12\,a^2x^2+24i\,ax+24\,\log\left(\frac{ax+i}{a}\right)}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*x^4 - 8*I*a^3*x^3 - 12*a^2*x^2 + 24*I*a*x + 24*log((a*x + I)/a
))/a^4
```

Sympy [A] time = 0.348497, size = 41, normalized size = 0.85

$$-\frac{x^4}{4}+\frac{2ix^3}{3a}+\frac{x^2}{a^2}-\frac{2ix}{a^3}-\frac{2\log\left(ax+i\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)*x**3,x)
```

```
[Out] -x**4/4 + 2*I*x**3/(3*a) + x**2/a**2 - 2*I*x/a**3 - 2*log(a*x + I)/a**4
```

Giac [A] time = 1.10319, size = 65, normalized size = 1.35

$$-\frac{3\,a^4x^4 - 8\,a^3ix^3 - 12\,a^2x^2 + 24\,aix}{12\,a^4} - \frac{2\,\log\left(ax + i\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*a^4*x^4 - 8*a^3*i*x^3 - 12*a^2*x^2 + 24*a*i*x)/a^4 - 2*log(a*x + i
)/a^4
```

3.12 $\int e^{2i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=39

$$\frac{2x}{a^2}-\frac{2i\log(ax+i)}{a^3}+\frac{ix^2}{a}-\frac{x^3}{3}$$

```
[Out] (2*x)/a^2 + (I*x^2)/a - x^3/3 - ((2*I)*Log[I + a*x])/a^3
```

Rubi [A] time = 0.0284318, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$\frac{2x}{a^2}-\frac{2i\log(ax+i)}{a^3}+\frac{ix^2}{a}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])*x^2,x]
```

```
[Out] (2*x)/a^2 + (I*x^2)/a - x^3/3 - ((2*I)*Log[I + a*x])/a^3
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{2i\tan^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1+iax)}{1-iax}\,dx\\ &= \int \left(\frac{2}{a^2}+\frac{2ix}{a}-x^2-\frac{2i}{a^2(i+ax)}\right)dx\\ &= \frac{2x}{a^2}+\frac{ix^2}{a}-\frac{x^3}{3}-\frac{2i\log(i+ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.0125104, size = 39, normalized size = 1.

$$\frac{2x}{a^2}-\frac{2i\log(ax+i)}{a^3}+\frac{ix^2}{a}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])*x^2,x]
```

```
[Out] (2*x)/a^2 + (I*x^2)/a - x^3/3 - ((2*I)*Log[I + a*x])/a^3
```

Maple [A] time = 0.036, size = 47, normalized size = 1.2

$$-\frac{x^3}{3}+\frac{ix^2}{a}+2\frac{x}{a^2}-\frac{i\ln\left(a^2x^2+1\right)}{a^3}-2\frac{\arctan(ax)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)*x^2,x)
```

```
[Out] -1/3*x^3+I*x^2/a+2*x/a^2-I/a^3*ln(a^2*x^2+1)-2/a^3*arctan(a*x)
```

Maxima [A] time = 1.52594, size = 63, normalized size = 1.62

$$-\frac{a^2x^3-3i\,ax^2-6\,x}{3\,a^2}-\frac{2\,\arctan(ax)}{a^3}-\frac{i\,\log\left(a^2x^2+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*x^3 - 3*I*a*x^2 - 6*x)/a^2 - 2*arctan(a*x)/a^3 - I*log(a^2*x^2 +
1)/a^3
```

Fricas [A] time = 1.59275, size = 88, normalized size = 2.26

$$-\frac{a^3x^3-3i\,a^2x^2-6\,ax+6i\,\log\left(\frac{ax+i}{a}\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(a^3*x^3 - 3*I*a^2*x^2 - 6*a*x + 6*I*log((a*x + I)/a))/a^3
```

Sympy [A] time = 0.359366, size = 31, normalized size = 0.79

$$-\frac{x^3}{3}+\frac{ix^2}{a}+\frac{2x}{a^2}-\frac{2i\log(ax+i)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)*x**2,x)
```

```
[Out] -x**3/3 + I*x**2/a + 2*x/a**2 - 2*I*log(a*x + I)/a**3
```

Giac [A] time = 1.09093, size = 53, normalized size = 1.36

$$-\frac{2\,i\log\left(ax+i\right)}{a^3}-\frac{a^3x^3-3\,a^2ix^2-6\,ax}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2,x, algorithm="giac")
```

```
[Out] -2*i*log(a*x + i)/a^3 - 1/3*(a^3*x^3 - 3*a^2*i*x^2 - 6*a*x)/a^3
```

3.13 $\int e^{2i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=29

$$\frac{2\log(ax+i)}{a^2}+\frac{2ix}{a}-\frac{x^2}{2}$$

```
[Out] ((2*I)*x)/a - x^2/2 + (2*Log[I + a*x])/a^2
```

Rubi [A] time = 0.0194471, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5062, 77}

$$\frac{2\log(ax+i)}{a^2}+\frac{2ix}{a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])*x,x]
```

```
[Out] ((2*I)*x)/a - x^2/2 + (2*Log[I + a*x])/a^2
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{2i\tan^{-1}(ax)}x\,dx &= \int \frac{x(1+iax)}{1-iax}\,dx\\ &= \int \left(\frac{2i}{a}-x+\frac{2}{a(i+ax)}\right)dx\\ &= \frac{2ix}{a}-\frac{x^2}{2}+\frac{2\log(i+ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.010063, size = 29, normalized size = 1.

$$\frac{2\log(ax+i)}{a^2}+\frac{2ix}{a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])*x,x]
```

```
[Out] ((2*I)*x)/a - x^2/2 + (2*Log[I + a*x])/a^2
```

Maple [A] time = 0.036, size = 38, normalized size = 1.3

$$-\frac{x^2}{2}+\frac{2\,ix}{a}+\frac{\ln\left(a^2x^2+1\right)}{a^2}-\frac{2\,i\arctan\left(ax\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)*x,x)
```

```
[Out] -1/2*x^2+2*I*x/a+1/a^2*ln(a^2*x^2+1)-2*I/a^2*arctan(a*x)
```

Maxima [A] time = 1.47589, size = 51, normalized size = 1.76

$$-\frac{ax^2-4i\,x}{2\,a}-\frac{2i\,\arctan\left(ax\right)}{a^2}+\frac{\log\left(a^2x^2+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x,x, algorithm="maxima")
```

```
[Out] -1/2*(a*x^2 - 4*I*x)/a - 2*I*arctan(a*x)/a^2 + log(a^2*x^2 + 1)/a^2
```

Fricas [A] time = 1.59842, size = 69, normalized size = 2.38

$$-\frac{a^2x^2-4i\,ax-4\,\log\left(\frac{ax+i}{a}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*x^2 - 4*I*a*x - 4*log((a*x + I)/a))/a^2
```

Sympy [A] time = 0.165334, size = 22, normalized size = 0.76

$$-\frac{x^2}{2}+\frac{2ix}{a}+\frac{2\log\left(ax+i\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)*x,x)
```

```
[Out] -x**2/2 + 2*I*x/a + 2*log(a*x + I)/a**2
```

Giac [A] time = 1.08694, size = 41, normalized size = 1.41

$$-\frac{a^2x^2 - 4\,aix}{2\,a^2} + \frac{2\,\log\left(ax + i\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*x^2 - 4*a*i*x)/a^2 + 2*log(a*x + i)/a^2
```

3.14 $\int e^{2i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=19

$$-x+\frac{2i\log(ax+i)}{a}$$

```
[Out] -x + ((2*I)*Log[I + a*x])/a
```

Rubi [A] time = 0.0091863, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5061, 43}

$$-x+\frac{2i\log(ax+i)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] -x + ((2*I)*Log[I + a*x])/a
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2i\tan^{-1}(ax)}\,dx &= \int \frac{1+iax}{1-iax}\,dx\\ &= \int \left(-1+\frac{2i}{i+ax}\right)dx\\ &= -x+\frac{2i\log(i+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0092926, size = 30, normalized size = 1.58

$$\frac{i\log\left(a^2x^2+1\right)}{a}+\frac{2\tan^{-1}(ax)}{a}-x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] -x + (2*ArcTan[a*x])/a + (I*Log[1 + a^2*x^2])/a
```

Maple [A] time = 0.036, size = 30, normalized size = 1.6

$$-x+\frac{i\ln\left(a^{2}x^{2}+1\right)}{a}+2\,\frac{\arctan\left(ax\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1),x)
```

```
[Out] -x+I/a*ln(a^2*x^2+1)+2*arctan(a*x)/a
```

Maxima [A] time = 1.50846, size = 38, normalized size = 2.

$$-x+\frac{2\,\arctan\left(ax\right)}{a}+\frac{i\,\log\left(a^{2}x^{2}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -x + 2*arctan(a*x)/a + I*log(a^2*x^2 + 1)/a
```

Fricas [A] time = 1.54591, size = 45, normalized size = 2.37

$$-\frac{ax-2i\,\log\left(\frac{ax+i}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -(a*x - 2*I*log((a*x + I)/a))/a
```

Sympy [A] time = 0.311358, size = 12, normalized size = 0.63

$$-x+\frac{2i\log\left(ax+i\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1),x)
```

```
[Out] -x + 2*I*log(a*x + I)/a
```

Giac [A] time = 1.1037, size = 22, normalized size = 1.16

$$-x+\frac{2\,i\log\left(ax+i\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -x + 2*i*log(a*x + i)/a
```

3.15 $\int \frac{e^{2i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=13

$$\log(x) - 2\log(ax + i)$$

```
[Out] Log[x] - 2*Log[I + a*x]
```

Rubi [A] time = 0.0194988, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 72}

$$\log(x) - 2\log(ax + i)$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])/x,x]

[Out] Log[x] - 2*Log[I + a*x]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2i\tan^{-1}(ax)}}{x}\,dx &= \int \frac{1+iax}{x(1-iax)}\,dx\\ &= \int \left(\frac{1}{x} - \frac{2a}{i+ax}\right)dx\\ &= \log(x) - 2\log(i+ax)\end{aligned}$$

Mathematica [A] time = 0.0058213, size = 13, normalized size = 1.

$$\log(x) - 2\log(ax + i)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/x,x]

[Out] Log[x] - 2*Log[I + a*x]
```

Maple [A] time = 0.041, size = 23, normalized size = 1.8

$$2 i \arctan(ax) - \ln\left(a^2x^2+1\right) + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/x,x)
```

```
[Out] 2*I*arctan(a*x)-ln(a^2*x^2+1)+ln(x)
```

Maxima [A] time = 1.50648, size = 28, normalized size = 2.15

$$2i \arctan(ax) - \log\left(a^2x^2+1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x,x, algorithm="maxima")
```

```
[Out] 2*I*arctan(a*x) - log(a^2*x^2 + 1) + log(x)
```

Fricas [A] time = 1.68503, size = 39, normalized size = 3.

$$\log(x) - 2\log\left(\frac{ax+i}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x,x, algorithm="fricas")
```

```
[Out] log(x) - 2*log((a*x + I)/a)
```

Sympy [A] time = 0.407838, size = 10, normalized size = 0.77

$$\log(x) - 2\log\left(x + \frac{i}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/x,x)
```

```
[Out] log(x) - 2*log(x + I/a)
```

Giac [A] time = 1.13246, size = 20, normalized size = 1.54

$$2 i^2 \log(ax+i) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x,x, algorithm="giac")
```

```
[Out] 2*i^2*log(a*x + i) + log(abs(x))
```

3.16 $\int \frac{e^{2 i \tan^{-1}(a x)}}{x^2} \, dx$

Optimal. Leaf size=26

$$2 i a \log(x) - 2 i a \log(a x + i) - \frac{1}{x}$$

```
[Out] -x^(-1) + (2*I)*a*Log[x] - (2*I)*a*Log[I + a*x]
```

Rubi [A] time = 0.0236566, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$2 i a \log(x) - 2 i a \log(a x + i) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])/x^2,x]
```

```
[Out] -x^(-1) + (2*I)*a*Log[x] - (2*I)*a*Log[I + a*x]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2 i \tan^{-1}(a x)}}{x^2} \, dx &= \int \frac{1 + i a x}{x^2 (1 - i a x)} \, dx \\
&= \int \left(\frac{1}{x^2} + \frac{2 i a}{x} - \frac{2 i a^2}{i + a x} \right) dx \\
&= -\frac{1}{x} + 2 i a \log(x) - 2 i a \log(i + a x)
\end{aligned}$$

Mathematica [A] time = 0.0085793, size = 26, normalized size = 1.

$$2 i a \log(x) - 2 i a \log(a x + i) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/x^2,x]
```

```
[Out] -x^(-1) + (2*I)*a*Log[x] - (2*I)*a*Log[I + a*x]
```

Maple [A] time = 0.043, size = 34, normalized size = 1.3

$$-ia\ln\left(a^2x^2+1\right)-2\,a\arctan\left(ax\right)-x^{-1}+2\,ia\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/x^2,x)
```

```
[Out] -I*a*ln(a^2*x^2+1)-2*a*arctan(a*x)-1/x+2*I*a*ln(x)
```

Maxima [A] time = 1.48702, size = 42, normalized size = 1.62

$$-2\,a\arctan\left(ax\right)-i\,a\log\left(a^2x^2+1\right)+2i\,a\log\left(x\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^2,x, algorithm="maxima")
```

```
[Out] -2*a*arctan(a*x) - I*a*log(a^2*x^2 + 1) + 2*I*a*log(x) - 1/x
```

Fricas [A] time = 1.79971, size = 69, normalized size = 2.65

$$\frac{2i\,ax\log\left(x\right)-2i\,ax\log\left(\frac{ax+i}{a}\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^2,x, algorithm="fricas")
```

```
[Out] (2*I*a*x*log(x) - 2*I*a*x*log((a*x + I)/a) - 1)/x
```

Sympy [A] time = 0.427577, size = 20, normalized size = 0.77

$$-2a\left(-i\log\left(x\right)+i\log\left(x+\frac{i}{a}\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/x**2,x)
```

```
[Out] -2*a*(-I*log(x) + I*log(x + I/a)) - 1/x
```

Giac [A] time = 1.09735, size = 31, normalized size = 1.19

$$-2\,ai\log\left(ax+i\right)+2\,ai\log\left(|x|\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^2,x, algorithm="giac")
```

```
[Out] -2*a*i*log(a*x + i) + 2*a*i*log(abs(x)) - 1/x
```

3.17 $\int \frac{e^{2i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=36

$$-2a^2\log(x)+2a^2\log(ax+i)-\frac{2ia}{x}-\frac{1}{2x^2}$$

```
[Out] -1/(2*x^2) - ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I + a*x]
```

Rubi [A] time = 0.0268669, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$-2a^2\log(x)+2a^2\log(ax+i)-\frac{2ia}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/(2*x^2) - ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I + a*x]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{1+iax}{x^3(1-iax)}\,dx\\ &= \int \left(\frac{1}{x^3}+\frac{2ia}{x^2}-\frac{2a^2}{x}+\frac{2a^3}{i+ax}\right)dx\\ &= -\frac{1}{2x^2}-\frac{2ia}{x}-2a^2\log(x)+2a^2\log(i+ax)\end{aligned}$$

Mathematica [A] time = 0.0098901, size = 36, normalized size = 1.

$$-2a^2\log(x)+2a^2\log(ax+i)-\frac{2ia}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/x^3,x]

[Out] -1/(2*x^2) - ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I + a*x]
```

Maple [A] time = 0.041, size = 45, normalized size = 1.3

$$-2\,ia^2\arctan(ax)+a^2\ln\left(a^2x^2+1\right)-\frac{1}{2\,x^2}-\frac{2\,ia}{x}-2\,a^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/x^3,x)

[Out] -2*I*a^2*arctan(a*x)+a^2*ln(a^2*x^2+1)-1/2/x^2-2*I*a/x-2*a^2*ln(x)
```

Maxima [A] time = 1.51449, size = 57, normalized size = 1.58

$$-2i\,a^2\arctan(ax)+a^2\log\left(a^2x^2+1\right)-2\,a^2\log(x)-\frac{4i\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^3,x, algorithm="maxima")

[Out] -2*I*a^2*arctan(a*x) + a^2*log(a^2*x^2 + 1) - 2*a^2*log(x) - 1/2*(4*I*a*x +
 1)/x^2
```

Fricas [A] time = 1.59904, size = 97, normalized size = 2.69

$$-\frac{4\,a^2x^2\log(x)-4\,a^2x^2\log\left(\frac{ax+i}{a}\right)+4i\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^3,x, algorithm="fricas")

[Out] -1/2*(4*a^2*x^2*log(x) - 4*a^2*x^2*log((a*x + I)/a) + 4*I*a*x + 1)/x^2
```

Sympy [A] time = 0.459619, size = 29, normalized size = 0.81

$$-2a^2\left(\log(x)-\log\left(x+\frac{i}{a}\right)\right)-\frac{4iax+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/x**3,x)

[Out] -2*a**2*(log(x) - log(x + I/a)) - (4*I*a*x + 1)/(2*x**2)
```

Giac [A] time = 1.11688, size = 43, normalized size = 1.19

$$2\,a^2\log\left(ax+i\right)-2\,a^2\log\left(|x|\right)-\frac{4\,aix+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^3,x, algorithm="giac")
```

```
[Out] 2*a^2*log(a*x + i) - 2*a^2*log(abs(x)) - 1/2*(4*a*i*x + 1)/x^2
```

3.18 $\int \frac{e^{2 i \tan^{-1}(ax)}}{x^4} \, dx$

Optimal. Leaf size=48

$$\frac{2a^2}{x} - 2ia^3\log(x) + 2ia^3\log(ax+i) - \frac{ia}{x^2} - \frac{1}{3x^3}$$

```
[Out] -1/(3*x^3) - (I*a)/x^2 + (2*a^2)/x - (2*I)*a^3*Log[x] + (2*I)*a^3*Log[I + a
*x]
```

Rubi [A] time = 0.0304261, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$\frac{2a^2}{x} - 2ia^3\log(x) + 2ia^3\log(ax+i) - \frac{ia}{x^2} - \frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/(3*x^3) - (I*a)/x^2 + (2*a^2)/x - (2*I)*a^3*Log[x] + (2*I)*a^3*Log[I + a
*x]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2i\tan^{-1}(ax)}}{x^4} \, dx &= \int \frac{1+iax}{x^4(1-iax)} \, dx \\
&= \int \left(\frac{1}{x^4} + \frac{2ia}{x^3} - \frac{2a^2}{x^2} - \frac{2ia^3}{x} + \frac{2ia^4}{i+ax}\right) dx \\
&= -\frac{1}{3x^3} - \frac{ia}{x^2} + \frac{2a^2}{x} - 2ia^3\log(x) + 2ia^3\log(i+ax)
\end{aligned}$$

Mathematica [A] time = 0.0124324, size = 48, normalized size = 1.

$$\frac{2a^2}{x} - 2ia^3\log(x) + 2ia^3\log(ax+i) - \frac{ia}{x^2} - \frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/(3*x^3) - (I*a)/x^2 + (2*a^2)/x - (2*I)*a^3*Log[x] + (2*I)*a^3*Log[I + a
*x]
```

Maple [A] time = 0.043, size = 55, normalized size = 1.2

$$ia^3\ln\left(a^2x^2+1\right)+2\,a^3\arctan\left(ax\right)-\frac{1}{3\,x^3}-2\,ia^3\ln\left(x\right)-\frac{ia}{x^2}+2\,\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/x^4,x)
```

```
[Out] I*a^3*ln(a^2*x^2+1)+2*a^3*arctan(a*x)-1/3/x^3-2*I*a^3*ln(x)-I*a/x^2+2*a^2/x
```

Maxima [A] time = 1.50283, size = 69, normalized size = 1.44

$$2\,a^3\arctan\left(ax\right)+i\,a^3\log\left(a^2x^2+1\right)-2i\,a^3\log\left(x\right)+\frac{6\,a^2x^2-3i\,ax-1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^4,x, algorithm="maxima")
```

```
[Out] 2*a^3*arctan(a*x) + I*a^3*log(a^2*x^2 + 1) - 2*I*a^3*log(x) + 1/3*(6*a^2*x^
2 - 3*I*a*x - 1)/x^3
```

Fricas [A] time = 1.71316, size = 119, normalized size = 2.48

$$\frac{-6i\,a^3x^3\log\left(x\right)+6i\,a^3x^3\log\left(\frac{ax+i}{a}\right)+6\,a^2x^2-3i\,ax-1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(-6*I*a^3*x^3*log(x) + 6*I*a^3*x^3*log((a*x + I)/a) + 6*a^2*x^2 - 3*I*a
*x - 1)/x^3
```

Sympy [A] time = 0.508047, size = 39, normalized size = 0.81

$$-2a^3\left(i\log\left(x\right)-i\log\left(x+\frac{i}{a}\right)\right)+\frac{6a^2x^2-3iax-1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/x**4,x)
```

```
[Out] -2*a**3*(I*log(x) - I*log(x + I/a)) + (6*a**2*x**2 - 3*I*a*x - 1)/(3*x**3)
```

Giac [A] time = 1.0904, size = 57, normalized size = 1.19

$$2\,a^3 i \log\left(ax+i\right) - 2\,a^3 i \log\left(|x|\right) + \frac{6\,a^2x^2 - 3\,aix - 1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^4,x, algorithm="giac")
```

```
[Out] 2*a^3*i*log(a*x + i) - 2*a^3*i*log(abs(x)) + 1/3*(6*a^2*x^2 - 3*a*i*x - 1)/
x^3
```

3.19 $\int e^{3i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=137

$$-\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{x^2\sqrt{a^2x^2+1}}{a^2}-\frac{9i(-3ax+2i)\sqrt{a^2x^2+1}}{8a^4}+\frac{27\sqrt{a^2x^2+1}}{4a^4}+\frac{(1+iax)^3}{a^4\sqrt{a^2x^2+1}}-\frac{51i\sinh^{-1}(ax)}{8a^4}$$

```
[Out] (1 + I*a*x)^3/(a^4*Sqrt[1 + a^2*x^2]) + (27*Sqrt[1 + a^2*x^2])/(4*a^4) - (x
^2*Sqrt[1 + a^2*x^2])/a^2 - ((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - (((9*I)/8)*(2
*I - 3*a*x)*Sqrt[1 + a^2*x^2])/a^4 - (((51*I)/8)*ArcSinh[a*x])/a^4
```

Rubi [A] time = 0.621367, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5060, 1633, 1593, 12, 852, 1635, 1815, 27, 743, 641, 215}

$$-\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{x^2\sqrt{a^2x^2+1}}{a^2}-\frac{9i(-3ax+2i)\sqrt{a^2x^2+1}}{8a^4}+\frac{27\sqrt{a^2x^2+1}}{4a^4}+\frac{(1+iax)^3}{a^4\sqrt{a^2x^2+1}}-\frac{51i\sinh^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])*x^3,x]
```

```
[Out] (1 + I*a*x)^3/(a^4*Sqrt[1 + a^2*x^2]) + (27*Sqrt[1 + a^2*x^2])/(4*a^4) - (x
^2*Sqrt[1 + a^2*x^2])/a^2 - ((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - (((9*I)/8)*(2
*I - 3*a*x)*Sqrt[1 + a^2*x^2])/a^4 - (((51*I)/8)*ArcSinh[a*x])/a^4
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{3i\tan^{-1}(ax)}x^3\,dx = \int \frac{x^3(1+iax)^2}{(1-iax)\sqrt{1+a^2x^2}}\,dx$$
$$= -\left((ia)\int \frac{\sqrt{1+a^2x^2}\left(\frac{ix^3}{a}-x^4\right)}{(1-iax)^2}\,dx\right)$$
$$= -\left((ia)\int \frac{\left(\frac{i}{a}-x\right)x^3\sqrt{1+a^2x^2}}{(1-iax)^2}\,dx\right)$$
$$= a^2\int \frac{x^3\left(1+a^2x^2\right)^{3/2}}{a^2(1-iax)^3}\,dx$$
$$= \int \frac{x^3\left(1+a^2x^2\right)^{3/2}}{(1-iax)^3}\,dx$$
$$= \int \frac{x^3(1+iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx$$
$$= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \int \frac{(1+iax)^2\left(\frac{3i}{a^3}-\frac{x}{a^2}-\frac{ix^2}{a}\right)}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int \frac{\frac{12i}{a}-28x-27iax^2+12a^2x^3}{\sqrt{1+a^2x^2}}\,dx}{4a^2}$$
$$= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int \frac{36ia-108a^2x-81ia^3x^2}{\sqrt{1+a^2x^2}}\,dx}{12a^4}$$
$$= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int -\frac{9ia(-2i+3ax)^2}{\sqrt{1+a^2x^2}}\,dx}{12a^4}$$
$$= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} + \frac{(3i)\int \frac{(-2i+3ax)^2}{\sqrt{1+a^2x^2}}\,dx}{4a^3}$$
$$= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i-3ax)\sqrt{1+a^2x^2}}{8a^4} + \frac{(3i)\int \frac{-17a^2-18ia^3x}{\sqrt{1+a^2x^2}}\,dx}{8a^5}$$
$$= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} + \frac{27\sqrt{1+a^2x^2}}{4a^4} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i-3ax)\sqrt{1+a^2x^2}}{8a^4} - \frac{(51i)\int}{}$$
$$= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} + \frac{27\sqrt{1+a^2x^2}}{4a^4} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i-3ax)\sqrt{1+a^2x^2}}{8a^4} - \frac{51i\sinh}{8a}$$

Mathematica [A] time = 0.0600751, size = 80, normalized size = 0.58

$$\sqrt{a^2x^2+1}\left(-\frac{x^2}{a^2}+\frac{19ix}{8a^3}+\frac{4i}{a^4(ax+i)}+\frac{6}{a^4}-\frac{ix^3}{4a}\right)-\frac{51i\sinh^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])*x^3,x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(6/a^4 + (((19*I)/8)*x)/a^3 - x^2/a^2 - ((I/4)*x^3)/a + (
4*I)/(a^4*(I + a*x))) - (((51*I)/8)*ArcSinh[a*x])/a^4
```

Maple [A] time = 0.079, size = 143, normalized size = 1.

$$-\frac{i}{4}ax^5\frac{1}{\sqrt{a^2x^2+1}}+\frac{\frac{17\,i}{8}x^3}{a}\frac{1}{\sqrt{a^2x^2+1}}+\frac{\frac{51\,i}{8}x}{a^3}\frac{1}{\sqrt{a^2x^2+1}}-\frac{\frac{51\,i}{8}}{a^3}\ln\left(a^2x\frac{1}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2}}-x^4\frac{1}{\sqrt{a^2x^2+1}}+5\frac{x^2}{a^2\sqrt{a^2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x)
```

```
[Out] -1/4*I*a*x^5/(a^2*x^2+1)^(1/2)+17/8*I/a*x^3/(a^2*x^2+1)^(1/2)+51/8*I/a^3*x/
(a^2*x^2+1)^(1/2)-51/8*I/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^
(1/2)-x^4/(a^2*x^2+1)^(1/2)+5*x^2/a^2/(a^2*x^2+1)^(1/2)+10/a^4/(a^2*x^2+1)^
(1/2)
```

Maxima [A] time = 1.01132, size = 170, normalized size = 1.24

$$-\frac{i\,ax^5}{4\,\sqrt{a^2x^2+1}}-\frac{x^4}{\sqrt{a^2x^2+1}}+\frac{17i\,x^3}{8\,\sqrt{a^2x^2+1}a}+\frac{5\,x^2}{\sqrt{a^2x^2+1}a^2}+\frac{51i\,x}{8\,\sqrt{a^2x^2+1}a^3}-\frac{51i\,\operatorname{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\,\sqrt{a^2}a^3}+\frac{10}{\sqrt{a^2x^2+1}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/4*I*a*x^5/sqrt(a^2*x^2 + 1) - x^4/sqrt(a^2*x^2 + 1) + 17/8*I*x^3/(sqrt(a
^2*x^2 + 1)*a) + 5*x^2/(sqrt(a^2*x^2 + 1)*a^2) + 51/8*I*x/(sqrt(a^2*x^2 + 1
)*a^3) - 51/8*I*arcsinh(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^3) + 10/(sqrt(a^2*x^2
 + 1)*a^4)
```

Fricas [A] time = 1.72582, size = 220, normalized size = 1.61

$$\frac{32i\,ax-51\,(-i\,ax+1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+\left(-2i\,a^4x^4-6\,a^3x^3+11i\,a^2x^2+29\,ax+80i\right)\sqrt{a^2x^2+1}-32}{8\left(a^5x+i\,a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(32*I*a*x - 51*(-I*a*x + 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + (-2*I*a^4*x
^4 - 6*a^3*x^3 + 11*I*a^2*x^2 + 29*a*x + 80*I)*sqrt(a^2*x^2 + 1) - 32)/(a^5
*x + I*a^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\,(iax+1)^3}{\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x**3,x)
```

```
[Out] Integral(x**3*(I*a*x + 1)**3/(a**2*x**2 + 1)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] undef
```

3.20 $\int e^{3i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=102

$$\frac{i(1+iax)^3}{a^3\sqrt{a^2x^2+1}}+\frac{i(3+iax)^2\sqrt{a^2x^2+1}}{3a^3}+\frac{(-3ax+28i)\sqrt{a^2x^2+1}}{6a^3}+\frac{11\sinh^{-1}(ax)}{2a^3}$$

```
[Out] (I*(1 + I*a*x)^3)/(a^3*Sqrt[1 + a^2*x^2]) + ((28*I - 3*a*x)*Sqrt[1 + a^2*x^
2])/(6*a^3) + ((I/3)*(3 + I*a*x)^2*Sqrt[1 + a^2*x^2])/a^3 + (11*ArcSinh[a*x
])/(2*a^3)
```

Rubi [A] time = 0.568457, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5060, 1633, 1593, 12, 852, 1635, 1654, 780, 215}

$$\frac{i(1+iax)^3}{a^3\sqrt{a^2x^2+1}}+\frac{i(3+iax)^2\sqrt{a^2x^2+1}}{3a^3}+\frac{(-3ax+28i)\sqrt{a^2x^2+1}}{6a^3}+\frac{11\sinh^{-1}(ax)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])*x^2,x]
```

```
[Out] (I*(1 + I*a*x)^3)/(a^3*Sqrt[1 + a^2*x^2]) + ((28*I - 3*a*x)*Sqrt[1 + a^2*x^
2])/(6*a^3) + ((I/3)*(3 + I*a*x)^2*Sqrt[1 + a^2*x^2])/a^3 + (11*ArcSinh[a*x
])/(2*a^3)
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
&& !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{3i\tan^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1+iax)^2}{(1-iax)\sqrt{1+a^2x^2}}\,dx\\
&= -\left((ia)\int \frac{\sqrt{1+a^2x^2}\left(\frac{ix^2}{a}-x^3\right)}{(1-iax)^2}\,dx\right)\\
&= -\left((ia)\int \frac{\left(\frac{i}{a}-x\right)x^2\sqrt{1+a^2x^2}}{(1-iax)^2}\,dx\right)\\
&= a^2\int \frac{x^2\left(1+a^2x^2\right)^{3/2}}{a^2(1-iax)^3}\,dx\\
&= \int \frac{x^2\left(1+a^2x^2\right)^{3/2}}{(1-iax)^3}\,dx\\
&= \int \frac{x^2(1+iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx\\
&= \frac{i(1+iax)^3}{a^3\sqrt{1+a^2x^2}} - \int \frac{\left(-\frac{3}{a^2}-\frac{ix}{a}\right)(1+iax)^2}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{i(1+iax)^3}{a^3\sqrt{1+a^2x^2}} + \frac{i(3+iax)^2\sqrt{1+a^2x^2}}{3a^3} + \frac{1}{3}\int \frac{\left(-\frac{3}{a^2}-\frac{ix}{a}\right)(-5-3iax)}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{i(1+iax)^3}{a^3\sqrt{1+a^2x^2}} + \frac{(28i-3ax)\sqrt{1+a^2x^2}}{6a^3} + \frac{i(3+iax)^2\sqrt{1+a^2x^2}}{3a^3} + \frac{11\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a^2}\\
&= \frac{i(1+iax)^3}{a^3\sqrt{1+a^2x^2}} + \frac{(28i-3ax)\sqrt{1+a^2x^2}}{6a^3} + \frac{i(3+iax)^2\sqrt{1+a^2x^2}}{3a^3} + \frac{11\sinh^{-1}(ax)}{2a^3}
\end{aligned}
$$

Mathematica [A] time = 0.0512884, size = 63, normalized size = 0.62

$$
\frac{33\sinh^{-1}(ax)+\frac{\sqrt{a^2x^2+1}\left(-2ia^3x^3-7a^2x^2+19iax-52\right)}{ax+i}}{6a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])*x^2,x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-52 + (19*I)*a*x - 7*a^2*x^2 - (2*I)*a^3*x^3))/(I + a*
x) + 33*ArcSinh[a*x])/(6*a^3)
```

Maple [A] time = 0.073, size = 123, normalized size = 1.2

$$
-\frac{i}{3}ax^4\frac{1}{\sqrt{a^2x^2+1}}+\frac{\frac{13i}{3}x^2}{a}\frac{1}{\sqrt{a^2x^2+1}}+\frac{\frac{26i}{3}}{a^3}\frac{1}{\sqrt{a^2x^2+1}}-\frac{3x^3}{2}\frac{1}{\sqrt{a^2x^2+1}}-\frac{11x}{2a^2}\frac{1}{\sqrt{a^2x^2+1}}+\frac{11}{2a^2}\ln\left(a^2x\frac{1}{\sqrt{a^2}}+\sqrt{a^2x^2+
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x)
```

```
[Out] -1/3*I*a*x^4/(a^2*x^2+1)^(1/2)+13/3*I/a*x^2/(a^2*x^2+1)^(1/2)+26/3*I/a^3/(a
^2*x^2+1)^(1/2)-3/2*x^3/(a^2*x^2+1)^(1/2)-11/2*x/a^2/(a^2*x^2+1)^(1/2)+11/2
```

```
/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Maxima [A] time = 1.02009, size = 144, normalized size = 1.41

$$-\frac{i\,ax^4}{3\,\sqrt{a^2x^2+1}}-\frac{3\,x^3}{2\,\sqrt{a^2x^2+1}}+\frac{13i\,x^2}{3\,\sqrt{a^2x^2+1}a}-\frac{11\,x}{2\,\sqrt{a^2x^2+1}a^2}+\frac{11\,\operatorname{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\,\sqrt{a^2}a^2}+\frac{26i}{3\,\sqrt{a^2x^2+1}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/3*I*a*x^4/sqrt(a^2*x^2 + 1) - 3/2*x^3/sqrt(a^2*x^2 + 1) + 13/3*I*x^2/(sq
rt(a^2*x^2 + 1)*a) - 11/2*x/(sqrt(a^2*x^2 + 1)*a^2) + 11/2*arcsinh(a^2*x/sq
rt(a^2))/(sqrt(a^2)*a^2) + 26/3*I/(sqrt(a^2*x^2 + 1)*a^3)
```

Fricas [A] time = 1.67389, size = 201, normalized size = 1.97

$$-\frac{24\,ax+(33\,ax+33i)\log\left(-ax+\sqrt{a^2x^2+1}\right)-\left(-2i\,a^3x^3-7\,a^2x^2+19i\,ax-52\right)\sqrt{a^2x^2+1}+24i}{6\left(a^4x+i\,a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(24*a*x + (33*a*x + 33*I)*log(-a*x + sqrt(a^2*x^2 + 1)) - (-2*I*a^3*x^
3 - 7*a^2*x^2 + 19*I*a*x - 52)*sqrt(a^2*x^2 + 1) + 24*I)/(a^4*x + I*a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\,(iax+1)^3}{\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x**2,x)
```

```
[Out] Integral(x**2*(I*a*x + 1)**3/(a**2*x**2 + 1)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] undef
```

3.21 $\int e^{3i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=92

$$-\frac{(a^2x^2+1)^{5/2}}{a^2(1-iax)^3}-\frac{3(a^2x^2+1)^{3/2}}{2a^2(1-iax)}-\frac{9\sqrt{a^2x^2+1}}{2a^2}+\frac{9i\sinh^{-1}(ax)}{2a^2}$$

```
[Out] (-9*Sqrt[1 + a^2*x^2])/(2*a^2) - (3*(1 + a^2*x^2)^(3/2))/(2*a^2*(1 - I*a*x)
) - (1 + a^2*x^2)^(5/2)/(a^2*(1 - I*a*x)^3) + (((9*I)/2)*ArcSinh[a*x])/a^2
```

Rubi [A] time = 0.325647, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5060, 1633, 1593, 12, 793, 665, 215}

$$-\frac{(a^2x^2+1)^{5/2}}{a^2(1-iax)^3}-\frac{3(a^2x^2+1)^{3/2}}{2a^2(1-iax)}-\frac{9\sqrt{a^2x^2+1}}{2a^2}+\frac{9i\sinh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])*x,x]
```

```
[Out] (-9*Sqrt[1 + a^2*x^2])/(2*a^2) - (3*(1 + a^2*x^2)^(3/2))/(2*a^2*(1 - I*a*x)
) - (1 + a^2*x^2)^(5/2)/(a^2*(1 - I*a*x)^3) + (((9*I)/2)*ArcSinh[a*x])/a^2
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
```

```
+ 1, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{3i\tan^{-1}(ax)}x\,dx &= \int \frac{x(1+iax)^2}{(1-iax)\sqrt{1+a^2x^2}}\,dx \\
&= -\left((ia)\int \frac{\left(\frac{ix}{a}-x^2\right)\sqrt{1+a^2x^2}}{(1-iax)^2}\,dx\right) \\
&= -\left((ia)\int \frac{\left(\frac{i}{a}-x\right)x\sqrt{1+a^2x^2}}{(1-iax)^2}\,dx\right) \\
&= a^2\int \frac{x\left(1+a^2x^2\right)^{3/2}}{a^2(1-iax)^3}\,dx \\
&= \int \frac{x\left(1+a^2x^2\right)^{3/2}}{(1-iax)^3}\,dx \\
&= -\frac{\left(1+a^2x^2\right)^{5/2}}{a^2(1-iax)^3}+\frac{(3i)\int \frac{\left(1+a^2x^2\right)^{3/2}}{(1-iax)^2}\,dx}{a} \\
&= -\frac{3\left(1+a^2x^2\right)^{3/2}}{2a^2(1-iax)}-\frac{\left(1+a^2x^2\right)^{5/2}}{a^2(1-iax)^3}+\frac{(9i)\int \frac{\sqrt{1+a^2x^2}}{1-iax}\,dx}{2a} \\
&= -\frac{9\sqrt{1+a^2x^2}}{2a^2}-\frac{3\left(1+a^2x^2\right)^{3/2}}{2a^2(1-iax)}-\frac{\left(1+a^2x^2\right)^{5/2}}{a^2(1-iax)^3}+\frac{(9i)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a} \\
&= -\frac{9\sqrt{1+a^2x^2}}{2a^2}-\frac{3\left(1+a^2x^2\right)^{3/2}}{2a^2(1-iax)}-\frac{\left(1+a^2x^2\right)^{5/2}}{a^2(1-iax)^3}+\frac{9i\sinh^{-1}(ax)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0594771, size = 54, normalized size = 0.59

$$-\frac{i\left(-9\sinh^{-1}(ax)+\frac{\sqrt{a^2x^2+1}\left(a^2x^2-5iax+14\right)}{ax+i}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])*x,x]
```

```
[Out] ((-I/2)*((Sqrt[1 + a^2*x^2]*(14 - (5*I)*a*x + a^2*x^2))/(I + a*x) - 9*ArcSi
nh[a*x]))/a^2
```

Maple [A] time = 0.071, size = 104, normalized size = 1.1

$$-\frac{i}{2}a x^{3}\frac{1}{\sqrt{a^{2}x^{2}+1}}-\frac{\frac{9i}{2}x}{a}\frac{1}{\sqrt{a^{2}x^{2}+1}}+\frac{\frac{9i}{2}}{a}\ln\left(a^{2}x\frac{1}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}+1}\right)\frac{1}{\sqrt{a^{2}}}-3\frac{x^{2}}{\sqrt{a^{2}x^{2}+1}}-7\frac{1}{a^{2}\sqrt{a^{2}x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x)
```

```
[Out] -1/2*I*a*x^3/(a^2*x^2+1)^(1/2)-9/2*I/a*x/(a^2*x^2+1)^(1/2)+9/2*I/a*ln(a^2*x
/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)-3*x^2/(a^2*x^2+1)^(1/2)-7/a^2/(
a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.03676, size = 119, normalized size = 1.29

$$-\frac{i\,a x^{3}}{2\,\sqrt{a^{2}x^{2}+1}}-\frac{3\,x^{2}}{\sqrt{a^{2}x^{2}+1}}-\frac{9i\,x}{2\,\sqrt{a^{2}x^{2}+1}a}+\frac{9i\,\operatorname{arsinh}\left(\frac{a^{2}x}{\sqrt{a^{2}}}\right)}{2\,\sqrt{a^{2}}a}-\frac{7}{\sqrt{a^{2}x^{2}+1}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] -1/2*I*a*x^3/sqrt(a^2*x^2 + 1) - 3*x^2/sqrt(a^2*x^2 + 1) - 9/2*I*x/(sqrt(a^
2*x^2 + 1)*a) + 9/2*I*arcsinh(a^2*x/sqrt(a^2))/(sqrt(a^2)*a) - 7/(sqrt(a^2*
x^2 + 1)*a^2)
```

Fricas [A] time = 1.71467, size = 176, normalized size = 1.91

$$\frac{-8i\,ax-9\,(i\,ax-1)\log\left(-ax+\sqrt{a^{2}x^{2}+1}\right)+\sqrt{a^{2}x^{2}+1}\left(-i\,a^{2}x^{2}-5\,ax-14i\right)+8}{2\,\left(a^{3}x+i\,a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(-8*I*a*x - 9*(I*a*x - 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + sqrt(a^2*x^2
+ 1)*(-I*a^2*x^2 - 5*a*x - 14*I) + 8)/(a^3*x + I*a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\,(iax+1)^{3}}{\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x,x)
```

```
[Out] Integral(x*(I*a*x + 1)**3/(a**2*x**2 + 1)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x, algorithm="giac")
```

```
[Out] undef
```

3.22 $\int e^{3i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=60

$$-\frac{2i(1+iax)^2}{a\sqrt{a^2x^2+1}}-\frac{3i\sqrt{a^2x^2+1}}{a}-\frac{3\sinh^{-1}(ax)}{a}$$

```
[Out] ((-2*I)*(1 + I*a*x)^2)/(a*Sqrt[1 + a^2*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2])/a
- (3*ArcSinh[a*x])/a
```

Rubi [A] time = 0.0448873, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5059, 853, 669, 641, 215}

$$-\frac{2i(1+iax)^2}{a\sqrt{a^2x^2+1}}-\frac{3i\sqrt{a^2x^2+1}}{a}-\frac{3\sinh^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] ((-2*I)*(1 + I*a*x)^2)/(a*Sqrt[1 + a^2*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2])/a
- (3*ArcSinh[a*x])/a
```

Rule 5059

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 - I*a*x)^((I*n + 1)/2)
/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2]), x] /; FreeQ[a, x] && Intege
rQ[(I*n - 1)/2]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{3i\tan^{-1}(ax)}\,dx &= \int \frac{(1+iax)^2}{(1-iax)\sqrt{1+a^2x^2}}\,dx\\
&= \int \frac{(1+iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx\\
&= -\frac{2i(1+iax)^2}{a\sqrt{1+a^2x^2}} - 3\int \frac{1+iax}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{2i(1+iax)^2}{a\sqrt{1+a^2x^2}} - \frac{3i\sqrt{1+a^2x^2}}{a} - 3\int \frac{1}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{2i(1+iax)^2}{a\sqrt{1+a^2x^2}} - \frac{3i\sqrt{1+a^2x^2}}{a} - \frac{3\sinh^{-1}(ax)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0324315, size = 42, normalized size = 0.7

$$
-\frac{3\sinh^{-1}(ax)}{a} + \frac{\sqrt{a^2x^2+1}\left(\frac{4}{ax+i}-i\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(-I + 4/(I + a*x)))/a - (3*ArcSinh[a*x])/a
```

Maple [A] time = 0.054, size = 81, normalized size = 1.4

$$
4\frac{x}{\sqrt{a^2x^2+1}} - iax^2\frac{1}{\sqrt{a^2x^2+1}} - \frac{5\,i}{a}\frac{1}{\sqrt{a^2x^2+1}} - 3\frac{1}{\sqrt{a^2}}\ln\left(\frac{a^2x}{\sqrt{a^2}} + \sqrt{a^2x^2+1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x)
```

```
[Out] 4*x/(a^2*x^2+1)^(1/2)-I*a*x^2/(a^2*x^2+1)^(1/2)-5*I/a/(a^2*x^2+1)^(1/2)-3*l
n(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Maxima [A] time = 0.99551, size = 89, normalized size = 1.48

$$
-\frac{i\,ax^2}{\sqrt{a^2x^2+1}} + \frac{4\,x}{\sqrt{a^2x^2+1}} - \frac{3\,\operatorname{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}} - \frac{5i}{\sqrt{a^2x^2+1}a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -I*a*x^2/sqrt(a^2*x^2 + 1) + 4*x/sqrt(a^2*x^2 + 1) - 3*arcsinh(a^2*x/sqrt(a
^2))/sqrt(a^2) - 5*I/(sqrt(a^2*x^2 + 1)*a)
```

Fricas [A] time = 1.6227, size = 146, normalized size = 2.43

$$\frac{4\,ax + (3\,ax + 3i)\log\left(-ax + \sqrt{a^2x^2+1}\right) + \sqrt{a^2x^2+1}(-i\,ax+5) + 4i}{a^2x + i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (4*a*x + (3*a*x + 3*I)*log(-a*x + sqrt(a^2*x^2 + 1)) + sqrt(a^2*x^2 + 1)*(-
I*a*x + 5) + 4*I)/(a^2*x + I*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(iax+1)^3}{\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((I*a*x + 1)**3/(a**2*x**2 + 1)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.23 $\int \frac{e^{3i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=51

$$\frac{4i\sqrt{a^2x^2+1}}{ax+i}-\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-i\sinh^{-1}(ax)$$

```
[Out] ((4*I)*Sqrt[1 + a^2*x^2])/(I + a*x) - I*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2
*x^2]]
```

Rubi [A] time = 0.663198, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5060, 6742, 215, 266, 63, 208, 651}

$$\frac{4i\sqrt{a^2x^2+1}}{ax+i}-\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-i\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])/x,x]
```

```
[Out] ((4*I)*Sqrt[1 + a^2*x^2])/(I + a*x) - I*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2
*x^2]]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3i\tan^{-1}(ax)}}{x}\,dx &= \int \frac{(1+iax)^2}{x(1-iax)\sqrt{1+a^2x^2}}\,dx\\
&= \int \left(-\frac{ia}{\sqrt{1+a^2x^2}}+\frac{1}{x\sqrt{1+a^2x^2}}-\frac{4a}{(i+ax)\sqrt{1+a^2x^2}}\right)dx\\
&= -\left((ia)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx\right)-(4a)\int \frac{1}{(i+ax)\sqrt{1+a^2x^2}}\,dx+\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\
&= \frac{4i\sqrt{1+a^2x^2}}{i+ax}-i\sinh^{-1}(ax)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= \frac{4i\sqrt{1+a^2x^2}}{i+ax}-i\sinh^{-1}(ax)+\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)}{a^2}\\
&= \frac{4i\sqrt{1+a^2x^2}}{i+ax}-i\sinh^{-1}(ax)-\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0391205, size = 55, normalized size = 1.08

$$\frac{4i\sqrt{a^2x^2+1}}{ax+i}-\log\left(\sqrt{a^2x^2+1}+1\right)-i\sinh^{-1}(ax)+\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/x,x]
```

```
[Out] ((4*I)*Sqrt[1 + a^2*x^2])/(I + a*x) - I*ArcSinh[a*x] + Log[x] - Log[1 + Sqr
t[1 + a^2*x^2]]
```

Maple [A] time = 0.064, size = 77, normalized size = 1.5

$$4\,iax\frac{1}{\sqrt{a^2x^2+1}}-ia\ln\left(a^2x\frac{1}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2}}+4\,\frac{1}{\sqrt{a^2x^2+1}}-\operatorname{Artanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x)
```

```
[Out] 4*I*a*x/(a^2*x^2+1)^(1/2)-I*a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)
^(1/2)+4/(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2+1)^(1/2))
```

Maxima [A] time = 0.999818, size = 82, normalized size = 1.61

$$\frac{4i\,ax}{\sqrt{a^2x^2+1}}-\frac{i\,a\,\mathrm{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}+\frac{4}{\sqrt{a^2x^2+1}}-\mathrm{arsinh}\left(\frac{1}{\sqrt{a^2}|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 4*I*a*x/sqrt(a^2*x^2 + 1) - I*a*arcsinh(a^2*x/sqrt(a^2))/sqrt(a^2) + 4/sqrt
(a^2*x^2 + 1) - arcsinh(1/(sqrt(a^2)*abs(x)))
```

Fricas [B] time = 1.72128, size = 250, normalized size = 4.9

$$\frac{4i\,ax-(ax+i)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+(i\,ax-1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+(ax+i)\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+}{ax+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (4*I*a*x - (a*x + I)*log(-a*x + sqrt(a^2*x^2 + 1) + 1) + (I*a*x - 1)*log(-a
*x + sqrt(a^2*x^2 + 1)) + (a*x + I)*log(-a*x + sqrt(a^2*x^2 + 1) - 1) + 4*I
*sqrt(a^2*x^2 + 1) - 4)/(a*x + I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(iax+1)^3}{x\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] Integral((I*a*x + 1)**3/(x*(a**2*x**2 + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x, algorithm="giac")
```

```
[Out] undef
```

3.24 $\int \frac{e^{3i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=63

$$-\frac{4a\sqrt{a^2x^2+1}}{ax+i}-\frac{\sqrt{a^2x^2+1}}{x}-3ia\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -(Sqrt[1 + a^2*x^2]/x) - (4*a*Sqrt[1 + a^2*x^2])/(I + a*x) - (3*I)*a*ArcTan
h[Sqrt[1 + a^2*x^2]]
```

Rubi [A] time = 0.559741, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5060, 6742, 264, 266, 63, 208, 651}

$$-\frac{4a\sqrt{a^2x^2+1}}{ax+i}-\frac{\sqrt{a^2x^2+1}}{x}-3ia\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])/x^2,x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/x) - (4*a*Sqrt[1 + a^2*x^2])/(I + a*x) - (3*I)*a*ArcTan
h[Sqrt[1 + a^2*x^2]]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1+iax)^2}{x^2(1-iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^2\sqrt{1+a^2x^2}} + \frac{3ia}{x\sqrt{1+a^2x^2}} - \frac{4ia^2}{(i+ax)\sqrt{1+a^2x^2}}\right) dx \\
&= (3ia)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx - \left(4ia^2\right)\int \frac{1}{(i+ax)\sqrt{1+a^2x^2}}\,dx + \int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{x} - \frac{4a\sqrt{1+a^2x^2}}{i+ax} + \frac{1}{2}(3ia)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{x} - \frac{4a\sqrt{1+a^2x^2}}{i+ax} + \frac{(3i)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a} \\
&= -\frac{\sqrt{1+a^2x^2}}{x} - \frac{4a\sqrt{1+a^2x^2}}{i+ax} - 3ia\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0444158, size = 61, normalized size = 0.97

$$\sqrt{a^2x^2+1}\left(-\frac{1}{x}-\frac{4a}{ax+i}\right)-3ia\log\left(\sqrt{a^2x^2+1}+1\right)+3ia\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/x^2,x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(-x^(-1) - (4*a)/(I + a*x)) + (3*I)*a*Log[x] - (3*I)*a*Lo
g[1 + Sqrt[1 + a^2*x^2]]
```

Maple [A] time = 0.069, size = 80, normalized size = 1.3

$$ia\frac{1}{\sqrt{a^2x^2+1}}-5\frac{a^2x}{\sqrt{a^2x^2+1}}-\frac{1}{x}\frac{1}{\sqrt{a^2x^2+1}}+3\,ia\left(\frac{1}{\sqrt{a^2x^2+1}}-\text{Artanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x)
```

```
[Out] I*a/(a^2*x^2+1)^(1/2)-5*a^2*x/(a^2*x^2+1)^(1/2)-1/x/(a^2*x^2+1)^(1/2)+3*I*a
*(1/(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.05147, size = 84, normalized size = 1.33

$$-\frac{5a^2x}{\sqrt{a^2x^2+1}}-3ia\operatorname{arsinh}\left(\frac{1}{\sqrt{a^2}|x|}\right)+\frac{4ia}{\sqrt{a^2x^2+1}}-\frac{1}{\sqrt{a^2x^2+1}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -5*a^2*x/sqrt(a^2*x^2 + 1) - 3*I*a*arcsinh(1/(sqrt(a^2)*abs(x))) + 4*I*a/sq
rt(a^2*x^2 + 1) - 1/(sqrt(a^2*x^2 + 1)*x)
```

Fricas [B] time = 1.79216, size = 247, normalized size = 3.92

$$-\frac{5a^2x^2+5iax+3\left(ia^2x^2-ax\right)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+3\left(-ia^2x^2+ax\right)\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+\sqrt{a^2x^2+1}(5}{ax^2+ix}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(5*a^2*x^2 + 5*I*a*x + 3*(I*a^2*x^2 - a*x)*log(-a*x + sqrt(a^2*x^2 + 1) +
1) + 3*(-I*a^2*x^2 + a*x)*log(-a*x + sqrt(a^2*x^2 + 1) - 1) + sqrt(a^2*x^2
+ 1)*(5*a*x + I))/(a*x^2 + I*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(iax+1)^3}{x^2\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] Integral((I*a*x + 1)**3/(x**2*(a**2*x**2 + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] undef
```

3.25 $\int \frac{e^{3 i \tan^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=92

$$-\frac{4ia^2\sqrt{a^2x^2+1}}{ax+i}-\frac{3ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -Sqrt[1 + a^2*x^2]/(2*x^2) - ((3*I)*a*Sqrt[1 + a^2*x^2])/x - ((4*I)*a^2*Sqr
t[1 + a^2*x^2])/(I + a*x) + (9*a^2*ArcTanh[Sqrt[1 + a^2*x^2]])/2
```

Rubi [A] time = 0.601464, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5060, 6742, 266, 51, 63, 208, 264, 651}

$$-\frac{4ia^2\sqrt{a^2x^2+1}}{ax+i}-\frac{3ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])/x^3,x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(2*x^2) - ((3*I)*a*Sqrt[1 + a^2*x^2])/x - ((4*I)*a^2*Sqr
t[1 + a^2*x^2])/(I + a*x) + (9*a^2*ArcTanh[Sqrt[1 + a^2*x^2]])/2
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1+iax)^2}{x^3(1-iax)\sqrt{1+a^2x^2}}\,dx\\
&= \int \left(\frac{1}{x^3\sqrt{1+a^2x^2}}+\frac{3ia}{x^2\sqrt{1+a^2x^2}}-\frac{4a^2}{x\sqrt{1+a^2x^2}}+\frac{4a^3}{(i+ax)\sqrt{1+a^2x^2}}\right)dx\\
&= (3ia)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx-\left(4a^2\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx+\left(4a^3\right)\int \frac{1}{(i+ax)\sqrt{1+a^2x^2}}\,dx+\int \frac{1}{x^3\sqrt{1+a^2x^2}}\\
&= -\frac{3ia\sqrt{1+a^2x^2}}{x}-\frac{4ia^2\sqrt{1+a^2x^2}}{i+ax}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx,x,x^2\right)-\left(2a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\right.\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{3ia\sqrt{1+a^2x^2}}{x}-\frac{4ia^2\sqrt{1+a^2x^2}}{i+ax}-4\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)-\frac{1}{4}a^2\,\text{Subst}\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{3ia\sqrt{1+a^2x^2}}{x}-\frac{4ia^2\sqrt{1+a^2x^2}}{i+ax}+4a^2\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx\right.\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{3ia\sqrt{1+a^2x^2}}{x}-\frac{4ia^2\sqrt{1+a^2x^2}}{i+ax}+\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0766212, size = 79, normalized size = 0.86

$$\sqrt{a^2x^2+1}\left(-\frac{4ia^2}{ax+i}-\frac{3ia}{x}-\frac{1}{2x^2}\right)+\frac{9}{2}a^2\log\left(\sqrt{a^2x^2+1}+1\right)-\frac{9}{2}a^2\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/x^3,x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(-1/(2*x^2) - ((3*I)*a)/x - ((4*I)*a^2)/(I + a*x)) - (9*a
^2*Log[x])/2 + (9*a^2*Log[1 + Sqrt[1 + a^2*x^2]])/2
```

Maple [A] time = 0.072, size = 105, normalized size = 1.1

$$-ia^3x\frac{1}{\sqrt{a^2x^2+1}}-\frac{1}{2x^2}\frac{1}{\sqrt{a^2x^2+1}}-\frac{9a^2}{2}\left(\frac{1}{\sqrt{a^2x^2+1}}-\operatorname{Artanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)+3ia\left(-\frac{1}{x}\frac{1}{\sqrt{a^2x^2+1}}-2\frac{a^2x}{\sqrt{a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x)
```

```
[Out] -I*a^3*x/(a^2*x^2+1)^(1/2)-1/2/x^2/(a^2*x^2+1)^(1/2)-9/2*a^2*(1/(a^2*x^2+1)
^(1/2)-arctanh(1/(a^2*x^2+1)^(1/2)))+3*I*a*(-1/x/(a^2*x^2+1)^(1/2)-2*a^2*x/
(a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.05043, size = 112, normalized size = 1.22

$$-\frac{7i\,a^3x}{\sqrt{a^2x^2+1}}+\frac{9}{2}a^2\operatorname{arsinh}\left(\frac{1}{\sqrt{a^2}|x|}\right)-\frac{9\,a^2}{2\sqrt{a^2x^2+1}}-\frac{3i\,a}{\sqrt{a^2x^2+1}x}-\frac{1}{2\sqrt{a^2x^2+1}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -7*I*a^3*x/sqrt(a^2*x^2 + 1) + 9/2*a^2*arcsinh(1/(sqrt(a^2)*abs(x))) - 9/2*
a^2/sqrt(a^2*x^2 + 1) - 3*I*a/(sqrt(a^2*x^2 + 1)*x) - 1/2/(sqrt(a^2*x^2 + 1
)*x^2)
```

Fricas [A] time = 1.71156, size = 294, normalized size = 3.2

$$\frac{-14i\,a^3x^3+14\,a^2x^2+9\left(a^3x^3+i\,a^2x^2\right)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-9\left(a^3x^3+i\,a^2x^2\right)\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+\sqrt{a}}{2\left(ax^3+i\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(-14*I*a^3*x^3 + 14*a^2*x^2 + 9*(a^3*x^3 + I*a^2*x^2)*log(-a*x + sqrt(a
^2*x^2 + 1) + 1) - 9*(a^3*x^3 + I*a^2*x^2)*log(-a*x + sqrt(a^2*x^2 + 1) - 1
) + sqrt(a^2*x^2 + 1)*(-14*I*a^2*x^2 + 5*a*x - I))/(a*x^3 + I*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(iax+1)^3}{x^3\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] Integral((I*a*x + 1)**3/(x**3*(a**2*x**2 + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

3.26 $\int \frac{e^{3i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=117

$$\frac{4a^3\sqrt{a^2x^2+1}}{ax+i}+\frac{14a^2\sqrt{a^2x^2+1}}{3x}-\frac{3ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{11}{2}ia^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -Sqrt[1 + a^2*x^2]/(3*x^3) - (((3*I)/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (14*a^2*
Sqrt[1 + a^2*x^2])/(3*x) + (4*a^3*Sqrt[1 + a^2*x^2])/(I + a*x) + ((11*I)/2)
*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rubi [A] time = 0.61597, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5060, 6742, 271, 264, 266, 51, 63, 208, 651}

$$\frac{4a^3\sqrt{a^2x^2+1}}{ax+i}+\frac{14a^2\sqrt{a^2x^2+1}}{3x}-\frac{3ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{11}{2}ia^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(3*x^3) - (((3*I)/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (14*a^2*
Sqrt[1 + a^2*x^2])/(3*x) + (4*a^3*Sqrt[1 + a^2*x^2])/(I + a*x) + ((11*I)/2)
*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{3i\tan^{-1}(ax)}}{x^4}\,dx = \int \frac{(1+iax)^2}{x^4(1-iax)\sqrt{1+a^2x^2}}\,dx$$

$$= \int \left(\frac{1}{x^4\sqrt{1+a^2x^2}} + \frac{3ia}{x^3\sqrt{1+a^2x^2}} - \frac{4a^2}{x^2\sqrt{1+a^2x^2}} - \frac{4ia^3}{x\sqrt{1+a^2x^2}} + \frac{4ia^4}{(i+ax)\sqrt{1+a^2x^2}}\right)dx$$

$$= (3ia)\int \frac{1}{x^3\sqrt{1+a^2x^2}}\,dx - \left(4a^2\right)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx - \left(4ia^3\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx + \left(4ia^4\right)\int \frac{1}{(i+ax)\sqrt{1}}$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{4a^2\sqrt{1+a^2x^2}}{x} + \frac{4a^3\sqrt{1+a^2x^2}}{i+ax} + \frac{1}{2}(3ia)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx, x, x^2\right) - \frac{1}{3}\left(2a^2\right)\int$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} - \frac{3ia\sqrt{1+a^2x^2}}{2x^2} + \frac{14a^2\sqrt{1+a^2x^2}}{3x} + \frac{4a^3\sqrt{1+a^2x^2}}{i+ax} - (4ia)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1}\right.$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} - \frac{3ia\sqrt{1+a^2x^2}}{2x^2} + \frac{14a^2\sqrt{1+a^2x^2}}{3x} + \frac{4a^3\sqrt{1+a^2x^2}}{i+ax} + 4ia^3\tanh^{-1}\left(\sqrt{1+a^2x^2}\right) - \frac{1}{2}(3ia)$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} - \frac{3ia\sqrt{1+a^2x^2}}{2x^2} + \frac{14a^2\sqrt{1+a^2x^2}}{3x} + \frac{4a^3\sqrt{1+a^2x^2}}{i+ax} + \frac{11}{2}ia^3\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)$$

Mathematica [A] time = 0.0754431, size = 89, normalized size = 0.76

$$\frac{1}{6}\left(\frac{\sqrt{a^2x^2+1}\left(52a^3x^3+19ia^2x^2+7ax-2i\right)}{x^3(ax+i)} + 33ia^3\log\left(\sqrt{a^2x^2+1}+1\right) - 33ia^3\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-2*I + 7*a*x + (19*I)*a^2*x^2 + 52*a^3*x^3))/(x^3*(I +
 a*x)) - (33*I)*a^3*Log[x] + (33*I)*a^3*Log[1 + Sqrt[1 + a^2*x^2]])/6
```

Maple [A] time = 0.07, size = 141, normalized size = 1.2

$$3\,ia\left(-\frac{1}{2\,x^2}\frac{1}{\sqrt{a^2x^2+1}}-\frac{3\,a^2}{2}\left(\frac{1}{\sqrt{a^2x^2+1}}-\mathrm{Artanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)\right)-\frac{13\,a^2}{3}\left(-\frac{1}{x}\frac{1}{\sqrt{a^2x^2+1}}-2\frac{a^2x}{\sqrt{a^2x^2+1}}\right)-\frac{1}{3\,x^3}\frac{}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x)
```

```
[Out] 3*I*a*(-1/2/x^2/(a^2*x^2+1)^(1/2)-3/2*a^2*(1/(a^2*x^2+1)^(1/2)-arctanh(1/(a
^2*x^2+1)^(1/2))))-13/3*a^2*(-1/x/(a^2*x^2+1)^(1/2)-2*a^2*x/(a^2*x^2+1)^(1/
2))-1/3/x^3/(a^2*x^2+1)^(1/2)-I*a^3*(1/(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2
+1)^(1/2)))
```

Maxima [A] time = 1.05337, size = 138, normalized size = 1.18

$$\frac{26\,a^4x}{3\,\sqrt{a^2x^2+1}}+\frac{11}{2}i\,a^3\,\mathrm{arsinh}\left(\frac{1}{\sqrt{a^2}|x|}\right)-\frac{11i\,a^3}{2\,\sqrt{a^2x^2+1}}+\frac{13\,a^2}{3\,\sqrt{a^2x^2+1}x}-\frac{3i\,a}{2\,\sqrt{a^2x^2+1}x^2}-\frac{1}{3\,\sqrt{a^2x^2+1}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 26/3*a^4*x/sqrt(a^2*x^2 + 1) + 11/2*I*a^3*arcsinh(1/(sqrt(a^2)*abs(x))) - 1
1/2*I*a^3/sqrt(a^2*x^2 + 1) + 13/3*a^2/(sqrt(a^2*x^2 + 1)*x) - 3/2*I*a/(sqr
t(a^2*x^2 + 1)*x^2) - 1/3/(sqrt(a^2*x^2 + 1)*x^3)
```

Fricas [A] time = 1.73378, size = 316, normalized size = 2.7

$$\frac{52\,a^4x^4+52i\,a^3x^3-33\left(-i\,a^4x^4+a^3x^3\right)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-33\left(i\,a^4x^4-a^3x^3\right)\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+}{6\left(ax^4+i\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(52*a^4*x^4 + 52*I*a^3*x^3 - 33*(-I*a^4*x^4 + a^3*x^3)*log(-a*x + sqrt(
a^2*x^2 + 1) + 1) - 33*(I*a^4*x^4 - a^3*x^3)*log(-a*x + sqrt(a^2*x^2 + 1) -
 1) + (52*a^3*x^3 + 19*I*a^2*x^2 + 7*a*x - 2*I)*sqrt(a^2*x^2 + 1))/(a*x^4 +
 I*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(iax+1)^3}{x^4\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] Integral((I*a*x + 1)**3/(x**4*(a**2*x**2 + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

3.27 $\int e^{4i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=65

$$-\frac{4x^2}{a^2}+\frac{12ix}{a^3}+\frac{4i}{a^4(ax+i)}+\frac{16\log(ax+i)}{a^4}-\frac{4ix^3}{3a}+\frac{x^4}{4}$$

```
[Out] ((12*I)*x)/a^3 - (4*x^2)/a^2 - (((4*I)/3)*x^3)/a + x^4/4 + (4*I)/(a^4*(I +
a*x)) + (16*Log[I + a*x])/a^4
```

Rubi [A] time = 0.0445987, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 88}

$$-\frac{4x^2}{a^2}+\frac{12ix}{a^3}+\frac{4i}{a^4(ax+i)}+\frac{16\log(ax+i)}{a^4}-\frac{4ix^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])*x^3,x]
```

```
[Out] ((12*I)*x)/a^3 - (4*x^2)/a^2 - (((4*I)/3)*x^3)/a + x^4/4 + (4*I)/(a^4*(I +
a*x)) + (16*Log[I + a*x])/a^4
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{4i\tan^{-1}(ax)}x^3\,dx &= \int \frac{x^3(1+iax)^2}{(1-iax)^2}\,dx\\ &= \int \left(\frac{12i}{a^3}-\frac{8x}{a^2}-\frac{4ix^2}{a}+x^3-\frac{4i}{a^3(i+ax)^2}+\frac{16}{a^3(i+ax)}\right)dx\\ &= \frac{12ix}{a^3}-\frac{4x^2}{a^2}-\frac{4ix^3}{3a}+\frac{x^4}{4}+\frac{4i}{a^4(i+ax)}+\frac{16\log(i+ax)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.042928, size = 65, normalized size = 1.

$$-\frac{4x^2}{a^2}+\frac{12ix}{a^3}+\frac{4i}{a^4(ax+i)}+\frac{16\log(ax+i)}{a^4}-\frac{4ix^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])*x^3,x]
```

```
[Out] ((12*I)*x)/a^3 - (4*x^2)/a^2 - (((4*I)/3)*x^3)/a + x^4/4 + (4*I)/(a^4*(I +
a*x)) + (16*Log[I + a*x])/a^4
```

Maple [A] time = 0.046, size = 70, normalized size = 1.1

$$\frac{x^4}{4} - \frac{\frac{4i}{3}x^3}{a} - 4\frac{x^2}{a^2} + \frac{12\,ix}{a^3} + \frac{4\,i}{a^4\,(ax+i)} + 8\frac{\ln\left(a^2x^2+1\right)}{a^4} - \frac{16\,i\arctan\left(ax\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x)
```

```
[Out] 1/4*x^4-4/3*I*x^3/a-4*x^2/a^2+12*I*x/a^3+4*I/a^4/(a*x+I)+8/a^4*ln(a^2*x^2+1
)-16*I/a^4*arctan(a*x)
```

Maxima [A] time = 1.53823, size = 104, normalized size = 1.6

$$-\frac{4\,(-i\,ax-1)}{a^6x^2+a^4} + \frac{3\,a^3x^4 - 16i\,a^2x^3 - 48\,ax^2 + 144i\,x}{12\,a^3} - \frac{16i\,\arctan\left(ax\right)}{a^4} + \frac{8\,\log\left(a^2x^2+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x, algorithm="maxima")
```

```
[Out] -4*(-I*a*x - 1)/(a^6*x^2 + a^4) + 1/12*(3*a^3*x^4 - 16*I*a^2*x^3 - 48*a*x^2
 + 144*I*x)/a^3 - 16*I*arctan(a*x)/a^4 + 8*log(a^2*x^2 + 1)/a^4
```

Fricas [A] time = 1.49774, size = 177, normalized size = 2.72

$$\frac{3\,a^5x^5 - 13i\,a^4x^4 - 32\,a^3x^3 + 96i\,a^2x^2 - 144\,ax + (192\,ax + 192i)\log\left(\frac{ax+i}{a}\right) + 48i}{12\left(a^5x + i\,a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^5*x^5 - 13*I*a^4*x^4 - 32*a^3*x^3 + 96*I*a^2*x^2 - 144*a*x + (192
*a*x + 192*I)*log((a*x + I)/a) + 48*I)/(a^5*x + I*a^4)
```

Sympy [A] time = 0.500013, size = 58, normalized size = 0.89

$$\frac{4ia}{a^6x + ia^5} + \frac{x^4}{4} - \frac{4ix^3}{3a} - \frac{4x^2}{a^2} + \frac{12ix}{a^3} + \frac{16\log\left(ax+i\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x**3,x)
```

```
[Out] 4*I*a/(a**6*x + I*a**5) + x**4/4 - 4*I*x**3/(3*a) - 4*x**2/a**2 + 12*I*x/a*
*3 + 16*log(a*x + I)/a**4
```

Giac [A] time = 1.09623, size = 85, normalized size = 1.31

$$\frac{16 \log(ax+i)}{a^4} + \frac{4i}{(ax+i)a^4} + \frac{3a^8x^4 - 16a^7ix^3 - 48a^6x^2 + 144a^5ix}{12a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x, algorithm="giac")
```

```
[Out] 16*log(a*x + i)/a^4 + 4*i/((a*x + i)*a^4) + 1/12*(3*a^8*x^4 - 16*a^7*i*x^3
- 48*a^6*x^2 + 144*a^5*i*x)/a^8
```

3.28 $\int e^{4 i \tan^{-1}(a x)} x^2 \, dx$

Optimal. Leaf size=53

$$-\frac{8x}{a^2}-\frac{4}{a^3(ax+i)}+\frac{12i\log(ax+i)}{a^3}-\frac{2ix^2}{a}+\frac{x^3}{3}$$

```
[Out] (-8*x)/a^2 - ((2*I)*x^2)/a + x^3/3 - 4/(a^3*(I + a*x)) + ((12*I)*Log[I + a*
x])/a^3
```

Rubi [A] time = 0.0399861, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 88}

$$-\frac{8x}{a^2}-\frac{4}{a^3(ax+i)}+\frac{12i\log(ax+i)}{a^3}-\frac{2ix^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])*x^2,x]
```

```
[Out] (-8*x)/a^2 - ((2*I)*x^2)/a + x^3/3 - 4/(a^3*(I + a*x)) + ((12*I)*Log[I + a*
x])/a^3
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int e^{4i\tan^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1+iax)^2}{(1-iax)^2}\,dx\\
&= \int \left(-\frac{8}{a^2}-\frac{4ix}{a}+x^2+\frac{4}{a^2(i+ax)^2}+\frac{12i}{a^2(i+ax)}\right)dx\\
&= -\frac{8x}{a^2}-\frac{2ix^2}{a}+\frac{x^3}{3}-\frac{4}{a^3(i+ax)}+\frac{12i\log(i+ax)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.0308238, size = 53, normalized size = 1.

$$-\frac{8x}{a^2}-\frac{4}{a^3(ax+i)}+\frac{12i\log(ax+i)}{a^3}-\frac{2ix^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])*x^2,x]
```

```
[Out] (-8*x)/a^2 - ((2*I)*x^2)/a + x^3/3 - 4/(a^3*(I + a*x)) + ((12*I)*Log[I + a*
x])/a^3
```

Maple [A] time = 0.046, size = 60, normalized size = 1.1

$$\frac{x^3}{3} - \frac{2\,ix^2}{a} - 8\,\frac{x}{a^2} - 4\,\frac{1}{a^3\,(ax+i)} + \frac{6\,i\ln\left(a^2x^2+1\right)}{a^3} + 12\,\frac{\arctan(ax)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x)
```

```
[Out] 1/3*x^3-2*I*x^2/a-8*x/a^2-4/a^3/(a*x+I)+6*I/a^3*ln(a^2*x^2+1)+12/a^3*arctan
(a*x)
```

Maxima [A] time = 1.49246, size = 92, normalized size = 1.74

$$-\frac{8\,ax-8i}{2\left(a^5x^2+a^3\right)} + \frac{a^2x^3-6i\,ax^2-24\,x}{3\,a^2} + \frac{12\,\arctan(ax)}{a^3} + \frac{6i\,\log\left(a^2x^2+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(8*a*x - 8*I)/(a^5*x^2 + a^3) + 1/3*(a^2*x^3 - 6*I*a*x^2 - 24*x)/a^2 +
 12*arctan(a*x)/a^3 + 6*I*log(a^2*x^2 + 1)/a^3
```

Fricas [A] time = 1.62085, size = 147, normalized size = 2.77

$$\frac{a^4x^4-5i\,a^3x^3-18\,a^2x^2-24i\,ax-36\,(-i\,ax+1)\log\left(\frac{ax+i}{a}\right)-12}{3\left(a^4x+i\,a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^4*x^4 - 5*I*a^3*x^3 - 18*a^2*x^2 - 24*I*a*x - 36*(-I*a*x + 1)*log((a
*x + I)/a) - 12)/(a^4*x + I*a^3)
```

Sympy [A] time = 0.490888, size = 46, normalized size = 0.87

$$-\frac{4a}{a^5x+ia^4} + \frac{x^3}{3} - \frac{2ix^2}{a} - \frac{8x}{a^2} + \frac{12i\log(ax+i)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x**2,x)
```

```
[Out] -4*a/(a**5*x + I*a**4) + x**3/3 - 2*I*x**2/a - 8*x/a**2 + 12*I*log(a*x + I)
/a**3
```

Giac [A] time = 1.11657, size = 72, normalized size = 1.36

$$\frac{12\,i\log\left(ax+i\right)}{a^3}-\frac{4}{(ax+i)a^3}+\frac{a^6x^3-6\,a^5ix^2-24\,a^4x}{3\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x, algorithm="giac")
```

```
[Out] 12*i*log(a*x + i)/a^3 - 4/((a*x + i)*a^3) + 1/3*(a^6*x^3 - 6*a^5*i*x^2 - 24
*a^4*x)/a^6
```

3.29 $\int e^{4i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=45

$$-\frac{4i}{a^2(ax+i)}-\frac{8\log(ax+i)}{a^2}-\frac{4ix}{a}+\frac{x^2}{2}$$

```
[Out] ((-4*I)*x)/a + x^2/2 - (4*I)/(a^2*(I + a*x)) - (8*Log[I + a*x])/a^2
```

Rubi [A] time = 0.0262917, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5062, 77}

$$-\frac{4i}{a^2(ax+i)}-\frac{8\log(ax+i)}{a^2}-\frac{4ix}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])*x,x]
```

```
[Out] ((-4*I)*x)/a + x^2/2 - (4*I)/(a^2*(I + a*x)) - (8*Log[I + a*x])/a^2
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int e^{4i\tan^{-1}(ax)}x\,dx &= \int \frac{x(1+iax)^2}{(1-iax)^2}\,dx\\
&= \int \left(-\frac{4i}{a}+x+\frac{4i}{a(i+ax)^2}-\frac{8}{a(i+ax)}\right)dx\\
&= -\frac{4ix}{a}+\frac{x^2}{2}-\frac{4i}{a^2(i+ax)}-\frac{8\log(i+ax)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.0240546, size = 45, normalized size = 1.

$$-\frac{4i}{a^2(ax+i)}-\frac{8\log(ax+i)}{a^2}-\frac{4ix}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])*x,x]
```

```
[Out] ((-4*I)*x)/a + x^2/2 - (4*I)/(a^2*(I + a*x)) - (8*Log[I + a*x])/a^2
```

Maple [A] time = 0.045, size = 53, normalized size = 1.2

$$\frac{x^2}{2}-\frac{4\,ix}{a}-\frac{4\,i}{a^2\,(ax+i)}-4\,\frac{\ln\left(a^2x^2+1\right)}{a^2}+\frac{8\,i\arctan\left(ax\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2*x,x)
```

```
[Out] 1/2*x^2-4*I*x/a-4*I/a^2/(a*x+I)-4/a^2*ln(a^2*x^2+1)+8*I/a^2*arctan(a*x)
```

Maxima [A] time = 1.50673, size = 81, normalized size = 1.8

$$-\frac{4\,(i\,ax+1)}{a^4x^2+a^2}+\frac{ax^2-8i\,x}{2\,a}+\frac{8i\,\arctan\left(ax\right)}{a^2}-\frac{4\,\log\left(a^2x^2+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x,x, algorithm="maxima")
```

```
[Out] -4*(I*a*x + 1)/(a^4*x^2 + a^2) + 1/2*(a*x^2 - 8*I*x)/a + 8*I*arctan(a*x)/a^
2 - 4*log(a^2*x^2 + 1)/a^2
```

Fricas [A] time = 1.57681, size = 127, normalized size = 2.82

$$\frac{a^3x^3-7i\,a^2x^2+8\,ax-(16\,ax+16i)\log\left(\frac{ax+i}{a}\right)-8i}{2\left(a^3x+i\,a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*x^3 - 7*I*a^2*x^2 + 8*a*x - (16*a*x + 16*I)*log((a*x + I)/a) - 8*I
)/(a^3*x + I*a^2)
```

Sympy [A] time = 0.483821, size = 37, normalized size = 0.82

$$-\frac{4ia}{a^4x+ia^3}+\frac{x^2}{2}-\frac{4ix}{a}-\frac{8\log\left(ax+i\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x,x)
```

```
[Out] -4*I*a/(a**4*x + I*a**3) + x**2/2 - 4*I*x/a - 8*log(a*x + I)/a**2
```

Giac [A] time = 1.11768, size = 61, normalized size = 1.36

$$-\frac{8\,\log\left(ax+i\right)}{a^2}-\frac{4\,i}{(ax+i)a^2}+\frac{a^4x^2-8\,a^3ix}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x,x, algorithm="giac")
```

```
[Out] -8*log(a*x + i)/a^2 - 4*i/((a*x + i)*a^2) + 1/2*(a^4*x^2 - 8*a^3*i*x)/a^4
```

3.30 $\int e^{4 i \tan^{-1}(a x)} \, dx$

Optimal. Leaf size=31

$$\frac{4}{a(ax+i)}-\frac{4i\log(ax+i)}{a}+x$$

```
[Out] x + 4/(a*(I + a*x)) - ((4*I)*Log[I + a*x])/a
```

Rubi [A] time = 0.0140897, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5061, 43}

$$\frac{4}{a(ax+i)}-\frac{4i\log(ax+i)}{a}+x$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x]),x]
```

```
[Out] x + 4/(a*(I + a*x)) - ((4*I)*Log[I + a*x])/a
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4i\tan^{-1}(ax)}\,dx &= \int \frac{(1+iax)^2}{(1-iax)^2}\,dx\\ &= \int \left(1-\frac{4}{(i+ax)^2}-\frac{4i}{i+ax}\right)dx\\ &= x+\frac{4}{a(i+ax)}-\frac{4i\log(i+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0204097, size = 42, normalized size = 1.35

$$-\frac{2i\log\left(a^2x^2+1\right)}{a}+\frac{4}{a(ax+i)}-\frac{4\tan^{-1}(ax)}{a}+x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((4*I)*ArcTan[a*x]),x]
```

```
[Out] x + 4/(a*(I + a*x)) - (4*ArcTan[a*x])/a - ((2*I)*Log[1 + a^2*x^2])/a
```

Maple [A] time = 0.043, size = 41, normalized size = 1.3

$$x+4\,\frac{1}{a\left(ax+i\right)}-\frac{2\,i\ln\left(a^{2}x^{2}+1\right)}{a}-4\,\frac{\arctan\left(ax\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2,x)
```

```
[Out] x+4/a/(a*x+I)-2*I/a*ln(a^2*x^2+1)-4*arctan(a*x)/a
```

Maxima [A] time = 1.52143, size = 61, normalized size = 1.97

$$x+\frac{8\,ax-8i}{2\left(a^{3}x^{2}+a\right)}-\frac{4\,\arctan\left(ax\right)}{a}-\frac{2i\,\log\left(a^{2}x^{2}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] x + 1/2*(8*a*x - 8*I)/(a^3*x^2 + a) - 4*arctan(a*x)/a - 2*I*log(a^2*x^2 + 1
)/a
```

Fricas [A] time = 1.53931, size = 95, normalized size = 3.06

$$\frac{a^{2}x^{2}+i\,ax-4\left(i\,ax-1\right)\log\left(\frac{ax+i}{a}\right)+4}{a^{2}x+i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] (a^2*x^2 + I*a*x - 4*(I*a*x - 1)*log((a*x + I)/a) + 4)/(a^2*x + I*a)
```

Sympy [A] time = 0.421508, size = 26, normalized size = 0.84

$$\frac{4a}{a^{3}x+ia^{2}}+x-\frac{4i\log\left(ax+i\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2,x)
```

```
[Out] 4*a/(a**3*x + I*a**2) + x - 4*I*log(a*x + I)/a
```

<u>Giac</u> [A] time = 1.12512, size = 35, normalized size = 1.13

$$x - \frac{4\,i\log\left(ax + i\right)}{a} + \frac{4}{(ax + i)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] x - 4*i*log(a*x + i)/a + 4/((a*x + i)*a)
```

3.31 $\int \frac{e^{4 i \tan^{-1}(a x)}}{x} \, dx$

Optimal. Leaf size=16

$$\log(x) + \frac{4i}{ax+i}$$

```
[Out] (4*I)/(I + a*x) + Log[x]
```

Rubi [A] time = 0.0215648, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 88}

$$\log(x) + \frac{4i}{ax+i}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])/x,x]
```

```
[Out] (4*I)/(I + a*x) + Log[x]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4 i \tan^{-1}(ax)}}{x} \, dx &= \int \frac{(1+iax)^2}{x(1-iax)^2} \, dx \\
&= \int \left(\frac{1}{x} - \frac{4ia}{(i+ax)^2}\right) dx \\
&= \frac{4i}{i+ax} + \log(x)
\end{aligned}$$

Mathematica [A] time = 0.0096094, size = 16, normalized size = 1.

$$\log(x) + \frac{4i}{ax+i}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/x,x]
```

```
[Out] (4*I)/(I + a*x) + Log[x]
```

Maple [A] time = 0.047, size = 15, normalized size = 0.9

$$\frac{4\,i}{ax+i}+\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/x,x)
```

```
[Out] 4*I/(a*x+I)+ln(x)
```

Maxima [A] time = 1.52583, size = 30, normalized size = 1.88

$$-\frac{4\,(-i\,ax-1)}{a^2x^2+1}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x,x, algorithm="maxima")
```

```
[Out] -4*(-I*a*x - 1)/(a^2*x^2 + 1) + log(x)
```

Fricas [A] time = 1.60104, size = 49, normalized size = 3.06

$$\frac{(ax+i)\log\left(x\right)+4i}{ax+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x,x, algorithm="fricas")
```

```
[Out] ((a*x + I)*log(x) + 4*I)/(a*x + I)
```

Sympy [A] time = 0.475003, size = 15, normalized size = 0.94

$$\frac{4ia}{a^2x+ia}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/x,x)
```

```
[Out] 4*I*a/(a**2*x + I*a) + log(x)
```

Giac [A] time = 1.10788, size = 19, normalized size = 1.19

$$\frac{4\,i}{ax+i}+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x,x, algorithm="giac")
```

```
[Out] 4*i/(a*x + i) + log(abs(x))
```

3.32 $\int \frac{e^{4 i \tan^{-1}(ax)}}{x^2} \, dx$

Optimal. Leaf size=38

$$-\frac{4a}{ax+i}+4ia\log(x)-4ia\log(ax+i)-\frac{1}{x}$$

```
[Out] -x^(-1) - (4*a)/(I + a*x) + (4*I)*a*Log[x] - (4*I)*a*Log[I + a*x]
```

Rubi [A] time = 0.030845, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 88}

$$-\frac{4a}{ax+i}+4ia\log(x)-4ia\log(ax+i)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])/x^2,x]
```

```
[Out] -x^(-1) - (4*a)/(I + a*x) + (4*I)*a*Log[x] - (4*I)*a*Log[I + a*x]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4 i \tan^{-1}(ax)}}{x^2} \, dx &= \int \frac{(1+iax)^2}{x^2(1-iax)^2} \, dx \\
&= \int \left(\frac{1}{x^2}+\frac{4ia}{x}+\frac{4a^2}{(i+ax)^2}-\frac{4ia^2}{i+ax}\right) dx \\
&= -\frac{1}{x}-\frac{4a}{i+ax}+4ia\log(x)-4ia\log(i+ax)
\end{aligned}$$

Mathematica [A] time = 0.0242619, size = 38, normalized size = 1.

$$-\frac{4a}{ax+i}+4ia\log(x)-4ia\log(ax+i)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/x^2,x]
```

```
[Out] -x^(-1) - (4*a)/(I + a*x) + (4*I)*a*Log[x] - (4*I)*a*Log[I + a*x]
```

Maple [A] time = 0.049, size = 45, normalized size = 1.2

$$-4\frac{a}{ax+i}-2\,ia\ln\left(a^2x^2+1\right)-4\,a\arctan\left(ax\right)-x^{-1}+4\,ia\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x)
```

```
[Out] -4*a/(a*x+I)-2*I*a*ln(a^2*x^2+1)-4*a*arctan(a*x)-1/x+4*I*a*ln(x)
```

Maxima [A] time = 1.53394, size = 72, normalized size = 1.89

$$-4\,a\arctan\left(ax\right)-2i\,a\log\left(a^2x^2+1\right)+4i\,a\log\left(x\right)-\frac{10\,a^2x^2-8i\,ax+2}{2\left(a^2x^3+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x, algorithm="maxima")
```

```
[Out] -4*a*arctan(a*x) - 2*I*a*log(a^2*x^2 + 1) + 4*I*a*log(x) - 1/2*(10*a^2*x^2
- 8*I*a*x + 2)/(a^2*x^3 + x)
```

Fricas [B] time = 1.66007, size = 131, normalized size = 3.45

$$-\frac{5\,ax+4\left(-i\,a^2x^2+ax\right)\log\left(x\right)+4\left(i\,a^2x^2-ax\right)\log\left(\frac{ax+i}{a}\right)+i}{ax^2+i\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x, algorithm="fricas")
```

```
[Out] -(5*a*x + 4*(-I*a^2*x^2 + a*x)*log(x) + 4*(I*a^2*x^2 - a*x)*log((a*x + I)/a
) + I)/(a*x^2 + I*x)
```

Sympy [A] time = 0.608917, size = 37, normalized size = 0.97

$$4a\left(i\log\left(x\right)-i\log\left(x+\frac{i}{a}\right)\right)-\frac{5a^2x+ia}{a^2x^2+iax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/x**2,x)
```

```
[Out] 4*a*(I*log(x) - I*log(x + I/a)) - (5*a**2*x + I*a)/(a**2*x**2 + I*a*x)
```

Giac [A] time = 1.10653, size = 50, normalized size = 1.32

$$-4\,ai\log\left(ax+i\right)+4\,ai\log\left(|x|\right)-\frac{5\,ax+i}{ax^{2}+ix}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x, algorithm="giac")
```

```
[Out] -4*a*i*log(a*x + i) + 4*a*i*log(abs(x)) - (5*a*x + i)/(a*x^2 + i*x)
```

3.33 $\int \frac{e^{4i\tan^{-1}(ax)}}{x^3} \, dx$

Optimal. Leaf size=52

$$-\frac{4ia^2}{ax+i} - 8a^2\log(x) + 8a^2\log(ax+i) - \frac{4ia}{x} - \frac{1}{2x^2}$$

```
[Out] -1/(2*x^2) - ((4*I)*a)/x - ((4*I)*a^2)/(I + a*x) - 8*a^2*Log[x] + 8*a^2*Log
[I + a*x]
```

Rubi [A] time = 0.0353444, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 88}

$$-\frac{4ia^2}{ax+i} - 8a^2\log(x) + 8a^2\log(ax+i) - \frac{4ia}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/(2*x^2) - ((4*I)*a)/x - ((4*I)*a^2)/(I + a*x) - 8*a^2*Log[x] + 8*a^2*Log
[I + a*x]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4i\tan^{-1}(ax)}}{x^3} \, dx &= \int \frac{(1+iax)^2}{x^3(1-iax)^2} \, dx \\
&= \int \left(\frac{1}{x^3} + \frac{4ia}{x^2} - \frac{8a^2}{x} + \frac{4ia^3}{(i+ax)^2} + \frac{8a^3}{i+ax}\right) dx \\
&= -\frac{1}{2x^2} - \frac{4ia}{x} - \frac{4ia^2}{i+ax} - 8a^2\log(x) + 8a^2\log(i+ax)
\end{aligned}$$

Mathematica [A] time = 0.0358388, size = 52, normalized size = 1.

$$-\frac{4ia^2}{ax+i} - 8a^2\log(x) + 8a^2\log(ax+i) - \frac{4ia}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/(2*x^2) - ((4*I)*a)/x - ((4*I)*a^2)/(I + a*x) - 8*a^2*Log[x] + 8*a^2*Log
[I + a*x]
```

Maple [A] time = 0.048, size = 60, normalized size = 1.2

$$\frac{-4\,ia^2}{ax+i}+4\,a^2\ln\left(a^2x^2+1\right)-8\,ia^2\arctan\left(ax\right)-\frac{1}{2\,x^2}-\frac{4\,ia}{x}-8\,a^2\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x)
```

```
[Out] -4*I*a^2/(a*x+I)+4*a^2*ln(a^2*x^2+1)-8*I*a^2*arctan(a*x)-1/2/x^2-4*I*a/x-8*
a^2*ln(x)
```

Maxima [A] time = 1.53399, size = 93, normalized size = 1.79

$$-8i\,a^2\arctan\left(ax\right)+4\,a^2\log\left(a^2x^2+1\right)-8\,a^2\log\left(x\right)+\frac{-16i\,a^3x^3-9\,a^2x^2-8i\,ax-1}{2\left(a^2x^4+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x, algorithm="maxima")
```

```
[Out] -8*I*a^2*arctan(a*x) + 4*a^2*log(a^2*x^2 + 1) - 8*a^2*log(x) + 1/2*(-16*I*a
^3*x^3 - 9*a^2*x^2 - 8*I*a*x - 1)/(a^2*x^4 + x^2)
```

Fricas [A] time = 1.71169, size = 171, normalized size = 3.29

$$\frac{-16i\,a^2x^2+7\,ax-16\left(a^3x^3+i\,a^2x^2\right)\log\left(x\right)+16\left(a^3x^3+i\,a^2x^2\right)\log\left(\frac{ax+i}{a}\right)-i}{2\left(ax^3+i\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(-16*I*a^2*x^2 + 7*a*x - 16*(a^3*x^3 + I*a^2*x^2)*log(x) + 16*(a^3*x^3
+ I*a^2*x^2)*log((a*x + I)/a) - I)/(a*x^3 + I*x^2)
```

Sympy [A] time = 0.685398, size = 51, normalized size = 0.98

$$8a^2\left(-\log\left(x\right)+\log\left(x+\frac{i}{a}\right)\right)-\frac{16ia^3x^2-7a^2x+ia}{2a^2x^3+2iax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/x**3,x)
```

```
[Out] 8*a**2*(-log(x) + log(x + I/a)) - (16*I*a**3*x**2 - 7*a**2*x + I*a)/(2*a**2
*x**3 + 2*I*a*x**2)
```

Giac [A] time = 1.09499, size = 63, normalized size = 1.21

$$8\,a^2\log\left(ax+i\right)-8\,a^2\log\left(|x|\right)-\frac{16\,a^2ix^2-7\,ax+i}{2\,(ax+i)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x, algorithm="giac")
```

```
[Out] 8*a^2*log(a*x + i) - 8*a^2*log(abs(x)) - 1/2*(16*a^2*i*x^2 - 7*a*x + i)/((a
*x + i)*x^2)
```

3.34 $\int \frac{e^{4i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=62

$$\frac{4a^3}{ax+i}+\frac{8a^2}{x}-12ia^3\log(x)+12ia^3\log(ax+i)-\frac{2ia}{x^2}-\frac{1}{3x^3}$$

```
[Out] -1/(3*x^3) - ((2*I)*a)/x^2 + (8*a^2)/x + (4*a^3)/(I + a*x) - (12*I)*a^3*Log
[x] + (12*I)*a^3*Log[I + a*x]
```

Rubi [A] time = 0.0390173, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 88}

$$\frac{4a^3}{ax+i}+\frac{8a^2}{x}-12ia^3\log(x)+12ia^3\log(ax+i)-\frac{2ia}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/(3*x^3) - ((2*I)*a)/x^2 + (8*a^2)/x + (4*a^3)/(I + a*x) - (12*I)*a^3*Log
[x] + (12*I)*a^3*Log[I + a*x]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4i\tan^{-1}(ax)}}{x^4}\,dx &= \int \frac{(1+iax)^2}{x^4(1-iax)^2}\,dx\\
&= \int \left(\frac{1}{x^4}+\frac{4ia}{x^3}-\frac{8a^2}{x^2}-\frac{12ia^3}{x}-\frac{4a^4}{(i+ax)^2}+\frac{12ia^4}{i+ax}\right)dx\\
&= -\frac{1}{3x^3}-\frac{2ia}{x^2}+\frac{8a^2}{x}+\frac{4a^3}{i+ax}-12ia^3\log(x)+12ia^3\log(i+ax)
\end{aligned}$$

Mathematica [A] time = 0.034766, size = 62, normalized size = 1.

$$\frac{4a^3}{ax+i}+\frac{8a^2}{x}-12ia^3\log(x)+12ia^3\log(ax+i)-\frac{2ia}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/(3*x^3) - ((2*I)*a)/x^2 + (8*a^2)/x + (4*a^3)/(I + a*x) - (12*I)*a^3*Log
[x] + (12*I)*a^3*Log[I + a*x]
```

Maple [A] time = 0.052, size = 68, normalized size = 1.1

$$4\,{\frac{{a}^{3}}{ax+i}}+6\,i{a}^{3}\ln \left({a}^{2}{x}^{2}+1 \right) +12\,{a}^{3}\arctan \left(ax \right) -{\frac{1}{3\,{x}^{3}}}-12\,i{a}^{3}\ln \left(x \right) -{\frac{2\,ia}{{x}^{2}}}+8\,{\frac{{a}^{2}}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x)
```

```
[Out] 4*a^3/(a*x+I)+6*I*a^3*ln(a^2*x^2+1)+12*a^3*arctan(a*x)-1/3/x^3-12*I*a^3*ln(
x)-2*I*a/x^2+8*a^2/x
```

Maxima [A] time = 1.51194, size = 104, normalized size = 1.68

$$12\,a^{3}\arctan\left(ax\right) + 6i\,a^{3}\log\left(a^{2}x^{2}+1\right) - 12i\,a^{3}\log\left(x\right) + \frac{72\,a^{4}x^{4} - 36i\,a^{3}x^{3} + 46\,a^{2}x^{2} - 12i\,ax - 2}{6\,\left(a^{2}x^{5} + x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x, algorithm="maxima")
```

```
[Out] 12*a^3*arctan(a*x) + 6*I*a^3*log(a^2*x^2 + 1) - 12*I*a^3*log(x) + 1/6*(72*a
^4*x^4 - 36*I*a^3*x^3 + 46*a^2*x^2 - 12*I*a*x - 2)/(a^2*x^5 + x^3)
```

Fricas [A] time = 1.78426, size = 189, normalized size = 3.05

$$\frac{36\,a^{3}x^{3} + 18i\,a^{2}x^{2} + 5\,ax - 36\,\left(i\,a^{4}x^{4} - a^{3}x^{3}\right)\log\left(x\right) - 36\,\left(-i\,a^{4}x^{4} + a^{3}x^{3}\right)\log\left(\frac{ax+i}{a}\right) - i}{3\,\left(ax^{4} + i\,x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(36*a^3*x^3 + 18*I*a^2*x^2 + 5*a*x - 36*(I*a^4*x^4 - a^3*x^3)*log(x) -
36*(-I*a^4*x^4 + a^3*x^3)*log((a*x + I)/a) - I)/(a*x^4 + I*x^3)
```

Sympy [A] time = 0.758275, size = 63, normalized size = 1.02

$$12a^{3}\left(-i\log\left(x\right) + i\log\left(x + \frac{i}{a}\right)\right) + \frac{36a^{4}x^{3} + 18ia^{3}x^{2} + 5a^{2}x - ia}{3a^{2}x^{4} + 3iax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/x**4,x)
```

```
[Out] 12*a**3*(-I*log(x) + I*log(x + I/a)) + (36*a**4*x**3 + 18*I*a**3*x**2 + 5*a
**2*x - I*a)/(3*a**2*x**4 + 3*I*a*x**3)
```

Giac [A] time = 1.10326, size = 80, normalized size = 1.29

$$12\,a^3 i\log\left(ax+i\right)-12\,a^3 i\log\left(|x|\right)+\frac{36\,a^3x^3+18\,a^2ix^2+5\,ax-i}{3\,(ax+i)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x, algorithm="giac")
```

```
[Out] 12*a^3*i*log(a*x + i) - 12*a^3*i*log(abs(x)) + 1/3*(36*a^3*x^3 + 18*a^2*i*x
^2 + 5*a*x - i)/((a*x + i)*x^3)
```

3.35 $\int e^{-i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=90

$$-\frac{ix^3\sqrt{a^2x^2+1}}{4a}+\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{(16-9iax)\sqrt{a^2x^2+1}}{24a^4}-\frac{3i\sinh^{-1}(ax)}{8a^4}$$

```
[Out] (x^2*Sqrt[1 + a^2*x^2])/(3*a^2) - ((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - ((16 -
(9*I)*a*x)*Sqrt[1 + a^2*x^2])/(24*a^4) - (((3*I)/8)*ArcSinh[a*x])/a^4
```

Rubi [A] time = 0.0664229, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5060, 833, 780, 215}

$$-\frac{ix^3\sqrt{a^2x^2+1}}{4a}+\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{(16-9iax)\sqrt{a^2x^2+1}}{24a^4}-\frac{3i\sinh^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(I*ArcTan[a*x]),x]
```

```
[Out] (x^2*Sqrt[1 + a^2*x^2])/(3*a^2) - ((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - ((16 -
(9*I)*a*x)*Sqrt[1 + a^2*x^2])/(24*a^4) - (((3*I)/8)*ArcSinh[a*x])/a^4
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-i\tan^{-1}(ax)}x^3\,dx = \int \frac{x^3(1-iax)}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{ix^3\sqrt{1+a^2x^2}}{4a} + \frac{\int \frac{x^2(3ia+4a^2x)}{\sqrt{1+a^2x^2}}\,dx}{4a^2}$$
$$= \frac{x^2\sqrt{1+a^2x^2}}{3a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} + \frac{\int \frac{x(-8a^2+9ia^3x)}{\sqrt{1+a^2x^2}}\,dx}{12a^4}$$
$$= \frac{x^2\sqrt{1+a^2x^2}}{3a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{(16-9iax)\sqrt{1+a^2x^2}}{24a^4} - \frac{(3i)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{8a^3}$$
$$= \frac{x^2\sqrt{1+a^2x^2}}{3a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{(16-9iax)\sqrt{1+a^2x^2}}{24a^4} - \frac{3i\sinh^{-1}(ax)}{8a^4}$$

Mathematica [A] time = 0.0431309, size = 56, normalized size = 0.62

$$\frac{\sqrt{a^2x^2+1}\left(-6ia^3x^3+8a^2x^2+9iax-16\right)-9i\sinh^{-1}(ax)}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^(I*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(-16 + (9*I)*a*x + 8*a^2*x^2 - (6*I)*a^3*x^3) - (9*I)*Ar
cSinh[a*x])/(24*a^4)
```

Maple [B] time = 0.156, size = 187, normalized size = 2.1

$$\frac{-\frac{i}{4}x}{a^3}\left(a^2x^2+1\right)^{\frac{3}{2}} + \frac{\frac{5i}{8}x}{a^3}\sqrt{a^2x^2+1} + \frac{\frac{5i}{8}}{a^3}\ln\left(a^2x\frac{1}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2}} + \frac{1}{3a^4}\left(a^2x^2+1\right)^{\frac{3}{2}} - \frac{1}{a^4}\sqrt{a^2\left(x-\frac{i}{a}\right)^2+2ia\left(x-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/4*I/a^3*x*(a^2*x^2+1)^(3/2)+5/8*I/a^3*x*(a^2*x^2+1)^(1/2)+5/8*I/a^3*ln(a
^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)+1/3/a^4*(a^2*x^2+1)^(3/2)-1
/a^4*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2)-I/a^3*ln((I*a+a^2*(x-I/a))/(a^2)^(
1/2)+(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)
```

Maxima [A] time = 1.49593, size = 103, normalized size = 1.14

$$-\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}x}{4\,a^3} + \frac{5i\sqrt{a^2x^2+1}x}{8\,a^3} + \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{3\,a^4} - \frac{3i\,\mathrm{arsinh}\,(ax)}{8\,a^4} - \frac{\sqrt{a^2x^2+1}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*I*(a^2*x^2 + 1)^(3/2)*x/a^3 + 5/8*I*sqrt(a^2*x^2 + 1)*x/a^3 + 1/3*(a^2
*x^2 + 1)^(3/2)/a^4 - 3/8*I*arcsinh(a*x)/a^4 - sqrt(a^2*x^2 + 1)/a^4
```

Fricas [A] time = 1.80407, size = 146, normalized size = 1.62

$$\frac{\left(-6i\,a^3x^3+8\,a^2x^2+9i\,ax-16\right)\sqrt{a^2x^2+1}+9i\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*((-6*I*a^3*x^3 + 8*a^2*x^2 + 9*I*a*x - 16)*sqrt(a^2*x^2 + 1) + 9*I*log
(-a*x + sqrt(a^2*x^2 + 1)))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{a^2x^2+1}}{iax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a**2*x**2 + 1)/(I*a*x + 1), x)
```

Giac [A] time = 1.11851, size = 99, normalized size = 1.1

$$-\frac{1}{24}\sqrt{a^2x^2+1}\left(\left(2\left(\frac{3\,ix}{a}-\frac{4}{a^2}\right)x-\frac{9\,i}{a^3}\right)x+\frac{16}{a^4}\right)+\frac{3\,i\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{8\,a^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(a^2*x^2 + 1)*((2*(3*i*x/a - 4/a^2)*x - 9*i/a^3)*x + 16/a^4) + 3/
8*i*log(-x*abs(a) + sqrt(a^2*x^2 + 1))/(a^3*abs(a))
```

3.36 $\int e^{-i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=75

$$-\frac{i\left(a^2x^2+1\right)^{3/2}}{3a^3}+\frac{x\sqrt{a^2x^2+1}}{2a^2}+\frac{i\sqrt{a^2x^2+1}}{a^3}-\frac{\sinh^{-1}(ax)}{2a^3}$$

```
[Out] (I*Sqrt[1 + a^2*x^2])/a^3 + (x*Sqrt[1 + a^2*x^2])/(2*a^2) - ((I/3)*(1 + a^2
*x^2)^(3/2))/a^3 - ArcSinh[a*x]/(2*a^3)
```

Rubi [A] time = 0.0473174, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5060, 797, 641, 195, 215}

$$-\frac{i\left(a^2x^2+1\right)^{3/2}}{3a^3}+\frac{x\sqrt{a^2x^2+1}}{2a^2}+\frac{i\sqrt{a^2x^2+1}}{a^3}-\frac{\sinh^{-1}(ax)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(I*ArcTan[a*x]),x]
```

```
[Out] (I*Sqrt[1 + a^2*x^2])/a^3 + (x*Sqrt[1 + a^2*x^2])/(2*a^2) - ((I/3)*(1 + a^2
*x^2)^(3/2))/a^3 - ArcSinh[a*x]/(2*a^3)
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-i\tan^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1-iax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\int \frac{1-iax}{\sqrt{1+a^2x^2}}\,dx}{a^2} + \frac{\int (1-iax)\sqrt{1+a^2x^2}\,dx}{a^2} \\
&= \frac{i\sqrt{1+a^2x^2}}{a^3} - \frac{i\left(1+a^2x^2\right)^{3/2}}{3a^3} - \frac{\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{a^2} + \frac{\int \sqrt{1+a^2x^2}\,dx}{a^2} \\
&= \frac{i\sqrt{1+a^2x^2}}{a^3} + \frac{x\sqrt{1+a^2x^2}}{2a^2} - \frac{i\left(1+a^2x^2\right)^{3/2}}{3a^3} - \frac{\sinh^{-1}(ax)}{a^3} + \frac{\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a^2} \\
&= \frac{i\sqrt{1+a^2x^2}}{a^3} + \frac{x\sqrt{1+a^2x^2}}{2a^2} - \frac{i\left(1+a^2x^2\right)^{3/2}}{3a^3} - \frac{\sinh^{-1}(ax)}{2a^3}
\end{aligned}
$$

Mathematica [A] time = 0.0311688, size = 46, normalized size = 0.61

$$\frac{-3\sinh^{-1}(ax) + \left(-2ia^2x^2 + 3ax + 4i\right)\sqrt{a^2x^2+1}}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^(I*ArcTan[a*x]),x]
```

```
[Out] ((4*I + 3*a*x - (2*I)*a^2*x^2)*Sqrt[1 + a^2*x^2] - 3*ArcSinh[a*x])/(6*a^3)
```

Maple [B] time = 0.077, size = 168, normalized size = 2.2

$$\frac{-\frac{i}{3}}{a^3}\left(a^2x^2+1\right)^{\frac{3}{2}} + \frac{x}{2a^2}\sqrt{a^2x^2+1} + \frac{1}{2a^2}\ln\left(a^2x\frac{1}{\sqrt{a^2}} + \sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2}} + \frac{i}{a^3}\sqrt{a^2\left(x-\frac{i}{a}\right)^2 + 2ia\left(x-\frac{i}{a}\right)} - \frac{1}{a^2}\ln\left(\left(ia\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/3*I*(a^2*x^2+1)^(3/2)/a^3+1/2*x*(a^2*x^2+1)^(1/2)/a^2+1/2/a^2*ln(a^2*x/(
a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)+I/a^3*(a^2*(x-I/a)^2+2*I*a*(x-I/a
))^(1/2)-1/a^2*ln((I*a+a^2*(x-I/a))/(a^2)^(1/2)+(a^2*(x-I/a)^2+2*I*a*(x-I/a
))^(1/2))/(a^2)^(1/2)
```

Maxima [A] time = 1.54917, size = 80, normalized size = 1.07

$$\frac{\sqrt{a^2x^2+1}x}{2a^2} - \frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}}{3a^3} - \frac{\operatorname{arsinh}(ax)}{2a^3} + \frac{i\sqrt{a^2x^2+1}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a^2*x^2 + 1)*x/a^2 - 1/3*I*(a^2*x^2 + 1)^(3/2)/a^3 - 1/2*arcsinh(a
*x)/a^3 + I*sqrt(a^2*x^2 + 1)/a^3
```

Fricas [A] time = 1.67148, size = 124, normalized size = 1.65

$$\frac{\sqrt{a^2x^2+1}\left(-2i\,a^2x^2+3\,ax+4i\right)+3\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(a^2*x^2 + 1)*(-2*I*a^2*x^2 + 3*a*x + 4*I) + 3*log(-a*x + sqrt(a^2
*x^2 + 1)))/a^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{a^2x^2+1}}{iax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a**2*x**2 + 1)/(I*a*x + 1), x)
```

Giac [A] time = 1.10836, size = 85, normalized size = 1.13

$$-\frac{1}{6}\sqrt{a^2x^2+1}\left(\left(\frac{2\,ix}{a}-\frac{3}{a^2}\right)x-\frac{4\,i}{a^3}\right)+\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{2\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(a^2*x^2 + 1)*((2*i*x/a - 3/a^2)*x - 4*i/a^3) + 1/2*log(-x*abs(a)
+ sqrt(a^2*x^2 + 1))/(a^2*abs(a))
```

3.37 $\int e^{-i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{a^2x^2+1}(2-iax)}{2a^2}+\frac{i\sinh^{-1}(ax)}{2a^2}$$

```
[Out] ((2 - I*a*x)*Sqrt[1 + a^2*x^2])/(2*a^2) + ((I/2)*ArcSinh[a*x])/a^2
```

Rubi [A] time = 0.0199573, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5060, 780, 215}

$$\frac{\sqrt{a^2x^2+1}(2-iax)}{2a^2}+\frac{i\sinh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(I*ArcTan[a*x]),x]
```

```
[Out] ((2 - I*a*x)*Sqrt[1 + a^2*x^2])/(2*a^2) + ((I/2)*ArcSinh[a*x])/a^2
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; FreeQ[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{-i\tan^{-1}(ax)}x\,dx &= \int \frac{x(1-iax)}{\sqrt{1+a^2x^2}}\,dx\\ &= \frac{(2-iax)\sqrt{1+a^2x^2}}{2a^2}+\frac{i\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a}\\ &= \frac{(2-iax)\sqrt{1+a^2x^2}}{2a^2}+\frac{i\sinh^{-1}(ax)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.0261351, size = 38, normalized size = 0.9

$$\frac{\sqrt{a^2x^2+1}(2-iax)+i\sinh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^(I*ArcTan[a*x]),x]
```

```
[Out] ((2 - I*a*x)*Sqrt[1 + a^2*x^2] + I*ArcSinh[a*x])/(2*a^2)
```

Maple [B] time = 0.069, size = 152, normalized size = 3.6

$$\frac{-\frac{i}{2}x}{a}\sqrt{a^2x^2+1}-\frac{\frac{i}{2}}{a}\ln\left(a^2x\frac{1}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2}}+\frac{1}{a^2}\sqrt{a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)}+\frac{i}{a}\ln\left(\left(ia+a^2\left(x-\frac{i}{a}\right)\right)\frac{1}{\sqrt{a^2}}+\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*I/a*x*(a^2*x^2+1)^(1/2)-1/2*I/a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2)
)/(a^2)^(1/2)+1/a^2*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2)+I/a*ln((I*a+a^2*(x-
I/a))/(a^2)^(1/2)+(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)
```

Maxima [A] time = 1.56325, size = 57, normalized size = 1.36

$$-\frac{i\sqrt{a^2x^2+1}x}{2\,a}+\frac{i\,\mathrm{arsinh}\,(ax)}{2\,a^2}+\frac{\sqrt{a^2x^2+1}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*sqrt(a^2*x^2 + 1)*x/a + 1/2*I*arcsinh(a*x)/a^2 + sqrt(a^2*x^2 + 1)/a
^2
```

Fricas [A] time = 1.76266, size = 103, normalized size = 2.45

$$\frac{\sqrt{a^2x^2+1}(-i\,ax+2)-i\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2*x^2 + 1)*(-I*a*x + 2) - I*log(-a*x + sqrt(a^2*x^2 + 1)))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\sqrt{a^2x^2+1}}{iax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a**2*x**2 + 1)/(I*a*x + 1), x)
```

Giac [A] time = 1.13438, size = 73, normalized size = 1.74

$$-\frac{1}{2}\sqrt{a^2x^2+1}\left(\frac{ix}{a}-\frac{2}{a^2}\right)-\frac{i\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{2\,a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a^2*x^2 + 1)*(i*x/a - 2/a^2) - 1/2*i*log(-x*abs(a) + sqrt(a^2*x^2
 + 1))/(a*abs(a))
```

3.38 $\int e^{-i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=29

$$\frac{\sinh^{-1}(ax)}{a}-\frac{i\sqrt{a^2x^2+1}}{a}$$

```
[Out] ((-I)*Sqrt[1 + a^2*x^2])/a + ArcSinh[a*x]/a
```

Rubi [A] time = 0.0094273, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5059, 641, 215}

$$\frac{\sinh^{-1}(ax)}{a}-\frac{i\sqrt{a^2x^2+1}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-I)*ArcTan[a*x]),x]
```

```
[Out] ((-I)*Sqrt[1 + a^2*x^2])/a + ArcSinh[a*x]/a
```

Rule 5059

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 - I*a*x)^((I*n + 1)/2)
/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2]), x] /; FreeQ[a, x] && Intege
rQ[(I*n - 1)/2]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{-i\tan^{-1}(ax)}\,dx &= \int \frac{1-iax}{\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{i\sqrt{1+a^2x^2}}{a}+\int \frac{1}{\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{i\sqrt{1+a^2x^2}}{a}+\frac{\sinh^{-1}(ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0129128, size = 26, normalized size = 0.9

$$\frac{\sinh^{-1}(ax)-i\sqrt{a^2x^2+1}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((-I)*ArcTan[a*x]),x]
```

```
[Out] ((-I)*Sqrt[1 + a^2*x^2] + ArcSinh[a*x])/a
```

Maple [B] time = 0.058, size = 97, normalized size = 3.3

$$-\frac{i}{a}\sqrt{a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)}+\ln\left(\left(ia+a^2\left(x-\frac{i}{a}\right)\right)\frac{1}{\sqrt{a^2}}+\sqrt{a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

```
[Out] -I/a*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2)+ln((I*a+a^2*(x-I/a))/(a^2)^(1/2)+(
a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)
```

Maxima [A] time = 1.56001, size = 34, normalized size = 1.17

$$\frac{\operatorname{arsinh}(ax)}{a}-\frac{i\sqrt{a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(a*x)/a - I*sqrt(a^2*x^2 + 1)/a
```

Fricas [A] time = 1.6649, size = 78, normalized size = 2.69

$$\frac{-i\sqrt{a^2x^2+1}-\log\left(-ax+\sqrt{a^2x^2+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(a^2*x^2 + 1) - log(-a*x + sqrt(a^2*x^2 + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a^2x^2+1}}{iax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2*x**2 + 1)/(I*a*x + 1), x)
```

Giac [A] time = 1.10608, size = 57, normalized size = 1.97

$$-\frac{\sqrt{a^2x^2+1}i}{a}-\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(a^2*x^2 + 1)*i/a - log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a)
```

3.39 $\int \frac{e^{-i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=25

$$-\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-i\sinh^{-1}(ax)$$

```
[Out] (-I)*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rubi [A] time = 0.036575, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5060, 844, 215, 266, 63, 208}

$$-\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-i\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*x),x]

[Out] (-I)*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-i\tan^{-1}(ax)}}{x}\,dx &= \int \frac{1-iax}{x\sqrt{1+a^2x^2}}\,dx\\ &= -\left((ia)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx\right)+\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\ &= -i\sinh^{-1}(ax)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\ &= -i\sinh^{-1}(ax)+\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)}{a^2}\\ &= -i\sinh^{-1}(ax)-\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0144069, size = 29, normalized size = 1.16

$$-\log\left(\sqrt{a^2x^2+1}+1\right)-i\sinh^{-1}(ax)+\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*x),x]
```

```
[Out] (-I)*ArcSinh[a*x] + Log[x] - Log[1 + Sqrt[1 + a^2*x^2]]
```

Maple [B] time = 0.078, size = 121, normalized size = 4.8

$$-\sqrt{a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)}-ia\ln\left(\left(ia+a^2\left(x-\frac{i}{a}\right)\right)\frac{1}{\sqrt{a^2}}+\sqrt{a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)}\right)\frac{1}{\sqrt{a^2}}+\sqrt{a^2x^2+1}-\mathrm{Artanl}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] -(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2)-I*a*ln((I*a+a^2*(x-I/a))/(a^2)^(1/2)+(
a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)+(a^2*x^2+1)^(1/2)-arctanh(1
/(a^2*x^2+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 1.78253, size = 143, normalized size = 5.72

$$-\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+i\log\left(-ax+\sqrt{a^2x^2+1}\right)+\log\left(-ax+\sqrt{a^2x^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -log(-a*x + sqrt(a^2*x^2 + 1) + 1) + I*log(-a*x + sqrt(a^2*x^2 + 1)) + log(
-a*x + sqrt(a^2*x^2 + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2x^2+1}}{x\,(iax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a**2*x**2 + 1)/(x*(I*a*x + 1)), x)
```

Giac [B] time = 1.13407, size = 92, normalized size = 3.68

$$\frac{ai\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|}-\log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right)+\log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] a*i*log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a) - log(abs(-x*abs(a) + sqrt(a^
2*x^2 + 1) + 1)) + log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) - 1))
```

3.40 $\int \frac{e^{-i\tan^{-1}(ax)}}{x^2} \, dx$

Optimal. Leaf size=38

$$-\frac{\sqrt{a^2x^2+1}}{x}+ia\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -(Sqrt[1 + a^2*x^2]/x) + I*a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rubi [A] time = 0.0369982, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5060, 807, 266, 63, 208}

$$-\frac{\sqrt{a^2x^2+1}}{x}+ia\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*x^2),x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/x) + I*a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{1-iax}{x^2\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{x} - (ia)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{x} - \frac{1}{2}(ia)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{1+a^2x^2}}{x} - \frac{i\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a}\\
&= -\frac{\sqrt{1+a^2x^2}}{x} + ia\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0262643, size = 47, normalized size = 1.24

$$
-\frac{\sqrt{a^2x^2+1}}{x} + ia\log\left(\sqrt{a^2x^2+1}+1\right) - ia\log(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*x^2),x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/x) - I*a*Log[x] + I*a*Log[1 + Sqrt[1 + a^2*x^2]]
```

Maple [B] time = 0.095, size = 194, normalized size = 5.1

$$
-\frac{1}{x}\left(a^2x^2+1\right)^{\frac{3}{2}} + a^2x\sqrt{a^2x^2+1} + a^2\ln\left(a^2x\frac{1}{\sqrt{a^2}} + \sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2}} + ia\sqrt{a^2\left(x-\frac{i}{a}\right)^2 + 2\,ia\left(x-\frac{i}{a}\right)} - a^2\ln\left(\left(ia + a^2\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] -1/x*(a^2*x^2+1)^(3/2)+a^2*x*(a^2*x^2+1)^(1/2)+a^2*ln(a^2*x/(a^2)^(1/2)+(a^
2*x^2+1)^(1/2))/(a^2)^(1/2)+I*a*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2)-a^2*ln(
(I*a+a^2*(x-I/a))/(a^2)^(1/2)+(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1
/2)+I*a*arctanh(1/(a^2*x^2+1)^(1/2))-I*a*(a^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)/((I*a*x + 1)*x^2), x)
```

Fricas [B] time = 1.66206, size = 153, normalized size = 4.03

$$\frac{i\,ax\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-i\,ax\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-ax-\sqrt{a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (I*a*x*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - I*a*x*log(-a*x + sqrt(a^2*x^2 +
1) - 1) - a*x - sqrt(a^2*x^2 + 1))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a^2x^2+1}}{x^2\,(iax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a**2*x**2 + 1)/(x**2*(I*a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] undef
```

3.41 $\int \frac{e^{-i\tan^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=63

$$\frac{ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -Sqrt[1 + a^2*x^2]/(2*x^2) + (I*a*Sqrt[1 + a^2*x^2])/x + (a^2*ArcTanh[Sqrt[
1 + a^2*x^2]])/2
```

Rubi [A] time = 0.05137, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5060, 835, 807, 266, 63, 208}

$$\frac{ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*x^3),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(2*x^2) + (I*a*Sqrt[1 + a^2*x^2])/x + (a^2*ArcTanh[Sqrt[
1 + a^2*x^2]])/2
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{1-iax}{x^3\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} - \frac{1}{2}\int \frac{2ia+a^2x}{x^2\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{ia\sqrt{1+a^2x^2}}{x} - \frac{1}{2}a^2\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{ia\sqrt{1+a^2x^2}}{x} - \frac{1}{4}a^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{ia\sqrt{1+a^2x^2}}{x} - \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{ia\sqrt{1+a^2x^2}}{x} + \frac{1}{2}a^2\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0407194, size = 57, normalized size = 0.9

$$\frac{1}{2}\left(\frac{(-1+2iax)\sqrt{a^2x^2+1}}{x^2}+a^2\log\left(\sqrt{a^2x^2+1}+1\right)+a^2(-\log(x))\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*x^3),x]
```

```
[Out] (((-1 + (2*I)*a*x)*Sqrt[1 + a^2*x^2])/x^2 - a^2*Log[x] + a^2*Log[1 + Sqrt[1
 + a^2*x^2]])/2
```

Maple [B] time = 0.079, size = 219, normalized size = 3.5

$$-\frac{1}{2x^2}\left(a^2x^2+1\right)^{\frac{3}{2}}+\frac{a^2}{2}\mathrm{Artanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)-\frac{a^2}{2}\sqrt{a^2x^2+1}+\frac{ia}{x}\left(a^2x^2+1\right)^{\frac{3}{2}}-ia^3x\sqrt{a^2x^2+1}-ia^3\ln\left(a^2x\frac{1}{\sqrt{a^2}}+\sqrt{a^2x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x)
```

```
[Out] -1/2/x^2*(a^2*x^2+1)^(3/2)+1/2*a^2*arctanh(1/(a^2*x^2+1)^(1/2))-1/2*a^2*(a^
2*x^2+1)^(1/2)+I*a/x*(a^2*x^2+1)^(3/2)-I*a^3*x*(a^2*x^2+1)^(1/2)-I*a^3*ln(a
```

```
^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)+a^2*(a^2*(x-I/a)^2+2*I*a*(x
-I/a))^(1/2)+I*a^3*ln((I*a+a^2*(x-I/a))/(a^2)^(1/2)+(a^2*(x-I/a)^2+2*I*a*(x
-I/a))^(1/2))/(a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)/((I*a*x + 1)*x^3), x)
```

Fricas [A] time = 1.81421, size = 196, normalized size = 3.11

$$\frac{a^2x^2\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-a^2x^2\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+2i\,a^2x^2+\sqrt{a^2x^2+1}(2i\,ax-1)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - a^2*x^2*log(-a*x + sqrt(a^
2*x^2 + 1) - 1) + 2*I*a^2*x^2 + sqrt(a^2*x^2 + 1)*(2*I*a*x - 1))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2x^2+1}}{x^3\,(iax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a**2*x**2 + 1)/(x**3*(I*a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

3.42 $\int \frac{e^{-i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=90

$$\frac{2a^2\sqrt{a^2x^2+1}}{3x}+\frac{ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{1}{2}ia^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -Sqrt[1 + a^2*x^2]/(3*x^3) + ((I/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (2*a^2*Sqrt[
1 + a^2*x^2])/(3*x) - (I/2)*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rubi [A] time = 0.0695705, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5060, 835, 807, 266, 63, 208}

$$\frac{2a^2\sqrt{a^2x^2+1}}{3x}+\frac{ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{1}{2}ia^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*x^4),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(3*x^3) + ((I/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (2*a^2*Sqrt[
1 + a^2*x^2])/(3*x) - (I/2)*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-i\tan^{-1}(ax)}}{x^4}\,dx &= \int \frac{1-iax}{x^4\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{1}{3}\int \frac{3ia+2a^2x}{x^3\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}}{3x^3}+\frac{ia\sqrt{1+a^2x^2}}{2x^2}+\frac{1}{6}\int \frac{-4a^2+3ia^3x}{x^2\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}}{3x^3}+\frac{ia\sqrt{1+a^2x^2}}{2x^2}+\frac{2a^2\sqrt{1+a^2x^2}}{3x}+\frac{1}{2}\left(ia^3\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}}{3x^3}+\frac{ia\sqrt{1+a^2x^2}}{2x^2}+\frac{2a^2\sqrt{1+a^2x^2}}{3x}+\frac{1}{4}\left(ia^3\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{1+a^2x^2}}{3x^3}+\frac{ia\sqrt{1+a^2x^2}}{2x^2}+\frac{2a^2\sqrt{1+a^2x^2}}{3x}+\frac{1}{2}(ia)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)\\ &= -\frac{\sqrt{1+a^2x^2}}{3x^3}+\frac{ia\sqrt{1+a^2x^2}}{2x^2}+\frac{2a^2\sqrt{1+a^2x^2}}{3x}-\frac{1}{2}ia^3\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0484899, size = 70, normalized size = 0.78

$$\frac{1}{6}\left(\frac{\sqrt{a^2x^2+1}\left(4a^2x^2+3iax-2\right)}{x^3}-3ia^3\log\left(\sqrt{a^2x^2+1}+1\right)+3ia^3\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*x^4),x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-2 + (3*I)*a*x + 4*a^2*x^2))/x^3 + (3*I)*a^3*Log[x] -
(3*I)*a^3*Log[1 + Sqrt[1 + a^2*x^2]])/6
```

Maple [B] time = 0.081, size = 237, normalized size = 2.6

$$\frac{\frac{i}{2}a}{x^2}\left(a^2x^2+1\right)^{\frac{3}{2}}-\frac{i}{2}a^3\mathrm{Artanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)+\frac{i}{2}a^3\sqrt{a^2x^2+1}+\frac{a^2}{x}\left(a^2x^2+1\right)^{\frac{3}{2}}-a^4x\sqrt{a^2x^2+1}-a^4\ln\left(a^2x\frac{1}{\sqrt{a^2}}+\sqrt{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x)
```

```
[Out] 1/2*I*a/x^2*(a^2*x^2+1)^(3/2)-1/2*I*a^3*arctanh(1/(a^2*x^2+1)^(1/2))+1/2*I*
a^3*(a^2*x^2+1)^(1/2)+a^2/x*(a^2*x^2+1)^(3/2)-a^4*x*(a^2*x^2+1)^(1/2)-a^4*l
n(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)-I*a^3*(a^2*(x-I/a)^2+2*I
*a*(x-I/a))^(1/2)+a^4*ln((I*a+a^2*(x-I/a))/(a^2)^(1/2)+(a^2*(x-I/a)^2+2*I*a
*(x-I/a))^(1/2))/(a^2)^(1/2)-1/3/x^3*(a^2*x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)/((I*a*x + 1)*x^4), x)
```

Fricas [A] time = 1.72583, size = 221, normalized size = 2.46

$$\frac{-3i\,a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+3i\,a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+4\,a^3x^3+\left(4\,a^2x^2+3i\,ax-2\right)\sqrt{a^2x^2+1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(-3*I*a^3*x^3*log(-a*x + sqrt(a^2*x^2 + 1) + 1) + 3*I*a^3*x^3*log(-a*x
+ sqrt(a^2*x^2 + 1) - 1) + 4*a^3*x^3 + (4*a^2*x^2 + 3*I*a*x - 2)*sqrt(a^2*x
^2 + 1))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2x^2+1}}{x^4\,(iax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(a**2*x**2 + 1)/(x**4*(I*a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

3.43 $\int \frac{e^{-i\tan^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=113

$$-\frac{2ia^3\sqrt{a^2x^2+1}}{3x}+\frac{3a^2\sqrt{a^2x^2+1}}{8x^2}+\frac{ia\sqrt{a^2x^2+1}}{3x^3}-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{3}{8}a^4\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -Sqrt[1 + a^2*x^2]/(4*x^4) + ((I/3)*a*Sqrt[1 + a^2*x^2])/x^3 + (3*a^2*Sqrt[
1 + a^2*x^2])/(8*x^2) - (((2*I)/3)*a^3*Sqrt[1 + a^2*x^2])/x - (3*a^4*ArcTan
h[Sqrt[1 + a^2*x^2]])/8
```

Rubi [A] time = 0.0891088, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5060, 835, 807, 266, 63, 208}

$$-\frac{2ia^3\sqrt{a^2x^2+1}}{3x}+\frac{3a^2\sqrt{a^2x^2+1}}{8x^2}+\frac{ia\sqrt{a^2x^2+1}}{3x^3}-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{3}{8}a^4\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*x^5),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(4*x^4) + ((I/3)*a*Sqrt[1 + a^2*x^2])/x^3 + (3*a^2*Sqrt[
1 + a^2*x^2])/(8*x^2) - (((2*I)/3)*a^3*Sqrt[1 + a^2*x^2])/x - (3*a^4*ArcTan
h[Sqrt[1 + a^2*x^2]])/8
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-i\tan^{-1}(ax)}}{x^5}\,dx &= \int \frac{1-iax}{x^5\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4}-\frac{1}{4}\int \frac{4ia+3a^2x}{x^4\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4}+\frac{ia\sqrt{1+a^2x^2}}{3x^3}+\frac{1}{12}\int \frac{-9a^2+8ia^3x}{x^3\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4}+\frac{ia\sqrt{1+a^2x^2}}{3x^3}+\frac{3a^2\sqrt{1+a^2x^2}}{8x^2}-\frac{1}{24}\int \frac{-16ia^3-9a^4x}{x^2\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4}+\frac{ia\sqrt{1+a^2x^2}}{3x^3}+\frac{3a^2\sqrt{1+a^2x^2}}{8x^2}-\frac{2ia^3\sqrt{1+a^2x^2}}{3x}+\frac{1}{8}\left(3a^4\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4}+\frac{ia\sqrt{1+a^2x^2}}{3x^3}+\frac{3a^2\sqrt{1+a^2x^2}}{8x^2}-\frac{2ia^3\sqrt{1+a^2x^2}}{3x}+\frac{1}{16}\left(3a^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x\right.\\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4}+\frac{ia\sqrt{1+a^2x^2}}{3x^3}+\frac{3a^2\sqrt{1+a^2x^2}}{8x^2}-\frac{2ia^3\sqrt{1+a^2x^2}}{3x}+\frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\right.\\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4}+\frac{ia\sqrt{1+a^2x^2}}{3x^3}+\frac{3a^2\sqrt{1+a^2x^2}}{8x^2}-\frac{2ia^3\sqrt{1+a^2x^2}}{3x}-\frac{3}{8}a^4\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0566261, size = 76, normalized size = 0.67

$$\frac{1}{24}\left(\frac{\sqrt{a^2x^2+1}\left(-16ia^3x^3+9a^2x^2+8iax-6\right)}{x^4}-9a^4\log\left(\sqrt{a^2x^2+1}+1\right)+9a^4\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*x^5),x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-6 + (8*I)*a*x + 9*a^2*x^2 - (16*I)*a^3*x^3))/x^4 + 9*
a^4*Log[x] - 9*a^4*Log[1 + Sqrt[1 + a^2*x^2]])/24
```

Maple [B] time = 0.082, size = 259, normalized size = 2.3

$$\frac{5a^2}{8x^2}\left(a^2x^2+1\right)^{\frac{3}{2}}-\frac{3a^4}{8}\text{Artanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)+\frac{3a^4}{8}\sqrt{a^2x^2+1}-\frac{1}{4x^4}\left(a^2x^2+1\right)^{\frac{3}{2}}-\frac{ia^3}{x}\left(a^2x^2+1\right)^{\frac{3}{2}}+ia^5x\sqrt{a^2x^2+1}+i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x)
```

```
[Out] 5/8*a^2/x^2*(a^2*x^2+1)^(3/2)-3/8*a^4*arctanh(1/(a^2*x^2+1)^(1/2))+3/8*a^4*
(a^2*x^2+1)^(1/2)-1/4/x^4*(a^2*x^2+1)^(3/2)-I*a^3/x*(a^2*x^2+1)^(3/2)+I*a^5
*x*(a^2*x^2+1)^(1/2)+I*a^5*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1
/2)-a^4*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2)-I*a^5*ln((I*a+a^2*(x-I/a))/(a^2
)^(1/2)+(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)+1/3*I*a/x^3*(a^2*x
^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)/((I*a*x + 1)*x^5), x)
```

Fricas [A] time = 1.78315, size = 243, normalized size = 2.15

$$-\frac{9\,a^4x^4\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-9\,a^4x^4\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+16i\,a^4x^4-\left(-16i\,a^3x^3+9\,a^2x^2+8i\,ax-6\right)\sqrt{}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(9*a^4*x^4*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 9*a^4*x^4*log(-a*x + s
qrt(a^2*x^2 + 1) - 1) + 16*I*a^4*x^4 - (-16*I*a^3*x^3 + 9*a^2*x^2 + 8*I*a*x
 - 6)*sqrt(a^2*x^2 + 1))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2x^2+1}}{x^5\,(iax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(a**2*x**2 + 1)/(x**5*(I*a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] undef
```

3.44 $\int e^{-2i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=49

$$\frac{x^2}{a^2}+\frac{2ix}{a^3}-\frac{2\log(-ax+i)}{a^4}-\frac{2ix^3}{3a}-\frac{x^4}{4}$$

```
[Out] ((2*I)*x)/a^3 + x^2/a^2 - (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I - a*x])/a^4
```

Rubi [A] time = 0.0352147, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$\frac{x^2}{a^2}+\frac{2ix}{a^3}-\frac{2\log(-ax+i)}{a^4}-\frac{2ix^3}{3a}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] ((2*I)*x)/a^3 + x^2/a^2 - (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I - a*x])/a^4
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{-2i\tan^{-1}(ax)}x^3\,dx &= \int \frac{x^3(1-iax)}{1+iax}\,dx\\ &= \int \left(\frac{2i}{a^3}+\frac{2x}{a^2}-\frac{2ix^2}{a}-x^3-\frac{2}{a^3(-i+ax)}\right)dx\\ &= \frac{2ix}{a^3}+\frac{x^2}{a^2}-\frac{2ix^3}{3a}-\frac{x^4}{4}-\frac{2\log(i-ax)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.0182823, size = 49, normalized size = 1.

$$\frac{x^2}{a^2}+\frac{2ix}{a^3}-\frac{2\log(-ax+i)}{a^4}-\frac{2ix^3}{3a}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] ((2*I)*x)/a^3 + x^2/a^2 - (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I - a*x])/a^4
```

Maple [A] time = 0.043, size = 55, normalized size = 1.1

$$-\frac{x^4}{4}-\frac{\frac{2\,i}{3}x^3}{a}+\frac{x^2}{a^2}+\frac{2\,ix}{a^3}-\frac{\ln\left(a^2x^2+1\right)}{a^4}-\frac{2\,i\arctan\left(ax\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+I*a*x)^2*(a^2*x^2+1),x)
```

```
[Out] -1/4*x^4-2/3*I*x^3/a+x^2/a^2+2*I*x/a^3-1/a^4*ln(a^2*x^2+1)-2*I/a^4*arctan(a
*x)
```

Maxima [A] time = 1.00334, size = 59, normalized size = 1.2

$$-\frac{i\left(-3i\,a^3x^4+8\,a^2x^3+12i\,ax^2-24\,x\right)}{12\,a^3}-\frac{2\,\log\left(i\,ax+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/12*I*(-3*I*a^3*x^4 + 8*a^2*x^3 + 12*I*a*x^2 - 24*x)/a^3 - 2*log(I*a*x +
1)/a^4
```

Fricas [A] time = 1.63158, size = 112, normalized size = 2.29

$$-\frac{3\,a^4x^4+8i\,a^3x^3-12\,a^2x^2-24i\,ax+24\,\log\left(\frac{ax-i}{a}\right)}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*x^4 + 8*I*a^3*x^3 - 12*a^2*x^2 - 24*I*a*x + 24*log((a*x - I)/a
))/a^4
```

Sympy [A] time = 0.360324, size = 41, normalized size = 0.84

$$-\frac{x^4}{4}-\frac{2ix^3}{3a}+\frac{x^2}{a^2}+\frac{2ix}{a^3}-\frac{2\log\left(ax-i\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+I*a*x)**2*(a**2*x**2+1),x)
```

```
[Out] -x**4/4 - 2*I*x**3/(3*a) + x**2/a**2 + 2*I*x/a**3 - 2*log(a*x - I)/a**4
```

Giac [B] time = 1.13513, size = 108, normalized size = 2.2

$$-\frac{(aix+1)^4\left(\frac{20\,i^2}{aix+1}-\frac{84\,i^4}{(aix+1)^3}-\frac{54\,i^2}{(aix+1)^2}+3\right)}{12\,a^4i^4}+\frac{2\,\log\left(\frac{1}{\sqrt{a^2x^2+1}|a|}\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/12*(a*i*x + 1)^4*(20*i^2/(a*i*x + 1) - 84*i^4/(a*i*x + 1)^3 - 54*i^2/(a*
i*x + 1)^2 + 3)/(a^4*i^4) + 2*log(1/(sqrt(a^2*x^2 + 1)*abs(a)))/a^4
```

3.45 $\int e^{-2i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=40

$$\frac{2x}{a^2}+\frac{2i\log(-ax+i)}{a^3}-\frac{ix^2}{a}-\frac{x^3}{3}$$

```
[Out] (2*x)/a^2 - (I*x^2)/a - x^3/3 + ((2*I)*Log[I - a*x])/a^3
```

Rubi [A] time = 0.0297327, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$\frac{2x}{a^2}+\frac{2i\log(-ax+i)}{a^3}-\frac{ix^2}{a}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] (2*x)/a^2 - (I*x^2)/a - x^3/3 + ((2*I)*Log[I - a*x])/a^3
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{-2i\tan^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1-iax)}{1+iax}\,dx\\ &= \int \left(\frac{2}{a^2}-\frac{2ix}{a}-x^2+\frac{2i}{a^2(-i+ax)}\right)dx\\ &= \frac{2x}{a^2}-\frac{ix^2}{a}-\frac{x^3}{3}+\frac{2i\log(i-ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.0125777, size = 40, normalized size = 1.

$$\frac{2x}{a^2}+\frac{2i\log(-ax+i)}{a^3}-\frac{ix^2}{a}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] (2*x)/a^2 - (I*x^2)/a - x^3/3 + ((2*I)*Log[I - a*x])/a^3
```

Maple [A] time = 0.046, size = 47, normalized size = 1.2

$$-\frac{x^3}{3} - \frac{ix^2}{a} + 2\frac{x}{a^2} + \frac{i\ln\left(a^2x^2+1\right)}{a^3} - 2\frac{\arctan(ax)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*a*x)^2*(a^2*x^2+1),x)
```

```
[Out] -1/3*x^3-I*x^2/a+2*x/a^2+I/a^3*ln(a^2*x^2+1)-2/a^3*arctan(a*x)
```

Maxima [A] time = 1.03743, size = 47, normalized size = 1.18

$$-\frac{a^2x^3 + 3i\,ax^2 - 6\,x}{3\,a^2} + \frac{2i\,\log(i\,ax+1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*x^3 + 3*I*a*x^2 - 6*x)/a^2 + 2*I*log(I*a*x + 1)/a^3
```

Fricas [A] time = 1.53685, size = 88, normalized size = 2.2

$$-\frac{a^3x^3 + 3i\,a^2x^2 - 6\,ax - 6i\,\log\left(\frac{ax-i}{a}\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/3*(a^3*x^3 + 3*I*a^2*x^2 - 6*a*x - 6*I*log((a*x - I)/a))/a^3
```

Sympy [A] time = 0.360423, size = 31, normalized size = 0.78

$$-\frac{x^3}{3} - \frac{ix^2}{a} + \frac{2x}{a^2} + \frac{2i\log(ax-i)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*a*x)**2*(a**2*x**2+1),x)
```

```
[Out] -x**3/3 - I*x**2/a + 2*x/a**2 + 2*I*log(a*x - I)/a**3
```

<u>Giac</u> [B] time = 1.09594, size = 92, normalized size = 2.3

$$-\frac{2\,i\log\left(\frac{1}{\sqrt{a^2x^2+1}|a|}\right)}{a^3}-\frac{(aix+1)^3\left(\frac{6\,i^2}{aix+1}-\frac{15\,i^2}{(aix+1)^2}+1\right)}{3\,a^3i^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -2*i*log(1/(sqrt(a^2*x^2 + 1)*abs(a)))/a^3 - 1/3*(a*i*x + 1)^3*(6*i^2/(a*i*
x + 1) - 15*i^2/(a*i*x + 1)^2 + 1)/(a^3*i^3)
```

3.46 $\int e^{-2i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=30

$$\frac{2\log(-ax+i)}{a^2}-\frac{2ix}{a}-\frac{x^2}{2}$$

```
[Out] ((-2*I)*x)/a - x^2/2 + (2*Log[I - a*x])/a^2
```

Rubi [A] time = 0.0210554, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5062, 77}

$$\frac{2\log(-ax+i)}{a^2}-\frac{2ix}{a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] ((-2*I)*x)/a - x^2/2 + (2*Log[I - a*x])/a^2
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{-2i\tan^{-1}(ax)}x\,dx&=\int\frac{x(1-iax)}{1+iax}\,dx\\&=\int\left(-\frac{2i}{a}-x+\frac{2}{a(-i+ax)}\right)dx\\&=-\frac{2ix}{a}-\frac{x^2}{2}+\frac{2\log(i-ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0101914, size = 30, normalized size = 1.

$$\frac{2\log(-ax+i)}{a^2}-\frac{2ix}{a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] ((-2*I)*x)/a - x^2/2 + (2*Log[I - a*x])/a^2
```

Maple [A] time = 0.041, size = 38, normalized size = 1.3

$$-\frac{x^2}{2}-\frac{2\,ix}{a}+\frac{\ln\left(a^2x^2+1\right)}{a^2}+\frac{2\,i\arctan\left(ax\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+I*a*x)^2*(a^2*x^2+1),x)
```

```
[Out] -1/2*x^2-2*I*x/a+1/a^2*ln(a^2*x^2+1)+2*I/a^2*arctan(a*x)
```

Maxima [A] time = 1.08784, size = 38, normalized size = 1.27

$$\frac{i\left(i\,ax^2-4\,x\right)}{2\,a}+\frac{2\,\log\left(i\,ax+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*I*(I*a*x^2 - 4*x)/a + 2*log(I*a*x + 1)/a^2
```

Fricas [A] time = 1.51843, size = 69, normalized size = 2.3

$$-\frac{a^2x^2+4i\,ax-4\,\log\left(\frac{ax-i}{a}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*x^2 + 4*I*a*x - 4*log((a*x - I)/a))/a^2
```

Sympy [A] time = 0.175426, size = 22, normalized size = 0.73

$$-\frac{x^2}{2}-\frac{2ix}{a}+\frac{2\log\left(ax-i\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)**2*(a**2*x**2+1),x)
```

```
[Out] -x**2/2 - 2*I*x/a + 2*log(a*x - I)/a**2
```

Giac [B] time = 1.11053, size = 78, normalized size = 2.6

$$\frac{i\left(\frac{4\,i\log\left(\frac{1}{\sqrt{a^2x^2+1}|a|}\right)}{a}+\frac{(aix+1)^2\left(i-\frac{6\,i}{aix+1}\right)}{ai^2}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*i*(4*i*log(1/(sqrt(a^2*x^2 + 1)*abs(a)))/a + (a*i*x + 1)^2*(i - 6*i/(a*
i*x + 1))/(a*i^2))/a
```

3.47 $\int e^{-2i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=20

$$-x-\frac{2i\log(-ax+i)}{a}$$

```
[Out] -x - ((2*I)*Log[I - a*x])/a
```

Rubi [A] time = 0.0101968, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5061, 43}

$$-x-\frac{2i\log(-ax+i)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-2*I)*ArcTan[a*x]),x]
```

```
[Out] -x - ((2*I)*Log[I - a*x])/a
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2i\tan^{-1}(ax)}\,dx &= \int \frac{1-iax}{1+iax}\,dx\\ &= \int \left(-1-\frac{2i}{-i+ax}\right)dx\\ &= -x-\frac{2i\log(i-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0088313, size = 30, normalized size = 1.5

$$-\frac{i\log\left(a^2x^2+1\right)}{a}+\frac{2\tan^{-1}(ax)}{a}-x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-2*I)*ArcTan[a*x]),x]
```

```
[Out] -x + (2*ArcTan[a*x])/a - (I*Log[1 + a^2*x^2])/a
```

Maple [A] time = 0.041, size = 30, normalized size = 1.5

$$-x-\frac{i\ln\left(a^2x^2+1\right)}{a}+2\frac{\arctan\left(ax\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1),x)
```

```
[Out] -x-I/a*ln(a^2*x^2+1)+2*arctan(a*x)/a
```

Maxima [A] time = 1.01925, size = 22, normalized size = 1.1

$$-x-\frac{2i\log\left(i\,ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -x - 2*I*log(I*a*x + 1)/a
```

Fricas [A] time = 1.52253, size = 45, normalized size = 2.25

$$-\frac{ax+2i\log\left(\frac{ax-i}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -(a*x + 2*I*log((a*x - I)/a))/a
```

Sympy [A] time = 0.330293, size = 14, normalized size = 0.7

$$-x-\frac{2i\log\left(ax-i\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1),x)
```

```
[Out] -x - 2*I*log(a*x - I)/a
```

Giac [B] time = 1.1094, size = 90, normalized size = 4.5

$$a^2\left(\frac{(aix+1)i}{a^3}+\frac{2\,i\log\left(\frac{1}{\sqrt{a^2x^2+1}|a|}\right)}{a^3}-\frac{i}{(aix+1)a^3}\right)+\frac{i}{(aix+1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
```

```
[Out] a^2*((a*i*x + 1)*i/a^3 + 2*i*log(1/(sqrt(a^2*x^2 + 1)*abs(a)))/a^3 - i/((a*
i*x + 1)*a^3)) + i/((a*i*x + 1)*a)
```

3.48 $\int \frac{e^{-2i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=14

$$\log(x) - 2\log(-ax + i)$$

[Out] Log[x] - 2*Log[I - a*x]

Rubi [A] time = 0.0200594, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 72}

$$\log(x) - 2\log(-ax + i)$$

Antiderivative was successfully verified.

[In] Int[1/(E^((2*I)*ArcTan[a*x])*x),x]

[Out] Log[x] - 2*Log[I - a*x]

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2i\tan^{-1}(ax)}}{x}\,dx &= \int \frac{1-iax}{x(1+iax)}\,dx\\ &= \int \left(\frac{1}{x} - \frac{2a}{-i+ax}\right) dx\\ &= \log(x) - 2\log(i - ax)\end{aligned}$$

Mathematica [A] time = 0.0069279, size = 14, normalized size = 1.

$$\log(x) - 2\log(-ax + i)$$

Antiderivative was successfully verified.

[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*x),x]

[Out] Log[x] - 2*Log[I - a*x]

Maple [A] time = 0.044, size = 23, normalized size = 1.6

$$-\ln\left(a^2x^2+1\right)-2\,i\arctan\left(ax\right)+\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x)
```

```
[Out] -ln(a^2*x^2+1)-2*I*arctan(a*x)+ln(x)
```

Maxima [A] time = 1.02635, size = 16, normalized size = 1.14

$$-2\,\log\left(i\,ax+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x, algorithm="maxima")
```

```
[Out] -2*log(I*a*x + 1) + log(x)
```

Fricas [A] time = 1.62398, size = 39, normalized size = 2.79

$$\log\left(x\right)-2\,\log\left(\frac{ax-i}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x, algorithm="fricas")
```

```
[Out] log(x) - 2*log((a*x - I)/a)
```

Sympy [A] time = 0.418123, size = 10, normalized size = 0.71

$$\log\left(x\right)-2\log\left(x-\frac{i}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/x,x)
```

```
[Out] log(x) - 2*log(x - I/a)
```

Giac [B] time = 1.09633, size = 63, normalized size = 4.5

$$-ai\left(\frac{i\log\left(-i+\frac{i}{aix+1}\right)}{a}+\frac{i\log\left(\frac{1}{\sqrt{a^2x^2+1}|a|}\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x, algorithm="giac")
```

```
[Out] -a*i*(i*log(-i + i/(a*i*x + 1))/a + i*log(1/(sqrt(a^2*x^2 + 1)*abs(a)))/a)
```

3.49 $\int \frac{e^{-2i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=27

$$-2ia\log(x)+2ia\log(-ax+i)-\frac{1}{x}$$

```
[Out] -x^(-1) - (2*I)*a*Log[x] + (2*I)*a*Log[I - a*x]
```

Rubi [A] time = 0.0255769, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$-2ia\log(x)+2ia\log(-ax+i)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*x^2),x]
```

```
[Out] -x^(-1) - (2*I)*a*Log[x] + (2*I)*a*Log[I - a*x]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{1-iax}{x^2(1+iax)}\,dx\\ &= \int \left(\frac{1}{x^2}-\frac{2ia}{x}+\frac{2ia^2}{-i+ax}\right)dx\\ &= -\frac{1}{x}-2ia\log(x)+2ia\log(i-ax)\end{aligned}$$

Mathematica [A] time = 0.0092211, size = 27, normalized size = 1.

$$-2ia\log(x)+2ia\log(-ax+i)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*x^2),x]
```

```
[Out] -x^(-1) - (2*I)*a*Log[x] + (2*I)*a*Log[I - a*x]
```

Maple [A] time = 0.044, size = 34, normalized size = 1.3

$$-2\,a\arctan(ax)+ia\ln\left(a^2x^2+1\right)-x^{-1}-2\,ia\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x)
```

```
[Out] -2*a*arctan(a*x)+I*a*ln(a^2*x^2+1)-1/x-2*I*a*ln(x)
```

Maxima [A] time = 1.05493, size = 46, normalized size = 1.7

$$2i\,a\log(i\,ax+1)-2i\,a\log(x)-\frac{ax-i}{ax^2-i\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x, algorithm="maxima")
```

```
[Out] 2*I*a*log(I*a*x + 1) - 2*I*a*log(x) - (a*x - I)/(a*x^2 - I*x)
```

Fricas [A] time = 1.74535, size = 70, normalized size = 2.59

$$\frac{-2i\,ax\log(x)+2i\,ax\log\left(\frac{ax-i}{a}\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x, algorithm="fricas")
```

```
[Out] (-2*I*a*x*log(x) + 2*I*a*x*log((a*x - I)/a) - 1)/x
```

Sympy [A] time = 0.420873, size = 20, normalized size = 0.74

$$-2a\left(i\log(x)-i\log\left(x-\frac{i}{a}\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/x**2,x)
```

```
[Out] -2*a*(I*log(x) - I*log(x - I/a)) - 1/x
```

Giac [A] time = 1.09886, size = 50, normalized size = 1.85

$$-2\,ai\log\left(-i+\frac{i}{aix+1}\right)+\frac{a}{i-\frac{i}{aix+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x, algorithm="giac")
```

```
[Out] -2*a*i*log(-i + i/(a*i*x + 1)) + a/(i - i/(a*i*x + 1))
```

3.50 $\int \frac{e^{-2i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=37

$$-2a^2\log(x) + 2a^2\log(-ax+i) + \frac{2ia}{x} - \frac{1}{2x^2}$$

```
[Out] -1/(2*x^2) + ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I - a*x]
```

Rubi [A] time = 0.0271134, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$-2a^2\log(x) + 2a^2\log(-ax+i) + \frac{2ia}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*x^3),x]
```

```
[Out] -1/(2*x^2) + ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I - a*x]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{1-iax}{x^3(1+iax)}\,dx\\
&= \int \left(\frac{1}{x^3} - \frac{2ia}{x^2} - \frac{2a^2}{x} + \frac{2a^3}{-i+ax}\right)dx\\
&= -\frac{1}{2x^2} + \frac{2ia}{x} - 2a^2\log(x) + 2a^2\log(i-ax)
\end{aligned}$$

Mathematica [A] time = 0.0107412, size = 37, normalized size = 1.

$$-2a^2\log(x) + 2a^2\log(-ax+i) + \frac{2ia}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*x^3),x]

[Out] -1/(2*x^2) + ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I - a*x]
```

Maple [A] time = 0.046, size = 45, normalized size = 1.2

$$2\,ia^2 \arctan(ax) + a^2 \ln\left(a^2x^2+1\right) - \frac{1}{2\,x^2} + \frac{2\,ia}{x} - 2\,a^2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x)

[Out] 2*I*a^2*arctan(a*x)+a^2*ln(a^2*x^2+1)-1/2/x^2+2*I*a/x-2*a^2*ln(x)
```

Maxima [A] time = 1.00913, size = 68, normalized size = 1.84

$$2\,a^2 \log(i\,ax+1) - 2\,a^2 \log(x) - \frac{4\,a^2x^2 - 3i\,ax + 1}{2i\,ax^3 + 2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x, algorithm="maxima")

[Out] 2*a^2*log(I*a*x + 1) - 2*a^2*log(x) - (4*a^2*x^2 - 3*I*a*x + 1)/(2*I*a*x^3
+ 2*x^2)
```

Fricas [A] time = 1.6601, size = 97, normalized size = 2.62

$$-\frac{4\,a^2x^2 \log(x) - 4\,a^2x^2 \log\left(\frac{ax-i}{a}\right) - 4i\,ax + 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x, algorithm="fricas")

[Out] -1/2*(4*a^2*x^2*log(x) - 4*a^2*x^2*log((a*x - I)/a) - 4*I*a*x + 1)/x^2
```

Sympy [A] time = 0.474097, size = 27, normalized size = 0.73

$$-2a^2\left(\log(x) - \log\left(x - \frac{i}{a}\right)\right) + \frac{4iax - 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/x**3,x)

[Out] -2*a**2*(log(x) - log(x - I/a)) + (4*I*a*x - 1)/(2*x**2)
```

Giac [B] time = 1.09567, size = 85, normalized size = 2.3

$$2\,a^{2}i^{2}\log\left(\frac{i^{2}}{aix+1}+1\right)+\frac{\frac{6\,a^{2}i^{2}}{aix+1}+5\,a^{2}}{2\left(i-\frac{i}{aix+1}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x, algorithm="giac")
```

```
[Out] 2*a^2*i^2*log(i^2/(a*i*x + 1) + 1) + 1/2*(6*a^2*i^2/(a*i*x + 1) + 5*a^2)/(i
 - i/(a*i*x + 1))^2
```

3.51 $\int \frac{e^{-2i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=49

$$\frac{2a^2}{x}+2ia^3\log(x)-2ia^3\log(-ax+i)+\frac{ia}{x^2}-\frac{1}{3x^3}$$

```
[Out] -1/(3*x^3) + (I*a)/x^2 + (2*a^2)/x + (2*I)*a^3*Log[x] - (2*I)*a^3*Log[I - a
*x]
```

Rubi [A] time = 0.0317706, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$\frac{2a^2}{x}+2ia^3\log(x)-2ia^3\log(-ax+i)+\frac{ia}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*x^4),x]
```

```
[Out] -1/(3*x^3) + (I*a)/x^2 + (2*a^2)/x + (2*I)*a^3*Log[x] - (2*I)*a^3*Log[I - a
*x]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2i\tan^{-1}(ax)}}{x^4}\,dx &= \int \frac{1-iax}{x^4(1+iax)}\,dx\\ &= \int \left(\frac{1}{x^4}-\frac{2ia}{x^3}-\frac{2a^2}{x^2}+\frac{2ia^3}{x}-\frac{2ia^4}{-i+ax}\right)dx\\ &= -\frac{1}{3x^3}+\frac{ia}{x^2}+\frac{2a^2}{x}+2ia^3\log(x)-2ia^3\log(i-ax)\end{aligned}$$

Mathematica [A] time = 0.0148524, size = 49, normalized size = 1.

$$\frac{2a^2}{x}+2ia^3\log(x)-2ia^3\log(-ax+i)+\frac{ia}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*x^4),x]
```

```
[Out] -1/(3*x^3) + (I*a)/x^2 + (2*a^2)/x + (2*I)*a^3*Log[x] - (2*I)*a^3*Log[I - a
*x]
```

Maple [A] time = 0.048, size = 55, normalized size = 1.1

$$2\,a^3\arctan(ax) - ia^3\ln\left(a^2x^2+1\right) - \frac{1}{3\,x^3} + \frac{ia}{x^2} + 2\,ia^3\ln(x) + 2\,\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x)
```

```
[Out] 2*a^3*arctan(a*x)-I*a^3*ln(a^2*x^2+1)-1/3/x^3+I*a/x^2+2*I*a^3*ln(x)+2*a^2/x
```

Maxima [A] time = 1.04681, size = 77, normalized size = 1.57

$$-2i\,a^3\log(i\,ax+1) + 2i\,a^3\log(x) + \frac{6i\,a^3x^3 + 3\,a^2x^2 + 2i\,ax - 1}{3i\,ax^4 + 3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x, algorithm="maxima")
```

```
[Out] -2*I*a^3*log(I*a*x + 1) + 2*I*a^3*log(x) + (6*I*a^3*x^3 + 3*a^2*x^2 + 2*I*a
*x - 1)/(3*I*a*x^4 + 3*x^3)
```

Fricas [A] time = 1.64331, size = 117, normalized size = 2.39

$$\frac{6i\,a^3x^3\log(x) - 6i\,a^3x^3\log\left(\frac{ax-i}{a}\right) + 6\,a^2x^2 + 3i\,ax - 1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(6*I*a^3*x^3*log(x) - 6*I*a^3*x^3*log((a*x - I)/a) + 6*a^2*x^2 + 3*I*a*
x - 1)/x^3
```

Sympy [A] time = 0.520236, size = 39, normalized size = 0.8

$$-2a^3\left(-i\log(x) + i\log\left(x - \frac{i}{a}\right)\right) + \frac{6a^2x^2 + 3iax - 1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/x**4,x)
```

```
[Out] -2*a**3*(-I*log(x) + I*log(x - I/a)) + (6*a**2*x**2 + 3*I*a*x - 1)/(3*x**3)
```

Giac [A] time = 1.10177, size = 104, normalized size = 2.12

$$2\,a^3 i \log\left(-i + \frac{i}{aix+1}\right) + \frac{\frac{24\,a^3 i^2}{aix+1} + 10\,a^3 - \frac{15\,a^3 i^2}{(aix+1)^2}}{3\left(i - \frac{i}{aix+1}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x, algorithm="giac")
```

```
[Out] 2*a^3*i*log(-i + i/(a*i*x + 1)) + 1/3*(24*a^3*i^2/(a*i*x + 1) + 10*a^3 - 15
*a^3*i^2/(a*i*x + 1)^2)/(i - i/(a*i*x + 1))^3
```

3.52 $\int e^{-3i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=137

$$\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{x^2\sqrt{a^2x^2+1}}{a^2}-\frac{9i(3ax+2i)\sqrt{a^2x^2+1}}{8a^4}+\frac{27\sqrt{a^2x^2+1}}{4a^4}+\frac{(1-iax)^3}{a^4\sqrt{a^2x^2+1}}+\frac{51i\sinh^{-1}(ax)}{8a^4}$$

```
[Out] (1 - I*a*x)^3/(a^4*Sqrt[1 + a^2*x^2]) + (27*Sqrt[1 + a^2*x^2])/(4*a^4) - (x
^2*Sqrt[1 + a^2*x^2])/a^2 + ((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - (((9*I)/8)*(2
*I + 3*a*x)*Sqrt[1 + a^2*x^2])/a^4 + (((51*I)/8)*ArcSinh[a*x])/a^4
```

Rubi [A] time = 0.62436, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5060, 1633, 1593, 12, 852, 1635, 1815, 27, 743, 641, 215}

$$\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{x^2\sqrt{a^2x^2+1}}{a^2}-\frac{9i(3ax+2i)\sqrt{a^2x^2+1}}{8a^4}+\frac{27\sqrt{a^2x^2+1}}{4a^4}+\frac{(1-iax)^3}{a^4\sqrt{a^2x^2+1}}+\frac{51i\sinh^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] (1 - I*a*x)^3/(a^4*Sqrt[1 + a^2*x^2]) + (27*Sqrt[1 + a^2*x^2])/(4*a^4) - (x
^2*Sqrt[1 + a^2*x^2])/a^2 + ((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - (((9*I)/8)*(2
*I + 3*a*x)*Sqrt[1 + a^2*x^2])/a^4 + (((51*I)/8)*ArcSinh[a*x])/a^4
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-3i\tan^{-1}(ax)}x^3\,dx = \int \frac{x^3(1-iax)^2}{(1+iax)\sqrt{1+a^2x^2}}\,dx$$
$$= (ia)\int \frac{\sqrt{1+a^2x^2}\left(-\frac{ix^3}{a}-x^4\right)}{(1+iax)^2}\,dx$$
$$= (ia)\int \frac{\left(-\frac{i}{a}-x\right)x^3\sqrt{1+a^2x^2}}{(1+iax)^2}\,dx$$
$$= a^2\int \frac{x^3\left(1+a^2x^2\right)^{3/2}}{a^2(1+iax)^3}\,dx$$
$$= \int \frac{x^3\left(1+a^2x^2\right)^{3/2}}{(1+iax)^3}\,dx$$
$$= \int \frac{x^3(1-iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx$$
$$= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} - \int \frac{(1-iax)^2\left(-\frac{3i}{a^3}-\frac{x}{a^2}+\frac{ix^2}{a}\right)}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int \frac{-\frac{12i}{a}-28x+27iax^2+12a^2x^3}{\sqrt{1+a^2x^2}}\,dx}{4a^2}$$
$$= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int \frac{-36ia-108a^2x+81ia^3x^2}{\sqrt{1+a^2x^2}}\,dx}{12a^4}$$
$$= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int \frac{9ia(2i+3ax)^2}{\sqrt{1+a^2x^2}}\,dx}{12a^4}$$
$$= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{(3i)\int \frac{(2i+3ax)^2}{\sqrt{1+a^2x^2}}\,dx}{4a^3}$$
$$= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i+3ax)\sqrt{1+a^2x^2}}{8a^4} - \frac{(3i)\int \frac{-17a^2+18ia^3x}{\sqrt{1+a^2x^2}}\,dx}{8a^5}$$
$$= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} + \frac{27\sqrt{1+a^2x^2}}{4a^4} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i+3ax)\sqrt{1+a^2x^2}}{8a^4} + \frac{(51i)\ldots}{}$$
$$= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} + \frac{27\sqrt{1+a^2x^2}}{4a^4} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i+3ax)\sqrt{1+a^2x^2}}{8a^4} + \frac{51i\,\mathrm{si}\ldots}{}$$

Mathematica [A] time = 0.0563022, size = 80, normalized size = 0.58

$$\sqrt{a^2x^2+1}\left(-\frac{x^2}{a^2}-\frac{19ix}{8a^3}-\frac{4i}{a^4(ax-i)}+\frac{6}{a^4}+\frac{ix^3}{4a}\right)+\frac{51i\sinh^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(6/a^4 - (((19*I)/8)*x)/a^3 - x^2/a^2 + ((I/4)*x^3)/a - (
4*I)/(a^4*(-I + a*x))) + (((51*I)/8)*ArcSinh[a*x])/a^4
```

Maple [B] time = 0.087, size = 296, normalized size = 2.2

$$\frac{\frac{i}{4}x}{a^3}\left(a^2x^2+1\right)^{\frac{3}{2}}+\frac{\frac{3i}{8}x}{a^3}\sqrt{a^2x^2+1}+\frac{\frac{3i}{8}}{a^3}\ln\left(a^2x\frac{1}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2}}-5\frac{1}{a^6}\left(a^2\left(x-\frac{i}{a}\right)^2+2ia\left(x-\frac{i}{a}\right)\right)^{5/2}\left(x-\frac{i}{a}\right)^{-2}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/4*I/a^3*x*(a^2*x^2+1)^(3/2)+3/8*I/a^3*x*(a^2*x^2+1)^(1/2)+3/8*I/a^3*ln(a^
2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)-5/a^6/(x-I/a)^2*(a^2*(x-I/a)
^2+2*I*a*(x-I/a))^(5/2)+4/a^4*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(3/2)+6*I/a^3*(
a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2)*x+6*I/a^3*ln((I*a+a^2*(x-I/a))/(a^2)^(1/
2)+(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)+I/a^7/(x-I/a)^3*(a^2*(x
-I/a)^2+2*I*a*(x-I/a))^(5/2)
```

Maxima [A] time = 1.55073, size = 292, normalized size = 2.13

$$\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{a^6x^2-2i\,a^5x-a^4}+\frac{3\left(a^2x^2+1\right)^{\frac{3}{2}}}{2i\,a^5x+2\,a^4}+\frac{6\sqrt{a^2x^2+1}}{i\,a^5x+a^4}+\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}x}{4\,a^3}+\frac{3i\sqrt{a^2x^2+1}x}{8\,a^3}-\frac{3i\sqrt{-a^2x^2+4i\,ax+3}x}{2\,a^3}-\frac{\left(a^2x^2+\cdots\right.}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] (a^2*x^2 + 1)^(3/2)/(a^6*x^2 - 2*I*a^5*x - a^4) + 3*(a^2*x^2 + 1)^(3/2)/(2*
I*a^5*x + 2*a^4) + 6*sqrt(a^2*x^2 + 1)/(I*a^5*x + a^4) + 1/4*I*(a^2*x^2 + 1
)^(3/2)*x/a^3 + 3/8*I*sqrt(a^2*x^2 + 1)*x/a^3 - 3/2*I*sqrt(-a^2*x^2 + 4*I*a
*x + 3)*x/a^3 - (a^2*x^2 + 1)^(3/2)/a^4 + 3/2*I*arcsin(I*a*x + 2)/a^4 + 63/
8*I*arcsinh(a*x)/a^4 + 9/2*sqrt(a^2*x^2 + 1)/a^4 - 3*sqrt(-a^2*x^2 + 4*I*a*
x + 3)/a^4
```

Fricas [A] time = 1.74957, size = 219, normalized size = 1.6

$$\frac{-32i\,ax-51\,(i\,ax+1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+\left(2i\,a^4x^4-6\,a^3x^3-11i\,a^2x^2+29\,ax-80i\right)\sqrt{a^2x^2+1}-32}{8\left(a^5x-i\,a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(-32*I*a*x - 51*(I*a*x + 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + (2*I*a^4*x^
4 - 6*a^3*x^3 - 11*I*a^2*x^2 + 29*a*x - 80*I)*sqrt(a^2*x^2 + 1) - 32)/(a^5*
x - I*a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.53 $\int e^{-3i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=102

$$-\frac{i(1-iax)^3}{a^3\sqrt{a^2x^2+1}}-\frac{i(3-iax)^2\sqrt{a^2x^2+1}}{3a^3}-\frac{(3ax+28i)\sqrt{a^2x^2+1}}{6a^3}+\frac{11\sinh^{-1}(ax)}{2a^3}$$

```
[Out] ((-I)*(1 - I*a*x)^3)/(a^3*Sqrt[1 + a^2*x^2]) - ((I/3)*(3 - I*a*x)^2*Sqrt[1
+ a^2*x^2])/a^3 - ((28*I + 3*a*x)*Sqrt[1 + a^2*x^2])/(6*a^3) + (11*ArcSinh[
a*x])/(2*a^3)
```

Rubi [A] time = 0.574355, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5060, 1633, 1593, 12, 852, 1635, 1654, 780, 215}

$$-\frac{i(1-iax)^3}{a^3\sqrt{a^2x^2+1}}-\frac{i(3-iax)^2\sqrt{a^2x^2+1}}{3a^3}-\frac{(3ax+28i)\sqrt{a^2x^2+1}}{6a^3}+\frac{11\sinh^{-1}(ax)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((3*I)*ArcTan[a*x]),x]
```



```
[Out] ((-I)*(1 - I*a*x)^3)/(a^3*Sqrt[1 + a^2*x^2]) - ((I/3)*(3 - I*a*x)^2*Sqrt[1
+ a^2*x^2])/a^3 - ((28*I + 3*a*x)*Sqrt[1 + a^2*x^2])/(6*a^3) + (11*ArcSinh[
a*x])/(2*a^3)
```


Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
&& !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-3i\tan^{-1}(ax)}x^2\,dx = \int \frac{x^2(1-iax)^2}{(1+iax)\sqrt{1+a^2x^2}}\,dx$$
$$= (ia)\int \frac{\sqrt{1+a^2x^2}\left(-\frac{ix^2}{a}-x^3\right)}{(1+iax)^2}\,dx$$
$$= (ia)\int \frac{\left(-\frac{i}{a}-x\right)x^2\sqrt{1+a^2x^2}}{(1+iax)^2}\,dx$$
$$= a^2\int \frac{x^2\left(1+a^2x^2\right)^{3/2}}{a^2(1+iax)^3}\,dx$$
$$= \int \frac{x^2\left(1+a^2x^2\right)^{3/2}}{(1+iax)^3}\,dx$$
$$= \int \frac{x^2(1-iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx$$
$$= -\frac{i(1-iax)^3}{a^3\sqrt{1+a^2x^2}} - \int \frac{\left(-\frac{3}{a^2}+\frac{ix}{a}\right)(1-iax)^2}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{i(1-iax)^3}{a^3\sqrt{1+a^2x^2}} - \frac{i(3-iax)^2\sqrt{1+a^2x^2}}{3a^3} + \frac{1}{3}\int \frac{\left(-\frac{3}{a^2}+\frac{ix}{a}\right)(-5+3iax)}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{i(1-iax)^3}{a^3\sqrt{1+a^2x^2}} - \frac{i(3-iax)^2\sqrt{1+a^2x^2}}{3a^3} - \frac{(28i+3ax)\sqrt{1+a^2x^2}}{6a^3} + \frac{11\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a^2}$$
$$= -\frac{i(1-iax)^3}{a^3\sqrt{1+a^2x^2}} - \frac{i(3-iax)^2\sqrt{1+a^2x^2}}{3a^3} - \frac{(28i+3ax)\sqrt{1+a^2x^2}}{6a^3} + \frac{11\sinh^{-1}(ax)}{2a^3}$$

Mathematica [A] time = 0.0490856, size = 63, normalized size = 0.62

$$\frac{33\sinh^{-1}(ax) + \frac{\sqrt{a^2x^2+1}\left(2ia^3x^3-7a^2x^2-19iax-52\right)}{ax-i}}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-52 - (19*I)*a*x - 7*a^2*x^2 + (2*I)*a^3*x^3))/(-I + a
*x) + 33*ArcSinh[a*x])/(6*a^3)
```

Maple [B] time = 0.079, size = 224, normalized size = 2.2

$$\frac{4i}{a^5}\left(a^2\left(x-\frac{i}{a}\right)^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}\left(x-\frac{i}{a}\right)^{-2} - \frac{\frac{11i}{3}}{a^3}\left(a^2\left(x-\frac{i}{a}\right)^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}} + \frac{11x}{2a^2}\sqrt{a^2\left(x-\frac{i}{a}\right)^2+2ia\left(x-\frac{i}{a}\right)} + \frac{11}{2a^{\cdot}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

```
[Out] 4*I/a^5/(x-I/a)^2*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(5/2)-11/3*I/a^3*(a^2*(x-I/
a)^2+2*I*a*(x-I/a))^(3/2)+11/2/a^2*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2)*x+11
```

```
/2/a^2*ln((I*a+a^2*(x-I/a))/(a^2)^(1/2)+(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2)
)/(a^2)^(1/2)+1/a^6/(x-I/a)^3*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(5/2)
```

Maxima [B] time = 1.5412, size = 244, normalized size = 2.39

$$-\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}}{a^5x^2-2i\,a^4x-a^3}-\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}}{i\,a^4x+a^3}-\frac{6i\,\sqrt{a^2x^2+1}}{i\,a^4x+a^3}-\frac{\sqrt{-a^2x^2+4i\,ax+3}x}{2\,a^2}+\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}}{3\,a^3}+\frac{\arcsin\left(i\,ax+2\right)}{2\,a^3}+\frac{6\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -I*(a^2*x^2 + 1)^(3/2)/(a^5*x^2 - 2*I*a^4*x - a^3) - I*(a^2*x^2 + 1)^(3/2)/
(I*a^4*x + a^3) - 6*I*sqrt(a^2*x^2 + 1)/(I*a^4*x + a^3) - 1/2*sqrt(-a^2*x^2
 + 4*I*a*x + 3)*x/a^2 + 1/3*I*(a^2*x^2 + 1)^(3/2)/a^3 + 1/2*arcsin(I*a*x +
2)/a^3 + 6*arcsinh(a*x)/a^3 - 3*I*sqrt(a^2*x^2 + 1)/a^3 + I*sqrt(-a^2*x^2 +
 4*I*a*x + 3)/a^3
```

Fricas [A] time = 1.66689, size = 200, normalized size = 1.96

$$-\frac{24\,ax+(33\,ax-33i)\log\left(-ax+\sqrt{a^2x^2+1}\right)-\left(2i\,a^3x^3-7\,a^2x^2-19i\,ax-52\right)\sqrt{a^2x^2+1}-24i}{6\left(a^4x-i\,a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(24*a*x + (33*a*x - 33*I)*log(-a*x + sqrt(a^2*x^2 + 1)) - (2*I*a^3*x^3
 - 7*a^2*x^2 - 19*I*a*x - 52)*sqrt(a^2*x^2 + 1) - 24*I)/(a^4*x - I*a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\left(a^2x^2+1\right)^{\frac{3}{2}}}{(iax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2*(a**2*x**2 + 1)**(3/2)/(I*a*x + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")

[Out] undef
```

3.54 $\int e^{-3i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=92

$$-\frac{(a^2x^2+1)^{5/2}}{a^2(1+iax)^3}-\frac{3(a^2x^2+1)^{3/2}}{2a^2(1+iax)}-\frac{9\sqrt{a^2x^2+1}}{2a^2}-\frac{9i\sinh^{-1}(ax)}{2a^2}$$

```
[Out] (-9*Sqrt[1 + a^2*x^2])/(2*a^2) - (3*(1 + a^2*x^2)^(3/2))/(2*a^2*(1 + I*a*x)
) - (1 + a^2*x^2)^(5/2)/(a^2*(1 + I*a*x)^3) - (((9*I)/2)*ArcSinh[a*x])/a^2
```

Rubi [A] time = 0.323644, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5060, 1633, 1593, 12, 793, 665, 215}

$$-\frac{(a^2x^2+1)^{5/2}}{a^2(1+iax)^3}-\frac{3(a^2x^2+1)^{3/2}}{2a^2(1+iax)}-\frac{9\sqrt{a^2x^2+1}}{2a^2}-\frac{9i\sinh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((3*I)*ArcTan[a*x]),x]
```


```
[Out] (-9*Sqrt[1 + a^2*x^2])/(2*a^2) - (3*(1 + a^2*x^2)^(3/2))/(2*a^2*(1 + I*a*x)
) - (1 + a^2*x^2)^(5/2)/(a^2*(1 + I*a*x)^3) - (((9*I)/2)*ArcSinh[a*x])/a^2
```


Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
```

```
+ 1, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{-3i\tan^{-1}(ax)}x\,dx &= \int \frac{x(1-iax)^2}{(1+iax)\sqrt{1+a^2x^2}}\,dx\\
&= (ia)\int \frac{\left(-\frac{ix}{a}-x^2\right)\sqrt{1+a^2x^2}}{(1+iax)^2}\,dx\\
&= (ia)\int \frac{\left(-\frac{i}{a}-x\right)x\sqrt{1+a^2x^2}}{(1+iax)^2}\,dx\\
&= a^2\int \frac{x\left(1+a^2x^2\right)^{3/2}}{a^2(1+iax)^3}\,dx\\
&= \int \frac{x\left(1+a^2x^2\right)^{3/2}}{(1+iax)^3}\,dx\\
&= -\frac{\left(1+a^2x^2\right)^{5/2}}{a^2(1+iax)^3}-\frac{(3i)\int \frac{\left(1+a^2x^2\right)^{3/2}}{(1+iax)^2}\,dx}{a}\\
&= -\frac{3\left(1+a^2x^2\right)^{3/2}}{2a^2(1+iax)}-\frac{\left(1+a^2x^2\right)^{5/2}}{a^2(1+iax)^3}-\frac{(9i)\int \frac{\sqrt{1+a^2x^2}}{1+iax}\,dx}{2a}\\
&= -\frac{9\sqrt{1+a^2x^2}}{2a^2}-\frac{3\left(1+a^2x^2\right)^{3/2}}{2a^2(1+iax)}-\frac{\left(1+a^2x^2\right)^{5/2}}{a^2(1+iax)^3}-\frac{(9i)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a}\\
&= -\frac{9\sqrt{1+a^2x^2}}{2a^2}-\frac{3\left(1+a^2x^2\right)^{3/2}}{2a^2(1+iax)}-\frac{\left(1+a^2x^2\right)^{5/2}}{a^2(1+iax)^3}-\frac{9i\sinh^{-1}(ax)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0396151, size = 60, normalized size = 0.65

$$\sqrt{a^2x^2+1}\left(\frac{4i}{a^2(ax-i)}-\frac{3}{a^2}+\frac{ix}{2a}\right)-\frac{9i\sinh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(-3/a^2 + ((I/2)*x)/a + (4*I)/(a^2*(-I + a*x))) - (((9*I)
/2)*ArcSinh[a*x])/a^2
```

Maple [B] time = 0.078, size = 226, normalized size = 2.5

$$3\frac{1}{a^4}\left(a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)\right)^{5/2}\left(x-\frac{i}{a}\right)^{-2}-3\frac{1}{a^2}\left(a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)\right)^{3/2}-\frac{\frac{9i}{2}x}{a}\sqrt{a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

```
[Out] 3/a^4/(x-I/a)^2*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(5/2)-3/a^2*(a^2*(x-I/a)^2+2*
I*a*(x-I/a))^(3/2)-9/2*I/a*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2)*x-9/2*I/a*ln
((I*a+a^2*(x-I/a))/(a^2)^(1/2)+(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(
1/2)-I/a^5/(x-I/a)^3*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(5/2)
```

Maxima [A] time = 1.55776, size = 151, normalized size = 1.64

$$-\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{a^4x^2-2i\,a^3x-a^2}-\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{2i\,a^3x+2\,a^2}-\frac{6\,\sqrt{a^2x^2+1}}{i\,a^3x+a^2}-\frac{9i\,\operatorname{arsinh}\left(ax\right)}{2\,a^2}-\frac{3\,\sqrt{a^2x^2+1}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -(a^2*x^2 + 1)^(3/2)/(a^4*x^2 - 2*I*a^3*x - a^2) - (a^2*x^2 + 1)^(3/2)/(2*I
*a^3*x + 2*a^2) - 6*sqrt(a^2*x^2 + 1)/(I*a^3*x + a^2) - 9/2*I*arcsinh(a*x)/
a^2 - 3/2*sqrt(a^2*x^2 + 1)/a^2
```

Fricas [A] time = 1.65666, size = 174, normalized size = 1.89

$$\frac{8i\,ax-9\left(-i\,ax-1\right)\log\left(-ax+\sqrt{a^2x^2+1}\right)+\sqrt{a^2x^2+1}\left(i\,a^2x^2-5\,ax+14i\right)+8}{2\left(a^3x-i\,a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(8*I*a*x - 9*(-I*a*x - 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + sqrt(a^2*x^2
+ 1)*(I*a^2*x^2 - 5*a*x + 14*I) + 8)/(a^3*x - I*a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\left(a^2x^2+1\right)^{\frac{3}{2}}}{\left(iax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*(a**2*x**2 + 1)**(3/2)/(I*a*x + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.55 $\int e^{-3i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=60

$$\frac{2i(1-iax)^2}{a\sqrt{a^2x^2+1}}+\frac{3i\sqrt{a^2x^2+1}}{a}-\frac{3\sinh^{-1}(ax)}{a}$$

```
[Out] ((2*I)*(1 - I*a*x)^2)/(a*Sqrt[1 + a^2*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2])/a -
 (3*ArcSinh[a*x])/a
```

Rubi [A] time = 0.0441652, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5059, 853, 669, 641, 215}

$$\frac{2i(1-iax)^2}{a\sqrt{a^2x^2+1}}+\frac{3i\sqrt{a^2x^2+1}}{a}-\frac{3\sinh^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-3*I)*ArcTan[a*x]),x]
```


```
[Out] ((2*I)*(1 - I*a*x)^2)/(a*Sqrt[1 + a^2*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2])/a -
 (3*ArcSinh[a*x])/a
```


Rule 5059

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 - I*a*x)^((I*n + 1)/2)
/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2]), x] /; FreeQ[a, x] && Intege
rQ[(I*n - 1)/2]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3i\tan^{-1}(ax)}\,dx &= \int \frac{(1-iax)^2}{(1+iax)\sqrt{1+a^2x^2}}\,dx\\
&= \int \frac{(1-iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx\\
&= \frac{2i(1-iax)^2}{a\sqrt{1+a^2x^2}} - 3\int \frac{1-iax}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{2i(1-iax)^2}{a\sqrt{1+a^2x^2}} + \frac{3i\sqrt{1+a^2x^2}}{a} - 3\int \frac{1}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{2i(1-iax)^2}{a\sqrt{1+a^2x^2}} + \frac{3i\sqrt{1+a^2x^2}}{a} - \frac{3\sinh^{-1}(ax)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0302173, size = 42, normalized size = 0.7

$$
-\frac{3\sinh^{-1}(ax)}{a} + \frac{\sqrt{a^2x^2+1}\left(\frac{4}{ax-i}+i\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((-3*I)*ArcTan[a*x]),x]

[Out] (Sqrt[1 + a^2*x^2]*(I + 4/(-I + a*x)))/a - (3*ArcSinh[a*x])/a
```

Maple [B] time = 0.055, size = 219, normalized size = 3.7

$$
-\frac{1}{a^4}\left(a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}\left(x-\frac{i}{a}\right)^{-3} - \frac{2\,i}{a^3}\left(a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}\left(x-\frac{i}{a}\right)^{-2} + \frac{2\,i}{a}\left(a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)

[Out] -1/a^4/(x-I/a)^3*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(5/2)-2*I/a^3/(x-I/a)^2*(a^2
*(x-I/a)^2+2*I*a*(x-I/a))^(5/2)+2*I/a*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(3/2)-3
*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2)*x-3*ln((I*a+a^2*(x-I/a))/(a^2)^(1/2)+(
a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)
```

Maxima [A] time = 1.506, size = 88, normalized size = 1.47

$$
\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}}{a^3x^2-2i\,a^2x-a} - \frac{3\,\operatorname{arsinh}(ax)}{a} + \frac{6i\sqrt{a^2x^2+1}}{i\,a^2x+a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] I*(a^2*x^2 + 1)^(3/2)/(a^3*x^2 - 2*I*a^2*x - a) - 3*arcsinh(a*x)/a + 6*I*sq
rt(a^2*x^2 + 1)/(I*a^2*x + a)
```

Fricas [A] time = 1.6467, size = 144, normalized size = 2.4

$$\frac{4\,ax + (3\,ax - 3i)\log\left(-ax + \sqrt{a^2x^2 + 1}\right) + \sqrt{a^2x^2 + 1}(i\,ax + 5) - 4i}{a^2x - i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (4*a*x + (3*a*x - 3*I)*log(-a*x + sqrt(a^2*x^2 + 1)) + sqrt(a^2*x^2 + 1)*(I
*a*x + 5) - 4*I)/(a^2*x - I*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2 + 1\right)^{\frac{3}{2}}}{(iax + 1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((a**2*x**2 + 1)**(3/2)/(I*a*x + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.56 $\int \frac{e^{-3i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=52

$$\frac{4i\sqrt{a^2x^2+1}}{-ax+i} - \tanh^{-1}\left(\sqrt{a^2x^2+1}\right) + i\sinh^{-1}(ax)$$

```
[Out] ((4*I)*Sqrt[1 + a^2*x^2])/(I - a*x) + I*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2
*x^2]]
```

Rubi [A] time = 0.572048, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5060, 6742, 215, 266, 63, 208, 651}

$$\frac{4i\sqrt{a^2x^2+1}}{-ax+i} - \tanh^{-1}\left(\sqrt{a^2x^2+1}\right) + i\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*x),x]
```

```
[Out] ((4*I)*Sqrt[1 + a^2*x^2])/(I - a*x) + I*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2
*x^2]]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3i\tan^{-1}(ax)}}{x}\,dx &= \int \frac{(1-iax)^2}{x(1+iax)\sqrt{1+a^2x^2}}\,dx\\
&= \int \left(\frac{ia}{\sqrt{1+a^2x^2}}+\frac{1}{x\sqrt{1+a^2x^2}}-\frac{4a}{(-i+ax)\sqrt{1+a^2x^2}}\right)dx\\
&= (ia)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx-(4a)\int \frac{1}{(-i+ax)\sqrt{1+a^2x^2}}\,dx+\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\
&= \frac{4i\sqrt{1+a^2x^2}}{i-ax}+i\sinh^{-1}(ax)+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= \frac{4i\sqrt{1+a^2x^2}}{i-ax}+i\sinh^{-1}(ax)+\frac{\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)}{a^2}\\
&= \frac{4i\sqrt{1+a^2x^2}}{i-ax}+i\sinh^{-1}(ax)-\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.040163, size = 55, normalized size = 1.06

$$
-\frac{4i\sqrt{a^2x^2+1}}{ax-i}-\log\left(\sqrt{a^2x^2+1}+1\right)+i\sinh^{-1}(ax)+\log(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*x),x]
```

```
[Out] ((-4*I)*Sqrt[1 + a^2*x^2])/(-I + a*x) + I*ArcSinh[a*x] + Log[x] - Log[1 + S
qrt[1 + a^2*x^2]]
```

Maple [B] time = 0.084, size = 257, normalized size = 4.9

$$
\frac{i}{a^3}\left(a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}\left(x-\frac{i}{a}\right)^{-3}-\frac{1}{a^2}\left(a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}\left(x-\frac{i}{a}\right)^{-2}+\frac{2}{3}\left(a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x)
```

```
[Out] I/a^3/(x-I/a)^3*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(5/2)-1/a^2/(x-I/a)^2*(a^2*(x
-I/a)^2+2*I*a*(x-I/a))^(5/2)+2/3*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(3/2)+I*a*(a
^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2)*x+I*a*ln((I*a+a^2*(x-I/a))/(a^2)^(1/2)+(a
```

```
^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)+1/3*(a^2*x^2+1)^(3/2)+(a^2*x
^2+1)^(1/2)-arctanh(1/(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{(i\,ax+1)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x), x)
```

Fricas [B] time = 1.73555, size = 252, normalized size = 4.85

$$\frac{-4i\,ax-(ax-i)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+(-i\,ax-1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+(ax-i)\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-4}{ax-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (-4*I*a*x - (a*x - I)*log(-a*x + sqrt(a^2*x^2 + 1) + 1) + (-I*a*x - 1)*log(
-a*x + sqrt(a^2*x^2 + 1)) + (a*x - I)*log(-a*x + sqrt(a^2*x^2 + 1) - 1) - 4
*I*sqrt(a^2*x^2 + 1) - 4)/(a*x - I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{x\,(iax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] Integral((a**2*x**2 + 1)**(3/2)/(x*(I*a*x + 1)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x, algorithm="giac")
```

```
[Out] undef
```

3.57 $\int \frac{e^{-3i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=64

$$\frac{4a\sqrt{a^2x^2+1}}{-ax+i}-\frac{\sqrt{a^2x^2+1}}{x}+3ia\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -(Sqrt[1 + a^2*x^2]/x) + (4*a*Sqrt[1 + a^2*x^2])/(I - a*x) + (3*I)*a*ArcTan
h[Sqrt[1 + a^2*x^2]]
```

Rubi [A] time = 0.552167, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5060, 6742, 264, 266, 63, 208, 651}

$$\frac{4a\sqrt{a^2x^2+1}}{-ax+i}-\frac{\sqrt{a^2x^2+1}}{x}+3ia\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*x^2),x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/x) + (4*a*Sqrt[1 + a^2*x^2])/(I - a*x) + (3*I)*a*ArcTan
h[Sqrt[1 + a^2*x^2]]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1-iax)^2}{x^2(1+iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^2\sqrt{1+a^2x^2}} - \frac{3ia}{x\sqrt{1+a^2x^2}} + \frac{4ia^2}{(-i+ax)\sqrt{1+a^2x^2}}\right) dx \\
&= -\left((3ia)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\right) + \left(4ia^2\right)\int \frac{1}{(-i+ax)\sqrt{1+a^2x^2}}\,dx + \int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{x} + \frac{4a\sqrt{1+a^2x^2}}{i-ax} - \frac{1}{2}(3ia)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{x} + \frac{4a\sqrt{1+a^2x^2}}{i-ax} - \frac{(3i)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a} \\
&= -\frac{\sqrt{1+a^2x^2}}{x} + \frac{4a\sqrt{1+a^2x^2}}{i-ax} + 3ia\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0475849, size = 61, normalized size = 0.95

$$\sqrt{a^2x^2+1}\left(-\frac{1}{x} - \frac{4a}{ax-i}\right) + 3ia\log\left(\sqrt{a^2x^2+1}+1\right) - 3ia\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*x^2),x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(-x^(-1) - (4*a)/(-I + a*x)) - (3*I)*a*Log[x] + (3*I)*a*L
og[1 + Sqrt[1 + a^2*x^2]]
```

Maple [B] time = 0.086, size = 305, normalized size = 4.8

$$ia\left(a^2\left(x-\frac{i}{a}\right)^2 + 2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}} - ia\left(a^2x^2+1\right)^{\frac{3}{2}} - \frac{3a^2x}{2}\sqrt{a^2\left(x-\frac{i}{a}\right)^2 + 2ia\left(x-\frac{i}{a}\right)} - \frac{3a^2}{2}\ln\left(\left(ia + a^2\left(x-\frac{i}{a}\right)\right)\frac{1}{\sqrt{a^2}} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x)
```

```
[Out] I*a*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(3/2)-I*a*(a^2*x^2+1)^(3/2)-3/2*a^2*(a^2*
(x-I/a)^2+2*I*a*(x-I/a))^(1/2)*x-3/2*a^2*ln((I*a+a^2*(x-I/a))/(a^2)^(1/2)+(
```

```
a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)+1/a^2/(x-I/a)^3*(a^2*(x-I/a
)^2+2*I*a*(x-I/a))^(5/2)+3*I*a*arctanh(1/(a^2*x^2+1)^(1/2))-3*I*a*(a^2*x^2+
1)^(1/2)-1/x*(a^2*x^2+1)^(5/2)+a^2*x*(a^2*x^2+1)^(3/2)+3/2*a^2*x*(a^2*x^2+1
)^(1/2)+3/2*a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{(i\,ax+1)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x^2), x)
```

Fricas [B] time = 1.64274, size = 247, normalized size = 3.86

$$-\frac{5\,a^2x^2-5i\,ax+3\left(-i\,a^2x^2-ax\right)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+3\left(i\,a^2x^2+ax\right)\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+\sqrt{a^2x^2+1}}{ax^2-i\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(5*a^2*x^2 - 5*I*a*x + 3*(-I*a^2*x^2 - a*x)*log(-a*x + sqrt(a^2*x^2 + 1) +
 1) + 3*(I*a^2*x^2 + a*x)*log(-a*x + sqrt(a^2*x^2 + 1) - 1) + sqrt(a^2*x^2
+ 1)*(5*a*x - I))/(a*x^2 - I*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] undef
```

3.58 $\int \frac{e^{-3i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=93

$$-\frac{4ia^2\sqrt{a^2x^2+1}}{-ax+i}+\frac{3ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -Sqrt[1 + a^2*x^2]/(2*x^2) + ((3*I)*a*Sqrt[1 + a^2*x^2])/x - ((4*I)*a^2*Sqr
t[1 + a^2*x^2])/(I - a*x) + (9*a^2*ArcTanh[Sqrt[1 + a^2*x^2]])/2
```

Rubi [A] time = 0.593131, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5060, 6742, 266, 51, 63, 208, 264, 651}

$$-\frac{4ia^2\sqrt{a^2x^2+1}}{-ax+i}+\frac{3ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*x^3),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(2*x^2) + ((3*I)*a*Sqrt[1 + a^2*x^2])/x - ((4*I)*a^2*Sqr
t[1 + a^2*x^2])/(I - a*x) + (9*a^2*ArcTanh[Sqrt[1 + a^2*x^2]])/2
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1-iax)^2}{x^3(1+iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^3\sqrt{1+a^2x^2}} - \frac{3ia}{x^2\sqrt{1+a^2x^2}} - \frac{4a^2}{x\sqrt{1+a^2x^2}} + \frac{4a^3}{(-i+ax)\sqrt{1+a^2x^2}}\right)dx \\
&= -\left((3ia)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx\right) - \left(4a^2\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx + \left(4a^3\right)\int \frac{1}{(-i+ax)\sqrt{1+a^2x^2}}\,dx + \int \frac{}{x^3
\\
&= \frac{3ia\sqrt{1+a^2x^2}}{x} - \frac{4ia^2\sqrt{1+a^2x^2}}{i-ax} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx, x, x^2\right) - \left(2a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a}
\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{3ia\sqrt{1+a^2x^2}}{x} - \frac{4ia^2\sqrt{1+a^2x^2}}{i-ax} - 4\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right) - \frac{1}{4}a^2\,\text{S}
\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{3ia\sqrt{1+a^2x^2}}{x} - \frac{4ia^2\sqrt{1+a^2x^2}}{i-ax} + 4a^2\tanh^{-1}\left(\sqrt{1+a^2x^2}\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x}{a}}
\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{3ia\sqrt{1+a^2x^2}}{x} - \frac{4ia^2\sqrt{1+a^2x^2}}{i-ax} + \frac{9}{2}a^2\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0740573, size = 79, normalized size = 0.85

$$\sqrt{a^2x^2+1}\left(\frac{4ia^2}{ax-i}+\frac{3ia}{x}-\frac{1}{2x^2}\right)+\frac{9}{2}a^2\log\left(\sqrt{a^2x^2+1}+1\right)-\frac{9}{2}a^2\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*x^3),x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(-1/(2*x^2) + ((3*I)*a)/x + ((4*I)*a^2)/(-I + a*x)) - (9*
a^2*Log[x])/2 + (9*a^2*Log[1 + Sqrt[1 + a^2*x^2]])/2
```

Maple [B] time = 0.095, size = 376, normalized size = 4.

$$-\frac{1}{2x^2}\left(a^2x^2+1\right)^{\frac{5}{2}}-\frac{3a^2}{2}\left(a^2x^2+1\right)^{\frac{3}{2}}-\frac{9a^2}{2}\sqrt{a^2x^2+1}+\frac{9a^2}{2}\text{Artanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)-\frac{9i}{2}a^3\ln\left(a^2x\frac{1}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x)
```

```
[Out] -1/2/x^2*(a^2*x^2+1)^(5/2)-3/2*a^2*(a^2*x^2+1)^(3/2)-9/2*a^2*(a^2*x^2+1)^(1
/2)+9/2*a^2*arctanh(1/(a^2*x^2+1)^(1/2))-9/2*I*a^3*ln(a^2*x/(a^2)^(1/2)+(a^
2*x^2+1)^(1/2))/(a^2)^(1/2)+3*I*a/x*(a^2*x^2+1)^(5/2)-I/a/(x-I/a)^3*(a^2*(x
-I/a)^2+2*I*a*(x-I/a))^(5/2)-9/2*I*a^3*x*(a^2*x^2+1)^(1/2)+9/2*I*a^3*(a^2*(
x-I/a)^2+2*I*a*(x-I/a))^(1/2)*x-1/(x-I/a)^2*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(
5/2)+3*a^2*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(3/2)-3*I*a^3*x*(a^2*x^2+1)^(3/2)+
9/2*I*a^3*ln((I*a+a^2*(x-I/a))/(a^2)^(1/2)+(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1
/2))/(a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{(i\,ax+1)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x^3), x)
```

Fricas [A] time = 1.6789, size = 292, normalized size = 3.14

$$\frac{14i\,a^3x^3+14\,a^2x^2+9\left(a^3x^3-i\,a^2x^2\right)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-9\left(a^3x^3-i\,a^2x^2\right)\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+\sqrt{a^2x^2+}}{2\left(ax^3-i\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(14*I*a^3*x^3 + 14*a^2*x^2 + 9*(a^3*x^3 - I*a^2*x^2)*log(-a*x + sqrt(a^
2*x^2 + 1) + 1) - 9*(a^3*x^3 - I*a^2*x^2)*log(-a*x + sqrt(a^2*x^2 + 1) - 1)
 + sqrt(a^2*x^2 + 1)*(14*I*a^2*x^2 + 5*a*x + I))/(a*x^3 - I*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

3.59 $\int \frac{e^{-3i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=118

$$-\frac{4a^3\sqrt{a^2x^2+1}}{-ax+i}+\frac{14a^2\sqrt{a^2x^2+1}}{3x}+\frac{3ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{11}{2}ia^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -Sqrt[1 + a^2*x^2]/(3*x^3) + (((3*I)/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (14*a^2*
Sqrt[1 + a^2*x^2])/(3*x) - (4*a^3*Sqrt[1 + a^2*x^2])/(I - a*x) - ((11*I)/2)
*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rubi [A] time = 0.603894, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5060, 6742, 271, 264, 266, 51, 63, 208, 651}

$$-\frac{4a^3\sqrt{a^2x^2+1}}{-ax+i}+\frac{14a^2\sqrt{a^2x^2+1}}{3x}+\frac{3ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{11}{2}ia^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*x^4),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(3*x^3) + (((3*I)/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (14*a^2*
Sqrt[1 + a^2*x^2])/(3*x) - (4*a^3*Sqrt[1 + a^2*x^2])/(I - a*x) - ((11*I)/2)
*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3i\tan^{-1}(ax)}}{x^4}\,dx &= \int \frac{(1-iax)^2}{x^4(1+iax)\sqrt{1+a^2x^2}}\,dx\\
&= \int \left(\frac{1}{x^4\sqrt{1+a^2x^2}} - \frac{3ia}{x^3\sqrt{1+a^2x^2}} - \frac{4a^2}{x^2\sqrt{1+a^2x^2}} + \frac{4ia^3}{x\sqrt{1+a^2x^2}} - \frac{4ia^4}{(-i+ax)\sqrt{1+a^2x^2}}\right)dx\\
&= -\left((3ia)\int \frac{1}{x^3\sqrt{1+a^2x^2}}\,dx\right) - \left(4a^2\right)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx + \left(4ia^3\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx - \left(4ia^4\right)\int \frac{}{(-}\\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{4a^2\sqrt{1+a^2x^2}}{x} - \frac{4a^3\sqrt{1+a^2x^2}}{i-ax} - \frac{1}{2}(3ia)\,\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx, x, x^2\right) - \frac{1}{3}\left(2a\right.\\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{3ia\sqrt{1+a^2x^2}}{2x^2} + \frac{14a^2\sqrt{1+a^2x^2}}{3x} - \frac{4a^3\sqrt{1+a^2x^2}}{i-ax} + (4ia)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x\right.\\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{3ia\sqrt{1+a^2x^2}}{2x^2} + \frac{14a^2\sqrt{1+a^2x^2}}{3x} - \frac{4a^3\sqrt{1+a^2x^2}}{i-ax} - 4ia^3\tanh^{-1}\left(\sqrt{1+a^2x^2}\right) + \frac{1}{2}(\\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{3ia\sqrt{1+a^2x^2}}{2x^2} + \frac{14a^2\sqrt{1+a^2x^2}}{3x} - \frac{4a^3\sqrt{1+a^2x^2}}{i-ax} - \frac{11}{2}ia^3\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0697248, size = 89, normalized size = 0.75

$$\frac{1}{6}\left(\frac{\sqrt{a^2x^2+1}\left(52a^3x^3-19ia^2x^2+7ax+2i\right)}{x^3(ax-i)} - 33ia^3\log\left(\sqrt{a^2x^2+1}+1\right) + 33ia^3\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*x^4),x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(2*I + 7*a*x - (19*I)*a^2*x^2 + 52*a^3*x^3))/(x^3*(-I +
 a*x)) + (33*I)*a^3*Log[x] - (33*I)*a^3*Log[1 + Sqrt[1 + a^2*x^2]])/6
```

Maple [B] time = 0.09, size = 392, normalized size = 3.3

$$-\frac{16 i}{3} a^3 \left(a^2 \left(x-\frac{i}{a}\right)^2+2 i a \left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}-\frac{11 i}{2} a^3 \text{Artanh}\left(\frac{1}{\sqrt{a^2 x^2+1}}\right)+8 a^4 \sqrt{a^2 \left(x-\frac{i}{a}\right)^2+2 i a \left(x-\frac{i}{a}\right)}\, x+8 \frac{a^4}{\sqrt{a^2}} \ln\left(\frac{1}{\sqrt{a^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x)
```

```
[Out] -16/3*I*a^3*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(3/2)-11/2*I*a^3*arctanh(1/(a^2*x
^2+1)^(1/2))+8*a^4*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2)*x+8*a^4*ln((I*a+a^2*
(x-I/a))/(a^2)^(1/2)+(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)-1/(x-
I/a)^3*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(5/2)+11/2*I*a^3*(a^2*x^2+1)^(1/2)+2*I
*a/(x-I/a)^2*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(5/2)+3/2*I*a/x^2*(a^2*x^2+1)^(5
/2)+11/6*I*a^3*(a^2*x^2+1)^(3/2)+16/3*a^2/x*(a^2*x^2+1)^(5/2)-16/3*a^4*x*(a
^2*x^2+1)^(3/2)-8*a^4*x*(a^2*x^2+1)^(1/2)-8*a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2+1)^(1/2))/(a^2)^(1/2)-1/3/x^3*(a^2*x^2+1)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2 x^2+1\right)^{\frac{3}{2}}}{(i a x+1)^3 x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x^4), x)
```

Fricas [A] time = 1.75583, size = 316, normalized size = 2.68

$$\frac{52 a^4 x^4-52 i a^3 x^3-33 \left(i a^4 x^4+a^3 x^3\right) \log\left(-a x+\sqrt{a^2 x^2+1}+1\right)-33 \left(-i a^4 x^4-a^3 x^3\right) \log\left(-a x+\sqrt{a^2 x^2+1}-1\right)+\left(52\right.}{6 \left(a x^4-i x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(52*a^4*x^4 - 52*I*a^3*x^3 - 33*(I*a^4*x^4 + a^3*x^3)*log(-a*x + sqrt(a
^2*x^2 + 1) + 1) - 33*(-I*a^4*x^4 - a^3*x^3)*log(-a*x + sqrt(a^2*x^2 + 1) -
 1) + (52*a^3*x^3 - 19*I*a^2*x^2 + 7*a*x + 2*I)*sqrt(a^2*x^2 + 1))/(a*x^4 -
 I*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

3.60 $\int \frac{e^{-3i\tan^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=139

$$\frac{4ia^4\sqrt{a^2x^2+1}}{-ax+i}-\frac{6ia^3\sqrt{a^2x^2+1}}{x}+\frac{19a^2\sqrt{a^2x^2+1}}{8x^2}+\frac{ia\sqrt{a^2x^2+1}}{x^3}-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{51}{8}a^4\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -Sqrt[1 + a^2*x^2]/(4*x^4) + (I*a*Sqrt[1 + a^2*x^2])/x^3 + (19*a^2*Sqrt[1 +
 a^2*x^2])/(8*x^2) - ((6*I)*a^3*Sqrt[1 + a^2*x^2])/x + ((4*I)*a^4*Sqrt[1 +
a^2*x^2])/(I - a*x) - (51*a^4*ArcTanh[Sqrt[1 + a^2*x^2]])/8
```

Rubi [A] time = 0.662225, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5060, 6742, 266, 51, 63, 208, 271, 264, 651}

$$\frac{4ia^4\sqrt{a^2x^2+1}}{-ax+i}-\frac{6ia^3\sqrt{a^2x^2+1}}{x}+\frac{19a^2\sqrt{a^2x^2+1}}{8x^2}+\frac{ia\sqrt{a^2x^2+1}}{x^3}-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{51}{8}a^4\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*x^5),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(4*x^4) + (I*a*Sqrt[1 + a^2*x^2])/x^3 + (19*a^2*Sqrt[1 +
 a^2*x^2])/(8*x^2) - ((6*I)*a^3*Sqrt[1 + a^2*x^2])/x + ((4*I)*a^4*Sqrt[1 +
a^2*x^2])/(I - a*x) - (51*a^4*ArcTanh[Sqrt[1 + a^2*x^2]])/8
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{-3i\tan^{-1}(ax)}}{x^5}\,dx = \int \frac{(1-iax)^2}{x^5(1+iax)\sqrt{1+a^2x^2}}\,dx$$

$$= \int \left(\frac{1}{x^5\sqrt{1+a^2x^2}} - \frac{3ia}{x^4\sqrt{1+a^2x^2}} - \frac{4a^2}{x^3\sqrt{1+a^2x^2}} + \frac{4ia^3}{x^2\sqrt{1+a^2x^2}} + \frac{4a^4}{x\sqrt{1+a^2x^2}} - \frac{4a^5}{(-i+ax)\sqrt{1+\cdots}}\right.$$

$$= -\left((3ia)\int \frac{1}{x^4\sqrt{1+a^2x^2}}\,dx\right) - \left(4a^2\right)\int \frac{1}{x^3\sqrt{1+a^2x^2}}\,dx + \left(4ia^3\right)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx + \left(4a^4\right)\int \cdots$$

$$= \frac{ia\sqrt{1+a^2x^2}}{x^3} - \frac{4ia^3\sqrt{1+a^2x^2}}{x} + \frac{4ia^4\sqrt{1+a^2x^2}}{i-ax} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+a^2x}}\,dx,x,x^2\right) - \left(2a^2\right)\text{Su}\cdots$$

$$= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{x^3} + \frac{2a^2\sqrt{1+a^2x^2}}{x^2} - \frac{6ia^3\sqrt{1+a^2x^2}}{x} + \frac{4ia^4\sqrt{1+a^2x^2}}{i-ax} - \frac{1}{8}\left(3a^2\right)\text{Subst}\cdots$$

$$= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{x^3} + \frac{19a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{6ia^3\sqrt{1+a^2x^2}}{x} + \frac{4ia^4\sqrt{1+a^2x^2}}{i-ax} - 4a^4\tanh^{-1}\left(\cdots\right.$$

$$= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{x^3} + \frac{19a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{6ia^3\sqrt{1+a^2x^2}}{x} + \frac{4ia^4\sqrt{1+a^2x^2}}{i-ax} - 6a^4\tanh^{-1}\left(\cdots\right.$$

$$= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{x^3} + \frac{19a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{6ia^3\sqrt{1+a^2x^2}}{x} + \frac{4ia^4\sqrt{1+a^2x^2}}{i-ax} - \frac{51}{8}a^4\tanh^{-1}\cdots$$

Mathematica [A] time = 0.0772052, size = 95, normalized size = 0.68

$$\frac{1}{8}\left(\frac{\sqrt{a^2x^2+1}\left(-80ia^4x^4-29a^3x^3-11ia^2x^2+6ax+2i\right)}{x^4(ax-i)} - 51a^4\log\left(\sqrt{a^2x^2+1}+1\right) + 51a^4\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*x^5),x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(2*I + 6*a*x - (11*I)*a^2*x^2 - 29*a^3*x^3 - (80*I)*a^4
*x^4))/(x^4*(-I + a*x)) + 51*a^4*Log[x] - 51*a^4*Log[1 + Sqrt[1 + a^2*x^2]]
)/8
```

Maple [B] time = 0.089, size = 416, normalized size = 3.

$$-\frac{1}{4x^{4}}\left(a^{2}x^{2}+1\right)^{\frac{5}{2}}+3a^{2}\left(a^{2}\left(x-\frac{i}{a}\right)^{2}+2ia\left(x-\frac{i}{a}\right)\right)^{5/2}\left(x-\frac{i}{a}\right)^{-2}+\frac{23a^{2}}{8x^{2}}\left(a^{2}x^{2}+1\right)^{\frac{5}{2}}-12ia^{5}\ln\left(\left(ia+a^{2}\left(x-\frac{i}{a}\right)\right)\frac{1}{\sqrt{a^{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x)
```

```
[Out] -1/4/x^4*(a^2*x^2+1)^(5/2)+3*a^2/(x-I/a)^2*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(5
/2)+23/8*a^2/x^2*(a^2*x^2+1)^(5/2)-12*I*a^5*ln((I*a+a^2*(x-I/a))/(a^2)^(1/2
)+(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)+12*I*a^5*ln(a^2*x/(a^2)^
(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)+12*I*a^5*x*(a^2*x^2+1)^(1/2)+I*a/(x-I/
a)^3*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(5/2)+I*a/x^3*(a^2*x^2+1)^(5/2)+8*I*a^5*
x*(a^2*x^2+1)^(3/2)-51/8*a^4*arctanh(1/(a^2*x^2+1)^(1/2))+51/8*a^4*(a^2*x^2
+1)^(1/2)-8*I*a^3/x*(a^2*x^2+1)^(5/2)+17/8*a^4*(a^2*x^2+1)^(3/2)-8*a^4*(a^2
*(x-I/a)^2+2*I*a*(x-I/a))^(3/2)-12*I*a^5*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2
)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(i\,ax+1\right)^{3}x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x^5), x)
```

Fricas [A] time = 1.77248, size = 346, normalized size = 2.49

$$\frac{-80i\,a^{5}x^{5}-80\,a^{4}x^{4}-\left(51\,a^{5}x^{5}-51i\,a^{4}x^{4}\right)\log\left(-ax+\sqrt{a^{2}x^{2}+1}+1\right)+\left(51\,a^{5}x^{5}-51i\,a^{4}x^{4}\right)\log\left(-ax+\sqrt{a^{2}x^{2}+1}-1\right)+}{8\left(ax^{5}-i\,x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(-80*I*a^5*x^5 - 80*a^4*x^4 - (51*a^5*x^5 - 51*I*a^4*x^4)*log(-a*x + sq
rt(a^2*x^2 + 1) + 1) + (51*a^5*x^5 - 51*I*a^4*x^4)*log(-a*x + sqrt(a^2*x^2
```

```
+ 1) - 1) + (-80*I*a^4*x^4 - 29*a^3*x^3 - 11*I*a^2*x^2 + 6*a*x + 2*I)*sqrt(
a^2*x^2 + 1))/(a*x^5 - I*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] undef
```

3.61 $\int e^{\frac{1}{2}i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=339

$$\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}-\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3}-\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3}-\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3}+\ldots$$

```
[Out] (((-3*I)/8)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a^3 - ((I/12)*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(5/4))/a^3 + (x*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(5/4))/(3*a
^2) + (((3*I)/8)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])
/(Sqrt[2]*a^3) - (((3*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a
*x)^(1/4)])/(Sqrt[2]*a^3) - (((3*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*
a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (((3
*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4
))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3)
```

Rubi [A] time = 0.227605, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5062, 90, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}-\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3}-\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3}-\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])*x^2,x]
```


```
[Out] (((-3*I)/8)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a^3 - ((I/12)*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(5/4))/a^3 + (x*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(5/4))/(3*a
^2) + (((3*I)/8)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])
/(Sqrt[2]*a^3) - (((3*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a
*x)^(1/4)])/(Sqrt[2]*a^3) - (((3*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*
a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (((3
*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4
))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3)
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{1}{2}i\tan^{-1}(ax)}x^2\,dx = \int \frac{x^2\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx$$

$$= \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} + \frac{\int \frac{\left(-1-\frac{iax}{2}\right)\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx}{3a^2}$$

$$= -\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{3\int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx}{8a^2}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{3\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,d}{16a^2}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{(3i)\,\mathrm{Subst}\left(\int \frac{x^2}{(2-x^4)}\right.}{4a}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{(3i)\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\right.}{4a^3}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\right.}{8a^3}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x}\right.}{16a}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\right.}{16\sqrt{2}a^3}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} + \frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+i}}\right.}{8\sqrt{2}a^3}$$

Mathematica [C] time = 0.0360453, size = 82, normalized size = 0.24

$$\frac{(1-iax)^{3/4}\left(\sqrt[4]{1+iax}\left(4ia^2x^2+5ax-i\right)-6i\sqrt[4]{2}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)\right)}{12a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])*x^2,x]
```

```
[Out] ((1 - I*a*x)^(3/4)*((1 + I*a*x)^(1/4)*(-I + 5*a*x + (4*I)*a^2*x^2) - (6*I)*
2^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, (1 - I*a*x)/2]))/(12*a^3)
```

Maple [F] time = 0.18, size = 0, normalized size = 0.

$$\int \sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Fricas [A] time = 1.84476, size = 645, normalized size = 1.9

$$-\frac{12\,a^3\sqrt{\frac{9i}{64\,a^6}}\log\left(\frac{8}{3}i\,a^3\sqrt{\frac{9i}{64\,a^6}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{9i}{64\,a^6}}\log\left(-\frac{8}{3}i\,a^3\sqrt{\frac{9i}{64\,a^6}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)+12\,a^3\sqrt{-\frac{9i}{64\,a^6}}\log\left(\frac{8}{3}i\,a^3\cdots\right.}{24\,a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*a^3*sqrt(9/64*I/a^6)*log(8/3*I*a^3*sqrt(9/64*I/a^6) + sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(9/64*I/a^6)*log(-8/3*I*a^3*sqrt(9/6
4*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*a^3*sqrt(-9/64*I/a^6)*
log(8/3*I*a^3*sqrt(-9/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12
*a^3*sqrt(-9/64*I/a^6)*log(-8/3*I*a^3*sqrt(-9/64*I/a^6) + sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I))) - (8*a^3*x^3 - 2*I*a^2*x^2 - a*x - 11*I)*sqrt(I*sqrt(a^
2*x^2 + 1)/(a*x + I)))/a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)*x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

3.62 $\int e^{\frac{1}{2}i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=295

$$\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}+\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}-\tan^{-1}\Big($$

```
[Out] ((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*a^2) + ((1 - I*a*x)^(3/4)*(1 + I*a
*x)^(5/4))/(2*a^2) - ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/
4)]/(4*Sqrt[2]*a^2) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1
/4)]/(4*Sqrt[2]*a^2) + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(
1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(8*Sqrt[2]*a^2) - Log[1 + Sqrt[1 - I*a
*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(8*Sqr
t[2]*a^2)
```

Rubi [A] time = 0.17861, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5062, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}+\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}-\tan^{-1}\Big($$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])*x,x]
```


```
[Out] ((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*a^2) + ((1 - I*a*x)^(3/4)*(1 + I*a
*x)^(5/4))/(2*a^2) - ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/
4)]/(4*Sqrt[2]*a^2) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1
/4)]/(4*Sqrt[2]*a^2) + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(
1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(8*Sqrt[2]*a^2) - Log[1 + Sqrt[1 - I*a
*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(8*Sqr
t[2]*a^2)
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{1}{2}i\tan^{-1}(ax)}x\,dx=\int\frac{x\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx$$
$$=\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{i\int\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx}{4a}$$
$$=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{i\int\frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx}{8a}$$
$$=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}+\frac{\operatorname{Subst}\left(\int\frac{x^2}{(2-x^4)^{3/4}}\,dx,x,\sqrt[4]{1-iax}\right)}{2a^2}$$
$$=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}+\frac{\operatorname{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a^2}$$
$$=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{\operatorname{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2}+\frac{\operatorname{Subst}\left(\int\frac{1+x^2}{1+x^4}\,d\right.}{4a^2}$$
$$=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}+\frac{\operatorname{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2}+\frac{\operatorname{Subst}\left(\int\frac{}{1+}\right.}{}$$
$$=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}+\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}-\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\right.}{8\sqrt{2}a^2}$$
$$=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+$$

Mathematica [C] time = 0.0206425, size = 63, normalized size = 0.21

$$\frac{(1-iax)^{3/4}\left(2\sqrt[4]{2}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)+3(1+iax)^{5/4}\right)}{6a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])*x,x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(3*(1 + I*a*x)^(5/4) + 2*2^(1/4)*Hypergeometric2F1[-1/4,
 3/4, 7/4, (1 - I*a*x)/2]))/(6*a^2)
```

Maple [F] time = 0.13, size = 0, normalized size = 0.

$$\int\sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Fricas [A] time = 1.68765, size = 597, normalized size = 2.02

$$-\frac{2\,a^2\sqrt{\frac{i}{16\,a^4}}\log\left(4\,a^2\sqrt{\frac{i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{i}{16\,a^4}}\log\left(-4\,a^2\sqrt{\frac{i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+2\,a^2\sqrt{-\frac{i}{16\,a^4}}\log\left(4\,a^2\sqrt{-\frac{i}{16\,a^4}}+\right.}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*sqrt(1/16*I/a^4)*log(4*a^2*sqrt(1/16*I/a^4) + sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I))) - 2*a^2*sqrt(1/16*I/a^4)*log(-4*a^2*sqrt(1/16*I/a^4) +
sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*a^2*sqrt(-1/16*I/a^4)*log(4*a^2*sq
rt(-1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(-1/16*I
/a^4)*log(-4*a^2*sqrt(-1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) -
 (2*a^2*x^2 - I*a*x + 3)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)*x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="giac")
```

```
[Out] integrate(x*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

3.63 $\int e^{\frac{1}{2}i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=268

$$\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{i\,\text{tar}}{}$$

```
[Out] (I*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a - (I*ArcTan[1 - (Sqrt[2]*(1 - I*a
*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*ArcTan[1 + (Sqrt[2]*(1 - I*
a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + ((I/2)*Log[1 + Sqrt[1 - I*a*x
]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2
]*a) - ((I/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x
)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rubi [A] time = 0.146986, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5061, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{i\,\text{tar}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] (I*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a - (I*ArcTan[1 - (Sqrt[2]*(1 - I*a
*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*ArcTan[1 + (Sqrt[2]*(1 - I*
a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + ((I/2)*Log[1 + Sqrt[1 - I*a*x
]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2
]*a) - ((I/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x
)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{2}i\tan^{-1}(ax)}dx &= \int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}dx\\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{1}{2}\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}dx\\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}dx, x, \sqrt[4]{1-iax}\right)}{a}\\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{x^2}{1+x^4}dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}\\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{i\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} + \frac{i\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}\\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} + \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} + \\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} - \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{i\,\text{Subst}\left(\int\right.}{}\\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{1}}\right.}{2\sqrt{2}a}\end{aligned}$$

Mathematica [C] time = 0.0233846, size = 41, normalized size = 0.15

$$-\frac{8ie^{\frac{5}{2}i\tan^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{5}{4},2,\frac{9}{4},-e^{2i\tan^{-1}(ax)}\right)}{5a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] (((-8*I)/5)*E^(((5*I)/2)*ArcTan[a*x])*Hypergeometric2F1[5/4, 2, 9/4, -E^((2
*I)*ArcTan[a*x])])/a
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Fricas [A] time = 1.699, size = 493, normalized size = 1.84

$$\frac{a\sqrt{\frac{i}{a^2}}\log\left(ia\sqrt{\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{\frac{i}{a^2}}\log\left(-ia\sqrt{\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+a\sqrt{-\frac{i}{a^2}}\log\left(ia\sqrt{-\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{-\frac{i}{a^2}}\log\left(\right.}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*sqrt(I/a^2)*log(I*a*sqrt(I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)
)) - a*sqrt(I/a^2)*log(-I*a*sqrt(I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) + a*sqrt(-I/a^2)*log(I*a*sqrt(-I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I))) - a*sqrt(-I/a^2)*log(-I*a*sqrt(-I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*
x + I))) + (2*a*x + 2*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

3.64 $\int \frac{e^{\frac{1}{2} i \tan^{-1}(ax)}}{x} dx$

Optimal. Leaf size=267

$$-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\sqrt{}$$

```
[Out] -2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2
]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 -
I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^
(1/4)] - Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/
4))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] +
(Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.173959, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5062, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])/x,x]
```

```
[Out] -2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2
]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 -
I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^
(1/4)] - Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/
4))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] +
(Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt[4]{1+iax}}{x\sqrt[4]{1-iax}}\,dx$$

$$= (ia)\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx + \int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= -\left(4\,\mathrm{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)\right) + 4\,\mathrm{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\left(2\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right) - 2\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 4\,\mathrm{Subst}\left(\int \frac{x^2}{1+\ldots}\right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 2\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\,\mathrm{Subst}\left(\int\ldots\right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{\mathrm{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\mathrm{Subst}\left(\int \frac{\ldots}{-1+\ldots}\right.}{\ldots}$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \ldots\right.}{\sqrt{2}}$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\tanh^{-1}\left(\ldots\right.$$

Mathematica [C] time = 0.0338281, size = 97, normalized size = 0.36

$$-\frac{2(1-iax)^{3/4}\left(\sqrt[4]{2}(1+iax)^{3/4}\mathrm{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)+2\mathrm{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{ax+i}{-ax+i}\right)\right)}{3(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x,x]
```

```
[Out] (-2*(1 - I*a*x)^(3/4)*(2^(1/4)*(1 + I*a*x)^(3/4)*Hypergeometric2F1[3/4, 3/4
, 7/4, (1 - I*a*x)/2] + 2*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x
```

```
)]))/(3*(1 + I*a*x)^(3/4))
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x, x)
```

Fricas [A] time = 1.70595, size = 660, normalized size = 2.47

$$\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 1/
2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 1/2
*sqrt(-4*I)*log(1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 1/2
*sqrt(-4*I)*log(-1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - lo
g(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - I*log(sqrt(I*sqrt(a^2*x^2 + 1)
/(a*x + I)) + I) + I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + log(sqr
t(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x, x)
```

3.65 $\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=92

$$-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}-ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -(((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x) - I*a*ArcTan[(1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)] - I*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rubi [A] time = 0.0338023, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5062, 94, 93, 212, 206, 203}

$$-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}-ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] -(((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x) - I*a*ArcTan[(1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)] - I*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{\sqrt[4]{1+iax}}{x^2\sqrt[4]{1-iax}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}+\frac{1}{2}(ia)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}+(2ia)\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}-(ia)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-(ia)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}-ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0149695, size = 71, normalized size = 0.77

$$-\frac{i(1-iax)^{3/4}\left(2ax\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{ax+i}{-ax+i}\right)+3ax-3i\right)}{3x(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x^2,x]

[Out] ((-I/3)*(1 - I*a*x)^(3/4)*(-3*I + 3*a*x + 2*a*x*Hypergeometric2F1[3/4, 1, 7
/4, (I + a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(3/4))
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)

[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^2, x)
```

Fricas [B] time = 1.63382, size = 363, normalized size = 3.95

$$\frac{-i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)-2\,(-i\,ax+1)\sqrt{}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + a*x*log(sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I)) + I) - a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I)) - I) + I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*(-I*a*x +
 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^2, x)
```

3.66 $\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=132

$$\frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2}-\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}$$

```
[Out] ((-I/4)*a*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - ((1 - I*a*x)^(3/4)*(1 +
I*a*x)^(5/4))/(2*x^2) + (a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
 + (a^2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rubi [A] time = 0.0426426, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5062, 96, 94, 93, 212, 206, 203}

$$\frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2}-\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] ((-I/4)*a*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - ((1 - I*a*x)^(3/4)*(1 +
I*a*x)^(5/4))/(2*x^2) + (a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
 + (a^2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{\sqrt[4]{1+iax}}{x^3\sqrt[4]{1-iax}}\,dx\\
&= -\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2}+\frac{1}{4}(ia)\int \frac{\sqrt[4]{1+iax}}{x^2\sqrt[4]{1-iax}}\,dx\\
&= -\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}-\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2}-\frac{1}{8}a^2\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx\\
&= -\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}-\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2}-\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\
&= -\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}-\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2}+\frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{1}{4}a^2\,\text{Su}\\
&= -\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}-\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2}+\frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0181263, size = 81, normalized size = 0.61

$$\frac{(1-iax)^{3/4}\left(2a^2x^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{ax+i}{-ax+i}\right)+9a^2x^2-15iax-6\right)}{12x^2(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(-6 - (15*I)*a*x + 9*a^2*x^2 + 2*a^2*x^2*Hypergeometric2
F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(12*x^2*(1 + I*a*x)^(3/4))
```

Maple [F] time = 0.141, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^3, x)
```

Fricas [A] time = 1.80993, size = 401, normalized size = 3.04

$$\frac{a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)-\left(6\,a^2\right.}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + I*a^2*x^2*log(s
qrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - I*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)) - I) - a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1
) - (6*a^2*x^2 + 2*I*a*x + 4)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^3, x)
```

3.67 $\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=170

$$\frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}+\frac{3}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{3}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}-\frac{(1-iax)^{3/4}}{3x}$$

```
[Out] -((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(3*x^3) - (((5*I)/12)*a*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(1/4))/x^2 + (11*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(24*x) + ((3*I)/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + ((3*I
)/8)*a^3*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rubi [A] time = 0.0628448, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5062, 99, 151, 12, 93, 212, 206, 203}

$$\frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}+\frac{3}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{3}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}-\frac{(1-iax)^{3/4}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])/x^4,x]
```



```
[Out] -((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(3*x^3) - (((5*I)/12)*a*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(1/4))/x^2 + (11*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(24*x) + ((3*I)/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + ((3*I
)/8)*a^3*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x^4}\,dx = \int \frac{\sqrt[4]{1+iax}}{x^4\sqrt[4]{1-iax}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} + \frac{1}{3}\int \frac{\frac{5ia}{2}-2a^2x}{x^3\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} - \frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} - \frac{1}{6}\int \frac{\frac{11a^2}{4}+\frac{5}{2}ia^3x}{x^2\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} - \frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} + \frac{1}{6}\int -\frac{9ia^3}{8x\sqrt[4]{1-iax}(1+}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} - \frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} - \frac{1}{16}\left(3ia^3\right)\int \frac{}{x\sqrt[4]{1-ia}}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} - \frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} - \frac{1}{4}\left(3ia^3\right)\mathrm{Subst}\left(\int \frac{}{-1}\right.$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} - \frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} + \frac{1}{8}\left(3ia^3\right)\mathrm{Subst}\left(\int \frac{}{1-}\right.$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} - \frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} + \frac{3}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right.$$

Mathematica [C] time = 0.0247147, size = 93, normalized size = 0.55

$$\frac{(1-iax)^{3/4}\left(6ia^3x^3\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{ax+i}{-ax+i}\right)+11ia^3x^3+21a^2x^2-18iax-8\right)}{24x^3(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(-8 - (18*I)*a*x + 21*a^2*x^2 + (11*I)*a^3*x^3 + (6*I)*a
^3*x^3*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1 + I
*a*x)^(3/4))
```

Maple [F] time = 0.14, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^4, x)
```

Fricas [A] time = 1.74123, size = 436, normalized size = 2.56

$$\frac{9i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-9\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+9\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-9i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(9*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 9*a^3*x^3*
log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 9*a^3*x^3*log(sqrt(I*sqrt(a^
2*x^2 + 1)/(a*x + I)) - I) - 9*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
+ I)) - 1) + (-22*I*a^3*x^3 + 2*a^2*x^2 - 4*I*a*x - 16)*sqrt(I*sqrt(a^2*x^2
```

```
+ 1)/(a*x + I)))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^4, x)
```

3.68 $\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=202

$$\frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2}+\frac{83ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{192x}-\frac{11}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{11}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{7ia(1-$$

```
[Out] -((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*x^4) - (((7*I)/24)*a*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(1/4))/x^3 + (29*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(96*x^2) + (((83*I)/192)*a^3*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - (11*
a^4*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (11*a^4*ArcTanh[(1 +
I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64
```

Rubi [A] time = 0.0790615, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5062, 99, 151, 12, 93, 212, 206, 203}

$$\frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2}+\frac{83ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{192x}-\frac{11}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{11}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{7ia(1-$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])/x^5,x]
```


```
[Out] -((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*x^4) - (((7*I)/24)*a*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(1/4))/x^3 + (29*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(96*x^2) + (((83*I)/192)*a^3*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - (11*
a^4*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (11*a^4*ArcTanh[(1 +
I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x^5}\,dx = \int \frac{\sqrt[4]{1+iax}}{x^5\sqrt[4]{1-iax}}\,dx$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{1}{4}\int \frac{\frac{7ia}{2}-3a^2x}{x^4\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3} - \frac{1}{12}\int \frac{\frac{29a^2}{4}+7ia^3x}{x^3\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3} + \frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2} + \frac{1}{24}\int \frac{-\frac{83ia^3}{8}+}{x^2\sqrt[4]{1-iax}(}$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3} + \frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2} + \frac{83ia^3(1-iax)^{3/4}\sqrt[4]{}}{192x}$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3} + \frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2} + \frac{83ia^3(1-iax)^{3/4}\sqrt[4]{}}{192x}$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3} + \frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2} + \frac{83ia^3(1-iax)^{3/4}\sqrt[4]{}}{192x}$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3} + \frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2} + \frac{83ia^3(1-iax)^{3/4}\sqrt[4]{}}{192x}$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3} + \frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2} + \frac{83ia^3(1-iax)^{3/4}\sqrt[4]{}}{192x}$$

Mathematica [C] time = 0.0278389, size = 99, normalized size = 0.49

$$-\frac{(1-iax)^{3/4}\left(22a^4x^4\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{ax+i}{-ax+i}\right)+83a^4x^4-141ia^3x^3-114a^2x^2+104iax+48\right)}{192x^4(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x^5,x]
```

```
[Out] -((1 - I*a*x)^(3/4)*(48 + (104*I)*a*x - 114*a^2*x^2 - (141*I)*a^3*x^3 + 83*
a^4*x^4 + 22*a^4*x^4*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/
(192*x^4*(1 + I*a*x)^(3/4))
```

Maple [F] time = 0.141, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^5, x)
```

Fricas [A] time = 1.79748, size = 463, normalized size = 2.29

$$-\frac{33\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+33i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-33i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-33\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-\right.}{384\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/384*(33*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 33*I*a^4*
x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 33*I*a^4*x^4*log(sqrt(I*
sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 33*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)
/(a*x + I)) - 1) - (166*a^4*x^4 + 50*I*a^3*x^3 + 4*a^2*x^2 - 16*I*a*x - 96)
*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^5, x)
```

3.69 $\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x^6}\,dx$

Optimal. Leaf size=240

$$\frac{269ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{960x^2}+\frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3}-\frac{611a^4(1-iax)^{3/4}\sqrt[4]{1+iax}}{1920x}-\frac{31}{128}ia^5\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{31}{128}$$

```
[Out] -((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(5*x^5) - (((9*I)/40)*a*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(1/4))/x^4 + (11*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(48*x^3) + (((269*I)/960)*a^3*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x^2 - (
611*a^4*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(1920*x) - ((31*I)/128)*a^5*Ar
cTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ((31*I)/128)*a^5*ArcTanh[(1 + I
*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rubi [A] time = 0.101977, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5062, 99, 151, 12, 93, 212, 206, 203}

$$\frac{269ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{960x^2}+\frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3}-\frac{611a^4(1-iax)^{3/4}\sqrt[4]{1+iax}}{1920x}-\frac{31}{128}ia^5\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{31}{128}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])/x^6,x]
```


```
[Out] -((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(5*x^5) - (((9*I)/40)*a*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(1/4))/x^4 + (11*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(48*x^3) + (((269*I)/960)*a^3*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x^2 - (
611*a^4*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(1920*x) - ((31*I)/128)*a^5*Ar
cTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ((31*I)/128)*a^5*ArcTanh[(1 + I
*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x^6}dx = \int \frac{\sqrt[4]{1+iax}}{x^6\sqrt[4]{1-iax}}dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} + \frac{1}{5}\int \frac{\frac{9ia}{2}-4a^2x}{x^5\sqrt[4]{1-iax}(1+iax)^{3/4}}dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} - \frac{1}{20}\int \frac{\frac{55a^2}{4}+\frac{27}{2}ia^3x}{x^4\sqrt[4]{1-iax}(1+iax)^{3/4}}dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{1}{60}\int \frac{-\frac{269ia^3}{8}+}{x^3\sqrt[4]{1-iax}(}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-iax)^{3/4}}{960x^2}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-iax)^{3/4}}{960x^2}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-iax)^{3/4}}{960x^2}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-iax)^{3/4}}{960x^2}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-iax)^{3/4}}{960x^2}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-iax)^{3/4}}{960x^2}$$

Mathematica [C] time = 0.0318243, size = 111, normalized size = 0.46

$$\frac{(1-iax)^{3/4}\left(-310ia^5x^5\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{ax+i}{-ax+i}\right)-611ia^5x^5-1149a^4x^4+978ia^3x^3+872a^2x^2-816iax-\right.}{1920x^5(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x^6,x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(-384 - (816*I)*a*x + 872*a^2*x^2 + (978*I)*a^3*x^3 - 11
49*a^4*x^4 - (611*I)*a^5*x^5 - (310*I)*a^5*x^5*Hypergeometric2F1[3/4, 1, 7/
4, (I + a*x)/(I - a*x)]))/(1920*x^5*(1 + I*a*x)^(3/4))
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \frac{1}{x^6}\sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^6, x)
```

Fricas [A] time = 1.74454, size = 497, normalized size = 2.07

$$\frac{-465i\,a^5x^5\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+465\,a^5x^5\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-465\,a^5x^5\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+465i\,a^5x^5\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right.}{3840\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/3840*(-465*I*a^5*x^5*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 465*a
^5*x^5*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 465*a^5*x^5*log(sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + 465*I*a^5*x^5*log(sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)) - 1) + (1222*I*a^5*x^5 - 146*a^4*x^4 + 196*I*a^3*x^3 + 16*
a^2*x^2 - 96*I*a*x - 768)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^6, x)
```

3.70 $\int e^{\frac{3}{2} i \tan^{-1}(ax)} x^3 \, dx$

Optimal. Leaf size=337

$$\frac{x^2 \sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4} - \frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{123\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 1\right)}{128\sqrt{2}a^4}$$

```
[Out] (-41*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(64*a^4) + (x^2*(1 - I*a*x)^(1/4)
*(1 + I*a*x)^(7/4))/(4*a^2) - ((1 - I*a*x)^(1/4)*(1 + I*a*x)^(7/4)*(11 + (4
*I)*a*x))/(32*a^4) + (123*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)])/(64*Sqrt[2]*a^4) - (123*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
 + I*a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (123*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 +
I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
- (123*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rubi [A] time = 0.214526, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5062, 100, 147, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{x^2 \sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4} - \frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{123\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 1\right)}{128\sqrt{2}a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])*x^3,x]
```


```
[Out] (-41*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(64*a^4) + (x^2*(1 - I*a*x)^(1/4)
*(1 + I*a*x)^(7/4))/(4*a^2) - ((1 - I*a*x)^(1/4)*(1 + I*a*x)^(7/4)*(11 + (4
*I)*a*x))/(32*a^4) + (123*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)])/(64*Sqrt[2]*a^4) - (123*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
 + I*a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (123*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 +
I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
- (123*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{3}{2}i\tan^{-1}(ax)}x^3\,dx = \int \frac{x^3(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx$$

$$= \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} + \frac{\int \frac{x(1+iax)^{3/4}\left(-2-\frac{3iax}{2}\right)}{(1-iax)^{3/4}}\,dx}{4a^2}$$

$$= \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4} + \frac{(41i)\int \frac{(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx}{64a^3}$$

$$= -\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4} + \frac{(123i)\int}{}$$

$$= -\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4} - \frac{123\,\mathrm{Subs}}{}$$

$$= -\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4} - \frac{123\,\mathrm{Subs}}{}$$

$$= -\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4} - \frac{123\,\mathrm{Subs}}{}$$

$$= -\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4} - \frac{123\,\mathrm{Subs}}{}$$

$$= -\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4} + \frac{123\log\Big(}{}$$

$$= -\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4} + \frac{123\tan^{-}}{}$$

Mathematica [C] time = 0.104815, size = 148, normalized size = 0.44

$$\frac{\sqrt[4]{1-iax}\left(-24\ 2^{3/4}\text{Hypergeometric2F1}\left(-\frac{11}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-iax)\right)+8\ 2^{3/4}\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-iax)\right)\right.}{4a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])*x^3,x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(a^2*x^2*(1 + I*a*x)^(3/4) + I*a^3*x^3*(1 + I*a*x)^(3/4)
 - 24*2^(3/4)*Hypergeometric2F1[-11/4, 1/4, 5/4, (1 - I*a*x)/2] + 8*2^(3/4)
*Hypergeometric2F1[-7/4, 1/4, 5/4, (1 - I*a*x)/2] + 2*2^(3/4)*Hypergeometri
c2F1[-3/4, 1/4, 5/4, (1 - I*a*x)/2]))/(4*a^4)
```

Maple [F] time = 0.175, size = 0, normalized size = 0.

$$\int \left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Fricas [A] time = 1.70543, size = 756, normalized size = 2.24

$$32\,a^4\sqrt{\frac{15129i}{4096\,a^8}}\log\left(\frac{64}{123}i\,a^4\sqrt{\frac{15129i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-32\,a^4\sqrt{\frac{15129i}{4096\,a^8}}\log\left(-\frac{64}{123}i\,a^4\sqrt{\frac{15129i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-32\,a^4\sqrt{-\frac{15129i}{4096\,a^8}}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/64*(32*a^4*sqrt(15129/4096*I/a^8)*log(64/123*I*a^4*sqrt(15129/4096*I/a^8)
 + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 32*a^4*sqrt(15129/4096*I/a^8)*log
(-64/123*I*a^4*sqrt(15129/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
) - 32*a^4*sqrt(-15129/4096*I/a^8)*log(64/123*I*a^4*sqrt(-15129/4096*I/a^8)
 + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 32*a^4*sqrt(-15129/4096*I/a^8)*lo
g(-64/123*I*a^4*sqrt(-15129/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) + (16*I*a^3*x^3 + 24*a^2*x^2 - 30*I*a*x - 63)*sqrt(a^2*x^2 + 1)*sqrt(I*
sqrt(a^2*x^2 + 1)/(a*x + I)))/a^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)*x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

3.71 $\int e^{\frac{3}{2}i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=339

$$\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}-\frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3}-\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3}+\frac{17i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3}-\frac{17i\log\left(\frac{\sqrt{1-}}{\sqrt{1+}}\right.}{}$$

```
[Out] (((-17*I)/24)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a^3 - ((I/4)*(1 - I*a*x)
^(1/4)*(1 + I*a*x)^(7/4))/a^3 + (x*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(7/4))/(3*
a^2) + (((17*I)/8)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)
])/(Sqrt[2]*a^3) - (((17*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (((17*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1
+ I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) -
(((17*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)
^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3)
```

Rubi [A] time = 0.209741, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5062, 90, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}-\frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3}-\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3}+\frac{17i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3}-\frac{17i\log\left(\frac{\sqrt{1-}}{\sqrt{1+}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])*x^2,x]
```


```
[Out] (((-17*I)/24)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a^3 - ((I/4)*(1 - I*a*x)
^(1/4)*(1 + I*a*x)^(7/4))/a^3 + (x*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(7/4))/(3*
a^2) + (((17*I)/8)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)
])/(Sqrt[2]*a^3) - (((17*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (((17*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1
+ I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) -
(((17*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)
^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3)
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
```

```
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{3}{2}i\tan^{-1}(ax)}x^2\,dx = \int \frac{x^2(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx$$

$$= \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} + \frac{\int \frac{(1+iax)^{3/4}\left(-1-\frac{3iax}{2}\right)}{(1-iax)^{3/4}}\,dx}{3a^2}$$

$$= -\frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} - \frac{17\int \frac{(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx}{24a^2}$$

$$= -\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} - \frac{17\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{16a^2}$$

$$= -\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} - \frac{(17i)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,d\right.}{4a^3}$$

$$= -\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} - \frac{(17i)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,\right.}{4a^3}$$

$$= -\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} - \frac{(17i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,\right.}{8a^3}$$

$$= -\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} - \frac{(17i)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x\ldots}\right.}{16a^3}$$

$$= -\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} + \frac{17i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\sqrt{\ldots}\right.}{16\sqrt{2}a^3}$$

$$= -\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} + \frac{17i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.}{8\sqrt{2}a^3}$$

Mathematica [C] time = 0.0377829, size = 82, normalized size = 0.24

$$\frac{\sqrt[4]{1-iax}\left((1+iax)^{3/4}\left(4ia^2x^2+7ax-3i\right)-34i2^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-iax)\right)\right)}{12a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])*x^2,x]
```

```
[Out] ((1 - I*a*x)^(1/4)*((1 + I*a*x)^(3/4)*(-3*I + 7*a*x + (4*I)*a^2*x^2) - (34*
I)*2^(3/4)*Hypergeometric2F1[-3/4, 1/4, 5/4, (1 - I*a*x)/2]))/(12*a^3)
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int \left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Fricas [A] time = 1.72108, size = 674, normalized size = 1.99

$$-\frac{12\,a^3\sqrt{\frac{289i}{64\,a^6}}\log\left(\frac{8}{17}\,a^3\sqrt{\frac{289i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{289i}{64\,a^6}}\log\left(-\frac{8}{17}\,a^3\sqrt{\frac{289i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{-\frac{289i}{64\,a^6}}\log\left(\frac{8}{17}\,a^3\cdots\right.}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*a^3*sqrt(289/64*I/a^6)*log(8/17*a^3*sqrt(289/64*I/a^6) + sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(289/64*I/a^6)*log(-8/17*a^3*sqrt
(289/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(-289/64
*I/a^6)*log(8/17*a^3*sqrt(-289/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I))) + 12*a^3*sqrt(-289/64*I/a^6)*log(-8/17*a^3*sqrt(-289/64*I/a^6) + sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I))) - sqrt(a^2*x^2 + 1)*(8*I*a^2*x^2 + 14*a*x -
 23*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)*x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

3.72 $\int e^{\frac{3}{2}i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=295

$$\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2}+\frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2}-\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}-\ 9\tan$$

```
[Out] (3*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(4*a^2) + ((1 - I*a*x)^(1/4)*(1 + I
*a*x)^(7/4))/(2*a^2) - (9*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)])/(4*Sqrt[2]*a^2) + (9*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)])/(4*Sqrt[2]*a^2) - (9*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x]
 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2) + (9*Log
[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a
*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rubi [A] time = 0.177046, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5062, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2}+\frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2}-\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}-\ 9\tan$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])*x,x]
```


```
[Out] (3*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(4*a^2) + ((1 - I*a*x)^(1/4)*(1 + I
*a*x)^(7/4))/(2*a^2) - (9*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)])/(4*Sqrt[2]*a^2) + (9*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)])/(4*Sqrt[2]*a^2) - (9*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x]
 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2) + (9*Log
[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a
*x)^(1/4)])/(8*Sqrt[2]*a^2)
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{3}{2}i\tan^{-1}(ax)}x\,dx = \int \frac{x(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx$$
$$= \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} - \frac{(3i)\int \frac{(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx}{4a}$$
$$= \frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} - \frac{(9i)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{8a}$$
$$= \frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} + \frac{9\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{2a^2}$$
$$= \frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} + \frac{9\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a^2}$$
$$= \frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} + \frac{9\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2} + \frac{9\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\right.}{4a}$$
$$= \frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} + \frac{9\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2} + \frac{9\,\mathrm{Subst}\left(\int\right.}{}$$
$$= \frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} - \frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} + \frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right.}{8\sqrt{2}}$$
$$= \frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} - \frac{9\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} + \frac{9\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}$$

Mathematica [C] time = 0.0163888, size = 61, normalized size = 0.21

$$\frac{\sqrt[4]{1-iax}\left(6\;2^{3/4}\mathrm{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-iax)\right)+(1+iax)^{7/4}\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])*x,x]
```

```
[Out] ((1 - I*a*x)^(1/4)*((1 + I*a*x)^(7/4) + 6*2^(3/4)*Hypergeometric2F1[-3/4, 1
/4, 5/4, (1 - I*a*x)/2]))/(2*a^2)
```

Maple [F] time = 0.141, size = 0, normalized size = 0.

$$\int \left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Fricas [A] time = 1.68012, size = 640, normalized size = 2.17

$$-\frac{2\,a^2\sqrt{\frac{81i}{16\,a^4}}\log\left(\frac{4}{9}i\,a^2\sqrt{\frac{81i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{81i}{16\,a^4}}\log\left(-\frac{4}{9}i\,a^2\sqrt{\frac{81i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{-\frac{81i}{16\,a^4}}\log\left(\frac{4}{9}i\,a^2\sqrt{-\frac{81i}{16\,a^4}}\right.}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*sqrt(81/16*I/a^4)*log(4/9*I*a^2*sqrt(81/16*I/a^4) + sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(81/16*I/a^4)*log(-4/9*I*a^2*sqrt(81/
16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(-81/16*I/a^4)
*log(4/9*I*a^2*sqrt(-81/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) +
2*a^2*sqrt(-81/16*I/a^4)*log(-4/9*I*a^2*sqrt(-81/16*I/a^4) + sqrt(I*sqrt(a^
2*x^2 + 1)/(a*x + I))) - sqrt(a^2*x^2 + 1)*(2*I*a*x + 5)*sqrt(I*sqrt(a^2*x^
2 + 1)/(a*x + I)))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)*x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="giac")
```

```
[Out] integrate(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

3.73 $\int e^{\frac{3}{2}i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=268

$$\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}-\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}+\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\cdots$$

```
[Out] (I*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a - ((3*I)*ArcTan[1 - (Sqrt[2]*(1 -
 I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + ((3*I)*ArcTan[1 + (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) - (((3*I)/2)*Log[1 + Sq
rt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/
4)])/(Sqrt[2]*a) + (((3*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sq
rt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rubi [A] time = 0.142655, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5061, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}-\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}+\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (I*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a - ((3*I)*ArcTan[1 - (Sqrt[2]*(1 -
 I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + ((3*I)*ArcTan[1 + (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) - (((3*I)/2)*Log[1 + Sq
rt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/
4)])/(Sqrt[2]*a) + (((3*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sq
rt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{3}{2}i\tan^{-1}(ax)}dx = \int \frac{(1+iax)^{3/4}}{(1-iax)^{3/4}}dx$$
$$= \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{3}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}dx$$
$$= \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{(6i)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}dx, x, \sqrt[4]{1-iax}\right)}{a}$$
$$= \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{(6i)\,\text{Subst}\left(\int \frac{1}{1+x^4}dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$
$$= \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{(3i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} + \frac{(3i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$
$$= \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{(3i)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} + \frac{(3i)\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}dx, x, \frac{\sqrt[4]{1-i}}{\sqrt[4]{1+i}}\right.}{2a}$$
$$= \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{(3i)\,\text{Sul}}{}$$
$$= \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\right.}{2\sqrt{2}a}$$

Mathematica [C] time = 0.0266814, size = 41, normalized size = 0.15

$$-\frac{8ie^{\frac{7}{2}i\tan^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{7}{4},2,\frac{11}{4},-e^{2i\tan^{-1}(ax)}\right)}{7a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (((-8*I)/7)*E^(((7*I)/2)*ArcTan[a*x])*Hypergeometric2F1[7/4, 2, 11/4, -E^((
2*I)*ArcTan[a*x])])/a
```

Maple [F] time = 0.127, size = 0, normalized size = 0.

$$\int \left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Fricas [A] time = 1.54523, size = 536, normalized size = 2.

$$\frac{a\sqrt{\frac{9i}{a^2}}\log\left(\frac{1}{3}\,a\sqrt{\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{\frac{9i}{a^2}}\log\left(-\frac{1}{3}\,a\sqrt{\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{-\frac{9i}{a^2}}\log\left(\frac{1}{3}\,a\sqrt{-\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+a\sqrt{-\frac{9i}{a^2}}\,\mathrm{lo}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*sqrt(9*I/a^2)*log(1/3*a*sqrt(9*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*
x + I))) - a*sqrt(9*I/a^2)*log(-1/3*a*sqrt(9*I/a^2) + sqrt(I*sqrt(a^2*x^2 +
 1)/(a*x + I))) - a*sqrt(-9*I/a^2)*log(1/3*a*sqrt(-9*I/a^2) + sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I))) + a*sqrt(-9*I/a^2)*log(-1/3*a*sqrt(-9*I/a^2) + sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*I*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^
2 + 1)/(a*x + I)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

3.74 $\int \frac{e^{\frac{3}{2} i \tan^{-1}(ax)}}{x} dx$

Optimal. Leaf size=267

$$\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\sqrt{2}$$

```
[Out] 2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - I
*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(
1/4)] + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4
))/(1 + I*a*x)^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (
Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.166098, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5062, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\sqrt{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])/x,x]
```

```
[Out] 2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - I
*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(
1/4)] + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4
))/(1 + I*a*x)^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (
Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x}\,dx = \int \frac{(1+iax)^{3/4}}{x(1-iax)^{3/4}}\,dx$$

$$= (ia)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx + \int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\left(4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)\right) + 4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 4\,\text{Subst}\left(\int \frac{1}{1+\cdots}\right.$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\,\text{Subst}\left(\int \cdots\right.$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\cdots}{-1+\cdots}\right.}{\cdots}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\cdots}{\cdots}\right.}{\sqrt{2}}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1\cdots}}{\sqrt[4]{1\cdots}}\right.$$

Mathematica [C] time = 0.0300315, size = 96, normalized size = 0.36

$$-2\,2^{3/4}\sqrt[4]{1-iax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-iax)\right) - \frac{4\sqrt[4]{1-iax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},-\frac{1-iax}{-1-iax}\right)}{\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])/x,x]
```

```
[Out] -2*2^(3/4)*(1 - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (1 - I*a*x)/2
] - (4*(1 - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1, 5/4, -((1 - I*a*x)/(-1 -
 I*a*x))])/(1 + I*a*x)^(1/4)
```

Maple [F] time = 0.139, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x, x)
```

Fricas [A] time = 1.56671, size = 671, normalized size = 2.51

$$\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) -
1/2*sqrt(4*I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) -
 1/2*sqrt(-4*I)*log(1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
 + 1/2*sqrt(-4*I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) - log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + I*log(sqrt(I*sqrt(a^2*
x^2 + 1)/(a*x + I)) + I) - I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) +
 log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x, x)
```

3.75 $\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=92

$$-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}+3ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-3ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -(((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x) + (3*I)*a*ArcTan[(1 + I*a*x)^(1/
4)/(1 - I*a*x)^(1/4)] - (3*I)*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)
]
```

Rubi [A] time = 0.0318131, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5062, 94, 93, 298, 203, 206}

$$-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}+3ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-3ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] -(((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x) + (3*I)*a*ArcTan[(1 + I*a*x)^(1/
4)/(1 - I*a*x)^(1/4)] - (3*I)*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)
]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1+iax)^{3/4}}{x^2(1-iax)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} + \frac{1}{2}(3ia)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx \\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} + (6ia)\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} - (3ia)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + (3ia)\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cdots\right) \\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} + 3ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 3ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0142082, size = 68, normalized size = 0.74

$$
-\frac{i\sqrt[4]{1-iax}\left(6ax\,\mathrm{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{ax+i}{-ax+i}\right)+ax-i\right)}{x\sqrt[4]{1+iax}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] ((-I)*(1 - I*a*x)^(1/4)*(-I + a*x + 6*a*x*Hypergeometric2F1[1/4, 1, 5/4, (I
 + a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(1/4))
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$
\int \frac{1}{x^2}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^2, x)
```

Fricas [B] time = 1.64969, size = 381, normalized size = 4.14

$$\frac{-3i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-3\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+3\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+3i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)-2\sqrt{a^2x^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(-3*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 3*a*x*log(sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 3*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I)) - I) + 3*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*s
qrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^2, x)
```

3.76 $\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=132

$$-\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2}-\frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x}$$

```
[Out] (((-3*I)/4)*a*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x - ((1 - I*a*x)^(1/4)*(
1 + I*a*x)^(7/4))/(2*x^2) - (9*a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/
4)])/4 + (9*a^2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rubi [A] time = 0.0423317, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5062, 96, 94, 93, 298, 203, 206}

$$-\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2}-\frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] (((-3*I)/4)*a*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x - ((1 - I*a*x)^(1/4)*(
1 + I*a*x)^(7/4))/(2*x^2) - (9*a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/
4)])/4 + (9*a^2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{3}{2} i \tan^{-1}(ax)}}{x^3} \, dx &= \int \frac{(1+iax)^{3/4}}{x^3(1-iax)^{3/4}} \, dx \\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2} + \frac{1}{4}(3ia) \int \frac{(1+iax)^{3/4}}{x^2(1-iax)^{3/4}} \, dx \\
&= -\frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2} - \frac{1}{8}\left(9a^2\right) \int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}} \, dx \\
&= -\frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2} - \frac{1}{2}\left(9a^2\right) \text{Subst}\left(\int \frac{x^2}{-1+x^4} \, dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2} + \frac{1}{4}\left(9a^2\right) \text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{1}{4}\left(9a\right. \\
&= -\frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2} - \frac{9}{4}a^2 \tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{9}{4}a^2 \tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0172755, size = 81, normalized size = 0.61

$$\frac{\sqrt[4]{1-iax}\left(18a^2x^2\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{ax+i}{-ax+i}\right)+5a^2x^2-7iax-2\right)}{4x^2\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-2 - (7*I)*a*x + 5*a^2*x^2 + 18*a^2*x^2*Hypergeometric2
F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(4*x^2*(1 + I*a*x)^(1/4))
```

Maple [F] time = 0.146, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^3, x)
```

Fricas [A] time = 1.71975, size = 423, normalized size = 3.2

$$\frac{9\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-9i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+9i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-9\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(9*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 9*I*a^2*x^2*l
og(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 9*I*a^2*x^2*log(sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I)) - I) - 9*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I)) - 1) - 2*sqrt(a^2*x^2 + 1)*(5*I*a*x + 2)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
 + I)))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^3, x)
```

3.77 $\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=170

$$\frac{23a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x}-\frac{17}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{17}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2}-\frac{\sqrt[4]{1-iax}}{}$$

```
[Out] -((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(3*x^3) - (((7*I)/12)*a*(1 - I*a*x)^
(1/4)*(1 + I*a*x)^(3/4))/x^2 + (23*a^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
/(24*x) - ((17*I)/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + ((17
*I)/8)*a^3*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rubi [A] time = 0.0632094, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5062, 99, 151, 12, 93, 298, 203, 206}

$$\frac{23a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x}-\frac{17}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{17}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2}-\frac{\sqrt[4]{1-iax}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] -((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(3*x^3) - (((7*I)/12)*a*(1 - I*a*x)^
(1/4)*(1 + I*a*x)^(3/4))/x^2 + (23*a^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
/(24*x) - ((17*I)/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + ((17
*I)/8)*a^3*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x^4}\,dx = \int \frac{(1+iax)^{3/4}}{x^4(1-iax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} + \frac{1}{3}\int \frac{\frac{7ia}{2}-2a^2x}{x^3(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} - \frac{1}{6}\int \frac{\frac{23a^2}{4}+\frac{7}{2}ia^3x}{x^2(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{23a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} + \frac{1}{6}\int -\frac{51ia^3}{8x(1-iax)^{3/4}\sqrt[4]{\ldots}}$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{23a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} - \frac{1}{16}\left(17ia^3\right)\int \frac{\ldots}{x(1-ia\ldots}$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{23a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} - \frac{1}{4}\left(17ia^3\right)\text{Subst}\left(\int -\ldots\right.$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{23a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} + \frac{1}{8}\left(17ia^3\right)\text{Subst}\left(\int \frac{\ldots}{1\ldots}\right.$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{23a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} - \frac{17}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+ia\ldots}}{\sqrt[4]{1-ia\ldots}}\right.$$

Mathematica [C] time = 0.0226513, size = 93, normalized size = 0.55

$$\frac{\sqrt[4]{1-iax}\left(102ia^3x^3\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{ax+i}{-ax+i}\right)+23ia^3x^3+37a^2x^2-22iax-8\right)}{24x^3\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-8 - (22*I)*a*x + 37*a^2*x^2 + (23*I)*a^3*x^3 + (102*I)
*a^3*x^3*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1 +
 I*a*x)^(1/4))
```

Maple [F] time = 0.139, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^4, x)
```

Fricas [A] time = 1.73878, size = 447, normalized size = 2.63

$$\frac{51i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+51\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-51\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-51i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right.}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(51*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 51*a^3*x^
3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 51*a^3*x^3*log(sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I)) - I) - 51*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I)) - 1) + (46*a^2*x^2 - 28*I*a*x - 16)*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt
```

```
(a^2*x^2 + 1)/(a*x + I)))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^4, x)
```

3.78 $\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=202

$$\frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2}+\frac{63ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{64x}+\frac{123}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{123}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{3ia\sqrt[4]{}}{}$$

```
[Out] -((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(4*x^4) - (((3*I)/8)*a*(1 - I*a*x)^(
1/4)*(1 + I*a*x)^(3/4))/x^3 + (15*a^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/
(32*x^2) + (((63*I)/64)*a^3*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x + (123*a
^4*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (123*a^4*ArcTanh[(1 +
I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64
```

Rubi [A] time = 0.0805202, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5062, 99, 151, 12, 93, 298, 203, 206}

$$\frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2}+\frac{63ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{64x}+\frac{123}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{123}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{3ia\sqrt[4]{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])/x^5,x]
```


```
[Out] -((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(4*x^4) - (((3*I)/8)*a*(1 - I*a*x)^(
1/4)*(1 + I*a*x)^(3/4))/x^3 + (15*a^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/
(32*x^2) + (((63*I)/64)*a^3*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x + (123*a
^4*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (123*a^4*ArcTanh[(1 +
I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x^5}\,dx = \int \frac{(1+iax)^{3/4}}{x^5(1-iax)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{1}{4}\int \frac{\frac{9ia}{2}-3a^2x}{x^4(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$
$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3} - \frac{1}{12}\int \frac{\frac{45a^2}{4}+9ia^3x}{x^3(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$
$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2} + \frac{1}{24}\int \frac{-\frac{189ia^3}{8}+}{x^2(1-iax)^{3/}}$$
$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-iax}(1+}{64x}$$
$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-iax}(1+}{64x}$$
$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-iax}(1+}{64x}$$
$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-iax}(1+}{64x}$$
$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-iax}(1+}{64x}$$

Mathematica [C] time = 0.028247, size = 99, normalized size = 0.49

$$-\frac{\sqrt[4]{1-iax}\left(246a^4x^4\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{ax+i}{-ax+i}\right)+63a^4x^4-93ia^3x^3-54a^2x^2+40iax+16\right)}{64x^4\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])/x^5,x]
```

```
[Out] -((1 - I*a*x)^(1/4)*(16 + (40*I)*a*x - 54*a^2*x^2 - (93*I)*a^3*x^3 + 63*a^4
*x^4 + 246*a^4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(6
4*x^4*(1 + I*a*x)^(1/4))
```

Maple [F] time = 0.141, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^5, x)
```

Fricas [A] time = 1.84434, size = 477, normalized size = 2.36

$$-\frac{123\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-123i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+123i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-123\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right.}{128\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/128*(123*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 123*I*a^
4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 123*I*a^4*x^4*log(sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 123*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2
+ 1)/(a*x + I)) - 1) - (126*I*a^3*x^3 + 60*a^2*x^2 - 48*I*a*x - 32)*sqrt(a^
2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^5, x)
```

3.79 $\int e^{\frac{5}{2}i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=373

$$-\frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2}-\frac{i(-452ax+521i)(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4}+\frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\right.}{128\sqrt{}}$$

```
[Out] (475*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(64*a^4) - ((4*I)*x^3*(1 + I*a*x)
^(5/4))/(a*(1 - I*a*x)^(1/4)) - (17*x^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(5/4)
)/(4*a^2) - ((I/96)*(521*I - 452*a*x)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(5/4))/
a^4 - (475*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(64*S
qrt[2]*a^4) + (475*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)
])/(64*Sqrt[2]*a^4) + (475*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[
2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4) - (475*Log[1 +
Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)])/(128*Sqrt[2]*a^4)
```

Rubi [A] time = 0.255284, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5062, 97, 153, 147, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2}-\frac{i(-452ax+521i)(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4}+\frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\right.}{128\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])*x^3,x]
```

```
[Out] (475*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(64*a^4) - ((4*I)*x^3*(1 + I*a*x)
^(5/4))/(a*(1 - I*a*x)^(1/4)) - (17*x^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(5/4)
)/(4*a^2) - ((I/96)*(521*I - 452*a*x)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(5/4))/
a^4 - (475*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(64*S
qrt[2]*a^4) + (475*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)
])/(64*Sqrt[2]*a^4) + (475*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[
2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4) - (475*Log[1 +
Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)])/(128*Sqrt[2]*a^4)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{5}{2}i\tan^{-1}(ax)}x^3\,dx = \int \frac{x^3(1+iax)^{5/4}}{(1-iax)^{5/4}}\,dx$$

$$= -\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} + \frac{(4i)\int \frac{x^2\sqrt[4]{1+iax}\left(3+\frac{17iax}{4}\right)}{\sqrt[4]{1-iax}}\,dx}{a}$$

$$= -\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} + \frac{i\int \frac{x\sqrt[4]{1+iax}\left(-\frac{17ia}{2}+\frac{113a^2x}{8}\right)}{\sqrt[4]{1-iax}}\,dx}{a^3}$$

$$= -\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4} - \frac{(475i)\int}{6}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)}{96a^4}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)}{96a^4}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)}{96a^4}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)}{96a^4}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)}{96a^4}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)}{96a^4}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)}{96a^4}$$

Mathematica [C] time = 0.0427348, size = 96, normalized size = 0.26

$$\frac{380\sqrt[4]{2}(1-iax)\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right) - \sqrt[4]{1+iax}(ax-i)^2\left(6a^2x^2-5iax+59\right)}{24a^4\sqrt[4]{1-iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])*x^3,x]
```

```
[Out] (-((1 + I*a*x)^(1/4)*(-I + a*x)^2*(59 - (5*I)*a*x + 6*a^2*x^2)) + 380*2^(1/
4)*(1 - I*a*x)*Hypergeometric2F1[-5/4, 3/4, 7/4, (1 - I*a*x)/2])/(24*a^4*(1
 - I*a*x)^(1/4))
```

Maple [F] time = 0.189, size = 0, normalized size = 0.

$$\int \left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Fricas [A] time = 1.7259, size = 759, normalized size = 2.03

$$-\frac{96\,a^4\sqrt{\frac{225625i}{4096\,a^8}}\log\left(\frac{64}{475}\,a^4\sqrt{\frac{225625i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-96\,a^4\sqrt{\frac{225625i}{4096\,a^8}}\log\left(-\frac{64}{475}\,a^4\sqrt{\frac{225625i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+96\,a^4\sqrt{-\frac{2256}{4096}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/192*(96*a^4*sqrt(225625/4096*I/a^8)*log(64/475*a^4*sqrt(225625/4096*I/a^
8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*a^4*sqrt(225625/4096*I/a^8)*
log(-64/475*a^4*sqrt(225625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) + 96*a^4*sqrt(-225625/4096*I/a^8)*log(64/475*a^4*sqrt(-225625/4096*I/a^
8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*a^4*sqrt(-225625/4096*I/a^8)
*log(-64/475*a^4*sqrt(-225625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I))) + (48*a^4*x^4 - 136*I*a^3*x^3 - 226*a^2*x^2 + 521*I*a*x - 2467)*sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)*x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

3.80 $\int e^{\frac{5}{2}i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=371

$$\frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3}+\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}+\frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3}+\frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3}+\frac{55i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\sqrt{2}\right.}{16\sqrt{2}a^3}$$

```
[Out] (((55*I)/8)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a^3 + (((11*I)/4)*(1 - I*a
*x)^(3/4)*(1 + I*a*x)^(5/4))/a^3 + ((2*I)*(1 + I*a*x)^(9/4))/(a^3*(1 - I*a*
x)^(1/4)) + ((I/3)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(9/4))/a^3 - (((55*I)/8)*A
rcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (
((55*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt
[2]*a^3) + (((55*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*
(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) - (((55*I)/16)*Log[1 +
 Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^
(1/4)])/(Sqrt[2]*a^3)
```

Rubi [A] time = 0.244422, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5062, 89, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3}+\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}+\frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3}+\frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3}+\frac{55i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\sqrt{2}\right.}{16\sqrt{2}a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])*x^2,x]
```


```
[Out] (((55*I)/8)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a^3 + (((11*I)/4)*(1 - I*a
*x)^(3/4)*(1 + I*a*x)^(5/4))/a^3 + ((2*I)*(1 + I*a*x)^(9/4))/(a^3*(1 - I*a*
x)^(1/4)) + ((I/3)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(9/4))/a^3 - (((55*I)/8)*A
rcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (
((55*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt
[2]*a^3) + (((55*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*
(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) - (((55*I)/16)*Log[1 +
 Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^
(1/4)])/(Sqrt[2]*a^3)
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
```

```
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{5}{2}i\tan^{-1}(ax)}x^2\,dx = \int \frac{x^2(1+iax)^{5/4}}{(1-iax)^{5/4}}\,dx$$

$$= \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} - \frac{(2i)\int \frac{(1+iax)^{5/4}\left(\frac{5ia}{2}-\frac{a^2x}{2}\right)}{\sqrt[4]{1-iax}}\,dx}{a^3}$$

$$= \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} + \frac{11\int \frac{(1+iax)^{5/4}}{\sqrt[4]{1-iax}}\,dx}{2a^2}$$

$$= \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} + \frac{55\int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx}{8a^2}$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} +$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} +$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} +$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} -$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} +$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} +$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} -$$

Mathematica [C] time = 0.0353617, size = 86, normalized size = 0.23

$$\frac{44\sqrt[4]{2}(ax+i)\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)-\sqrt[4]{1+iax}(ax-i)^2(ax+7i)}{3a^3\sqrt[4]{1-iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])*x^2,x]
```

```
[Out] (-((1 + I*a*x)^(1/4)*(-I + a*x)^2*(7*I + a*x)) + 44*2^(1/4)*(I + a*x)*Hyper
geometric2F1[-5/4, 3/4, 7/4, (1 - I*a*x)/2])/(3*a^3*(1 - I*a*x)^(1/4))
```

Maple [F] time = 0.153, size = 0, normalized size = 0.

$$\int \left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Fricas [A] time = 1.78517, size = 689, normalized size = 1.86

$$\frac{12\,a^3\sqrt{\frac{3025i}{64\,a^6}}\log\left(\frac{8}{55}i\,a^3\sqrt{\frac{3025i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{3025i}{64\,a^6}}\log\left(-\frac{8}{55}i\,a^3\sqrt{\frac{3025i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+12\,a^3\sqrt{-\frac{3025i}{64\,a^6}}\log\left(\frac{8}{55}i\,a^3\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*a^3*sqrt(3025/64*I/a^6)*log(8/55*I*a^3*sqrt(3025/64*I/a^6) + sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(3025/64*I/a^6)*log(-8/55*I*a^
3*sqrt(3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*a^3*sqrt(
-3025/64*I/a^6)*log(8/55*I*a^3*sqrt(-3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 +
 1)/(a*x + I))) - 12*a^3*sqrt(-3025/64*I/a^6)*log(-8/55*I*a^3*sqrt(-3025/64
```

```
*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (8*a^3*x^3 - 26*I*a^2*x^2
- 61*a*x - 287*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)*x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{i\,ax + 1}{\sqrt{a^2x^2 + 1}}\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

3.81 $\int e^{\frac{5}{2} i \tan^{-1}(ax)} x \, dx$

Optimal. Leaf size=324

$$-\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}-\frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}-\frac{25\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{25\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\cdots\right.}{8\sqrt{2}a^2\cdots}$$

```
[Out] (-25*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*a^2) - (5*(1 - I*a*x)^(3/4)*(1
 + I*a*x)^(5/4))/(2*a^2) - (2*(1 + I*a*x)^(9/4))/(a^2*(1 - I*a*x)^(1/4)) +
(25*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(4*Sqrt[2]*a
^2) - (25*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(4*Sqr
t[2]*a^2) - (25*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a
*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2) + (25*Log[1 + Sqrt[1 - I*a*x
]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt
[2]*a^2)
```

Rubi [A] time = 0.209591, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5062, 78, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}-\frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}-\frac{25\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{25\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\cdots\right.}{8\sqrt{2}a^2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])*x,x]
```

```
[Out] (-25*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*a^2) - (5*(1 - I*a*x)^(3/4)*(1
 + I*a*x)^(5/4))/(2*a^2) - (2*(1 + I*a*x)^(9/4))/(a^2*(1 - I*a*x)^(1/4)) +
(25*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(4*Sqrt[2]*a
^2) - (25*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(4*Sqr
t[2]*a^2) - (25*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a
*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2) + (25*Log[1 + Sqrt[1 - I*a*x
]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt
[2]*a^2)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{5}{2}i\tan^{-1}(ax)}x\,dx = \int \frac{x(1+iax)^{5/4}}{(1-iax)^{5/4}}\,dx$$

$$= -\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} + \frac{(5i)\int \frac{(1+iax)^{5/4}}{\sqrt[4]{1-iax}}\,dx}{a}$$

$$= -\frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} + \frac{(25i)\int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx}{4a}$$

$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} + \frac{(25i)\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx}{8a}$$

$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} - \frac{25\,\mathrm{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1}\right.}{2a^2}$$

$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} - \frac{25\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a^2}$$

$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} + \frac{25\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2}$$

$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} - \frac{25\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.}{8a^2}$$

$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} - \frac{25\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}$$

$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} + \frac{25\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} - 25\,\mathrm{t}$$

Mathematica [C] time = 0.0343106, size = 72, normalized size = 0.22

$$\frac{2\left(20i\sqrt[4]{2}(ax+i)\mathrm{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)-3(1+iax)^{9/4}\right)}{3a^2\sqrt[4]{1-iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])*x,x]
```

```
[Out] (2*(-3*(1 + I*a*x)^(9/4) + (20*I)*2^(1/4)*(I + a*x)*Hypergeometric2F1[-5/4,
 3/4, 7/4, (1 - I*a*x)/2]))/(3*a^2*(1 - I*a*x)^(1/4))
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int \left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Fricas [A] time = 1.7549, size = 637, normalized size = 1.97

$$\frac{2\,a^2\sqrt{\frac{625i}{16\,a^4}}\log\left(\frac{4}{25}\,a^2\sqrt{\frac{625i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{625i}{16\,a^4}}\log\left(-\frac{4}{25}\,a^2\sqrt{\frac{625i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+2\,a^2\sqrt{-\frac{625i}{16\,a^4}}\log\left(\frac{4}{25}\,a^2\sqrt{-\frac{6}{1}}\right.}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*sqrt(625/16*I/a^4)*log(4/25*a^2*sqrt(625/16*I/a^4) + sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(625/16*I/a^4)*log(-4/25*a^2*sqrt(625
/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*a^2*sqrt(-625/16*I/a^
4)*log(4/25*a^2*sqrt(-625/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
- 2*a^2*sqrt(-625/16*I/a^4)*log(-4/25*a^2*sqrt(-625/16*I/a^4) + sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I))) - (2*a^2*x^2 - 9*I*a*x + 43)*sqrt(I*sqrt(a^2*x^2
+ 1)/(a*x + I)))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)*x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="giac")
```

```
[Out] integrate(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

3.82 $\int e^{\frac{5}{2} i \tan^{-1}(ax)}\, dx$

Optimal. Leaf size=299

$$-\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}-\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}-\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}+\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}+\frac{5i\tan^{-1}}{}$$

```
[Out] ((-5*I)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a - ((4*I)*(1 + I*a*x)^(5/4))/
(a*(1 - I*a*x)^(1/4)) + ((5*I)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)])/(Sqrt[2]*a) - ((5*I)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/
(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) - (((5*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1
 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (
((5*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1
/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rubi [A] time = 0.178278, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5061, 47, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}-\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}-\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}+\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}+\frac{5i\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((-5*I)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a - ((4*I)*(1 + I*a*x)^(5/4))/
(a*(1 - I*a*x)^(1/4)) + ((5*I)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)])/(Sqrt[2]*a) - ((5*I)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/
(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) - (((5*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1
 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (
((5*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1
/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{5}{2}i\tan^{-1}(ax)}\,dx = \int \frac{(1+iax)^{5/4}}{(1-iax)^{5/4}}\,dx$$
$$= -\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - 5\int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx$$
$$= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{5}{2}\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$
$$= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{(10i)\,\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)}{a}$$
$$= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{(10i)\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$
$$= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} + \frac{(5i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} - \frac{(5i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\right.}{a}$$
$$= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{(5i)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} - \frac{(5i)\,\text{Subst}\left(\int \right.}{}$$
$$= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \right.}{2\sqrt{2}a}$$
$$= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} + \frac{5i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \frac{5i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \ldots$$

Mathematica [C] time = 0.0379242, size = 41, normalized size = 0.14

$$-\frac{8ie^{\frac{9}{2}i\tan^{-1}(ax)}\text{Hypergeometric2F1}\left(2,\frac{9}{4},\frac{13}{4},-e^{2i\tan^{-1}(ax)}\right)}{9a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (((-8*I)/9)*E^(((9*I)/2)*ArcTan[a*x])*Hypergeometric2F1[2, 9/4, 13/4, -E^((
2*I)*ArcTan[a*x])])/a
```

Maple [F] time = 0.13, size = 0, normalized size = 0.

$$\int \left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Fricas [A] time = 1.64298, size = 549, normalized size = 1.84

$$-\frac{a\sqrt{\frac{25i}{a^2}}\log\left(\frac{1}{5}ia\sqrt{\frac{25i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{\frac{25i}{a^2}}\log\left(-\frac{1}{5}ia\sqrt{\frac{25i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+a\sqrt{-\frac{25i}{a^2}}\log\left(\frac{1}{5}ia\sqrt{-\frac{25i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2*(a*sqrt(25*I/a^2)*log(1/5*I*a*sqrt(25*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1
)/(a*x + I))) - a*sqrt(25*I/a^2)*log(-1/5*I*a*sqrt(25*I/a^2) + sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I))) + a*sqrt(-25*I/a^2)*log(1/5*I*a*sqrt(-25*I/a^2) +
sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - a*sqrt(-25*I/a^2)*log(-1/5*I*a*sqrt(
-25*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (2*a*x + 18*I)*sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

3.83 $\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=293

$$\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{1\cdots}}\right.$$

```
[Out] (8*(1 + I*a*x)^(1/4))/(1 - I*a*x)^(1/4) - 2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I
*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] -
 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + Log[1 + Sqrt[1 - I*a*x]/S
qrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] - L
og[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.218881, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5062, 98, 21, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{1\cdots}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])/x,x]
```

```
[Out] (8*(1 + I*a*x)^(1/4))/(1 - I*a*x)^(1/4) - 2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I
*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] -
 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + Log[1 + Sqrt[1 - I*a*x]/S
qrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] - L
og[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)]/Sqrt[2]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x}\,dx = \int \frac{(1+iax)^{5/4}}{x(1-iax)^{5/4}}\,dx$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} + \frac{(4i)\int \frac{-\frac{ia}{4}-\frac{a^2x}{4}}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx}{a}$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} + \int \frac{(1-iax)^{3/4}}{x(1+iax)^{3/4}}\,dx$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - (ia)\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx + \int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} + 4\,\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right) + 4\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 4\,\text{Subst}\Bigg(\int$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + 2\,\text{S}$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\text{Sub}}{}$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\log\Big(1+\sqrt{}}{}$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\,\text{ta}$$

Mathematica [C] time = 0.036743, size = 112, normalized size = 0.38

$$\frac{4\left(3\sqrt[4]{2}(1+iax)^{3/4}\text{Hypergeometric2F1}\left(-\frac{1}{4},-\frac{1}{4},\frac{3}{4},\frac{1}{2}(1-iax)\right)+(-1+iax)\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{ax+i}{-ax+i}\right)+3ia\right.}{3\sqrt[4]{1-iax}(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])/x,x]
```

```
[Out] (4*(3 + (3*I)*a*x + 3*2^(1/4)*(1 + I*a*x)^(3/4)*Hypergeometric2F1[-1/4, -1/
4, 3/4, (1 - I*a*x)/2] + (-1 + I*a*x)*Hypergeometric2F1[3/4, 1, 7/4, (I + a
*x)/(I - a*x)]))/(3*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x, x)
```

Fricas [A] time = 1.69264, size = 716, normalized size = 2.44

$$-\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 1
/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 1/
2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 1/
2*sqrt(-4*I)*log(-1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 8
*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I)) + 1) - I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + I*log(sqrt(I*sq
rt(a^2*x^2 + 1)/(a*x + I)) - I) + log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) -
 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x, x)
```

3.84 $\int \frac{e^{\frac{5}{2} i \tan^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=121

$$-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}+\frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-5ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-5ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] ((10*I)*a*(1 + I*a*x)^(1/4))/(1 - I*a*x)^(1/4) - (1 + I*a*x)^(5/4)/(x*(1 -
I*a*x)^(1/4)) - (5*I)*a*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - (5*I)
*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rubi [A] time = 0.0381298, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5062, 94, 93, 212, 206, 203}

$$-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}+\frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-5ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-5ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])/x^2,x]
```


```
[Out] ((10*I)*a*(1 + I*a*x)^(1/4))/(1 - I*a*x)^(1/4) - (1 + I*a*x)^(5/4)/(x*(1 -
I*a*x)^(1/4)) - (5*I)*a*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - (5*I)
*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
```

```
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1+iax)^{5/4}}{x^2(1-iax)^{5/4}}\,dx\\
&= -\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}+\frac{1}{2}(5ia)\int \frac{\sqrt[4]{1+iax}}{x(1-iax)^{5/4}}\,dx\\
&= \frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}+\frac{1}{2}(5ia)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx\\
&= \frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}+(10ia)\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\
&= \frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}-(5ia)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-(5ia)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\right.\\
&= \frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}-5ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-5ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0205397, size = 87, normalized size = 0.72

$$\frac{-10ax(ax+i)\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{ax+i}{-ax+i}\right)-3\left(9a^2x^2-8iax+1\right)}{3x\sqrt[4]{1-iax}(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] (-3*(1 - (8*I)*a*x + 9*a^2*x^2) - 10*a*x*(I + a*x)*Hypergeometric2F1[3/4, 1
, 7/4, (I + a*x)/(I - a*x)])/(3*x*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
```

Maple [F] time = 0.15, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^2, x)
```

Fricas [A] time = 1.63947, size = 377, normalized size = 3.12

$$\frac{-5i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+5\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-5\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+5i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)-2\,(-9}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(-5*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 5*a*x*log(sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 5*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I)) - I) + 5*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*(
-9*I*a*x + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^2, x)
```

3.85 $\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=163

$$-\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}}+\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}-\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}$$

```
[Out] (-25*a^2*(1 + I*a*x)^(1/4))/(2*(1 - I*a*x)^(1/4)) - (((5*I)/4)*a*(1 + I*a*x
)^(5/4))/(x*(1 - I*a*x)^(1/4)) - (1 + I*a*x)^(9/4)/(2*x^2*(1 - I*a*x)^(1/4)
) + (25*a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4 + (25*a^2*ArcTan
h[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rubi [A] time = 0.0505436, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5062, 96, 94, 93, 212, 206, 203}

$$-\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}}+\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}-\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])/x^3,x]
```


```
[Out] (-25*a^2*(1 + I*a*x)^(1/4))/(2*(1 - I*a*x)^(1/4)) - (((5*I)/4)*a*(1 + I*a*x
)^(5/4))/(x*(1 - I*a*x)^(1/4)) - (1 + I*a*x)^(9/4)/(2*x^2*(1 - I*a*x)^(1/4)
) + (25*a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4 + (25*a^2*ArcTan
h[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1+iax)^{5/4}}{x^3(1-iax)^{5/4}}\,dx\\
&= -\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}+\frac{1}{4}(5ia)\int \frac{(1+iax)^{5/4}}{x^2(1-iax)^{5/4}}\,dx\\
&= -\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}-\frac{1}{8}\left(25a^2\right)\int \frac{\sqrt[4]{1+iax}}{x(1-iax)^{5/4}}\,dx\\
&= -\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}}-\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}-\frac{1}{8}\left(25a^2\right)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx\\
&= -\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}}-\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}-\frac{1}{2}\left(25a^2\right)\mathrm{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\
&= -\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}}-\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}+\frac{1}{4}\left(25a^2\right)\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\\
&= -\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}}-\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}+\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.
\end{aligned}
$$

Mathematica [C] time = 0.0232869, size = 99, normalized size = 0.61

$$
\frac{50a^2x^2(1-iax)\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{ax+i}{-ax+i}\right)-129ia^3x^3-102a^2x^2-33iax-6}{12x^2\sqrt[4]{1-iax}(1+iax)^{3/4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] (-6 - (33*I)*a*x - 102*a^2*x^2 - (129*I)*a^3*x^3 + 50*a^2*x^2*(1 - I*a*x)*H
ypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)])/(12*x^2*(1 - I*a*x)^(1/
```

```
4)*(1 + I*a*x)^(3/4))
```

Maple [F] time = 0.146, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^3, x)
```

Fricas [A] time = 1.7383, size = 420, normalized size = 2.58

$$\frac{25\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+25i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-25i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-25\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right.}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(25*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 25*I*a^2*x^2
*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 25*I*a^2*x^2*log(sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I)) - I) - 25*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a
*x + I)) - 1) - (86*a^2*x^2 + 18*I*a*x + 4)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I)))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^3, x)
```

3.86 $\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=203

$$-\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}}+\frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}}+\frac{55}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{55}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}$$

```
[Out] (((-287*I)/24)*a^3*(1 + I*a*x)^(1/4))/(1 - I*a*x)^(1/4) - (1 + I*a*x)^(1/4)
/(3*x^3*(1 - I*a*x)^(1/4)) - (((13*I)/12)*a*(1 + I*a*x)^(1/4))/(x^2*(1 - I*
a*x)^(1/4)) + (61*a^2*(1 + I*a*x)^(1/4))/(24*x*(1 - I*a*x)^(1/4)) + ((55*I)
/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + ((55*I)/8)*a^3*ArcTan
h[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rubi [A] time = 0.0819834, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5062, 98, 151, 155, 12, 93, 212, 206, 203}

$$-\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}}+\frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}}+\frac{55}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{55}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])/x^4,x]
```


```
[Out] (((-287*I)/24)*a^3*(1 + I*a*x)^(1/4))/(1 - I*a*x)^(1/4) - (1 + I*a*x)^(1/4)
/(3*x^3*(1 - I*a*x)^(1/4)) - (((13*I)/12)*a*(1 + I*a*x)^(1/4))/(x^2*(1 - I*
a*x)^(1/4)) + (61*a^2*(1 + I*a*x)^(1/4))/(24*x*(1 - I*a*x)^(1/4)) + ((55*I)
/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + ((55*I)/8)*a^3*ArcTan
h[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x^4}\,dx = \int \frac{(1+iax)^{5/4}}{x^4(1-iax)^{5/4}}\,dx$$
$$= -\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{1}{3}\int \frac{-\frac{13ia}{2}+6a^2x}{x^3(1-iax)^{5/4}(1+iax)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{1}{6}\int \frac{-\frac{61a^2}{4}-13ia^3x}{x^2(1-iax)^{5/4}(1+iax)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} - \frac{1}{6}\int \frac{\frac{165ia^3}{8}-\frac{61a^4x}{4}}{x(1-iax)^{5/4}(1+iax)^{3/4}}\,dx$$
$$= -\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} - \frac{i\int \frac{165a^4}{16x\sqrt[4]{1-iax}(1+iax)^{3/4}}dx}{3a}$$
$$= -\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} - \frac{1}{16}\left(55ia^3\right)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax}$$
$$= -\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} - \frac{1}{4}\left(55ia^3\right)\text{Subst}\left(\int \frac{1}{-1+x^4}dx\right.$$
$$= -\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} + \frac{1}{8}\left(55ia^3\right)\text{Subst}\left(\int \frac{1}{1-x^2}dx,\right.$$
$$= -\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} + \frac{55}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{55}{8}i$$

Mathematica [C] time = 0.0298097, size = 106, normalized size = 0.52

$$\frac{110a^3x^3(ax+i)\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{ax+i}{-ax+i}\right)+287a^4x^4-226ia^3x^3+87a^2x^2-34iax-8}{24x^3\sqrt[4]{1-iax}(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] (-8 - (34*I)*a*x + 87*a^2*x^2 - (226*I)*a^3*x^3 + 287*a^4*x^4 + 110*a^3*x^3
*(I + a*x)*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)])/(24*x^3*(1
- I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^4, x)
```

Fricas [A] time = 1.7023, size = 452, normalized size = 2.23

$$\frac{165i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-165\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+165\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-165i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right.}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(165*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 165*a^3*
x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 165*a^3*x^3*log(sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I)) - I) - 165*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 +
1)/(a*x + I)) - 1) + (-574*I*a^3*x^3 + 122*a^2*x^2 - 52*I*a*x - 16)*sqrt(I*
sqrt(a^2*x^2 + 1)/(a*x + I)))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^4, x)
```

3.87 $\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=233

$$\frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}}+\frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}}+\frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}}-\frac{475}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{475}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{17ia\sqrt[4]{1}}{24x^3\sqrt[4]{1}}$$

```
[Out] (2467*a^4*(1 + I*a*x)^(1/4))/(192*(1 - I*a*x)^(1/4)) - (1 + I*a*x)^(1/4)/(4
*x^4*(1 - I*a*x)^(1/4)) - (((17*I)/24)*a*(1 + I*a*x)^(1/4))/(x^3*(1 - I*a*x
)^(1/4)) + (113*a^2*(1 + I*a*x)^(1/4))/(96*x^2*(1 - I*a*x)^(1/4)) + (((521*
I)/192)*a^3*(1 + I*a*x)^(1/4))/(x*(1 - I*a*x)^(1/4)) - (475*a^4*ArcTan[(1 +
 I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (475*a^4*ArcTanh[(1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)])/64
```

Rubi [A] time = 0.0996443, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5062, 98, 151, 155, 12, 93, 212, 206, 203}

$$\frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}}+\frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}}+\frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}}-\frac{475}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{475}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{17ia\sqrt[4]{1}}{24x^3\sqrt[4]{1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])/x^5,x]
```


```
[Out] (2467*a^4*(1 + I*a*x)^(1/4))/(192*(1 - I*a*x)^(1/4)) - (1 + I*a*x)^(1/4)/(4
*x^4*(1 - I*a*x)^(1/4)) - (((17*I)/24)*a*(1 + I*a*x)^(1/4))/(x^3*(1 - I*a*x
)^(1/4)) + (113*a^2*(1 + I*a*x)^(1/4))/(96*x^2*(1 - I*a*x)^(1/4)) + (((521*
I)/192)*a^3*(1 + I*a*x)^(1/4))/(x*(1 - I*a*x)^(1/4)) - (475*a^4*ArcTan[(1 +
 I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (475*a^4*ArcTanh[(1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)])/64
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x^5}\,dx = \int \frac{(1+iax)^{5/4}}{x^5(1-iax)^{5/4}}\,dx$$

$$= -\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{1}{4}\int \frac{-\frac{17ia}{2}+8a^2x}{x^4(1-iax)^{5/4}(1+iax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{1}{12}\int \frac{-\frac{113a^2}{4}-\frac{51}{2}ia^3x}{x^3(1-iax)^{5/4}(1+iax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}} - \frac{1}{24}\int \frac{\frac{521ia^3}{8}-\frac{113a^4x}{2}}{x^2(1-iax)^{5/4}(1+iax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}} + \frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}} + \frac{1}{24}\int \frac{\frac{1425a^4}{16}+\frac{521}{8}ia^5x}{x(1-iax)^{5/4}(1+iax)^{3/4}}$$

$$= \frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}} + \frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}} + \frac{i\int -\frac{14}{32x\sqrt[4]{1-i}}}{12}$$

$$= \frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}} + \frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}} + \frac{1}{128}\left(475a^4\right)$$

$$= \frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}} + \frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}} + \frac{1}{32}\left(475a^4\right)$$

$$= \frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}} + \frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}} - \frac{1}{64}\left(475a^4\right)$$

$$= \frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}} + \frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}} - \frac{475}{64}a^4\tan^{-1}$$

Mathematica [C] time = 0.034015, size = 118, normalized size = 0.51

$$\frac{950ia^4x^4(ax+i)\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{ax+i}{-ax+i}\right)+2467ia^5x^5+1946a^4x^4+747ia^3x^3+362a^2x^2-184iax-48}{192x^4\sqrt[4]{1-iax}(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])/x^5,x]
```

```
[Out] (-48 - (184*I)*a*x + 362*a^2*x^2 + (747*I)*a^3*x^3 + 1946*a^4*x^4 + (2467*I
)*a^5*x^5 + (950*I)*a^4*x^4*(I + a*x)*Hypergeometric2F1[3/4, 1, 7/4, (I + a
*x)/(I - a*x)])/(192*x^4*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^5, x)
```

Fricas [A] time = 2.07907, size = 482, normalized size = 2.07

$$-\frac{1425\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+1425i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-1425i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-1425\,a^4x^4\log\left(\right.}{384\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/384*(1425*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 1425*I*
a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 1425*I*a^4*x^4*log(s
qrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 1425*a^4*x^4*log(sqrt(I*sqrt(a^2*
x^2 + 1)/(a*x + I)) - 1) - (4934*a^4*x^4 + 1042*I*a^3*x^3 + 452*a^2*x^2 - 2
72*I*a*x - 96)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^5, x)
```

3.88 $\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=337

$$\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4}-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}-\frac{11\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}a^4}\cdot$$

```
[Out] (-11*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(64*a^4) + (x^2*(1 - I*a*x)^(5/4)
*(1 + I*a*x)^(3/4))/(4*a^2) - ((1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4)*(25 - (4
*I)*a*x))/(96*a^4) - (11*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)
^(1/4)])/(64*Sqrt[2]*a^4) + (11*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
 I*a*x)^(1/4)])/(64*Sqrt[2]*a^4) - (11*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a
*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4) + (
11*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rubi [A] time = 0.213259, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5062, 100, 147, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4}-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}-\frac{11\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}a^4}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] (-11*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(64*a^4) + (x^2*(1 - I*a*x)^(5/4)
*(1 + I*a*x)^(3/4))/(4*a^2) - ((1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4)*(25 - (4
*I)*a*x))/(96*a^4) - (11*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)
^(1/4)])/(64*Sqrt[2]*a^4) + (11*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
 I*a*x)^(1/4)])/(64*Sqrt[2]*a^4) - (11*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a
*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4) + (
11*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x^3\,dx=\int\frac{x^3\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx$$

$$=\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}+\frac{\int\frac{x\left(-2+\frac{iax}{2}\right)\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{4a^2}$$

$$=\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4}-\frac{(11i)\int\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{64a^3}$$

$$=-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}+\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4}-\frac{(11i)\int}{}$$

$$=-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}+\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4}+\frac{11\,\mathrm{Sub}}{}$$

$$=-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}+\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4}+\frac{11\,\mathrm{Sub}}{}$$

$$=-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}+\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4}+\frac{11\,\mathrm{Sub}}{}$$

$$=-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}+\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4}+\frac{11\,\mathrm{Sub}}{}$$

$$=-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}+\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4}-\frac{11\log(}{}$$

$$=-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}+\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4}-\frac{11\tan^{}}{}$$

Mathematica [C] time = 0.100572, size = 127, normalized size = 0.38

$$\frac{(1-iax)^{5/4}\left(4\,2^{3/4}\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-iax)\right)-12\,2^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-iax)\right)+5\right.}{20a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(5/4)*(5*a^2*x^2*(1 + I*a*x)^(3/4) + 4*2^(3/4)*Hypergeometric2
F1[-7/4, 5/4, 9/4, (1 - I*a*x)/2] - 12*2^(3/4)*Hypergeometric2F1[-3/4, 5/4,
 9/4, (1 - I*a*x)/2] + 5*2^(3/4)*Hypergeometric2F1[1/4, 5/4, 9/4, (1 - I*a*
x)/2]))/(20*a^4)
```

Maple [F] time = 0.146, size = 0, normalized size = 0.

$$\int x^3 \frac{1}{\sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Fricas [A] time = 2.16295, size = 733, normalized size = 2.18

$$-\frac{96\,a^4\sqrt{\frac{121i}{4096\,a^8}}\log\left(\frac{64}{11}i\,a^4\sqrt{\frac{121i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-96\,a^4\sqrt{\frac{121i}{4096\,a^8}}\log\left(-\frac{64}{11}i\,a^4\sqrt{\frac{121i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-96\,a^4\sqrt{-\frac{121i}{4096\,a^8}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/192*(96*a^4*sqrt(121/4096*I/a^8)*log(64/11*I*a^4*sqrt(121/4096*I/a^8) +
sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*a^4*sqrt(121/4096*I/a^8)*log(-64/
11*I*a^4*sqrt(121/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*a
^4*sqrt(-121/4096*I/a^8)*log(64/11*I*a^4*sqrt(-121/4096*I/a^8) + sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I))) + 96*a^4*sqrt(-121/4096*I/a^8)*log(-64/11*I*a^4*
sqrt(-121/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (-48*I*a^3*x
^3 + 56*a^2*x^2 + 58*I*a*x - 83)*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)
/(a*x + I)))/a^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

3.89 $\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=339

$$\frac{x(1+iax)^{3/4}(1-iax)^{5/4}}{3a^2}+\frac{i(1+iax)^{3/4}(1-iax)^{5/4}}{12a^3}+\frac{3i(1+iax)^{3/4}\sqrt[4]{1-iax}}{8a^3}+\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3}-\frac{3i\log(}{}$$

```
[Out] (((3*I)/8)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a^3 + ((I/12)*(1 - I*a*x)^(
5/4)*(1 + I*a*x)^(3/4))/a^3 + (x*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4))/(3*a^
2) + (((3*I)/8)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/
(Sqrt[2]*a^3) - (((3*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*
x)^(1/4)])/(Sqrt[2]*a^3) + (((3*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a
*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) - (((3*
I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3)
```

Rubi [A] time = 0.21572, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5062, 90, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{x(1+iax)^{3/4}(1-iax)^{5/4}}{3a^2}+\frac{i(1+iax)^{3/4}(1-iax)^{5/4}}{12a^3}+\frac{3i(1+iax)^{3/4}\sqrt[4]{1-iax}}{8a^3}+\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3}-\frac{3i\log(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] (((3*I)/8)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a^3 + ((I/12)*(1 - I*a*x)^(
5/4)*(1 + I*a*x)^(3/4))/a^3 + (x*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4))/(3*a^
2) + (((3*I)/8)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/
(Sqrt[2]*a^3) - (((3*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*
x)^(1/4)])/(Sqrt[2]*a^3) + (((3*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a
*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) - (((3*
I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
```

```
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x^2\,dx = \int \frac{x^2\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx$$

$$= \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} + \frac{\int \frac{\left(-1+\frac{iax}{2}\right)\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{3a^2}$$

$$= \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{3\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{8a^2}$$

$$= \frac{3i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{3\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+ia\cdots}}}{16a^2}$$

$$= \frac{3i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{(3i)\,\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{3i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1}{1+\cdots}\right.}{4a\cdots}$$

$$= \frac{3i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1-\cdots}{1+\cdots}\right.}{8a\cdots}$$

$$= \frac{3i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{(3i)\,\mathrm{Subst}\left(\int \frac{\cdots}{1-\cdots}\right.}{1\cdots}$$

$$= \frac{3i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} + \frac{3i\log\left(1+\frac{\sqrt{1-ia\cdots}}{\sqrt{1+ia\cdots}}\right.}{16\sqrt{2}\cdots}$$

$$= \frac{3i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} + \frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\cdots}{\sqrt[4]{\cdots}}\right.}{8\sqrt{2}a^3}$$

Mathematica [C] time = 0.0264357, size = 73, normalized size = 0.22

$$\frac{(1-iax)^{5/4}\left(5(1+iax)^{3/4}(4ax+i)-9i2^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-iax)\right)\right)}{60a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(5/4)*(5*(1 + I*a*x)^(3/4)*(I + 4*a*x) - (9*I)*2^(3/4)*Hyperge
ometric2F1[1/4, 5/4, 9/4, (1 - I*a*x)/2]))/(60*a^3)
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int x^2\frac{1}{\sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)

[Out] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Fricas [A] time = 2.21384, size = 648, normalized size = 1.91

$$-\frac{12\,a^3\sqrt{\frac{9i}{64\,a^6}}\log\left(\frac{8}{3}\,a^3\sqrt{\frac{9i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{9i}{64\,a^6}}\log\left(-\frac{8}{3}\,a^3\sqrt{\frac{9i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{-\frac{9i}{64\,a^6}}\log\left(\frac{8}{3}\,a^3\sqrt{-\frac{9i}{64\,a^6}}\right.}{24\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")

[Out] -1/24*(12*a^3*sqrt(9/64*I/a^6)*log(8/3*a^3*sqrt(9/64*I/a^6) + sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(9/64*I/a^6)*log(-8/3*a^3*sqrt(9/64*I/
a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(-9/64*I/a^6)*log(
8/3*a^3*sqrt(-9/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*a^3*s
qrt(-9/64*I/a^6)*log(-8/3*a^3*sqrt(-9/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/
(a*x + I))) - sqrt(a^2*x^2 + 1)*(-8*I*a^2*x^2 + 10*a*x + 11*I)*sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I)))/a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

3.90 $\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=295

$$\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}+\frac{(1+iax)^{3/4}\sqrt[4]{1-iax}}{4a^2}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{\tan^{-1}\left(1-\right.}{4\sqrt{}}$$

```
[Out] ((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(4*a^2) + ((1 - I*a*x)^(5/4)*(1 + I*a
*x)^(3/4))/(2*a^2) + ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/
4)]/(4*Sqrt[2]*a^2) - ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1
/4)]/(4*Sqrt[2]*a^2) + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(
1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(8*Sqrt[2]*a^2) - Log[1 + Sqrt[1 - I*a
*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(8*Sqr
t[2]*a^2)
```

Rubi [A] time = 0.172965, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5062, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}+\frac{(1+iax)^{3/4}\sqrt[4]{1-iax}}{4a^2}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{\tan^{-1}\left(1-\right.}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(4*a^2) + ((1 - I*a*x)^(5/4)*(1 + I*a
*x)^(3/4))/(2*a^2) + ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/
4)]/(4*Sqrt[2]*a^2) - ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1
/4)]/(4*Sqrt[2]*a^2) + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(
1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(8*Sqrt[2]*a^2) - Log[1 + Sqrt[1 - I*a
*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(8*Sqr
t[2]*a^2)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x\,dx=\int \frac{x\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx$$
$$=\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2}+\frac{i\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{4a}$$
$$=\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2}+\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2}+\frac{i\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{8a}$$
$$=\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2}+\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx,x,\sqrt[4]{1-iax}\right)}{2a^2}$$
$$=\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2}+\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2}-\frac{\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a^2}$$
$$=\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2}+\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2}-\frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2}-\frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,\right.}{4a^2}$$
$$=\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2}+\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2}-\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2}-\frac{\text{Subst}\left(\int \frac{}{1+\sqrt{2}}\right.}{}$$
$$=\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2}+\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2}+\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}-\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\right.}{8\sqrt{2}a^2}$$
$$=\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2}+\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}-\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\ldots$$

Mathematica [C] time = 0.0210959, size = 63, normalized size = 0.21

$$\frac{(1-iax)^{5/4}\left(5(1+iax)^{3/4}-2^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-iax)\right)\right)}{10a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(5/4)*(5*(1 + I*a*x)^(3/4) - 2^(3/4)*Hypergeometric2F1[1/4, 5/
4, 9/4, (1 - I*a*x)/2]))/(10*a^2)
```

Maple [F] time = 0.141, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Fricas [A] time = 2.15505, size = 618, normalized size = 2.09

$$\frac{2\,a^2\sqrt{\frac{i}{16\,a^4}}\log\left(4i\,a^2\sqrt{\frac{i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{i}{16\,a^4}}\log\left(-4i\,a^2\sqrt{\frac{i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{-\frac{i}{16\,a^4}}\log\left(4i\,a^2\sqrt{-\frac{i}{16}}\right.}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*sqrt(1/16*I/a^4)*log(4*I*a^2*sqrt(1/16*I/a^4) + sqrt(I*sqrt(a^2*
x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(1/16*I/a^4)*log(-4*I*a^2*sqrt(1/16*I/a^4)
 + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(-1/16*I/a^4)*log(4*I*a
^2*sqrt(-1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*a^2*sqrt(-1
/16*I/a^4)*log(-4*I*a^2*sqrt(-1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I))) + sqrt(a^2*x^2 + 1)*(-2*I*a*x + 3)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)
))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

3.91 $\int e^{-\frac{1}{2}i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=268

$$-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}-\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{i\tan^{-}}{}$$

```
[Out] ((-I)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a - (I*ArcTan[1 - (Sqrt[2]*(1 -
I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*ArcTan[1 + (Sqrt[2]*(1 -
 I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) - ((I/2)*Log[1 + Sqrt[1 - I*
a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqr
t[2]*a) + ((I/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*
a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rubi [A] time = 0.146362, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5061, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}-\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{i\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-I/2)*ArcTan[a*x]),x]
```


```
[Out] ((-I)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a - (I*ArcTan[1 - (Sqrt[2]*(1 -
I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*ArcTan[1 + (Sqrt[2]*(1 -
 I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) - ((I/2)*Log[1 + Sqrt[1 - I*
a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqr
t[2]*a) + ((I/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*
a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```


Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}i\tan^{-1}(ax)}dx = \int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}dx$$
$$= -\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{1}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}dx$$
$$= -\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}dx, x, \sqrt[4]{1-iax}\right)}{a}$$
$$= -\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{1}{1+x^4}dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$
$$= -\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{i\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} + \frac{i\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$
$$= -\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} + \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} - \cdots$$
$$= -\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{i\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$
$$= -\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1\cdots}}{\sqrt[4]{1+\cdots}}\right.}{2\sqrt{2}a}$$

Mathematica [C] time = 0.0280596, size = 41, normalized size = 0.15

$$-\frac{8ie^{\frac{3}{2}i\tan^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2i\tan^{-1}(ax)}\right)}{3a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-I/2)*ArcTan[a*x]),x]
```

```
[Out] (((-8*I)/3)*E^(((3*I)/2)*ArcTan[a*x])*Hypergeometric2F1[3/4, 2, 7/4, -E^((2
*I)*ArcTan[a*x])])/a
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Fricas [A] time = 2.02765, size = 493, normalized size = 1.84

$$\frac{a\sqrt{\frac{i}{a^2}}\log\left(a\sqrt{\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{\frac{i}{a^2}}\log\left(-a\sqrt{\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{-\frac{i}{a^2}}\log\left(a\sqrt{-\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+a\sqrt{-\frac{i}{a^2}}\log\Big(}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*sqrt(I/a^2)*log(a*sqrt(I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
 - a*sqrt(I/a^2)*log(-a*sqrt(I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
- a*sqrt(-I/a^2)*log(a*sqrt(-I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
+ a*sqrt(-I/a^2)*log(-a*sqrt(-I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
 - 2*I*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

3.92 $\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=267

$$-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)+\sqrt{2}\,\mathrm{t}$$

```
[Out] 2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - I
*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(
1/4)] - Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4
))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (
Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.160957, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5062, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)+\sqrt{2}\,\mathrm{t}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((I/2)*ArcTan[a*x])*x),x]
```

```
[Out] 2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - I
*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(
1/4)] - Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4
))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (
Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\,dx$$

$$= -\left((ia)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx\right) + \int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= 4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right) + 4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 4\,\text{Subst}\left(\int \frac{1}{1+x^4}\right.$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + 2\,\text{Subst}\left(\int \frac{1-}{1-}\right.$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2}{-1+\sqrt{2}x}\right.}{}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1+}}\right.}{\sqrt{2}}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.$$

Mathematica [C] time = 0.0277312, size = 96, normalized size = 0.36

$$2\,2^{3/4}\sqrt[4]{1-iax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-iax)\right) - \frac{4\sqrt[4]{1-iax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},-\frac{1-iax}{-1-iax}\right)}{\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((I/2)*ArcTan[a*x])*x),x]
```

```
[Out] 2*2^(3/4)*(1 - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (1 - I*a*x)/2]
 - (4*(1 - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1, 5/4, -((1 - I*a*x)/(-1 -
I*a*x))])/(1 + I*a*x)^(1/4)
```

Maple [F] time = 0.139, size = 0, normalized size = 0.

$$\int \frac{1}{x} \frac{1}{\sqrt{(1 + iax)\frac{1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

Fricas [A] time = 2.01266, size = 672, normalized size = 2.52

$$-\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(4*I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) +
 1/2*sqrt(4*I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
+ 1/2*sqrt(-4*I)*log(1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
) - 1/2*sqrt(-4*I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I))) - log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + I*log(sqrt(I*sqrt(a^2
*x^2 + 1)/(a*x + I)) + I) - I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I)
+ log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

3.93 $\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=92

$$-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}-ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -(((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x) - I*a*ArcTan[(1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)] + I*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rubi [A] time = 0.0351591, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5062, 94, 93, 298, 203, 206}

$$-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}-ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((I/2)*ArcTan[a*x])*x^2),x]
```

```
[Out] -(((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x) - I*a*ArcTan[(1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)] + I*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{\sqrt[4]{1-iax}}{x^2\sqrt[4]{1+iax}}\,dx\\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} - \frac{1}{2}(ia)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx\\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} - (2ia)\operatorname{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} + (ia)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - (ia)\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} - ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0152455, size = 69, normalized size = 0.75

$$\frac{i\sqrt[4]{1-iax}\left(2ax\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{ax+i}{-ax+i}\right)-ax+i\right)}{x\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((I/2)*ArcTan[a*x])*x^2),x]
```

```
[Out] (I*(1 - I*a*x)^(1/4)*(I - a*x + 2*a*x*Hypergeometric2F1[1/4, 1, 5/4, (I + a
*x)/(I - a*x)]))/(x*(1 + I*a*x)^(1/4))
```

Maple [F] time = 0.136, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

Fricas [B] time = 2.104, size = 369, normalized size = 4.01

$$\frac{iax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-iax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)-2\sqrt{a^2x^2+1}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + a*x*log(sqrt(I*sq
rt(a^2*x^2 + 1)/(a*x + I)) + I) - a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
)) - I) - I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*sqrt(a^2*x
^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

3.94 $\int \frac{e^{-\frac{1}{2} i \tan^{-1}(ax)}}{x^3} \, dx$

Optimal. Leaf size=132

$$-\frac{1}{4} a^2 \tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{1}{4} a^2 \tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2}+\frac{ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x}$$

```
[Out] ((I/4)*a*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x - ((1 - I*a*x)^(5/4)*(1 + I
*a*x)^(3/4))/(2*x^2) - (a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
+ (a^2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rubi [A] time = 0.0431289, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5062, 96, 94, 93, 298, 203, 206}

$$-\frac{1}{4} a^2 \tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{1}{4} a^2 \tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2}+\frac{ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((I/2)*ArcTan[a*x])*x^3),x]
```


```
[Out] ((I/4)*a*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x - ((1 - I*a*x)^(5/4)*(1 + I
*a*x)^(3/4))/(2*x^2) - (a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
+ (a^2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{\sqrt[4]{1-iax}}{x^3\sqrt[4]{1+iax}}\,dx\\
&= -\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} - \frac{1}{4}(ia)\int \frac{\sqrt[4]{1-iax}}{x^2\sqrt[4]{1+iax}}\,dx\\
&= \frac{ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} - \frac{1}{8}a^2\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx\\
&= \frac{ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} - \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\
&= \frac{ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} + \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{1}{4}a^2\,\text{S}\\
&= \frac{ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} - \frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+ia}}{\sqrt[4]{1-ia}}\right.
\end{aligned}
$$

Mathematica [C] time = 0.0206996, size = 81, normalized size = 0.61

$$\frac{\sqrt[4]{1-iax}\left(2a^2x^2\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{ax+i}{-ax+i}\right)-3a^2x^2+iax-2\right)}{4x^2\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((I/2)*ArcTan[a*x])*x^3),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-2 + I*a*x - 3*a^2*x^2 + 2*a^2*x^2*Hypergeometric2F1[1/
4, 1, 5/4, (I + a*x)/(I - a*x)]))/(4*x^2*(1 + I*a*x)^(1/4))
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} \frac{1}{\sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

Fricas [A] time = 2.00042, size = 413, normalized size = 3.13

$$\frac{a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)-2\sqrt{a^2x^2}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - I*a^2*x^2*log(s
qrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + I*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)) - I) - a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1
) - 2*sqrt(a^2*x^2 + 1)*(-3*I*a*x + 2)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

3.95 $\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=170

$$\frac{11a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x}+\frac{3}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{3}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2}-\frac{\sqrt[4]{1-iax}(1+i}{3x^3}$$

```
[Out] -((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(3*x^3) + (((5*I)/12)*a*(1 - I*a*x)^
(1/4)*(1 + I*a*x)^(3/4))/x^2 + (11*a^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
/(24*x) + ((3*I)/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ((3*I
)/8)*a^3*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rubi [A] time = 0.0690398, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5062, 99, 151, 12, 93, 298, 203, 206}

$$\frac{11a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x}+\frac{3}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{3}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2}-\frac{\sqrt[4]{1-iax}(1+i}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((I/2)*ArcTan[a*x])*x^4),x]
```

```
[Out] -((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(3*x^3) + (((5*I)/12)*a*(1 - I*a*x)^
(1/4)*(1 + I*a*x)^(3/4))/x^2 + (11*a^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
/(24*x) + ((3*I)/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ((3*I
)/8)*a^3*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x^4}\,dx = \int \frac{\sqrt[4]{1-iax}}{x^4\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} + \frac{1}{3}\int \frac{-\frac{5ia}{2}-2a^2x}{x^3(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} + \frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} - \frac{1}{6}\int \frac{\frac{11a^2}{4}-\frac{5}{2}ia^3x}{x^2(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} + \frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{11a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} + \frac{1}{6}\int \frac{9ia \cdots}{8x(1-iax)^{3/\cdots}}$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} + \frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{11a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} + \frac{1}{16}\left(3ia^3\right)\int \frac{\cdots}{x(1-\cdots}$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} + \frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{11a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} + \frac{1}{4}\left(3ia^3\right)\text{Subst}\Big(\cdots$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} + \frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{11a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} - \frac{1}{8}\left(3ia^3\right)\text{Subst}\Big(\cdots$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} + \frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{11a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} + \frac{3}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+\cdots}}{\sqrt[4]{1-\cdots}}\right.$$

Mathematica [C] time = 0.0237386, size = 92, normalized size = 0.54

$$\frac{\sqrt[4]{1-iax}\left(-18ia^3x^3\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{ax+i}{-ax+i}\right)+11ia^3x^3+a^2x^2+2iax-8\right)}{24x^3\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((I/2)*ArcTan[a*x])*x^4),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-8 + (2*I)*a*x + a^2*x^2 + (11*I)*a^3*x^3 - (18*I)*a^3*
x^3*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1 + I*a*
x)^(1/4))
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\frac{1}{\sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

Fricas [A] time = 2.09924, size = 443, normalized size = 2.61

$$\frac{-9i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-9\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+9\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+9i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)+}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(-9*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 9*a^3*x^3
*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 9*a^3*x^3*log(sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I)) - I) + 9*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
```

```
 + I)) - 1) + (22*a^2*x^2 + 20*I*a*x - 16)*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^
2*x^2 + 1)/(a*x + I)))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x^4*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

3.96 $\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=202

$$\frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2}-\frac{83ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{192x}+\frac{11}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{11}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{7ia\sqrt[4]{1-ia}}{2}$$

```
[Out] -((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(4*x^4) + (((7*I)/24)*a*(1 - I*a*x)^
(1/4)*(1 + I*a*x)^(3/4))/x^3 + (29*a^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
/(96*x^2) - (((83*I)/192)*a^3*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x + (11*
a^4*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (11*a^4*ArcTanh[(1 +
I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64
```

Rubi [A] time = 0.0807455, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5062, 99, 151, 12, 93, 298, 203, 206}

$$\frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2}-\frac{83ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{192x}+\frac{11}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{11}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{7ia\sqrt[4]{1-ia}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((I/2)*ArcTan[a*x])*x^5),x]
```

```
[Out] -((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(4*x^4) + (((7*I)/24)*a*(1 - I*a*x)^
(1/4)*(1 + I*a*x)^(3/4))/x^3 + (29*a^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
/(96*x^2) - (((83*I)/192)*a^3*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x + (11*
a^4*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (11*a^4*ArcTanh[(1 +
I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x^5}\,dx = \int \frac{\sqrt[4]{1-iax}}{x^5\sqrt[4]{1+iax}}\,dx$$
$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{1}{4}\int \frac{-\frac{7ia}{2} - 3a^2x}{x^4(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$
$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3} - \frac{1}{12}\int \frac{\frac{29a^2}{4} - 7ia^3x}{x^3(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$
$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2} + \frac{1}{24}\int \frac{\frac{83ia^3}{8} + 2\cdots}{x^2(1-iax)^{3/4}\cdots}$$
$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2} - \frac{83ia^3\sqrt[4]{1-iax}(1+ia\cdots}{192x}$$
$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2} - \frac{83ia^3\sqrt[4]{1-iax}(1+ia\cdots}{192x}$$
$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2} - \frac{83ia^3\sqrt[4]{1-iax}(1+ia\cdots}{192x}$$
$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2} - \frac{83ia^3\sqrt[4]{1-iax}(1+ia\cdots}{192x}$$
$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2} - \frac{83ia^3\sqrt[4]{1-iax}(1+ia\cdots}{192x}$$

Mathematica [C] time = 0.0312894, size = 99, normalized size = 0.49

$$\frac{\sqrt[4]{1-iax}\left(-66a^4x^4\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{ax+i}{-ax+i}\right) + 83a^4x^4 - 25ia^3x^3 + 2a^2x^2 + 8iax - 48\right)}{192x^4\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((I/2)*ArcTan[a*x])*x^5),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-48 + (8*I)*a*x + 2*a^2*x^2 - (25*I)*a^3*x^3 + 83*a^4*x
^4 - 66*a^4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(192*
x^4*(1 + I*a*x)^(1/4))
```

Maple [F] time = 0.146, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\frac{1}{\sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^5*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

Fricas [A] time = 2.09505, size = 475, normalized size = 2.35

$$-\frac{33\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-33i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+33i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-33\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right.}{384\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/384*(33*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 33*I*a^4*
x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 33*I*a^4*x^4*log(sqrt(I*
sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 33*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)
/(a*x + I)) - 1) - (-166*I*a^3*x^3 + 116*a^2*x^2 + 112*I*a*x - 96)*sqrt(a^2
*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(1/(x^5*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

3.97 $\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=337

$$\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}-\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4}-\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4}+\frac{123\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}a^4}$$

```
[Out] (-41*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(64*a^4) + (x^2*(1 - I*a*x)^(7/4)
*(1 + I*a*x)^(1/4))/(4*a^2) - ((1 - I*a*x)^(7/4)*(1 + I*a*x)^(1/4)*(11 - (4
*I)*a*x))/(32*a^4) - (123*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)])/(64*Sqrt[2]*a^4) + (123*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
 + I*a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (123*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 +
I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
- (123*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rubi [A] time = 0.21698, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5062, 100, 147, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}-\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4}-\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4}+\frac{123\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (-41*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(64*a^4) + (x^2*(1 - I*a*x)^(7/4)
*(1 + I*a*x)^(1/4))/(4*a^2) - ((1 - I*a*x)^(7/4)*(1 + I*a*x)^(1/4)*(11 - (4
*I)*a*x))/(32*a^4) - (123*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)])/(64*Sqrt[2]*a^4) + (123*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
 + I*a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (123*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 +
I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
- (123*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
```

```
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x^3\,dx = \int \frac{x^3(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx$$

$$= \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} + \frac{\int \frac{x(1-iax)^{3/4}\left(-2+\frac{3iax}{2}\right)}{(1+iax)^{3/4}}\,dx}{4a^2}$$

$$= \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} - \frac{(41i)\int \frac{(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx}{64a^3}$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} - \frac{(123i)\int \frac{}{\sqrt[4]{1}}}{12}$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} + \frac{123\,\mathrm{Subst}}{}$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} + \frac{123\,\mathrm{Subst}}{}$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} - \frac{123\,\mathrm{Subst}}{}$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} + \frac{123\,\mathrm{Subst}}{}$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} + \frac{123\log\left(1-\right.}{}$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} - \frac{123\tan^{-1}\left(\right.}{64}$$

Mathematica [C] time = 0.102481, size = 127, normalized size = 0.38

$$\frac{(1-iax)^{7/4}\left(12\sqrt[4]{2}\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}(1-iax)\right)-20\sqrt[4]{2}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}(1-iax)\right)+7\right.}{28a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(7/4)*(7*a^2*x^2*(1 + I*a*x)^(1/4) + 12*2^(1/4)*Hypergeometric
2F1[-5/4, 7/4, 11/4, (1 - I*a*x)/2] - 20*2^(1/4)*Hypergeometric2F1[-1/4, 7/
4, 11/4, (1 - I*a*x)/2] + 7*2^(1/4)*Hypergeometric2F1[3/4, 7/4, 11/4, (1 -
I*a*x)/2]))/(28*a^4)
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int x^3 \left((1 + iax)\frac{1}{\sqrt{a^2x^2 + 1}}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Fricas [A] time = 2.04081, size = 740, normalized size = 2.2

$$-\frac{32\,a^4\sqrt{\frac{15129i}{4096\,a^8}}\log\left(\frac{64}{123}\,a^4\sqrt{\frac{15129i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-32\,a^4\sqrt{\frac{15129i}{4096\,a^8}}\log\left(-\frac{64}{123}\,a^4\sqrt{\frac{15129i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+32\,a^4\sqrt{-\frac{15129}{4096\,a^{8}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(32*a^4*sqrt(15129/4096*I/a^8)*log(64/123*a^4*sqrt(15129/4096*I/a^8)
+ sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 32*a^4*sqrt(15129/4096*I/a^8)*log(
-64/123*a^4*sqrt(15129/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) +
 32*a^4*sqrt(-15129/4096*I/a^8)*log(64/123*a^4*sqrt(-15129/4096*I/a^8) + sq
rt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 32*a^4*sqrt(-15129/4096*I/a^8)*log(-64
/123*a^4*sqrt(-15129/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (
16*a^4*x^4 + 40*I*a^3*x^3 - 54*a^2*x^2 - 93*I*a*x + 63)*sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)))/a^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

3.98 $\int e^{-\frac{3}{2} i \tan^{-1}(ax)} x^2 \, dx$

Optimal. Leaf size=339

$$\frac{x\sqrt[4]{1+iax}(1-iax)^{7/4}}{3a^2}+\frac{i\sqrt[4]{1+iax}(1-iax)^{7/4}}{4a^3}+\frac{17i\sqrt[4]{1+iax}(1-iax)^{3/4}}{24a^3}-\frac{17i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3}+\frac{17i\log\left(\right.}{}$$

```
[Out] (((17*I)/24)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a^3 + ((I/4)*(1 - I*a*x)^
(7/4)*(1 + I*a*x)^(1/4))/a^3 + (x*(1 - I*a*x)^(7/4)*(1 + I*a*x)^(1/4))/(3*a
^2) + (((17*I)/8)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]
)/(Sqrt[2]*a^3) - (((17*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)])/(Sqrt[2]*a^3) - (((17*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 +
 I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (
((17*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^
(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3)
```

Rubi [A] time = 0.219318, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5062, 90, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{x\sqrt[4]{1+iax}(1-iax)^{7/4}}{3a^2}+\frac{i\sqrt[4]{1+iax}(1-iax)^{7/4}}{4a^3}+\frac{17i\sqrt[4]{1+iax}(1-iax)^{3/4}}{24a^3}-\frac{17i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3}+\frac{17i\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(((3*I)/2)*ArcTan[a*x]),x]
```


```
[Out] (((17*I)/24)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a^3 + ((I/4)*(1 - I*a*x)^
(7/4)*(1 + I*a*x)^(1/4))/a^3 + (x*(1 - I*a*x)^(7/4)*(1 + I*a*x)^(1/4))/(3*a
^2) + (((17*I)/8)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]
)/(Sqrt[2]*a^3) - (((17*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)])/(Sqrt[2]*a^3) - (((17*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 +
 I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (
((17*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^
(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3)
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x^2\,dx = \int \frac{x^2(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx$$

$$= \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} + \frac{\int \frac{(1-iax)^{3/4}\left(-1+\frac{3iax}{2}\right)}{(1+iax)^{3/4}}\,dx}{3a^2}$$

$$= \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} - \frac{17\int \frac{(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx}{24a^2}$$

$$= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} - \frac{17\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,d}{16a^2}$$

$$= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} - \frac{(17i)\,\mathrm{Subst}\left(\int \frac{x^2}{(2-x^4}\right.}{4a}$$

$$= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} - \frac{(17i)\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\right.}{4a^3}$$

$$= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} + \frac{(17i)\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\right.}{8a^3}$$

$$= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} - \frac{(17i)\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}}\right.}{16a}$$

$$= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} - \frac{17i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right.}{16\sqrt{2}a^3}$$

$$= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} + \frac{17i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1+}}\right.}{8\sqrt{2}a^3}$$

Mathematica [C] time = 0.0285041, size = 73, normalized size = 0.22

$$\frac{(1-iax)^{7/4}\left(7\sqrt[4]{1+iax}(4ax+3i)-17i\sqrt[4]{2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}(1-iax)\right)\right)}{84a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(7/4)*(7*(1 + I*a*x)^(1/4)*(3*I + 4*a*x) - (17*I)*2^(1/4)*Hype
rgeometric2F1[3/4, 7/4, 11/4, (1 - I*a*x)/2]))/(84*a^3)
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int x^2 \left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Fricas [A] time = 2.05278, size = 678, normalized size = 2.

$$-\frac{12\,a^3\sqrt{\frac{289i}{64\,a^6}}\log\left(\frac{8}{17}i\,a^3\sqrt{\frac{289i}{64\,a^6}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{289i}{64\,a^6}}\log\left(-\frac{8}{17}i\,a^3\sqrt{\frac{289i}{64\,a^6}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)+12\,a^3\sqrt{-\frac{289i}{64\,a^6}}\log\left(\frac{8}{17}i\,a^3\sqrt{\cdots}\right.}{24\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(12*a^3*sqrt(289/64*I/a^6)*log(8/17*I*a^3*sqrt(289/64*I/a^6) + sqrt(I
*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(289/64*I/a^6)*log(-8/17*I*a^3*
sqrt(289/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*a^3*sqrt(-28
9/64*I/a^6)*log(8/17*I*a^3*sqrt(-289/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I))) - 12*a^3*sqrt(-289/64*I/a^6)*log(-8/17*I*a^3*sqrt(-289/64*I/a^6)
 + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (8*a^3*x^3 + 22*I*a^2*x^2 - 37*a*
x - 23*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

3.99 $\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=295

$$\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2a^2}+\frac{3\sqrt[4]{1+iax}(1-iax)^{3/4}}{4a^2}-\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{9\tan^{-1}}{}$$

```
[Out] (3*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*a^2) + ((1 - I*a*x)^(7/4)*(1 + I
*a*x)^(1/4))/(2*a^2) + (9*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)])/(4*Sqrt[2]*a^2) - (9*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)])/(4*Sqrt[2]*a^2) - (9*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x]
 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2) + (9*Log
[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a
*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rubi [A] time = 0.177515, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5062, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2a^2}+\frac{3\sqrt[4]{1+iax}(1-iax)^{3/4}}{4a^2}-\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{9\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (3*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*a^2) + ((1 - I*a*x)^(7/4)*(1 + I
*a*x)^(1/4))/(2*a^2) + (9*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)])/(4*Sqrt[2]*a^2) - (9*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)])/(4*Sqrt[2]*a^2) - (9*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x]
 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2) + (9*Log
[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a
*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x\,dx = \int \frac{x(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx$$
$$= \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} + \frac{(3i)\int \frac{(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx}{4a}$$
$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} + \frac{(9i)\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx}{8a}$$
$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} - \frac{9\,\mathrm{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)}{2a^2}$$
$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} - \frac{9\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a^2}$$
$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} + \frac{9\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2} - \frac{9\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,d\right.}{4a^2}$$
$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} - \frac{9\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2} - \frac{9\,\mathrm{Subst}\left(\int \frac{}{1+}\right.}{}$$
$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} - \frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} + \frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\right.}{8\sqrt{2}a^2}$$
$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} + \frac{9\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} - \frac{9\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} -$$

Mathematica [C] time = 0.0144996, size = 63, normalized size = 0.21

$$\frac{(1-iax)^{7/4}\left(7\sqrt[4]{1+iax}-3\sqrt[4]{2}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}(1-iax)\right)\right)}{14a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(7/4)*(7*(1 + I*a*x)^(1/4) - 3*2^(1/4)*Hypergeometric2F1[3/4,
7/4, 11/4, (1 - I*a*x)/2]))/(14*a^2)
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int x\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Fricas [A] time = 2.13422, size = 620, normalized size = 2.1

$$\frac{2\,a^2\sqrt{\frac{81i}{16\,a^4}}\log\left(\frac{4}{9}\,a^2\sqrt{\frac{81i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{81i}{16\,a^4}}\log\left(-\frac{4}{9}\,a^2\sqrt{\frac{81i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+2\,a^2\sqrt{-\frac{81i}{16\,a^4}}\log\left(\frac{4}{9}\,a^2\sqrt{-\frac{81i}{16\,a^4}}\right.}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*sqrt(81/16*I/a^4)*log(4/9*a^2*sqrt(81/16*I/a^4) + sqrt(I*sqrt(a^
2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(81/16*I/a^4)*log(-4/9*a^2*sqrt(81/16*I/
a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*a^2*sqrt(-81/16*I/a^4)*log(
4/9*a^2*sqrt(-81/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*s
qrt(-81/16*I/a^4)*log(-4/9*a^2*sqrt(-81/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1
)/(a*x + I))) - (2*a^2*x^2 + 7*I*a*x - 5)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
)))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

3.100 $\int e^{-\frac{3}{2}i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=268

$$-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{3i}{}$$

```
[Out] ((-I)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a - ((3*I)*ArcTan[1 - (Sqrt[2]*(
1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + ((3*I)*ArcTan[1 + (Sqrt
[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (((3*I)/2)*Log[1 +
 Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^
(1/4)])/(Sqrt[2]*a) - (((3*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] +
(Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rubi [A] time = 0.150413, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5061, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{3i}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((-3*I)/2)*ArcTan[a*x]),x]
```


```
[Out] ((-I)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a - ((3*I)*ArcTan[1 - (Sqrt[2]*(
1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + ((3*I)*ArcTan[1 + (Sqrt
[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (((3*I)/2)*Log[1 +
 Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^
(1/4)])/(Sqrt[2]*a) - (((3*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] +
(Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```


Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{-\frac{3}{2}i\tan^{-1}(ax)}\,dx=\int\frac{(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx$$
$$=-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{3}{2}\int\frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$
$$=-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{(6i)\,\mathrm{Subst}\left(\int\frac{x^2}{(2-x^4)^{3/4}}\,dx,x,\sqrt[4]{1-iax}\right)}{a}$$
$$=-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{(6i)\,\mathrm{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$
$$=-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}-\frac{(3i)\,\mathrm{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}+\frac{(3i)\,\mathrm{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$
$$=-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{(3i)\,\mathrm{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a}+\frac{(3i)\,\mathrm{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-ia}}{\sqrt[4]{1+ia}}\right.}{2a}$$
$$=-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}-\frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}+\frac{(3i)\,\mathrm{Sub}}{}$$
$$=-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\right.}{2\sqrt{2}a}$$

Mathematica [C] time = 0.0387365, size = 39, normalized size = 0.15

$$-\frac{8ie^{\frac{1}{2}i\tan^{-1}(ax)}\mathrm{Hypergeometric2F1}\left(\frac{1}{4},2,\frac{5}{4},-e^{2i\tan^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((-3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((-8*I)*E^((I/2)*ArcTan[a*x])*Hypergeometric2F1[1/4, 2, 5/4, -E^((2*I)*ArcT
an[a*x])])/a
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(-3/2), x)
```

Fricas [A] time = 2.05208, size = 536, normalized size = 2.

$$\frac{a\sqrt{\frac{9i}{a^2}}\log\left(\frac{1}{3}ia\sqrt{\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{\frac{9i}{a^2}}\log\left(-\frac{1}{3}ia\sqrt{\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+a\sqrt{-\frac{9i}{a^2}}\log\left(\frac{1}{3}ia\sqrt{-\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{-}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*sqrt(9*I/a^2)*log(1/3*I*a*sqrt(9*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I))) - a*sqrt(9*I/a^2)*log(-1/3*I*a*sqrt(9*I/a^2) + sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I))) + a*sqrt(-9*I/a^2)*log(1/3*I*a*sqrt(-9*I/a^2) + sqrt(I*
sqrt(a^2*x^2 + 1)/(a*x + I))) - a*sqrt(-9*I/a^2)*log(-1/3*I*a*sqrt(-9*I/a^2
) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (2*a*x + 2*I)*sqrt(I*sqrt(a^2*x^
2 + 1)/(a*x + I)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(-3/2), x)
```

3.101 $\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=267

$$\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)+\sqrt{2}\tan$$

```
[Out] -2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2
]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 -
I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^
(1/4)] + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/
4))/(1 + I*a*x)^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] +
(Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.1681, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5062, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)+\sqrt{2}\tan$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x),x]
```

```
[Out] -2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2
]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 -
I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^
(1/4)] + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/
4))/(1 + I*a*x)^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] +
(Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x}\,dx = \int \frac{(1-iax)^{3/4}}{x(1+iax)^{3/4}}\,dx$$

$$= -\left((ia)\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx\right) + \int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= 4\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right) + 4\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 4\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1}\right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}}{-1+\sqrt{}}\right.}{}$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}}{}\right.}{\sqrt{2}}$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.$$

Mathematica [C] time = 0.0275994, size = 97, normalized size = 0.36

$$\frac{2(1-iax)^{3/4}\left(\sqrt[4]{2}(1+iax)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)-2\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{ax+i}{-ax+i}\right)\right)}{3(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x),x]
```

```
[Out] (2*(1 - I*a*x)^(3/4)*(2^(1/4)*(1 + I*a*x)^(3/4)*Hypergeometric2F1[3/4, 3/4,
 7/4, (1 - I*a*x)/2] - 2*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)
```

```
]))/(3*(1 + I*a*x)^(3/4))
```

Maple [F] time = 0.146, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

Fricas [A] time = 2.11226, size = 662, normalized size = 2.48

$$-\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 1
/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 1/
2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 1/
2*sqrt(-4*I)*log(-1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - l
og(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - I*log(sqrt(I*sqrt(a^2*x^2 + 1
)/(a*x + I)) + I) + I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + log(sq
rt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(1/(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

3.102 $\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=92

$$-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}+3ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+3ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -(((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x) + (3*I)*a*ArcTan[(1 + I*a*x)^(1/
4)/(1 - I*a*x)^(1/4)] + (3*I)*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)
]
```

Rubi [A] time = 0.0317273, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5062, 94, 93, 212, 206, 203}

$$-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}+3ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+3ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x^2),x]
```

```
[Out] -(((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x) + (3*I)*a*ArcTan[(1 + I*a*x)^(1/
4)/(1 - I*a*x)^(1/4)] + (3*I)*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)
]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1-iax)^{3/4}}{x^2(1+iax)^{3/4}}\,dx\\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} - \frac{1}{2}(3ia)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx\\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} - (6ia)\,\mathrm{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} + (3ia)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + (3ia)\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.\\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} + 3ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 3ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0160334, size = 69, normalized size = 0.75

$$\frac{i(1-iax)^{3/4}\left(2ax\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{ax+i}{-ax+i}\right)-ax+i\right)}{x(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x^2),x]
```

```
[Out] (I*(1 - I*a*x)^(3/4)*(I - a*x + 2*a*x*Hypergeometric2F1[3/4, 1, 7/4, (I + a
*x)/(I - a*x)]))/(x*(1 + I*a*x)^(3/4))
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

Fricas [B] time = 2.03938, size = 373, normalized size = 4.05

$$\frac{3i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-3\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+3\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-3i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)-2\,(-i\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 3*a*x*log(sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 3*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a
*x + I)) - I) - 3*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*(-
I*a*x + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

3.103 $\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=132

$$\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2}+\frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}$$

```
[Out] (((3*I)/4)*a*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - ((1 - I*a*x)^(7/4)*(1
 + I*a*x)^(1/4))/(2*x^2) + (9*a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4
)])/4 + (9*a^2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rubi [A] time = 0.0425153, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5062, 96, 94, 93, 212, 206, 203}

$$\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2}+\frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x^3),x]
```

```
[Out] (((3*I)/4)*a*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - ((1 - I*a*x)^(7/4)*(1
 + I*a*x)^(1/4))/(2*x^2) + (9*a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4
)])/4 + (9*a^2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1-iax)^{3/4}}{x^3(1+iax)^{3/4}}\,dx\\
&= -\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2} - \frac{1}{4}(3ia)\int \frac{(1-iax)^{3/4}}{x^2(1+iax)^{3/4}}\,dx\\
&= \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2} - \frac{1}{8}\left(9a^2\right)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx\\
&= \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2} - \frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\
&= \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2} + \frac{1}{4}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{4}\\
&= \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2} + \frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right.
\end{aligned}$$

Mathematica [C] time = 0.0189489, size = 81, normalized size = 0.61

$$\frac{(1-iax)^{3/4}\left(6a^2x^2\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{ax+i}{-ax+i}\right)-5a^2x^2+3iax-2\right)}{4x^2(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x^3),x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(-2 + (3*I)*a*x - 5*a^2*x^2 + 6*a^2*x^2*Hypergeometric2F
1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(4*x^2*(1 + I*a*x)^(3/4))
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

Fricas [A] time = 2.1067, size = 414, normalized size = 3.14

$$\frac{9\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+9i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-9i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-9\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)+(}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(9*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 9*I*a^2*x^2*l
og(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 9*I*a^2*x^2*log(sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I)) - I) - 9*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I)) - 1) + (10*a^2*x^2 + 14*I*a*x - 4)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
)/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \left(\frac{i a x + 1}{\sqrt{a^{2} x^{2} + 1}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

3.104 $\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=170

$$\frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}-\frac{17}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{17}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}-\frac{(1-iax)^{3/4}\cdots}{3x^3}$$

```
[Out] -((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(3*x^3) + (((7*I)/12)*a*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(1/4))/x^2 + (23*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(24*x) - ((17*I)/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ((17
*I)/8)*a^3*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rubi [A] time = 0.0645408, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5062, 99, 151, 12, 93, 212, 206, 203}

$$\frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}-\frac{17}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{17}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}-\frac{(1-iax)^{3/4}\cdots}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x^4),x]
```

```
[Out] -((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(3*x^3) + (((7*I)/12)*a*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(1/4))/x^2 + (23*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(24*x) - ((17*I)/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ((17
*I)/8)*a^3*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x^4}\,dx = \int \frac{(1-iax)^{3/4}}{x^4(1+iax)^{3/4}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} + \frac{1}{3}\int \frac{-\frac{7ia}{2}-2a^2x}{x^3\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} + \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} - \frac{1}{6}\int \frac{\frac{23a^2}{4}-\frac{7}{2}ia^3x}{x^2\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} + \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} + \frac{1}{6}\int \frac{51i\ldots}{8x\sqrt[4]{1-iax}(\ldots}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} + \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} + \frac{1}{16}\left(17ia^3\right)\int \frac{\ldots}{x\sqrt[4]{\ldots}}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} + \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} + \frac{1}{4}\left(17ia^3\right)\mathrm{Subst}\Big(\ldots$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} + \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} - \frac{1}{8}\left(17ia^3\right)\mathrm{Subst}\Big(\ldots$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} + \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} - \frac{17}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1\ldots}}{\sqrt[4]{1\ldots}}\right.$$

Mathematica [C] time = 0.0234939, size = 93, normalized size = 0.55

$$\frac{(1-iax)^{3/4}\left(-34ia^3x^3\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{ax+i}{-ax+i}\right)+23ia^3x^3+9a^2x^2+6iax-8\right)}{24x^3(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x^4),x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(-8 + (6*I)*a*x + 9*a^2*x^2 + (23*I)*a^3*x^3 - (34*I)*a^
3*x^3*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1 + I*
a*x)^(3/4))
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

Fricas [A] time = 2.09351, size = 446, normalized size = 2.62

$$\frac{-51i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+51\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-51\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+51i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-\right.}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(-51*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 51*a^3*x
^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 51*a^3*x^3*log(sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I)) - I) + 51*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/
(a*x + I)) - 1) + (-46*I*a^3*x^3 + 74*a^2*x^2 + 44*I*a*x - 16)*sqrt(I*sqrt(
```

```
a^2*x^2 + 1)/(a*x + I)))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x^4*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

3.105 $\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=202

$$\frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2}-\frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{64x}-\frac{123}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{123}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{3ia(1-i}{}$$

```
[Out] -((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*x^4) + (((3*I)/8)*a*(1 - I*a*x)^(
3/4)*(1 + I*a*x)^(1/4))/x^3 + (15*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/
(32*x^2) - (((63*I)/64)*a^3*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - (123*a
^4*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (123*a^4*ArcTanh[(1 +
I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64
```

Rubi [A] time = 0.0812171, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5062, 99, 151, 12, 93, 212, 206, 203}

$$\frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2}-\frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{64x}-\frac{123}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{123}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{3ia(1-i}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x^5),x]
```

```
[Out] -((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*x^4) + (((3*I)/8)*a*(1 - I*a*x)^(
3/4)*(1 + I*a*x)^(1/4))/x^3 + (15*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/
(32*x^2) - (((63*I)/64)*a^3*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - (123*a
^4*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (123*a^4*ArcTanh[(1 +
I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x^5}dx = \int \frac{(1-iax)^{3/4}}{x^5(1+iax)^{3/4}}dx$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{1}{4}\int \frac{-\frac{9ia}{2}-3a^2x}{x^4\sqrt[4]{1-iax}(1+iax)^{3/4}}dx$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} - \frac{1}{12}\int \frac{\frac{45a^2}{4}-9ia^3x}{x^3\sqrt[4]{1-iax}(1+iax)^{3/4}}dx$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} + \frac{1}{24}\int \frac{\frac{189ia^3}{8}+4\ldots}{x^2\sqrt[4]{1-iax}(1-\ldots}$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} - \frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1-\ldots}}{64x}$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} - \frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1-\ldots}}{64x}$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} - \frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1-\ldots}}{64x}$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} - \frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1-\ldots}}{64x}$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} - \frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1-\ldots}}{64x}$$

Mathematica [C] time = 0.0323301, size = 99, normalized size = 0.49

$$\frac{(1-iax)^{3/4}\left(-82a^4x^4\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{ax+i}{-ax+i}\right)+63a^4x^4-33ia^3x^3+6a^2x^2+8iax-16\right)}{64x^4(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x^5),x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(-16 + (8*I)*a*x + 6*a^2*x^2 - (33*I)*a^3*x^3 + 63*a^4*x
^4 - 82*a^4*x^4*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(64*x
^4*(1 + I*a*x)^(3/4))
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^5*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

Fricas [A] time = 2.10922, size = 473, normalized size = 2.34

$$-\frac{123\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+123i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-123i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-123\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a}}{a}}\right.}{128\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/128*(123*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 123*I*a^
4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 123*I*a^4*x^4*log(sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 123*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2
+ 1)/(a*x + I)) - 1) + (126*a^4*x^4 + 186*I*a^3*x^3 - 108*a^2*x^2 - 80*I*a*
x + 32)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(1/(x^5*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

3.106 $\int e^{-\frac{5}{2} i \tan^{-1}(ax)} x^3 \, dx$

Optimal. Leaf size=373

$$-\frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{96a^4}+\frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}}{}\right.}{128\sqrt{2}a^4}$$

```
[Out] ((4*I)*x^3*(1 - I*a*x)^(5/4))/(a*(1 + I*a*x)^(1/4)) + (475*(1 - I*a*x)^(1/4
)*(1 + I*a*x)^(3/4))/(64*a^4) - (17*x^2*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4)
)/(4*a^2) - ((I/96)*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4)*(521*I + 452*a*x))/
a^4 + (475*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(64*S
qrt[2]*a^4) - (475*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)
])/(64*Sqrt[2]*a^4) + (475*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[
2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4) - (475*Log[1 +
Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)])/(128*Sqrt[2]*a^4)
```

Rubi [A] time = 0.254462, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5062, 97, 153, 147, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{96a^4}+\frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}}{}\right.}{128\sqrt{2}a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(((5*I)/2)*ArcTan[a*x]),x]
```



```
[Out] ((4*I)*x^3*(1 - I*a*x)^(5/4))/(a*(1 + I*a*x)^(1/4)) + (475*(1 - I*a*x)^(1/4
)*(1 + I*a*x)^(3/4))/(64*a^4) - (17*x^2*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4)
)/(4*a^2) - ((I/96)*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4)*(521*I + 452*a*x))/
a^4 + (475*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(64*S
qrt[2]*a^4) - (475*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)
])/(64*Sqrt[2]*a^4) + (475*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[
2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4) - (475*Log[1 +
Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)])/(128*Sqrt[2]*a^4)
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x^3\,dx = \int \frac{x^3(1-iax)^{5/4}}{(1+iax)^{5/4}}\,dx$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} - \frac{(4i)\int \frac{x^2\sqrt[4]{1-iax}\left(3-\frac{17iax}{4}\right)}{\sqrt[4]{1+iax}}\,dx}{a}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i\int \frac{x\sqrt[4]{1-iax}\left(\frac{17ia}{2}+\frac{113a^2x}{8}\right)}{\sqrt[4]{1+iax}}\,dx}{a^3}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax)^{3/4}(521i+452ax)}{96a^4} + \frac{(475i}{}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax}{96}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax}{96}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax}{96}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax}{96}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax}{96}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax}{96}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax}{96}$$

Mathematica [C] time = 0.0412575, size = 100, normalized size = 0.27

$$-\frac{\sqrt[4]{1-iax}(ax+i)^2\left(3\left(6a^2x^2+5iax+59\right)-95\,2^{3/4}\sqrt[4]{1+iax}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}(1-iax)\right)\right)}{72a^4\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] -((1 - I*a*x)^(1/4)*(I + a*x)^2*(3*(59 + (5*I)*a*x + 6*a^2*x^2) - 95*2^(3/4
)*(1 + I*a*x)^(1/4)*Hypergeometric2F1[1/4, 9/4, 13/4, (1 - I*a*x)/2]))/(72*
a^4*(1 + I*a*x)^(1/4))
```

Maple [F] time = 0.179, size = 0, normalized size = 0.

$$\int x^3\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Fricas [A] time = 2.06684, size = 876, normalized size = 2.35

$$96\left(a^5x-i\,a^4\right)\sqrt{\frac{225625i}{4096\,a^8}}\log\left(\frac{64}{475}i\,a^4\sqrt{\frac{225625i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-96\left(a^5x-i\,a^4\right)\sqrt{\frac{225625i}{4096\,a^8}}\log\left(-\frac{64}{475}i\,a^4\sqrt{\frac{225625i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(96*(a^5*x - I*a^4)*sqrt(225625/4096*I/a^8)*log(64/475*I*a^4*sqrt(225
625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*(a^5*x - I*a^4)
*sqrt(225625/4096*I/a^8)*log(-64/475*I*a^4*sqrt(225625/4096*I/a^8) + sqrt(I
*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*(a^5*x - I*a^4)*sqrt(-225625/4096*I/a^8
)*log(64/475*I*a^4*sqrt(-225625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
 + I))) + 96*(a^5*x - I*a^4)*sqrt(-225625/4096*I/a^8)*log(-64/475*I*a^4*sqr
t(-225625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (48*I*a^4*x^
4 - 136*a^3*x^3 - 226*I*a^2*x^2 + 521*a*x - 2467*I)*sqrt(a^2*x^2 + 1)*sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I)))/(a^5*x - I*a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

3.107 $\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=371

$$-\frac{i(1+iax)^{3/4}(1-iax)^{9/4}}{3a^3}-\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}-\frac{11i(1+iax)^{3/4}(1-iax)^{5/4}}{4a^3}-\frac{55i(1+iax)^{3/4}\sqrt[4]{1-iax}}{8a^3}-\frac{55i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.}{16\sqrt{2}a^3}$$

```
[Out] ((-2*I)*(1 - I*a*x)^(9/4))/(a^3*(1 + I*a*x)^(1/4)) - (((55*I)/8)*(1 - I*a*x
)^(1/4)*(1 + I*a*x)^(3/4))/a^3 - (((11*I)/4)*(1 - I*a*x)^(5/4)*(1 + I*a*x)^
(3/4))/a^3 - ((I/3)*(1 - I*a*x)^(9/4)*(1 + I*a*x)^(3/4))/a^3 - (((55*I)/8)*
ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) +
(((55*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqr
t[2]*a^3) - (((55*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (((55*I)/16)*Log[1
+ Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)
^(1/4)])/(Sqrt[2]*a^3)
```

Rubi [A] time = 0.241132, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5062, 89, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{i(1+iax)^{3/4}(1-iax)^{9/4}}{3a^3}-\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}-\frac{11i(1+iax)^{3/4}(1-iax)^{5/4}}{4a^3}-\frac{55i(1+iax)^{3/4}\sqrt[4]{1-iax}}{8a^3}-\frac{55i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.}{16\sqrt{2}a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(((5*I)/2)*ArcTan[a*x]),x]
```


```
[Out] ((-2*I)*(1 - I*a*x)^(9/4))/(a^3*(1 + I*a*x)^(1/4)) - (((55*I)/8)*(1 - I*a*x
)^(1/4)*(1 + I*a*x)^(3/4))/a^3 - (((11*I)/4)*(1 - I*a*x)^(5/4)*(1 + I*a*x)^
(3/4))/a^3 - ((I/3)*(1 - I*a*x)^(9/4)*(1 + I*a*x)^(3/4))/a^3 - (((55*I)/8)*
ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) +
(((55*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqr
t[2]*a^3) - (((55*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (((55*I)/16)*Log[1
+ Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)
^(1/4)])/(Sqrt[2]*a^3)
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x^2\,dx = \int \frac{x^2(1-iax)^{5/4}}{(1+iax)^{5/4}}\,dx$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} + \frac{(2i)\int \frac{(1-iax)^{5/4}\left(-\frac{5ia}{2}-\frac{a^2x}{2}\right)}{\sqrt[4]{1+iax}}\,dx}{a^3}$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} + \frac{11\int \frac{(1-iax)^{5/4}}{\sqrt[4]{1+iax}}\,dx}{2a^2}$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} + \frac{55\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{8a^2}$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} +$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} +$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} +$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} +$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} +$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} -$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} -$$

Mathematica [C] time = 0.0302484, size = 91, normalized size = 0.25

$$-\frac{\sqrt[4]{1-iax}(ax+i)^2\left(11i2^{3/4}\sqrt[4]{1+iax}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}(1-iax)\right)+3ax-21i\right)}{9a^3\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] -((1 - I*a*x)^(1/4)*(I + a*x)^2*(-21*I + 3*a*x + (11*I)*2^(3/4)*(1 + I*a*x)
^(1/4)*Hypergeometric2F1[1/4, 9/4, 13/4, (1 - I*a*x)/2]))/(9*a^3*(1 + I*a*x
)^(1/4))
```

Maple [F] time = 0.173, size = 0, normalized size = 0.

$$\int x^2\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Fricas [A] time = 1.86518, size = 783, normalized size = 2.11

$$12\left(a^4x-i\,a^3\right)\sqrt{\frac{3025i}{64\,a^6}}\log\left(\frac{8}{55}\,a^3\sqrt{\frac{3025i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\left(a^4x-i\,a^3\right)\sqrt{\frac{3025i}{64\,a^6}}\log\left(-\frac{8}{55}\,a^3\sqrt{\frac{3025i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\left(a^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(12*(a^4*x - I*a^3)*sqrt(3025/64*I/a^6)*log(8/55*a^3*sqrt(3025/64*I/a^
6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*(a^4*x - I*a^3)*sqrt(3025/64
*I/a^6)*log(-8/55*a^3*sqrt(3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
```

```
 I))) - 12*(a^4*x - I*a^3)*sqrt(-3025/64*I/a^6)*log(8/55*a^3*sqrt(-3025/64*
I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*(a^4*x - I*a^3)*sqrt(-30
25/64*I/a^6)*log(-8/55*a^3*sqrt(-3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/
(a*x + I))) + (8*I*a^3*x^3 - 26*a^2*x^2 - 61*I*a*x - 287)*sqrt(a^2*x^2 + 1)
*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/(a^4*x - I*a^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

3.108 $\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=324

$$-\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}-\frac{5(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}-\frac{25(1+iax)^{3/4}\sqrt[4]{1-iax}}{4a^2}-\frac{25\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{25\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right.}{8\sqrt{}}$$

```
[Out] (-2*(1 - I*a*x)^(9/4))/(a^2*(1 + I*a*x)^(1/4)) - (25*(1 - I*a*x)^(1/4)*(1 +
 I*a*x)^(3/4))/(4*a^2) - (5*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4))/(2*a^2) -
(25*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(4*Sqrt[2]*a
^2) + (25*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(4*Sqr
t[2]*a^2) - (25*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a
*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2) + (25*Log[1 + Sqrt[1 - I*a*x
]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt
[2]*a^2)
```

Rubi [A] time = 0.205937, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5062, 78, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}-\frac{5(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}-\frac{25(1+iax)^{3/4}\sqrt[4]{1-iax}}{4a^2}-\frac{25\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{25\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right.}{8\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (-2*(1 - I*a*x)^(9/4))/(a^2*(1 + I*a*x)^(1/4)) - (25*(1 - I*a*x)^(1/4)*(1 +
 I*a*x)^(3/4))/(4*a^2) - (5*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4))/(2*a^2) -
(25*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(4*Sqrt[2]*a
^2) + (25*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(4*Sqr
t[2]*a^2) - (25*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a
*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2) + (25*Log[1 + Sqrt[1 - I*a*x
]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt
[2]*a^2)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x\,dx = \int \frac{x(1-iax)^{5/4}}{(1+iax)^{5/4}}\,dx$$
$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{(5i)\int \frac{(1-iax)^{5/4}}{\sqrt[4]{1+iax}}\,dx}{a}$$
$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{(25i)\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{4a}$$
$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{(25i)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{8a}$$
$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{25\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{\ldots}\right.}{2a^2}$$
$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{25\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1\ldots}}{\sqrt[4]{1\ldots}}\right.}{2a^2}$$
$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{25\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1\ldots}}{\sqrt[4]{1\ldots}}\right.}{4a^2}$$
$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{25\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x\ldots\right.}{8a^2}$$
$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{25\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-\ldots}}{\sqrt[4]{1+i\ldots}}\right.}{8\sqrt{2}a^2}$$
$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{25\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} + \ldots$$

Mathematica [C] time = 0.027655, size = 63, normalized size = 0.19

$$\frac{2(1-iax)^{9/4}\left(5\,2^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}(1-iax)\right)-\frac{9}{\sqrt[4]{1+iax}}\right)}{9a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (2*(1 - I*a*x)^(9/4)*(-9/(1 + I*a*x)^(1/4) + 5*2^(3/4)*Hypergeometric2F1[1/
4, 9/4, 13/4, (1 - I*a*x)/2]))/(9*a^2)
```

Maple [F] time = 0.173, size = 0, normalized size = 0.

$$\int x\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Fricas [A] time = 1.78464, size = 757, normalized size = 2.34

$$-\frac{2\left(a^3x-i\,a^2\right)\sqrt{\frac{625i}{16\,a^4}}\log\left(\frac{4}{25}i\,a^2\sqrt{\frac{625i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\left(a^3x-i\,a^2\right)\sqrt{\frac{625i}{16\,a^4}}\log\left(-\frac{4}{25}i\,a^2\sqrt{\frac{625i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\left(a^3x-i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^3*x - I*a^2)*sqrt(625/16*I/a^4)*log(4/25*I*a^2*sqrt(625/16*I/a^4
) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*(a^3*x - I*a^2)*sqrt(625/16*I/
a^4)*log(-4/25*I*a^2*sqrt(625/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) - 2*(a^3*x - I*a^2)*sqrt(-625/16*I/a^4)*log(4/25*I*a^2*sqrt(-625/16*I/a
^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*(a^3*x - I*a^2)*sqrt(-625/16
*I/a^4)*log(-4/25*I*a^2*sqrt(-625/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
 + I))) - sqrt(a^2*x^2 + 1)*(2*I*a^2*x^2 - 9*a*x + 43*I)*sqrt(I*sqrt(a^2*x^
2 + 1)/(a*x + I)))/(a^3*x - I*a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

3.109 $\int e^{-\frac{5}{2}i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=299

$$\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}+\frac{5i(1+iax)^{3/4}\sqrt[4]{1-iax}}{a}+\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}+\frac{5i\tan^{-1}\left(1-\right.}{\sqrt{2}}$$

```
[Out] ((4*I)*(1 - I*a*x)^(5/4))/(a*(1 + I*a*x)^(1/4)) + ((5*I)*(1 - I*a*x)^(1/4)*
(1 + I*a*x)^(3/4))/a + ((5*I)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)])/(Sqrt[2]*a) - ((5*I)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(
1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (((5*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1
+ I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) - ((
(5*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/
4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rubi [A] time = 0.171722, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5061, 47, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}+\frac{5i(1+iax)^{3/4}\sqrt[4]{1-iax}}{a}+\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}+\frac{5i\tan^{-1}\left(1-\right.}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((-5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((4*I)*(1 - I*a*x)^(5/4))/(a*(1 + I*a*x)^(1/4)) + ((5*I)*(1 - I*a*x)^(1/4)*
(1 + I*a*x)^(3/4))/a + ((5*I)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)])/(Sqrt[2]*a) - ((5*I)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(
1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (((5*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1
+ I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) - ((
(5*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/
4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{5}{2}i\tan^{-1}(ax)}\,dx = \int \frac{(1-iax)^{5/4}}{(1+iax)^{5/4}}\,dx$$
$$= \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} - 5\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx$$
$$= \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{5}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$
$$= \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{(10i)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{a}$$
$$= \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{(10i)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$
$$= \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{(5i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} - \frac{(5i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x \ldots\right.}{a}$$
$$= \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{(5i)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} - \frac{(5i)\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\ldots}\right.}{2a}$$
$$= \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} - \frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1\ldots}}{\sqrt[4]{1+\ldots}}\right.}{2\sqrt{2}a}$$
$$= \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{5i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \frac{5i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{5i\log\ldots}{}$$

Mathematica [C] time = 0.0465898, size = 39, normalized size = 0.13

$$\frac{8ie^{-\frac{1}{2}i\tan^{-1}(ax)}\text{Hypergeometric2F1}\left(-\frac{1}{4},2,\frac{3}{4},-e^{2i\tan^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((-5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((8*I)*Hypergeometric2F1[-1/4, 2, 3/4, -E^((2*I)*ArcTan[a*x])])/(a*E^((I/2)
*ArcTan[a*x]))
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int \left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(-5/2), x)
```

Fricas [A] time = 1.70973, size = 644, normalized size = 2.15

$$-\frac{(a^2x-ia)\sqrt{\frac{25i}{a^2}}\log\left(\frac{1}{5}a\sqrt{\frac{25i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-(a^2x-ia)\sqrt{\frac{25i}{a^2}}\log\left(-\frac{1}{5}a\sqrt{\frac{25i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-(a^2x-ia)\sqrt{-\frac{25i}{a^2}}\log\Big(}{2\,(a^2x-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2*((a^2*x - I*a)*sqrt(25*I/a^2)*log(1/5*a*sqrt(25*I/a^2) + sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I))) - (a^2*x - I*a)*sqrt(25*I/a^2)*log(-1/5*a*sqrt(25*I
/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (a^2*x - I*a)*sqrt(-25*I/a^2
)*log(1/5*a*sqrt(-25*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (a^2*x
 - I*a)*sqrt(-25*I/a^2)*log(-1/5*a*sqrt(-25*I/a^2) + sqrt(I*sqrt(a^2*x^2 +
1)/(a*x + I))) + 2*sqrt(a^2*x^2 + 1)*(-I*a*x - 9)*sqrt(I*sqrt(a^2*x^2 + 1)/
(a*x + I)))/(a^2*x - I*a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(-5/2), x)
```

3.110 $\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=293

$$\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+i}}\right.$$

```
[Out] (8*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4) + 2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I
*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] -
 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + Log[1 + Sqrt[1 - I*a*x]/S
qrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] - L
og[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.218536, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5062, 98, 21, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+i}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x),x]
```

```
[Out] (8*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4) + 2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I
*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] -
 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + Log[1 + Sqrt[1 - I*a*x]/S
qrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] - L
og[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)]/Sqrt[2]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
```

```
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x}\,dx = \int \frac{(1-iax)^{5/4}}{x(1+iax)^{5/4}}\,dx$$
$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(4i)\int \frac{\frac{ia}{4}-\frac{a^2x}{4}}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{a}$$
$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + \int \frac{(1+iax)^{3/4}}{x(1-iax)^{3/4}}\,dx$$
$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + (ia)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx + \int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$
$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - 4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right) + 4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$
$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 4\,\text{Sub}$$
$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) -$$
$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} +$$
$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\log\left(1\right.}{}$$
$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) -$$

Mathematica [C] time = 0.0581778, size = 106, normalized size = 0.36

$$\frac{\sqrt[4]{1-iax}\left(-20\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{ax+i}{-ax+i}\right) + 2^{3/4}(1-iax)\sqrt[4]{1+iax}\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-iax)\right.\right.}{5\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(20 - 20*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a
*x)] + 2^(3/4)*(1 - I*a*x)*(1 + I*a*x)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/
4, (1 - I*a*x)/2]))/(5*(1 + I*a*x)^(1/4))
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

Fricas [A] time = 1.86325, size = 879, normalized size = 3.

$$\sqrt{4i}(ax-i)\log\left(\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-\sqrt{4i}(ax-i)\log\left(-\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-\sqrt{-4i}(ax-i)\log\left(\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] (sqrt(4*I)*(a*x - I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I))) - sqrt(4*I)*(a*x - I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/
(a*x + I))) - sqrt(-4*I)*(a*x - I)*log(1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I))) + sqrt(-4*I)*(a*x - I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I))) - (2*a*x - 2*I)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I)) + 1) - 2*(-I*a*x - 1)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I
) - 2*(I*a*x + 1)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + (2*a*x - 2
*I)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 16*I*sqrt(a^2*x^2 + 1)*s
qrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/(2*a*x - 2*I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(1/(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

3.111 $\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=121

$$-\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}}-\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-5ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+5ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] ((-10*I)*a*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4) - (1 - I*a*x)^(5/4)/(x*(1 +
 I*a*x)^(1/4)) - (5*I)*a*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + (5*I
)*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rubi [A] time = 0.0392875, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5062, 94, 93, 298, 203, 206}

$$-\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}}-\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-5ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+5ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x^2),x]
```

```
[Out] ((-10*I)*a*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4) - (1 - I*a*x)^(5/4)/(x*(1 +
 I*a*x)^(1/4)) - (5*I)*a*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + (5*I
)*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1-iax)^{5/4}}{x^2(1+iax)^{5/4}}\,dx \\
&= -\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} - \frac{1}{2}(5ia)\int \frac{\sqrt[4]{1-iax}}{x(1+iax)^{5/4}}\,dx \\
&= -\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} - \frac{1}{2}(5ia)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx \\
&= -\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} - (10ia)\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} + (5ia)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - (5ia)\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\right. \\
&= -\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} - 5ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 5ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0165903, size = 69, normalized size = 0.57

$$\frac{i\sqrt[4]{1-iax}\left(10ax\,\mathrm{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{ax+i}{-ax+i}\right)-9ax+i\right)}{x\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x^2),x]
```

```
[Out] (I*(1 - I*a*x)^(1/4)*(I - 9*a*x + 10*a*x*Hypergeometric2F1[1/4, 1, 5/4, (I
 + a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(1/4))
```

Maple [F] time = 0.175, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

Fricas [B] time = 1.81714, size = 491, normalized size = 4.06

$$-\frac{\sqrt{a^2x^2+1}(18\,ax-2i)\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}+5\left(-i\,a^2x^2-ax\right)\log\left(\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}+1\right)-\left(5\,a^2x^2-5i\,ax\right)\log\left(\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}+i\right)+\left(5\,a^2x\right.}{2\left(ax^2-i\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(a^2*x^2 + 1)*(18*a*x - 2*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
+ 5*(-I*a^2*x^2 - a*x)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - (5*a^
2*x^2 - 5*I*a*x)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + (5*a^2*x^2
- 5*I*a*x)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + 5*(I*a^2*x^2 + a*
x)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1))/(a*x^2 - I*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

3.112 $\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=163

$$-\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}}-\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}}+\frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}}$$

```
[Out] (-25*a^2*(1 - I*a*x)^(1/4))/(2*(1 + I*a*x)^(1/4)) + (((5*I)/4)*a*(1 - I*a*x
)^(5/4))/(x*(1 + I*a*x)^(1/4)) - (1 - I*a*x)^(9/4)/(2*x^2*(1 + I*a*x)^(1/4)
) - (25*a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4 + (25*a^2*ArcTan
h[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rubi [A] time = 0.0504077, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5062, 96, 94, 93, 298, 203, 206}

$$-\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}}-\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}}+\frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x^3),x]
```


```
[Out] (-25*a^2*(1 - I*a*x)^(1/4))/(2*(1 + I*a*x)^(1/4)) + (((5*I)/4)*a*(1 - I*a*x
)^(5/4))/(x*(1 + I*a*x)^(1/4)) - (1 - I*a*x)^(9/4)/(2*x^2*(1 + I*a*x)^(1/4)
) - (25*a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4 + (25*a^2*ArcTan
h[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1-iax)^{5/4}}{x^3(1+iax)^{5/4}}\,dx \\
&= -\frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} - \frac{1}{4}(5ia)\int \frac{(1-iax)^{5/4}}{x^2(1+iax)^{5/4}}\,dx \\
&= \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} - \frac{1}{8}\left(25a^2\right)\int \frac{\sqrt[4]{1-iax}}{x(1+iax)^{5/4}}\,dx \\
&= -\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}} + \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} - \frac{1}{8}\left(25a^2\right)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx \\
&= -\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}} + \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} - \frac{1}{2}\left(25a^2\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}} + \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} + \frac{1}{4}\left(25a^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{1}{4} \\
&= -\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}} + \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} - \frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+ia}}{\sqrt[4]{1-ia}}\right.
\end{aligned}
$$

Mathematica [C] time = 0.020729, size = 81, normalized size = 0.5

$$
\frac{\sqrt[4]{1-iax}\left(50a^2x^2\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{ax+i}{-ax+i}\right) - 43a^2x^2 + 9iax - 2\right)}{4x^2\sqrt[4]{1+iax}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x^3),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-2 + (9*I)*a*x - 43*a^2*x^2 + 50*a^2*x^2*Hypergeometric
2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(4*x^2*(1 + I*a*x)^(1/4))
```

Maple [F] time = 0.172, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

Fricas [B] time = 1.79016, size = 543, normalized size = 3.33

$$\frac{\sqrt{a^2x^2+1}\left(86i\,a^2x^2+18\,ax+4i\right)\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}+25\left(a^3x^3-i\,a^2x^2\right)\log\left(\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}+1\right)+\left(-25i\,a^3x^3-25\,a^2x^2\right)\log\left(\sqrt{\cdot}\right.}{8\left(ax^3-i\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(a^2*x^2 + 1)*(86*I*a^2*x^2 + 18*a*x + 4*I)*sqrt(I*sqrt(a^2*x^2 +
1)/(a*x + I)) + 25*(a^3*x^3 - I*a^2*x^2)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
+ I)) + 1) + (-25*I*a^3*x^3 - 25*a^2*x^2)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
 + I)) + I) + (25*I*a^3*x^3 + 25*a^2*x^2)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
 + I)) - I) - 25*(a^3*x^3 - I*a^2*x^2)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I)) - 1))/(a*x^3 - I*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

3.113 $\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=203

$$\frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}}+\frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}}+\frac{55}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{55}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}}-\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}$$

```
[Out] (((287*I)/24)*a^3*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4) - (1 - I*a*x)^(1/4)/
(3*x^3*(1 + I*a*x)^(1/4)) + (((13*I)/12)*a*(1 - I*a*x)^(1/4))/(x^2*(1 + I*a
*x)^(1/4)) + (61*a^2*(1 - I*a*x)^(1/4))/(24*x*(1 + I*a*x)^(1/4)) + ((55*I)/
8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ((55*I)/8)*a^3*ArcTanh
[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rubi [A] time = 0.0789914, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5062, 98, 151, 155, 12, 93, 298, 203, 206}

$$\frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}}+\frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}}+\frac{55}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{55}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}}-\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x^4),x]
```

```
[Out] (((287*I)/24)*a^3*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4) - (1 - I*a*x)^(1/4)/
(3*x^3*(1 + I*a*x)^(1/4)) + (((13*I)/12)*a*(1 - I*a*x)^(1/4))/(x^2*(1 + I*a
*x)^(1/4)) + (61*a^2*(1 - I*a*x)^(1/4))/(24*x*(1 + I*a*x)^(1/4)) + ((55*I)/
8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ((55*I)/8)*a^3*ArcTanh
[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x^4}\,dx = \int \frac{(1-iax)^{5/4}}{x^4(1+iax)^{5/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} - \frac{1}{3}\int \frac{\frac{13ia}{2}+6a^2x}{x^3(1-iax)^{3/4}(1+iax)^{5/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{1}{6}\int \frac{-\frac{61a^2}{4}+13ia^3x}{x^2(1-iax)^{3/4}(1+iax)^{5/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} - \frac{1}{6}\int \frac{-\frac{165ia^3}{8}-\frac{61a^4x}{4}}{x(1-iax)^{3/4}(1+iax)^{5/4}}\,dx$$

$$= \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} + \frac{i\int \frac{165a^4}{16x(1-iax)^{3/4}\sqrt[4]{1+iax}}dx}{3a}$$

$$= \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} + \frac{1}{16}\left(55ia^3\right)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{}}$$

$$= \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} + \frac{1}{4}\left(55ia^3\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\right.$$

$$= \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} - \frac{1}{8}\left(55ia^3\right)\text{Subst}\left(\int \frac{1}{1-x^2}\right.$$

$$= \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} + \frac{55}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{5}{8}$$

Mathematica [C] time = 0.0249633, size = 93, normalized size = 0.46

$$\frac{\sqrt[4]{1-iax}\left(-330ia^3x^3\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{ax+i}{-ax+i}\right)+287ia^3x^3+61a^2x^2+26iax-8\right)}{24x^3\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x^4),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-8 + (26*I)*a*x + 61*a^2*x^2 + (287*I)*a^3*x^3 - (330*I
)*a^3*x^3*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1
+ I*a*x)^(1/4))
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

Fricas [A] time = 1.79354, size = 574, normalized size = 2.83

$$\frac{\left(574\,a^3x^3-122i\,a^2x^2+52\,ax+16i\right)\sqrt{a^2x^2+1}\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-165\left(i\,a^4x^4+a^3x^3\right)\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-\left(165\,a^4x^4-165i\,a^{}\right.}{48\left(ax^4-i\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*((574*a^3*x^3 - 122*I*a^2*x^2 + 52*a*x + 16*I)*sqrt(a^2*x^2 + 1)*sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 165*(I*a^4*x^4 + a^3*x^3)*log(sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I)) + 1) - (165*a^4*x^4 - 165*I*a^3*x^3)*log(sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I)) + I) + (165*a^4*x^4 - 165*I*a^3*x^3)*log(sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I)) - I) - 165*(-I*a^4*x^4 - a^3*x^3)*log(sqrt(I*sq
rt(a^2*x^2 + 1)/(a*x + I)) - 1))/(a*x^4 - I*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x^4*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

3.114 $\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=233

$$\frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}}+\frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}}-\frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}}+\frac{475}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{475}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{17ia}{24x^3}$$

```
[Out] (2467*a^4*(1 - I*a*x)^(1/4))/(192*(1 + I*a*x)^(1/4)) - (1 - I*a*x)^(1/4)/(4
*x^4*(1 + I*a*x)^(1/4)) + (((17*I)/24)*a*(1 - I*a*x)^(1/4))/(x^3*(1 + I*a*x
)^(1/4)) + (113*a^2*(1 - I*a*x)^(1/4))/(96*x^2*(1 + I*a*x)^(1/4)) - (((521*
I)/192)*a^3*(1 - I*a*x)^(1/4))/(x*(1 + I*a*x)^(1/4)) + (475*a^4*ArcTan[(1 +
 I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (475*a^4*ArcTanh[(1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)])/64
```

Rubi [A] time = 0.0989286, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5062, 98, 151, 155, 12, 93, 298, 203, 206}

$$\frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}}+\frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}}-\frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}}+\frac{475}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{475}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{17ia}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x^5),x]
```

```
[Out] (2467*a^4*(1 - I*a*x)^(1/4))/(192*(1 + I*a*x)^(1/4)) - (1 - I*a*x)^(1/4)/(4
*x^4*(1 + I*a*x)^(1/4)) + (((17*I)/24)*a*(1 - I*a*x)^(1/4))/(x^3*(1 + I*a*x
)^(1/4)) + (113*a^2*(1 - I*a*x)^(1/4))/(96*x^2*(1 + I*a*x)^(1/4)) - (((521*
I)/192)*a^3*(1 - I*a*x)^(1/4))/(x*(1 + I*a*x)^(1/4)) + (475*a^4*ArcTan[(1 +
 I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (475*a^4*ArcTanh[(1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)])/64
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x^5}\,dx = \int \frac{(1-iax)^{5/4}}{x^5(1+iax)^{5/4}}\,dx$$
$$= -\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} - \frac{1}{4}\int \frac{\frac{17ia}{2}+8a^2x}{x^4(1-iax)^{3/4}(1+iax)^{5/4}}\,dx$$
$$= -\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{1}{12}\int \frac{-\frac{113a^2}{4}+\frac{51}{2}ia^3x}{x^3(1-iax)^{3/4}(1+iax)^{5/4}}\,dx$$
$$= -\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{1}{24}\int \frac{-\frac{521ia^3}{8}-\frac{113a^4x}{2}}{x^2(1-iax)^{3/4}(1+iax)^{5/4}}\,dx$$
$$= -\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} + \frac{1}{24}\int \frac{\frac{1425a^4}{16}-\frac{521}{8}ia^5x}{x(1-iax)^{3/4}(1+iax)^{5/4}}\,dx$$
$$= \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} - \frac{i\int \ldots}{32x(1\ldots}$$
$$= \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} + \frac{1}{128}(47\ldots$$
$$= \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} + \frac{1}{32}(475\ldots$$
$$= \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} - \frac{1}{64}(475\ldots$$
$$= \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} + \frac{475}{64}a^4\,\mathrm{t}\ldots$$

Mathematica [C] time = 0.0325974, size = 99, normalized size = 0.42

$$\frac{\sqrt[4]{1-iax}\left(-2850a^4x^4\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{ax+i}{-ax+i}\right)+2467a^4x^4-521ia^3x^3+226a^2x^2+136iax-48\right)}{192x^4\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x^5),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-48 + (136*I)*a*x + 226*a^2*x^2 - (521*I)*a^3*x^3 + 246
7*a^4*x^4 - 2850*a^4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)
]))/(192*x^4*(1 + I*a*x)^(1/4))
```

Maple [F] time = 0.179, size = 0, normalized size = 0.

$$\int \frac{1}{x^5}\left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5 \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^5*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

Fricas [A] time = 1.75454, size = 610, normalized size = 2.62

$$\frac{\left(-4934i\, a^4x^4 - 1042\, a^3x^3 - 452i\, a^2x^2 + 272\, ax + 96i\right)\sqrt{a^2x^2+1}\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}} - \left(1425\, a^5x^5 - 1425i\, a^4x^4\right)\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}} + \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/384*((-4934*I*a^4*x^4 - 1042*a^3*x^3 - 452*I*a^2*x^2 + 272*a*x + 96*I)*sq
rt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - (1425*a^5*x^5 - 1425*
I*a^4*x^4)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 1425*(-I*a^5*x^5
- a^4*x^4)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 1425*(I*a^5*x^5 +
 a^4*x^4)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + (1425*a^5*x^5 - 14
25*I*a^4*x^4)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1))/(a*x^5 - I*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5 \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(1/(x^5*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

3.115 $\int e^{\frac{1}{3}i\tan^{-1}(x)}x^2\,dx$

Optimal. Leaf size=319

$$\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}-\frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6}-\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix}-\frac{19i\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{108\sqrt{3}}+\frac{19i\log\left(\ldots\right.}{\ldots}$$

```
[Out] ((-19*I)/54)*(1 - I*x)^(5/6)*(1 + I*x)^(1/6) - (I/18)*(1 - I*x)^(5/6)*(1 +
I*x)^(7/6) + ((1 - I*x)^(5/6)*(1 + I*x)^(7/6)*x)/3 + ((19*I)/162)*ArcTan[Sq
rt[3] - (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] - ((19*I)/162)*ArcTan[Sqrt[3]
+ (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] - ((19*I)/81)*ArcTan[(1 - I*x)^(1/6)
/(1 + I*x)^(1/6)] - (((19*I)/108)*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3) -
 (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)])/Sqrt[3] + (((19*I)/108)*Log[1
+ (1 - I*x)^(1/3)/(1 + I*x)^(1/3) + (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/
6)])/Sqrt[3]
```

Rubi [A] time = 0.384128, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {5062, 90, 80, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}-\frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6}-\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix}-\frac{19i\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{108\sqrt{3}}+\frac{19i\log\left(\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/3)*ArcTan[x])*x^2,x]
```

```
[Out] ((-19*I)/54)*(1 - I*x)^(5/6)*(1 + I*x)^(1/6) - (I/18)*(1 - I*x)^(5/6)*(1 +
I*x)^(7/6) + ((1 - I*x)^(5/6)*(1 + I*x)^(7/6)*x)/3 + ((19*I)/162)*ArcTan[Sq
rt[3] - (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] - ((19*I)/162)*ArcTan[Sqrt[3]
+ (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] - ((19*I)/81)*ArcTan[(1 - I*x)^(1/6)
/(1 + I*x)^(1/6)] - (((19*I)/108)*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3) -
 (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)])/Sqrt[3] + (((19*I)/108)*Log[1
+ (1 - I*x)^(1/3)/(1 + I*x)^(1/3) + (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/
6)])/Sqrt[3]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{3}i\tan^{-1}(x)}x^2\,dx = \int \frac{\sqrt[6]{1+ix}x^2}{\sqrt[6]{1-ix}}\,dx$$

$$= \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x + \frac{1}{3}\int \frac{\left(-1-\frac{ix}{3}\right)\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\,dx$$

$$= -\frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{54}\int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\,dx$$

$$= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{162}\int \frac{1}{\sqrt[6]{1-ix}(1+$$

$$= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{27}i\,\mathrm{Subst}\left(\int \frac{}{\left(2-\right.}$$

$$= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{27}i\,\mathrm{Subst}\left(\int \frac{x^4}{1+x}\right.$$

$$= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{81}i\,\mathrm{Subst}\left(\int \frac{1}{1+x}\right.$$

$$= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{81}i\tan^{-1}\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right.$$

$$= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{81}i\tan^{-1}\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right.$$

$$= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x + \frac{19}{162}i\tan^{-1}\left(\sqrt{3}-\right.$$

Mathematica [C] time = 0.0344584, size = 73, normalized size = 0.23

$$\frac{1}{90}(1-ix)^{5/6}\left(5\sqrt[6]{1+ix}\left(6ix^2+7x-i\right)-38i\sqrt[6]{2}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}-\frac{ix}{2}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/3)*ArcTan[x])*x^2,x]
```

```
[Out] ((1 - I*x)^(5/6)*(5*(1 + I*x)^(1/6)*(-I + 7*x + (6*I)*x^2) - (38*I)*2^(1/6)
*Hypergeometric2F1[-1/6, 5/6, 11/6, 1/2 - (I/2)*x]))/90
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \sqrt[3]{(1+ix)\frac{1}{\sqrt{x^2+1}}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

Fricas [A] time = 1.76186, size = 695, normalized size = 2.18

$$-\frac{19}{324}\left(-i\sqrt{3}+1\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)-\frac{19}{324}\left(-i\sqrt{3}-1\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)-\frac{19}{324}\left(i\sqrt{3}+1\right)\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="fricas")
```

```
[Out] -19/324*(-I*sqrt(3) + 1)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3)
+ 1/2*I) - 19/324*(-I*sqrt(3) - 1)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x +
I))^(1/3) - 1/2*I) - 19/324*(I*sqrt(3) + 1)*log(-1/2*sqrt(3) + (I*sqrt(x^2
+ 1)/(x + I))^(1/3) + 1/2*I) - 19/324*(I*sqrt(3) - 1)*log(-1/2*sqrt(3) + (I
*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2*I) + 1/324*(108*x^3 - 18*I*x^2 - 6*x -
132*I)*(I*sqrt(x^2 + 1)/(x + I))^(1/3) - 19/162*log((I*sqrt(x^2 + 1)/(x + I
))^(1/3) + I) + 19/162*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) - I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)*x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

3.116 $\int e^{\frac{1}{3}i\tan^{-1}(x)}x\,dx$

Optimal. Leaf size=278

$$\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}+\frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{12\sqrt{3}}-\frac{\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{12\sqrt{3}}-\frac{1}{18}\tan^{-1}\left(\sqrt{3}-\right.$$

```
[Out] ((1 - I*x)^(5/6)*(1 + I*x)^(1/6))/6 + ((1 - I*x)^(5/6)*(1 + I*x)^(7/6))/2 -
 ArcTan[Sqrt[3] - (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)]/18 + ArcTan[Sqrt[3]
+ (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)]/18 + ArcTan[(1 - I*x)^(1/6)/(1 + I*x
)^(1/6)]/9 + Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3) - (Sqrt[3]*(1 - I*x)^(
1/6))/(1 + I*x)^(1/6)]/(12*Sqrt[3]) - Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/
3) + (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)]/(12*Sqrt[3])
```

Rubi [A] time = 0.338368, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5062, 80, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}+\frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{12\sqrt{3}}-\frac{\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{12\sqrt{3}}-\frac{1}{18}\tan^{-1}\left(\sqrt{3}-\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/3)*ArcTan[x])*x,x]
```

```
[Out] ((1 - I*x)^(5/6)*(1 + I*x)^(1/6))/6 + ((1 - I*x)^(5/6)*(1 + I*x)^(7/6))/2 -
 ArcTan[Sqrt[3] - (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)]/18 + ArcTan[Sqrt[3]
+ (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)]/18 + ArcTan[(1 - I*x)^(1/6)/(1 + I*x
)^(1/6)]/9 + Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3) - (Sqrt[3]*(1 - I*x)^(
1/6))/(1 + I*x)^(1/6)]/(12*Sqrt[3]) - Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/
3) + (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)]/(12*Sqrt[3])
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{3}i\tan^{-1}(x)}x\,dx=\int\frac{\sqrt[6]{1+ix}x}{\sqrt[6]{1-ix}}\,dx$$
$$=\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}-\frac{1}{6}i\int\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\,dx$$
$$=\frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}-\frac{1}{18}i\int\frac{1}{\sqrt[6]{1-ix}(1+ix)^{5/6}}\,dx$$
$$=\frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}+\frac{1}{3}\operatorname{Subst}\left(\int\frac{x^4}{\left(2-x^6\right)^{5/6}}\,dx,x,\sqrt[6]{1-ix}\right)$$
$$=\frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}+\frac{1}{3}\operatorname{Subst}\left(\int\frac{x^4}{1+x^6}\,dx,x,\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)$$
$$=\frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}+\frac{1}{9}\operatorname{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{9}\operatorname{Subst}\left(\int\frac{-}{1-}\right.$$
$$=\frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}+\frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{36}\operatorname{Subst}\left(\int\frac{1}{1-\sqrt{3}x+x^2}\,dx,x\right.$$
$$=\frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}+\frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)}{12\sqrt{3}}-\frac{\log}{}$$
$$=\frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}-\frac{1}{18}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{18}\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right.$$

Mathematica [C] time = 0.022786, size = 57, normalized size = 0.21

$$\frac{1}{10}(1-ix)^{5/6}\left(2\sqrt[6]{2}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}-\frac{ix}{2}\right)+5(1+ix)^{7/6}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/3)*ArcTan[x])*x,x]
```

```
[Out] ((1 - I*x)^(5/6)*(5*(1 + I*x)^(7/6) + 2*2^(1/6)*Hypergeometric2F1[-1/6, 5/6
, 11/6, 1/2 - (I/2)*x]))/10
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int\sqrt[3]{(1+ix)\frac{1}{\sqrt{x^2+1}}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

Fricas [A] time = 1.78683, size = 645, normalized size = 2.32

$$-\frac{1}{36}\left(\sqrt{3}+i\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)-\frac{1}{36}\left(\sqrt{3}-i\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)+\frac{1}{36}\left(\sqrt{3}-i\right)\log\left(-\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x, algorithm="fricas")
```

```
[Out] -1/36*(sqrt(3) + I)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2
*I) - 1/36*(sqrt(3) - I)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3)
- 1/2*I) + 1/36*(sqrt(3) - I)*log(-1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^
(1/3) + 1/2*I) + 1/36*(sqrt(3) + I)*log(-1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x
+ I))^(1/3) - 1/2*I) + 1/6*(3*x^2 - I*x + 4)*(I*sqrt(x^2 + 1)/(x + I))^(1/3
) - 1/18*I*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + I) + 1/18*I*log((I*sqrt(x^
2 + 1)/(x + I))^(1/3) - I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)*x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x, algorithm="giac")
```

```
[Out] integrate(x*((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

3.117 $\int e^{\frac{1}{3}i\tan^{-1}(x)}\,dx$

Optimal. Leaf size=262

$$i(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{i\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{2\sqrt{3}}-\frac{i\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{2\sqrt{3}}-\frac{1}{3}i\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{3}i\tan^{-1}\left(\right.$$

```
[Out] I*(1 - I*x)^(5/6)*(1 + I*x)^(1/6) - (I/3)*ArcTan[Sqrt[3] - (2*(1 - I*x)^(1/
6))/(1 + I*x)^(1/6)] + (I/3)*ArcTan[Sqrt[3] + (2*(1 - I*x)^(1/6))/(1 + I*x)
^(1/6)] + ((2*I)/3)*ArcTan[(1 - I*x)^(1/6)/(1 + I*x)^(1/6)] + ((I/2)*Log[1
+ (1 - I*x)^(1/3)/(1 + I*x)^(1/3) - (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/
6)])/Sqrt[3] - ((I/2)*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3) + (Sqrt[3]*(1
 - I*x)^(1/6))/(1 + I*x)^(1/6)])/Sqrt[3]
```

Rubi [A] time = 0.310024, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5061, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$i(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{i\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{2\sqrt{3}}-\frac{i\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{2\sqrt{3}}-\frac{1}{3}i\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{3}i\tan^{-1}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/3)*ArcTan[x]),x]
```

```
[Out] I*(1 - I*x)^(5/6)*(1 + I*x)^(1/6) - (I/3)*ArcTan[Sqrt[3] - (2*(1 - I*x)^(1/
6))/(1 + I*x)^(1/6)] + (I/3)*ArcTan[Sqrt[3] + (2*(1 - I*x)^(1/6))/(1 + I*x)
^(1/6)] + ((2*I)/3)*ArcTan[(1 - I*x)^(1/6)/(1 + I*x)^(1/6)] + ((I/2)*Log[1
+ (1 - I*x)^(1/3)/(1 + I*x)^(1/3) - (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/
6)])/Sqrt[3] - ((I/2)*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3) + (Sqrt[3]*(1
 - I*x)^(1/6))/(1 + I*x)^(1/6)])/Sqrt[3]
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{\frac{1}{3}i\tan^{-1}(x)}\,dx &= \int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\,dx \\
&= i(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{1}{3}\int \frac{1}{\sqrt[6]{1-ix}(1+ix)^{5/6}}\,dx \\
&= i(1-ix)^{5/6}\sqrt[6]{1+ix}+2i\,\text{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\,dx,x,\sqrt[6]{1-ix}\right) \\
&= i(1-ix)^{5/6}\sqrt[6]{1+ix}+2i\,\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx,x,\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) \\
&= i(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{2}{3}i\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{2}{3}i\,\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx,x,\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right. \\
&= i(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{2}{3}i\tan^{-1}\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{6}i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx,x,\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{6}i\,\text{Subst}\left(\int\right. \\
&= i(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{2}{3}i\tan^{-1}\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)}{2\sqrt{3}}-\frac{i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)}{2\sqrt{3}} \\
&= i(1-ix)^{5/6}\sqrt[6]{1+ix}-\frac{1}{3}i\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{3}i\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{2}{3}i\tan^{-1}\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+
\end{aligned}
$$

Mathematica [C] time = 0.0167512, size = 34, normalized size = 0.13

$$-\frac{12}{7}ie^{\frac{7}{3}i\tan^{-1}(x)}\text{Hypergeometric2F1}\left(\frac{7}{6},2,\frac{13}{6},-e^{2i\tan^{-1}(x)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/3)*ArcTan[x]),x]
```

```
[Out] ((-12*I)/7)*E^(((7*I)/3)*ArcTan[x])*Hypergeometric2F1[7/6, 2, 13/6, -E^((2*
I)*ArcTan[x])]
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \sqrt[3]{(1+ix)\frac{1}{\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3),x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

Fricas [A] time = 1.69093, size = 636, normalized size = 2.43

$$\frac{1}{6}\left(-i\sqrt{3}+1\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)+\frac{1}{6}\left(-i\sqrt{3}-1\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)+\frac{1}{6}\left(i\sqrt{3}+1\right)\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*(-I*sqrt(3) + 1)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/
2*I) + 1/6*(-I*sqrt(3) - 1)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/
3) - 1/2*I) + 1/6*(I*sqrt(3) + 1)*log(-1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x +
I))^(1/3) + 1/2*I) + 1/6*(I*sqrt(3) - 1)*log(-1/2*sqrt(3) + (I*sqrt(x^2 + 1
)/(x + I))^(1/3) - 1/2*I) + 1/6*(6*x + 6*I)*(I*sqrt(x^2 + 1)/(x + I))^(1/3)
 + 1/3*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + I) - 1/3*log((I*sqrt(x^2 + 1)/
(x + I))^(1/3) - I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

3.118 $\int \frac{e^{\frac{1}{3}i\tan^{-1}(x)}}{x}\,dx$

Optimal. Leaf size=430

$$-\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)+\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)-\frac{1}{2}\log$$

```
[Out] ArcTan[Sqrt[3] - (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] - ArcTan[Sqrt[3] + (2
*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] + Sqrt[3]*ArcTan[(1 - (2*(1 + I*x)^(1/6)
)/(1 - I*x)^(1/6))/Sqrt[3]] - Sqrt[3]*ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1 -
I*x)^(1/6))/Sqrt[3]] - 2*ArcTan[(1 - I*x)^(1/6)/(1 + I*x)^(1/6)] - 2*ArcTan
h[(1 + I*x)^(1/6)/(1 - I*x)^(1/6)] - (Sqrt[3]*Log[1 + (1 - I*x)^(1/3)/(1 +
I*x)^(1/3) - (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)])/2 + (Sqrt[3]*Log[1
 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3) + (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1
/6)])/2 + Log[1 - (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*
x)^(1/3)]/2 - Log[1 + (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1
- I*x)^(1/3)]/2
```

Rubi [A] time = 0.458226, antiderivative size = 430, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {5062, 105, 63, 331, 295, 634, 618, 204, 628, 203, 93, 210, 206}

$$-\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)+\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)-\frac{1}{2}\log$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/3)*ArcTan[x])/x,x]
```


```
[Out] ArcTan[Sqrt[3] - (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] - ArcTan[Sqrt[3] + (2
*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] + Sqrt[3]*ArcTan[(1 - (2*(1 + I*x)^(1/6)
)/(1 - I*x)^(1/6))/Sqrt[3]] - Sqrt[3]*ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1 -
I*x)^(1/6))/Sqrt[3]] - 2*ArcTan[(1 - I*x)^(1/6)/(1 + I*x)^(1/6)] - 2*ArcTan
h[(1 + I*x)^(1/6)/(1 - I*x)^(1/6)] - (Sqrt[3]*Log[1 + (1 - I*x)^(1/3)/(1 +
I*x)^(1/3) - (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)])/2 + (Sqrt[3]*Log[1
 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3) + (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1
/6)])/2 + Log[1 - (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*
x)^(1/3)]/2 - Log[1 + (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1
- I*x)^(1/3)]/2
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
```

```
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{3} i \tan^{-1}(x)}}{x}\,dx &= \int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}x}\,dx \\
&= i\int \frac{1}{\sqrt[6]{1-ix}(1+ix)^{5/6}}\,dx + \int \frac{1}{\sqrt[6]{1-ix}(1+ix)^{5/6}x}\,dx \\
&= -\left(6\,\text{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\,dx, x, \sqrt[6]{1-ix}\right)\right) + 6\,\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) \\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)\right) - 2\,\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - 2\,\text{Subst}\left(\int \frac{1+}{1+x-}\right. \\
&= -2\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sqrt[6]{1+}}{\sqrt[6]{1-}}\right. \\
&= -2\tan^{-1}\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{2}\log\left(1-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) - \frac{1}{2}\log\left(1+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\right. \\
&= \sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right) - \sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right) - 2\tan^{-1}\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{2}\sqrt{3}\,\text{lo} \\
&= \tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) - \tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right) - \sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right.
\end{aligned}$$

Mathematica [C] time = 0.0305776, size = 90, normalized size = 0.21

$$-\frac{3(1-ix)^{5/6}\left(\sqrt[6]{2}(1+ix)^{5/6}\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}-\frac{ix}{2}\right)+2\text{Hypergeometric2F1}\left(\frac{5}{6},1,\frac{11}{6},\frac{x+i}{-x+i}\right)\right)}{5(1+ix)^{5/6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/3)*ArcTan[x])/x,x]
```

```
[Out] (-3*(1 - I*x)^(5/6)*(2^(1/6)*(1 + I*x)^(5/6)*Hypergeometric2F1[5/6, 5/6, 11
/6, 1/2 - (I/2)*x] + 2*Hypergeometric2F1[5/6, 1, 11/6, (I + x)/(I - x)]))/(
5*(1 + I*x)^(5/6))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt[3]{(1+ix)\frac{1}{\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x, x)
```

Fricas [A] time = 1.7904, size = 1108, normalized size = 2.58

$$\frac{1}{2}\left(\sqrt{3}+i\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)+\frac{1}{2}\left(\sqrt{3}-i\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)+\frac{1}{2}\left(-i\sqrt{3}-1\right)\log\left(\frac{1}{2}i\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(3) + I)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2*I
) + 1/2*(sqrt(3) - I)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1
/2*I) + 1/2*(-I*sqrt(3) - 1)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^
(1/3) + 1/2) + 1/2*(-I*sqrt(3) + 1)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x
 + I))^(1/3) - 1/2) + 1/2*(I*sqrt(3) - 1)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2
+ 1)/(x + I))^(1/3) + 1/2) + 1/2*(I*sqrt(3) + 1)*log(-1/2*I*sqrt(3) + (I*sq
rt(x^2 + 1)/(x + I))^(1/3) - 1/2) - 1/2*(sqrt(3) - I)*log(-1/2*sqrt(3) + (I
*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2*I) - 1/2*(sqrt(3) + I)*log(-1/2*sqrt(3)
 + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2*I) - log((I*sqrt(x^2 + 1)/(x + I))
^(1/3) + 1) + I*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + I) - I*log((I*sqrt(x^
2 + 1)/(x + I))^(1/3) - I) + log((I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x, x)
```

3.119 $\int \frac{e^{\frac{1}{3}i\tan^{-1}(x)}}{x^2}\,dx$

Optimal. Leaf size=253

$$-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}+\frac{1}{6}i\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)-\frac{1}{6}i\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)+\frac{i\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{i\tan}{}$$

```
[Out] -(((1 - I*x)^(5/6)*(1 + I*x)^(1/6))/x) + (I*ArcTan[(1 - (2*(1 + I*x)^(1/6))
/(1 - I*x)^(1/6))/Sqrt[3]])/Sqrt[3] - (I*ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1
 - I*x)^(1/6))/Sqrt[3]])/Sqrt[3] - ((2*I)/3)*ArcTanh[(1 + I*x)^(1/6)/(1 - I
*x)^(1/6)] + (I/6)*Log[1 - (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3
)/(1 - I*x)^(1/3)] - (I/6)*Log[1 + (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I
*x)^(1/3)/(1 - I*x)^(1/3)]
```

Rubi [A] time = 0.155645, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5062, 94, 93, 210, 634, 618, 204, 628, 206}

$$-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}+\frac{1}{6}i\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)-\frac{1}{6}i\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)+\frac{i\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{i\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/3)*ArcTan[x])/x^2,x]
```

```
[Out] -(((1 - I*x)^(5/6)*(1 + I*x)^(1/6))/x) + (I*ArcTan[(1 - (2*(1 + I*x)^(1/6))
/(1 - I*x)^(1/6))/Sqrt[3]])/Sqrt[3] - (I*ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1
 - I*x)^(1/6))/Sqrt[3]])/Sqrt[3] - ((2*I)/3)*ArcTanh[(1 + I*x)^(1/6)/(1 - I
*x)^(1/6)] + (I/6)*Log[1 - (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3
)/(1 - I*x)^(1/3)] - (I/6)*Log[1 + (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I
*x)^(1/3)/(1 - I*x)^(1/3)]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}i\tan^{-1}(x)}}{x^2}\,dx = \int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}x^2}\,dx$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} + \frac{1}{3}i\int \frac{1}{\sqrt[6]{1-ix}(1+ix)^{5/6}x}\,dx$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} + 2i\,\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} - \frac{2}{3}i\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{2}{3}i\,\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right.$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} - \frac{2}{3}i\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{6}i\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{6}i\,\text{Subst}$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} - \frac{2}{3}i\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{6}i\log\left(1-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) - \frac{1}{6}i\log\left(1+\frac{\sqrt[6]{1+}}{\sqrt[6]{1-}}\right.$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} + \frac{i\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{i\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}i\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{6}i\log\left(1-\right.$$

Mathematica [C] time = 0.013165, size = 64, normalized size = 0.25

$$-\frac{i(1-ix)^{5/6}\left(2x\text{Hypergeometric2F1}\left(\frac{5}{6},1,\frac{11}{6},\frac{x+i}{-x+i}\right)+5x-5i\right)}{5(1+ix)^{5/6}x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/3)*ArcTan[x])/x^2,x]
```

```
[Out] ((-I/5)*(1 - I*x)^(5/6)*(-5*I + 5*x + 2*x*Hypergeometric2F1[5/6, 1, 11/6, (
I + x)/(I - x)]))/((1 + I*x)^(5/6)*x)
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt[3]{(1+ix)\frac{1}{\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^2, x)
```

Fricas [A] time = 1.73545, size = 635, normalized size = 2.51

$$(\sqrt{3}x - ix)\log\left(\frac{1}{2}i\sqrt{3} + \left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}} + \frac{1}{2}\right) + (\sqrt{3}x + ix)\log\left(\frac{1}{2}i\sqrt{3} + \left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}} - \frac{1}{2}\right) - (\sqrt{3}x + ix)\log\left(-\frac{1}{2}i\sqrt{3} + \left(\frac{i\sqrt{x^2+1}}{x+i}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*((sqrt(3)*x - I*x)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3)
+ 1/2) + (sqrt(3)*x + I*x)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1
/3) - 1/2) - (sqrt(3)*x + I*x)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I
))^(1/3) + 1/2) - (sqrt(3)*x - I*x)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(
x + I))^(1/3) - 1/2) - 2*I*x*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1) + 2*I
*x*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1) - 6*(-I*x + 1)*(I*sqrt(x^2 + 1)
/(x + I))^(1/3))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^2, x)
```

3.120 $\int \frac{e^{\frac{1}{3} i \tan^{-1}(x)}}{x^3} \, dx$

Optimal. Leaf size=280

$$-\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2}-\frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x}-\frac{1}{36}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)+\frac{1}{36}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)--$$

```
[Out] -((1 - I*x)^(5/6)*(1 + I*x)^(7/6))/(2*x^2) - ((I/6)*(1 - I*x)^(5/6)*(1 + I*
x)^(1/6))/x - ArcTan[(1 - (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]]/(6*
Sqrt[3]) + ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]]/(6*Sqr
t[3]) + ArcTanh[(1 + I*x)^(1/6)/(1 - I*x)^(1/6)]/9 - Log[1 - (1 + I*x)^(1/6
)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*x)^(1/3)]/36 + Log[1 + (1 + I*x)
^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*x)^(1/3)]/36
```

Rubi [A] time = 0.176115, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5062, 96, 94, 93, 210, 634, 618, 204, 628, 206}

$$-\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2}-\frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x}-\frac{1}{36}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)+\frac{1}{36}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)--$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/3)*ArcTan[x])/x^3,x]
```


```
[Out] -((1 - I*x)^(5/6)*(1 + I*x)^(7/6))/(2*x^2) - ((I/6)*(1 - I*x)^(5/6)*(1 + I*
x)^(1/6))/x - ArcTan[(1 - (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]]/(6*
Sqrt[3]) + ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]]/(6*Sqr
t[3]) + ArcTanh[(1 + I*x)^(1/6)/(1 - I*x)^(1/6)]/9 - Log[1 - (1 + I*x)^(1/6
)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*x)^(1/3)]/36 + Log[1 + (1 + I*x)
^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*x)^(1/3)]/36
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{1}{3} i \tan^{-1}(x)}}{x^3}\,dx &= \int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}x^3}\,dx \\
&= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} + \frac{1}{6}i\int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}x^2}\,dx \\
&= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} - \frac{1}{18}\int \frac{1}{\sqrt[6]{1-ix}(1+ix)^{5/6}x}\,dx \\
&= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) \\
&= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} + \frac{1}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{9}\text{Subst}\left(\int \frac{}{1}\right. \\
&= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} + \frac{1}{9}\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{36}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x\right. \\
&= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} + \frac{1}{9}\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{36}\log\left(1 - \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}} + \frac{\sqrt[3]{1+}}{\sqrt[3]{1-}}\right. \\
&= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{1}{9}\tanh^{-1}\left(\right.
\end{aligned}
$$

Mathematica [C] time = 0.0167851, size = 72, normalized size = 0.26

$$
\frac{(1-ix)^{5/6}\left(2x^2\text{Hypergeometric2F1}\left(\frac{5}{6},1,\frac{11}{6},\frac{x+i}{-x+i}\right)+5\left(4x^2-7ix-3\right)\right)}{30(1+ix)^{5/6}x^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/3)*ArcTan[x])/x^3,x]
```

```
[Out] ((1 - I*x)^(5/6)*(5*(-3 - (7*I)*x + 4*x^2) + 2*x^2*Hypergeometric2F1[5/6, 1
, 11/6, (I + x)/(I - x)]))/(30*(1 + I*x)^(5/6)*x^2)
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$
\int \frac{1}{x^3}\sqrt[3]{(1+ix)\frac{1}{\sqrt{x^2+1}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^3, x)
```

Fricas [A] time = 1.73347, size = 672, normalized size = 2.4

$$2\,x^2\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+1\right)-2\,x^2\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-1\right)+\left(i\sqrt{3}x^2+x^2\right)\log\left(\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}\right)+\left(i\sqrt{3}x^2-x^2\right)\log\left(\frac{1}{2}i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/36*(2*x^2*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1) - 2*x^2*log((I*sqrt(x^
2 + 1)/(x + I))^(1/3) - 1) + (I*sqrt(3)*x^2 + x^2)*log(1/2*I*sqrt(3) + (I*s
qrt(x^2 + 1)/(x + I))^(1/3) + 1/2) + (I*sqrt(3)*x^2 - x^2)*log(1/2*I*sqrt(3
) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2) + (-I*sqrt(3)*x^2 + x^2)*log(-1/
2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2) + (-I*sqrt(3)*x^2 - x^
2)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2) - 6*(4*x^2 +
 I*x + 3)*(I*sqrt(x^2 + 1)/(x + I))^(1/3))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^3, x)
```

3.121 $\int \frac{e^{\frac{1}{3}i\tan^{-1}(x)}}{x^4}\,dx$

Optimal. Leaf size=319

$$-\frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2}-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}+\frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x}-\frac{19}{324}i\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)+\frac{19}{324}i\log\left(\frac{\sqrt[3]{}}{\sqrt[3]{}}\right.$$

```
[Out] -((1 - I*x)^(5/6)*(1 + I*x)^(1/6))/(3*x^3) - (((7*I)/18)*(1 - I*x)^(5/6)*(1
 + I*x)^(1/6))/x^2 + (11*(1 - I*x)^(5/6)*(1 + I*x)^(1/6))/(27*x) - (((19*I)
/54)*ArcTan[(1 - (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]])/Sqrt[3] + (
((19*I)/54)*ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]])/Sqrt
[3] + ((19*I)/81)*ArcTanh[(1 + I*x)^(1/6)/(1 - I*x)^(1/6)] - ((19*I)/324)*L
og[1 - (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*x)^(1/3)] +
 ((19*I)/324)*Log[1 + (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1
- I*x)^(1/3)]
```

Rubi [A] time = 0.200022, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5062, 99, 151, 12, 93, 210, 634, 618, 204, 628, 206}

$$-\frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2}-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}+\frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x}-\frac{19}{324}i\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)+\frac{19}{324}i\log\left(\frac{\sqrt[3]{}}{\sqrt[3]{}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/3)*ArcTan[x])/x^4,x]
```


```
[Out] -((1 - I*x)^(5/6)*(1 + I*x)^(1/6))/(3*x^3) - (((7*I)/18)*(1 - I*x)^(5/6)*(1
 + I*x)^(1/6))/x^2 + (11*(1 - I*x)^(5/6)*(1 + I*x)^(1/6))/(27*x) - (((19*I)
/54)*ArcTan[(1 - (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]])/Sqrt[3] + (
((19*I)/54)*ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]])/Sqrt
[3] + ((19*I)/81)*ArcTanh[(1 + I*x)^(1/6)/(1 - I*x)^(1/6)] - ((19*I)/324)*L
og[1 - (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*x)^(1/3)] +
 ((19*I)/324)*Log[1 + (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1
- I*x)^(1/3)]
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
```

```
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}i\tan^{-1}(x)}}{x^4}\,dx = \int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}x^4}\,dx$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} + \frac{1}{3}\int \frac{\frac{7i}{3}-2x}{\sqrt[6]{1-ix}(1+ix)^{5/6}x^3}\,dx$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} - \frac{1}{6}\int \frac{\frac{22}{9}+\frac{7ix}{3}}{\sqrt[6]{1-ix}(1+ix)^{5/6}x^2}\,dx$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} + \frac{1}{6}\int -\frac{19i}{27\sqrt[6]{1-ix}(1+ix)^{5/6}x}\,d$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} - \frac{19}{162}i\int \frac{1}{\sqrt[6]{1-ix}(1+ix)^{5/6}x}\,dx$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} - \frac{19}{27}i\,\mathrm{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \right.$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} + \frac{19}{81}i\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \right.$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} + \frac{19}{81}i\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{19}{324}i$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} + \frac{19}{81}i\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{19}{324}i$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} - \frac{19i\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{54\sqrt{3}} + \frac{19i\tan}{}$$

Mathematica [C] time = 0.022007, size = 81, normalized size = 0.25

$$\frac{(1-ix)^{5/6}\left(38ix^3\text{Hypergeometric2F1}\left(\frac{5}{6},1,\frac{11}{6},\frac{x+i}{-x+i}\right)+5\left(22ix^3+43x^2-39ix-18\right)\right)}{270(1+ix)^{5/6}x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/3)*ArcTan[x])/x^4,x]
```

```
[Out] ((1 - I*x)^(5/6)*(5*(-18 - (39*I)*x + 43*x^2 + (22*I)*x^3) + (38*I)*x^3*Hyp
ergeometric2F1[5/6, 1, 11/6, (I + x)/(I - x)]))/(270*(1 + I*x)^(5/6)*x^3)
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\sqrt[3]{(1+ix)\frac{1}{\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^4, x)
```

Fricas [A] time = 1.73565, size = 733, normalized size = 2.3

$$38ix^3\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+1\right)-38ix^3\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-1\right)-\left(19\sqrt{3}x^3-19ix^3\right)\log\left(\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}\right)-\left(19\sqrt{3}x^3+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x, algorithm="fricas")
```

```
[Out] 1/324*(38*I*x^3*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1) - 38*I*x^3*log((I*
sqrt(x^2 + 1)/(x + I))^(1/3) - 1) - (19*sqrt(3)*x^3 - 19*I*x^3)*log(1/2*I*s
qrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2) - (19*sqrt(3)*x^3 + 19*I*x^
3)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2) + (19*sqrt(3)
*x^3 + 19*I*x^3)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2
) + (19*sqrt(3)*x^3 - 19*I*x^3)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x +
I))^(1/3) - 1/2) + (-132*I*x^3 + 6*x^2 - 18*I*x - 108)*(I*sqrt(x^2 + 1)/(x
+ I))^(1/3))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x, algorithm="giac")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^4, x)
```

3.122 $\int e^{\frac{2}{3}i\tan^{-1}(x)}x^2\,dx$

Optimal. Leaf size=177

$$\frac{1}{3}(1-ix)^{2/3}x(1+ix)^{4/3}-\frac{1}{9}i(1-ix)^{2/3}(1+ix)^{4/3}-\frac{11}{27}i(1-ix)^{2/3}\sqrt[3]{1+ix}+\frac{11}{27}i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)+\frac{11}{81}i\log(1+ix)+\frac{22i}{}$$

```
[Out] ((-11*I)/27)*(1 - I*x)^(2/3)*(1 + I*x)^(1/3) - (I/9)*(1 - I*x)^(2/3)*(1 + I
*x)^(4/3) + ((1 - I*x)^(2/3)*(1 + I*x)^(4/3)*x)/3 + (((22*I)/27)*ArcTan[1/S
qrt[3] - (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))])/Sqrt[3] + ((11*I)/
27)*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3)] + ((11*I)/81)*Log[1 + I*x]
```

Rubi [A] time = 0.0546673, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5062, 90, 80, 50, 60}

$$\frac{1}{3}(1-ix)^{2/3}x(1+ix)^{4/3}-\frac{1}{9}i(1-ix)^{2/3}(1+ix)^{4/3}-\frac{11}{27}i(1-ix)^{2/3}\sqrt[3]{1+ix}+\frac{11}{27}i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)+\frac{11}{81}i\log(1+ix)+\frac{22i}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((2*I)/3)*ArcTan[x])*x^2,x]
```


```
[Out] ((-11*I)/27)*(1 - I*x)^(2/3)*(1 + I*x)^(1/3) - (I/9)*(1 - I*x)^(2/3)*(1 + I
*x)^(4/3) + ((1 - I*x)^(2/3)*(1 + I*x)^(4/3)*x)/3 + (((22*I)/27)*ArcTan[1/S
qrt[3] - (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))])/Sqrt[3] + ((11*I)/
27)*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3)] + ((11*I)/81)*Log[1 + I*x]
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rubi steps

$$\begin{aligned}
\int e^{\frac{2}{3}i\tan^{-1}(x)}x^2\,dx &= \int \frac{\sqrt[3]{1+ix}x^2}{\sqrt[3]{1-ix}}\,dx \\
&= \frac{1}{3}(1-ix)^{2/3}(1+ix)^{4/3}x+\frac{1}{3}\int \frac{\left(-1-\frac{2ix}{3}\right)\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\,dx \\
&= -\frac{1}{9}i(1-ix)^{2/3}(1+ix)^{4/3}+\frac{1}{3}(1-ix)^{2/3}(1+ix)^{4/3}x-\frac{11}{27}\int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\,dx \\
&= -\frac{11}{27}i(1-ix)^{2/3}\sqrt[3]{1+ix}-\frac{1}{9}i(1-ix)^{2/3}(1+ix)^{4/3}+\frac{1}{3}(1-ix)^{2/3}(1+ix)^{4/3}x-\frac{22}{81}\int \frac{1}{\sqrt[3]{1-ix}(1+ix} \\
&= -\frac{11}{27}i(1-ix)^{2/3}\sqrt[3]{1+ix}-\frac{1}{9}i(1-ix)^{2/3}(1+ix)^{4/3}+\frac{1}{3}(1-ix)^{2/3}(1+ix)^{4/3}x+\frac{22i\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1}}{\sqrt{3}\sqrt[3]{}}\right.}{27\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.0321483, size = 73, normalized size = 0.41

$$\frac{1}{18}(1-ix)^{2/3}\left(2\sqrt[3]{1+ix}\left(3ix^2+4x-i\right)-11i\sqrt[3]{2}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{1}{2}-\frac{ix}{2}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x])*x^2,x]
```

```
[Out] ((1 - I*x)^(2/3)*(2*(1 + I*x)^(1/3)*(-I + 4*x + (3*I)*x^2) - (11*I)*2^(1/3)
*Hypergeometric2F1[-1/3, 2/3, 5/3, 1/2 - (I/2)*x]))/18
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \left((1+ix)\frac{1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x)
```

[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="maxima")

[Out] integrate(x^2*((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)

Fricas [A] time = 1.68868, size = 397, normalized size = 2.24

$$-\frac{1}{81}\left(11\sqrt{3}+11i\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right)+\frac{1}{81}\left(11\sqrt{3}-11i\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right)+\frac{1}{81}\left(27x^3-9ix\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="fricas")

[Out] -1/81*(11*sqrt(3) + 11*I)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(
3) - 1/2) + 1/81*(11*sqrt(3) - 11*I)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) -
1/2*I*sqrt(3) - 1/2) + 1/81*(27*x^3 - 9*I*x^2 - 6*x - 42*I)*(I*sqrt(x^2 + 1
)/(x + I))^(2/3) + 22/81*I*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)*x**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="giac")

[Out] integrate(x^2*((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)

3.123 $\int e^{\frac{2}{3}i\tan^{-1}(x)}x\,dx$

Optimal. Leaf size=140

$$\frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3}+\frac{1}{3}(1-ix)^{2/3}\sqrt[3]{1+ix}-\frac{1}{3}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{9}\log(1+ix)-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}}$$

```
[Out] ((1 - I*x)^(2/3)*(1 + I*x)^(1/3))/3 + ((1 - I*x)^(2/3)*(1 + I*x)^(4/3))/2 -
 (2*ArcTan[1/Sqrt[3] - (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))])/(3*S
qrt[3]) - Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3)]/3 - Log[1 + I*x]/9
```

Rubi [A] time = 0.0359681, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5062, 80, 50, 60}

$$\frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3}+\frac{1}{3}(1-ix)^{2/3}\sqrt[3]{1+ix}-\frac{1}{3}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{9}\log(1+ix)-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((2*I)/3)*ArcTan[x])*x,x]
```

```
[Out] ((1 - I*x)^(2/3)*(1 + I*x)^(1/3))/3 + ((1 - I*x)^(2/3)*(1 + I*x)^(4/3))/2 -
 (2*ArcTan[1/Sqrt[3] - (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))])/(3*S
qrt[3]) - Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3)]/3 - Log[1 + I*x]/9
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
```

```
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rubi steps

$$\int e^{\frac{2}{3}i\tan^{-1}(x)}x\,dx = \int \frac{\sqrt[3]{1+ix}x}{\sqrt[3]{1-ix}}\,dx$$
$$= \frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3} - \frac{1}{3}i\int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\,dx$$
$$= \frac{1}{3}(1-ix)^{2/3}\sqrt[3]{1+ix} + \frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3} - \frac{2}{9}i\int \frac{1}{\sqrt[3]{1-ix}(1+ix)^{2/3}}\,dx$$
$$= \frac{1}{3}(1-ix)^{2/3}\sqrt[3]{1+ix} + \frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3} - \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}} - \frac{1}{3}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right) - \frac{1}{9}\log$$

Mathematica [C] time = 0.0198766, size = 54, normalized size = 0.39

$$\frac{1}{2}(1-ix)^{2/3}\left(\sqrt[3]{2}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{1}{2}-\frac{ix}{2}\right)+(1+ix)^{4/3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x])*x,x]
```

```
[Out] ((1 - I*x)^(2/3)*((1 + I*x)^(4/3) + 2^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/
3, 1/2 - (I/2)*x]))/2
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \left((1+ix)\frac{1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

Fricas [A] time = 1.6965, size = 362, normalized size = 2.59

$$-\frac{1}{9}\left(i\sqrt{3}-1\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right)-\frac{1}{9}\left(-i\sqrt{3}-1\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right)+\frac{1}{6}\left(3x^2-2ix+5\right)\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x, algorithm="fricas")
```

```
[Out] -1/9*(I*sqrt(3) - 1)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3) -
1/2) - 1/9*(-I*sqrt(3) - 1)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sqr
t(3) - 1/2) + 1/6*(3*x^2 - 2*I*x + 5)*(I*sqrt(x^2 + 1)/(x + I))^(2/3) - 2/9
*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)*x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x, algorithm="giac")
```

```
[Out] integrate(x*((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

3.124 $\int e^{\frac{2}{3}i\tan^{-1}(x)}\,dx$

Optimal. Leaf size=116

$$i(1-ix)^{2/3}\sqrt[3]{1+ix}-i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{3}i\log(1+ix)-\frac{2i\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}}$$

```
[Out] I*(1 - I*x)^(2/3)*(1 + I*x)^(1/3) - ((2*I)*ArcTan[1/Sqrt[3] - (2*(1 - I*x)^
(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))])/Sqrt[3] - I*Log[1 + (1 - I*x)^(1/3)/(1 +
 I*x)^(1/3)] - (I/3)*Log[1 + I*x]
```

Rubi [A] time = 0.0205023, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5061, 50, 60}

$$i(1-ix)^{2/3}\sqrt[3]{1+ix}-i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{3}i\log(1+ix)-\frac{2i\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((2*I)/3)*ArcTan[x]),x]
```

```
[Out] I*(1 - I*x)^(2/3)*(1 + I*x)^(1/3) - ((2*I)*ArcTan[1/Sqrt[3] - (2*(1 - I*x)^
(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))])/Sqrt[3] - I*Log[1 + (1 - I*x)^(1/3)/(1 +
 I*x)^(1/3)] - (I/3)*Log[1 + I*x]
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{2}{3}i\tan^{-1}(x)}dx &= \int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}dx\\ &= i(1-ix)^{2/3}\sqrt[3]{1+ix}+\frac{2}{3}\int \frac{1}{\sqrt[3]{1-ix}(1+ix)^{2/3}}dx\\ &= i(1-ix)^{2/3}\sqrt[3]{1+ix}-\frac{2i\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}}-i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{3}i\log(1+ix)\end{aligned}$$

Mathematica [C] time = 0.0172518, size = 34, normalized size = 0.29

$$-\frac{3}{2}ie^{\frac{8}{3}i\tan^{-1}(x)}\text{Hypergeometric2F1}\left(\frac{4}{3},2,\frac{7}{3},-e^{2i\tan^{-1}(x)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x]),x]
```

```
[Out] ((-3*I)/2)*E^(((8*I)/3)*ArcTan[x])*Hypergeometric2F1[4/3, 2, 7/3, -E^((2*I)
*ArcTan[x])]
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \left((1+ix)\frac{1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3),x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

Fricas [A] time = 1.70289, size = 346, normalized size = 2.98

$$\frac{1}{3}\left(\sqrt{3}+i\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right)-\frac{1}{3}\left(\sqrt{3}-i\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right)+\frac{1}{3}(3x+3i)\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(3) + I)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3) - 1/2
) - 1/3*(sqrt(3) - I)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sqrt(3) -
 1/2) + 1/3*(3*x + 3*I)*(I*sqrt(x^2 + 1)/(x + I))^(2/3) - 2/3*I*log((I*sqrt
(x^2 + 1)/(x + I))^(2/3) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

3.125 $\int \frac{e^{\frac{2}{3}i\tan^{-1}(x)}}{x}\,dx$

Optimal. Leaf size=163

$$\frac{3}{2}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)+\frac{3}{2}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)+\frac{1}{2}\log(1+ix)-\frac{\log(x)}{2}+\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)+\sqrt{3}\tan^{-1}$$

```
[Out] Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))] +
 Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))]
+ (3*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3)])/2 + (3*Log[(1 - I*x)^(1/3) -
 (1 + I*x)^(1/3)])/2 + Log[1 + I*x]/2 - Log[x]/2
```

Rubi [A] time = 0.0403384, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5062, 105, 60, 91}

$$\frac{3}{2}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)+\frac{3}{2}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)+\frac{1}{2}\log(1+ix)-\frac{\log(x)}{2}+\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)+\sqrt{3}\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((2*I)/3)*ArcTan[x])/x,x]
```

```
[Out] Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))] +
 Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))]
+ (3*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3)])/2 + (3*Log[(1 - I*x)^(1/3) -
 (1 + I*x)^(1/3)])/2 + Log[1 + I*x]/2 - Log[x]/2
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
```

```
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\frac{2}{3}i\tan^{-1}(x)}}{x}\,dx &= \int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}x}\,dx\\ &= i\int \frac{1}{\sqrt[3]{1-ix}(1+ix)^{2/3}}\,dx + \int \frac{1}{\sqrt[3]{1-ix}(1+ix)^{2/3}x}\,dx\\ &= \sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)+\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)+\frac{3}{2}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)+\frac{3}{2}\log\left(\sqrt[3]{1-i\right.\end{aligned}$$

Mathematica [C] time = 0.0297735, size = 90, normalized size = 0.55

$$-\frac{3(1-ix)^{2/3}\left(\sqrt[3]{2}(1+ix)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{1}{2}-\frac{ix}{2}\right)+2\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},\frac{x+i}{-x+i}\right)\right)}{4(1+ix)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x])/x,x]
```

```
[Out] (-3*(1 - I*x)^(2/3)*(2^(1/3)*(1 + I*x)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/
3, 1/2 - (I/2)*x] + 2*Hypergeometric2F1[2/3, 1, 5/3, (I + x)/(I - x)]))/(4*
(1 + I*x)^(2/3))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left((1+ix)\frac{1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x, algorithm="maxima")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x, x)
```

Fricas [A] time = 1.65603, size = 423, normalized size = 2.6

$$\frac{1}{2}\left(i\sqrt{3}-1\right)\log\left(\frac{\sqrt{3}(ix-1)+x+2i\sqrt{x^2+1}\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+i}{2x+2i}\right)+\frac{1}{2}\left(-i\sqrt{3}-1\right)\log\left(\frac{\sqrt{3}(-ix+1)+x+2i\sqrt{x^2+1}\left(\frac{i\sqrt{x}}{x\cdot}\right.}{2x+2i}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(I*sqrt(3) - 1)*log((sqrt(3)*(I*x - 1) + x + 2*I*sqrt(x^2 + 1)*(I*sqrt(
x^2 + 1)/(x + I))^(1/3) + I)/(2*x + 2*I)) + 1/2*(-I*sqrt(3) - 1)*log((sqrt(
3)*(-I*x + 1) + x + 2*I*sqrt(x^2 + 1)*(I*sqrt(x^2 + 1)/(x + I))^(1/3) + I)/
(2*x + 2*I)) + log(-(x - I*sqrt(x^2 + 1)*(I*sqrt(x^2 + 1)/(x + I))^(1/3) +
I)/(x + I))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x, algorithm="giac")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x, x)
```

3.126 $\int \frac{e^{\frac{2}{3}i\tan^{-1}(x)}}{x^2}\,dx$

Optimal. Leaf size=111

$$-\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x}+i\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)-\frac{1}{3}i\log(x)+\frac{2i\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}}$$

```
[Out] -(((1 - I*x)^(2/3)*(1 + I*x)^(1/3))/x) + ((2*I)*ArcTan[1/Sqrt[3] + (2*(1 -
I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))])/Sqrt[3] + I*Log[(1 - I*x)^(1/3) - (
1 + I*x)^(1/3)] - (I/3)*Log[x]
```

Rubi [A] time = 0.0292463, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5062, 94, 91}

$$-\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x}+i\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)-\frac{1}{3}i\log(x)+\frac{2i\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((2*I)/3)*ArcTan[x])/x^2,x]
```

```
[Out] -(((1 - I*x)^(2/3)*(1 + I*x)^(1/3))/x) + ((2*I)*ArcTan[1/Sqrt[3] + (2*(1 -
I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))])/Sqrt[3] + I*Log[(1 - I*x)^(1/3) - (
1 + I*x)^(1/3)] - (I/3)*Log[x]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rubi steps

$$\int \frac{e^{\frac{2}{3}i\tan^{-1}(x)}}{x^2}\,dx = \int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}x^2}\,dx$$
$$= -\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x} + \frac{2}{3}i\int \frac{1}{\sqrt[3]{1-ix}(1+ix)^{2/3}x}\,dx$$
$$= -\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x} + \frac{2i\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}} + i\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right) - \frac{1}{3}i\log(x)$$

Mathematica [C] time = 0.0115087, size = 59, normalized size = 0.53

$$-\frac{i(1-ix)^{2/3}\left(x\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},\frac{x+i}{-x+i}\right)+x-i\right)}{(1+ix)^{2/3}x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x])/x^2,x]
```

```
[Out] ((-I)*(1 - I*x)^(2/3)*(-I + x + x*Hypergeometric2F1[2/3, 1, 5/3, (I + x)/(I
 - x)]))/((1 + I*x)^(2/3)*x)
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left((1+ix)\frac{1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x^2, x)
```

Fricas [A] time = 1.69989, size = 352, normalized size = 3.17

$$\frac{\left(\sqrt{3}x-ix\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+\frac{1}{2}i\sqrt{3}+\frac{1}{2}\right)-\left(\sqrt{3}x+ix\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-\frac{1}{2}i\sqrt{3}+\frac{1}{2}\right)+2ix\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-1\right)-3(-i}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*((sqrt(3)*x - I*x)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3)
+ 1/2) - (sqrt(3)*x + I*x)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sqrt
(3) + 1/2) + 2*I*x*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1) - 3*(-I*x + 1)*
(I*sqrt(x^2 + 1)/(x + I))^(2/3))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x^2, x)
```

3.127 $\int \frac{e^{\frac{2}{3}i\tan^{-1}(x)}}{x^3}\,dx$

Optimal. Leaf size=142

$$-\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2}-\frac{i(1-ix)^{2/3}\sqrt[3]{1+ix}}{3x}-\frac{1}{3}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)+\frac{\log(x)}{9}-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}}$$

```
[Out] -((1 - I*x)^(2/3)*(1 + I*x)^(4/3))/(2*x^2) - ((I/3)*(1 - I*x)^(2/3)*(1 + I*
x)^(1/3))/x - (2*ArcTan[1/Sqrt[3] + (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^
(1/3))])/(3*Sqrt[3]) - Log[(1 - I*x)^(1/3) - (1 + I*x)^(1/3)]/3 + Log[x]/9
```

Rubi [A] time = 0.0398932, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5062, 96, 94, 91}

$$-\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2}-\frac{i(1-ix)^{2/3}\sqrt[3]{1+ix}}{3x}-\frac{1}{3}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)+\frac{\log(x)}{9}-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((2*I)/3)*ArcTan[x])/x^3,x]
```

```
[Out] -((1 - I*x)^(2/3)*(1 + I*x)^(4/3))/(2*x^2) - ((I/3)*(1 - I*x)^(2/3)*(1 + I*
x)^(1/3))/x - (2*ArcTan[1/Sqrt[3] + (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^
(1/3))])/(3*Sqrt[3]) - Log[(1 - I*x)^(1/3) - (1 + I*x)^(1/3)]/3 + Log[x]/9
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\frac{2}{3}i\tan^{-1}(x)}}{x^3}\,dx &= \int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}x^3}\,dx\\ &= -\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2}+\frac{1}{3}i\int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}x^2}\,dx\\ &= -\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2}-\frac{i(1-ix)^{2/3}\sqrt[3]{1+ix}}{3x}-\frac{2}{9}\int \frac{1}{\sqrt[3]{1-ix}(1+ix)^{2/3}x}\,dx\\ &= -\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2}-\frac{i(1-ix)^{2/3}\sqrt[3]{1+ix}}{3x}-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}}-\frac{1}{3}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)+\end{aligned}$$

Mathematica [C] time = 0.0142606, size = 69, normalized size = 0.49

$$\frac{(1-ix)^{2/3}\left(2x^2\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},\frac{x+i}{-x+i}\right)+5x^2-8ix-3\right)}{6(1+ix)^{2/3}x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x])/x^3,x]
```

```
[Out] ((1 - I*x)^(2/3)*(-3 - (8*I)*x + 5*x^2 + 2*x^2*Hypergeometric2F1[2/3, 1, 5/
3, (I + x)/(I - x)]))/(6*(1 + I*x)^(2/3)*x^2)
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left((1+ix)\frac{1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x^3, x)
```

Fricas [A] time = 1.68321, size = 387, normalized size = 2.73

$$-\frac{4\,x^{2}\log\left(\left(\frac{i\,\sqrt{x^{2}+1}}{x+i}\right)^{\frac{2}{3}}-1\right)+2\left(-i\,\sqrt{3}x^{2}-x^{2}\right)\log\left(\left(\frac{i\,\sqrt{x^{2}+1}}{x+i}\right)^{\frac{2}{3}}+\frac{1}{2}i\,\sqrt{3}+\frac{1}{2}\right)+2\left(i\,\sqrt{3}x^{2}-x^{2}\right)\log\left(\left(\frac{i\,\sqrt{x^{2}+1}}{x+i}\right)^{\frac{2}{3}}-\frac{1}{2}i\,\sqrt{3}+\ldots\right.}{18\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/18*(4*x^2*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1) + 2*(-I*sqrt(3)*x^2 -
 x^2)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3) + 1/2) + 2*(I*sqr
t(3)*x^2 - x^2)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sqrt(3) + 1/2)
+ 3*(5*x^2 + 2*I*x + 3)*(I*sqrt(x^2 + 1)/(x + I))^(2/3))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,x+1}{\sqrt{x^{2}+1}}\right)^{\frac{2}{3}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x^3, x)
```

3.128 $\int e^{\frac{1}{4}i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=741

$$\frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2}-\frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3}-\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3}-\frac{11i\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right.}{256a^3}$$

```
[Out] (((-11*I)/32)*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/a^3 - ((I/24)*(1 - I*a*x
)^(7/8)*(1 + I*a*x)^(9/8))/a^3 + (x*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(9/8))/(3
*a^2) + (((11*I)/128)*Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 -
 I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/a^3 + (((11*I)/128)*S
qrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/(1 + I*a
*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/a^3 - (((11*I)/128)*Sqrt[2 + Sqrt[2]]*ArcTan
[(Sqrt[2 - Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqr
t[2]]])/a^3 - (((11*I)/128)*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (
2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/a^3 - (((11*I)/
256)*Sqrt[2 - Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[
2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a^3 + (((11*I)/256)*Sqr
t[2 - Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 - Sqrt
[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a^3 - (((11*I)/256)*Sqrt[2 + Sq
rt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1
- I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a^3 + (((11*I)/256)*Sqrt[2 + Sqrt[2]]*L
og[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)
^(1/8))/(1 + I*a*x)^(1/8)])/a^3
```

Rubi [A] time = 0.733391, antiderivative size = 741, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5062, 90, 80, 50, 63, 331, 299, 1122, 1169, 634, 618, 204, 628}

$$\frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2}-\frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3}-\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3}-\frac{11i\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right.}{256a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/4)*ArcTan[a*x])*x^2,x]
```

```
[Out] (((-11*I)/32)*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/a^3 - ((I/24)*(1 - I*a*x
)^(7/8)*(1 + I*a*x)^(9/8))/a^3 + (x*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(9/8))/(3
*a^2) + (((11*I)/128)*Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 -
 I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/a^3 + (((11*I)/128)*S
qrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/(1 + I*a
*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/a^3 - (((11*I)/128)*Sqrt[2 + Sqrt[2]]*ArcTan
[(Sqrt[2 - Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqr
t[2]]])/a^3 - (((11*I)/128)*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (
2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/a^3 - (((11*I)/
256)*Sqrt[2 - Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[
2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a^3 + (((11*I)/256)*Sqr
t[2 - Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 - Sqrt
[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a^3 - (((11*I)/256)*Sqrt[2 + Sq
rt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1
- I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a^3 + (((11*I)/256)*Sqrt[2 + Sqrt[2]]*L
og[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)
```

```
^(1/8))/(1 + I*a*x)^(1/8)])/a^3
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{1}{4}i\tan^{-1}(ax)}x^2\,dx = \int \frac{x^2\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx$$

$$= \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} + \frac{\int \frac{\left(-1-\frac{iax}{4}\right)\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx}{3a^2}$$

$$= -\frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{11\int \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx}{32a^2}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{11\int \frac{1}{\sqrt[8]{1-iax}(1+iax)}}{128a^2}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{(11i)\,\text{Subst}\Big(\int}{}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{(11i)\,\text{Subst}\Big(\int}{1}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{(11i)\,\text{Subst}\Big(\int}{}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} + \frac{(11i)\,\text{Subst}\Big(\int}{}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{(11i)\,\text{Subst}\Big(\int}{64}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{\Big(11i\sqrt{\frac{1}{2}}\big(3-2\sqrt{}}{}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{11i\sqrt{2-\sqrt{2}}\log}{}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} + \frac{11i\sqrt{2+\sqrt{2}}\tan}{1}$$

Mathematica [C] time = 0.0379116, size = 83, normalized size = 0.11

$$\frac{(1-iax)^{7/8}\left(7\sqrt[8]{1+iax}\left(8ia^2x^2+9ax-i\right)-66i\sqrt[8]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{7}{8},\frac{15}{8},\frac{1}{2}(1-iax)\right)\right)}{168a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/4)*ArcTan[a*x])*x^2,x]
```

```
[Out] ((1 - I*a*x)^(7/8)*(7*(1 + I*a*x)^(1/8)*(-I + 9*a*x + (8*I)*a^2*x^2) - (66*
I)*2^(1/8)*Hypergeometric2F1[-1/8, 7/8, 15/8, (1 - I*a*x)/2]))/(168*a^3)
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \sqrt[4]{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

Fricas [A] time = 1.90122, size = 1531, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="fricas")
```

```
[Out] 1/96*(96*I*a^3*(14641/268435456*I/a^12)^(1/4)*log(128/11*a^3*(14641/2684354
56*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 96*a^3*(14641/2
68435456*I/a^12)^(1/4)*log(128/11*I*a^3*(14641/268435456*I/a^12)^(1/4) + (I
*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 96*a^3*(14641/268435456*I/a^12)^(1/4
)*log(-128/11*I*a^3*(14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(
a*x + I))^(1/4)) - 96*I*a^3*(14641/268435456*I/a^12)^(1/4)*log(-128/11*a^3*
(14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 9
6*I*a^3*(-14641/268435456*I/a^12)^(1/4)*log(128/11*a^3*(-14641/268435456*I/
a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 96*a^3*(-14641/26843
5456*I/a^12)^(1/4)*log(128/11*I*a^3*(-14641/268435456*I/a^12)^(1/4) + (I*sq
rt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 96*a^3*(-14641/268435456*I/a^12)^(1/4)*
log(-128/11*I*a^3*(-14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a
*x + I))^(1/4)) - 96*I*a^3*(-14641/268435456*I/a^12)^(1/4)*log(-128/11*a^3*
(-14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) +
(32*a^3*x^3 - 4*I*a^2*x^2 - a*x - 37*I)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/
4))/a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)*x**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.129 $\int e^{\frac{1}{4}i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=689

$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2}+\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2}+\frac{\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{64a^2}-\frac{\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-}}{}\right.}{64a^2}$$

```
[Out] ((1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/(8*a^2) + ((1 - I*a*x)^(7/8)*(1 + I*a
*x)^(9/8))/(2*a^2) - (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 -
 I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(32*a^2) - (Sqrt[2 -
Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8
))/Sqrt[2 - Sqrt[2]]])/(32*a^2) + (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[
2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(32*a^2)
 + (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1
+ I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/(32*a^2) + (Sqrt[2 - Sqrt[2]]*Log[1 + (
1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/
(1 + I*a*x)^(1/8)])/(64*a^2) - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)
/(1 + I*a*x)^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8
)])/(64*a^2) + (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/
4) - (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/(64*a^2) - (
Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 + S
qrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/(64*a^2)
```

Rubi [A] time = 0.494931, antiderivative size = 689, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {5062, 80, 50, 63, 331, 299, 1122, 1169, 634, 618, 204, 628}

$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2}+\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2}+\frac{\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{64a^2}-\frac{\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-}}{}\right.}{64a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/4)*ArcTan[a*x])*x,x]
```


```
[Out] ((1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/(8*a^2) + ((1 - I*a*x)^(7/8)*(1 + I*a
*x)^(9/8))/(2*a^2) - (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 -
 I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(32*a^2) - (Sqrt[2 -
Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8
))/Sqrt[2 - Sqrt[2]]])/(32*a^2) + (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[
2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(32*a^2)
 + (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1
+ I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/(32*a^2) + (Sqrt[2 - Sqrt[2]]*Log[1 + (
1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/
(1 + I*a*x)^(1/8)])/(64*a^2) - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)
/(1 + I*a*x)^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8
)])/(64*a^2) + (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/
4) - (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/(64*a^2) - (
Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 + S
qrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/(64*a^2)
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
```

```
(d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{1}{4}i\tan^{-1}(ax)}x\,dx = \int \frac{x\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx$$
$$= \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} - \frac{i\int \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx}{8a}$$
$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} - \frac{i\int \frac{1}{\sqrt[8]{1-iax}(1+iax)^{7/8}}\,dx}{32a}$$
$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\text{Subst}\left(\int \frac{x^6}{(2-x^8)^{7/8}}\,dx, x, \sqrt[8]{1-iax}\right)}{4a^2}$$
$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{4a^2}$$
$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8\sqrt{2}a^2} - \frac{\text{Subst}\left(\int \frac{}{1}\right.}{}$$
$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} - \frac{\text{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8\sqrt{2}a^2} + \frac{\text{Subst}\left(\int \frac{}{1}\right.}{}$$
$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-(1-\sqrt{2})x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{16\sqrt{2\left(2-\sqrt{2}\right)}a^2} + \frac{\text{Subst}}{}$$
$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-i}}{\sqrt[8]{1+i}}\right.}{32a^2}$$
$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2} - \frac{\sqrt{2}}{}$$
$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} - \frac{\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{32a^2} - \frac{\sqrt{2-\sqrt{2}}\tan}{}$$

Mathematica [C] time = 0.0219907, size = 63, normalized size = 0.09

$$\frac{(1-iax)^{7/8}\left(2\sqrt[8]{2}\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{7}{8},\frac{15}{8},\frac{1}{2}(1-iax)\right)+7(1+iax)^{9/8}\right)}{14a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/4)*ArcTan[a*x])*x,x]
```

```
[Out] ((1 - I*a*x)^(7/8)*(7*(1 + I*a*x)^(9/8) + 2*2^(1/8)*Hypergeometric2F1[-1/8,
 7/8, 15/8, (1 - I*a*x)/2]))/(14*a^2)
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \sqrt[4]{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

Fricas [A] time = 1.89723, size = 1304, normalized size = 1.89

$$-\frac{8\,a^2\left(\frac{i}{1048576\,a^8}\right)^{\frac{1}{4}}\log\left(32\,a^2\left(\frac{i}{1048576\,a^8}\right)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)+8i\,a^2\left(\frac{i}{1048576\,a^8}\right)^{\frac{1}{4}}\log\left(32i\,a^2\left(\frac{i}{1048576\,a^8}\right)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)-8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a^2*(1/1048576*I/a^8)^(1/4)*log(32*a^2*(1/1048576*I/a^8)^(1/4) + (I
*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 8*I*a^2*(1/1048576*I/a^8)^(1/4)*log(
32*I*a^2*(1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) -
 8*I*a^2*(1/1048576*I/a^8)^(1/4)*log(-32*I*a^2*(1/1048576*I/a^8)^(1/4) + (I
*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 8*a^2*(1/1048576*I/a^8)^(1/4)*log(-3
2*a^2*(1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 8*
a^2*(-1/1048576*I/a^8)^(1/4)*log(32*a^2*(-1/1048576*I/a^8)^(1/4) + (I*sqrt(
a^2*x^2 + 1)/(a*x + I))^(1/4)) + 8*I*a^2*(-1/1048576*I/a^8)^(1/4)*log(32*I*
a^2*(-1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 8*I
*a^2*(-1/1048576*I/a^8)^(1/4)*log(-32*I*a^2*(-1/1048576*I/a^8)^(1/4) + (I*s
qrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 8*a^2*(-1/1048576*I/a^8)^(1/4)*log(-32
*a^2*(-1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - (4
*a^2*x^2 - I*a*x + 5)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)*x,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.130 $\int e^{\frac{1}{4}i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=674

$$\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}+\frac{i\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{8a}-\frac{i\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{8a}+\frac{i\sqrt{2+}}{}$$

```
[Out] (I*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/a - ((I/4)*Sqrt[2 + Sqrt[2]]*ArcTan
[(Sqrt[2 - Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqr
t[2]]])/a - ((I/4)*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - I*
a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/a + ((I/4)*Sqrt[2 + Sqrt
[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/S
qrt[2 + Sqrt[2]]])/a + ((I/4)*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] +
 (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/a + ((I/8)*Sq
rt[2 - Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 - Sqr
t[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a - ((I/8)*Sqrt[2 - Sqrt[2]]*L
og[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)
^(1/8))/(1 + I*a*x)^(1/8)])/a + ((I/8)*Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x
)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*
x)^(1/8)])/a - ((I/8)*Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*
x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a
```

Rubi [A] time = 0.433852, antiderivative size = 674, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5061, 50, 63, 331, 299, 1122, 1169, 634, 618, 204, 628}

$$\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}+\frac{i\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{8a}-\frac{i\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{8a}+\frac{i\sqrt{2+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/4)*ArcTan[a*x]),x]
```


```
[Out] (I*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/a - ((I/4)*Sqrt[2 + Sqrt[2]]*ArcTan
[(Sqrt[2 - Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqr
t[2]]])/a - ((I/4)*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - I*
a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/a + ((I/4)*Sqrt[2 + Sqrt
[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/S
qrt[2 + Sqrt[2]]])/a + ((I/4)*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] +
 (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/a + ((I/8)*Sq
rt[2 - Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 - Sqr
t[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a - ((I/8)*Sqrt[2 - Sqrt[2]]*L
og[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)
^(1/8))/(1 + I*a*x)^(1/8)])/a + ((I/8)*Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x
)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*
x)^(1/8)])/a - ((I/8)*Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*
x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a
```


Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^{\frac{1}{4} i \tan^{-1}(ax)}\, dx &= \int \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\, dx \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{1}{4}\int \frac{1}{\sqrt[8]{1-iax}(1+iax)^{7/8}}\, dx \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{(2i)\,\mathrm{Subst}\left(\int \frac{x^6}{\left(2-x^8\right)^{7/8}}\, dx, x, \sqrt[8]{1-iax}\right)}{a} \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{(2i)\,\mathrm{Subst}\left(\int \frac{x^6}{1+x^8}\, dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{a} \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{i\,\mathrm{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\, dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{\sqrt{2}a} - \frac{i\,\mathrm{Subst}\left(\int \frac{x^4}{1+\sqrt{2}x^2+x^4}\, dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{\sqrt{2}a} \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} - \frac{i\,\mathrm{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\, dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{\sqrt{2}a} + \frac{i\,\mathrm{Subst}\left(\int \frac{1+\sqrt{2}x^2}{1+\sqrt{2}x^2+x^4}\, dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{\sqrt{2}a} \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{i\,\mathrm{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\, dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}a} + \frac{i\,\mathrm{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\, dx, x, \frac{\sqrt[8]{1}}{\sqrt[8]{1}}\right.}{2\sqrt{2\left(2-\sqrt{2}\right)}a} \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{\left(i\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\, dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{4a} + \frac{\left(i\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\right)\mathrm{Su}}{} \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{i\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a} - \frac{i\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-}}{}\right.}{8a} \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} - \frac{i\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{4a} - \frac{i\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{4a} + \frac{i\sqrt{2+}}{}
\end{aligned}
$$

Mathematica [C] time = 0.0217621, size = 41, normalized size = 0.06

$$
-\frac{16ie^{\frac{9}{4}i\tan^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{9}{8},2,\frac{17}{8},-e^{2i\tan^{-1}(ax)}\right)}{9a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/4)*ArcTan[a*x]),x]
```

```
[Out] (((-16*I)/9)*E^(((9*I)/4)*ArcTan[a*x])*Hypergeometric2F1[9/8, 2, 17/8, -E^(
(2*I)*ArcTan[a*x])])/a
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \sqrt[4]{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

Fricas [A] time = 1.79819, size = 1115, normalized size = 1.65

$$-i\,a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}\log\left(4\,a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)+a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}\log\left(4i\,a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)-a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}\log\left(-4i\,a\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x, algorithm="fricas")
```

```
[Out] (-I*a*(1/256*I/a^4)^(1/4)*log(4*a*(1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 + 1
)/(a*x + I))^(1/4)) + a*(1/256*I/a^4)^(1/4)*log(4*I*a*(1/256*I/a^4)^(1/4) +
 (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - a*(1/256*I/a^4)^(1/4)*log(-4*I*a*
(1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + I*a*(1/256*I
/a^4)^(1/4)*log(-4*a*(1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^
(1/4)) - I*a*(-1/256*I/a^4)^(1/4)*log(4*a*(-1/256*I/a^4)^(1/4) + (I*sqrt(a^
2*x^2 + 1)/(a*x + I))^(1/4)) + a*(-1/256*I/a^4)^(1/4)*log(4*I*a*(-1/256*I/a
^4)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - a*(-1/256*I/a^4)^(1/4)
*log(-4*I*a*(-1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) +
 I*a*(-1/256*I/a^4)^(1/4)*log(-4*a*(-1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 +
```

```
1)/(a*x + I))^(1/4)) + (a*x + I)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x, algorithm="giac")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

3.131 $\int \frac{e^{\frac{1}{4}i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=859

$$-2\tan^{-1}\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt{2+\sqrt{2}}}\right)+\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt{2-\sqrt{2}}}\right)-\sqrt{2+\sqrt{2}}\tan$$

```
[Out] -2*ArcTan[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)] + Sqrt[2 + Sqrt[2]]*ArcTan[(
Sqrt[2 - Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[
2]]] + Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/
(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]] - Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 -
Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]] - Sq
rt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*
x)^(1/8))/Sqrt[2 - Sqrt[2]]] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 + I*a*x)^(1/8
))/(1 - I*a*x)^(1/8)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 -
 I*a*x)^(1/8)] - 2*ArcTanh[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)] - (Sqrt[2 -
 Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 - Sqrt[2]]*
(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2 + (Sqrt[2 - Sqrt[2]]*Log[1 + (1 -
I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 +
 I*a*x)^(1/8)])/2 - (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x
)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2 + (Sq
rt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 + Sqr
t[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2 + Log[1 - (Sqrt[2]*(1 + I*a*
x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/Sqrt[2]
- Log[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)
/(1 - I*a*x)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.545886, antiderivative size = 859, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 20, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.25, Rules used = {5062, 105, 63, 331, 299, 1122, 1169, 634, 618, 204, 628, 93, 214, 212, 206, 203, 211, 1165, 1162, 617}

$$-2\tan^{-1}\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt{2+\sqrt{2}}}\right)+\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt{2-\sqrt{2}}}\right)-\sqrt{2+\sqrt{2}}\tan$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/4)*ArcTan[a*x])/x,x]
```

```
[Out] -2*ArcTan[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)] + Sqrt[2 + Sqrt[2]]*ArcTan[(
Sqrt[2 - Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[
2]]] + Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/
(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]] - Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 -
Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]] - Sq
rt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*
x)^(1/8))/Sqrt[2 - Sqrt[2]]] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 + I*a*x)^(1/8
))/(1 - I*a*x)^(1/8)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 -
 I*a*x)^(1/8)] - 2*ArcTanh[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)] - (Sqrt[2 -
 Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 - Sqrt[2]]*
(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2 + (Sqrt[2 - Sqrt[2]]*Log[1 + (1 -
I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 +
 I*a*x)^(1/8)])/2 - (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x
```

```
)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2 + (Sq
rt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 + Sqr
t[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2 + Log[1 - (Sqrt[2]*(1 + I*a*
x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/Sqrt[2]
- Log[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)
/(1 - I*a*x)^(1/4)]/Sqrt[2]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
```

```
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}i\tan^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt[8]{1+iax}}{x\sqrt[8]{1-iax}}\,dx$$
$$= (ia)\int \frac{1}{\sqrt[8]{1-iax}(1+iax)^{7/8}}\,dx + \int \frac{1}{x\sqrt[8]{1-iax}(1+iax)^{7/8}}\,dx$$
$$= -\left(8\,\text{Subst}\left(\int \frac{x^6}{\left(2-x^8\right)^{7/8}}\,dx, x, \sqrt[8]{1-iax}\right)\right) + 8\,\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$
$$= -\left(4\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)\right) - 4\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 8\,\text{Subst}\left(\int \frac{x^6}{1+\cdots}\right.$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2\,\text{Subst}\left(\int \frac{1-\cdots}{1+\cdots}\right.$$
$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\cdots}{-1\cdots}\right.}{\cdots}$$
$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\cdots\right.}{\sqrt{2}}$$
$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2\tanh^{-1}\left(\cdots\right.$$
$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2\tanh^{-1}\left(\cdots\right.$$
$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right) + \sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1\cdots}}{\sqrt[8]{1+\cdots}}}{\sqrt{2-\sqrt{2}}}\right.$$

Mathematica [C] time = 0.033823, size = 97, normalized size = 0.11

$$-\frac{4(1-iax)^{7/8}\left(\sqrt[8]{2}(1+iax)^{7/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{7}{8},\frac{15}{8},\frac{1}{2}(1-iax)\right)+2\text{Hypergeometric2F1}\left(\frac{7}{8},1,\frac{15}{8},\frac{ax+i}{-ax+i}\right)\right)}{7(1+iax)^{7/8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/4)*ArcTan[a*x])/x,x]
```

```
[Out] (-4*(1 - I*a*x)^(7/8)*(2^(1/8)*(1 + I*a*x)^(7/8)*Hypergeometric2F1[7/8, 7/8
, 15/8, (1 - I*a*x)/2] + 2*Hypergeometric2F1[7/8, 1, 15/8, (I + a*x)/(I - a
*x)]))/(7*(1 + I*a*x)^(7/8))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt[4]{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x)

[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="maxima")

[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4)/x, x)
```

Fricas [A] time = 1.8875, size = 1423, normalized size = 1.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="fricas")

[Out] -1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) +
 1/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4))
- 1/2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)
) + 1/2*sqrt(-4*I)*log(-1/2*sqrt(-4*I) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1
/4)) + I^(1/4)*log(I^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + I*I^(
1/4)*log(I*I^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - I*I^(1/4)*log
(-I*I^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - I^(1/4)*log(-I^(1/4)
 + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + (-I)^(1/4)*log((-I)^(1/4) + (I*
sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + I*(-I)^(1/4)*log(I*(-I)^(1/4) + (I*sq
rt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - I*(-I)^(1/4)*log(-I*(-I)^(1/4) + (I*sqr
t(a^2*x^2 + 1)/(a*x + I))^(1/4)) - (-I)^(1/4)*log(-(-I)^(1/4) + (I*sqrt(a^2
*x^2 + 1)/(a*x + I))^(1/4)) - log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + 1
) - I*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + I) + I*log((I*sqrt(a^2*x^
2 + 1)/(a*x + I))^(1/4) - I) + log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) -
1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)/x,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.132 $\int \frac{e^{\frac{1}{4} i \tan^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=328

$$-\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}+\frac{ia\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{4\sqrt{2}}-\frac{ia\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{4\sqrt{2}}-\frac{1}{2}ia\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)+\frac{ia\tan\ldots}{}$$

```
[Out] -(((1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/x) - (I/2)*a*ArcTan[(1 + I*a*x)^(1/
8)/(1 - I*a*x)^(1/8)] + ((I/2)*a*ArcTan[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1
- I*a*x)^(1/8)])/Sqrt[2] - ((I/2)*a*ArcTan[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/
(1 - I*a*x)^(1/8)])/Sqrt[2] - (I/2)*a*ArcTanh[(1 + I*a*x)^(1/8)/(1 - I*a*x)
^(1/8)] + ((I/4)*a*Log[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) +
(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/Sqrt[2] - ((I/4)*a*Log[1 + (Sqrt[2]*(
1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])
/Sqrt[2]
```

Rubi [A] time = 0.132148, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5062, 94, 93, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}+\frac{ia\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{4\sqrt{2}}-\frac{ia\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{4\sqrt{2}}-\frac{1}{2}ia\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)+\frac{ia\tan\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/4)*ArcTan[a*x])/x^2,x]
```


```
[Out] -(((1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/x) - (I/2)*a*ArcTan[(1 + I*a*x)^(1/
8)/(1 - I*a*x)^(1/8)] + ((I/2)*a*ArcTan[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1
- I*a*x)^(1/8)])/Sqrt[2] - ((I/2)*a*ArcTan[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/
(1 - I*a*x)^(1/8)])/Sqrt[2] - (I/2)*a*ArcTanh[(1 + I*a*x)^(1/8)/(1 - I*a*x)
^(1/8)] + ((I/4)*a*Log[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) +
(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/Sqrt[2] - ((I/4)*a*Log[1 + (Sqrt[2]*(
1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])
/Sqrt[2]
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}i\tan^{-1}(ax)}}{x^2}\,dx = \int \frac{\sqrt[8]{1+iax}}{x^2\sqrt[8]{1-iax}}\,dx$$
$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} + \frac{1}{4}(ia)\int \frac{1}{x\sqrt[8]{1-iax}(1+iax)^{7/8}}\,dx$$
$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} + (2ia)\,\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$
$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} - (ia)\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - (ia)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+ia}}{\sqrt[8]{1-ia}}\right.$$
$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} - \frac{1}{2}(ia)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{1}{2}(ia)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[8]{1}}{\sqrt[8]{1}}\right.$$
$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} - \frac{1}{2}ia\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{1}{2}ia\tanh^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{1}{4}(ia)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x}\right.$$
$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} - \frac{1}{2}ia\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{1}{2}ia\tanh^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{ia\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+\frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.}{4\sqrt{2}}$$
$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} - \frac{1}{2}ia\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{ia\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{2\sqrt{2}} - \frac{ia\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{2\sqrt{2}}$$

Mathematica [C] time = 0.0162501, size = 71, normalized size = 0.22

$$-\frac{i(1-iax)^{7/8}\left(2ax\,\text{Hypergeometric2F1}\left(\frac{7}{8},1,\frac{15}{8},\frac{ax+i}{-ax+i}\right)+7ax-7i\right)}{7x(1+iax)^{7/8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/4)*ArcTan[a*x])/x^2,x]
```

```
[Out] ((-I/7)*(1 - I*a*x)^(7/8)*(-7*I + 7*a*x + 2*a*x*Hypergeometric2F1[7/8, 1, 1
5/8, (I + a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(7/8))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt[4]{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4)/x^2, x)
```

Fricas [A] time = 1.82177, size = 819, normalized size = 2.5

$$-i\,ax\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+1\right)+ax\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+i\right)-ax\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}-i\right)+i\,ax\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}-1\right)+\sqrt{i\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(-I*a*x*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + 1) + a*x*log((I*sqr
t(a^2*x^2 + 1)/(a*x + I))^(1/4) + I) - a*x*log((I*sqrt(a^2*x^2 + 1)/(a*x +
I))^(1/4) - I) + I*a*x*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) - 1) + sqr
t(I*a^2)*x*log((a*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + I*sqrt(I*a^2))/a)
 - sqrt(I*a^2)*x*log((a*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) - I*sqrt(I*a^
2))/a) + sqrt(-I*a^2)*x*log((a*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + I*sq
rt(-I*a^2))/a) - sqrt(-I*a^2)*x*log((a*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4
) - I*sqrt(-I*a^2))/a) - 4*(-I*a*x + 1)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/
4))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)/x**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.133 $\int \frac{e^{\frac{1}{4}i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=364

$$-\frac{a^2\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{32\sqrt{2}}+\frac{a^2\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{32\sqrt{2}}+\frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)-\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right.}{16\sqrt{2}}$$

```
[Out] ((-I/8)*a*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/x - ((1 - I*a*x)^(7/8)*(1 +
I*a*x)^(9/8))/(2*x^2) + (a^2*ArcTan[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)])/1
6 - (a^2*ArcTan[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8)])/(16*Sqr
t[2]) + (a^2*ArcTan[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8)])/(16
*Sqrt[2]) + (a^2*ArcTanh[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)])/16 - (a^2*Lo
g[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1
- I*a*x)^(1/4)])/(32*Sqrt[2]) + (a^2*Log[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1
 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/(32*Sqrt[2])
```

Rubi [A] time = 0.161089, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5062, 96, 94, 93, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a^2\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{32\sqrt{2}}+\frac{a^2\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{32\sqrt{2}}+\frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)-\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right.}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/4)*ArcTan[a*x])/x^3,x]
```

```
[Out] ((-I/8)*a*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/x - ((1 - I*a*x)^(7/8)*(1 +
I*a*x)^(9/8))/(2*x^2) + (a^2*ArcTan[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)])/1
6 - (a^2*ArcTan[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8)])/(16*Sqr
t[2]) + (a^2*ArcTan[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8)])/(16
*Sqrt[2]) + (a^2*ArcTanh[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)])/16 - (a^2*Lo
g[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1
- I*a*x)^(1/4)])/(32*Sqrt[2]) + (a^2*Log[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1
 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/(32*Sqrt[2])
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}i\tan^{-1}(ax)}}{x^3}\,dx = \int \frac{\sqrt[8]{1+iax}}{x^3\sqrt[8]{1-iax}}\,dx$$
$$= -\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{8}(ia)\int \frac{\sqrt[8]{1+iax}}{x^2\sqrt[8]{1-iax}}\,dx$$
$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} - \frac{1}{32}a^2\int \frac{1}{x\sqrt[8]{1-iax}(1+iax)^{7/8}}\,dx$$
$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} - \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$
$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{8}a^2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{1}{8}a^2$$
$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{16}a^2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{1}{16}$$
$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{1}{16}a^2\tanh^{-1}\left(\frac{\sqrt[8]{1+}}{\sqrt[8]{1-}}\right.$$
$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{1}{16}a^2\tanh^{-1}\left(\frac{\sqrt[8]{1+}}{\sqrt[8]{1-}}\right.$$
$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1-}}{\sqrt[8]{1-i}}\right.}{16\sqrt{2}}$$

Mathematica [C] time = 0.0215134, size = 84, normalized size = 0.23

$$\frac{(1-iax)^{7/8}\left(2a^2x^2\text{Hypergeometric2F1}\left(\frac{7}{8},1,\frac{15}{8},\frac{ax+i}{-ax+i}\right)+7\left(5a^2x^2-9iax-4\right)\right)}{56x^2(1+iax)^{7/8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/4)*ArcTan[a*x])/x^3,x]
```

```
[Out] ((1 - I*a*x)^(7/8)*(7*(-4 - (9*I)*a*x + 5*a^2*x^2) + 2*a^2*x^2*Hypergeometr
ic2F1[7/8, 1, 15/8, (I + a*x)/(I - a*x)]))/(56*x^2*(1 + I*a*x)^(7/8))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt[4]{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4)/x^3, x)
```

Fricas [A] time = 1.83024, size = 883, normalized size = 2.43

$$a^2x^2\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+1\right)+i\,a^2x^2\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+i\right)-i\,a^2x^2\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}-i\right)-a^2x^2\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}-1\right)+\sqrt{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="fricas")
```

```
[Out] 1/32*(a^2*x^2*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + 1) + I*a^2*x^2*lo
g((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + I) - I*a^2*x^2*log((I*sqrt(a^2*x^
2 + 1)/(a*x + I))^(1/4) - I) - a^2*x^2*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^
(1/4) - 1) + sqrt(I*a^4)*x^2*log((a^2*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)
 + sqrt(I*a^4))/a^2) - sqrt(I*a^4)*x^2*log((a^2*(I*sqrt(a^2*x^2 + 1)/(a*x +
 I))^(1/4) - sqrt(I*a^4))/a^2) + sqrt(-I*a^4)*x^2*log((a^2*(I*sqrt(a^2*x^2
+ 1)/(a*x + I))^(1/4) + sqrt(-I*a^4))/a^2) - sqrt(-I*a^4)*x^2*log((a^2*(I*s
qrt(a^2*x^2 + 1)/(a*x + I))^(1/4) - sqrt(-I*a^4))/a^2) - (20*a^2*x^2 + 4*I*
a*x + 16)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)/x**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.134 $\int e^{6i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=114

$$\frac{2\left(2m^2+4m+3\right)x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,iax)}{m+1}+\frac{4ix^{m+1}\left(a\left(m^2+3m+3\right)x+i(m+1)^2\right)}{(m+1)(1-iax)^2}-\frac{(1+ia}{(m+1)(}$$

```
[Out] -((x^(1 + m)*(1 + I*a*x)^2)/((1 + m)*(1 - I*a*x)^2)) + ((4*I)*x^(1 + m)*(I*
(1 + m)^2 + a*(3 + 3*m + m^2)*x))/((1 + m)*(1 - I*a*x)^2) + (2*(3 + 4*m + 2
*m^2)*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, I*a*x])/(1 + m)
```

Rubi [A] time = 0.0950837, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5062, 100, 145, 64}

$$\frac{2\left(2m^2+4m+3\right)x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,iax)}{m+1}+\frac{4ix^{m+1}\left(a\left(m^2+3m+3\right)x+i(m+1)^2\right)}{(m+1)(1-iax)^2}-\frac{(1+ia}{(m+1)(}$$

Antiderivative was successfully verified.

```
[In] Int[E^((6*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] -((x^(1 + m)*(1 + I*a*x)^2)/((1 + m)*(1 - I*a*x)^2)) + ((4*I)*x^(1 + m)*(I*
(1 + m)^2 + a*(3 + 3*m + m^2)*x))/((1 + m)*(1 - I*a*x)^2) + (2*(3 + 4*m + 2
*m^2)*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, I*a*x])/(1 + m)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int e^{6i\tan^{-1}(ax)}x^m\,dx = \int \frac{x^m(1+iax)^3}{(1-iax)^3}\,dx$$

$$= -\frac{x^{1+m}(1+iax)^2}{(1+m)(1-iax)^2} + \frac{i\int \frac{x^m(1+iax)\left(-2ia(1+m)+2a^2(3+m)x\right)}{(1-iax)^3}\,dx}{a(1+m)}$$

$$= -\frac{x^{1+m}(1+iax)^2}{(1+m)(1-iax)^2} + \frac{4ix^{1+m}\left(i(1+m)^2+a\left(3+3m+m^2\right)x\right)}{(1+m)(1-iax)^2} + \left(2\left(3+4m+2m^2\right)\right)\int \frac{x^m}{1-iax}$$

$$= -\frac{x^{1+m}(1+iax)^2}{(1+m)(1-iax)^2} + \frac{4ix^{1+m}\left(i(1+m)^2+a\left(3+3m+m^2\right)x\right)}{(1+m)(1-iax)^2} + \frac{2\left(3+4m+2m^2\right)x^{1+m}\,{}_2F_1(1,1}{1+m}$$

Mathematica [A] time = 0.0449684, size = 94, normalized size = 0.82

$$\frac{x^{m+1}\left(2\left(2m^2+4m+3\right)(ax+i)^2\text{Hypergeometric2F1}(1,m+1,m+2,iax)-a^2x^2+m^2(4-4iax)+4m(2-3iax)-\right.}{(m+1)(ax+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((6*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(5 - (10*I)*a*x - a^2*x^2 + 4*m*(2 - (3*I)*a*x) + m^2*(4 - (4*I)
*a*x) + 2*(3 + 4*m + 2*m^2)*(I + a*x)^2*Hypergeometric2F1[1, 1 + m, 2 + m,
I*a*x]))/((1 + m)*(I + a*x)^2)
```

Maple [C] time = 0.625, size = 748, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x)
```

```
[Out] 1/4*(a^2)^(-1/2-1/2*m)*(1/2/(1+m)*x^(1+m)*(a^2)^(1/2+1/2*m)*(-a^2*m^2*x^2+2
*a^2*m*x^2+3*a^2*x^2-m^2+4*m+5)/(a^2*x^2+1)^2+4/(1+m)*x^(1+m)*(a^2)^(1/2+1/
2*m)*(1/16*m^3-3/16*m^2-1/16*m+3/16)*LerchPhi(-a^2*x^2,1,1/2+1/2*m))+3/2*I/
a*(a^2)^(-1/2*m)*(1/2*x^m*(a^2)^(1/2*m)*(a^2*m*x^2+m-2)/(a^2*x^2+1)^2-1/4*x
^m*(a^2)^(1/2*m)*(-2+m)*m*LerchPhi(-a^2*x^2,1,1/2*m))-15/4*(a^2)^(-1/2-1/2*
m)*(1/2*x^(1+m)*(a^2)^(3/2+1/2*m)*(a^2*m*x^2+a^2*x^2+m-1)/(a^2*x^2+1)^2/a^2
-1/4*x^(1+m)*(a^2)^(3/2+1/2*m)*(1+m)*(-1+m)/a^2*LerchPhi(-a^2*x^2,1,1/2+1/2
*m))-5*I/a*(a^2)^(-1/2*m)*(-1/2*x^m*(a^2)^(1/2*m)*(a^2*m*x^2+4*a^2*x^2+m+2)
/(a^2*x^2+1)^2+1/4*x^m*(a^2)^(1/2*m)*(2+m)*m*LerchPhi(-a^2*x^2,1,1/2*m))+15
/4*(a^2)^(-1/2-1/2*m)*(-1/2*x^(1+m)*(a^2)^(5/2+1/2*m)*(a^2*m*x^2+5*a^2*x^2+
m+3)/a^4/(a^2*x^2+1)^2+1/4*x^(1+m)*(a^2)^(5/2+1/2*m)*(m^2+4*m+3)/a^4*LerchP
hi(-a^2*x^2,1,1/2+1/2*m))+3/2*I/a*(a^2)^(-1/2*m)*(1/2*x^m*(a^2)^(1/2*m)*(8*
a^4*x^4+a^2*m^2*x^2+8*a^2*m*x^2+16*a^2*x^2+m^2+6*m+8)/(a^2*x^2+1)^2/m-1/4*x
```

```
^m*(a^2)^(1/2*m)*(m^2+6*m+8)*LerchPhi(-a^2*x^2,1,1/2*m))-1/4*(a^2)^(-1/2-1/
2*m)*(1/2*x^(1+m)*(a^2)^(7/2+1/2*m)*(8*a^4*x^4+a^2*m^2*x^2+10*a^2*m*x^2+25*
a^2*x^2+m^2+8*m+15)/(a^2*x^2+1)^2/(1+m)/a^6-1/4*x^(1+m)*(a^2)^(7/2+1/2*m)*(
m^2+8*m+15)/a^6*LerchPhi(-a^2*x^2,1,1/2+1/2*m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)^6 x^m}{\left(a^2x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x, algorithm="maxima")
```

```
[Out] integrate((I*a*x + 1)^6*x^m/(a^2*x^2 + 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(a^3x^3-3i\,a^2x^2-3\,ax+i\right)x^m}{a^3x^3+3i\,a^2x^2-3\,ax-i},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x, algorithm="fricas")
```

```
[Out] integral(-(a^3*x^3 - 3*I*a^2*x^2 - 3*a*x + I)*x^m/(a^3*x^3 + 3*I*a^2*x^2 -
3*a*x - I), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\,(iax+1)^6}{\left(a^2x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**6/(a**2*x**2+1)**3*x**m,x)
```

```
[Out] Integral(x**m*(I*a*x + 1)**6/(a**2*x**2 + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)^6 x^m}{\left(a^2x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x, algorithm="giac")
```

```
[Out] integrate((I*a*x + 1)^6*x^m/(a^2*x^2 + 1)^3, x)
```

3.135 $\int e^{4i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=50

$$-4x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,iax)+\frac{4x^{m+1}}{1-iax}+\frac{x^{m+1}}{m+1}$$

```
[Out] x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 - I*a*x) - 4*x^(1 + m)*Hypergeometric2
F1[1, 1 + m, 2 + m, I*a*x]
```

Rubi [A] time = 0.0420944, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5062, 89, 80, 64}

$$-4x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,iax)+\frac{4x^{m+1}}{1-iax}+\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 - I*a*x) - 4*x^(1 + m)*Hypergeometric2
F1[1, 1 + m, 2 + m, I*a*x]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int e^{4i\tan^{-1}(ax)}x^m\,dx = \int \frac{x^m(1+iax)^2}{(1-iax)^2}\,dx$$
$$= \frac{4x^{1+m}}{1-iax} + \frac{\int \frac{x^m\left(-a^2(3+4m)-ia^3x\right)}{1-iax}\,dx}{a^2}$$
$$= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1-iax} - (4(1+m))\int \frac{x^m}{1-iax}\,dx$$
$$= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1-iax} - 4x^{1+m}\,{}_2F_1(1,1+m;2+m;iax)$$

Mathematica [A] time = 0.0213528, size = 58, normalized size = 1.16

$$\frac{x^{m+1}(-4(m+1)(ax+i)\text{Hypergeometric2F1}(1,m+1,m+2,iax)+ax+4im+5i)}{(m+1)(ax+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(5*I + (4*I)*m + a*x - 4*(1 + m)*(I + a*x)*Hypergeometric2F1[1,
1 + m, 2 + m, I*a*x]))/((1 + m)*(I + a*x))
```

Maple [C] time = 0.516, size = 417, normalized size = 8.3

$$\frac{1}{2}\left(a^2\right)^{-\frac{1}{2}-\frac{m}{2}}\left(2\frac{x^{1+m}\left(a^2\right)^{1/2+m/2}}{2a^2x^2+2}+2\frac{x^{1+m}\left(a^2\right)^{1/2+m/2}\left(-1/4\,m^2+1/4\right)\text{LerchPhi}\left(-a^2x^2,1,1/2+m/2\right)}{1+m}\right)+\frac{2i}{a}\left(a^2\right)^{-\frac{m}{2}}\left(\frac{x}{(2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x)
```

```
[Out] 1/2*(a^2)^(-1/2-1/2*m)*(2*x^(1+m)*(a^2)^(1/2+1/2*m)/(2*a^2*x^2+2)+2/(1+m)*x
^(1+m)*(a^2)^(1/2+1/2*m)*(-1/4*m^2+1/4)*LerchPhi(-a^2*x^2,1,1/2+1/2*m))+2*I
/a*(a^2)^(-1/2*m)*(1/(2+m)*x^m*(a^2)^(1/2*m)*(-m-2)/(a^2*x^2+1)+1/2*x^m*(a^
2)^(1/2*m)*m*LerchPhi(-a^2*x^2,1,1/2*m))-3*(a^2)^(-1/2-1/2*m)*(1/(3+m)*x^(1
+m)*(a^2)^(3/2+1/2*m)*(-3-m)/a^2/(a^2*x^2+1)+1/2*x^(1+m)*(a^2)^(3/2+1/2*m)*
(1+m)/a^2*LerchPhi(-a^2*x^2,1,1/2+1/2*m))-2*I/a*(a^2)^(-1/2*m)*(x^m*(a^2)^(
1/2*m)*(2*a^2*x^2+m+2)/(a^2*x^2+1)/m-1/2*x^m*(a^2)^(1/2*m)*(2+m)*LerchPhi(-
a^2*x^2,1,1/2*m))+1/2*(a^2)^(-1/2-1/2*m)*(x^(1+m)*(a^2)^(5/2+1/2*m)*(2*a^2*
x^2+m+3)/(a^2*x^2+1)/a^4/(1+m)-1/2*x^(1+m)*(a^2)^(5/2+1/2*m)*(3+m)/a^4*Lerc
hPhi(-a^2*x^2,1,1/2+1/2*m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)^4x^m}{\left(a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x, algorithm="maxima")
```

```
[Out] integrate((I*a*x + 1)^4*x^m/(a^2*x^2 + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2x^2-2i\,ax-1\right)x^m}{a^2x^2+2i\,ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - 2*I*a*x - 1)*x^m/(a^2*x^2 + 2*I*a*x - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^m\,(iax+1)^4}{\left(a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x**m,x)
```

```
[Out] Integral(x**m*(I*a*x + 1)**4/(a**2*x**2 + 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(i\,ax+1)^4x^m}{\left(a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x, algorithm="giac")
```

```
[Out] integrate((I*a*x + 1)^4*x^m/(a^2*x^2 + 1)^2, x)
```

3.136 $\int e^{2i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=39

$$-\frac{x^{m+1}}{m+1}+\frac{2x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,iax)}{m+1}$$

```
[Out] -(x^(1 + m)/(1 + m)) + (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, I*a*
x])/(1 + m)
```

Rubi [A] time = 0.0226873, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5062, 80, 64}

$$-\frac{x^{m+1}}{m+1}+\frac{2x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,iax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] -(x^(1 + m)/(1 + m)) + (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, I*a*
x])/(1 + m)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int e^{2i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1+iax)}{1-iax}\,dx\\ &= -\frac{x^{1+m}}{1+m}+2\int \frac{x^m}{1-iax}\,dx\\ &= -\frac{x^{1+m}}{1+m}+\frac{2x^{1+m}\,{}_2F_1(1,1+m;2+m;iax)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.0066885, size = 29, normalized size = 0.74

$$\frac{x^{m+1}(-1+2\text{Hypergeometric2F1}(1,m+1,m+2,iax))}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(-1 + 2*Hypergeometric2F1[1, 1 + m, 2 + m, I*a*x]))/(1 + m)
```

Maple [C] time = 0.463, size = 175, normalized size = 4.5

$$\frac{x^{1+m}}{1+m}\left(\frac{1}{2}+\frac{m}{2}\right)\text{LerchPhi}\left(-a^2x^2,1,\frac{1}{2}+\frac{m}{2}\right)+\frac{i}{a}\left(a^2\right)^{-\frac{m}{2}}\left(2\frac{x^m\left(a^2\right)^{m/2}}{m}+\frac{x^m\left(-m-2\right)}{2+m}\left(a^2\right)^{\frac{m}{2}}\text{LerchPhi}\left(-a^2x^2,1,\frac{m}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)*x^m,x)
```

```
[Out] 1/(1+m)*x^(1+m)*(1/2+1/2*m)*LerchPhi(-a^2*x^2,1,1/2+1/2*m)+I/a*(a^2)^(-1/2*
m)*(2*x^m*(a^2)^(1/2*m)/m+1/(2+m)*x^m*(a^2)^(1/2*m)*(-m-2)*LerchPhi(-a^2*x^
2,1,1/2*m))-1/2*(a^2)^(-1/2-1/2*m)*(2*x^(1+m)*(a^2)^(3/2+1/2*m)/(1+m)/a^2+1
/(3+m)*x^(1+m)*(a^2)^(3/2+1/2*m)*(-3-m)/a^2*LerchPhi(-a^2*x^2,1,1/2+1/2*m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)^2x^m}{a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^m,x, algorithm="maxima")
```

```
[Out] integrate((I*a*x + 1)^2*x^m/(a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ax-i)x^m}{ax+i},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^m,x, algorithm="fricas")
```

```
[Out] integral(-(a*x - I)*x^m/(a*x + I), x)
```

Sympy [B] time = 3.93472, size = 269, normalized size = 6.9

$$-\frac{a^2mx^3x^m\Phi\left(a^2x^2e^{i\pi},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}-\frac{3a^2x^3x^m\Phi\left(a^2x^2e^{i\pi},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{iamx^2x^m\Phi\left(a^2x^2e^{i\pi},1,\frac{m}{2}+1\right)\Gamma\left(\right.}{2\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)*x**m,x)
```

```
[Out] -a**2*m*x**3*x**m*lerchphi(a**2*x**2*exp_polar(I*pi), 1, m/2 + 3/2)*gamma(m
/2 + 3/2)/(4*gamma(m/2 + 5/2)) - 3*a**2*x**3*x**m*lerchphi(a**2*x**2*exp_po
lar(I*pi), 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*gamma(m/2 + 5/2)) + I*a*m*x**2
*x**m*lerchphi(a**2*x**2*exp_polar(I*pi), 1, m/2 + 1)*gamma(m/2 + 1)/(2*gam
ma(m/2 + 2)) + I*a*x**2*x**m*lerchphi(a**2*x**2*exp_polar(I*pi), 1, m/2 + 1
)*gamma(m/2 + 1)/gamma(m/2 + 2) + m*x*x**m*lerchphi(a**2*x**2*exp_polar(I*p
i), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*gamma(m/2 + 3/2)) + x*x**m*lerchphi(a
**2*x**2*exp_polar(I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*gamma(m/2 + 3/2
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)^2x^m}{a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^m,x, algorithm="giac")
```

```
[Out] integrate((I*a*x + 1)^2*x^m/(a^2*x^2 + 1), x)
```

3.137 $\int e^{-2i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=39

$$-\frac{x^{m+1}}{m+1}+\frac{2x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-iax)}{m+1}$$

```
[Out] -(x^(1 + m)/(1 + m)) + (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-I)
*a*x])/(1 + m)
```

Rubi [A] time = 0.0231997, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5062, 80, 64}

$$-\frac{x^{m+1}}{m+1}+\frac{2x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-iax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] -(x^(1 + m)/(1 + m)) + (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-I)
*a*x])/(1 + m)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int e^{-2i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1-iax)}{1+iax}\,dx\\ &= -\frac{x^{1+m}}{1+m}+2\int \frac{x^m}{1+iax}\,dx\\ &= -\frac{x^{1+m}}{1+m}+\frac{2x^{1+m}\,{}_2F_1(1,1+m;2+m;-iax)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.0082643, size = 29, normalized size = 0.74

$$\frac{x^{m+1}(-1+2\text{Hypergeometric2F1}(1,m+1,m+2,-iax))}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] (x^(1 + m)*(-1 + 2*Hypergeometric2F1[1, 1 + m, 2 + m, (-I)*a*x]))/(1 + m)
```

Maple [C] time = 0.382, size = 158, normalized size = 4.1

$$\frac{i(ia)^{-m}}{a}\left(\frac{x^m(ia)^m\left(-a^2mx^2-iamx-m^2-2iax-3m-2\right)}{m(1+m)(1+iax)}+x^m(ia)^m(2+m)\,\text{LerchPhi}(-iax,1,m)\right)-\frac{i(ia)^{-m}}{a}\left(\frac{x^m(ia)}{(1+m}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1+I*a*x)^2*(a^2*x^2+1),x)
```

```
[Out] I*(I*a)^(-m)/a*(x^m*(I*a)^m*(-a^2*m*x^2-I*a*m*x-m^2-2*I*a*x-3*m-2)/(1+m)/m/
(1+I*a*x)+x^m*(I*a)^m*(2+m)*LerchPhi(-I*a*x,1,m))-I*(I*a)^(-m)/a*(1/(1+m)*x
^m*(I*a)^m*(-1-m)/(1+I*a*x)+x^m*(I*a)^m*m*LerchPhi(-I*a*x,1,m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2x^2+1\right)x^m}{(i\,ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)*x^m/(I*a*x + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ax+i)x^m}{ax-i},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-(a*x + I)*x^m/(a*x - I), x)
```

Sympy [B] time = 31.5172, size = 578, normalized size = 14.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(1+I*a*x)**2*(a**2*x**2+1),x)
```

```
[Out] a**2*(-I*a*m**2*x**4*x**m*lerchphi(a*x*exp_polar(3*I*pi/2), 1, m + 3)*gamma
(m + 3)/(I*a*x*gamma(m + 4) + gamma(m + 4)) - 5*I*a*m*x**4*x**m*lerchphi(a*
x*exp_polar(3*I*pi/2), 1, m + 3)*gamma(m + 3)/(I*a*x*gamma(m + 4) + gamma(m
 + 4)) - 6*I*a*x**4*x**m*lerchphi(a*x*exp_polar(3*I*pi/2), 1, m + 3)*gamma(
m + 3)/(I*a*x*gamma(m + 4) + gamma(m + 4)) - m**2*x**3*x**m*lerchphi(a*x*ex
p_polar(3*I*pi/2), 1, m + 3)*gamma(m + 3)/(I*a*x*gamma(m + 4) + gamma(m + 4
)) - 5*m*x**3*x**m*lerchphi(a*x*exp_polar(3*I*pi/2), 1, m + 3)*gamma(m + 3)
/(I*a*x*gamma(m + 4) + gamma(m + 4)) + m*x**3*x**m*gamma(m + 3)/(I*a*x*gamm
a(m + 4) + gamma(m + 4)) - 6*x**3*x**m*lerchphi(a*x*exp_polar(3*I*pi/2), 1,
 m + 3)*gamma(m + 3)/(I*a*x*gamma(m + 4) + gamma(m + 4)) + 3*x**3*x**m*gamm
a(m + 3)/(I*a*x*gamma(m + 4) + gamma(m + 4))) - I*a*m**2*x**2*x**m*lerchphi
(a*x*exp_polar(3*I*pi/2), 1, m + 1)*gamma(m + 1)/(I*a*x*gamma(m + 2) + gamm
a(m + 2)) - I*a*m*x**2*x**m*lerchphi(a*x*exp_polar(3*I*pi/2), 1, m + 1)*gam
ma(m + 1)/(I*a*x*gamma(m + 2) + gamma(m + 2)) - m**2*x*x**m*lerchphi(a*x*ex
p_polar(3*I*pi/2), 1, m + 1)*gamma(m + 1)/(I*a*x*gamma(m + 2) + gamma(m + 2
)) - m*x*x**m*lerchphi(a*x*exp_polar(3*I*pi/2), 1, m + 1)*gamma(m + 1)/(I*a
*x*gamma(m + 2) + gamma(m + 2)) + m*x*x**m*gamma(m + 1)/(I*a*x*gamma(m + 2)
 + gamma(m + 2)) + x*x**m*gamma(m + 1)/(I*a*x*gamma(m + 2) + gamma(m + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2x^2+1)x^m}{(iax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 + 1)*x^m/(I*a*x + 1)^2, x)
```

3.138 $\int e^{-4i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=50

$$-4x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-iax)+\frac{4x^{m+1}}{1+iax}+\frac{x^{m+1}}{m+1}$$

```
[Out] x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 + I*a*x) - 4*x^(1 + m)*Hypergeometric2
F1[1, 1 + m, 2 + m, (-I)*a*x]
```

Rubi [A] time = 0.0390302, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5062, 89, 80, 64}

$$-4x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-iax)+\frac{4x^{m+1}}{1+iax}+\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((4*I)*ArcTan[a*x]),x]
```

```
[Out] x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 + I*a*x) - 4*x^(1 + m)*Hypergeometric2
F1[1, 1 + m, 2 + m, (-I)*a*x]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int e^{-4i\tan^{-1}(ax)}x^m\,dx = \int \frac{x^m(1-iax)^2}{(1+iax)^2}\,dx$$
$$= \frac{4x^{1+m}}{1+iax} + \frac{\int \frac{x^m\left(-a^2(3+4m)+ia^3x\right)}{1+iax}\,dx}{a^2}$$
$$= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1+iax} - (4(1+m))\int \frac{x^m}{1+iax}\,dx$$
$$= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1+iax} - 4x^{1+m}\,{}_2F_1(1,1+m;2+m;-iax)$$

Mathematica [A] time = 0.0207909, size = 58, normalized size = 1.16

$$\frac{x^{m+1}(-4(m+1)(ax-i)\text{Hypergeometric2F1}(1,m+1,m+2,-iax)+ax-4im-5i)}{(m+1)(ax-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/E^((4*I)*ArcTan[a*x]),x]
```

```
[Out] (x^(1 + m)*(-5*I - (4*I)*m + a*x - 4*(1 + m)*(-I + a*x)*Hypergeometric2F1[1
, 1 + m, 2 + m, (-I)*a*x]))/((1 + m)*(-I + a*x))
```

Maple [C] time = 0.536, size = 428, normalized size = 8.6

$$\frac{-\frac{i}{6}(ia)^{-m}}{a}\left(\frac{x^m(ia)^m\left(a^2x^2m^4+6\,a^4x^4m+11\,a^2x^2m^3-126\,iamx-2\,iaxm^4+46\,a^2x^2m^2-72\,iax-m^4+24\,ia^3x^3+90\right.}{m(1+m)(1+iax)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x)
```

```
[Out] -1/6*I*(I*a)^(-m)/a*(x^m*(I*a)^m*(a^2*x^2*m^4+6*a^4*x^4*m+11*a^2*x^2*m^3-12
6*I*a*m*x-2*I*a*x*m^4+46*a^2*x^2*m^2-72*I*a*x-m^4+24*I*a^3*x^3+90*a^2*m*x^2
-21*I*a*x*m^3-10*m^3+72*a^2*x^2-79*I*a*x*m^2-35*m^2+6*I*a^3*x^3*m-50*m-24)/
(1+m)/m/(1+I*a*x)^3+x^m*(I*a)^m*(m^3+9*m^2+26*m+24)*LerchPhi(-I*a*x,1,m))+1
/3*I*(I*a)^(-m)/a*(-x^m*(I*a)^m*(-a^2*x^2*m^2-4*a^2*m*x^2+2*I*a*x*m^2-6*a^2
*x^2+7*I*a*m*x+m^2+6*I*a*x+3*m+2)/(1+I*a*x)^3+x^m*(I*a)^m*m*(m^2+3*m+2)*Ler
chPhi(-I*a*x,1,m))-1/6*I*(I*a)^(-m)/a*(-x^m*(I*a)^m*(-a^2*x^2*m^2+2*a^2*m*x
^2+2*I*a*x*m^2-5*I*a*m*x+m^2-3*m+2)/(1+I*a*x)^3+x^m*(I*a)^m*(m^2-3*m+2)*m*L
erchPhi(-I*a*x,1,m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2+1\right)^2 x^m}{(i\,ax+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^2*x^m/(I*a*x + 1)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2x^2+2i\,ax-1\right)x^m}{a^2x^2-2i\,ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*I*a*x - 1)*x^m/(a^2*x^2 - 2*I*a*x - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(1+I*a*x)**4*(a**2*x**2+1)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2x^2+1\right)^2x^m}{(i\,ax+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 + 1)^2*x^m/(I*a*x + 1)^4, x)
```

3.139 $\int e^{-6i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=115

$$\frac{2\left(2m^2+4m+3\right)x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-iax)}{m+1}+\frac{4ix^{m+1}\left(-a\left(m^2+3m+3\right)x+i(m+1)^2\right)}{(m+1)(1+iax)^2}-\frac{(}{(m}$$

```
[Out] -((x^(1 + m)*(1 - I*a*x)^2)/((1 + m)*(1 + I*a*x)^2)) + ((4*I)*x^(1 + m)*(I*
(1 + m)^2 - a*(3 + 3*m + m^2)*x))/((1 + m)*(1 + I*a*x)^2) + (2*(3 + 4*m + 2
*m^2)*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-I)*a*x])/(1 + m)
```

Rubi [A] time = 0.0897069, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5062, 100, 145, 64}

$$\frac{2\left(2m^2+4m+3\right)x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-iax)}{m+1}+\frac{4ix^{m+1}\left(-a\left(m^2+3m+3\right)x+i(m+1)^2\right)}{(m+1)(1+iax)^2}-\frac{(}{(m}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((6*I)*ArcTan[a*x]),x]
```

```
[Out] -((x^(1 + m)*(1 - I*a*x)^2)/((1 + m)*(1 + I*a*x)^2)) + ((4*I)*x^(1 + m)*(I*
(1 + m)^2 - a*(3 + 3*m + m^2)*x))/((1 + m)*(1 + I*a*x)^2) + (2*(3 + 4*m + 2
*m^2)*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-I)*a*x])/(1 + m)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int e^{-6i\tan^{-1}(ax)}x^m\,dx = \int \frac{x^m(1-iax)^3}{(1+iax)^3}\,dx$$

$$= -\frac{x^{1+m}(1-iax)^2}{(1+m)(1+iax)^2} - \frac{i\int \frac{x^m(1-iax)\left(2ia(1+m)+2a^2(3+m)x\right)}{(1+iax)^3}\,dx}{a(1+m)}$$

$$= -\frac{x^{1+m}(1-iax)^2}{(1+m)(1+iax)^2} + \frac{4ix^{1+m}\left(i(1+m)^2-a\left(3+3m+m^2\right)x\right)}{(1+m)(1+iax)^2} + \left(2\left(3+4m+2m^2\right)\right)\int \frac{x^m}{1+iax}\,dx$$

$$= -\frac{x^{1+m}(1-iax)^2}{(1+m)(1+iax)^2} + \frac{4ix^{1+m}\left(i(1+m)^2-a\left(3+3m+m^2\right)x\right)}{(1+m)(1+iax)^2} + \frac{2\left(3+4m+2m^2\right)x^{1+m}\,{}_2F_1(1,1+\ldots}{1+m}$$

Mathematica [A] time = 0.0344049, size = 94, normalized size = 0.82

$$\frac{x^{m+1}\left(2\left(2m^2+4m+3\right)(ax-i)^2\text{Hypergeometric2F1}(1,m+1,m+2,-iax)-a^2x^2+m^2(4+4iax)+4m(2+3iax)+1\ldots\right)}{(m+1)(ax-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/E^((6*I)*ArcTan[a*x]),x]
```

```
[Out] (x^(1 + m)*(5 + (10*I)*a*x - a^2*x^2 + 4*m*(2 + (3*I)*a*x) + m^2*(4 + (4*I)
*a*x) + 2*(3 + 4*m + 2*m^2)*(-I + a*x)^2*Hypergeometric2F1[1, 1 + m, 2 + m,
 (-I)*a*x]))/((1 + m)*(-I + a*x)^2)
```

Maple [C] time = 0.657, size = 1196, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x)
```

```
[Out] 1/120*I*(I*a)^(-m)/a*(x^m*(I*a)^m*(-720-1764*m-1624*m^2+7200*a^2*x^2-21*m^5
-m^6+14400*a^2*m*x^2-3600*a^4*x^4-4200*a^4*x^4*m+1112*a^2*x^2*m^4+4911*a^2*
x^2*m^3+11722*a^2*x^2*m^2-735*m^3-175*m^4+6*a^2*x^2*m^6-120*a^6*x^6*m+129*a
^2*x^2*m^5-720*I*a^5*x^5+7200*I*a^3*x^3-3600*I*a*x-a^4*x^4*m^6-22*a^4*x^4*m
^5-197*a^4*x^4*m^4-932*a^4*x^4*m^3-2556*a^4*x^4*m^2+4*I*a^3*x^3*m^6+87*I*a^
3*x^3*m^5+764*I*a^3*x^3*m^4-4*I*a*x*m^6-120*I*a^5*x^5*m+3483*I*a^3*x^3*m^3-
85*I*a*x*m^5+8802*I*a^3*x^3*m^2-720*I*a*x*m^4+12000*I*a^3*x^3*m-3095*I*a*x*
m^3-7076*I*a*x*m^2-8100*I*a*m*x)/(1+m)/m/(1+I*a*x)^5+x^m*(I*a)^m*(m^5+20*m^
4+155*m^3+580*m^2+1044*m+720)*LerchPhi(-I*a*x,1,m))-1/40*I*(I*a)^(-m)/a*(-x
^m*(I*a)^m*(24+50*m+35*m^2-240*I*a^3*x^3+120*I*a*x-240*a^2*x^2-392*a^2*m*x^
2+120*a^4*x^4+96*a^4*x^4*m+149*I*a*x*m^2+226*I*a*m*x-6*a^2*x^2*m^4-63*a^2*x
^2*m^3-239*a^2*x^2*m^2+10*m^3+m^4+a^4*x^4*m^4+11*a^4*x^4*m^3+46*a^4*x^4*m^2
-4*I*a^3*x^3*m^4-43*I*a^3*x^3*m^3-171*I*a^3*x^3*m^2+4*I*a*x*m^4-312*I*a^3*x
```

```
^3*m+41*I*a*x*m^3)/(1+I*a*x)^5+x^m*(I*a)^m*m*(m^4+10*m^3+35*m^2+50*m+24)*Le
rchPhi(-I*a*x,1,m))+1/40*I*(I*a)^(-m)/a*(-x^m*(I*a)^m*(a^4*x^4*m^4+a^4*x^4*
m^3+4*I*a*x*m^4-4*a^4*x^4*m^2-4*I*a*m*x-6*a^2*x^2*m^4-4*a^4*x^4*m+19*I*a^3*
x^3*m^2-3*a^2*x^2*m^3+20*I*a*x-3*I*a^3*x^3*m^3+31*a^2*x^2*m^2-21*I*a*x*m^2+
m^4+18*a^2*m*x^2-4*I*a^3*x^3*m^4-40*a^2*x^2+I*a*x*m^3-5*m^2+18*I*a^3*x^3*m+
4)/(1+I*a*x)^5+x^m*(I*a)^m*(m^2-3*m+2)*m*(m^2+3*m+2)*LerchPhi(-I*a*x,1,m))-
1/120*I*(I*a)^(-m)/a*(-x^m*(I*a)^m*(a^4*x^4*m^4-9*a^4*x^4*m^3+4*I*a*x*m^4+2
6*a^4*x^4*m^2-154*I*a*m*x-6*a^2*x^2*m^4-24*a^4*x^4*m-111*I*a^3*x^3*m^2+57*a
^2*x^2*m^3+37*I*a^3*x^3*m^3+129*I*a*x*m^2-179*a^2*x^2*m^2-4*I*a^3*x^3*m^4+m
^4+188*a^2*m*x^2-39*I*a*x*m^3-10*m^3+108*I*a^3*x^3*m+35*m^2-50*m+24)/(1+I*a
*x)^5+x^m*(I*a)^m*(m^4-10*m^3+35*m^2-50*m+24)*m*LerchPhi(-I*a*x,1,m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2+1\right)^3 x^m}{(i\,ax+1)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^3*x^m/(I*a*x + 1)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(a^3x^3+3i\,a^2x^2-3\,ax-i\right)x^m}{a^3x^3-3i\,a^2x^2-3\,ax+i},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^3*x^3 + 3*I*a^2*x^2 - 3*a*x - I)*x^m/(a^3*x^3 - 3*I*a^2*x^2 -
3*a*x + I), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(1+I*a*x)**6*(a**2*x**2+1)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2+1\right)^3 x^m}{(i\,ax+1)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 + 1)^3*x^m/(I*a*x + 1)^6, x)
```

3.140 $\int e^{3i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=159

$$-\frac{3x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)}{m+1}+\frac{4x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)}{m+1}-\frac{iax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m+2}+\frac{4iax^{m+2}\,{}_2F_1}{}$$

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1
+ m) - (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^
2)])/(2 + m) + (4*x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, -(
a^2*x^2)])/(1 + m) + ((4*I)*a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (
4 + m)/2, -(a^2*x^2)])/(2 + m)
```

Rubi [A] time = 0.774235, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5060, 6742, 364, 850, 808}

$$-\frac{3x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}+\frac{4x^{m+1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}-\frac{iax^{m+2}\text{Hy}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1
+ m) - (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^
2)])/(2 + m) + (4*x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, -(
a^2*x^2)])/(1 + m) + ((4*I)*a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (
4 + m)/2, -(a^2*x^2)])/(2 + m)
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{3i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1+iax)^2}{(1-iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int\left(-\frac{3x^m}{\sqrt{1+a^2x^2}}-\frac{iax^{1+m}}{\sqrt{1+a^2x^2}}+\frac{4x^m}{(1-iax)\sqrt{1+a^2x^2}}\right)dx \\
&= -\left(3\int\frac{x^m}{\sqrt{1+a^2x^2}}\,dx\right)+4\int\frac{x^m}{(1-iax)\sqrt{1+a^2x^2}}\,dx-(ia)\int\frac{x^{1+m}}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m}-\frac{iax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+m}+4\int\frac{x^m(1+iax)}{\left(1+a^2x^2\right)^{3/2}}\,dx \\
&= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m}-\frac{iax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+m}+4\int\frac{x^m}{\left(1+a^2x^2\right)^{3/2}}\,dx+(4ia
\\
&= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m}-\frac{iax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+m}+\frac{4x^{1+m}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2
}{1+m}
\end{aligned}$$

Mathematica [C] time = 0.0770909, size = 113, normalized size = 0.71

$$-\frac{i\sqrt{1-iax}\sqrt{ax-i}x^{m+1}\left(F_1\left(m+1;-\frac{1}{2},\frac{1}{2};m+2;-iax,iax\right)-2F_1\left(m+1;-\frac{1}{2},\frac{3}{2};m+2;-iax,iax\right)\right)}{(m+1)\sqrt{1+iax}\sqrt{ax+i}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] ((-I)*x^(1 + m)*Sqrt[1 - I*a*x]*Sqrt[-I + a*x]*(AppellF1[1 + m, -1/2, 1/2,
2 + m, (-I)*a*x, I*a*x] - 2*AppellF1[1 + m, -1/2, 3/2, 2 + m, (-I)*a*x, I*a
*x]))/((1 + m)*Sqrt[1 + I*a*x]*Sqrt[I + a*x])
```

Maple [A] time = 0.438, size = 146, normalized size = 0.9

$$\frac{x^{1+m}}{1+m}{}_2F_1(\frac{3}{2},\frac{1}{2}+\frac{m}{2};\frac{3}{2}+\frac{m}{2};-a^2x^2)+\frac{3\,iax^{2+m}}{2+m}{}_2F_1(\frac{3}{2},1+\frac{m}{2};2+\frac{m}{2};-a^2x^2)-3\,\frac{a^2x^{3+m}{}_2F_1(3/2,3/2+m/2;5/2+m/2;}{3+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x)
```

```
[Out] x^(1+m)*hypergeom([3/2,1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)+3*I*a/(2+m)*x
^(2+m)*hypergeom([3/2,1+1/2*m],[2+1/2*m],-a^2*x^2)-3*a^2/(3+m)*x^(3+m)*hype
rgeom([3/2,3/2+1/2*m],[5/2+1/2*m],-a^2*x^2)-I*a^3/(4+m)*x^(4+m)*hypergeom([
3/2,2+1/2*m],[3+1/2*m],-a^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)^3 x^m}{(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate((I*a*x + 1)^3*x^m/(a^2*x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{a^2x^2+1}(-i\,ax-1)x^m}{a^2x^2+2i\,ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*(-I*a*x - 1)*x^m/(a^2*x^2 + 2*I*a*x - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\,(iax+1)^3}{(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x**m,x)
```

```
[Out] Integral(x**m*(I*a*x + 1)**3/(a**2*x**2 + 1)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)^3 x^m}{(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x, algorithm="giac")
```

```
[Out] integrate((I*a*x + 1)^3*x^m/(a^2*x^2 + 1)^(3/2), x)
```

3.141 $\int e^{i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=79

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)}{m+1}+\frac{iax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m+2}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1 + m
) + (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)]
)/(2 + m)
```

Rubi [A] time = 0.0407399, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5060, 808, 364}

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}+\frac{iax^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1 + m
) + (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)]
)/(2 + m)
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{i\tan^{-1}(ax)}x^m\,dx = \int \frac{x^m(1+iax)}{\sqrt{1+a^2x^2}}\,dx$$
$$= (ia)\int \frac{x^{1+m}}{\sqrt{1+a^2x^2}}\,dx + \int \frac{x^m}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m} + \frac{iax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+m}$$

Mathematica [C] time = 0.0369974, size = 85, normalized size = 1.08

$$\frac{i\sqrt{1-iax}\sqrt{ax-i}x^{m+1}F_1\left(m+1;-\frac{1}{2},\frac{1}{2};m+2;-iax,iax\right)}{(m+1)\sqrt{1+iax}\sqrt{ax+i}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a*x])*x^m,x]
```

```
[Out] (I*x^(1 + m)*Sqrt[1 - I*a*x]*Sqrt[-I + a*x]*AppellF1[1 + m, -1/2, 1/2, 2 +
m, (-I)*a*x, I*a*x])/((1 + m)*Sqrt[1 + I*a*x]*Sqrt[I + a*x])
```

Maple [A] time = 0.293, size = 71, normalized size = 0.9

$$\frac{x^{1+m}}{1+m}{}_2\mathrm{F}_1(\frac{1}{2},\frac{1}{2}+\frac{m}{2};\frac{3}{2}+\frac{m}{2};-a^2x^2)+\frac{iax^{2+m}}{2+m}{}_2\mathrm{F}_1(\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};-a^2x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x)
```

```
[Out] x^(1+m)*hypergeom([1/2,1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)+I*a*x^(2+m)*h
ypergeom([1/2,1+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)x^m}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate((I*a*x + 1)*x^m/sqrt(a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{i\sqrt{a^2x^2+1}x^m}{ax+i},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(I*sqrt(a^2*x^2 + 1)*x^m/(a*x + I), x)
```

Sympy [A] time = 3.36593, size = 95, normalized size = 1.2

$$\frac{iax^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|a^2x^2e^{i\pi}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{xx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|a^2x^2e^{i\pi}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**m,x)
```

```
[Out] I*a*x**2*x**m*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,), a**2*x**2*ex
p_polar(I*pi))/(2*gamma(m/2 + 2)) + x*x**m*gamma(m/2 + 1/2)*hyper((1/2, m/2
 + 1/2), (m/2 + 3/2,), a**2*x**2*exp_polar(I*pi))/(2*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)x^m}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x, algorithm="giac")
```

```
[Out] integrate((I*a*x + 1)*x^m/sqrt(a^2*x^2 + 1), x)
```

3.142 $\int e^{-i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=79

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)}{m+1}-\frac{iax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m+2}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1 + m
) - (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)]
)/(2 + m)
```

Rubi [A] time = 0.0403945, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5060, 808, 364}

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}-\frac{iax^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(I*ArcTan[a*x]),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1 + m
) - (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)]
)/(2 + m)
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{-i\tan^{-1}(ax)}x^m\,dx = \int \frac{x^m(1-iax)}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\left((ia)\int \frac{x^{1+m}}{\sqrt{1+a^2x^2}}\,dx\right) + \int \frac{x^m}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m} - \frac{iax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+m}$$

Mathematica [C] time = 0.0409323, size = 85, normalized size = 1.08

$$-\frac{i\sqrt{1+iax}\sqrt{ax+i}x^{m+1}F_1\left(m+1;\frac{1}{2},-\frac{1}{2};m+2;-iax,iax\right)}{(m+1)\sqrt{1-iax}\sqrt{ax-i}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/E^(I*ArcTan[a*x]),x]
```

```
[Out] ((-I)*x^(1 + m)*Sqrt[1 + I*a*x]*Sqrt[I + a*x]*AppellF1[1 + m, 1/2, -1/2, 2
+ m, (-I)*a*x, I*a*x])/((1 + m)*Sqrt[1 - I*a*x]*Sqrt[-I + a*x])
```

Maple [F] time = 0.414, size = 0, normalized size = 0.

$$\int \frac{x^m}{1+iax}\sqrt{a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2x^2+1}x^m}{i\,ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)*x^m/(I*a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{i\sqrt{a^2x^2+1}x^m}{ax-i},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-I*sqrt(a^2*x^2 + 1)*x^m/(a*x - I), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \sqrt{a^2x^2+1}}{iax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(a**2*x**2 + 1)/(I*a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2x^2+1}\,x^m}{i\,ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)*x^m/(I*a*x + 1), x)
```

3.143 $\int e^{-3i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=159

$$-\frac{3x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)}{m+1}+\frac{4x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)}{m+1}+\frac{iax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m+2}-\frac{4iax^{m+2}\,{}_2F_1\left(\frac{3}{2},\right.}{m}$$

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1
+ m) + (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^
2)])/(2 + m) + (4*x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, -(
a^2*x^2)])/(1 + m) - ((4*I)*a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (
4 + m)/2, -(a^2*x^2)])/(2 + m)
```

Rubi [A] time = 0.704954, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5060, 6742, 364, 850, 808}

$$-\frac{3x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}+\frac{4x^{m+1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}+\frac{iax^{m+2}\text{Hype}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1
+ m) + (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^
2)])/(2 + m) + (4*x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, -(
a^2*x^2)])/(1 + m) - ((4*I)*a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (
4 + m)/2, -(a^2*x^2)])/(2 + m)
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int e^{-3i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1-iax)^2}{(1+iax)\sqrt{1+a^2x^2}}\,dx\\ &= \int \left(-\frac{3x^m}{\sqrt{1+a^2x^2}}+\frac{iax^{1+m}}{\sqrt{1+a^2x^2}}+\frac{4x^m}{(1+iax)\sqrt{1+a^2x^2}}\right)dx\\ &= -\left(3\int \frac{x^m}{\sqrt{1+a^2x^2}}\,dx\right)+4\int \frac{x^m}{(1+iax)\sqrt{1+a^2x^2}}\,dx+(ia)\int \frac{x^{1+m}}{\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m}+\frac{iax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+m}+4\int \frac{x^m(1-iax)}{\left(1+a^2x^2\right)^{3/2}}\,dx\\ &= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m}+\frac{iax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+m}+4\int \frac{x^m}{\left(1+a^2x^2\right)^{3/2}}\,dx-\\ &= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m}+\frac{iax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+m}+\frac{4x^{1+m}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2}\right.}{1+m}\end{aligned}$$

Mathematica [C] time = 0.0713403, size = 113, normalized size = 0.71

$$\frac{i\sqrt{1+iax}\sqrt{ax+i}x^{m+1}\left(F_1\left(m+1;\frac{1}{2},-\frac{1}{2};m+2;-iax,iax\right)-2F_1\left(m+1;\frac{3}{2},-\frac{1}{2};m+2;-iax,iax\right)\right)}{(m+1)\sqrt{1-iax}\sqrt{ax-i}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] (I*x^(1 + m)*Sqrt[1 + I*a*x]*Sqrt[I + a*x]*(AppellF1[1 + m, 1/2, -1/2, 2 +
m, (-I)*a*x, I*a*x] - 2*AppellF1[1 + m, 3/2, -1/2, 2 + m, (-I)*a*x, I*a*x])
)/((1 + m)*Sqrt[1 - I*a*x]*Sqrt[-I + a*x])
```

Maple [F] time = 0.41, size = 0, normalized size = 0.

$$\int \frac{x^m}{(1+iax)^3}\left(a^2x^2+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}x^m}{(iax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)*x^m/(I*a*x + 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2x^2+1}(iax-1)x^m}{a^2x^2-2iax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*(I*a*x - 1)*x^m/(a^2*x^2 - 2*I*a*x - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}x^m}{(iax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)*x^m/(I*a*x + 1)^3, x)
```

3.144 $\int e^{\frac{5}{2}i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}F_1\left(m+1;\frac{5}{4},-\frac{5}{4};m+2;iax,-iax\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, 5/4, -5/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rubi [A] time = 0.0271013, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{5}{4},-\frac{5}{4};m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 5/4, -5/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\frac{5}{2}i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1+iax)^{5/4}}{(1-iax)^{5/4}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{5}{4},-\frac{5}{4};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.206884, size = 0, normalized size = 0.

$$\int e^{\frac{5}{2}i\tan^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])*x^m,x]
```

```
[Out] Integrate[E^(((5*I)/2)*ArcTan[a*x])*x^m, x]
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int \left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(ax-i)x^m\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}}{ax+i}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(-(a*x - I)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))/(a*x + I), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

3.145 $\int e^{\frac{3}{2}i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}F_1\left(m+1;\frac{3}{4},-\frac{3}{4};m+2;iax,-iax\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, 3/4, -3/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rubi [A] time = 0.0260395, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{3}{4},-\frac{3}{4};m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 3/4, -3/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\frac{3}{2}i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{3}{4},-\frac{3}{4};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.182232, size = 0, normalized size = 0.

$$\int e^{\frac{3}{2}i\tan^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])*x^m,x]
```

```
[Out] Integrate[E^(((3*I)/2)*ArcTan[a*x])*x^m, x]
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i\sqrt{a^2x^2+1}x^m\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}}{ax+i}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(I*sqrt(a^2*x^2 + 1)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))/(a*x +
 I), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

3.146 $\int e^{\frac{1}{2}i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{4},-\frac{1}{4};m+2;iax,-iax\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/4, -1/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rubi [A] time = 0.0260548, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{4},-\frac{1}{4};m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/4, -1/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{2}i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{1}{4},-\frac{1}{4};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.17397, size = 0, normalized size = 0.

$$\int e^{\frac{1}{2}i\tan^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])*x^m,x]
```

```
[Out] Integrate[E^((I/2)*ArcTan[a*x])*x^m, x]
```

Maple [F] time = 0.126, size = 0, normalized size = 0.

$$\int \sqrt{(1 + iax)\frac{1}{\sqrt{a^2x^2 + 1}}}\,x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\frac{i\,ax + 1}{\sqrt{a^2x^2 + 1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^m \sqrt{\frac{i\,\sqrt{a^2x^2 + 1}}{ax + i}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

3.147 $\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}F_1\left(m+1;-\frac{1}{4},\frac{1}{4};m+2;iax,-iax\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, -1/4, 1/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rubi [A] time = 0.0256764, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;-\frac{1}{4},\frac{1}{4};m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -1/4, 1/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;-\frac{1}{4},\frac{1}{4};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.185523, size = 0, normalized size = 0.

$$\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^m/E^((I/2)*ArcTan[a*x]), x]
```

Maple [F] time = 0.134, size = 0, normalized size = 0.

$$\int x^m \frac{1}{\sqrt{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{i\sqrt{a^2x^2+1}x^m\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}}{ax-i}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-I*sqrt(a^2*x^2 + 1)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))/(a*x
- I), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

3.148 $\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}F_1\left(m+1;-\frac{3}{4},\frac{3}{4};m+2;iax,-iax\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, -3/4, 3/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rubi [A] time = 0.0254629, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;-\frac{3}{4},\frac{3}{4};m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -3/4, 3/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;-\frac{3}{4},\frac{3}{4};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.199196, size = 0, normalized size = 0.

$$\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^m/E^(((3*I)/2)*ArcTan[a*x]), x]
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int x^m \left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ax+i)x^m\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}}{ax-i}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*x + I)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))/(a*x - I), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

3.149 $\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}F_1\left(m+1;-\frac{5}{4},\frac{5}{4};m+2;iax,-iax\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, -5/4, 5/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rubi [A] time = 0.0256187, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;-\frac{5}{4},\frac{5}{4};m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -5/4, 5/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1-iax)^{5/4}}{(1+iax)^{5/4}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;-\frac{5}{4},\frac{5}{4};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.231573, size = 0, normalized size = 0.

$$\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^m/E^(((5*I)/2)*ArcTan[a*x]), x]
```

Maple [F] time = 0.164, size = 0, normalized size = 0.

$$\int x^m \left((1+iax)\frac{1}{\sqrt{a^2x^2+1}}\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2x^2+1}(i\,ax-1)x^m\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}}{a^2x^2-2i\,ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*(I*a*x - 1)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I))/(a^2*x^2 - 2*I*a*x - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

3.150 $\int e^{\frac{2}{3}\tan^{-1}(x)}x^m\,dx$

Optimal. Leaf size=38

$$\frac{x^{m+1}F_1\left(m+1;-\frac{i}{3},\frac{i}{3};m+2;ix,-ix\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, -I/3, I/3, 2 + m, I*x, (-I)*x])/(1 + m)
```

Rubi [A] time = 0.0232761, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;-\frac{i}{3},\frac{i}{3};m+2;ix,-ix\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcTan[x])/3)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -I/3, I/3, 2 + m, I*x, (-I)*x])/(1 + m)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\frac{2}{3}\tan^{-1}(x)}x^m\,dx &= \int (1-ix)^{\frac{i}{3}}(1+ix)^{-\frac{i}{3}}x^m\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;-\frac{i}{3},\frac{i}{3};2+m;ix,-ix\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.19496, size = 0, normalized size = 0.

$$\int e^{\frac{2}{3}\tan^{-1}(x)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^((2*ArcTan[x])/3)*x^m,x]
```

```
[Out] Integrate[E^((2*ArcTan[x])/3)*x^m, x]
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int \mathrm{e}^{\frac{2\arctan(x)}{3}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2/3*arctan(x))*x^m,x)
```

```
[Out] int(exp(2/3*arctan(x))*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m e^{\left(\frac{2}{3}\arctan(x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2/3*arctan(x))*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*e^(2/3*arctan(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^m e^{\left(\frac{2}{3}\arctan(x)\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2/3*arctan(x))*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*e^(2/3*arctan(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m e^{\frac{2\,\mathrm{atan}\,(x)}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2/3*atan(x))*x**m,x)
```

```
[Out] Integral(x**m*exp(2*atan(x)/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m e^{\left(\frac{2}{3}\arctan(x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2/3*arctan(x))*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*e^(2/3*arctan(x)), x)
```

3.151 $\int e^{\frac{1}{3}\tan^{-1}(x)} x^m \, dx$

Optimal. Leaf size=38

$$\frac{x^{m+1} F_1\left(m+1; -\frac{i}{6}, \frac{i}{6}; m+2; ix, -ix\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, -I/6, I/6, 2 + m, I*x, (-I)*x])/(1 + m)
```

Rubi [A] time = 0.023389, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5062, 133}

$$\frac{x^{m+1} F_1\left(m+1; -\frac{i}{6}, \frac{i}{6}; m+2; ix, -ix\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTan[x]/3)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -I/6, I/6, 2 + m, I*x, (-I)*x])/(1 + m)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{3}\tan^{-1}(x)} x^m \, dx &= \int (1-ix)^{\frac{i}{6}} (1+ix)^{-\frac{i}{6}} x^m \, dx \\ &= \frac{x^{1+m} F_1\left(1+m; -\frac{i}{6}, \frac{i}{6}; 2+m; ix, -ix\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.198047, size = 0, normalized size = 0.

$$\int e^{\frac{1}{3}\tan^{-1}(x)} x^m \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(ArcTan[x]/3)*x^m,x]
```

```
[Out] Integrate[E^(ArcTan[x]/3)*x^m, x]
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int \mathrm{e}^{\frac{\arctan(x)}{3}} x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1/3*arctan(x))*x^m,x)
```

```
[Out] int(exp(1/3*arctan(x))*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m e^{\left(\frac{1}{3}\,\arctan(x)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/3*arctan(x))*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*e^(1/3*arctan(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^m e^{\left(\frac{1}{3}\,\arctan(x)\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/3*arctan(x))*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*e^(1/3*arctan(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m e^{\frac{\operatorname{atan}(x)}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/3*atan(x))*x**m,x)
```

```
[Out] Integral(x**m*exp(atan(x)/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m e^{\left(\frac{1}{3}\,\arctan(x)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/3*arctan(x))*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*e^(1/3*arctan(x)), x)
```

3.152 $\int e^{\frac{1}{4}i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{8},-\frac{1}{8};m+2;iax,-iax\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/8, -1/8, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rubi [A] time = 0.0246622, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{8},-\frac{1}{8};m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/4)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/8, -1/8, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{4}i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{1}{8},-\frac{1}{8};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.171496, size = 0, normalized size = 0.

$$\int e^{\frac{1}{4}i\tan^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^((I/4)*ArcTan[a*x])*x^m,x]
```

```
[Out] Integrate[E^((I/4)*ArcTan[a*x])*x^m, x]
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \sqrt[4]{(1+iax)\frac{1}{\sqrt{a^2x^2+1}}}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^m \left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)*x**m,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.153 $\int e^{in\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=40

$$\frac{x^{m+1}F_1\left(m+1;\frac{n}{2},-\frac{n}{2};m+2;iax,-iax\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, n/2, -n/2, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rubi [A] time = 0.0280571, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{n}{2},-\frac{n}{2};m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, n/2, -n/2, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int e^{in\tan^{-1}(ax)}x^m\,dx &= \int x^m(1-iax)^{-n/2}(1+iax)^{n/2}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{n}{2},-\frac{n}{2};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.220206, size = 0, normalized size = 0.

$$\int e^{in\tan^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(I*n*ArcTan[a*x])*x^m,x]
```

```
[Out] Integrate[E^(I*n*ArcTan[a*x])*x^m, x]
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int e^{in\arctan(ax)}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))*x^m,x)
```

```
[Out] int(exp(I*n*arctan(a*x))*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m e^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*e^(I*n*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m/(-(a*x + I)/(a*x - I))^(1/2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m e^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*e^(I*n*arctan(a*x)), x)
```

3.154 $\int e^{in\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=171

$$-\frac{2^{\frac{n}{2}-2}n\left(n^2+8\right)(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{3a^4(2-n)}-\frac{(1+iax)^{\frac{n+2}{2}}\left(2ianx+n^2+6\right)(1-iax)^{1-\frac{n}{2}}}{24a^4}+\frac{x^2(1+$$

```
[Out] (x^2*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(4*a^2) - ((1 - I*a*x)^
(1 - n/2)*(1 + I*a*x)^((2 + n)/2)*(6 + n^2 + (2*I)*a*n*x))/(24*a^4) - (2^(-
2 + n/2)*n*(8 + n^2)*(1 - I*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2,
 2 - n/2, (1 - I*a*x)/2])/(3*a^4*(2 - n))
```

Rubi [A] time = 0.113763, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5062, 100, 147, 69}

$$-\frac{2^{\frac{n}{2}-2}n\left(n^2+8\right)(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{3a^4(2-n)}-\frac{(1+iax)^{\frac{n+2}{2}}\left(2ianx+n^2+6\right)(1-iax)^{1-\frac{n}{2}}}{24a^4}+\frac{x^2(1+$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])*x^3,x]
```


```
[Out] (x^2*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(4*a^2) - ((1 - I*a*x)^
(1 - n/2)*(1 + I*a*x)^((2 + n)/2)*(6 + n^2 + (2*I)*a*n*x))/(24*a^4) - (2^(-
2 + n/2)*n*(8 + n^2)*(1 - I*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2,
 2 - n/2, (1 - I*a*x)/2])/(3*a^4*(2 - n))
```


Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{in\tan^{-1}(ax)}x^3\,dx = \int x^3(1-iax)^{-n/2}(1+iax)^{n/2}\,dx$$

$$= \frac{x^2(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{4a^2} + \frac{\int x(1-iax)^{-n/2}(1+iax)^{n/2}(-2-ianx)\,dx}{4a^2}$$

$$= \frac{x^2(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{4a^2} - \frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}\left(6+n^2+2ianx\right)}{24a^4} + \frac{\left(in\left(8+n^2\right)\right)\int(1-iax)^{-}\cdots}{24a^3}$$

$$= \frac{x^2(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{4a^2} - \frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}\left(6+n^2+2ianx\right)}{24a^4} - \frac{2^{-2+\frac{n}{2}}n\left(8+n^2\right)(1-iax)\cdots}{\cdots}$$

Mathematica [A] time = 0.180215, size = 210, normalized size = 1.23

$$\frac{(ax+i)(1-iax)^{-n/2}\left((n-2)\left(a^2x^2(ax-i)(1+iax)^{n/2}-i2^{\frac{n}{2}+1}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)\right)-i2^{\frac{n}{2}+3}n\,{}_2F_1\left(-\frac{n}{2}-2,1-\cdots\right.\right.}{4a^4(n-2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*n*ArcTan[a*x])*x^3,x]
```

```
[Out] ((I + a*x)*((-I)*2^(3 + n/2)*n*Hypergeometric2F1[-2 - n/2, 1 - n/2, 2 - n/2
, (1 - I*a*x)/2] + I*2^(3 + n/2)*(-1 + n)*Hypergeometric2F1[-1 - n/2, 1 - n
/2, 2 - n/2, (1 - I*a*x)/2] + (-2 + n)*(a^2*x^2*(1 + I*a*x)^(n/2)*(-I + a*x
) - I*2^(1 + n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - I*a*x)/2])
))/(4*a^4*(-2 + n)*(1 - I*a*x)^(n/2))
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int e^{in\arctan(ax)}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))*x^3,x)
```

```
[Out] int(exp(I*n*arctan(a*x))*x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 e^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*e^(I*n*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3/(-(a*x + I)/(a*x - I))^(1/2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))*x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 e^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*e^(I*n*arctan(a*x)), x)
```

3.155 $\int e^{in\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=159

$$-\frac{i2^{n/2}\left(n^2+2\right)(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{3a^3(2-n)}-\frac{in(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{6a^3}+\frac{x(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{3a^2}$$

```
[Out] ((-I/6)*n*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/a^3 + (x*(1 - I*a*
x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(3*a^2) - ((I/3)*2^(n/2)*(2 + n^2)*(1
 - I*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - I*a*x)/2
])/(a^3*(2 - n))
```

Rubi [A] time = 0.0857674, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5062, 90, 80, 69}

$$-\frac{i2^{n/2}\left(n^2+2\right)(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{3a^3(2-n)}-\frac{in(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{6a^3}+\frac{x(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])*x^2,x]
```

```
[Out] ((-I/6)*n*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/a^3 + (x*(1 - I*a*
x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(3*a^2) - ((I/3)*2^(n/2)*(2 + n^2)*(1
 - I*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - I*a*x)/2
])/(a^3*(2 - n))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{in\tan^{-1}(ax)}x^2\,dx = \int x^2(1-iax)^{-n/2}(1+iax)^{n/2}\,dx$$
$$= \frac{x(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3a^2} + \frac{\int (1-iax)^{-n/2}(1+iax)^{n/2}(-1-ianx)\,dx}{3a^2}$$
$$= -\frac{in(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6a^3} + \frac{x(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3a^2} - \frac{(2+n^2)\int (1-iax)^{-n/2}(1+iax)^{n/2}\,dx}{6a^2}$$
$$= -\frac{in(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6a^3} + \frac{x(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3a^2} - \frac{i2^{n/2}(2+n^2)(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},\right.}{3a^3(2-n)}$$

Mathematica [A] time = 0.0494213, size = 116, normalized size = 0.73

$$\frac{(ax+i)(1-iax)^{-n/2}\left(2^{\frac{n}{2}+1}(n^2+2)\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)+(n-2)(ax-i)(2ax-in)(1+iax)^{n/2}\right)}{6a^3(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*n*ArcTan[a*x])*x^2,x]
```

```
[Out] ((I + a*x)*((-2 + n)*(1 + I*a*x)^(n/2)*(-I + a*x)*((-I)*n + 2*a*x) + 2^(1 +
 n/2)*(2 + n^2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - I*a*x)/2]))/
(6*a^3*(-2 + n)*(1 - I*a*x)^(n/2))
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int e^{in\arctan(ax)}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))*x^2,x)
```

```
[Out] int(exp(I*n*arctan(a*x))*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 e^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*e^(I*n*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2/(-(a*x + I)/(a*x - I))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 e^{in\,\text{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))*x**2,x)
```

```
[Out] Integral(x**2*exp(I*n*atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 e^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*e^(I*n*arctan(a*x)), x)
```

3.156 $\int e^{in\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=107

$$\frac{2^{n/2}n(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{a^2(2-n)}+\frac{(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{2a^2}$$

```
[Out] ((1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(2*a^2) + (2^(n/2)*n*(1 - I
*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - I*a*x)/2])/(
a^2*(2 - n))
```

Rubi [A] time = 0.0458557, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5062, 80, 69}

$$\frac{2^{n/2}n(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{a^2(2-n)}+\frac{(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])*x,x]
```

```
[Out] ((1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(2*a^2) + (2^(n/2)*n*(1 - I
*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - I*a*x)/2])/(
a^2*(2 - n))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{in\tan^{-1}(ax)}x\,dx = \int x(1-iax)^{-n/2}(1+iax)^{n/2}\,dx$$
$$= \frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{2a^2} - \frac{(in)\int(1-iax)^{-n/2}(1+iax)^{n/2}\,dx}{2a}$$
$$= \frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{2a^2} + \frac{2^{n/2}n(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{a^2(2-n)}$$

Mathematica [A] time = 0.0245457, size = 105, normalized size = 0.98

$$\frac{(ax+i)(1-iax)^{-n/2}\left(i2^{\frac{n}{2}+1}n\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)+(n-2)(ax-i)(1+iax)^{n/2}\right)}{2a^2(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*n*ArcTan[a*x])*x,x]
```

```
[Out] ((I + a*x)*((-2 + n)*(1 + I*a*x)^(n/2)*(-I + a*x) + I*2^(1 + n/2)*n*Hyperge
ometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - I*a*x)/2]))/(2*a^2*(-2 + n)*(1 - I*
a*x)^(n/2))
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int e^{in\arctan(ax)}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))*x,x)
```

```
[Out] int(exp(I*n*arctan(a*x))*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x,x, algorithm="maxima")
```

```
[Out] integrate(x*e^(I*n*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x,x, algorithm="fricas")
```

```
[Out] integral(x/(-(a*x + I)/(a*x - I))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{in\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))*x,x)
```

```
[Out] Integral(x*exp(I*n*atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x,x, algorithm="giac")
```

```
[Out] integrate(x*e^(I*n*arctan(a*x)), x)
```

3.157 $\int e^{in\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=71

$$\frac{i2^{\frac{n}{2}+1}(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{a(2-n)}$$

```
[Out] (I*2^(1 + n/2)*(1 - I*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n
/2, (1 - I*a*x)/2])/(a*(2 - n))
```

Rubi [A] time = 0.0127803, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5061, 69}

$$\frac{i2^{\frac{n}{2}+1}(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{a(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x]),x]
```

```
[Out] (I*2^(1 + n/2)*(1 - I*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n
/2, (1 - I*a*x)/2])/(a*(2 - n))
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{in\tan^{-1}(ax)}\,dx &= \int (1-iax)^{-n/2}(1+iax)^{n/2}\,dx\\ &= \frac{i2^{1+\frac{n}{2}}(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{a(2-n)}\end{aligned}$$

Mathematica [A] time = 0.0291691, size = 53, normalized size = 0.75

$$-\frac{4ie^{i(n+2)\tan^{-1}(ax)}\,{}_2F_1\left(2,\frac{n}{2}+1;\frac{n}{2}+2;-e^{2i\tan^{-1}(ax)}\right)}{a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*n*ArcTan[a*x]),x]
```

```
[Out] ((-4*I)*E^(I*(2 + n)*ArcTan[a*x])*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E
^((2*I)*ArcTan[a*x])])/(a*(2 + n))
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \mathrm{e}^{in\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x)),x)
```

```
[Out] int(exp(I*n*arctan(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(e^(I*n*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}\,n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral(1/((-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{in\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x)),x)
```

```
[Out] Integral(exp(I*n*atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x)),x, algorithm="giac")
```

```
[Out] integrate(e^(I*n*arctan(a*x)), x)
```

3.158 $\int \frac{e^{in\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=125

$$\frac{2(1-iax)^{-n/2}(1+iax)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{1-iax}{iax+1}\right)}{n}-\frac{2^{\frac{n}{2}+1}(1-iax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{n}$$

```
[Out] (2*(1 + I*a*x)^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (1 - I*a*x)/(1 + I
*a*x)])/(n*(1 - I*a*x)^(n/2)) - (2^(1 + n/2)*Hypergeometric2F1[-n/2, -n/2,
1 - n/2, (1 - I*a*x)/2])/(n*(1 - I*a*x)^(n/2))
```

Rubi [A] time = 0.0503532, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5062, 105, 69, 131}

$$\frac{2(1-iax)^{-n/2}(1+iax)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{1-iax}{iax+1}\right)}{n}-\frac{2^{\frac{n}{2}+1}(1-iax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])/x,x]
```

```
[Out] (2*(1 + I*a*x)^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (1 - I*a*x)/(1 + I
*a*x)])/(n*(1 - I*a*x)^(n/2)) - (2^(1 + n/2)*Hypergeometric2F1[-n/2, -n/2,
1 - n/2, (1 - I*a*x)/2])/(n*(1 - I*a*x)^(n/2))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
```

```
f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{in\tan^{-1}(ax)}}{x}\,dx = \int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x}\,dx$$
$$= -\left((ia)\int (1-iax)^{-1-\frac{n}{2}}(1+iax)^{n/2}\,dx\right) + \int \frac{(1-iax)^{-1-\frac{n}{2}}(1+iax)^{n/2}}{x}\,dx$$
$$= \frac{2(1-iax)^{-n/2}(1+iax)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{1-iax}{1+iax}\right)}{n} - \frac{2^{1+\frac{n}{2}}(1-iax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{n}$$

Mathematica [A] time = 0.0257732, size = 106, normalized size = 0.85

$$\frac{2(1-iax)^{-n/2}\left((1+iax)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{ax+i}{i-ax}\right)-2^{n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-iax)\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*n*ArcTan[a*x])/x,x]
```

```
[Out] (2*((1 + I*a*x)^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (I + a*x)/(I - a*
x)] - 2^(n/2)*Hypergeometric2F1[-n/2, -n/2, 1 - n/2, (1 - I*a*x)/2]))/(n*(1
 - I*a*x)^(n/2))
```

Maple [F] time = 0.166, size = 0, normalized size = 0.

$$\int \frac{e^{in\arctan(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))/x,x)
```

```
[Out] int(exp(I*n*arctan(a*x))/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(i\,n\arctan(ax))}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x,x, algorithm="maxima")
```

```
[Out] integrate(e^(I*n*arctan(a*x))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x,x, algorithm="fricas")
```

```
[Out] integral(1/(x*(-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{in\operatorname{atan}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))/x,x)
```

```
[Out] Integral(exp(I*n*atan(a*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(in\arctan(ax))}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x,x, algorithm="giac")
```

```
[Out] integrate(e^(I*n*arctan(a*x))/x, x)
```

3.159 $\int \frac{e^{in\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=79

$$-\frac{4ia(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n-2}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-iax}{iax+1}\right)}{2-n}$$

```
[Out] ((-4*I)*a*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((-2 + n)/2)*Hypergeometric2F1[
2, 1 - n/2, 2 - n/2, (1 - I*a*x)/(1 + I*a*x)])/(2 - n)
```

Rubi [A] time = 0.0307808, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5062, 131}

$$-\frac{4ia(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n-2}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-iax}{iax+1}\right)}{2-n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])/x^2,x]
```

```
[Out] ((-4*I)*a*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((-2 + n)/2)*Hypergeometric2F1[
2, 1 - n/2, 2 - n/2, (1 - I*a*x)/(1 + I*a*x)])/(2 - n)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{in\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x^2}\,dx\\ &= -\frac{4ia(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-iax}{1+iax}\right)}{2-n}\end{aligned}$$

Mathematica [A] time = 0.0154725, size = 82, normalized size = 1.04

$$-\frac{2ia(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n}{2}-1}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};-\frac{1-iax}{-iax-1}\right)}{1-\frac{n}{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*n*ArcTan[a*x])/x^2,x]
```

```
[Out] ((-2*I)*a*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^(-1 + n/2)*Hypergeometric2F1[2,
 1 - n/2, 2 - n/2, -((1 - I*a*x)/(-1 - I*a*x))])/(1 - n/2)
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int \frac{e^{in\arctan(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))/x^2,x)
```

```
[Out] int(exp(I*n*arctan(a*x))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(i\,n\arctan(ax))}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(I*n*arctan(a*x))/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{x^2\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*(-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{in\,\mathrm{atan}\,(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))/x**2,x)
```

```
[Out] Integral(exp(I*n*atan(a*x))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(i\,n\arctan(ax))}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate(e^(I*n*arctan(a*x))/x^2, x)
```

3.160 $\int \frac{e^{in\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=120

$$\frac{2a^2n(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n-2}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-iax}{iax+1}\right)}{2-n}-\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{2x^2}$$

```
[Out] -((1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(2*x^2) + (2*a^2*n*(1 - I*
a*x)^(1 - n/2)*(1 + I*a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n
/2, (1 - I*a*x)/(1 + I*a*x)])/(2 - n)
```

Rubi [A] time = 0.0492065, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5062, 96, 131}

$$\frac{2a^2n(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n-2}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-iax}{iax+1}\right)}{2-n}-\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])/x^3,x]
```

```
[Out] -((1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(2*x^2) + (2*a^2*n*(1 - I*
a*x)^(1 - n/2)*(1 + I*a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n
/2, (1 - I*a*x)/(1 + I*a*x)])/(2 - n)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{in\tan^{-1}(ax)}}{x^3}\,dx = \int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x^3}\,dx$$
$$= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{2x^2} + \frac{1}{2}(ian)\int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x^2}\,dx$$
$$= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{2x^2} + \frac{2a^2n(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-iax}{1+iax}\right)}{2-n}$$

Mathematica [A] time = 0.0301376, size = 114, normalized size = 0.95

$$\frac{(ax+i)(1-iax)^{-n/2}(1+iax)^{n/2}\left(4a^2nx^2\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{ax+i}{i-ax}\right)-(n-2)(ax-i)^2\right)}{2(n-2)x^2(ax-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*n*ArcTan[a*x])/x^3,x]
```

```
[Out] ((1 + I*a*x)^(n/2)*(I + a*x)*(-((-2 + n)*(-I + a*x)^2) + 4*a^2*n*x^2*Hyperg
eometric2F1[2, 1 - n/2, 2 - n/2, (I + a*x)/(I - a*x)]))/(2*(-2 + n)*x^2*(1
- I*a*x)^(n/2)*(-I + a*x))
```

Maple [F] time = 0.179, size = 0, normalized size = 0.

$$\int \frac{e^{in\arctan(ax)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))/x^3,x)
```

```
[Out] int(exp(I*n*arctan(a*x))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(i\,n\arctan(ax))}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(e^(I*n*arctan(a*x))/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^3\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x^3*(-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{in\operatorname{atan}(ax)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))/x**3,x)
```

```
[Out] Integral(exp(I*n*atan(a*x))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(in\arctan(ax))}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate(e^(I*n*arctan(a*x))/x^3, x)
```

3.161 $\int \frac{e^{in\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=171

$$\frac{2ia^3\left(n^2+2\right)(1+iax)^{\frac{n-2}{2}}(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-iax}{iax+1}\right)}{3(2-n)}-\frac{ian(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{6x^2}-\frac{(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{3x^3}$$

```
[Out] -((1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(3*x^3) - ((I/6)*a*n*(1 -
I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/x^2 + (((2*I)/3)*a^3*(2 + n^2)*(1
 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2,
2 - n/2, (1 - I*a*x)/(1 + I*a*x)])/(2 - n)
```

Rubi [A] time = 0.0720683, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5062, 129, 151, 12, 131}

$$\frac{2ia^3\left(n^2+2\right)(1+iax)^{\frac{n-2}{2}}(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-iax}{iax+1}\right)}{3(2-n)}-\frac{ian(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{6x^2}-\frac{(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])/x^4,x]
```

```
[Out] -((1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(3*x^3) - ((I/6)*a*n*(1 -
I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/x^2 + (((2*I)/3)*a^3*(2 + n^2)*(1
 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2,
2 - n/2, (1 - I*a*x)/(1 + I*a*x)])/(2 - n)
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{in\tan^{-1}(ax)}}{x^4}\,dx = \int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x^4}\,dx$$

$$= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3x^3} - \frac{1}{3}\int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}\left(-ian+a^2x\right)}{x^3}\,dx$$

$$= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3x^3} - \frac{ian(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6x^2} - \frac{1}{6}\int \frac{a^2\left(2+n^2\right)(1-iax)^{-n/2}(1+iax)^{n/}}{x^2}$$

$$= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3x^3} - \frac{ian(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6x^2} - \frac{1}{6}\left(a^2\left(2+n^2\right)\right)\int \frac{(1-iax)^{-n/2}(1+iax}{x^2}$$

$$= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3x^3} - \frac{ian(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6x^2} + \frac{2ia^3\left(2+n^2\right)(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{1}{2}(-2-}}{3(2-n)}$$

Mathematica [A] time = 0.0545292, size = 119, normalized size = 0.7

$$-\frac{(ax+i)(1-iax)^{-n/2}(1+iax)^{\frac{n-2}{2}}\left(4a^3\left(n^2+2\right)x^3\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{ax+i}{i-ax}\right)-(n-2)(ax-i)^2(anx-2i)\right)}{6(n-2)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*n*ArcTan[a*x])/x^4,x]
```

```
[Out] -((1 + I*a*x)^((-2 + n)/2)*(I + a*x)*(-((-2 + n)*(-I + a*x)^2*(-2*I + a*n*x
)) + 4*a^3*(2 + n^2)*x^3*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, (I + a*x)/(
I - a*x)]))/(6*(-2 + n)*x^3*(1 - I*a*x)^(n/2))
```

Maple [F] time = 0.19, size = 0, normalized size = 0.

$$\int \frac{e^{in\arctan(ax)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))/x^4,x)
```

```
[Out] int(exp(I*n*arctan(a*x))/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(i\,n\arctan(ax))}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^4,x, algorithm="maxima")
```

```
[Out] integrate(e^(I*n*arctan(a*x))/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^4\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral(1/(x^4*(-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(i\,n\arctan(ax))}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate(e^(I*n*arctan(a*x))/x^4, x)
```

3.162 $\int e^{i\tan^{-1}(a+bx)}x^4\,dx$

Optimal. Leaf size=276

$$\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}\left(-2\left(-36a^2-14ia+13\right)bx-96a^3-86ia^2+114a+19i\right)}{120b^5}+\frac{\left(8ia^4-16a^3-24ia^2+12a\right.}{}$$

```
[Out] ((3*I + 12*a - (24*I)*a^2 - 16*a^3 + (8*I)*a^4)*Sqrt[1 - I*a - I*b*x]*Sqrt[
1 + I*a + I*b*x])/(8*b^5) - ((I + 8*a)*x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a +
 I*b*x)^(3/2))/(20*b^3) + (x^3*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2
))/(5*b^2) + (Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(19*I + 114*a -
 (86*I)*a^2 - 96*a^3 - 2*(13 - (14*I)*a - 36*a^2)*b*x))/(120*b^5) + ((3 - (
12*I)*a - 24*a^2 + (16*I)*a^3 + 8*a^4)*ArcSinh[a + b*x])/(8*b^5)
```

Rubi [A] time = 0.201435, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5095, 100, 153, 147, 50, 53, 619, 215}

$$\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}\left(-2\left(-36a^2-14ia+13\right)bx-96a^3-86ia^2+114a+19i\right)}{120b^5}+\frac{\left(8ia^4-16a^3-24ia^2+12a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a + b*x])*x^4,x]
```

```
[Out] ((3*I + 12*a - (24*I)*a^2 - 16*a^3 + (8*I)*a^4)*Sqrt[1 - I*a - I*b*x]*Sqrt[
1 + I*a + I*b*x])/(8*b^5) - ((I + 8*a)*x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a +
 I*b*x)^(3/2))/(20*b^3) + (x^3*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2
))/(5*b^2) + (Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(19*I + 114*a -
 (86*I)*a^2 - 96*a^3 - 2*(13 - (14*I)*a - 36*a^2)*b*x))/(120*b^5) + ((3 - (
12*I)*a - 24*a^2 + (16*I)*a^3 + 8*a^4)*ArcSinh[a + b*x])/(8*b^5)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{i\tan^{-1}(a+bx)}x^4\,dx = \int \frac{x^4\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx$$
$$= \frac{x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2} + \frac{\int \frac{x^2\sqrt{1+ia+ibx}\left(-3(1+a^2)-(i+8a)bx\right)}{\sqrt{1-ia-ibx}}dx}{5b^2}$$
$$= -\frac{(i+8a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3} + \frac{x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2} + \frac{\int \frac{x\sqrt{1+ia+ibx}(-2(i-}{}}{}$$
$$= -\frac{(i+8a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3} + \frac{x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2} + \frac{\sqrt{1-ia-ibx}(1+}{}$$
$$= \frac{\left(3i+12a-24ia^2-16a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{(i+8a)x^2\sqrt{1-ia-ibx}(1+ia+}{20b^3}$$
$$= \frac{\left(3i+12a-24ia^2-16a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{(i+8a)x^2\sqrt{1-ia-ibx}(1+ia+}{20b^3}$$
$$= \frac{\left(3i+12a-24ia^2-16a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{(i+8a)x^2\sqrt{1-ia-ibx}(1+ia+}{20b^3}$$
$$= \frac{\left(3i+12a-24ia^2-16a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{(i+8a)x^2\sqrt{1-ia-ibx}(1+ia+}{20b^3}$$

Mathematica [A] time = 0.49234, size = 217, normalized size = 0.79

$$\frac{i\sqrt{a^2+2abx+b^2x^2+1}\left(2a^2\left(12b^2x^2-65ibx-166\right)+a^3(-24bx+250i)+24a^4+a\left(-24b^3x^3+70ib^2x^2+116bx-27\right.\right.}{120b^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a + b*x])*x^4,x]
```

```
[Out] ((I/120)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(64 + 24*a^4 + (45*I)*b*x - 32*b
^2*x^2 - (30*I)*b^3*x^3 + 24*b^4*x^4 + a^3*(250*I - 24*b*x) + 2*a^2*(-166 -
 (65*I)*b*x + 12*b^2*x^2) + a*(-275*I + 116*b*x + (70*I)*b^2*x^2 - 24*b^3*x
^3)))/b^5 + ((-1)^(1/4)*(3 - (12*I)*a - 24*a^2 + (16*I)*a^3 + 8*a^4)*Sqrt[(
-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]]
)/(4*b^(11/2))
```

Maple [B] time = 0.138, size = 656, normalized size = 2.4

$$\frac{-\frac{4i}{15}x^2}{b^3}\sqrt{b^2x^2+2xab+a^2+1} - \frac{3x}{8b^4}\sqrt{b^2x^2+2xab+a^2+1} + \frac{3}{8b^4}\ln\left((b^2x+ab)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+2xab+a^2+1}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x)
```

```
[Out] -4/15*I/b^3*x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/8/b^4*x*(b^2*x^2+2*a*b*x+a^
2+1)^(1/2)+3/8/b^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
)/(b^2)^(1/2)-25/12*a^3/b^5*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+55/24*a/b^5*(b^2*
```

```
x^2+2*a*b*x+a^2+1)^(1/2)-83/30*I/b^5*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+2*I/
b^4*a^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/
2)+1/5*I/b^5*a^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+8/15*I/b^5*(b^2*x^2+2*a*b*x+
a^2+1)^(1/2)-3/2*I/b^4*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)
^(1/2))/(b^2)^(1/2)+1/4*x^3/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/5*I/b*x^4*(
b^2*x^2+2*a*b*x+a^2+1)^(1/2)-7/12*a/b^3*x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1
3/12*a^2/b^4*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a^4/b^4*ln((b^2*x+a*b)/(b^2)^(
1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-3*a^2/b^4*ln((b^2*x+a*b)/(b
^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/5*I/b^3*a^2*x^2*(b^2
*x^2+2*a*b*x+a^2+1)^(1/2)-1/5*I/b^2*a*x^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+29/
30*I/b^4*a*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/5*I/b^4*a^3*x*(b^2*x^2+2*a*b*x
+a^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78394, size = 520, normalized size = 1.88

$$\frac{186i\,a^5 - 1345\,a^4 - 1730i\,a^3 + 1320\,a^2 - \left(960\,a^4 + 1920i\,a^3 - 2880\,a^2 - 1440i\,a + 360\right)\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x, algorithm="fricas")
```

```
[Out] 1/960*(186*I*a^5 - 1345*a^4 - 1730*I*a^3 + 1320*a^2 - (960*a^4 + 1920*I*a^3
 - 2880*a^2 - 1440*I*a + 360)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)) + (192*I*b^4*x^4 - 48*(4*I*a - 5)*b^3*x^3 + (192*I*a^2 - 560*a - 256*I
)*b^2*x^2 + 192*I*a^4 - 2000*a^3 + (-192*I*a^3 + 1040*a^2 + 928*I*a - 360)*
b*x - 2656*I*a^2 + 2200*a + 512*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 300*
I*a)/b^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\,(ia + ibx + 1)}{\sqrt{a^2 + 2abx + b^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)*x**4,x)
```

```
[Out] Integral(x**4*(I*a + I*b*x + 1)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

Giac [A] time = 1.15667, size = 289, normalized size = 1.05

$$\frac{1}{120}\sqrt{(bx+a)^2+1}\left(\left(2\left(3\left(\frac{4\,ix}{b}-\frac{4\,ab^7i-5\,b^7}{b^9}\right)x+\frac{12\,a^2b^6i-35\,ab^6-16\,b^6i}{b^9}\right)x-\frac{24\,a^3b^5i-130\,a^2b^5-116\,ab^5i+45}{b^9}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x, algorithm="giac")
```

```
[Out] 1/120*sqrt((b*x + a)^2 + 1)*((2*(3*(4*i*x/b - (4*a*b^7*i - 5*b^7)/b^9)*x +
(12*a^2*b^6*i - 35*a*b^6 - 16*b^6*i)/b^9)*x - (24*a^3*b^5*i - 130*a^2*b^5 -
 116*a*b^5*i + 45*b^5)/b^9)*x + (24*a^4*b^4*i - 250*a^3*b^4 - 332*a^2*b^4*i
 + 275*a*b^4 + 64*b^4*i)/b^9) - 1/8*(8*a^4 + 16*a^3*i - 24*a^2 - 12*a*i + 3
)*log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^4*abs(b))
```

3.163 $\int e^{i\tan^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=201

$$-\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}\left(-18a^2+2(6a+i)bx-10ia+7\right)}{24b^4}-\frac{\left(8ia^3-12a^2-12ia+3\right)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+}}{8b^4}$$

```
[Out] -((3 - (12*I)*a - 12*a^2 + (8*I)*a^3)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a +
I*b*x])/(8*b^4) + (x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(4*b^
2) - (Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(7 - (10*I)*a - 18*a^2
+ 2*(I + 6*a)*b*x))/(24*b^4) + ((3*I + 12*a - (12*I)*a^2 - 8*a^3)*ArcSinh[a
 + b*x])/(8*b^4)
```

Rubi [A] time = 0.192415, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 100, 147, 50, 53, 619, 215}

$$-\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}\left(-18a^2+2(6a+i)bx-10ia+7\right)}{24b^4}-\frac{\left(8ia^3-12a^2-12ia+3\right)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a + b*x])*x^3,x]
```

```
[Out] -((3 - (12*I)*a - 12*a^2 + (8*I)*a^3)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a +
I*b*x])/(8*b^4) + (x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(4*b^
2) - (Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(7 - (10*I)*a - 18*a^2
+ 2*(I + 6*a)*b*x))/(24*b^4) + ((3*I + 12*a - (12*I)*a^2 - 8*a^3)*ArcSinh[a
 + b*x])/(8*b^4)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
```

```
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{i\tan^{-1}(a+bx)}x^3\,dx = \int \frac{x^3\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx$$

$$= \frac{x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} + \frac{\int \frac{x\sqrt{1+ia+ibx}\left(-2\left(1+a^2\right)-(i+6a)bx\right)}{\sqrt{1-ia-ibx}}\,dx}{4b^2}$$

$$= \frac{x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} - \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\left(7-10ia-18a^2+2(i+6a)bx\right)}{24b^4}$$

$$= -\frac{\left(3-12ia-12a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} - \frac{\sqrt{1}}{}$$

$$= -\frac{\left(3-12ia-12a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} - \frac{\sqrt{1}}{}$$

$$= -\frac{\left(3-12ia-12a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} - \frac{\sqrt{1}}{}$$

$$= -\frac{\left(3-12ia-12a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} - \frac{\sqrt{1}}{}$$

Mathematica [A] time = 0.272815, size = 176, normalized size = 0.88

$$\frac{\sqrt{b}\sqrt{a^2+2abx+b^2x^2+1}\left(a^2(44+6ibx)-6ia^3+a\left(-6ib^2x^2-20bx+39i\right)+6ib^3x^3+8b^2x^2-9ibx-16\right)-6\sqrt[4]{-1}\left(8a\right.}{24b^{9/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a + b*x])*x^3,x]
```

```
[Out] (Sqrt[b]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-16 - (6*I)*a^3 - (9*I)*b*x + 8
*b^2*x^2 + (6*I)*b^3*x^3 + a^2*(44 + (6*I)*b*x) + a*(39*I - 20*b*x - (6*I)*
b^2*x^2)) - 6*(-1)^(1/4)*(-3*I - 12*a + (12*I)*a^2 + 8*a^3)*Sqrt[(-I)*b]*Ar
cSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(24*b^(
9/2))
```

Maple [B] time = 0.117, size = 465, normalized size = 2.3

$$\frac{\frac{3i}{8}}{b^3}\ln\left((b^2x+ab)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+2xab+a^2+1}\right)\frac{1}{\sqrt{b^2}}+\frac{\frac{i}{4}a^2x}{b^3}\sqrt{b^2x^2+2xab+a^2+1}-\frac{\frac{i}{4}ax^2}{b^2}\sqrt{b^2x^2+2xab+a^2+1}-\frac{3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x)
```

```
[Out] 3/8*I/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(
1/2)+1/4*I/b^3*a^2*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/4*I/b^2*a*x^2*(b^2*x^2
+2*a*b*x+a^2+1)^(1/2)-3/8*I/b^3*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+13/8*I/b^4*
a*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/4*I/b^4*a^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
-3/2*I/b^3*a^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b
^2)^(1/2)+1/4*I/b*x^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/3*x^2/b^2*(b^2*x^2+2*
a*b*x+a^2+1)^(1/2)-5/6*a/b^3*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+11/6*a^2/b^4*(
b^2*x^2+2*a*b*x+a^2+1)^(1/2)-a^3/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+3/2*a/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x
^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-2/3/b^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71922, size = 386, normalized size = 1.92

$$\frac{-45i\,a^4+224\,a^3+192i\,a^2+\left(192\,a^3+288i\,a^2-288\,a-72i\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+\left(48i\,b^3x^3-16\,(3i\right.}{192\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/192*(-45*I*a^4 + 224*a^3 + 192*I*a^2 + (192*a^3 + 288*I*a^2 - 288*a - 72*
I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (48*I*b^3*x^3 - 16*(
```

```
3*I*a - 4)*b^2*x^2 - 48*I*a^3 + (48*I*a^2 - 160*a - 72*I)*b*x + 352*a^2 + 3
12*I*a - 128)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 72*a)/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (ia + ibx + 1)}{\sqrt{a^2 + 2abx + b^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)*x**3,x)
```

```
[Out] Integral(x**3*(I*a + I*b*x + 1)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

Giac [A] time = 1.17295, size = 220, normalized size = 1.09

$$\frac{1}{24}\sqrt{(bx+a)^2+1}\left(\left(2\left(\frac{3\,ix}{b}-\frac{3\,ab^5i-4\,b^5}{b^7}\right)x+\frac{6\,a^2b^4i-20\,ab^4-9\,b^4i}{b^7}\right)x-\frac{6\,a^3b^3i-44\,a^2b^3-39\,ab^3i+16\,b^3}{b^7}\right)+\frac{(8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/24*sqrt((b*x + a)^2 + 1)*((2*(3*i*x/b - (3*a*b^5*i - 4*b^5)/b^7)*x + (6*a
^2*b^4*i - 20*a*b^4 - 9*b^4*i)/b^7)*x - (6*a^3*b^3*i - 44*a^2*b^3 - 39*a*b^
3*i + 16*b^3)/b^7) + 1/8*(8*a^3 + 12*a^2*i - 12*a - 3*i)*log(-a*b - (x*abs(
b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^3*abs(b))
```

3.164 $\int e^{i \tan^{-1}(a+bx)} x^2 \, dx$

Optimal. Leaf size=171

$$-\frac{(-2ia^2+2a+i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2b^3}-\frac{(-2a^2-2ia+1)\sinh^{-1}(a+bx)}{2b^3}+\frac{x\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{3b^2}-($$

```
[Out] -((I + 2*a - (2*I)*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^3
) - ((I + 4*a)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(6*b^3) + (x*
Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(3*b^2) - ((1 - (2*I)*a - 2*
a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rubi [A] time = 0.125008, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 90, 80, 50, 53, 619, 215}

$$-\frac{(-2ia^2+2a+i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2b^3}-\frac{(-2a^2-2ia+1)\sinh^{-1}(a+bx)}{2b^3}+\frac{x\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{3b^2}-($$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a + b*x])*x^2,x]
```


```
[Out] -((I + 2*a - (2*I)*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^3
) - ((I + 4*a)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(6*b^3) + (x*
Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(3*b^2) - ((1 - (2*I)*a - 2*
a^2)*ArcSinh[a + b*x])/(2*b^3)
```


Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{i\tan^{-1}(a+bx)}x^2\,dx = \int \frac{x^2\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx$$

$$= \frac{x\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{3b^2} + \frac{\int \frac{\sqrt{1+ia+ibx}\left(-1-a^2-(i+4a)bx\right)}{\sqrt{1-ia-ibx}}\,dx}{3b^2}$$

$$= -\frac{(i+4a)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} + \frac{x\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{3b^2} - \frac{\left(1-2ia-2a^2\right)\int \frac{\sqrt{1}}{\sqrt{1}}}{2b^2}$$

$$= -\frac{\left(i+2a-2ia^2\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} - \frac{(i+4a)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} + \frac{x\sqrt{1-ia}}{}$$

$$= -\frac{\left(i+2a-2ia^2\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} - \frac{(i+4a)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} + \frac{x\sqrt{1-ia}}{}$$

$$= -\frac{\left(i+2a-2ia^2\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} - \frac{(i+4a)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} + \frac{x\sqrt{1-ia}}{}$$

$$= -\frac{\left(i+2a-2ia^2\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} - \frac{(i+4a)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} + \frac{x\sqrt{1-ia}}{}$$

Mathematica [A] time = 0.134284, size = 135, normalized size = 0.79

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}\left(2ia^2+a(-9-2ibx)+2ib^2x^2+3bx-4i\right)}{6b^3} + \frac{\sqrt[4]{-1}\left(2a^2+2ia-1\right)\sqrt{-ib}\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+bx}}{\sqrt{-ib}}\right.}{b^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-4*I + (2*I)*a^2 + 3*b*x + (2*I)*b^2*x^
2 + a*(-9 - (2*I)*b*x)))/(6*b^3) + ((-1)^(1/4)*(-1 + (2*I)*a + 2*a^2)*Sqrt[
```

```
(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]
])/b^(7/2)
```

Maple [B] time = 0.117, size = 302, normalized size = 1.8

$$\frac{\frac{i}{3}x^2}{b}\sqrt{b^2x^2+2xab+a^2+1}-\frac{\frac{i}{3}ax}{b^2}\sqrt{b^2x^2+2xab+a^2+1}+\frac{\frac{i}{3}a^2}{b^3}\sqrt{b^2x^2+2xab+a^2+1}+\frac{ia}{b^2}\ln\left((b^2x+ab)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x)
```

```
[Out] 1/3*I/b*x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/3*I/b^2*a*x*(b^2*x^2+2*a*b*x+a^
2+1)^(1/2)+1/3*I/b^3*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+I/b^2*a*ln((b^2*x+a*
b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-2/3*I/b^3*(b^2*x^
2+2*a*b*x+a^2+1)^(1/2)+1/2*x/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*a/b^3*(b
^2*x^2+2*a*b*x+a^2+1)^(1/2)+a^2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a
*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+
2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69255, size = 278, normalized size = 1.63

$$\frac{7i\,a^3-21\,a^2-12\left(2\,a^2+2i\,a-1\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+\sqrt{b^2x^2+2\,abx+a^2+1}\left(8i\,b^2x^2-4\,(2i\,a-3)\right.}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/24*(7*I*a^3 - 21*a^2 - 12*(2*a^2 + 2*I*a - 1)*log(-b*x - a + sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(8*I*b^2*x^2 - 4
*(2*I*a - 3)*b*x + 8*I*a^2 - 36*a - 16*I) - 9*I*a)/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\,(ia+ibx+1)}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)*x**2,x)
```

```
[Out] Integral(x**2*(I*a + I*b*x + 1)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

Giac [A] time = 1.15091, size = 158, normalized size = 0.92

$$\frac{1}{6}\sqrt{(bx+a)^2+1}\left(\left(\frac{2\,ix}{b}-\frac{2\,ab^3i-3\,b^3}{b^5}\right)x+\frac{2\,a^2b^2i-9\,ab^2-4\,b^2i}{b^5}\right)-\frac{\left(2\,a^2+2\,ai-1\right)\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+\right.\right.}{2\,b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt((b*x + a)^2 + 1)*((2*i*x/b - (2*a*b^3*i - 3*b^3)/b^5)*x + (2*a^2*b
^2*i - 9*a*b^2 - 4*b^2*i)/b^5) - 1/2*(2*a^2 + 2*a*i - 1)*log(-a*b - (x*abs(
b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^2*abs(b))
```

3.165 $\int e^{i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b^2}+\frac{(1-2ia)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2b^2}-\frac{(2a+i)\sinh^{-1}(a+bx)}{2b^2}$$

```
[Out] ((1 - (2*I)*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^2) + (Sqrt
[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(2*b^2) - ((I + 2*a)*ArcSinh[a +
 b*x])/(2*b^2)
```

Rubi [A] time = 0.0753336, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5095, 80, 50, 53, 619, 215}

$$\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b^2}+\frac{(1-2ia)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2b^2}-\frac{(2a+i)\sinh^{-1}(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a + b*x])*x,x]
```

```
[Out] ((1 - (2*I)*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^2) + (Sqrt
[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(2*b^2) - ((I + 2*a)*ArcSinh[a +
 b*x])/(2*b^2)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{i\tan^{-1}(a+bx)}x\,dx = \int \frac{x\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx$$
$$= \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(i+2a)\int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx}{2b}$$
$$= \frac{(1-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} + \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(i+2a)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+i}}}{2b}$$
$$= \frac{(1-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} + \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(i+2a)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2ab}}}{2b}$$
$$= \frac{(1-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} + \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(i+2a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\right.}{4b^3}$$
$$= \frac{(1-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} + \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(i+2a)\sinh^{-1}(a+bx)}{2b^2}$$

Mathematica [A] time = 0.113457, size = 108, normalized size = 0.98

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}(-ia+ibx+2)}{2b^2} + \frac{(-1)^{3/4}(2a+i)\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{\sqrt{-ib}b^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a + b*x])*x,x]
```

```
[Out] ((2 - I*a + I*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/(2*b^2) + ((-1)^(3/4)
*(I + 2*a)*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)
*b]])/(Sqrt[(-I)*b]*b^(3/2))
```

Maple [A] time = 0.109, size = 171, normalized size = 1.6

$$\frac{\frac{i}{2}x}{b}\sqrt{b^2x^2+2\,xab+a^2+1} - \frac{\frac{i}{2}a}{b^2}\sqrt{b^2x^2+2\,xab+a^2+1} - \frac{\frac{i}{2}}{b}\ln\left((b^2x+ab)\frac{1}{\sqrt{b^2}} + \sqrt{b^2x^2+2\,xab+a^2+1}\right)\frac{1}{\sqrt{b^2}} + \frac{1}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x)
```

```
[Out] 1/2*I/b*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*I/b^2*a*(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)-1/2*I/b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b
^2)^(1/2)+1/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-a/b*ln((b^2*x+a*b)/(b^2)^(1/2
)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64975, size = 200, normalized size = 1.82

$$\frac{-3i\,a^2 + (8\,a + 4i)\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}(4i\,bx - 4i\,a + 8) + 4\,a}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] 1/8*(-3*I*a^2 + (8*a + 4*I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(4*I*b*x - 4*I*a + 8) + 4*a)/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,(ia + ibx + 1)}{\sqrt{a^2 + 2abx + b^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)*x,x)
```

```
[Out] Integral(x*(I*a + I*b*x + 1)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

Giac [A] time = 1.1554, size = 103, normalized size = 0.94

$$\frac{1}{2}\sqrt{(bx + a)^2 + 1}\left(\frac{ix}{b} - \frac{abi - 2\,b}{b^3}\right) + \frac{(2\,a + i)\log\left(-ab - \left(x|b| - \sqrt{(bx + a)^2 + 1}\right)|b|\right)}{2\,b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt((b*x + a)^2 + 1)*(i*x/b - (a*b*i - 2*b)/b^3) + 1/2*(2*a + i)*log(-
a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b*abs(b))
```

3.166 $\int e^{i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=52

$$\frac{\sinh^{-1}(a+bx)}{b}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}$$

```
[Out] (I*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[a + b*x]/b
```

Rubi [A] time = 0.0337835, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5093, 50, 53, 619, 215}

$$\frac{\sinh^{-1}(a+bx)}{b}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a + b*x]),x]
```

```
[Out] (I*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[a + b*x]/b
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{i\tan^{-1}(a+bx)}\,dx &= \int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx \\
&= \frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} + \int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx \\
&= \frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} + \int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx \\
&= \frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{2b^2} \\
&= \frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} + \frac{\sinh^{-1}(a+bx)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0177386, size = 28, normalized size = 0.54

$$
\frac{\sinh^{-1}(a+bx)+i\sqrt{(a+bx)^2+1}}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a + b*x]),x]
```

```
[Out] (I*Sqrt[1 + (a + b*x)^2] + ArcSinh[a + b*x])/b
```

Maple [A] time = 0.088, size = 69, normalized size = 1.3

$$
\frac{i}{b}\sqrt{b^2x^2+2\,xab+a^2+1}+\ln\left((b^2x+ab)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+2\,xab+a^2+1}\right)\frac{1}{\sqrt{b^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] I/b*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b
*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67551, size = 144, normalized size = 2.77

$$\frac{i\,a + 2i\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1} - 2\,\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(I*a + 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*log(-b*x - a + sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

Sympy [A] time = 2.80244, size = 36, normalized size = 0.69

$$\begin{cases} \frac{i\sqrt{(a+bx)^2+1}+\operatorname{asinh}(a+bx)}{b} & \text{for } b \neq 0 \\ \frac{x(ia+1)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] Piecewise(((I*sqrt((a + b*x)**2 + 1) + asinh(a + b*x))/b, Ne(b, 0)), (x*(I*
a + 1)/sqrt(a**2 + 1), True))
```

Giac [A] time = 1.16162, size = 69, normalized size = 1.33

$$\frac{\sqrt{(bx + a)^2 + 1}i}{b} - \frac{\log\left(-ab - \left(x|b| - \sqrt{(bx + a)^2 + 1}\right)|b|\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt((b*x + a)^2 + 1)*i/b - log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*a
bs(b))/abs(b)
```

3.167 $\int \frac{e^{i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=89

$$i\sinh^{-1}(a+bx)-\frac{2\sqrt{-a+i}\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{a+i}}$$

```
[Out] I*ArcSinh[a + b*x] - (2*Sqrt[I - a]*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b
*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/Sqrt[I + a]
```

Rubi [A] time = 0.0751899, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 105, 53, 619, 215, 93, 208}

$$i\sinh^{-1}(a+bx)-\frac{2\sqrt{-a+i}\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{a+i}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a + b*x])/x,x]
```

```
[Out] I*ArcSinh[a + b*x] - (2*Sqrt[I - a]*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b
*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/Sqrt[I + a]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{i\tan^{-1}(a+bx)}}{x}\,dx &= \int \frac{\sqrt{1+ia+ibx}}{x\sqrt{1-ia-ibx}}\,dx \\
&= -\left((-1-ia)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx\right)+(ib)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx \\
&= (2(1+ia))\,\mathrm{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)+(ib)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2ab
\end{aligned}$$

$$= -\frac{2\sqrt{i-a}\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i+a}}+\frac{i\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{2b}$$

$$= i\sinh^{-1}(a+bx)-\frac{2\sqrt{i-a}\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i+a}}$$

Mathematica [A] time = 0.134819, size = 132, normalized size = 1.48

$$\frac{2\tan^{-1}\left(\frac{\sqrt{-i(a+bx+i)}}{\sqrt{\frac{a+i}{a-i}}\sqrt{ia+ibx+1}}\right)}{\sqrt{\frac{a+i}{a-i}}}+\frac{2(-1)^{3/4}\sqrt{-ib}\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{\sqrt{b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a + b*x])/x,x]
```

```
[Out] (2*(-1)^(3/4)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a +
b*x)])/Sqrt[(-I)*b]])/Sqrt[b] + (2*ArcTan[Sqrt[(-I)*(I + a + b*x)]/(Sqrt[(I
 + a)/(-I + a)]*Sqrt[1 + I*a + I*b*x])])/Sqrt[(I + a)/(-I + a)]
```

Maple [B] time = 0.105, size = 157, normalized size = 1.8

$$ib\ln\left((b^2x+ab)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+2xab+a^2+1}\right)\frac{1}{\sqrt{b^2}}-ia\ln\left(\frac{1}{x}\left(2a^2+2+2xab+2\sqrt{a^2+1}\sqrt{b^2x^2+2xab+a^2+1}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x)
```

```
[Out] I*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I
/(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^
(1/2))/x)*a-1/(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.69298, size = 402, normalized size = 4.52

$$\frac{1}{2}\sqrt{-\frac{4\,a-4i}{a+i}}\log\left(-bx+\frac{1}{2}\,(i\,a-1)\sqrt{-\frac{4\,a-4i}{a+i}}+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\frac{1}{2}\sqrt{-\frac{4\,a-4i}{a+i}}\log\left(-bx+\frac{1}{2}\,(-i\,a+1)\sqrt{-\frac{4}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-(4*a - 4*I)/(a + I))*log(-b*x + 1/2*(I*a - 1)*sqrt(-(4*a - 4*I)/(
a + I)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 1/2*sqrt(-(4*a - 4*I)/(a + I
))*log(-b*x + 1/2*(-I*a + 1)*sqrt(-(4*a - 4*I)/(a + I)) + sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)) - I*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ia+ibx+1}{x\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)/x,x)
```

```
[Out] Integral((I*a + I*b*x + 1)/(x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.168 $\int \frac{e^{i \tan^{-1}(a+bx)}}{x^2} dx$

Optimal. Leaf size=130

$$\frac{2ib \tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}(a+i)^{3/2}} - \frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1-ia)x}$$

```
[Out] -((Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 - I*a)*x)) + ((2*I)*b*A
rcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*
x])])/(Sqrt[I - a]*(I + a)^(3/2))
```

Rubi [A] time = 0.0640981, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5095, 94, 93, 208}

$$\frac{2ib \tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}(a+i)^{3/2}} - \frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1-ia)x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a + b*x])/x^2,x]
```


```
[Out] -((Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 - I*a)*x)) + ((2*I)*b*A
rcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*
x])])/(Sqrt[I - a]*(I + a)^(3/2))
```


Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{i\tan^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{\sqrt{1+ia+ibx}}{x^2\sqrt{1-ia-ibx}}\,dx\\ &= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{(1-ia)x} - \frac{b\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{i+a}\\ &= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{(1-ia)x} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{i+a}\\ &= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{(1-ia)x} + \frac{2ib\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i-a}(i+a)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0819284, size = 115, normalized size = 0.88

$$-i\left(\frac{\sqrt{a^2+2abx+b^2x^2+1}}{ax+ix} + \frac{2b\tan^{-1}\left(\frac{\sqrt{-i(a+bx+i)}}{\sqrt{\frac{a+i}{a-i}}\sqrt{ia+ibx+1}}\right)}{(-1+ia)^{3/2}\sqrt{1+ia}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a + b*x])/x^2,x]
```

```
[Out] (-I)*(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/(I*x + a*x) + (2*b*ArcTan[Sqrt[(-I)
*(I + a + b*x)]/(Sqrt[(I + a)/(-I + a)]*Sqrt[1 + I*a + I*b*x])])/((-1 + I*a
)^(3/2)*Sqrt[1 + I*a]))
```

Maple [B] time = 0.109, size = 236, normalized size = 1.8

$$\frac{-ia}{\left(a^2+1\right)x}\sqrt{b^2x^2+2\,xab+a^2+1} - \frac{1}{\left(a^2+1\right)x}\sqrt{b^2x^2+2\,xab+a^2+1} + ia^2b\ln\left(\frac{1}{x}\left(2\,a^2+2+2\,xab+2\,\sqrt{a^2+1}\sqrt{b^2x^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x)
```

```
[Out] -I/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a
^2+1)^(1/2)+I*a^2*b/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*
x^2+2*a*b*x+a^2+1)^(1/2))/x)+a*b/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1
)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-I*b/(a^2+1)^(1/2)*ln((2*a^2+2+2*x
*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.7045, size = 545, normalized size = 4.19

$$-\frac{2\,(a+i)\sqrt{\frac{b^2}{a^4+2i\,a^3+2i\,a-1}}\,x\log\left(-\frac{b^2x-\sqrt{b^2x^2+2\,abx+a^2+1}\,b+\left(a^3+i\,a^2+a+i\right)\sqrt{\frac{b^2}{a^4+2i\,a^3+2i\,a-1}}}{b}\right)-2\,(a+i)\sqrt{\frac{b^2}{a^4+2i\,a^3+2i\,a-1}}\,x\log\left(-\frac{b^2x-\sqrt{b}\cdots}{\cdots}\right.}{(2\,a+2i)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*(a + I)*sqrt(b^2/(a^4 + 2*I*a^3 + 2*I*a - 1))*x*log(-(b^2*x - sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*b + (a^3 + I*a^2 + a + I)*sqrt(b^2/(a^4 + 2*I*a^3
+ 2*I*a - 1)))/b) - 2*(a + I)*sqrt(b^2/(a^4 + 2*I*a^3 + 2*I*a - 1))*x*log(-
(b^2*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b - (a^3 + I*a^2 + a + I)*sqrt(b
^2/(a^4 + 2*I*a^3 + 2*I*a - 1)))/b) + 2*I*b*x + 2*I*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1))/((2*a + 2*I)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ia+ibx+1}{x^2\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)/x**2,x)
```

```
[Out] Integral((I*a + I*b*x + 1)/(x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.169 $\int \frac{e^{i\tan^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=201

$$-\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2\left(a^2+1\right)x^2}+\frac{(1+2ia)b^2\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}(a+i)^{5/2}}-\frac{(1+2ia)b\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(-a+i)(a+i)^2x}$$

```
[Out] -((1 + (2*I)*a)*b*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*(I - a)*(
I + a)^2*x) - (Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(2*(1 + a^2)*
x^2) + ((1 + (2*I)*a)*b^2*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt
[I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a)^(3/2)*(I + a)^(5/2))
```

Rubi [A] time = 0.137747, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5095, 96, 94, 93, 208}

$$-\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2\left(a^2+1\right)x^2}+\frac{(1+2ia)b^2\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}(a+i)^{5/2}}-\frac{(1+2ia)b\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(-a+i)(a+i)^2x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a + b*x])/x^3,x]
```

```
[Out] -((1 + (2*I)*a)*b*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*(I - a)*(
I + a)^2*x) - (Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(2*(1 + a^2)*
x^2) + ((1 + (2*I)*a)*b^2*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt
[I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a)^(3/2)*(I + a)^(5/2))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{i\tan^{-1}(a+bx)}}{x^3}\,dx = \int \frac{\sqrt{1+ia+ibx}}{x^3\sqrt{1-ia-ibx}}\,dx$$

$$= -\frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2\left(1+a^2\right)x^2} + \frac{((i-2a)b)\int \frac{\sqrt{1+ia+ibx}}{x^2\sqrt{1-ia-ibx}}\,dx}{2\left(1+a^2\right)}$$

$$= -\frac{(i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2(1-ia)\left(1+a^2\right)x} - \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2\left(1+a^2\right)x^2} - \frac{\left((i-2a)b^2\right)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{}}}{2(i+a)\left(1+a^2\right.}$$

$$= -\frac{(i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2(1-ia)\left(1+a^2\right)x} - \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2\left(1+a^2\right)x^2} - \frac{\left((i-2a)b^2\right)\text{Subst}\left(\int \frac{}{-1}\right.}{(i+a)}$$

$$= -\frac{(i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2(1-ia)\left(1+a^2\right)x} - \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2\left(1+a^2\right)x^2} + \frac{(1+2ia)b^2\tanh^{-1}\left(\frac{\sqrt{i+a}}{\sqrt{i-a}}\right.}{(i-a)^{3/2}(i+a)^{5}}$$

Mathematica [A] time = 0.118302, size = 149, normalized size = 0.74

$$\frac{\frac{2(2a-i)b^2\tan^{-1}\left(\frac{\sqrt{-i(a+bx+i)}}{\sqrt{\frac{a+i}{a-i}}\sqrt{ia+ibx+1}}\right)}{\sqrt{-1+ia}\sqrt{1+ia}} - \frac{i\left(a^2-abx+2ibx+1\right)\sqrt{a^2+2abx+b^2x^2+1}}{x^2}}{2(a-i)(a+i)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a + b*x])/x^3,x]
```

```
[Out] (((-I)*(1 + a^2 + (2*I)*b*x - a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x^2
 + (2*(-I + 2*a)*b^2*ArcTan[Sqrt[(-I)*(I + a + b*x)]/(Sqrt[(I + a)/(-I + a)
]*Sqrt[1 + I*a + I*b*x])]])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a]))/(2*(-I + a)*(I +
 a)^2)
```

Maple [B] time = 0.113, size = 405, normalized size = 2.

$$\frac{-\frac{i}{2}a}{\left(a^2+1\right)x^2}\sqrt{b^2x^2+2xab+a^2+1} - \frac{1}{\left(2a^2+2\right)x^2}\sqrt{b^2x^2+2xab+a^2+1} + \frac{\frac{3i}{2}a^2b}{\left(a^2+1\right)^2x}\sqrt{b^2x^2+2xab+a^2+1} + \frac{}{2\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x)
```

```
[Out] -1/2*I/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-1/2/(a^2+1)/x^2*(b^2*x^2
+2*a*b*x+a^2+1)^(1/2)+3/2*I*a^2*b/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
+3/2*a*b/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*I*a^3*b^2/(a^2+1)^(5
/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-3
/2*a^2*b^2/(a^2+1)^(5/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b
*x+a^2+1)^(1/2))/x)+3/2*I*b^2/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(
1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)*a+1/2*b^2/(a^2+1)^(3/2)*ln((2*a^2+2+
2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-I*b/(a^2+1)/x*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.83042, size = 1103, normalized size = 5.49

$$(ia+2)b^2x^2 + \sqrt{\frac{(4a^2-4ia-1)b^4}{a^8+2ia^7+2a^6+6ia^5+6ia^3-2a^2+2ia-1}}\left(a^3+ia^2+a+i\right)x^2\log\left(-\frac{(2a-i)b^3x-\sqrt{b^2x^2+2abx+a^2+1}(2a-i)b^2+\left(a^5+ia^4+2a^3+2ia^2\right.}{(2a-i)b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] ((I*a + 2)*b^2*x^2 + sqrt((4*a^2 - 4*I*a - 1)*b^4/(a^8 + 2*I*a^7 + 2*a^6 +
6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1))*(a^3 + I*a^2 + a + I)*x^2*log(-((2*
a - I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a - I)*b^2 + (a^5 + I*a
^4 + 2*a^3 + 2*I*a^2 + a + I)*sqrt((4*a^2 - 4*I*a - 1)*b^4/(a^8 + 2*I*a^7 +
 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1)))/((2*a - I)*b^2)) - sqrt((
4*a^2 - 4*I*a - 1)*b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 +
 2*I*a - 1))*(a^3 + I*a^2 + a + I)*x^2*log(-((2*a - I)*b^3*x - sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*(2*a - I)*b^2 - (a^5 + I*a^4 + 2*a^3 + 2*I*a^2 + a +
I)*sqrt((4*a^2 - 4*I*a - 1)*b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3
- 2*a^2 + 2*I*a - 1)))/((2*a - I)*b^2)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*((I*a + 2)*b*x - I*a^2 - I))/((2*a^3 + 2*I*a^2 + 2*a + 2*I)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ia+ibx+1}{x^3\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)/x**3,x)
```

```
[Out] Integral((I*a + I*b*x + 1)/(x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.170 $\int \frac{e^{i\tan^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=283

$$\frac{\left(-2a^2+9ia+4\right)b^2\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{6(1-ia)\left(a^2+1\right)^2x}+\frac{\left(2a-i\left(1-2a^2\right)\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{5/2}(a+i)^{7/2}}-\frac{(-2a+3i)b\sqrt{-ia-ib}}{6(1-ia)\left(a\right.}$$

```
[Out] -(Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(3*(1 - I*a)*x^3) - ((3*I -
2*a)*b*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(6*(1 - I*a)*(1 + a^2)*
x^2) + ((4 + (9*I)*a - 2*a^2)*b^2*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*
x])/(6*(1 - I*a)*(1 + a^2)^2*x) + ((2*a - I*(1 - 2*a^2))*b^3*ArcTanh[(Sqrt[
I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a
)^(5/2)*(I + a)^(7/2))
```

Rubi [A] time = 0.183258, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5095, 99, 151, 12, 93, 208}

$$\frac{\left(-2a^2+9ia+4\right)b^2\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{6(1-ia)\left(a^2+1\right)^2x}+\frac{\left(2a-i\left(1-2a^2\right)\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{5/2}(a+i)^{7/2}}-\frac{(-2a+3i)b\sqrt{-ia-ib}}{6(1-ia)\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a + b*x])/x^4,x]
```

```
[Out] -(Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(3*(1 - I*a)*x^3) - ((3*I -
2*a)*b*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(6*(1 - I*a)*(1 + a^2)*
x^2) + ((4 + (9*I)*a - 2*a^2)*b^2*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*
x])/(6*(1 - I*a)*(1 + a^2)^2*x) + ((2*a - I*(1 - 2*a^2))*b^3*ArcTanh[(Sqrt[
I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a
)^(5/2)*(I + a)^(7/2))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{i\tan^{-1}(a+bx)}}{x^4}\,dx = \int \frac{\sqrt{1+ia+ibx}}{x^4\sqrt{1-ia-ibx}}\,dx$$

$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1-ia)x^3} + \frac{\int \frac{(3i-2a)b-2b^2x}{x^3\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{3(1-ia)}$$

$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1-ia)x^3} - \frac{(3i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)x^2} - \frac{\int \frac{\left(4+9ia-2a^2\right)b^2+(3i-2a)b^3x}{x^2\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{6(1-ia)\left(1+a^2\right)}$$

$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1-ia)x^3} - \frac{(3i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)x^2} + \frac{\left(4+9ia-2a^2\right)b^2\sqrt{1-ia}}{6(1-ia)\left(1+\right.}$$

$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1-ia)x^3} - \frac{(3i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)x^2} + \frac{\left(4+9ia-2a^2\right)b^2\sqrt{1-ia}}{6(1-ia)\left(1+\right.}$$

$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1-ia)x^3} - \frac{(3i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)x^2} + \frac{\left(4+9ia-2a^2\right)b^2\sqrt{1-ia}}{6(1-ia)\left(1+\right.}$$

$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1-ia)x^3} - \frac{(3i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)x^2} + \frac{\left(4+9ia-2a^2\right)b^2\sqrt{1-ia}}{6(1-ia)\left(1+\right.}$$

Mathematica [A] time = 0.262788, size = 242, normalized size = 0.86

$$\frac{(1+4ia)bx(a+bx-i)\sqrt{a^2+2abx+b^2x^2+1}+2(1-ia)(a-i)(a+bx-i)\sqrt{a^2+2abx+b^2x^2+1}+\frac{3\left(2a^2-2ia-1\right)b^2x^2\left(\sqrt{-1}\right.}{}}{6\left(a^2+1\right)^2x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a + b*x])/x^4,x]
```

```
[Out] (2*(1 - I*a)*(-I + a)*(-I + a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (1
 + (4*I)*a)*b*x*(-I + a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (3*(-1 -
 (2*I)*a + 2*a^2)*b^2*x^2*(Sqrt[-1 + I*a]*Sqrt[1 + I*a]*Sqrt[1 + a^2 + 2*a*
b*x + b^2*x^2] - (2*I)*b*x*ArcTan[Sqrt[(-I)*(I + a + b*x)]/(Sqrt[(I + a)/(-
I + a)]*Sqrt[1 + I*a + I*b*x])]))/((-1 + I*a)^(3/2)*Sqrt[1 + I*a]))/(6*(1 +
 a^2)^2*x^3)
```

Maple [B] time = 0.118, size = 611, normalized size = 2.2

$$\frac{-\frac{i}{2}b}{\left(a^2+1\right)x^2}\sqrt{b^2x^2+2xab+a^2+1}+\frac{\frac{5i}{6}a^2b}{\left(a^2+1\right)^2x^2}\sqrt{b^2x^2+2xab+a^2+1}-3ib^3a^2\ln\left(\frac{1}{x}\left(2a^2+2+2xab+2\sqrt{a^2+1}\sqrt{b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x)
```

```
[Out] -1/2*I*b/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+5/6*I*a^2*b/(a^2+1)^2/x^
2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*I*b^3*a^2/(a^2+1)^(5/2)*ln((2*a^2+2+2*x*a
*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+1/2*I*b^3/(a^2+1)^(3/2
)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-5/2
*I*a^3*b^2/(a^2+1)^3/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/3/(a^2+1)/x^3*(b^2*x
^2+2*a*b*x+a^2+1)^(1/2)+5/2*I*a^4*b^3/(a^2+1)^(7/2)*ln((2*a^2+2+2*x*a*b+2*(
a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+5/6*a*b/(a^2+1)^2/x^2*(b^2*x
^2+2*a*b*x+a^2+1)^(1/2)-1/3*I/(a^2+1)/x^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-5
/2*a^2*b^2/(a^2+1)^3/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+13/6*I*b^2*a/(a^2+1)^2
/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+5/2*a^3*b^3/(a^2+1)^(7/2)*ln((2*a^2+2+2*x*
a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-3/2*a*b^3/(a^2+1)^(5/
2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+2/
3*b^2/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.82361, size = 1767, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] ((-2*I*a^2 - 9*a + 4*I)*b^3*x^3 - sqrt((4*a^4 - 8*I*a^3 - 8*a^2 + 4*I*a + 1
)*b^6/(a^12 + 2*I*a^11 + 4*a^10 + 10*I*a^9 + 5*a^8 + 20*I*a^7 + 20*I*a^5 -
5*a^4 + 10*I*a^3 - 4*a^2 + 2*I*a - 1))*(3*a^5 + 3*I*a^4 + 6*a^3 + 6*I*a^2 +
 3*a + 3*I)*x^3*log(-((2*a^2 - 2*I*a - 1)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*(2*a^2 - 2*I*a - 1)*b^3 + (a^7 + I*a^6 + 3*a^5 + 3*I*a^4 + 3*a^3 +
 3*I*a^2 + a + I)*sqrt((4*a^4 - 8*I*a^3 - 8*a^2 + 4*I*a + 1)*b^6/(a^12 + 2*
I*a^11 + 4*a^10 + 10*I*a^9 + 5*a^8 + 20*I*a^7 + 20*I*a^5 - 5*a^4 + 10*I*a^3
 - 4*a^2 + 2*I*a - 1)))/((2*a^2 - 2*I*a - 1)*b^3)) + sqrt((4*a^4 - 8*I*a^3
- 8*a^2 + 4*I*a + 1)*b^6/(a^12 + 2*I*a^11 + 4*a^10 + 10*I*a^9 + 5*a^8 + 20*
I*a^7 + 20*I*a^5 - 5*a^4 + 10*I*a^3 - 4*a^2 + 2*I*a - 1))*(3*a^5 + 3*I*a^4
+ 6*a^3 + 6*I*a^2 + 3*a + 3*I)*x^3*log(-((2*a^2 - 2*I*a - 1)*b^4*x - sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a^2 - 2*I*a - 1)*b^3 - (a^7 + I*a^6 + 3*a^5
+ 3*I*a^4 + 3*a^3 + 3*I*a^2 + a + I)*sqrt((4*a^4 - 8*I*a^3 - 8*a^2 + 4*I*a
+ 1)*b^6/(a^12 + 2*I*a^11 + 4*a^10 + 10*I*a^9 + 5*a^8 + 20*I*a^7 + 20*I*a^5
 - 5*a^4 + 10*I*a^3 - 4*a^2 + 2*I*a - 1)))/((2*a^2 - 2*I*a - 1)*b^3)) + ((-
2*I*a^2 - 9*a + 4*I)*b^2*x^2 - 2*I*a^4 + (2*I*a^3 + 3*a^2 + 2*I*a + 3)*b*x
- 4*I*a^2 - 2*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((6*a^5 + 6*I*a^4 + 12*
a^3 + 12*I*a^2 + 6*a + 6*I)*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ia + ibx + 1}{x^4\sqrt{a^2 + 2abx + b^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)/x**4,x)
```

```
[Out] Integral((I*a + I*b*x + 1)/(x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.171 $\int e^{2i\tan^{-1}(a+bx)}x^4\,dx$

Optimal. Leaf size=92

$$\frac{2(1-ia)x^3}{3b^2}+\frac{i(a+i)^2x^2}{b^3}-\frac{2(1-ia)^3x}{b^4}+\frac{2i(a+i)^4\log(a+bx+i)}{b^5}+\frac{ix^4}{2b}-\frac{x^5}{5}$$

```
[Out] (-2*(1 - I*a)^3*x)/b^4 + (I*(I + a)^2*x^2)/b^3 + (2*(1 - I*a)*x^3)/(3*b^2)
+ ((I/2)*x^4)/b - x^5/5 + ((2*I)*(I + a)^4*Log[I + a + b*x])/b^5
```

Rubi [A] time = 0.0828601, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$\frac{2(1-ia)x^3}{3b^2}+\frac{i(a+i)^2x^2}{b^3}-\frac{2(1-ia)^3x}{b^4}+\frac{2i(a+i)^4\log(a+bx+i)}{b^5}+\frac{ix^4}{2b}-\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x])*x^4,x]
```

```
[Out] (-2*(1 - I*a)^3*x)/b^4 + (I*(I + a)^2*x^2)/b^3 + (2*(1 - I*a)*x^3)/(3*b^2)
+ ((I/2)*x^4)/b - x^5/5 + ((2*I)*(I + a)^4*Log[I + a + b*x])/b^5
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))))
```

Rubi steps

$$\begin{aligned}\int e^{2i\tan^{-1}(a+bx)}x^4\,dx&=\int\frac{x^4(1+ia+ibx)}{1-ia-ibx}\,dx\\&=\int\left(\frac{2(-1+ia)^3}{b^4}+\frac{2i(i+a)^2x}{b^3}+\frac{2(1-ia)x^2}{b^2}+\frac{2ix^3}{b}-x^4+\frac{2i(i+a)^4}{b^4(i+a+bx)}\right)dx\\&=-\frac{2(1-ia)^3x}{b^4}+\frac{i(i+a)^2x^2}{b^3}+\frac{2(1-ia)x^3}{3b^2}+\frac{ix^4}{2b}-\frac{x^5}{5}+\frac{2i(i+a)^4\log(i+a+bx)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.066813, size = 92, normalized size = 1.

$$\frac{2(1-ia)x^3}{3b^2}+\frac{i(a+i)^2x^2}{b^3}-\frac{2(1-ia)^3x}{b^4}+\frac{2i(a+i)^4\log(a+bx+i)}{b^5}+\frac{ix^4}{2b}-\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])*x^4,x]
```

```
[Out] (-2*(1 - I*a)^3*x)/b^4 + (I*(I + a)^2*x^2)/b^3 + (2*(1 - I*a)*x^3)/(3*b^2)
+ ((I/2)*x^4)/b - x^5/5 + ((2*I)*(I + a)^4*Log[I + a + b*x])/b^5
```

Maple [B] time = 0.039, size = 347, normalized size = 3.8

$$-\frac{x^5}{5} - \frac{8\,ia}{b^5}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\right) + \frac{8\,ia^3}{b^5}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\right) - \frac{\frac{2\,i}{3}x^3a}{b^2} - \frac{ix^2}{b^3} + \frac{2\,x^3}{3\,b^2} + \frac{6\,iax}{b^4} - 2\,\frac{ax^2}{b^3} + \frac{i\ln\left(b^2x^2 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x)
```

```
[Out] -1/5*x^5-8*I/b^5*arctan(1/2*(2*b^2*x+2*a*b)/b)*a+8*I/b^5*arctan(1/2*(2*b^2*
x+2*a*b)/b)*a^3-2/3*I/b^2*x^3*a-I/b^3*x^2+2/3/b^2*x^3+6*I/b^4*a*x-2/b^3*x^2
*a+I/b^5*ln(b^2*x^2+2*a*b*x+a^2+1)+6/b^4*a^2*x-2/b^4*x-2*I/b^4*a^3*x+I/b^5*
ln(b^2*x^2+2*a*b*x+a^2+1)*a^4-4/b^5*ln(b^2*x^2+2*a*b*x+a^2+1)*a^3+I/b^3*x^2
*a^2+4/b^5*ln(b^2*x^2+2*a*b*x+a^2+1)*a+1/2*I*x^4/b+2/b^5*arctan(1/2*(2*b^2*
x+2*a*b)/b)*a^4-6*I/b^5*ln(b^2*x^2+2*a*b*x+a^2+1)*a^2-12/b^5*arctan(1/2*(2*
b^2*x+2*a*b)/b)*a^2+2/b^5*arctan(1/2*(2*b^2*x+2*a*b)/b)
```

Maxima [B] time = 1.49865, size = 203, normalized size = 2.21

$$-\frac{6\,b^4x^5 - 15i\,b^3x^4 + 20\,(i\,a-1)b^2x^3 - \left(30i\,a^2 - 60\,a - 30i\right)bx^2 - \left(-60i\,a^3 + 180\,a^2 + 180i\,a - 60\right)x}{30\,b^4} + \frac{\left(2\,a^4 + 8i\,a^3 - 1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x, algorithm="maxima")
```

```
[Out] -1/30*(6*b^4*x^5 - 15*I*b^3*x^4 + 20*(I*a - 1)*b^2*x^3 - (30*I*a^2 - 60*a -
 30*I)*b*x^2 - (-60*I*a^3 + 180*a^2 + 180*I*a - 60)*x)/b^4 + (2*a^4 + 8*I*a
^3 - 12*a^2 - 8*I*a + 2)*arctan((b^2*x + a*b)/b)/b^5 + (I*a^4 - 4*a^3 - 6*I
*a^2 + 4*a + I)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^5
```

Fricas [A] time = 1.57646, size = 284, normalized size = 3.09

$$-\frac{6\,b^5x^5 - 15i\,b^4x^4 + 20\,(i\,a-1)b^3x^3 - \left(30i\,a^2 - 60\,a - 30i\right)b^2x^2 - \left(-60i\,a^3 + 180\,a^2 + 180i\,a - 60\right)bx - \left(60i\,a^4 - 240\,a\right.}{30\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x, algorithm="fricas")
```

```
[Out] -1/30*(6*b^5*x^5 - 15*I*b^4*x^4 + 20*(I*a - 1)*b^3*x^3 - (30*I*a^2 - 60*a -
 30*I)*b^2*x^2 - (-60*I*a^3 + 180*a^2 + 180*I*a - 60)*b*x - (60*I*a^4 - 240
*a^3 - 360*I*a^2 + 240*a + 60*I)*log((b*x + a + I)/b))/b^5
```

Sympy [B] time = 13.4752, size = 2315, normalized size = 25.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)*x**4,x)
```

```
[Out] -x**5/5 + x**4*(I*a**8 - 8*a**7 - 28*I*a**6 + 56*a**5 + 70*I*a**4 - 56*a**3
 - 28*I*a**2 + 8*a + I)/(2*a**8*b + 16*I*a**7*b - 56*a**6*b - 112*I*a**5*b
+ 140*a**4*b + 112*I*a**3*b - 56*a**2*b - 16*I*a*b + 2*b) - x**3*(2*I*a**17
 - 34*a**16 - 272*I*a**15 + 1360*a**14 + 4760*I*a**13 - 12376*a**12 - 24752
*I*a**11 + 38896*a**10 + 48620*I*a**9 - 48620*a**8 - 38896*I*a**7 + 24752*a
**6 + 12376*I*a**5 - 4760*a**4 - 1360*I*a**3 + 272*a**2 + 34*I*a - 2)/(3*a*
*16*b**2 + 48*I*a**15*b**2 - 360*a**14*b**2 - 1680*I*a**13*b**2 + 5460*a**1
2*b**2 + 13104*I*a**11*b**2 - 24024*a**10*b**2 - 34320*I*a**9*b**2 + 38610*
a**8*b**2 + 34320*I*a**7*b**2 - 24024*a**6*b**2 - 13104*I*a**5*b**2 + 5460*
a**4*b**2 + 1680*I*a**3*b**2 - 360*a**2*b**2 - 48*I*a*b**2 + 3*b**2) + x**2
*(I*a**26 - 26*a**25 - 325*I*a**24 + 2600*a**23 + 14950*I*a**22 - 65780*a**
21 - 230230*I*a**20 + 657800*a**19 + 1562275*I*a**18 - 3124550*a**17 - 5311
735*I*a**16 + 7726160*a**15 + 9657700*I*a**14 - 10400600*a**13 - 9657700*I*
a**12 + 7726160*a**11 + 5311735*I*a**10 - 3124550*a**9 - 1562275*I*a**8 + 6
57800*a**7 + 230230*I*a**6 - 65780*a**5 - 14950*I*a**4 + 2600*a**3 + 325*I*
a**2 - 26*a - I)/(a**24*b**3 + 24*I*a**23*b**3 - 276*a**22*b**3 - 2024*I*a*
*21*b**3 + 10626*a**20*b**3 + 42504*I*a**19*b**3 - 134596*a**18*b**3 - 3461
04*I*a**17*b**3 + 735471*a**16*b**3 + 1307504*I*a**15*b**3 - 1961256*a**14*
b**3 - 2496144*I*a**13*b**3 + 2704156*a**12*b**3 + 2496144*I*a**11*b**3 - 1
961256*a**10*b**3 - 1307504*I*a**9*b**3 + 735471*a**8*b**3 + 346104*I*a**7*
b**3 - 134596*a**6*b**3 - 42504*I*a**5*b**3 + 10626*a**4*b**3 + 2024*I*a**3
*b**3 - 276*a**2*b**3 - 24*I*a*b**3 + b**3) - x*(2*I*a**35 - 70*a**34 - 119
0*I*a**33 + 13090*a**32 + 104720*I*a**31 - 649264*a**30 - 3246320*I*a**29 +
 13449040*a**28 + 47071640*I*a**27 - 141214920*a**26 - 367158792*I*a**25 +
834451800*a**24 + 1668903600*I*a**23 - 2952675600*a**22 - 4639918800*I*a**2
1 + 6495886320*a**20 + 8119857900*I*a**19 - 9075135300*a**18 - 9075135300*I
*a**17 + 8119857900*a**16 + 6495886320*I*a**15 - 4639918800*a**14 - 2952675
600*I*a**13 + 1668903600*a**12 + 834451800*I*a**11 - 367158792*a**10 - 1412
14920*I*a**9 + 47071640*a**8 + 13449040*I*a**7 - 3246320*a**6 - 649264*I*a*
*5 + 104720*a**4 + 13090*I*a**3 - 1190*a**2 - 70*I*a + 2)/(a**32*b**4 + 32*
I*a**31*b**4 - 496*a**30*b**4 - 4960*I*a**29*b**4 + 35960*a**28*b**4 + 2013
76*I*a**27*b**4 - 906192*a**26*b**4 - 3365856*I*a**25*b**4 + 10518300*a**24
*b**4 + 28048800*I*a**23*b**4 - 64512240*a**22*b**4 - 129024480*I*a**21*b**
4 + 225792840*a**20*b**4 + 347373600*I*a**19*b**4 - 471435600*a**18*b**4 -
565722720*I*a**17*b**4 + 601080390*a**16*b**4 + 565722720*I*a**15*b**4 - 47
1435600*a**14*b**4 - 347373600*I*a**13*b**4 + 225792840*a**12*b**4 + 129024
480*I*a**11*b**4 - 64512240*a**10*b**4 - 28048800*I*a**9*b**4 + 10518300*a*
*8*b**4 + 3365856*I*a**7*b**4 - 906192*a**6*b**4 - 201376*I*a**5*b**4 + 359
60*a**4*b**4 + 4960*I*a**3*b**4 - 496*a**2*b**4 - 32*I*a*b**4 + b**4) + 2*(
I*a**44 - 44*a**43 - 946*I*a**42 + 13244*a**41 + 135751*I*a**40 - 1086008*a
**39 - 7059052*I*a**38 + 38320568*a**37 + 177232627*I*a**36 - 708930508*a**
35 - 2481256778*I*a**34 + 7669339132*a**33 + 21090682613*I*a**32 - 51915526
432*a**31 - 114955808528*I*a**30 + 229911617056*a**29 + 416714805914*I*a**2
8 - 686353797976*a**27 - 1029530696964*I*a**26 + 1408831480056*a**25 + 1761
039350070*I*a**24 - 2012616400080*a**23 - 2104098963720*I*a**22 + 201261640
0080*a**21 + 1761039350070*I*a**20 - 1408831480056*a**19 - 1029530696964*I*
a**18 + 686353797976*a**17 + 416714805914*I*a**16 - 229911617056*a**15 - 11
4955808528*I*a**14 + 51915526432*a**13 + 21090682613*I*a**12 - 7669339132*a
**11 - 2481256778*I*a**10 + 708930508*a**9 + 177232627*I*a**8 - 38320568*a*
*7 - 7059052*I*a**6 + 1086008*a**5 + 135751*I*a**4 - 13244*a**3 - 946*I*a**
2 + 44*a + I)*log(-a**41 - 41*I*a**40 + 820*a**39 + 10660*I*a**38 - 101270*
```

```
a**37 - 749398*I*a**36 + 4496388*a**35 + 22481940*I*a**34 - 95548245*a**33
- 350343565*I*a**32 + 1121099408*a**31 + 3159461968*I*a**30 - 7898654920*a*
*29 - 17620076360*I*a**28 + 35240152720*a**27 + 63432274896*I*a**26 - 10307
7446706*a**25 - 151584480450*I*a**24 + 202112640600*a**23 + 244662670200*I*
a**22 - 269128937220*a**21 - 269128937220*I*a**20 + 244662670200*a**19 + 20
2112640600*I*a**18 - 151584480450*a**17 - 103077446706*I*a**16 + 6343227489
6*a**15 + 35240152720*I*a**14 - 17620076360*a**13 - 7898654920*I*a**12 + 31
59461968*a**11 + 1121099408*I*a**10 - 350343565*a**9 - 95548245*I*a**8 + 22
481940*a**7 + 4496388*I*a**6 - 749398*a**5 - 101270*I*a**4 + 10660*a**3 + 8
20*I*a**2 - 41*a + x*(-a**40*b - 40*I*a**39*b + 780*a**38*b + 9880*I*a**37*
b - 91390*a**36*b - 658008*I*a**35*b + 3838380*a**34*b + 18643560*I*a**33*b
 - 76904685*a**32*b - 273438880*I*a**31*b + 847660528*a**30*b + 2311801440*
I*a**29*b - 5586853480*a**28*b - 12033222880*I*a**27*b + 23206929840*a**26*
b + 40225345056*I*a**25*b - 62852101650*a**24*b - 88732378800*I*a**23*b + 1
13380261800*a**22*b + 131282408400*I*a**21*b - 137846528820*a**20*b - 13128
2408400*I*a**19*b + 113380261800*a**18*b + 88732378800*I*a**17*b - 62852101
650*a**16*b - 40225345056*I*a**15*b + 23206929840*a**14*b + 12033222880*I*a
**13*b - 5586853480*a**12*b - 2311801440*I*a**11*b + 847660528*a**10*b + 27
3438880*I*a**9*b - 76904685*a**8*b - 18643560*I*a**7*b + 3838380*a**6*b + 6
58008*I*a**5*b - 91390*a**4*b - 9880*I*a**3*b + 780*a**2*b + 40*I*a*b - b)
- I)/(b**5*(a**40 + 40*I*a**39 - 780*a**38 - 9880*I*a**37 + 91390*a**36 + 6
58008*I*a**35 - 3838380*a**34 - 18643560*I*a**33 + 76904685*a**32 + 2734388
80*I*a**31 - 847660528*a**30 - 2311801440*I*a**29 + 5586853480*a**28 + 1203
3222880*I*a**27 - 23206929840*a**26 - 40225345056*I*a**25 + 62852101650*a**
24 + 88732378800*I*a**23 - 113380261800*a**22 - 131282408400*I*a**21 + 1378
46528820*a**20 + 131282408400*I*a**19 - 113380261800*a**18 - 88732378800*I*
a**17 + 62852101650*a**16 + 40225345056*I*a**15 - 23206929840*a**14 - 12033
222880*I*a**13 + 5586853480*a**12 + 2311801440*I*a**11 - 847660528*a**10 -
273438880*I*a**9 + 76904685*a**8 + 18643560*I*a**7 - 3838380*a**6 - 658008*
I*a**5 + 91390*a**4 + 9880*I*a**3 - 780*a**2 - 40*I*a + 1))
```

Giac [A] time = 1.11349, size = 176, normalized size = 1.91

$$\frac{2\left(a^4i - 4a^3 - 6a^2i + 4a + i\right)\log\left(bx + a + i\right)}{b^5} - \frac{6b^5x^5 - 15b^4ix^4 + 20ab^3ix^3 - 30a^2b^2ix^2 + 60a^3bix - 20b^3x^3 + 60a}{30b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x, algorithm="giac")
```

```
[Out] 2*(a^4*i - 4*a^3 - 6*a^2*i + 4*a + i)*log(b*x + a + i)/b^5 - 1/30*(6*b^5*x^
5 - 15*b^4*i*x^4 + 20*a*b^3*i*x^3 - 30*a^2*b^2*i*x^2 + 60*a^3*b*i*x - 20*b^
3*x^3 + 60*a*b^2*x^2 + 30*b^2*i*x^2 - 180*a^2*b*x - 180*a*b*i*x + 60*b*x)/b
^5
```

3.172 $\int e^{2i\tan^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=72

$$\frac{(1-ia)x^2}{b^2}+\frac{2i(a+i)^2x}{b^3}-\frac{2(1-ia)^3\log(a+bx+i)}{b^4}+\frac{2ix^3}{3b}-\frac{x^4}{4}$$

```
[Out] ((2*I)*(I + a)^2*x)/b^3 + ((1 - I*a)*x^2)/b^2 + (((2*I)/3)*x^3)/b - x^4/4 -
 (2*(1 - I*a)^3*Log[I + a + b*x])/b^4
```

Rubi [A] time = 0.0586489, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$\frac{(1-ia)x^2}{b^2}+\frac{2i(a+i)^2x}{b^3}-\frac{2(1-ia)^3\log(a+bx+i)}{b^4}+\frac{2ix^3}{3b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x])*x^3,x]
```

```
[Out] ((2*I)*(I + a)^2*x)/b^3 + ((1 - I*a)*x^2)/b^2 + (((2*I)/3)*x^3)/b - x^4/4 -
 (2*(1 - I*a)^3*Log[I + a + b*x])/b^4
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{2i\tan^{-1}(a+bx)}x^3\,dx &= \int \frac{x^3(1+ia+ibx)}{1-ia-ibx}\,dx\\ &= \int \left(\frac{2i(i+a)^2}{b^3}+\frac{2(1-ia)x}{b^2}+\frac{2ix^2}{b}-x^3+\frac{2(-1+ia)^3}{b^3(i+a+bx)}\right)dx\\ &= \frac{2i(i+a)^2x}{b^3}+\frac{(1-ia)x^2}{b^2}+\frac{2ix^3}{3b}-\frac{x^4}{4}-\frac{2(1-ia)^3\log(i+a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.0575975, size = 72, normalized size = 1.

$$\frac{(1-ia)x^2}{b^2}+\frac{2i(a+i)^2x}{b^3}-\frac{2(1-ia)^3\log(a+bx+i)}{b^4}+\frac{2ix^3}{3b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])*x^3,x]
```

```
[Out] ((2*I)*(I + a)^2*x)/b^3 + ((1 - I*a)*x^2)/b^2 + (((2*I)/3)*x^3)/b - x^4/4 -
 (2*(1 - I*a)^3*Log[I + a + b*x])/b^4
```

Maple [B] time = 0.038, size = 255, normalized size = 3.5

$$-\frac{x^4}{4}+\frac{\frac{2i}{3}x^3}{b}-\frac{ix^2a}{b^2}+\frac{2\,ia^2x}{b^3}+\frac{x^2}{b^2}-\frac{2\,ix}{b^3}-4\,\frac{ax}{b^3}-\frac{i\ln\left(b^2x^2+2\,xab+a^2+1\right)a^3}{b^4}+\frac{3\,i\ln\left(b^2x^2+2\,xab+a^2+1\right)a}{b^4}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x)
```

```
[Out] -1/4*x^4+2/3*I*x^3/b-I/b^2*x^2*a+2*I/b^3*a^2*x+1/b^2*x^2-2*I/b^3*x-4/b^3*a*
x-I/b^4*ln(b^2*x^2+2*a*b*x+a^2+1)*a^3+3*I/b^4*ln(b^2*x^2+2*a*b*x+a^2+1)*a+3
/b^4*ln(b^2*x^2+2*a*b*x+a^2+1)*a^2-1/b^4*ln(b^2*x^2+2*a*b*x+a^2+1)-6*I/b^4*
arctan(1/2*(2*b^2*x+2*a*b)/b)*a^2-2/b^4*arctan(1/2*(2*b^2*x+2*a*b)/b)*a^3+2
*I/b^4*arctan(1/2*(2*b^2*x+2*a*b)/b)+6/b^4*arctan(1/2*(2*b^2*x+2*a*b)/b)*a
```

Maxima [B] time = 1.45958, size = 159, normalized size = 2.21

$$-\frac{3\,b^3x^4-8i\,b^2x^3+12\,(i\,a-1)bx^2-\left(24i\,a^2-48\,a-24i\right)x}{12\,b^3}-\frac{\left(2\,a^3+6i\,a^2-6\,a-2i\right)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^4}+\frac{\left(-i\,a^3+3\,a^2\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^3*x^4 - 8*I*b^2*x^3 + 12*(I*a - 1)*b*x^2 - (24*I*a^2 - 48*a - 24
*I)*x)/b^3 - (2*a^3 + 6*I*a^2 - 6*a - 2*I)*arctan((b^2*x + a*b)/b)/b^4 + (-
I*a^3 + 3*a^2 + 3*I*a - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4
```

Fricas [A] time = 1.55231, size = 201, normalized size = 2.79

$$-\frac{3\,b^4x^4-8i\,b^3x^3+12\,(i\,a-1)b^2x^2-\left(24i\,a^2-48\,a-24i\right)bx-\left(-24i\,a^3+72\,a^2+72i\,a-24\right)\log\left(\frac{bx+a+i}{b}\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^4*x^4 - 8*I*b^3*x^3 + 12*(I*a - 1)*b^2*x^2 - (24*I*a^2 - 48*a -
24*I)*b*x - (-24*I*a^3 + 72*a^2 + 72*I*a - 24)*log((b*x + a + I)/b))/b^4
```

Sympy [B] time = 6.43326, size = 1212, normalized size = 16.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)*x**3,x)
```

```
[Out] -x**4/4 + x**3*(2*I*a**6 - 12*a**5 - 30*I*a**4 + 40*a**3 + 30*I*a**2 - 12*a
 - 2*I)/(3*a**6*b + 18*I*a**5*b - 45*a**4*b - 60*I*a**3*b + 45*a**2*b + 18*
I*a*b - 3*b) - x**2*(I*a**13 - 13*a**12 - 78*I*a**11 + 286*a**10 + 715*I*a*
*9 - 1287*a**8 - 1716*I*a**7 + 1716*a**6 + 1287*I*a**5 - 715*a**4 - 286*I*a
**3 + 78*a**2 + 13*I*a - 1)/(a**12*b**2 + 12*I*a**11*b**2 - 66*a**10*b**2 -
 220*I*a**9*b**2 + 495*a**8*b**2 + 792*I*a**7*b**2 - 924*a**6*b**2 - 792*I*
a**5*b**2 + 495*a**4*b**2 + 220*I*a**3*b**2 - 66*a**2*b**2 - 12*I*a*b**2 +
b**2) + x*(2*I*a**20 - 40*a**19 - 380*I*a**18 + 2280*a**17 + 9690*I*a**16 -
 31008*a**15 - 77520*I*a**14 + 155040*a**13 + 251940*I*a**12 - 335920*a**11
 - 369512*I*a**10 + 335920*a**9 + 251940*I*a**8 - 155040*a**7 - 77520*I*a**
6 + 31008*a**5 + 9690*I*a**4 - 2280*a**3 - 380*I*a**2 + 40*a + 2*I)/(a**18*
b**3 + 18*I*a**17*b**3 - 153*a**16*b**3 - 816*I*a**15*b**3 + 3060*a**14*b**
3 + 8568*I*a**13*b**3 - 18564*a**12*b**3 - 31824*I*a**11*b**3 + 43758*a**10
*b**3 + 48620*I*a**9*b**3 - 43758*a**8*b**3 - 31824*I*a**7*b**3 + 18564*a**
6*b**3 + 8568*I*a**5*b**3 - 3060*a**4*b**3 - 816*I*a**3*b**3 + 153*a**2*b**
3 + 18*I*a*b**3 - b**3) + 2*(-I*a**27 + 27*a**26 + 351*I*a**25 - 2925*a**24
 - 17550*I*a**23 + 80730*a**22 + 296010*I*a**21 - 888030*a**20 - 2220075*I*
a**19 + 4686825*a**18 + 8436285*I*a**17 - 13037895*a**16 - 17383860*I*a**15
 + 20058300*a**14 + 20058300*I*a**13 - 17383860*a**12 - 13037895*I*a**11 +
8436285*a**10 + 4686825*I*a**9 - 2220075*a**8 - 888030*I*a**7 + 296010*a**6
 + 80730*I*a**5 - 17550*a**4 - 2925*I*a**3 + 351*a**2 + 27*I*a - 1)*log(-a*
*25 - 25*I*a**24 + 300*a**23 + 2300*I*a**22 - 12650*a**21 - 53130*I*a**20 +
 177100*a**19 + 480700*I*a**18 - 1081575*a**17 - 2042975*I*a**16 + 3268760*
a**15 + 4457400*I*a**14 - 5200300*a**13 - 5200300*I*a**12 + 4457400*a**11 +
 3268760*I*a**10 - 2042975*a**9 - 1081575*I*a**8 + 480700*a**7 + 177100*I*a
**6 - 53130*a**5 - 12650*I*a**4 + 2300*a**3 + 300*I*a**2 - 25*a + x*(-a**24
*b - 24*I*a**23*b + 276*a**22*b + 2024*I*a**21*b - 10626*a**20*b - 42504*I*
a**19*b + 134596*a**18*b + 346104*I*a**17*b - 735471*a**16*b - 1307504*I*a*
*15*b + 1961256*a**14*b + 2496144*I*a**13*b - 2704156*a**12*b - 2496144*I*a
**11*b + 1961256*a**10*b + 1307504*I*a**9*b - 735471*a**8*b - 346104*I*a**7
*b + 134596*a**6*b + 42504*I*a**5*b - 10626*a**4*b - 2024*I*a**3*b + 276*a*
*2*b + 24*I*a*b - b) - I)/(b**4*(a**24 + 24*I*a**23 - 276*a**22 - 2024*I*a*
*21 + 10626*a**20 + 42504*I*a**19 - 134596*a**18 - 346104*I*a**17 + 735471*
a**16 + 1307504*I*a**15 - 1961256*a**14 - 2496144*I*a**13 + 2704156*a**12 +
 2496144*I*a**11 - 1961256*a**10 - 1307504*I*a**9 + 735471*a**8 + 346104*I*
a**7 - 134596*a**6 - 42504*I*a**5 + 10626*a**4 + 2024*I*a**3 - 276*a**2 - 2
4*I*a + 1))
```

Giac [A] time = 1.09609, size = 119, normalized size = 1.65

$$-\frac{2\left(a^3 i - 3\,a^2 - 3\,ai + 1\right)\log\left(bx + a + i\right)}{b^4} - \frac{3\,b^4x^4 - 8\,b^3ix^3 + 12\,ab^2ix^2 - 24\,a^2bix - 12\,b^2x^2 + 48\,abx + 24\,bix}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x, algorithm="giac")
```

```
[Out] -2*(a^3*i - 3*a^2 - 3*a*i + 1)*log(b*x + a + i)/b^4 - 1/12*(3*b^4*x^4 - 8*b
^3*i*x^3 + 12*a*b^2*i*x^2 - 24*a^2*b*i*x - 12*b^2*x^2 + 48*a*b*x + 24*b*i*x
)/b^4
```

3.173 $\int e^{2i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=54

$$\frac{2(1-ia)x}{b^2}+\frac{2i(a+i)^2\log(a+bx+i)}{b^3}+\frac{ix^2}{b}-\frac{x^3}{3}$$

```
[Out] (2*(1 - I*a)*x)/b^2 + (I*x^2)/b - x^3/3 + ((2*I)*(I + a)^2*Log[I + a + b*x]
)/b^3
```

Rubi [A] time = 0.0478082, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$\frac{2(1-ia)x}{b^2}+\frac{2i(a+i)^2\log(a+bx+i)}{b^3}+\frac{ix^2}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (2*(1 - I*a)*x)/b^2 + (I*x^2)/b - x^3/3 + ((2*I)*(I + a)^2*Log[I + a + b*x]
)/b^3
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{2i\tan^{-1}(a+bx)}x^2\,dx&=\int\frac{x^2(1+ia+ibx)}{1-ia-ibx}\,dx\\&=\int\left(-\frac{2i(i+a)}{b^2}+\frac{2ix}{b}-x^2+\frac{2i(i+a)^2}{b^2(i+a+bx)}\right)dx\\&=\frac{2(1-ia)x}{b^2}+\frac{ix^2}{b}-\frac{x^3}{3}+\frac{2i(i+a)^2\log(i+a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0318999, size = 54, normalized size = 1.

$$\frac{2(1-ia)x}{b^2}+\frac{2i(a+i)^2\log(a+bx+i)}{b^3}+\frac{ix^2}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (2*(1 - I*a)*x)/b^2 + (I*x^2)/b - x^3/3 + ((2*I)*(I + a)^2*Log[I + a + b*x]
)/b^3
```

Maple [B] time = 0.037, size = 176, normalized size = 3.3

$$-\frac{x^3}{3}+\frac{ix^2}{b}-\frac{2\,iax}{b^2}+2\,\frac{x}{b^2}+\frac{i\ln\left(b^2x^2+2\,xab+a^2+1\right)a^2}{b^3}-\frac{i\ln\left(b^2x^2+2\,xab+a^2+1\right)}{b^3}-2\,\frac{\ln\left(b^2x^2+2\,xab+a^2+1\right)a}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x)
```

```
[Out] -1/3*x^3+I*x^2/b-2*I/b^2*a*x+2/b^2*x+I/b^3*ln(b^2*x^2+2*a*b*x+a^2+1)*a^2-I/
b^3*ln(b^2*x^2+2*a*b*x+a^2+1)-2/b^3*ln(b^2*x^2+2*a*b*x+a^2+1)*a+4*I/b^3*arc
tan(1/2*(2*b^2*x+2*a*b)/b)*a+2/b^3*arctan(1/2*(2*b^2*x+2*a*b)/b)*a^2-2/b^3*
arctan(1/2*(2*b^2*x+2*a*b)/b)
```

Maxima [B] time = 1.50306, size = 117, normalized size = 2.17

$$-\frac{b^2x^3-3i\,bx^2+6\,(i\,a-1)x}{3\,b^2}+\frac{2\left(a^2+2i\,a-1\right)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^3}+\frac{\left(i\,a^2-2\,a-i\right)\log\left(b^2x^2+2\,abx+a^2+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b^2*x^3 - 3*I*b*x^2 + 6*(I*a - 1)*x)/b^2 + 2*(a^2 + 2*I*a - 1)*arctan
((b^2*x + a*b)/b)/b^3 + (I*a^2 - 2*a - I)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/
b^3
```

Fricas [A] time = 1.61974, size = 132, normalized size = 2.44

$$-\frac{b^3x^3-3i\,b^2x^2+6\,(i\,a-1)bx-\left(6i\,a^2-12\,a-6i\right)\log\left(\frac{bx+a+i}{b}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 - 3*I*b^2*x^2 + 6*(I*a - 1)*b*x - (6*I*a^2 - 12*a - 6*I)*log(
(b*x + a + I)/b))/b^3
```

Sympy [B] time = 2.6935, size = 513, normalized size = 9.5

$$-\frac{x^3}{3}+\frac{x^2\left(ia^4-4a^3-6ia^2+4a+i\right)}{a^4b+4ia^3b-6a^2b-4iab+b}-\frac{x\left(2ia^9-18a^8-72ia^7+168a^6+252ia^5-252a^4-168ia^3+72a^2+18ia-2\right)}{a^8b^2+8ia^7b^2-28a^6b^2-56ia^5b^2+70a^4b^2+56ia^3b^2-28a^2b^2-8iab^2+b^2}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)*x**2,x)
```

```
[Out] -x**3/3 + x**2*(I*a**4 - 4*a**3 - 6*I*a**2 + 4*a + I)/(a**4*b + 4*I*a**3*b
- 6*a**2*b - 4*I*a*b + b) - x*(2*I*a**9 - 18*a**8 - 72*I*a**7 + 168*a**6 +
252*I*a**5 - 252*a**4 - 168*I*a**3 + 72*a**2 + 18*I*a - 2)/(a**8*b**2 + 8*I
*a**7*b**2 - 28*a**6*b**2 - 56*I*a**5*b**2 + 70*a**4*b**2 + 56*I*a**3*b**2
- 28*a**2*b**2 - 8*I*a*b**2 + b**2) + 2*(I*a**14 - 14*a**13 - 91*I*a**12 +
364*a**11 + 1001*I*a**10 - 2002*a**9 - 3003*I*a**8 + 3432*a**7 + 3003*I*a**
6 - 2002*a**5 - 1001*I*a**4 + 364*a**3 + 91*I*a**2 - 14*a - I)*log(-a**13 -
 13*I*a**12 + 78*a**11 + 286*I*a**10 - 715*a**9 - 1287*I*a**8 + 1716*a**7 +
 1716*I*a**6 - 1287*a**5 - 715*I*a**4 + 286*a**3 + 78*I*a**2 - 13*a + x*(-a
**12*b - 12*I*a**11*b + 66*a**10*b + 220*I*a**9*b - 495*a**8*b - 792*I*a**7
*b + 924*a**6*b + 792*I*a**5*b - 495*a**4*b - 220*I*a**3*b + 66*a**2*b + 12
*I*a*b - b) - I)/(b**3*(a**12 + 12*I*a**11 - 66*a**10 - 220*I*a**9 + 495*a*
*8 + 792*I*a**7 - 924*a**6 - 792*I*a**5 + 495*a**4 + 220*I*a**3 - 66*a**2 -
 12*I*a + 1))
```

Giac [A] time = 1.09059, size = 77, normalized size = 1.43

$$\frac{2\left(a^2 i - 2\,a - i\right)\log\left(bx + a + i\right)}{b^3} - \frac{b^3x^3 - 3\,b^2ix^2 + 6\,abix - 6\,bx}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*i - 2*a - i)*log(b*x + a + i)/b^3 - 1/3*(b^3*x^3 - 3*b^2*i*x^2 + 6*a
*b*i*x - 6*b*x)/b^3
```

3.174 $\int e^{2i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=37

$$\frac{2(1-ia)\log(a+bx+i)}{b^2}+\frac{2ix}{b}-\frac{x^2}{2}$$

```
[Out] ((2*I)*x)/b - x^2/2 + (2*(1 - I*a)*Log[I + a + b*x])/b^2
```

Rubi [A] time = 0.0300955, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5095, 77}

$$\frac{2(1-ia)\log(a+bx+i)}{b^2}+\frac{2ix}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x])*x,x]
```

```
[Out] ((2*I)*x)/b - x^2/2 + (2*(1 - I*a)*Log[I + a + b*x])/b^2
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{2i\tan^{-1}(a+bx)}x\,dx &= \int \frac{x(1+ia+ibx)}{1-ia-ibx}\,dx\\ &= \int \left(\frac{2i}{b}-x+\frac{2(1-ia)}{b(i+a+bx)}\right)dx\\ &= \frac{2ix}{b}-\frac{x^2}{2}+\frac{2(1-ia)\log(i+a+bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0192281, size = 37, normalized size = 1.

$$\frac{2(1-ia)\log(a+bx+i)}{b^2}+\frac{2ix}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])*x,x]
```

```
[Out] ((2*I)*x)/b - x^2/2 + (2*(1 - I*a)*Log[I + a + b*x])/b^2
```

Maple [B] time = 0.038, size = 107, normalized size = 2.9

$$-\frac{x^2}{2}+\frac{2\,ix}{b}-\frac{i\ln\left(b^2x^2+2\,xab+a^2+1\right)a}{b^2}+\frac{\ln\left(b^2x^2+2\,xab+a^2+1\right)}{b^2}-\frac{2\,i}{b^2}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\right)-2\,\frac{a}{b^2}\arctan\left(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x)
```

```
[Out] -1/2*x^2+2*I*x/b-I/b^2*ln(b^2*x^2+2*a*b*x+a^2+1)*a+1/b^2*ln(b^2*x^2+2*a*b*x
+a^2+1)-2*I/b^2*arctan(1/2*(2*b^2*x+2*a*b)/b)-2/b^2*arctan(1/2*(2*b^2*x+2*a
*b)/b)*a
```

Maxima [B] time = 1.46228, size = 89, normalized size = 2.41

$$-\frac{bx^2-4i\,x}{2\,b}-\frac{(2\,a+2i)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^2}+\frac{(-i\,a+1)\log\left(b^2x^2+2\,abx+a^2+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 - 4*I*x)/b - (2*a + 2*I)*arctan((b^2*x + a*b)/b)/b^2 + (-I*a +
1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2
```

Fricas [A] time = 1.87842, size = 88, normalized size = 2.38

$$-\frac{b^2x^2-4i\,bx+4\,(i\,a-1)\log\left(\frac{bx+a+i}{b}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - 4*I*b*x + 4*(I*a - 1)*log((b*x + a + I)/b))/b^2
```

Sympy [B] time = 1.0342, size = 148, normalized size = 4.

$$-\frac{x^2}{2}+\frac{x\left(2ia^2-4a-2i\right)}{a^2b+2iab-b}+\frac{2\left(-ia^5+5a^4+10ia^3-10a^2-5ia+1\right)\log\left(-a^5-5ia^4+10a^3+10ia^2-5a+x\left(-a^4b-4\right.\right.}{b^2\left(a^4+4ia^3-6a^2-4ia+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)*x,x)
```

```
[Out] -x**2/2 + x*(2*I*a**2 - 4*a - 2*I)/(a**2*b + 2*I*a*b - b) + 2*(-I*a**5 + 5*
a**4 + 10*I*a**3 - 10*a**2 - 5*I*a + 1)*log(-a**5 - 5*I*a**4 + 10*a**3 + 10
*I*a**2 - 5*a + x*(-a**4*b - 4*I*a**3*b + 6*a**2*b + 4*I*a*b - b) - I)/(b**
2*(a**4 + 4*I*a**3 - 6*a**2 - 4*I*a + 1))
```

Giac [A] time = 1.09022, size = 49, normalized size = 1.32

$$-\frac{2\,(ai-1)\log\left(bx+a+i\right)}{b^2}-\frac{b^2x^2-4\,bix}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x, algorithm="giac")
```

```
[Out] -2*(a*i - 1)*log(b*x + a + i)/b^2 - 1/2*(b^2*x^2 - 4*b*i*x)/b^2
```

3.175 $\int e^{2i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=20

$$-x+\frac{2i\log(a+bx+i)}{b}$$

```
[Out] -x + ((2*I)*Log[I + a + b*x])/b
```

Rubi [A] time = 0.0118928, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5093, 43}

$$-x+\frac{2i\log(a+bx+i)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] -x + ((2*I)*Log[I + a + b*x])/b
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2i\tan^{-1}(a+bx)}\,dx &= \int \frac{1+ia+ibx}{1-ia-ibx}\,dx\\ &= \int \left(-1+\frac{2i}{i+a+bx}\right)dx\\ &= -x+\frac{2i\log(i+a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0112383, size = 32, normalized size = 1.6

$$\frac{i\log\left((a+bx)^2+1\right)}{b}+\frac{2\tan^{-1}(a+bx)}{b}-x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] -x + (2*ArcTan[a + b*x])/b + (I*Log[1 + (a + b*x)^2])/b
```

Maple [B] time = 0.039, size = 51, normalized size = 2.6

$$-x+\frac{i\ln\left(b^2x^2+2\,xab+a^2+1\right)}{b}+2\,\frac{1}{b}\arctan\left(1/2\,\frac{2\,b^2x+2\,ab}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2),x)
```

```
[Out] -x+I/b*ln(b^2*x^2+2*a*b*x+a^2+1)+2/b*arctan(1/2*(2*b^2*x+2*a*b)/b)
```

Maxima [B] time = 1.45703, size = 62, normalized size = 3.1

$$-x+\frac{2\arctan\left(\frac{b^2x+ab}{b}\right)}{b}+\frac{i\log\left(b^2x^2+2\,abx+a^2+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -x + 2*arctan((b^2*x + a*b)/b)/b + I*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

Fricas [A] time = 1.73152, size = 50, normalized size = 2.5

$$-\frac{bx-2i\log\left(\frac{bx+a+i}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -(b*x - 2*I*log((b*x + a + I)/b))/b
```

Sympy [A] time = 0.372951, size = 14, normalized size = 0.7

$$-x+\frac{2i\log\left(a+bx+i\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2),x)
```

```
[Out] -x + 2*I*log(a + b*x + I)/b
```

Giac [A] time = 1.11232, size = 23, normalized size = 1.15

$$-x + \frac{2\,i\log\left(bx + a + i\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -x + 2*i*log(b*x + a + i)/b
```

3.176 $\int \frac{e^{2i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=38

$$\frac{(-a+i)\log(x)}{a+i}-\frac{2\log(a+bx+i)}{1-ia}$$

```
[Out] ((I - a)*Log[x])/(I + a) - (2*Log[I + a + b*x])/(1 - I*a)
```

Rubi [A] time = 0.0339896, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 72}

$$\frac{(-a+i)\log(x)}{a+i}-\frac{2\log(a+bx+i)}{1-ia}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x])/x,x]
```

```
[Out] ((I - a)*Log[x])/(I + a) - (2*Log[I + a + b*x])/(1 - I*a)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2i\tan^{-1}(a+bx)}}{x}\,dx &= \int \frac{1+ia+ibx}{x(1-ia-ibx)}\,dx\\ &= \int \left(\frac{i-a}{(i+a)x}-\frac{2ib}{(i+a)(i+a+bx)}\right)dx\\ &= \frac{(i-a)\log(x)}{i+a}-\frac{2\log(i+a+bx)}{1-ia}\end{aligned}$$

Mathematica [A] time = 0.0180426, size = 31, normalized size = 0.82

$$-\frac{2i\log(a+bx+i)+(a-i)\log(x)}{a+i}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])/x,x]
```

```
[Out] -(((-I + a)*Log[x] + (2*I)*Log[I + a + b*x])/(I + a))
```

Maple [B] time = 0.043, size = 149, normalized size = 3.9

$$\frac{-i\ln\left(b^2x^2+2\,xab+a^2+1\right)a}{a^2+1}-\frac{\ln\left(b^2x^2+2\,xab+a^2+1\right)}{a^2+1}+\frac{2\,i}{a^2+1}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\right)-2\,\frac{a}{a^2+1}\arctan\left(1/2\,\frac{2\,i}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x)
```

```
[Out] -I/(a^2+1)*ln(b^2*x^2+2*a*b*x+a^2+1)*a-1/(a^2+1)*ln(b^2*x^2+2*a*b*x+a^2+1)+
2*I/(a^2+1)*arctan(1/2*(2*b^2*x+2*a*b)/b)-2/(a^2+1)*arctan(1/2*(2*b^2*x+2*a
*b)/b)*a+2*I/(a^2+1)*ln(x)*a-1/(a^2+1)*ln(x)*a^2+1/(a^2+1)*ln(x)
```

Maxima [B] time = 1.47925, size = 108, normalized size = 2.84

$$-\frac{(2\,a-2i)\arctan\left(\frac{b^2x+ab}{b}\right)}{a^2+1}-\frac{(i\,a+1)\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^2+1}-\frac{\left(a^2-2i\,a-1\right)\log\left(x\right)}{a^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] -(2*a - 2*I)*arctan((b^2*x + a*b)/b)/(a^2 + 1) - (I*a + 1)*log(b^2*x^2 + 2*
a*b*x + a^2 + 1)/(a^2 + 1) - (a^2 - 2*I*a - 1)*log(x)/(a^2 + 1)
```

Fricas [A] time = 1.81157, size = 73, normalized size = 1.92

$$-\frac{(a-i)\log\left(x\right)+2i\,\log\left(\frac{bx+a+i}{b}\right)}{a+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x, algorithm="fricas")
```

```
[Out] -((a - I)*log(x) + 2*I*log((b*x + a + I)/b))/(a + I)
```

Sympy [B] time = 2.54089, size = 1538, normalized size = 40.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)/x,x)
```

```
[Out] (-sqrt((-a**8 + 4*I*a**7 + 4*a**6 + 4*I*a**5 + 10*a**4 - 4*I*a**3 + 4*a**2
- 4*I*a - 1)*(-a**8 + 12*I*a**7 + 60*a**6 - 164*I*a**5 - 270*a**4 + 276*I*a
```

```
**3 + 172*a**2 - 60*I*a - 9))/(2*(a**8 - 4*I*a**7 - 4*a**6 - 4*I*a**5 - 10*
a**4 + 4*I*a**3 - 4*a**2 + 4*I*a + 1)) - 1/2)*log(x + (-sqrt((-a**8 + 4*I*a
**7 + 4*a**6 + 4*I*a**5 + 10*a**4 - 4*I*a**3 + 4*a**2 - 4*I*a - 1)*(-a**8 +
 12*I*a**7 + 60*a**6 - 164*I*a**5 - 270*a**4 + 276*I*a**3 + 172*a**2 - 60*I
*a - 9))/(2*(a**8 - 4*I*a**7 - 4*a**6 - 4*I*a**5 - 10*a**4 + 4*I*a**3 - 4*a
**2 + 4*I*a + 1)) - 1/2)*(a**11 - 9*I*a**10 - 31*a**9 + 47*I*a**8 + 10*a**7
 + 70*I*a**6 + 98*a**5 - 34*I*a**4 + 37*a**3 - 45*I*a**2 - 19*a + 3*I)/(a**
10*b - 14*I*a**9*b - 85*a**8*b + 296*I*a**7*b + 658*a**6*b - 980*I*a**5*b -
 994*a**4*b + 680*I*a**3*b + 301*a**2*b - 78*I*a*b - 9*b) + (a**25 - 29*I*a
**24 - 396*a**23 + 3388*I*a**22 + 20378*a**21 - 91602*I*a**20 - 319116*a**1
9 + 880764*I*a**18 + 1948887*a**17 - 3465467*I*a**16 - 4901848*a**15 + 5325
624*I*a**14 + 3970316*a**13 - 967708*I*a**12 + 2488392*a**11 - 4876008*I*a*
*10 - 5388609*a**9 + 4348701*I*a**8 + 2724068*a**7 - 1346548*I*a**6 - 52378
2*a**5 + 157646*I*a**4 + 35524*a**3 - 5652*I*a**2 - 567*a + 27*I)/(a**24*b
- 32*I*a**23*b - 484*a**22*b + 4608*I*a**21*b + 31026*a**20*b - 157344*I*a*
*19*b - 624948*a**18*b + 1995456*I*a**17*b + 5216127*a**16*b - 11307584*I*a
**15*b - 20514376*a**14*b + 31338752*I*a**13*b + 40461564*a**12*b - 4421740
8*I*a**11*b - 40876296*a**10*b + 31876224*I*a**9*b + 20859663*a**8*b - 1136
1696*I*a**7*b - 5089492*a**6*b + 1842944*I*a**5*b + 526066*a**4*b - 113952*
I*a**3*b - 17604*a**2*b + 1728*I*a*b + 81*b)) + (sqrt((-a**8 + 4*I*a**7 + 4
*a**6 + 4*I*a**5 + 10*a**4 - 4*I*a**3 + 4*a**2 - 4*I*a - 1)*(-a**8 + 12*I*a
**7 + 60*a**6 - 164*I*a**5 - 270*a**4 + 276*I*a**3 + 172*a**2 - 60*I*a - 9)
)/(2*(a**8 - 4*I*a**7 - 4*a**6 - 4*I*a**5 - 10*a**4 + 4*I*a**3 - 4*a**2 + 4
*I*a + 1)) - 1/2)*log(x + (sqrt((-a**8 + 4*I*a**7 + 4*a**6 + 4*I*a**5 + 10*
a**4 - 4*I*a**3 + 4*a**2 - 4*I*a - 1)*(-a**8 + 12*I*a**7 + 60*a**6 - 164*I*
a**5 - 270*a**4 + 276*I*a**3 + 172*a**2 - 60*I*a - 9))/(2*(a**8 - 4*I*a**7
- 4*a**6 - 4*I*a**5 - 10*a**4 + 4*I*a**3 - 4*a**2 + 4*I*a + 1)) - 1/2)*(a**
11 - 9*I*a**10 - 31*a**9 + 47*I*a**8 + 10*a**7 + 70*I*a**6 + 98*a**5 - 34*I
*a**4 + 37*a**3 - 45*I*a**2 - 19*a + 3*I)/(a**10*b - 14*I*a**9*b - 85*a**8*
b + 296*I*a**7*b + 658*a**6*b - 980*I*a**5*b - 994*a**4*b + 680*I*a**3*b +
301*a**2*b - 78*I*a*b - 9*b) + (a**25 - 29*I*a**24 - 396*a**23 + 3388*I*a**
22 + 20378*a**21 - 91602*I*a**20 - 319116*a**19 + 880764*I*a**18 + 1948887*
a**17 - 3465467*I*a**16 - 4901848*a**15 + 5325624*I*a**14 + 3970316*a**13 -
 967708*I*a**12 + 2488392*a**11 - 4876008*I*a**10 - 5388609*a**9 + 4348701*
I*a**8 + 2724068*a**7 - 1346548*I*a**6 - 523782*a**5 + 157646*I*a**4 + 3552
4*a**3 - 5652*I*a**2 - 567*a + 27*I)/(a**24*b - 32*I*a**23*b - 484*a**22*b
+ 4608*I*a**21*b + 31026*a**20*b - 157344*I*a**19*b - 624948*a**18*b + 1995
456*I*a**17*b + 5216127*a**16*b - 11307584*I*a**15*b - 20514376*a**14*b + 3
1338752*I*a**13*b + 40461564*a**12*b - 44217408*I*a**11*b - 40876296*a**10*
b + 31876224*I*a**9*b + 20859663*a**8*b - 11361696*I*a**7*b - 5089492*a**6*
b + 1842944*I*a**5*b + 526066*a**4*b - 113952*I*a**3*b - 17604*a**2*b + 172
8*I*a*b + 81*b))
```

Giac [A] time = 1.11587, size = 49, normalized size = 1.29

$$-\frac{2\,bi\log\left(bx+a+i\right)}{ab+bi}-\frac{(a-i)\log\left(|x|\right)}{a+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x, algorithm="giac")
```

```
[Out] -2*b*i*log(b*x + a + i)/(a*b + b*i) - (a - i)*log(abs(x))/(a + i)
```

3.177 $\int \frac{e^{2i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=55

$$-\frac{2ib\log(x)}{(a+i)^2}+\frac{2ib\log(a+bx+i)}{(a+i)^2}-\frac{-a+i}{(a+i)x}$$

```
[Out] -((I - a)/((I + a)*x)) - ((2*I)*b*Log[x])/(I + a)^2 + ((2*I)*b*Log[I + a +
b*x])/(I + a)^2
```

Rubi [A] time = 0.0413953, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$-\frac{2ib\log(x)}{(a+i)^2}+\frac{2ib\log(a+bx+i)}{(a+i)^2}-\frac{-a+i}{(a+i)x}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] -((I - a)/((I + a)*x)) - ((2*I)*b*Log[x])/(I + a)^2 + ((2*I)*b*Log[I + a +
b*x])/(I + a)^2
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2i\tan^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{1+ia+ibx}{x^2(1-ia-ibx)}\,dx\\
&= \int \left(\frac{i-a}{(i+a)x^2}-\frac{2ib}{(i+a)^2x}+\frac{2ib^2}{(i+a)^2(i+a+bx)}\right)dx\\
&= -\frac{i-a}{(i+a)x}-\frac{2ib\log(x)}{(i+a)^2}+\frac{2ib\log(i+a+bx)}{(i+a)^2}
\end{aligned}$$

Mathematica [A] time = 0.0246957, size = 39, normalized size = 0.71

$$\frac{a^2+2ibx\log(a+bx+i)-2ibx\log(x)+1}{(a+i)^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] (1 + a^2 - (2*I)*b*x*Log[x] + (2*I)*b*x*Log[I + a + b*x])/((I + a)^2*x)
```

Maple [B] time = 0.044, size = 260, normalized size = 4.7

$$\frac{ib\ln\left(b^2x^2+2\,xab+a^2+1\right)a^2}{\left(a^2+1\right)^2}-\frac{ib\ln\left(b^2x^2+2\,xab+a^2+1\right)}{\left(a^2+1\right)^2}+2\frac{b\ln\left(b^2x^2+2\,xab+a^2+1\right)a}{\left(a^2+1\right)^2}-\frac{4\,iba}{\left(a^2+1\right)^2}\arctan\left(\frac{2\,b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x)
```

```
[Out] I*b/(a^2+1)^2*ln(b^2*x^2+2*a*b*x+a^2+1)*a^2-I*b/(a^2+1)^2*ln(b^2*x^2+2*a*b*
x+a^2+1)+2*b/(a^2+1)^2*ln(b^2*x^2+2*a*b*x+a^2+1)*a-4*I*b/(a^2+1)^2*arctan(1
/2*(2*b^2*x+2*a*b)/b)*a+2*b/(a^2+1)^2*arctan(1/2*(2*b^2*x+2*a*b)/b)*a^2-2*b
/(a^2+1)^2*arctan(1/2*(2*b^2*x+2*a*b)/b)-2*I/(a^2+1)/x*a+1/(a^2+1)/x*a^2-1/
(a^2+1)/x-2*I*b/(a^2+1)^2*ln(x)*a^2+2*I*b/(a^2+1)^2*ln(x)-4*b/(a^2+1)^2*ln(
x)*a
```

Maxima [B] time = 1.51218, size = 169, normalized size = 3.07

$$\frac{2\left(a^2-2i\,a-1\right)b\arctan\left(\frac{b^2x+ab}{b}\right)}{a^4+2\,a^2+1}+\frac{\left(i\,a^2+2\,a-i\right)b\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^4+2\,a^2+1}+\frac{\left(-2i\,a^2-4\,a+2i\right)b\log\left(x\right)}{a^4+2\,a^2+1}+\frac{a^2-2i\,a}{\left(a^2+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] 2*(a^2 - 2*I*a - 1)*b*arctan((b^2*x + a*b)/b)/(a^4 + 2*a^2 + 1) + (I*a^2 +
2*a - I)*b*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^4 + 2*a^2 + 1) + (-2*I*a^2 -
 4*a + 2*I)*b*log(x)/(a^4 + 2*a^2 + 1) + (a^2 - 2*I*a - 1)/((a^2 + 1)*x)
```

Fricas [A] time = 1.93706, size = 111, normalized size = 2.02

$$\frac{-2i\,bx\log\left(x\right)+2i\,bx\log\left(\frac{bx+a+i}{b}\right)+a^2+1}{\left(a^2+2i\,a-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] (-2*I*b*x*log(x) + 2*I*b*x*log((b*x + a + I)/b) + a^2 + 1)/((a^2 + 2*I*a -
1)*x)
```

Sympy [B] time = 8.1939, size = 3550, normalized size = 64.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)/x**2,x)
```

```
[Out] 2*b*sqrt((-a**23 + 23*I*a**22 + 253*a**21 - 1771*I*a**20 - 8855*a**19 + 336
49*I*a**18 + 100947*a**17 - 245157*I*a**16 - 490314*a**15 + 817190*I*a**14
+ 1144066*a**13 - 1352078*I*a**12 - 1352078*a**11 + 1144066*I*a**10 + 81719
0*a**9 - 490314*I*a**8 - 245157*a**7 + 100947*I*a**6 + 33649*a**5 - 8855*I*
a**4 - 1771*a**3 + 253*I*a**2 + 23*a - I)/(a**27 - 19*I*a**26 - 167*a**25 +
 893*I*a**24 + 3198*a**23 - 7866*I*a**22 - 12650*a**21 + 9614*I*a**20 - 103
73*a**19 + 43263*I*a**18 + 62491*a**17 - 37145*I*a**16 + 29716*a**15 - 8914
8*I*a**14 - 89148*a**13 + 29716*I*a**12 - 37145*a**11 + 62491*I*a**10 + 432
63*a**9 - 10373*I*a**8 + 9614*a**7 - 12650*I*a**6 - 7866*a**5 + 3198*I*a**4
 + 893*a**3 - 167*I*a**2 - 19*a + I))*log(-2*b*sqrt((-a**23 + 23*I*a**22 +
253*a**21 - 1771*I*a**20 - 8855*a**19 + 33649*I*a**18 + 100947*a**17 - 2451
57*I*a**16 - 490314*a**15 + 817190*I*a**14 + 1144066*a**13 - 1352078*I*a**1
2 - 1352078*a**11 + 1144066*I*a**10 + 817190*a**9 - 490314*I*a**8 - 245157*
a**7 + 100947*I*a**6 + 33649*a**5 - 8855*I*a**4 - 1771*a**3 + 253*I*a**2 +
23*a - I)/(a**27 - 19*I*a**26 - 167*a**25 + 893*I*a**24 + 3198*a**23 - 7866
*I*a**22 - 12650*a**21 + 9614*I*a**20 - 10373*a**19 + 43263*I*a**18 + 62491
*a**17 - 37145*I*a**16 + 29716*a**15 - 89148*I*a**14 - 89148*a**13 + 29716*
I*a**12 - 37145*a**11 + 62491*I*a**10 + 43263*a**9 - 10373*I*a**8 + 9614*a*
*7 - 12650*I*a**6 - 7866*a**5 + 3198*I*a**4 + 893*a**3 - 167*I*a**2 - 19*a
+ I))*(I*a**33 + 27*a**32 - 348*I*a**31 - 2844*a**30 + 16500*I*a**29 + 7203
6*a**28 - 244412*I*a**27 - 654588*a**26 + 1384344*I*a**25 + 2262000*a**24 -
 2646540*I*a**23 - 1560780*a**22 - 1560780*I*a**21 - 5882940*a**20 + 900450
0*I*a**19 + 8364180*a**18 - 3421710*I*a**17 + 3421710*a**16 - 8364180*I*a**
15 - 9004500*a**14 + 5882940*I*a**13 + 1560780*a**12 + 1560780*I*a**11 + 26
46540*a**10 - 2262000*I*a**9 - 1384344*a**8 + 654588*I*a**7 + 244412*a**6 -
 72036*I*a**5 - 16500*a**4 + 2844*I*a**3 + 348*a**2 - 27*I*a - 1)/(4*a**30*
b**2 - 120*I*a**29*b**2 - 1740*a**28*b**2 + 16240*I*a**27*b**2 + 109620*a**
26*b**2 - 570024*I*a**25*b**2 - 2375100*a**24*b**2 + 8143200*I*a**23*b**2 +
 23411700*a**22*b**2 - 57228600*I*a**21*b**2 - 120180060*a**20*b**2 + 21850
9200*I*a**19*b**2 + 345972900*a**18*b**2 - 479039400*I*a**17*b**2 - 5816907
00*a**16*b**2 + 620470080*I*a**15*b**2 + 581690700*a**14*b**2 - 479039400*I
*a**13*b**2 - 345972900*a**12*b**2 + 218509200*I*a**11*b**2 + 120180060*a**
10*b**2 - 57228600*I*a**9*b**2 - 23411700*a**8*b**2 + 8143200*I*a**7*b**2 +
 2375100*a**6*b**2 - 570024*I*a**5*b**2 - 109620*a**4*b**2 + 16240*I*a**3*b
**2 + 1740*a**2*b**2 - 120*I*a*b**2 - 4*b**2) + x + (a**50 - 48*I*a**49 - 1
127*a**48 + 17248*I*a**47 + 193452*a**46 - 1695008*I*a**45 - 12076932*a**44
 + 71916768*I*a**43 + 365077482*a**42 - 1603477568*I*a**41 - 6163366902*a**
40 + 20918093728*I*a**39 + 63127818572*a**38 - 170333048928*I*a**37 - 41265
2088772*a**36 + 900331830048*I*a**35 + 1772528290407*a**34 - 3151161405168*
I*a**33 - 5054988087457*a**32 + 7297426411968*I*a**31 + 9425842448792*a**30
 - 10772391370048*I*a**29 - 10649977831752*a**28 + 8643460269248*I*a**27 +
4861946401452*a**26 + 4861946401452*a**24 - 8643460269248*I*a**23 - 1064997
7831752*a**22 + 10772391370048*I*a**21 + 9425842448792*a**20 - 729742641196
8*I*a**19 - 5054988087457*a**18 + 3151161405168*I*a**17 + 1772528290407*a**
16 - 900331830048*I*a**15 - 412652088772*a**14 + 170333048928*I*a**13 + 631
27818572*a**12 - 20918093728*I*a**11 - 6163366902*a**10 + 1603477568*I*a**9
 + 365077482*a**8 - 71916768*I*a**7 - 12076932*a**6 + 1695008*I*a**5 + 1934
52*a**4 - 17248*I*a**3 - 1127*a**2 + 48*I*a + 1)/(2*a**49*b - 98*I*a**48*b
- 2352*a**47*b + 36848*I*a**46*b + 423752*a**45*b - 3813768*I*a**44*b - 279
67632*a**43*b + 171801168*I*a**42*b + 901956132*a**41*b - 4108911268*I*a**4
0*b - 16435645072*a**39*b + 58271832528*I*a**38*b + 184527469672*a**37*b -
```

525193567528*I*a**36*b - 1350497745072*a**35*b + 3151161405168*I*a**34*b +
6696217985982*a**33*b - 12998540796318*I*a**32*b - 23108516971232*a**31*b +
37703369795168*I*a**30*b + 56555054692752*a**29*b - 78099837432848*I*a**28
*b - 99399793096352*a**27*b + 116686713634848*I*a**26*b + 126410606437752*a
25*b - 126410606437752*I*a24*b - 116686713634848*a**23*b + 993997930963
52*I*a**22*b + 78099837432848*a**21*b - 56555054692752*I*a**20*b - 37703369
795168*a**19*b + 23108516971232*I*a**18*b + 12998540796318*a**17*b - 669621
7985982*I*a**16*b - 3151161405168*a**15*b + 1350497745072*I*a**14*b + 52519
3567528*a**13*b - 184527469672*I*a**12*b - 58271832528*a**11*b + 1643564507
2*I*a**10*b + 4108911268*a**9*b - 901956132*I*a**8*b - 171801168*a**7*b + 2
7967632*I*a**6*b + 3813768*a**5*b - 423752*I*a**4*b - 36848*a**3*b + 2352*I
*a**2*b + 98*a*b - 2*I*b)) - 2*b*sqrt((-a**23 + 23*I*a**22 + 253*a**21 - 17
71*I*a**20 - 8855*a**19 + 33649*I*a**18 + 100947*a**17 - 245157*I*a**16 - 4
90314*a**15 + 817190*I*a**14 + 1144066*a**13 - 1352078*I*a**12 - 1352078*a*
*11 + 1144066*I*a**10 + 817190*a**9 - 490314*I*a**8 - 245157*a**7 + 100947*
I*a**6 + 33649*a**5 - 8855*I*a**4 - 1771*a**3 + 253*I*a**2 + 23*a - I)/(a**
27 - 19*I*a**26 - 167*a**25 + 893*I*a**24 + 3198*a**23 - 7866*I*a**22 - 126
50*a**21 + 9614*I*a**20 - 10373*a**19 + 43263*I*a**18 + 62491*a**17 - 37145
*I*a**16 + 29716*a**15 - 89148*I*a**14 - 89148*a**13 + 29716*I*a**12 - 3714
5*a**11 + 62491*I*a**10 + 43263*a**9 - 10373*I*a**8 + 9614*a**7 - 12650*I*a
6 - 7866*a5 + 3198*I*a**4 + 893*a**3 - 167*I*a**2 - 19*a + I))*log(2*b*
sqrt((-a**23 + 23*I*a**22 + 253*a**21 - 1771*I*a**20 - 8855*a**19 + 33649*I
*a**18 + 100947*a**17 - 245157*I*a**16 - 490314*a**15 + 817190*I*a**14 + 11
44066*a**13 - 1352078*I*a**12 - 1352078*a**11 + 1144066*I*a**10 + 817190*a*
*9 - 490314*I*a**8 - 245157*a**7 + 100947*I*a**6 + 33649*a**5 - 8855*I*a**4
 - 1771*a**3 + 253*I*a**2 + 23*a - I)/(a**27 - 19*I*a**26 - 167*a**25 + 893
*I*a**24 + 3198*a**23 - 7866*I*a**22 - 12650*a**21 + 9614*I*a**20 - 10373*a
19 + 43263*I*a18 + 62491*a**17 - 37145*I*a**16 + 29716*a**15 - 89148*I*
a**14 - 89148*a**13 + 29716*I*a**12 - 37145*a**11 + 62491*I*a**10 + 43263*a
9 - 10373*I*a8 + 9614*a**7 - 12650*I*a**6 - 7866*a**5 + 3198*I*a**4 + 8
93*a**3 - 167*I*a**2 - 19*a + I))*(I*a**33 + 27*a**32 - 348*I*a**31 - 2844*
a**30 + 16500*I*a**29 + 72036*a**28 - 244412*I*a**27 - 654588*a**26 + 13843
44*I*a**25 + 2262000*a**24 - 2646540*I*a**23 - 1560780*a**22 - 1560780*I*a*
*21 - 5882940*a**20 + 9004500*I*a**19 + 8364180*a**18 - 3421710*I*a**17 + 3
421710*a**16 - 8364180*I*a**15 - 9004500*a**14 + 5882940*I*a**13 + 1560780*
a**12 + 1560780*I*a**11 + 2646540*a**10 - 2262000*I*a**9 - 1384344*a**8 + 6
54588*I*a**7 + 244412*a**6 - 72036*I*a**5 - 16500*a**4 + 2844*I*a**3 + 348*
a**2 - 27*I*a - 1)/(4*a**30*b**2 - 120*I*a**29*b**2 - 1740*a**28*b**2 + 162
40*I*a**27*b**2 + 109620*a**26*b**2 - 570024*I*a**25*b**2 - 2375100*a**24*b
2 + 8143200*I*a23*b**2 + 23411700*a**22*b**2 - 57228600*I*a**21*b**2 -
120180060*a**20*b**2 + 218509200*I*a**19*b**2 + 345972900*a**18*b**2 - 4790
39400*I*a**17*b**2 - 581690700*a**16*b**2 + 620470080*I*a**15*b**2 + 581690
700*a**14*b**2 - 479039400*I*a**13*b**2 - 345972900*a**12*b**2 + 218509200*
I*a**11*b**2 + 120180060*a**10*b**2 - 57228600*I*a**9*b**2 - 23411700*a**8*
b**2 + 8143200*I*a**7*b**2 + 2375100*a**6*b**2 - 570024*I*a**5*b**2 - 10962
0*a**4*b**2 + 16240*I*a**3*b**2 + 1740*a**2*b**2 - 120*I*a*b**2 - 4*b**2) +
 x + (a**50 - 48*I*a**49 - 1127*a**48 + 17248*I*a**47 + 193452*a**46 - 1695
008*I*a**45 - 12076932*a**44 + 71916768*I*a**43 + 365077482*a**42 - 1603477
568*I*a**41 - 6163366902*a**40 + 20918093728*I*a**39 + 63127818572*a**38 -
170333048928*I*a**37 - 412652088772*a**36 + 900331830048*I*a**35 + 17725282
90407*a**34 - 3151161405168*I*a**33 - 5054988087457*a**32 + 7297426411968*I
*a**31 + 9425842448792*a**30 - 10772391370048*I*a**29 - 10649977831752*a**2
8 + 8643460269248*I*a**27 + 4861946401452*a**26 + 4861946401452*a**24 - 864
3460269248*I*a**23 - 10649977831752*a**22 + 10772391370048*I*a**21 + 942584
2448792*a**20 - 7297426411968*I*a**19 - 5054988087457*a**18 + 3151161405168
*I*a**17 + 1772528290407*a**16 - 900331830048*I*a**15 - 412652088772*a**14
+ 170333048928*I*a**13 + 63127818572*a**12 - 20918093728*I*a**11 - 61633669
02*a**10 + 1603477568*I*a**9 + 365077482*a**8 - 71916768*I*a**7 - 12076932*
a**6 + 1695008*I*a**5 + 193452*a**4 - 17248*I*a**3 - 1127*a**2 + 48*I*a + 1

```
)/(2*a**49*b - 98*I*a**48*b - 2352*a**47*b + 36848*I*a**46*b + 423752*a**45
*b - 3813768*I*a**44*b - 27967632*a**43*b + 171801168*I*a**42*b + 901956132
*a**41*b - 4108911268*I*a**40*b - 16435645072*a**39*b + 58271832528*I*a**38
*b + 184527469672*a**37*b - 525193567528*I*a**36*b - 1350497745072*a**35*b
+ 3151161405168*I*a**34*b + 6696217985982*a**33*b - 12998540796318*I*a**32*
b - 23108516971232*a**31*b + 37703369795168*I*a**30*b + 56555054692752*a**2
9*b - 78099837432848*I*a**28*b - 99399793096352*a**27*b + 116686713634848*I
*a**26*b + 126410606437752*a**25*b - 126410606437752*I*a**24*b - 1166867136
34848*a**23*b + 99399793096352*I*a**22*b + 78099837432848*a**21*b - 5655505
4692752*I*a**20*b - 37703369795168*a**19*b + 23108516971232*I*a**18*b + 129
98540796318*a**17*b - 6696217985982*I*a**16*b - 3151161405168*a**15*b + 135
0497745072*I*a**14*b + 525193567528*a**13*b - 184527469672*I*a**12*b - 5827
1832528*a**11*b + 16435645072*I*a**10*b + 4108911268*a**9*b - 901956132*I*a
**8*b - 171801168*a**7*b + 27967632*I*a**6*b + 3813768*a**5*b - 423752*I*a*
*4*b - 36848*a**3*b + 2352*I*a**2*b + 98*a*b - 2*I*b)) + (-a**5 + 5*I*a**4
+ 10*a**3 - 10*I*a**2 - 5*a + I)/(x*(-a**5 + 3*I*a**4 + 2*a**3 + 2*I*a**2 +
 3*a - I))
```

Giac [A] time = 1.09192, size = 92, normalized size = 1.67

$$-\frac{2\,b^2\log\left(bx+a+i\right)}{a^2bi-2\,ab-bi}+\frac{2\,b\log\left(|x|\right)}{a^2i-2\,a-i}-\frac{\left(a^2i+i\right)i}{(a+i)^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x, algorithm="giac")
```

```
[Out] -2*b^2*log(b*x + a + i)/(a^2*b*i - 2*a*b - b*i) + 2*b*log(abs(x))/(a^2*i -
2*a - i) - (a^2*i + i)*i/((a + i)^2*x)
```

3.178 $\int \frac{e^{2i\tan^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=76

$$-\frac{2b^2\log(x)}{(1-ia)^3}+\frac{2b^2\log(a+bx+i)}{(1-ia)^3}+\frac{2ib}{(a+i)^2x}-\frac{-a+i}{2(a+i)x^2}$$

```
[Out] -(I - a)/(2*(I + a)*x^2) + ((2*I)*b)/((I + a)^2*x) - (2*b^2*Log[x])/(1 - I*
a)^3 + (2*b^2*Log[I + a + b*x])/(1 - I*a)^3
```

Rubi [A] time = 0.0492248, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$-\frac{2b^2\log(x)}{(1-ia)^3}+\frac{2b^2\log(a+bx+i)}{(1-ia)^3}+\frac{2ib}{(a+i)^2x}-\frac{-a+i}{2(a+i)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x])/x^3,x]
```

```
[Out] -(I - a)/(2*(I + a)*x^2) + ((2*I)*b)/((I + a)^2*x) - (2*b^2*Log[x])/(1 - I*
a)^3 + (2*b^2*Log[I + a + b*x])/(1 - I*a)^3
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2i\tan^{-1}(a+bx)}}{x^3}\,dx &= \int \frac{1+ia+ibx}{x^3(1-ia-ibx)}\,dx\\
&= \int \left(\frac{i-a}{(i+a)x^3}-\frac{2ib}{(i+a)^2x^2}+\frac{2ib^2}{(i+a)^3x}-\frac{2ib^3}{(i+a)^3(i+a+bx)}\right)dx\\
&= -\frac{i-a}{2(i+a)x^2}+\frac{2ib}{(i+a)^2x}-\frac{2b^2\log(x)}{(1-ia)^3}+\frac{2b^2\log(i+a+bx)}{(1-ia)^3}
\end{aligned}$$

Mathematica [A] time = 0.0324949, size = 63, normalized size = 0.83

$$\frac{(a+i)\left(a^2+4ibx+1\right)-4ib^2x^2\log(a+bx+i)+4ib^2x^2\log(x)}{2(a+i)^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])/x^3,x]
```

```
[Out] ((I + a)*(1 + a^2 + (4*I)*b*x) + (4*I)*b^2*x^2*Log[x] - (4*I)*b^2*x^2*Log[I
 + a + b*x])/(2*(I + a)^3*x^2)
```

Maple [B] time = 0.048, size = 406, normalized size = 5.3

$$\frac{-ia}{\left(a^2+1\right)x^2}-\frac{2\,ib^2}{\left(a^2+1\right)^3}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\right)-3\,\frac{b^2\ln\left(b^2x^2+2\,xab+a^2+1\right)a^2}{\left(a^2+1\right)^3}+\frac{b^2\ln\left(b^2x^2+2\,xab+a^2+1\right)}{\left(a^2+1\right)^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x)
```

```
[Out] -I/(a^2+1)/x^2*a-2*I*b^2/(a^2+1)^3*arctan(1/2*(2*b^2*x+2*a*b)/b)-3*b^2/(a^2
+1)^3*ln(b^2*x^2+2*a*b*x+a^2+1)*a^2+b^2/(a^2+1)^3*ln(b^2*x^2+2*a*b*x+a^2+1)
+3*I*b^2/(a^2+1)^3*ln(b^2*x^2+2*a*b*x+a^2+1)*a-2*b^2/(a^2+1)^3*arctan(1/2*(
2*b^2*x+2*a*b)/b)*a^3+6*I*b^2/(a^2+1)^3*arctan(1/2*(2*b^2*x+2*a*b)/b)*a^2+6
*b^2/(a^2+1)^3*arctan(1/2*(2*b^2*x+2*a*b)/b)*a-I*b^2/(a^2+1)^3*ln(b^2*x^2+2
*a*b*x+a^2+1)*a^3+1/2/(a^2+1)/x^2*a^2-1/2/(a^2+1)/x^2+2*I*b^2/(a^2+1)^3*ln(
x)*a^3+2*I*b/(a^2+1)^2/x*a^2+4*b/(a^2+1)^2/x*a-2*I*b/(a^2+1)^2/x-6*I*b^2/(a
^2+1)^3*ln(x)*a+6*b^2/(a^2+1)^3*ln(x)*a^2-2*b^2/(a^2+1)^3*ln(x)
```

Maxima [B] time = 1.49249, size = 254, normalized size = 3.34

$$-\frac{\left(2\,a^3-6i\,a^2-6\,a+2i\right)b^2\arctan\left(\frac{b^2x+ab}{b}\right)}{a^6+3\,a^4+3\,a^2+1}+\frac{\left(-2i\,a^3-6\,a^2+6i\,a+2\right)b^2\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\left(a^6+3\,a^4+3\,a^2+1\right)}+\frac{\left(2i\,a^3+6\,a^2\right.}{a^6+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x, algorithm="maxima")
```

```
[Out] -(2*a^3 - 6*I*a^2 - 6*a + 2*I)*b^2*arctan((b^2*x + a*b)/b)/(a^6 + 3*a^4 + 3
*a^2 + 1) + 1/2*(-2*I*a^3 - 6*a^2 + 6*I*a + 2)*b^2*log(b^2*x^2 + 2*a*b*x +
a^2 + 1)/(a^6 + 3*a^4 + 3*a^2 + 1) + (2*I*a^3 + 6*a^2 - 6*I*a - 2)*b^2*log(
x)/(a^6 + 3*a^4 + 3*a^2 + 1) + 1/2*(a^4 - 2*I*a^3 + (4*I*a^2 + 8*a - 4*I)*b
*x - 2*I*a - 1)/((a^4 + 2*a^2 + 1)*x^2)
```

Fricas [A] time = 1.93852, size = 181, normalized size = 2.38

$$\frac{4i\,b^2x^2\log\left(x\right)-4i\,b^2x^2\log\left(\frac{bx+a+i}{b}\right)+a^3-4\left(-i\,a+1\right)bx+i\,a^2+a+i}{\left(2\,a^3+6i\,a^2-6\,a-2i\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x, algorithm="fricas")
```

```
[Out] (4*I*b^2*x^2*log(x) - 4*I*b^2*x^2*log((b*x + a + I)/b) + a^3 - 4*(-I*a + 1)
*b*x + I*a^2 + a + I)/((2*a^3 + 6*I*a^2 - 6*a - 2*I)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13591, size = 132, normalized size = 1.74

$$\frac{2\,b^3\log\left(bx+a+i\right)}{a^3bi-3\,a^2b-3\,abi+b}-\frac{2\,b^2\log\left(|x|\right)}{a^3i-3\,a^2-3\,ai+1}+\frac{a^3i-a^2+ai-4\,(ab+bi)x-1}{2\,(a+i)^3ix^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x, algorithm="giac")
```

```
[Out] 2*b^3*log(b*x + a + i)/(a^3*b*i - 3*a^2*b - 3*a*b*i + b) - 2*b^2*log(abs(x)
)/(a^3*i - 3*a^2 - 3*a*i + 1) + 1/2*(a^3*i - a^2 + a*i - 4*(a*b + b*i)*x -
1)/((a + i)^3*i*x^2)
```

3.179 $\int \frac{e^{2i\tan^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=93

$$\frac{2b^2}{(1-ia)^3x}-\frac{2ib^3\log(x)}{(a+i)^4}+\frac{2ib^3\log(a+bx+i)}{(a+i)^4}+\frac{ib}{(a+i)^2x^2}-\frac{-a+i}{3(a+i)x^3}$$

```
[Out] -(I - a)/(3*(I + a)*x^3) + (I*b)/((I + a)^2*x^2) + (2*b^2)/((1 - I*a)^3*x)
- ((2*I)*b^3*Log[x])/(I + a)^4 + ((2*I)*b^3*Log[I + a + b*x])/(I + a)^4
```

Rubi [A] time = 0.0564335, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$\frac{2b^2}{(1-ia)^3x}-\frac{2ib^3\log(x)}{(a+i)^4}+\frac{2ib^3\log(a+bx+i)}{(a+i)^4}+\frac{ib}{(a+i)^2x^2}-\frac{-a+i}{3(a+i)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x])/x^4,x]
```

```
[Out] -(I - a)/(3*(I + a)*x^3) + (I*b)/((I + a)^2*x^2) + (2*b^2)/((1 - I*a)^3*x)
- ((2*I)*b^3*Log[x])/(I + a)^4 + ((2*I)*b^3*Log[I + a + b*x])/(I + a)^4
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2i\tan^{-1}(a+bx)}}{x^4}\,dx &= \int \frac{1+ia+ibx}{x^4(1-ia-ibx)}\,dx\\ &= \int \left(\frac{i-a}{(i+a)x^4}-\frac{2ib}{(i+a)^2x^3}+\frac{2ib^2}{(i+a)^3x^2}-\frac{2ib^3}{(i+a)^4x}+\frac{2ib^4}{(i+a)^4(i+a+bx)}\right)dx\\ &= -\frac{i-a}{3(i+a)x^3}+\frac{ib}{(i+a)^2x^2}+\frac{2b^2}{(1-ia)^3x}-\frac{2ib^3\log(x)}{(i+a)^4}+\frac{2ib^3\log(i+a+bx)}{(i+a)^4}\end{aligned}$$

Mathematica [A] time = 0.0460624, size = 88, normalized size = 0.95

$$\frac{(a+i)\left(a^3+ia^2+3iabx+a-6ib^2x^2-3bx+i\right)+6ib^3x^3\log(a+bx+i)-6ib^3x^3\log(x)}{3(a+i)^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])/x^4,x]
```

```
[Out] ((I + a)*(I + a + I*a^2 + a^3 - 3*b*x + (3*I)*a*b*x - (6*I)*b^2*x^2) - (6*I
)*b^3*x^3*Log[x] + (6*I)*b^3*x^3*Log[I + a + b*x])/(3*(I + a)^4*x^3)
```

Maple [B] time = 0.052, size = 560, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x)
```

```
[Out] 2*b^2/(a^2+1)^3/x+1/3/(a^2+1)/x^3*a^2+2*b^3/(a^2+1)^4*arctan(1/2*(2*b^2*x+2
*a*b)/b)-2*I*b^2/(a^2+1)^3/x*a^3+6*I*b^2/(a^2+1)^3/x*a+I*b^3/(a^2+1)^4*ln(b
^2*x^2+2*a*b*x+a^2+1)*a^4-6*I*b^3/(a^2+1)^4*ln(b^2*x^2+2*a*b*x+a^2+1)*a^2-8
*I*b^3/(a^2+1)^4*arctan(1/2*(2*b^2*x+2*a*b)/b)*a^3+8*I*b^3/(a^2+1)^4*arctan
(1/2*(2*b^2*x+2*a*b)/b)*a+I*b/(a^2+1)^2/x^2*a^2-2*I*b^3/(a^2+1)^4*ln(x)*a^4
+12*I*b^3/(a^2+1)^4*ln(x)*a^2-8*b^3/(a^2+1)^4*ln(x)*a^3+8*b^3/(a^2+1)^4*ln(
x)*a-2*I*b^3/(a^2+1)^4*ln(x)+2*b/(a^2+1)^2/x^2*a-I*b/(a^2+1)^2/x^2-6*b^2/(a
^2+1)^3/x*a^2-2/3*I/(a^2+1)/x^3*a-4*b^3/(a^2+1)^4*ln(b^2*x^2+2*a*b*x+a^2+1)
*a+2*b^3/(a^2+1)^4*arctan(1/2*(2*b^2*x+2*a*b)/b)*a^4-12*b^3/(a^2+1)^4*arcta
n(1/2*(2*b^2*x+2*a*b)/b)*a^2+4*b^3/(a^2+1)^4*ln(b^2*x^2+2*a*b*x+a^2+1)*a^3+
I*b^3/(a^2+1)^4*ln(b^2*x^2+2*a*b*x+a^2+1)-1/3/(a^2+1)/x^3
```

Maple [B] time = 1.532, size = 355, normalized size = 3.82

$$\frac{\left(2\,a^4-8i\,a^3-12\,a^2+8i\,a+2\right)b^3\arctan\left(\frac{b^2x+ab}{b}\right)}{a^8+4\,a^6+6\,a^4+4\,a^2+1}+\frac{\left(i\,a^4+4\,a^3-6i\,a^2-4\,a+i\right)b^3\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^8+4\,a^6+6\,a^4+4\,a^2+1}+\frac{\left(-2i\,a^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x, algorithm="maxima")
```

```
[Out] (2*a^4 - 8*I*a^3 - 12*a^2 + 8*I*a + 2)*b^3*arctan((b^2*x + a*b)/b)/(a^8 + 4
*a^6 + 6*a^4 + 4*a^2 + 1) + (I*a^4 + 4*a^3 - 6*I*a^2 - 4*a + I)*b^3*log(b^2
*x^2 + 2*a*b*x + a^2 + 1)/(a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1) + (-2*I*a^4 - 8
*a^3 + 12*I*a^2 + 8*a - 2*I)*b^3*log(x)/(a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1) +
 1/3*(a^6 - 2*I*a^5 - (6*I*a^3 + 18*a^2 - 18*I*a - 6)*b^2*x^2 + a^4 - 4*I*a
^3 - (-3*I*a^4 - 6*a^3 - 6*a + 3*I)*b*x - a^2 - 2*I*a - 1)/((a^6 + 3*a^4 +
3*a^2 + 1)*x^3)
```

Fricas [A] time = 1.91116, size = 247, normalized size = 2.66

$$\frac{-6i\,b^3x^3\log\left(x\right)+6i\,b^3x^3\log\left(\frac{bx+a+i}{b}\right)-6\left(i\,a-1\right)b^2x^2+a^4+2i\,a^3+\left(3i\,a^2-6\,a-3i\right)bx+2i\,a-1}{\left(3\,a^4+12i\,a^3-18\,a^2-12i\,a+3\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x, algorithm="fricas")
```

```
[Out] (-6*I*b^3*x^3*log(x) + 6*I*b^3*x^3*log((b*x + a + I)/b) - 6*(I*a - 1)*b^2*x
^2 + a^4 + 2*I*a^3 + (3*I*a^2 - 6*a - 3*I)*b*x + 2*I*a - 1)/((3*a^4 + 12*I*
a^3 - 18*a^2 - 12*I*a + 3)*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09622, size = 184, normalized size = 1.98

$$-\frac{2\,b^4\log\left(bx+a+i\right)}{a^4bi-4\,a^3b-6\,a^2bi+4\,ab+bi}+\frac{2\,b^3\log\left(|x|\right)}{a^4i-4\,a^3-6\,a^2i+4\,a+i}+\frac{a^4i-2\,a^3+6\left(ab^2+b^2i\right)x^2-3\left(a^2b+2\,abi-b\right)x-}{3\left(a+i\right)^4ix^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x, algorithm="giac")
```

```
[Out] -2*b^4*log(b*x + a + i)/(a^4*b*i - 4*a^3*b - 6*a^2*b*i + 4*a*b + b*i) + 2*b
^3*log(abs(x))/(a^4*i - 4*a^3 - 6*a^2*i + 4*a + i) + 1/3*(a^4*i - 2*a^3 + 6
*(a*b^2 + b^2*i)*x^2 - 3*(a^2*b + 2*a*b*i - b)*x - 2*a - i)/((a + i)^4*i*x^
3)
```

3.180 $\int e^{3i\tan^{-1}(a+bx)}x^4\,dx$

Optimal. Leaf size=324

$$-\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}\left(2\left(-52ia^2+118a+61i\right)bx+112ia^3-422a^2-458ia+163\right)}{40b^5}-\frac{3\left(8ia^4-48a^3-88ia^2\right.}{}$$

```
[Out] (-3*(19*I + 68*a - (88*I)*a^2 - 48*a^3 + (8*I)*a^4)*Sqrt[1 - I*a - I*b*x]*S
qrt[1 + I*a + I*b*x])/(8*b^5) - ((2*I)*x^4*(1 + I*a + I*b*x)^(3/2))/(b*Sqrt
[1 - I*a - I*b*x]) + (3*(17*I + 16*a)*x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a +
I*b*x)^(3/2))/(20*b^3) - (11*x^3*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3
/2))/(5*b^2) - ((I/40)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(163 -
 (458*I)*a - 422*a^2 + (112*I)*a^3 + 2*(61*I + 118*a - (52*I)*a^2)*b*x))/b^
5 - (3*(19 - (68*I)*a - 88*a^2 + (48*I)*a^3 + 8*a^4)*ArcSinh[a + b*x])/(8*b
^5)
```

Rubi [A] time = 0.26693, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5095, 97, 153, 147, 50, 53, 619, 215}

$$-\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}\left(2\left(-52ia^2+118a+61i\right)bx+112ia^3-422a^2-458ia+163\right)}{40b^5}-\frac{3\left(8ia^4-48a^3-88ia^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x])*x^4,x]
```



```
[Out] (-3*(19*I + 68*a - (88*I)*a^2 - 48*a^3 + (8*I)*a^4)*Sqrt[1 - I*a - I*b*x]*S
qrt[1 + I*a + I*b*x])/(8*b^5) - ((2*I)*x^4*(1 + I*a + I*b*x)^(3/2))/(b*Sqrt
[1 - I*a - I*b*x]) + (3*(17*I + 16*a)*x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a +
I*b*x)^(3/2))/(20*b^3) - (11*x^3*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3
/2))/(5*b^2) - ((I/40)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(163 -
 (458*I)*a - 422*a^2 + (112*I)*a^3 + 2*(61*I + 118*a - (52*I)*a^2)*b*x))/b^
5 - (3*(19 - (68*I)*a - 88*a^2 + (48*I)*a^3 + 8*a^4)*ArcSinh[a + b*x])/(8*b
^5)
```


Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
```

```
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{3i\tan^{-1}(a+bx)}x^4\,dx = \int \frac{x^4(1+ia+ibx)^{3/2}}{(1-ia-ibx)^{3/2}}\,dx$$

$$= -\frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} + \frac{(2i)\int \frac{x^3\sqrt{1+ia+ibx}\left(4(1+ia)+\frac{11ibx}{2}\right)}{\sqrt{1-ia-ibx}}dx}{b}$$

$$= -\frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{11x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2} + \frac{(2i)\int \frac{x^2\sqrt{1+ia+ibx}\left(-\frac{33}{2}(i-a)(1-ia)b+\frac{3}{2}(17-\right.}{\sqrt{1-ia-ibx}}}{5b^3}$$

$$= -\frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} + \frac{3(17i+16a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3} - \frac{11x^3\sqrt{1-ia-ibx}(1+ia+}{5b^2}$$

$$= -\frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} + \frac{3(17i+16a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3} - \frac{11x^3\sqrt{1-ia-ibx}(1+ia+}{5b^2}$$

$$= -\frac{3\left(19i+68a-88ia^2-48a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} + \frac{3(17i+}{}$$

$$= -\frac{3\left(19i+68a-88ia^2-48a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} + \frac{3(17i+}{}$$

$$= -\frac{3\left(19i+68a-88ia^2-48a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} + \frac{3(17i+}{}$$

$$= -\frac{3\left(19i+68a-88ia^2-48a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} + \frac{3(17i+}{}$$

Mathematica [A] time = 0.447303, size = 249, normalized size = 0.77

$$\frac{3(-1)^{3/4}\left(8a^4+48ia^3-88a^2-68ia+19\right)\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{4\sqrt{-ib}b^{9/2}} - \frac{\sqrt{ia+ibx+1}\left(-ia^2\left(52b^2x^2-422ibx+2599\right)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])*x^4,x]
```

```
[Out] -(Sqrt[1 + I*a + I*b*x]*(448*I + (418*I)*a^4 + 8*a^5 + 163*b*x + (61*I)*b^2
*x^2 - 34*b^3*x^3 - (22*I)*b^4*x^4 + 8*b^5*x^5 + (14*I)*a^3*(121*I + 8*b*x)
 - I*a^2*(2599 - (422*I)*b*x + 52*b^2*x^2) + a*(1763 - (458*I)*b*x + 118*b^
2*x^2 + (32*I)*b^3*x^3)))/(40*b^5*Sqrt[(-I)*(I + a + b*x)]) + (3*(-1)^(3/4)
*(19 - (68*I)*a - 88*a^2 + (48*I)*a^3 + 8*a^4)*ArcSinh[((1/2 + I/2)*Sqrt[b]
*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*Sqrt[(-I)*b]*b^(9/2))
```

Maple [B] time = 0.131, size = 933, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x)
```

```
[Out] 501/10*I/b^4*a^3*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-57/8/b^4*ln((b^2*x+a*b)/(b
^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-18*I/b^4*a^3*ln((b^2*x
+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-29/10*I/b^2*a*
x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-367/10*I/b^4*a*x/(b^2*x^2+2*a*b*x+a^2+1)^
(1/2)+51/2*I/b^4*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
)/(b^2)^(1/2)+89/10*I/b^3*a^2*x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/5*I/b^4*a
^5*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*a^4/b^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^
2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+33*a^2/b^4*ln((b^2*x+a*b)/(b^2)^(1/
2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+19/8*x^3/b^2/(b^2*x^2+2*a*b*x
+a^2+1)^(1/2)+57/8/b^4*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-181/8*a^3/b^5/(b^2*x
^2+2*a*b*x+a^2+1)^(1/2)+41/4*a^5/b^5/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-263/8*a/
b^5/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-56/5*I/b^5/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+
1/4/b*a*x^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*x^3/b^2/(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)*a^2+3/2*a^3/b^3*x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/5*I*b*x^6/(b^2
*x^2+2*a*b*x+a^2+1)^(1/2)-1/5*I*a*x^5/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+7/5*I/b
*x^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-28/5*I/b^3*x^2/(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)+209/10*I/b^5*a^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/5*I/b^5*a^6/(b^2*x^2+
2*a*b*x+a^2+1)^(1/2)+319/10*I/b^5*a^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-103/8*a
/b^3*x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-527/8*a^2/b^4*x/(b^2*x^2+2*a*b*x+a^2
+1)^(1/2)+53/4*a^4/b^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-3/4*x^5/(b^2*x^2+2*a
*b*x+a^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05581, size = 822, normalized size = 2.54

$$\frac{-62i\,a^6 + 2687\,a^5 + 11575i\,a^4 - 20350\,a^3 + \left(-62i\,a^5 + 2625\,a^4 + 8950i\,a^3 - 11400\,a^2 - 6340i\,a + 1280\right)bx - 17740i\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x, algorithm="fricas")
```

```
[Out] (-62*I*a^6 + 2687*a^5 + 11575*I*a^4 - 20350*a^3 + (-62*I*a^5 + 2625*a^4 + 8
950*I*a^3 - 11400*a^2 - 6340*I*a + 1280)*b*x - 17740*I*a^2 + (960*a^5 + 672
0*I*a^4 - 16320*a^3 + (960*a^4 + 5760*I*a^3 - 10560*a^2 - 8160*I*a + 2280)*
b*x - 18720*I*a^2 + 10440*a + 2280*I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)) + (-64*I*b^5*x^5 - 176*b^4*x^4 + (256*a + 272*I)*b^3*x^3 - 64*
I*a^5 - 8*(52*a^2 + 118*I*a - 61)*b^2*x^2 + 3344*a^4 + 13552*I*a^3 + (896*a
^3 + 3376*I*a^2 - 3664*a - 1304*I)*b*x - 20792*a^2 - 14104*I*a + 3584)*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1) + 7620*a + 1280*I)/(320*b^6*x + (320*a + 320*
I)*b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)*x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16915, size = 525, normalized size = 1.62

$$-\frac{1}{40}\sqrt{(bx+a)^2+1}\left(\left(2\left(\left(\frac{4\,ix}{b}-\frac{4\,ab^{17}i-15\,b^{17}}{b^{19}}\right)x+\frac{4\,a^2b^{16}i-35\,ab^{16}-32\,b^{16}i}{b^{19}}\right)x-\frac{8\,a^3b^{15}i-130\,a^2b^{15}-252\,ab^{15}i+1}{b^{19}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x, algorithm="giac")
```

```
[Out] -1/40*sqrt((b*x + a)^2 + 1)*((2*((4*i*x/b - (4*a*b^17*i - 15*b^17)/b^19)*x
+ (4*a^2*b^16*i - 35*a*b^16 - 32*b^16*i)/b^19)*x - (8*a^3*b^15*i - 130*a^2*
b^15 - 252*a*b^15*i + 125*b^15)/b^19)*x + (8*a^4*b^14*i - 250*a^3*b^14 - 80
4*a^2*b^14*i + 835*a*b^14 + 288*b^14*i)/b^19) + 1/8*(8*a^4 + 48*a^3*i - 88*
a^2 - 68*a*i + 19)*log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b +
 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b*i + 2*a^2*b*i + (x*abs(b) - sqrt(
(b*x + a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b
) + 4*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a*i*abs(b) - a*b - (x*abs(b) - sqr
t((b*x + a)^2 + 1))*abs(b))/(b^4*abs(b)) + 1/4*(8*a^4*abs(b) + 48*a^3*i*abs
(b) - 88*a^2*abs(b) - 68*a*i*abs(b) + 19*abs(b))*log(96*b^5)/b^6
```

3.181 $\int e^{3i\tan^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=249

$$-\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}\left(-22ia^2-2(11-10ia)bx+54a+29i\right)}{8b^4}+\frac{3\left(8ia^3-36a^2-44ia+17\right)\sqrt{-ia-ibx+1}}{8b^4}$$

```
[Out] (3*(17 - (44*I)*a - 36*a^2 + (8*I)*a^3)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a
+ I*b*x])/(8*b^4) - ((2*I)*x^3*(1 + I*a + I*b*x)^(3/2))/(b*Sqrt[1 - I*a - I
*b*x]) - (9*x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(4*b^2) - ((
I/8)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(29*I + 54*a - (22*I)*a^
2 - 2*(11 - (10*I)*a)*b*x))/b^4 - (3*(17*I + 44*a - (36*I)*a^2 - 8*a^3)*Arc
Sinh[a + b*x])/(8*b^4)
```

Rubi [A] time = 0.242333, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5095, 97, 153, 147, 50, 53, 619, 215}

$$-\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}\left(-22ia^2-2(11-10ia)bx+54a+29i\right)}{8b^4}+\frac{3\left(8ia^3-36a^2-44ia+17\right)\sqrt{-ia-ibx+1}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x])*x^3,x]
```


```
[Out] (3*(17 - (44*I)*a - 36*a^2 + (8*I)*a^3)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a
+ I*b*x])/(8*b^4) - ((2*I)*x^3*(1 + I*a + I*b*x)^(3/2))/(b*Sqrt[1 - I*a - I
*b*x]) - (9*x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(4*b^2) - ((
I/8)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(29*I + 54*a - (22*I)*a^
2 - 2*(11 - (10*I)*a)*b*x))/b^4 - (3*(17*I + 44*a - (36*I)*a^2 - 8*a^3)*Arc
Sinh[a + b*x])/(8*b^4)
```


Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
```

```
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{3i\tan^{-1}(a+bx)}x^3\,dx = \int \frac{x^3(1+ia+ibx)^{3/2}}{(1-ia-ibx)^{3/2}}\,dx$$

$$= -\frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} + \frac{(2i)\int \frac{x^2\sqrt{1+ia+ibx}\left(3(1+ia)+\frac{9ibx}{2}\right)}{\sqrt{1-ia-ibx}}\,dx}{b}$$

$$= -\frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} + \frac{i\int \frac{x\sqrt{1+ia+ibx}\left(-9i(1+a^2)b+\frac{3}{2}(11-10ia)b^2}{\sqrt{1-ia-ibx}}}{2b^3}$$

$$= -\frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} - \frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\left(29\right.}{8b}$$

$$= \frac{3\left(17-44ia-36a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{1-ia-}}{}$$

$$= \frac{3\left(17-44ia-36a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{1-ia-}}{}$$

$$= \frac{3\left(17-44ia-36a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{1-ia-}}{}$$

$$= \frac{3\left(17-44ia-36a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{1-ia-}}{}$$

Mathematica [A] time = 0.261418, size = 201, normalized size = 0.81

$$\frac{\sqrt{ia+ibx+1}\left(a^2(-233+22ibx)+2a^4+78ia^3-ia\left(10b^2x^2-54ibx+237\right)-2b^4x^4+6ib^3x^3+11b^2x^2-29ibx+80\right)}{8b^4\sqrt{-i(a+bx+i)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])*x^3,x]
```

```
[Out] (Sqrt[1 + I*a + I*b*x]*(80 + (78*I)*a^3 + 2*a^4 - (29*I)*b*x + 11*b^2*x^2 +
 (6*I)*b^3*x^3 - 2*b^4*x^4 + a^2*(-233 + (22*I)*b*x) - I*a*(237 - (54*I)*b*
x + 10*b^2*x^2)))/(8*b^4*Sqrt[(-I)*(I + a + b*x)]) + (3*(-1)^(1/4)*(-17*I -
 44*a + (36*I)*a^2 + 8*a^3)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[
(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*b^(9/2))
```

Maple [B] time = 0.129, size = 711, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x)
```

```
[Out] 1/2*a^2/b^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-19/2*a^4/b^4/(b^2*x^2+2*a*b*x+a^2
+1)^(1/2)+53/2*a/b^3*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-25/2*a^3/b^3/(b^2*x^2+
2*a*b*x+a^2+1)^(1/2)*x-33/2*a/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b
```

```
*x+a^2+1)^(1/2))/(b^2)^(1/2)+5*x^2/b^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2/b*
a*x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*x^2/b^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)*a^2+3*a^3/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/
(b^2)^(1/2)+17/8*I/b*x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/4*I/b^4*a^5/(b^2*x
^2+2*a*b*x+a^2+1)^(1/2)-265/8*I/b^3*a^2*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-155
/8*I/b^4*a^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-x^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)-1/4*I*b*x^5/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/4*I/b^3*a^4*x/(b^2*x^2+2*a*b*
x+a^2+1)^(1/2)+27/2*I/b^3*a^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a
^2+1)^(1/2))/(b^2)^(1/2)-53/8*I/b^2*a*x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-157
/8*I/b^4*a/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-51/8*I/b^3*ln((b^2*x+a*b)/(b^2)^(1
/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/4*I*a*x^4/(b^2*x^2+2*a*b*x
+a^2+1)^(1/2)+51/8*I/b^3*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+10/b^4/(b^2*x^2+2*
a*b*x+a^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07708, size = 633, normalized size = 2.54

$$\frac{15i\,a^5 - 495\,a^4 - 1664i\,a^3 + \left(15i\,a^4 - 480\,a^3 - 1184i\,a^2 + 968\,a + 256i\right)bx + 2152\,a^2 - \left(192\,a^4 + 1056i\,a^3 + \left(192\,a^3 + 864\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] (15*I*a^5 - 495*a^4 - 1664*I*a^3 + (15*I*a^4 - 480*a^3 - 1184*I*a^2 + 968*a
 + 256*I)*b*x + 2152*a^2 - (192*a^4 + 1056*I*a^3 + (192*a^3 + 864*I*a^2 - 1
056*a - 408*I)*b*x - 1920*a^2 - 1464*I*a + 408)*log(-b*x - a + sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)) + (-16*I*b^4*x^4 - 48*b^3*x^3 + (80*a + 88*I)*b^2*x^
2 + 16*I*a^4 - 624*a^3 - 8*(22*a^2 + 54*I*a - 29)*b*x - 1864*I*a^2 + 1896*a
 + 640*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1224*I*a - 256)/(64*b^5*x + (
64*a + 64*I)*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (ia + ibx + 1)^3}{\left(a^2 + 2abx + b^2x^2 + 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)*x**3,x)
```

```
[Out] Integral(x**3*(I*a + I*b*x + 1)**3/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2),
 x)
```

Giac [A] time = 1.18536, size = 446, normalized size = 1.79

$$-\frac{1}{8}\sqrt{(bx+a)^2+1}\left(\left(2\left(\frac{ix}{b}-\frac{ab^{11}i-4\,b^{11}}{b^{13}}\right)x+\frac{2\,a^2b^{10}i-20\,ab^{10}-19\,b^{10}i}{b^{13}}\right)x-\frac{2\,a^3b^9i-44\,a^2b^9-93\,ab^9i+48\,b^9}{b^{13}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] -1/8*sqrt((b*x + a)^2 + 1)*((2*(i*x/b - (a*b^11*i - 4*b^11)/b^13)*x + (2*a^
2*b^10*i - 20*a*b^10 - 19*b^10*i)/b^13)*x - (2*a^3*b^9*i - 44*a^2*b^9 - 93*
a*b^9*i + 48*b^9)/b^13) - 1/8*(8*a^3 + 36*a^2*i - 44*a - 17*i)*log(3*(x*abs
(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + 2*(x*abs(b) - sqrt((b*x + a)^2
 + 1))^2*b*i + 2*a^2*b*i + (x*abs(b) - sqrt((b*x + a)^2 + 1))^3*abs(b) + 3*
(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) + 4*(x*abs(b) - sqrt((b*x + a
)^2 + 1))*a*i*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^
3*abs(b)) - 1/4*(8*a^3*abs(b) + 36*a^2*i*abs(b) - 44*a*abs(b) - 17*i*abs(b)
)*log(96*b^4)/b^5
```

3.182 $\int e^{3i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=227

$$\frac{\left(-6ia^2+18a+11i\right)\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{6b^3}+\frac{\left(-6ia^2+18a+11i\right)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2b^3}+\frac{\left(-6a^2-18ia+\right.}{}$$

```
[Out] ((11*I + 18*a - (6*I)*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*
b^3) + ((11*I + 18*a - (6*I)*a^2)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(
3/2))/(6*b^3) - (I*(I + a)^2*(1 + I*a + I*b*x)^(5/2))/(b^3*Sqrt[1 - I*a - I
*b*x]) + ((I/3)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(5/2))/b^3 + ((11 -
 (18*I)*a - 6*a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rubi [A] time = 0.169239, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 89, 80, 50, 53, 619, 215}

$$\frac{\left(-6ia^2+18a+11i\right)\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{6b^3}+\frac{\left(-6ia^2+18a+11i\right)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2b^3}+\frac{\left(-6a^2-18ia+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] ((11*I + 18*a - (6*I)*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*
b^3) + ((11*I + 18*a - (6*I)*a^2)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(
3/2))/(6*b^3) - (I*(I + a)^2*(1 + I*a + I*b*x)^(5/2))/(b^3*Sqrt[1 - I*a - I
*b*x]) + ((I/3)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(5/2))/b^3 + ((11 -
 (18*I)*a - 6*a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{3i\tan^{-1}(a+bx)}x^2\,dx = \int \frac{x^2(1+ia+ibx)^{3/2}}{(1-ia-ibx)^{3/2}}\,dx$$

$$= -\frac{i(i+a)^2(1+ia+ibx)^{5/2}}{b^3\sqrt{1-ia-ibx}} - \frac{i\int \frac{(1+ia+ibx)^{3/2}\left((3-2ia)(i+a)b-b^2x\right)}{\sqrt{1-ia-ibx}}\,dx}{b^3}$$

$$= -\frac{i(i+a)^2(1+ia+ibx)^{5/2}}{b^3\sqrt{1-ia-ibx}} + \frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{5/2}}{3b^3} + \frac{\left(11-18ia-6a^2\right)\int \frac{(1+ia+ibx)^{3/2}}{\sqrt{1-ia-ibx}}\,d}{3b^2}$$

$$= \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} - \frac{i(i+a)^2(1+ia+ibx)^{5/2}}{b^3\sqrt{1-ia-ibx}} + \frac{i\sqrt{1-ia-ibx}(1}{3b^3}$$

$$= \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}(1+ia+ibx)^{3}}{6b^3}$$

$$= \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}(1+ia+ibx)^{3}}{6b^3}$$

$$= \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}(1+ia+ibx)^{3}}{6b^3}$$

$$= \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}(1+ia+ibx)^{3}}{6b^3}$$

Mathematica [A] time = 0.236446, size = 160, normalized size = 0.7

$$\frac{\sqrt{ia+ibx+1}\left(-2a^3-53ia^2+a(103-16ibx)-2b^3x^3+7ib^2x^2+19bx+52i\right)}{6b^3\sqrt{-i(a+bx+i)}} + \frac{(-1)^{3/4}\left(6a^2+18ia-11\right)\sinh^{-1}\left(\left(\frac{1}{2}+\right.\right.}{\sqrt{-ib}b^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (Sqrt[1 + I*a + I*b*x]*(52*I - (53*I)*a^2 - 2*a^3 + 19*b*x + (7*I)*b^2*x^2
- 2*b^3*x^3 + a*(103 - (16*I)*b*x)))/(6*b^3*Sqrt[(-I)*(I + a + b*x)]) + ((-
1)^(3/4)*(-11 + (18*I)*a + 6*a^2)*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I
 + a + b*x)])/Sqrt[(-I)*b]])/(Sqrt[(-I)*b]*b^(5/2))
```

Maple [B] time = 0.12, size = 519, normalized size = 2.3

$$\frac{17a}{2b^3}\frac{1}{\sqrt{b^2x^2+2xab+a^2+1}}+\frac{17a^3}{2b^3}\frac{1}{\sqrt{b^2x^2+2xab+a^2+1}}-\frac{i}{3}ax^3\frac{1}{\sqrt{b^2x^2+2xab+a^2+1}}-\frac{9ia}{b^2}\ln\left((b^2x+ab)\frac{1}{\sqrt{b^2}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x)
```

```
[Out] 17/2*a/b^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+17/2*a^3/b^3/(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)-1/3*I*a*x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-9*I/b^2*a*ln((b^2*x+a*b)
/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+26/3*I/b^3/(b^2*x^2
+2*a*b*x+a^2+1)^(1/2)+11/2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+
a^2+1)^(1/2))/(b^2)^(1/2)-11/2*x/b^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/3*I/b^
2*a^3*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/3*I*b*x^4/(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)+53/3*I/b^2*a*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+25/3*I/b^3*a^2/(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)-1/3*I/b^3*a^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+13/3*I/b*x^
2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*a^2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x
^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+3/2/b*a*x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)-3/2*x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+23/2*a^2/b^2/(b^2*x^2+2*a*b*x+a^2
+1)^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01836, size = 489, normalized size = 2.15

$$\frac{-7ia^4+166a^3+\left(-7ia^3+159a^2+249ia-96\right)bx+408ia^2+\left(72a^3+12\left(6a^2+18ia-11\right)bx+288ia^2-348a-132i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] (-7*I*a^4 + 166*a^3 + (-7*I*a^3 + 159*a^2 + 249*I*a - 96)*b*x + 408*I*a^2 +
 (72*a^3 + 12*(6*a^2 + 18*I*a - 11)*b*x + 288*I*a^2 - 348*a - 132*I)*log(-b
*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (-8*I*b^3*x^3 - 28*b^2*x^2 -
8*I*a^3 + (64*a + 76*I)*b*x + 212*a^2 + 412*I*a - 208)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1) - 345*a - 96*I)/(24*b^4*x + (24*a + 24*I)*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (ia + ibx + 1)^3}{\left(a^2 + 2abx + b^2x^2 + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)*x**2,x)
```

```
[Out] Integral(x**2*(I*a + I*b*x + 1)**3/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2),
 x)
```

Giac [A] time = 1.16247, size = 375, normalized size = 1.65

$$-\frac{1}{6}\sqrt{(bx+a)^2+1}\left(\left(\frac{2\,ix}{b}-\frac{2\,ab^6i-9\,b^6}{b^8}\right)x+\frac{2\,a^2b^5i-27\,ab^5-28\,b^5i}{b^8}\right)+\frac{\left(6\,a^2+18\,ai-11\right)\log\left(3\left(x|b|-\sqrt{(bx+a)^2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] -1/6*sqrt((b*x + a)^2 + 1)*((2*i*x/b - (2*a*b^6*i - 9*b^6)/b^8)*x + (2*a^2*
b^5*i - 27*a*b^5 - 28*b^5*i)/b^8) + 1/6*(6*a^2 + 18*a*i - 11)*log(3*(x*abs(
b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + 2*(x*abs(b) - sqrt((b*x + a)^2
+ 1))^2*b*i + 2*a^2*b*i + (x*abs(b) - sqrt((b*x + a)^2 + 1))^3*abs(b) + 3*(
x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) + 4*(x*abs(b) - sqrt((b*x + a)
^2 + 1))*a*i*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^2
*abs(b)) + 1/3*(6*a^2*abs(b) + 18*a*i*abs(b) - 11*abs(b))*log(8*b^3)/b^4
```

3.183 $\int e^{3i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=163

$$-\frac{(1-ia)(ia+ibx+1)^{5/2}}{b^2\sqrt{-ia-ibx+1}}-\frac{(3-2ia)\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b^2}-\frac{3(3-2ia)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2b^2}+\frac{3(2a+3i}{}$$

```
[Out] (-3*(3 - (2*I)*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^2) - ((
3 - (2*I)*a)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(2*b^2) - ((1 -
 I*a)*(1 + I*a + I*b*x)^(5/2))/(b^2*Sqrt[1 - I*a - I*b*x]) + (3*(3*I + 2*a)
*ArcSinh[a + b*x])/(2*b^2)
```

Rubi [A] time = 0.119658, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5095, 78, 50, 53, 619, 215}

$$-\frac{(1-ia)(ia+ibx+1)^{5/2}}{b^2\sqrt{-ia-ibx+1}}-\frac{(3-2ia)\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b^2}-\frac{3(3-2ia)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2b^2}+\frac{3(2a+3i}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x])*x,x]
```

```
[Out] (-3*(3 - (2*I)*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^2) - ((
3 - (2*I)*a)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(2*b^2) - ((1 -
 I*a)*(1 + I*a + I*b*x)^(5/2))/(b^2*Sqrt[1 - I*a - I*b*x]) + (3*(3*I + 2*a)
*ArcSinh[a + b*x])/(2*b^2)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
```

```
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{3i\tan^{-1}(a+bx)}x\,dx = \int \frac{x(1+ia+ibx)^{3/2}}{(1-ia-ibx)^{3/2}}\,dx$$

$$= -\frac{(1-ia)(1+ia+ibx)^{5/2}}{b^2\sqrt{1-ia-ibx}} + \frac{(3i+2a)\int \frac{(1+ia+ibx)^{3/2}}{\sqrt{1-ia-ibx}}\,dx}{b}$$

$$= -\frac{(3-2ia)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(1-ia)(1+ia+ibx)^{5/2}}{b^2\sqrt{1-ia-ibx}} + \frac{(3(3i+2a))\int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx}{2b}$$

$$= -\frac{3(3-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3-2ia)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(1-ia)(1+i}{b^2\sqrt{1-ia}}$$

$$= -\frac{3(3-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3-2ia)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(1-ia)(1+i}{b^2\sqrt{1-ia}}$$

$$= -\frac{3(3-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3-2ia)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(1-ia)(1+i}{b^2\sqrt{1-ia}}$$

$$= -\frac{3(3-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3-2ia)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(1-ia)(1+i}{b^2\sqrt{1-ia}}$$

Mathematica [A] time = 0.163109, size = 132, normalized size = 0.81

$$\frac{\sqrt{ia+ibx+1}\left(a^2+15ia-b^2x^2+5ibx-14\right)}{2b^2\sqrt{-i(a+bx+i)}} + \frac{3\sqrt[4]{-1}(2a+3i)\sqrt{-ib}\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{b^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])*x,x]
```

```
[Out] (Sqrt[1 + I*a + I*b*x]*(-14 + (15*I)*a + a^2 + (5*I)*b*x - b^2*x^2))/(2*b^2
*Sqrt[(-I)*(I + a + b*x)]) + (3*(-1)^(1/4)*(3*I + 2*a)*Sqrt[(-I)*b]*ArcSinh
[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(5/2)
```

Maple [B] time = 0.116, size = 358, normalized size = 2.2

$$-7\frac{1}{b^2\sqrt{b^2x^2+2xab+a^2+1}} - 7\frac{a^2}{b^2\sqrt{b^2x^2+2xab+a^2+1}} + \frac{\frac{i}{2}a^2x}{b}\frac{1}{\sqrt{b^2x^2+2xab+a^2+1}} - 3\frac{x^2}{\sqrt{b^2x^2+2xab+a^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x)
```

```
[Out] -7/b^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-7*a^2/b^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)+1/2*I/b*a^2*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*x^2/(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)-1/2*I*a*x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*I/b^2*a/(b^2*x^2+2*a*b
*x+a^2+1)^(1/2)-9/2*I/b*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+9/2*I/b*ln((b^2*x+a
*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*I/b^2*a^3/(b
^2*x^2+2*a*b*x+a^2+1)^(1/2)-10*a/b/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-1/2*I*b*
x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*a/b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2
+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03503, size = 366, normalized size = 2.25

$$\frac{3i\,a^3 + \left(3i\,a^2 - 44\,a - 32i\right)bx - 47\,a^2 - \left((24\,a + 36i)bx + 24\,a^2 + 60i\,a - 36\right)\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) + \sqrt{}}{8\,b^3x + (8\,a + 8i)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] (3*I*a^3 + (3*I*a^2 - 44*a - 32*I)*b*x - 47*a^2 - ((24*a + 36*I)*b*x + 24*a
^2 + 60*I*a - 36)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*(-4*I*b^2*x^2 + 4*I*a^2 - 20*b*x - 60*a - 56*I
) - 76*I*a + 32)/(8*b^3*x + (8*a + 8*I)*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,(ia + ibx + 1)^3}{\left(a^2 + 2abx + b^2x^2 + 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)*x,x)
```

```
[Out] Integral(x*(I*a + I*b*x + 1)**3/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

Giac [B] time = 1.15689, size = 319, normalized size = 1.96

$$-\frac{1}{2}\sqrt{(bx+a)^2+1}\left(\frac{ix}{b}-\frac{ab^2i-6\,b^2}{b^4}\right)-\frac{(2\,a+3\,i)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+2\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt((b*x + a)^2 + 1)*(i*x/b - (a*b^2*i - 6*b^2)/b^4) - 1/2*(2*a + 3*i
)*log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + 2*(x*abs(b) - sq
rt((b*x + a)^2 + 1))^2*b*i + 2*a^2*b*i + (x*abs(b) - sqrt((b*x + a)^2 + 1))
^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) + 4*(x*abs(b) -
 sqrt((b*x + a)^2 + 1))*a*i*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1
))*abs(b))/(b*abs(b)) - (2*a*abs(b) + 3*i*abs(b))*log(24*b^2)/b^3
```

3.184 $\int e^{3i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=94

$$-\frac{2i(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}-\frac{3i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}-\frac{3\sinh^{-1}(a+bx)}{b}$$

```
[Out] ((-3*I)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b - ((2*I)*(1 + I*a +
I*b*x)^(3/2))/(b*Sqrt[1 - I*a - I*b*x]) - (3*ArcSinh[a + b*x])/b
```

Rubi [A] time = 0.0418089, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5093, 47, 50, 53, 619, 215}

$$-\frac{2i(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}-\frac{3i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}-\frac{3\sinh^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-3*I)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b - ((2*I)*(1 + I*a +
I*b*x)^(3/2))/(b*Sqrt[1 - I*a - I*b*x]) - (3*ArcSinh[a + b*x])/b
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{3i\tan^{-1}(a+bx)}\,dx &= \int \frac{(1+ia+ibx)^{3/2}}{(1-ia-ibx)^{3/2}}\,dx\\
&= -\frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - 3\int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx\\
&= -\frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - 3\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx\\
&= -\frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - 3\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx\\
&= -\frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{2b^2}\\
&= -\frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{3\sinh^{-1}(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0393388, size = 45, normalized size = 0.48

$$-\frac{3\sinh^{-1}(a+bx)}{b} + \frac{\sqrt{(a+bx)^2+1}\left(\frac{4}{a+bx+i}-i\right)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] (Sqrt[1 + (a + b*x)^2]*(-I + 4/(I + a + b*x)))/b - (3*ArcSinh[a + b*x])/b
```

Maple [B] time = 0.092, size = 362, normalized size = 3.9

$$\frac{-5\,i}{b}\frac{1}{\sqrt{b^2x^2+2\,xab+a^2+1}}+3\,\frac{a}{b\sqrt{b^2x^2+2\,xab+a^2+1}}+3\,\frac{a^2x}{\sqrt{b^2x^2+2\,xab+a^2+1}}+3\,\frac{a^3}{b\sqrt{b^2x^2+2\,xab+a^2+1}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x)
```

```
[Out] -5*I/b/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3/b*a/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*
a^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+3/b*a^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+2
*(1+I*a)^3*(2*b^2*x+2*a*b)/(4*b^2*(a^2+1)-4*a^2*b^2)/(b^2*x^2+2*a*b*x+a^2+1
)^(1/2)+I*a^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+I/b*a^4/(b^2*x^2+2*a*b*x+a^2+
```

```
1)^(1/2)-I*b*x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-5*I*a*x/(b^2*x^2+2*a*b*x+a^2
+1)^(1/2)+3*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-4*I/b*a^2/(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)-3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^
(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.16026, size = 265, normalized size = 2.82

$$\frac{(-i\,a+8)bx - i\,a^2 + (6\,bx+6\,a+6i)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right) + \sqrt{b^2x^2+2\,abx+a^2+1}(-2i\,bx-2i\,a+10}{2\,b^2x+(2\,a+2i)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((-I*a + 8)*b*x - I*a^2 + (6*b*x + 6*a + 6*I)*log(-b*x - a + sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-2*I*b*x - 2*I*a
+ 10) + 9*a + 8*I)/(2*b^2*x + (2*a + 2*I)*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia+ibx+1)^3}{\left(a^2+2abx+b^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2),x)
```

```
[Out] Integral((I*a + I*b*x + 1)**3/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

Giac [B] time = 1.16944, size = 262, normalized size = 2.79

$$-\frac{\sqrt{(bx+a)^2+1}i}{b} + \frac{\log\left(3\left(x|b| - \sqrt{(bx+a)^2+1}\right)^2 ab + a^3b + 2\left(x|b| - \sqrt{(bx+a)^2+1}\right)^2 bi + 2\,a^2bi + \left(x|b| - \sqrt{(bx+a)^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt((b*x + a)^2 + 1)*i/b + log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b
 + a^3*b + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b*i + 2*a^2*b*i + (x*abs(
b) - sqrt((b*x + a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))
*a^2*abs(b) + 4*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a*i*abs(b) - a*b - (x*ab
s(b) - sqrt((b*x + a)^2 + 1))*abs(b))/abs(b) + 2*abs(b)*log(12*b)/b^2
```

3.185 $\int \frac{e^{3i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=134

$$\frac{4\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}-i\sinh^{-1}(a+bx)-\frac{2(-a+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(a+i)^{3/2}}$$

```
[Out] (4*Sqrt[1 + I*a + I*b*x])/((1 - I*a)*Sqrt[1 - I*a - I*b*x]) - I*ArcSinh[a +
 b*x] - (2*(I - a)^(3/2)*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[
I - a]*Sqrt[1 - I*a - I*b*x])])/(I + a)^(3/2)
```

Rubi [A] time = 0.0950191, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5095, 98, 157, 53, 619, 215, 93, 208}

$$\frac{4\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}-i\sinh^{-1}(a+bx)-\frac{2(-a+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(a+i)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x])/x,x]
```

```
[Out] (4*Sqrt[1 + I*a + I*b*x])/((1 - I*a)*Sqrt[1 - I*a - I*b*x]) - I*ArcSinh[a +
 b*x] - (2*(I - a)^(3/2)*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[
I - a]*Sqrt[1 - I*a - I*b*x])])/(I + a)^(3/2)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
```

```
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3i\tan^{-1}(a+bx)}}{x}\,dx &= \int \frac{(1+ia+ibx)^{3/2}}{x(1-ia-ibx)^{3/2}}\,dx \\
&= \frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}} - \frac{2\int \frac{\frac{1}{2}i(i-a)^2b-\frac{1}{2}(1-ia)b^2x}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{(i+a)b} \\
&= \frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}} - \frac{(i-a)^2\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{1-ia} - (ib)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx \\
&= \frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}} - \frac{\left(2(i-a)^2\right)\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{1-ia} - (ib)\int \frac{}{\sqrt{(1-ia)(1}} \\
&= \frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}} - \frac{2(i-a)^{3/2}\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i+a)^{3/2}} - \frac{i\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right.}{2b} \\
&= \frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}} - i\sinh^{-1}(a+bx) - \frac{2(i-a)^{3/2}\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i+a)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.713648, size = 229, normalized size = 1.71

$$
\frac{2\sqrt[4]{-1}(-ib)^{3/2}\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{b^{3/2}} + \frac{2i\left(2\sqrt{-1+ia}\sqrt{1+ia}\sqrt{ia+ibx+1} - (a-i)^2\sqrt{-i(a+bx+i)}\tan^{-1}\left(\frac{\sqrt{-i}}{\sqrt{\frac{a+i}{a-i}}}\right.\right.}{\sqrt{-1+ia}\sqrt{1+ia}(a+i)\sqrt{-i(a+bx+i)}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])/x,x]
```

```
[Out] (2*(-1)^(1/4)*((-I)*b)^(3/2)*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a
+ b*x)])/Sqrt[(-I)*b]])/b^(3/2) + ((2*I)*(2*Sqrt[-1 + I*a]*Sqrt[1 + I*a]*Sq
rt[1 + I*a + I*b*x] - (-I + a)^2*Sqrt[(-I)*(I + a + b*x)]*ArcTan[Sqrt[(-I)*
(I + a + b*x)]/(Sqrt[(I + a)/(-I + a)]*Sqrt[1 + I*a + I*b*x])]))/(Sqrt[-1 +
 I*a]*Sqrt[1 + I*a]*(I + a)*Sqrt[(-I)*(I + a + b*x)])
```

Maple [B] time = 0.11, size = 818, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x)
```

```
[Out] I*b*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*I/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)*a-4*a^2/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*a^4/(a^2+1)/(b^2*x^2+2*a
*b*x+a^2+1)^(1/2)+3/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*
x^2+2*a*b*x+a^2+1)^(1/2))/x)*a^2+I*a^4*b/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)*x+1/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*a^3*b/(a^2+1)/(b^2*x^2+2*a*
b*x+a^2+1)^(1/2)*x-a*b/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+6*I*(1+I*a)^
2*b*(2*b^2*x+2*a*b)/(4*b^2*(a^2+1)-4*a^2*b^2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
-4*I/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3-3*I/(a^2+1)^(3/2)*ln((2*a^2+
2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)*a+3/(b^2*x^2+2*
a*b*x+a^2+1)^(1/2)+2*I*a/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*I*a^2*b/(a^2+1)/(b
^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+2*I*b*a^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+I*a
^5/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-I*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^
2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+3*b*a/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
*x+I/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/x)*a^3+2*I*a^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*a^2/(b^2*x^2+2*a*
b*x+a^2+1)^(1/2)-1/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x
^2+2*a*b*x+a^2+1)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.50094, size = 983, normalized size = 7.34

$$-\frac{\left((a+i)bx+a^2+2i\,a-1\right)\sqrt{-\frac{4\,a^3-12i\,a^2-12\,a+4i}{a^3+3i\,a^2-3\,a-i}}\log\left(-\frac{(2\,a-2i)bx-\sqrt{b^2x^2+2\,abx+a^2+1}(2\,a-2i)-\left(i\,a^2-2\,a-i\right)\sqrt{-\frac{4\,a^3-12i\,a^2-12\,a+4i}{a^3+3i\,a^2-3\,a-i}}}{2\,a-2i}\right)-\left((a+i)b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -(((a + I)*b*x + a^2 + 2*I*a - 1)*sqrt(-(4*a^3 - 12*I*a^2 - 12*a + 4*I)/(a^
3 + 3*I*a^2 - 3*a - I))*log(-((2*a - 2*I)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*(2*a - 2*I) - (I*a^2 - 2*a - I)*sqrt(-(4*a^3 - 12*I*a^2 - 12*a + 4*I
)/(a^3 + 3*I*a^2 - 3*a - I)))/(2*a - 2*I)) - ((a + I)*b*x + a^2 + 2*I*a - 1
)*sqrt(-(4*a^3 - 12*I*a^2 - 12*a + 4*I)/(a^3 + 3*I*a^2 - 3*a - I))*log(-((2
*a - 2*I)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a - 2*I) - (-I*a^2 + 2
*a + I)*sqrt(-(4*a^3 - 12*I*a^2 - 12*a + 4*I)/(a^3 + 3*I*a^2 - 3*a - I)))/(
2*a - 2*I)) + 8*b*x + (2*(-I*a + 1)*b*x - 2*I*a^2 + 4*a + 2*I)*log(-b*x - a
 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 8*a + 8*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1) + 8*I)/((2*a + 2*I)*b*x + 2*a^2 + 4*I*a - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia + ibx + 1)^3}{x\left(a^2 + 2abx + b^2x^2 + 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)/x,x)
```

```
[Out] Integral((I*a + I*b*x + 1)**3/(x*(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)),
x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.186 $\int \frac{e^{3i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=176

$$-\frac{(ia+ibx+1)^{3/2}}{(1-ia)x\sqrt{-ia-ibx+1}}-\frac{6ib\sqrt{ia+ibx+1}}{(a+i)^2\sqrt{-ia-ibx+1}}+\frac{6i\sqrt{-a+i}b\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(a+i)^{5/2}}$$

```
[Out] ((-6*I)*b*Sqrt[1 + I*a + I*b*x])/((I + a)^2*Sqrt[1 - I*a - I*b*x]) - (1 + I
*a + I*b*x)^(3/2)/((1 - I*a)*x*Sqrt[1 - I*a - I*b*x]) + ((6*I)*Sqrt[I - a]*
b*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I
*b*x])])/(I + a)^(5/2)
```

Rubi [A] time = 0.0833436, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5095, 94, 93, 208}

$$-\frac{(ia+ibx+1)^{3/2}}{(1-ia)x\sqrt{-ia-ibx+1}}-\frac{6ib\sqrt{ia+ibx+1}}{(a+i)^2\sqrt{-ia-ibx+1}}+\frac{6i\sqrt{-a+i}b\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(a+i)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x])/x^2,x]
```


```
[Out] ((-6*I)*b*Sqrt[1 + I*a + I*b*x])/((I + a)^2*Sqrt[1 - I*a - I*b*x]) - (1 + I
*a + I*b*x)^(3/2)/((1 - I*a)*x*Sqrt[1 - I*a - I*b*x]) + ((6*I)*Sqrt[I - a]*
b*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I
*b*x])])/(I + a)^(5/2)
```


Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3i\tan^{-1}(a+bx)}}{x^2}\,dx = \int \frac{(1+ia+ibx)^{3/2}}{x^2(1-ia-ibx)^{3/2}}\,dx$$
$$= -\frac{(1+ia+ibx)^{3/2}}{(1-ia)x\sqrt{1-ia-ibx}} - \frac{(3b)\int \frac{\sqrt{1+ia+ibx}}{x(1-ia-ibx)^{3/2}}\,dx}{i+a}$$
$$= -\frac{6ib\sqrt{1+ia+ibx}}{(i+a)^2\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{3/2}}{(1-ia)x\sqrt{1-ia-ibx}} - \frac{(3(i-a)b)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{(i+a)^2}$$
$$= -\frac{6ib\sqrt{1+ia+ibx}}{(i+a)^2\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{3/2}}{(1-ia)x\sqrt{1-ia-ibx}} - \frac{(6(i-a)b)\,\mathrm{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia}}{\sqrt{1-ia}}\right.}{(i+a)^2}$$
$$= -\frac{6ib\sqrt{1+ia+ibx}}{(i+a)^2\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{3/2}}{(1-ia)x\sqrt{1-ia-ibx}} + \frac{6i\sqrt{i-a}b\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i+a)^{5/2}}$$

Mathematica [A] time = 0.15052, size = 143, normalized size = 0.81

$$\frac{\frac{\sqrt{ia+ibx+1}\left(a^2+abx-5ibx+1\right)}{x\sqrt{-i(a+bx+i)}}+\frac{6(a-i)b\tan^{-1}\left(\frac{\sqrt{-i(a+bx+i)}}{\sqrt{\frac{a+i}{a-i}}\sqrt{ia+ibx+1}}\right)}{\sqrt{-1+ia}\sqrt{1+ia}}}{(a+i)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] ((Sqrt[1 + I*a + I*b*x]*(1 + a^2 - (5*I)*b*x + a*b*x))/(x*Sqrt[(-I)*(I + a
+ b*x)]) + (6*(-I + a)*b*ArcTan[Sqrt[(-I)*(I + a + b*x)]/(Sqrt[(I + a)/(-I
+ a)]*Sqrt[1 + I*a + I*b*x])]))/(Sqrt[-1 + I*a]*Sqrt[1 + I*a]))/(I + a)^2
```

Maple [B] time = 0.117, size = 1358, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x)
```

```
[Out] 12/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b^2*a^2-3*I/(a^2+1)/x/(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)*a+12*I*a^4*b/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-9*
I*a^2*b/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*I*a^6*b/(a^2+1)^2/(b^2*x^
2+2*a*b*x+a^2+1)^(1/2)-3*I*a^4*b/(a^2+1)^(5/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1
)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+9*I*a^2*b/(a^2+1)^(5/2)*ln((2*a^2
+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+I/(a^2+1)/x/(b
^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3-9*a^4*b^2/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)*x+I*b^2*a/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+3*a^2*b^2/(a^2+1)^2/(b^2*x
^2+2*a*b*x+a^2+1)^(1/2)*x+I*b/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/(a^2+1)/x/(b^
```

```
2*x^2+2*a*b*x+a^2+1)^(1/2)+5*I/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^4*b+
3*I*a^2*b/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/x)-6*I*a*b^2*(2*b^2*x+2*a*b)/(4*b^2*(a^2+1)-4*a^2*b^2)/(b^2
*x^2+2*a*b*x+a^2+1)^(1/2)+5*I/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b^2*a
^3-9*I/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b^2*a+9*I*a^3*b^2/(a^2+1)^2/
(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-3*I*a^5*b^2/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)*x-12*I/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^2*b-2/(a^2+1)/(b^2*
x^2+2*a*b*x+a^2+1)^(1/2)*x*b^2-8/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a*b+
3/(a^2+1)/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^2-9*a^5*b/(a^2+1)^2/(b^2*x^2+2*
a*b*x+a^2+1)^(1/2)-9*a^3*b/(a^2+1)^(5/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2
)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+12/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)*a^3*b+I*b*a^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*a*b/(a^2+1)^2/(b^2*x^2+2*a*
b*x+a^2+1)^(1/2)+12*a^3*b/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+6*a*b/(a^
2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2
))/x)-6*b^2*(2*b^2*x+2*a*b)/(4*b^2*(a^2+1)-4*a^2*b^2)/(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)+3*a*b/(a^2+1)^(5/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2
*a*b*x+a^2+1)^(1/2))/x)+3*I*b/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*I*b/(
a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.40508, size = 1030, normalized size = 5.85

$$-\frac{2(-ia-5)b^2x^2-\left(2ia^2+8a+10i\right)bx-\left(\left(a^2+2ia-1\right)bx^2+\left(a^3+3ia^2-3a-i\right)x\right)\sqrt{\frac{(36a-36i)b^2}{a^5+5ia^4-10a^3-10ia^2+5a+i}}\log\left(-\frac{6b^2}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*(-I*a - 5)*b^2*x^2 - (2*I*a^2 + 8*a + 10*I)*b*x - ((a^2 + 2*I*a - 1)*b*
x^2 + (a^3 + 3*I*a^2 - 3*a - I)*x)*sqrt((36*a - 36*I)*b^2/(a^5 + 5*I*a^4 -
10*a^3 - 10*I*a^2 + 5*a + I))*log(-1/6*(6*b^2*x + (a^3 + 3*I*a^2 - 3*a - I)
*sqrt((36*a - 36*I)*b^2/(a^5 + 5*I*a^4 - 10*a^3 - 10*I*a^2 + 5*a + I)) - 6*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b)/b) + ((a^2 + 2*I*a - 1)*b*x^2 + (a^3 +
 3*I*a^2 - 3*a - I)*x)*sqrt((36*a - 36*I)*b^2/(a^5 + 5*I*a^4 - 10*a^3 - 10*
I*a^2 + 5*a + I))*log(-1/6*(6*b^2*x - (a^3 + 3*I*a^2 - 3*a - I)*sqrt((36*a
- 36*I)*b^2/(a^5 + 5*I*a^4 - 10*a^3 - 10*I*a^2 + 5*a + I)) - 6*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*b)/b) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*(-I*a -
5)*b*x - 2*I*a^2 - 2*I))/(2*(a^2 + 2*I*a - 1)*b*x^2 + (2*a^3 + 6*I*a^2 - 6*
a - 2*I)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia+ibx+1)^3}{x^2\left(a^2+2abx+b^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)/x**2,x)
```

```
[Out] Integral((I*a + I*b*x + 1)**3/(x**2*(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] undef
```

3.187 $\int \frac{e^{3i\tan^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=264

$$-\frac{(ia+ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{-ia-ibx+1}}+\frac{3(-2a+3i)b^2\sqrt{ia+ibx+1}}{(1+ia)(a+i)^3\sqrt{-ia-ibx+1}}+\frac{3(3+2ia)b^2\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}(a+i)^{7/2}}+\frac{(-2a+3i)b(}{2(1+ia)(a+i)}$$

```
[Out] (3*(3*I - 2*a)*b^2*Sqrt[1 + I*a + I*b*x])/((1 + I*a)*(I + a)^3*Sqrt[1 - I*a
 - I*b*x]) + ((3*I - 2*a)*b*(1 + I*a + I*b*x)^(3/2))/(2*(1 + I*a)*(I + a)^2
*x*Sqrt[1 - I*a - I*b*x]) - (1 + I*a + I*b*x)^(5/2)/(2*(1 + a^2)*x^2*Sqrt[1
 - I*a - I*b*x]) + (3*(3 + (2*I)*a)*b^2*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a +
 I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/(Sqrt[I - a]*(I + a)^(7/2))
```

Rubi [A] time = 0.157934, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5095, 96, 94, 93, 208}

$$-\frac{(ia+ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{-ia-ibx+1}}+\frac{3(-2a+3i)b^2\sqrt{ia+ibx+1}}{(1+ia)(a+i)^3\sqrt{-ia-ibx+1}}+\frac{3(3+2ia)b^2\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}(a+i)^{7/2}}+\frac{(-2a+3i)b(}{2(1+ia)(a+i)}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x])/x^3,x]
```

```
[Out] (3*(3*I - 2*a)*b^2*Sqrt[1 + I*a + I*b*x])/((1 + I*a)*(I + a)^3*Sqrt[1 - I*a
 - I*b*x]) + ((3*I - 2*a)*b*(1 + I*a + I*b*x)^(3/2))/(2*(1 + I*a)*(I + a)^2
*x*Sqrt[1 - I*a - I*b*x]) - (1 + I*a + I*b*x)^(5/2)/(2*(1 + a^2)*x^2*Sqrt[1
 - I*a - I*b*x]) + (3*(3 + (2*I)*a)*b^2*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a +
 I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/(Sqrt[I - a]*(I + a)^(7/2))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3i\tan^{-1}(a+bx)}}{x^3}\,dx = \int \frac{(1+ia+ibx)^{3/2}}{x^3(1-ia-ibx)^{3/2}}\,dx$$
$$= -\frac{(1+ia+ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1-ia-ibx}} + \frac{((3i-2a)b)\int \frac{(1+ia+ibx)^{3/2}}{x^2(1-ia-ibx)^{3/2}}\,dx}{2\left(1+a^2\right)}$$
$$= -\frac{(3i-2a)b(1+ia+ibx)^{3/2}}{2(1-ia)\left(1+a^2\right)x\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1-ia-ibx}} - \frac{\left(3(3i-2a)b^2\right)\int \frac{\sqrt{1+ia+ibx}}{x(1-ia-ibx)^{3/2}}\,dx}{2(i+a)\left(1+a^2\right)}$$
$$= -\frac{3(3i-2a)b^2\sqrt{1+ia+ibx}}{(i-a)(1-ia)^3\sqrt{1-ia-ibx}} - \frac{(3i-2a)b(1+ia+ibx)^{3/2}}{2(1-ia)\left(1+a^2\right)x\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1-ia-ibx}} +$$
$$= -\frac{3(3i-2a)b^2\sqrt{1+ia+ibx}}{(i-a)(1-ia)^3\sqrt{1-ia-ibx}} - \frac{(3i-2a)b(1+ia+ibx)^{3/2}}{2(1-ia)\left(1+a^2\right)x\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1-ia-ibx}} +$$
$$= -\frac{3(3i-2a)b^2\sqrt{1+ia+ibx}}{(i-a)(1-ia)^3\sqrt{1-ia-ibx}} - \frac{(3i-2a)b(1+ia+ibx)^{3/2}}{2(1-ia)\left(1+a^2\right)x\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1-ia-ibx}} +$$

Mathematica [A] time = 0.206453, size = 180, normalized size = 0.68

$$\frac{\frac{\sqrt{ia+ibx+1}\left(a^3+ia^2-ab^2x^2+5iabx+a+14ib^2x^2-5bx+i\right)}{x^2\sqrt{-i(a+bx+i)}} - \frac{6(2a-3i)b^2\tan^{-1}\left(\frac{\sqrt{-i(a+bx+i)}}{\sqrt{\frac{a+i}{a-i}}\sqrt{ia+ibx+1}}\right)}{\sqrt{-1+ia}\sqrt{1+ia}}}{2(a+i)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])/x^3,x]
```

```
[Out] ((Sqrt[1 + I*a + I*b*x]*(I + a + I*a^2 + a^3 - 5*b*x + (5*I)*a*b*x + (14*I)
*b^2*x^2 - a*b^2*x^2))/(x^2*Sqrt[(-I)*(I + a + b*x)]) - (6*(-3*I + 2*a)*b^2
*ArcTan[Sqrt[(-I)*(I + a + b*x)]/(Sqrt[(I + a)/(-I + a)]*Sqrt[1 + I*a + I*b
*x])])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a]))/(2*(I + a)^3)
```

Maple [B] time = 0.116, size = 1955, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x)
```

```
[Out] 9*I/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b^3*a^2+3*I*b/(a^2+1)/x/(b^2*x^
2+2*a*b*x+a^2+1)^(1/2)*a^2-5/2*I*a^4*b/(a^2+1)^2/x/(b^2*x^2+2*a*b*x+a^2+1)^
(1/2)+15/2*I*a^2*b/(a^2+1)^2/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+15/2*I*a^6*b^3
/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-45/2*I*a^4*b^3/(a^2+1)^3/(b^2*x^
2+2*a*b*x+a^2+1)^(1/2)*x-31/2*I*a^4*b^3/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)*x+57/2*I*a^2*b^3/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-15/2*a^3*b/
(a^2+1)^2/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-75/2*a^3*b^3/(a^2+1)^2/(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)*x+5/2*a*b/(a^2+1)^2/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-15/
2*a^3*b^3/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+13/2*a*b^3/(a^2+1)^2/(b
^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-2*I*b^3*(2*b^2*x+2*a*b)/(4*b^2*(a^2+1)-4*a^2*
b^2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-21/2*I*b^2/(a^2+1)^(5/2)*ln((2*a^2+2+2*x
*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)*a^3-27/2*I*b^2/(a^2+
1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+39*I*a^3*b^2/(a^2+1)^2/(b^2*x^2+2*a*b*
x+a^2+1)^(1/2)+27/2*I*b^2/(a^2+1)^(5/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)
*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)*a-9*I/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)*a*b^2+9*I/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3*b^2+3*I*b^2/(a^2+1)
^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x
)*a-3*I*b/(a^2+1)/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-6*I/(a^2+1)/(b^2*x^2+2*a*
b*x+a^2+1)^(1/2)*x*b^3+6*b/(a^2+1)/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+15/(a^
2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b^3*a+45/2*a^5*b^3/(a^2+1)^3/(b^2*x^2+
2*a*b*x+a^2+1)^(1/2)*x+15/2*I*a^5*b^2/(a^2+1)^(7/2)*ln((2*a^2+2+2*x*a*b+2*(
a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-45/2*I*a^3*b^2/(a^2+1)^(7/2)
*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+1/2*
I/(a^2+1)/x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3-3/2*I/(a^2+1)/x^2/(b^2*x^2+
2*a*b*x+a^2+1)^(1/2)*a-30*I*a^5*b^2/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
+45/2*I*a^3*b^2/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+15/2*I*a^7*b^2/(a^2
+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*b^2/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)-3/2*b^2/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2/(a^2+1)/x^2/(b^2*x
^2+2*a*b*x+a^2+1)^(1/2)+3*b^2/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(
1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+3/2*b^2/(a^2+1)^(5/2)*ln((2*a^2+2+2*
x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-31/2*I*a^5*b^2/(a^2
+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+15/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)*a^2*b^2-15/2*a^2*b^2/(a^2+1)^(7/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b
^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-45/2*a^2*b^2/(a^2+1)^(5/2)*ln((2*a^2+2+2*x*
a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+3/2/(a^2+1)/x^2/(b^2*
x^2+2*a*b*x+a^2+1)^(1/2)*a^2+45/2*a^6*b^2/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)+45/2*a^4*b^2/(a^2+1)^(7/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*
x^2+2*a*b*x+a^2+1)^(1/2))/x)-75/2*a^4*b^2/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)+15/2*a^2*b^2/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-30*a^4*b^2/(a^2
+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+29*a^2*b^2/(a^2+1)^2/(b^2*x^2+2*a*b*x+a
^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.39899, size = 1462, normalized size = 5.54

$$(-i\,a-14)b^3x^3+\left(-i\,a^2-13\,a-14i\right)b^2x^2-3\left(\left(a^3+3i\,a^2-3\,a-i\right)bx^3+\left(a^4+4i\,a^3-6\,a^2-4i\,a+1\right)x^2\right)\sqrt{\frac{\left(\right.}{a^8+6i\,a^7-14\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] ((-I*a - 14)*b^3*x^3 + (-I*a^2 - 13*a - 14*I)*b^2*x^2 - 3*((a^3 + 3*I*a^2 -
 3*a - I)*b*x^3 + (a^4 + 4*I*a^3 - 6*a^2 - 4*I*a + 1)*x^2)*sqrt((4*a^2 - 12
*I*a - 9)*b^4/(a^8 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*
a - 1))*log(-((6*a - 9*I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a -
9*I)*b^2 + 3*(a^5 + 3*I*a^4 - 2*a^3 + 2*I*a^2 - 3*a - I)*sqrt((4*a^2 - 12*I
*a - 9)*b^4/(a^8 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a
- 1)))/((6*a - 9*I)*b^2)) + 3*((a^3 + 3*I*a^2 - 3*a - I)*b*x^3 + (a^4 + 4*I
*a^3 - 6*a^2 - 4*I*a + 1)*x^2)*sqrt((4*a^2 - 12*I*a - 9)*b^4/(a^8 + 6*I*a^7
 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1))*log(-((6*a - 9*I)*b^
3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a - 9*I)*b^2 - 3*(a^5 + 3*I*a^4
- 2*a^3 + 2*I*a^2 - 3*a - I)*sqrt((4*a^2 - 12*I*a - 9)*b^4/(a^8 + 6*I*a^7 -
 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1)))/((6*a - 9*I)*b^2)) +
((-I*a - 14)*b^2*x^2 + I*a^3 - (5*a + 5*I)*b*x - a^2 + I*a - 1)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))/((2*a^3 + 6*I*a^2 - 6*a - 2*I)*b*x^3 + (2*a^4 + 8*I
*a^3 - 12*a^2 - 8*I*a + 2)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia+ibx+1)^3}{x^3\left(a^2+2abx+b^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)/x**3,x)
```

```
[Out] Integral((I*a + I*b*x + 1)**3/(x**3*(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

3.188 $\int \frac{e^{3i\tan^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=338

$$\frac{\left(-2a^2+51ia+52\right)b^3\sqrt{ia+ibx+1}}{6(-a+i)(a+i)^4\sqrt{-ia-ibx+1}}-\frac{\left(-6ia^2-18a+11i\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}(a+i)^{9/2}}+\frac{(19+16ia)b^2\sqrt{ia+ibx+1}}{6(-a+i)(a+i)^3x\sqrt{-ia-ibx+}}$$

```
[Out] ((52 + (51*I)*a - 2*a^2)*b^3*Sqrt[1 + I*a + I*b*x])/(6*(I - a)*(I + a)^4*Sq
rt[1 - I*a - I*b*x]) - ((I - a)*Sqrt[1 + I*a + I*b*x])/(3*(I + a)*x^3*Sqrt[
1 - I*a - I*b*x]) + (((7*I)/6)*b*Sqrt[1 + I*a + I*b*x])/((I + a)^2*x^2*Sqrt
[1 - I*a - I*b*x]) + ((19 + (16*I)*a)*b^2*Sqrt[1 + I*a + I*b*x])/(6*(I - a)
*(I + a)^3*x*Sqrt[1 - I*a - I*b*x]) - ((11*I - 18*a - (6*I)*a^2)*b^3*ArcTan
h[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])
/((I - a)^(3/2)*(I + a)^(9/2))
```

Rubi [A] time = 0.269927, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 98, 151, 152, 12, 93, 208}

$$\frac{\left(-2a^2+51ia+52\right)b^3\sqrt{ia+ibx+1}}{6(-a+i)(a+i)^4\sqrt{-ia-ibx+1}}-\frac{\left(-6ia^2-18a+11i\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}(a+i)^{9/2}}+\frac{(19+16ia)b^2\sqrt{ia+ibx+1}}{6(-a+i)(a+i)^3x\sqrt{-ia-ibx+}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x])/x^4,x]
```

```
[Out] ((52 + (51*I)*a - 2*a^2)*b^3*Sqrt[1 + I*a + I*b*x])/(6*(I - a)*(I + a)^4*Sq
rt[1 - I*a - I*b*x]) - ((I - a)*Sqrt[1 + I*a + I*b*x])/(3*(I + a)*x^3*Sqrt[
1 - I*a - I*b*x]) + (((7*I)/6)*b*Sqrt[1 + I*a + I*b*x])/((I + a)^2*x^2*Sqrt
[1 - I*a - I*b*x]) + ((19 + (16*I)*a)*b^2*Sqrt[1 + I*a + I*b*x])/(6*(I - a)
*(I + a)^3*x*Sqrt[1 - I*a - I*b*x]) - ((11*I - 18*a - (6*I)*a^2)*b^3*ArcTan
h[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])
/((I - a)^(3/2)*(I + a)^(9/2))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3i\tan^{-1}(a+bx)}}{x^4}dx = \int \frac{(1+ia+ibx)^{3/2}}{x^4(1-ia-ibx)^{3/2}}dx$$
$$= -\frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} - \frac{\int \frac{-7(i-a)b+6b^2x}{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}dx}{3(1-ia)}$$
$$= -\frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}} + \frac{\int \frac{-(19+35ia-16a^2)b^2-14(i-a)b^3x}{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}dx}{6(1-ia)(1+a^2)}$$
$$= -\frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}} + \frac{(19i-16a)b^2\sqrt{1+ia+ibx}}{6(i-a)(1-ia)^3x\sqrt{1-ia-ibx}} - \frac{\int \frac{3(i-a)(11+}{x}}{}$$
$$= \frac{(52+51ia-2a^2)b^3\sqrt{1+ia+ibx}}{6(i-a)(i+a)^4\sqrt{1-ia-ibx}} - \frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}} + \frac{(19i-}{6(i-a)(}$$
$$= \frac{(52+51ia-2a^2)b^3\sqrt{1+ia+ibx}}{6(i-a)(i+a)^4\sqrt{1-ia-ibx}} - \frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}} + \frac{(19i-}{6(i-a)(}$$
$$= \frac{(52+51ia-2a^2)b^3\sqrt{1+ia+ibx}}{6(i-a)(i+a)^4\sqrt{1-ia-ibx}} - \frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}} + \frac{(19i-}{6(i-a)(}$$
$$= \frac{(52+51ia-2a^2)b^3\sqrt{1+ia+ibx}}{6(i-a)(i+a)^4\sqrt{1-ia-ibx}} - \frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}} + \frac{(19i-}{6(i-a)(}$$

Mathematica [A] time = 0.420622, size = 277, normalized size = 0.82

$$-\frac{-i(6a^2-18ia-11)b^2x^2\left(i\sqrt{-1+ia}\sqrt{ia+ibx+1}(a^2+abx-5ibx+1)+6\sqrt{1+ia}bx\sqrt{-i(a+bx+i)}\tan^{-1}\left(\frac{\sqrt{-i(a+bx+}}{\sqrt{\frac{a+i}{a-i}}\sqrt{ia+ibx}}\right.\right.}{6(-1+ia)^{5/2}(a^2+1)^2x^3\sqrt{-i(a+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])/x^4,x]
```

```
[Out] -(2*(-1 + I*a)^(3/2)*(1 + I*a)*(I + a)^2*(1 + I*a + I*b*x)^(5/2) + (3*I - 4
*a)*(-1 + I*a)^(5/2)*b*x*(1 + I*a + I*b*x)^(5/2) - I*(-11 - (18*I)*a + 6*a^
2)*b^2*x^2*(I*Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x]*(1 + a^2 - (5*I)*b*x + a
*b*x) + 6*Sqrt[1 + I*a]*b*x*Sqrt[(-I)*(I + a + b*x)]*ArcTan[Sqrt[(-I)*(I +
a + b*x)]/(Sqrt[(I + a)/(-I + a)]*Sqrt[1 + I*a + I*b*x])])))/(6*(-1 + I*a)^(
5/2)*(1 + a^2)^2*x^3*Sqrt[(-I)*(I + a + b*x)])
```

Maple [B] time = 0.119, size = 2624, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x)
```

```
[Out] -1/3/(a^2+1)/x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*I*b/(a^2+1)/x^2/(b^2*x^2
+2*a*b*x+a^2+1)^(1/2)+3*I*b^2/(a^2+1)/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-260
```

```
/3*a^3*b^3/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+205/2*a^5*b^3/(a^2+1)^3/
(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-56*a^3*b^3/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^
(1/2)+62/3*b^3/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-9/2*I*b^3/(a^2+1)^
2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+9/2*I*b^3/(a^2+1)^(5/2)*ln((2*a^2+2+2*x*a*b
+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+35/2*a^3*b^3/(a^2+1)^(9/
2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-15
/2*a*b^3/(a^2+1)^(7/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x
+a^2+1)^(1/2))/x)+3*b^2/(a^2+1)/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+6*b^4/(a^2+
1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+6*b^3/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)*a+I*b^3/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*
b*x+a^2+1)^(1/2))/x)-I*b^3/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/(a^2+1)/
x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^2-105/2*a^7*b^3/(a^2+1)^4/(b^2*x^2+2*a*
b*x+a^2+1)^(1/2)-105/2*a^5*b^3/(a^2+1)^(9/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^
(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+4/3*b^2/(a^2+1)^2/x/(b^2*x^2+2*a*b*
x+a^2+1)^(1/2)+8/3*b^4/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-56*a^2*b^4
/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-19*a^2*b^2/(a^2+1)^2/x/(b^2*x^2+
2*a*b*x+a^2+1)^(1/2)+205/2*a^4*b^4/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*
x+3*b/(a^2+1)/x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-110*I*a^4*b^3/(a^2+1)^3/(
b^2*x^2+2*a*b*x+a^2+1)^(1/2)+125/3*I*a^6*b^3/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2
+1)^(1/2)+7*I*b^3/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^2-27/2*I*b^3/(a^2
+1)^(5/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
)/x)*a^2-45*I*a^2*b^3/(a^2+1)^(7/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+45*I*a^2*b^3/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)+30*I*a^4*b^3/(a^2+1)^(7/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-187/6*I*a^4*b^3/(a^2+1)^2/(b^2*x^2+2*a*b*x+a
^2+1)^(1/2)+41*I*a^2*b^3/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-35/2*I*a^8
*b^3/(a^2+1)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-35/2*I*a^6*b^3/(a^2+1)^(9/2)*l
n((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+105/2*
I*a^4*b^3/(a^2+1)^(9/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/x)+1/3*I/(a^2+1)/x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3-I/(a
^2+1)/x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+70*I*a^6*b^3/(a^2+1)^4/(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)-105/2*I*a^4*b^3/(a^2+1)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
-18*a*b^3/(a^2+1)^(5/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/x)-35/2*a^3*b^3/(a^2+1)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+70*
a^5*b^3/(a^2+1)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+15/2*a*b^3/(a^2+1)^3/(b^2*x
^2+2*a*b*x+a^2+1)^(1/2)-35/6*a^2*b^2/(a^2+1)^3/x/(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)+35/2*a^4*b^4/(a^2+1)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-115/6*a^2*b^4/(a
^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+3/2*I*b/(a^2+1)/x^2/(b^2*x^2+2*a*b*
x+a^2+1)^(1/2)*a^2-53/6*I*a^3*b^2/(a^2+1)^2/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
+23/2*I*a*b^2/(a^2+1)^2/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+125/3*I*a^5*b^4/(a^
2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-80*I*a^3*b^4/(a^2+1)^3/(b^2*x^2+2*a*
b*x+a^2+1)^(1/2)*x-7/6*I*a^4*b/(a^2+1)^2/x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+
7/2*I*a^2*b/(a^2+1)^2/x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+35/6*I*a^5*b^2/(a^2
+1)^3/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-35/2*I*a^3*b^2/(a^2+1)^3/x/(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)-35/2*I*a^7*b^4/(a^2+1)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*
x+105/2*I*a^5*b^4/(a^2+1)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+7*I*b^4/(a^2+1)
/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*a+55/2*I*a*b^4/(a^2+1)^2/(b^2*x^2+2*a*b*x+
a^2+1)^(1/2)*x-187/6*I*a^3*b^4/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+13
5/2*a^3*b^3/(a^2+1)^(7/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*
b*x+a^2+1)^(1/2))/x)-105/2*a^6*b^4/(a^2+1)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*
x-7/2*a^3*b/(a^2+1)^2/x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+35/2*a^4*b^2/(a^2+1
)^3/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+7/6*a*b/(a^2+1)^2/x^2/(b^2*x^2+2*a*b*x+
a^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.62691, size = 2215, normalized size = 6.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] ((2*I*a^2 + 51*a - 52*I)*b^4*x^4 + (2*I*a^3 + 49*a^2 - I*a + 52)*b^3*x^3 +
sqrt((36*a^4 - 216*I*a^3 - 456*a^2 + 396*I*a + 121)*b^6/(a^12 + 6*I*a^11 -
12*a^10 - 2*I*a^9 - 27*a^8 - 36*I*a^7 - 36*I*a^5 + 27*a^4 - 2*I*a^3 + 12*a^
2 + 6*I*a - 1))*((3*a^5 + 9*I*a^4 - 6*a^3 + 6*I*a^2 - 9*a - 3*I)*b*x^4 + (3
*a^6 + 12*I*a^5 - 15*a^4 - 15*a^2 - 12*I*a + 3)*x^3)*log(-((6*a^2 - 18*I*a
- 11)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a^2 - 18*I*a - 11)*b^3 +
 (a^7 + 3*I*a^6 - a^5 + 5*I*a^4 - 5*a^3 + I*a^2 - 3*a - I)*sqrt((36*a^4 - 2
16*I*a^3 - 456*a^2 + 396*I*a + 121)*b^6/(a^12 + 6*I*a^11 - 12*a^10 - 2*I*a^
9 - 27*a^8 - 36*I*a^7 - 36*I*a^5 + 27*a^4 - 2*I*a^3 + 12*a^2 + 6*I*a - 1)))
/((6*a^2 - 18*I*a - 11)*b^3)) - sqrt((36*a^4 - 216*I*a^3 - 456*a^2 + 396*I*
a + 121)*b^6/(a^12 + 6*I*a^11 - 12*a^10 - 2*I*a^9 - 27*a^8 - 36*I*a^7 - 36*
I*a^5 + 27*a^4 - 2*I*a^3 + 12*a^2 + 6*I*a - 1))*((3*a^5 + 9*I*a^4 - 6*a^3 +
 6*I*a^2 - 9*a - 3*I)*b*x^4 + (3*a^6 + 12*I*a^5 - 15*a^4 - 15*a^2 - 12*I*a
+ 3)*x^3)*log(-((6*a^2 - 18*I*a - 11)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)*(6*a^2 - 18*I*a - 11)*b^3 - (a^7 + 3*I*a^6 - a^5 + 5*I*a^4 - 5*a^3 + I
*a^2 - 3*a - I)*sqrt((36*a^4 - 216*I*a^3 - 456*a^2 + 396*I*a + 121)*b^6/(a^
12 + 6*I*a^11 - 12*a^10 - 2*I*a^9 - 27*a^8 - 36*I*a^7 - 36*I*a^5 + 27*a^4 -
 2*I*a^3 + 12*a^2 + 6*I*a - 1)))/((6*a^2 - 18*I*a - 11)*b^3)) + ((2*I*a^2 +
 51*a - 52*I)*b^3*x^3 + 2*I*a^5 + (16*a^2 - 3*I*a + 19)*b^2*x^2 - 2*a^4 + 4
*I*a^3 - (7*a^3 + 7*I*a^2 + 7*a + 7*I)*b*x - 4*a^2 + 2*I*a - 2)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))/((6*a^5 + 18*I*a^4 - 12*a^3 + 12*I*a^2 - 18*a - 6*I
)*b*x^4 + (6*a^6 + 24*I*a^5 - 30*a^4 - 30*a^2 - 24*I*a + 6)*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

3.189 $\int e^{-i\tan^{-1}(a+bx)}x^4\,dx$

Optimal. Leaf size=276

$$-\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(2\left(-36a^2+14ia+13\right)bx+96a^3-86ia^2-114a+19i\right)}{120b^5}-\frac{\left(8ia^4+16a^3-24ia^2-12a+\right.}{8}$$

```
[Out] -((3*I - 12*a - (24*I)*a^2 + 16*a^3 + (8*I)*a^4)*Sqrt[1 - I*a - I*b*x]*Sqrt
[1 + I*a + I*b*x])/(8*b^5) + ((I - 8*a)*x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1
+ I*a + I*b*x])/(20*b^3) + (x^3*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*
x])/(5*b^2) - ((1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(19*I - 114*a
- (86*I)*a^2 + 96*a^3 + 2*(13 + (14*I)*a - 36*a^2)*b*x))/(120*b^5) + ((3 +
(12*I)*a - 24*a^2 - (16*I)*a^3 + 8*a^4)*ArcSinh[a + b*x])/(8*b^5)
```

Rubi [A] time = 0.224759, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5095, 100, 153, 147, 50, 53, 619, 215}

$$-\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(2\left(-36a^2+14ia+13\right)bx+96a^3-86ia^2-114a+19i\right)}{120b^5}-\frac{\left(8ia^4+16a^3-24ia^2-12a+\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] -((3*I - 12*a - (24*I)*a^2 + 16*a^3 + (8*I)*a^4)*Sqrt[1 - I*a - I*b*x]*Sqrt
[1 + I*a + I*b*x])/(8*b^5) + ((I - 8*a)*x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1
+ I*a + I*b*x])/(20*b^3) + (x^3*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*
x])/(5*b^2) - ((1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(19*I - 114*a
- (86*I)*a^2 + 96*a^3 + 2*(13 + (14*I)*a - 36*a^2)*b*x))/(120*b^5) + ((3 +
(12*I)*a - 24*a^2 - (16*I)*a^3 + 8*a^4)*ArcSinh[a + b*x])/(8*b^5)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-i\tan^{-1}(a+bx)}x^4\,dx = \int \frac{x^4\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx$$

$$= \frac{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} + \frac{\int \frac{x^2\sqrt{1-ia-ibx}\left(-3\left(1+a^2\right)+(i-8a)bx\right)}{\sqrt{1+ia+ibx}}dx}{5b^2}$$

$$= \frac{(i-8a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} + \frac{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} + \frac{\int \frac{x\sqrt{1-ia-ibx}(2(i-8a)(i-}{\sqrt{1}}$$

$$= \frac{(i-8a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} + \frac{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+}}{}$$

$$= -\frac{\left(3i-12a-24ia^2+16a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} + \frac{(i-8a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia}}{20b^3}$$

$$= -\frac{\left(3i-12a-24ia^2+16a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} + \frac{(i-8a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia}}{20b^3}$$

$$= -\frac{\left(3i-12a-24ia^2+16a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} + \frac{(i-8a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia}}{20b^3}$$

$$= -\frac{\left(3i-12a-24ia^2+16a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} + \frac{(i-8a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia}}{20b^3}$$

Mathematica [A] time = 0.758265, size = 248, normalized size = 0.9

$$\frac{i\sqrt{ia+ibx+1}\left(a^2\left(-84b^2x^2-346ibx+57\right)+2a^3(72bx-41i)+24ia^5+226a^4+a\left(64b^3x^3+154ib^2x^2-346bx-211i\right)+\right.}{120b^5\sqrt{-i(a+bx+i)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] ((I/120)*Sqrt[1 + I*a + I*b*x]*(-64 + 226*a^4 + (24*I)*a^5 + (109*I)*b*x +
77*b^2*x^2 - (62*I)*b^3*x^3 - 54*b^4*x^4 + (24*I)*b^5*x^5 + 2*a^3*(-41*I +
72*b*x) + a^2*(57 - (346*I)*b*x - 84*b^2*x^2) + a*(-211*I - 346*b*x + (154*
I)*b^2*x^2 + 64*b^3*x^3)))/(b^5*Sqrt[(-I)*(I + a + b*x)]) + ((-1)^(1/4)*(-3
*I + 12*a + (24*I)*a^2 - 16*a^3 - (8*I)*a^4)*ArcSinh[((1/2 + I/2)*Sqrt[b]*S
qrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*Sqrt[(-I)*b]*b^(9/2))
```

Maple [B] time = 0.223, size = 1208, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] 7/15*I/b^5*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-4/b^5*((x-(I-a)/b)^2*b^2+2*I*(x-(I
-a)/b)*b)^(1/2)*a^3+4/b^5*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a+1/b
^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*
b)^(1/2))/(b^2)^(1/2)-I/b^5*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)+3*a
```

```
^3/b^5*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-5/8/b^4*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)-5/8/b^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(
1/2)-5/2*I/b^4*a*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+2*I/b^4*a^3*x*(b^2*x^2+2*a
*b*x+a^2+1)^(1/2)-5/2*I/b^4*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a
^2+1)^(1/2))/(b^2)^(1/2)-4*I/b^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(
I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)*a^3+4*I/b^4*ln((I*b+(x-
(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2
)^(1/2)*a+3/5*I/b^4*a*x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-5/8*a/b^5*(b^2*x^2+2*
a*b*x+a^2+1)^(1/2)+1/4/b^4*x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-13/12/b^5*a*(b^2
*x^2+2*a*b*x+a^2+1)^(3/2)+2*I/b^4*a^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2
*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/b^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)
+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)*a^4-6/b^4*ln((I*b
+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/
(b^2)^(1/2)*a^2-I/b^5*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a^4+6*I/b
^5*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a^2-6/5*I/b^5*a^2*(b^2*x^2+2
*a*b*x+a^2+1)^(3/2)-5/2*I/b^5*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+2*I/b^5*a^4
*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*a^2/b^4*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/
5*I/b^3*x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+3*a^2/b^4*ln((b^2*x+a*b)/(b^2)^(1
/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Maxima [B] time = 1.63244, size = 616, normalized size = 2.23

$$\frac{2i\sqrt{b^2x^2+2\,abx+a^2+1}a^3x}{b^4}-\frac{i\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}x^2}{5\,b^3}+\frac{a^4\operatorname{arsinh}(bx+a)}{b^5}+\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}a^4}{b^5}+\frac{3i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3*x/b^4 - 1/5*I*(b^2*x^2 + 2*a*b*x
+ a^2 + 1)^(3/2)*x^2/b^3 + a^4*arcsinh(b*x + a)/b^5 + I*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)*a^4/b^5 + 3/5*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a*x/b^4
+ 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*x/b^4 - 2*I*a^3*arcsinh(b*x + a)/
b^5 - 6/5*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/b^5 - sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*a^3/b^5 + 1/4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x/b^4 -
5/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b^4 - 3*a^2*arcsinh(b*x + a)/b^
5 - 13/12*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/b^5 + 7/2*I*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)*a^2/b^5 - 5/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^4 +
3/2*I*a*arcsinh(b*x + a)/b^5 + 7/15*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/b
^5 + 27/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^5 + 3/8*arcsinh(b*x + a)/b^
5 - I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^5
```

Fricas [A] time = 2.36044, size = 524, normalized size = 1.9

$$\frac{-186i\,a^5-1345\,a^4+1730i\,a^3+1320\,a^2-\left(960\,a^4-1920i\,a^3-2880\,a^2+1440i\,a+360\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,ab}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/960*(-186*I*a^5 - 1345*a^4 + 1730*I*a^3 + 1320*a^2 - (960*a^4 - 1920*I*a^
3 - 2880*a^2 + 1440*I*a + 360)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)) + (-192*I*b^4*x^4 - 48*(-4*I*a - 5)*b^3*x^3 + (-192*I*a^2 - 560*a + 2
```

```
56*I)*b^2*x^2 - 192*I*a^4 - 2000*a^3 + (192*I*a^3 + 1040*a^2 - 928*I*a - 36
0)*b*x + 2656*I*a^2 + 2200*a - 512*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 3
00*I*a)/b^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12711, size = 289, normalized size = 1.05

$$-\frac{1}{120}\sqrt{(bx+a)^2+1}\left(\left(2\left(3\left(\frac{4\,ix}{b}-\frac{4\,ab^7i+5\,b^7}{b^9}\right)x+\frac{12\,a^2b^6i+35\,ab^6-16\,b^6i}{b^9}\right)x-\frac{24\,a^3b^5i+130\,a^2b^5-116\,ab^5i-45\,b}{b^9}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/120*sqrt((b*x + a)^2 + 1)*((2*(3*(4*i*x/b - (4*a*b^7*i + 5*b^7)/b^9)*x +
 (12*a^2*b^6*i + 35*a*b^6 - 16*b^6*i)/b^9)*x - (24*a^3*b^5*i + 130*a^2*b^5
- 116*a*b^5*i - 45*b^5)/b^9)*x + (24*a^4*b^4*i + 250*a^3*b^4 - 332*a^2*b^4*
i - 275*a*b^4 + 64*b^4*i)/b^9) - 1/8*(8*a^4 - 16*a^3*i - 24*a^2 + 12*a*i +
3)*log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^4*abs(b))
```

3.190 $\int e^{-i\tan^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=201

$$-\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-18a^2-2(-6a+i)bx+10ia+7\right)}{24b^4}-\frac{\left(-8ia^3-12a^2+12ia+3\right)\sqrt{-ia-ibx+1}\sqrt{ia+}}{8b^4}$$

```
[Out] -((3 + (12*I)*a - 12*a^2 - (8*I)*a^3)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a +
I*b*x])/(8*b^4) + (x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(4*b^
2) - ((1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(7 + (10*I)*a - 18*a^2
- 2*(I - 6*a)*b*x))/(24*b^4) - ((3*I - 12*a - (12*I)*a^2 + 8*a^3)*ArcSinh[a
 + b*x])/(8*b^4)
```

Rubi [A] time = 0.194344, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 100, 147, 50, 53, 619, 215}

$$-\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-18a^2-2(-6a+i)bx+10ia+7\right)}{24b^4}-\frac{\left(-8ia^3-12a^2+12ia+3\right)\sqrt{-ia-ibx+1}\sqrt{ia+}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] -((3 + (12*I)*a - 12*a^2 - (8*I)*a^3)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a +
I*b*x])/(8*b^4) + (x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(4*b^
2) - ((1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(7 + (10*I)*a - 18*a^2
- 2*(I - 6*a)*b*x))/(24*b^4) - ((3*I - 12*a - (12*I)*a^2 + 8*a^3)*ArcSinh[a
 + b*x])/(8*b^4)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
```

```
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-i\tan^{-1}(a+bx)}x^3\,dx = \int \frac{x^3\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx$$

$$= \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} + \frac{\int \frac{x\sqrt{1-ia-ibx}\left(-2\left(1+a^2\right)+(i-6a)bx\right)}{\sqrt{1+ia+ibx}}\,dx}{4b^2}$$

$$= \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}\left(7+10ia-18a^2-2(i-6a)bx\right)}{24b^4} -$$

$$= -\frac{\left(3+12ia-12a^2-8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{(1-ia}{}$$

$$= -\frac{\left(3+12ia-12a^2-8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{(1-ia}{}$$

$$= -\frac{\left(3+12ia-12a^2-8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{(1-ia}{}$$

$$= -\frac{\left(3+12ia-12a^2-8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{(1-ia}{}$$

Mathematica [A] time = 0.472465, size = 202, normalized size = 1.

$$\frac{\sqrt{ia+ibx+1}\left(5a^2(1-6ibx)+6a^4-38ia^3+ia\left(18b^2x^2+50ibx-23\right)-6b^4x^4-14ib^3x^3+17b^2x^2+25ibx-16\right)}{24b^4\sqrt{-i(a+bx+i)}} + \frac{(-1)^{3/}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] (Sqrt[1 + I*a + I*b*x]*(-16 - (38*I)*a^3 + 6*a^4 + (25*I)*b*x + 17*b^2*x^2
- (14*I)*b^3*x^3 - 6*b^4*x^4 + 5*a^2*(1 - (6*I)*b*x) + I*a*(-23 + (50*I)*b*
x + 18*b^2*x^2)))/(24*b^4*Sqrt[(-I)*(I + a + b*x)]) + ((-1)^(3/4)*(-3 - (12
*I)*a + 12*a^2 + (8*I)*a^3)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[
(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*b^(9/2))
```

Maple [B] time = 0.122, size = 894, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] -3/2*I/b^3*a^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b
^2)^(1/2)+3*I/b^3*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2
*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)*a^2-3/2*I/b^4*a^3*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2)+I/b^4*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a^3-3/2*I/b^3*a
^2*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+5/8*I/b^3*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)-3/2*a/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)
^(1/2)+3/b^4*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a^2-3/2/b^4*a^2*(b
^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/3/b^4*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-I/b^3*ln(
(I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/
2))/(b^2)^(1/2)-1/b^3*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b
^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)*a^3+3/b^3*ln((I*b+(x-(I-a)/b)*b^2)
/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)*a+5/8
*I/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2
)+5/8*I/b^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+3/4*I/b^4*a*(b^2*x^2+2*a*b*x+a^
2+1)^(3/2)-1/4*I/b^3*x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-3/2/b^3*a*x*(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)-3*I/b^4*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a-1
/b^4*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)
```

Maxima [B] time = 1.56296, size = 416, normalized size = 2.07

$$-\frac{3i\sqrt{b^2x^2+2\,abx+a^2+1}a^2x}{2\,b^3}-\frac{a^3\operatorname{arsinh}(bx+a)}{b^4}-\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}a^3}{2\,b^4}-\frac{i\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}x}{4\,b^3}-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*x/b^3 - a^3*arcsinh(b*x + a)/b
^4 - 1/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3/b^4 - 1/4*I*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2)*x/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b^3
 + 3/2*I*a^2*arcsinh(b*x + a)/b^4 + 3/4*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/
2)*a/b^4 + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/b^4 + 5/8*I*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)*x/b^3 + 3/2*a*arcsinh(b*x + a)/b^4 + 1/3*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^(3/2)/b^4 - 19/8*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/
b^4 - 3/8*I*arcsinh(b*x + a)/b^4 - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4
```

Fricas [A] time = 2.23941, size = 389, normalized size = 1.94

$$\frac{45i\,a^4 + 224\,a^3 - 192i\,a^2 + \left(192\,a^3 - 288i\,a^2 - 288\,a + 72i\right)\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) + \left(-48i\,b^3x^3 - 16\,(-3\right.}{192\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/192*(45*I*a^4 + 224*a^3 - 192*I*a^2 + (192*a^3 - 288*I*a^2 - 288*a + 72*I
)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (-48*I*b^3*x^3 - 16*(
-3*I*a - 4)*b^2*x^2 + 48*I*a^3 + (-48*I*a^2 - 160*a + 72*I)*b*x + 352*a^2 -
 312*I*a - 128)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 72*a)/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{a^2 + 2abx + b^2x^2 + 1}}{ia + ibx + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(I*a + I*b*x + 1), x)
```

Giac [A] time = 1.13196, size = 220, normalized size = 1.09

$$-\frac{1}{24}\sqrt{(bx+a)^2+1}\left(\left(2\left(\frac{3\,ix}{b} - \frac{3\,ab^5i + 4\,b^5}{b^7}\right)x + \frac{6\,a^2b^4i + 20\,ab^4 - 9\,b^4i}{b^7}\right)x - \frac{6\,a^3b^3i + 44\,a^2b^3 - 39\,ab^3i - 16\,b^3}{b^7}\right) + \frac{(8\,a^{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt((b*x + a)^2 + 1)*((2*(3*i*x/b - (3*a*b^5*i + 4*b^5)/b^7)*x + (6*
a^2*b^4*i + 20*a*b^4 - 9*b^4*i)/b^7)*x - (6*a^3*b^3*i + 44*a^2*b^3 - 39*a*b
^3*i - 16*b^3)/b^7) + 1/8*(8*a^3 - 12*a^2*i - 12*a + 3*i)*log(-a*b - (x*abs
(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^3*abs(b))
```

3.191 $\int e^{-i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=171

$$\frac{\left(-2ia^2-2a+i\right)\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{2b^3}-\frac{\left(-2a^2+2ia+1\right)\sinh^{-1}(a+bx)}{2b^3}+\frac{x\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{3b^2}+$$

```
[Out] ((I - 2*a - (2*I)*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^3)
 + ((I - 4*a)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(6*b^3) + (x*(
1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(3*b^2) - ((1 + (2*I)*a - 2*a
^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rubi [A] time = 0.125868, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 90, 80, 50, 53, 619, 215}

$$\frac{\left(-2ia^2-2a+i\right)\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{2b^3}-\frac{\left(-2a^2+2ia+1\right)\sinh^{-1}(a+bx)}{2b^3}+\frac{x\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{3b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(I*ArcTan[a + b*x]),x]
```


```
[Out] ((I - 2*a - (2*I)*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^3)
 + ((I - 4*a)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(6*b^3) + (x*(
1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(3*b^2) - ((1 + (2*I)*a - 2*a
^2)*ArcSinh[a + b*x])/(2*b^3)
```


Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-i\tan^{-1}(a+bx)}x^2\,dx = \int \frac{x^2\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx$$

$$= \frac{x(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{3b^2} + \frac{\int \frac{\sqrt{1-ia-ibx}\left(-1-a^2+(i-4a)bx\right)}{\sqrt{1+ia+ibx}}\,dx}{3b^2}$$

$$= \frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} + \frac{x(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{3b^2} - \frac{\left(1+2ia-2a^2\right)\int \frac{\sqrt{1-ia-}}{\sqrt{1+ia+}}}{2b^2}$$

$$= -\frac{\left(2a-i\left(1-2a^2\right)\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} + \frac{x(1-ia}{}$$

$$= -\frac{\left(2a-i\left(1-2a^2\right)\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} + \frac{x(1-ia}{}$$

$$= -\frac{\left(2a-i\left(1-2a^2\right)\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} + \frac{x(1-ia}{}$$

$$= -\frac{\left(2a-i\left(1-2a^2\right)\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} + \frac{x(1-ia}{}$$

Mathematica [A] time = 0.28138, size = 162, normalized size = 0.95

$$\frac{i\sqrt{ia+ibx+1}\left(2ia^3+7a^2+a(8bx+5i)+2ib^3x^3-5b^2x^2-7ibx+4\right)}{6b^3\sqrt{-i(a+bx+i)}} + \frac{\sqrt[4]{-1}\left(2a^2-2ia-1\right)\sqrt{-ib}\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+}}{\sqrt{-ib}}\right.}{b^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] ((I/6)*Sqrt[1 + I*a + I*b*x]*(4 + 7*a^2 + (2*I)*a^3 - (7*I)*b*x - 5*b^2*x^2
 + (2*I)*b^3*x^3 + a*(5*I + 8*b*x)))/(b^3*Sqrt[(-I)*(I + a + b*x)]) + ((-1)
```

```
^(1/4)*(-1 - (2*I)*a + 2*a^2)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqr
t[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(7/2)
```

Maple [B] time = 0.124, size = 605, normalized size = 3.5

$$\frac{-\frac{i}{3}}{b^{3}}\left(b^{2}x^{2}+2\,xab+a^{2}+1\right)^{\frac{3}{2}}+\frac{iax}{b^{2}}\sqrt{b^{2}x^{2}+2\,xab+a^{2}+1}+\frac{ia^{2}}{b^{3}}\sqrt{b^{2}x^{2}+2\,xab+a^{2}+1}+\frac{ia}{b^{2}}\ln\left((b^{2}x+ab)\frac{1}{\sqrt{b^{2}}}+\sqrt{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] -1/3*I/b^3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+I/b^2*a*x*(b^2*x^2+2*a*b*x+a^2+1)^
(1/2)+I/b^3*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+I/b^2*a*ln((b^2*x+a*b)/(b^2)^
(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*x/b^2*(b^2*x^2+2*a*b*x
+a^2+1)^(1/2)+1/2*a/b^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2/b^2*ln((b^2*x+a*b
)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I/b^3*((x-(I-a)/b)
^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a^2+I/b^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b
)*b)^(1/2)-2/b^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a+1/b^2*ln((I*
b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))
/(b^2)^(1/2)*a^2-1/b^2*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*
b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)-2*I/b^2*ln((I*b+(x-(I-a)/b)*b^2)/
(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)*a
```

Maxima [A] time = 1.51059, size = 217, normalized size = 1.27

$$\frac{i\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}ax}{b^{2}}+\frac{a^{2}\operatorname{arsinh}(bx+a)}{b^{3}}+\frac{\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}x}{2\,b^{2}}-\frac{i\,a\operatorname{arsinh}(bx+a)}{b^{3}}-\frac{i\left(b^{2}x^{2}+2\,abx+\right.}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b^2 + a^2*arcsinh(b*x + a)/b^3 + 1/
2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - I*a*arcsinh(b*x + a)/b^3 - 1/3*
I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*a/b^3 - 1/2*arcsinh(b*x + a)/b^3 + I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)/b^3
```

Fricas [A] time = 2.28399, size = 282, normalized size = 1.65

$$\frac{-7i\,a^{3}-21\,a^{2}-12\left(2\,a^{2}-2i\,a-1\right)\log\left(-bx-a+\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\right)+\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\left(-8i\,b^{2}x^{2}-4\,(-\right.}{24\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(-7*I*a^3 - 21*a^2 - 12*(2*a^2 - 2*I*a - 1)*log(-b*x - a + sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-8*I*b^2*x^2 -
```

```
4*(-2*I*a - 3)*b*x - 8*I*a^2 - 36*a + 16*I) + 9*I*a)/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{a^2 + 2abx + b^2x^2 + 1}}{ia + ibx + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(I*a + I*b*x + 1), x)
```

Giac [A] time = 1.12717, size = 158, normalized size = 0.92

$$-\frac{1}{6}\sqrt{(bx+a)^2+1}\left(\left(\frac{2\,ix}{b} - \frac{2\,ab^3i + 3\,b^3}{b^5}\right)x + \frac{2\,a^2b^2i + 9\,ab^2 - 4\,b^2i}{b^5}\right) - \frac{\left(2\,a^2 - 2\,ai - 1\right)\log\left(-ab - \left(x|b| - \sqrt{(bx+a)^2 + 1}\right)|b|\right)}{2\,b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt((b*x + a)^2 + 1)*((2*i*x/b - (2*a*b^3*i + 3*b^3)/b^5)*x + (2*a^2*
b^2*i + 9*a*b^2 - 4*b^2*i)/b^5) - 1/2*(2*a^2 - 2*a*i - 1)*log(-a*b - (x*abs
(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^2*abs(b))
```

3.192 $\int e^{-i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{2b^2}+\frac{(1+2ia)\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{2b^2}+\frac{(-2a+i)\sinh^{-1}(a+bx)}{2b^2}$$

```
[Out] ((1 + (2*I)*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^2) + ((1 -
 I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(2*b^2) + ((I - 2*a)*ArcSinh[a +
 b*x])/(2*b^2)
```

Rubi [A] time = 0.0759064, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5095, 80, 50, 53, 619, 215}

$$\frac{\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{2b^2}+\frac{(1+2ia)\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{2b^2}+\frac{(-2a+i)\sinh^{-1}(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] ((1 + (2*I)*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^2) + ((1 -
 I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(2*b^2) + ((I - 2*a)*ArcSinh[a +
 b*x])/(2*b^2)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{-i\tan^{-1}(a+bx)}x\,dx &= \int \frac{x\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx\\
&= \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2}+\frac{(i-2a)\int \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx}{2b}\\
&= \frac{(1+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2}+\frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2}+\frac{(i-2a)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}}{2b}\\
&= \frac{(1+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2}+\frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2}+\frac{(i-2a)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+}}}{2b}\\
&= \frac{(1+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2}+\frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2}+\frac{(i-2a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,d\right.}{4b^3}\\
&= \frac{(1+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2}+\frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2}+\frac{(i-2a)\sinh^{-1}(a+bx)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.129312, size = 131, normalized size = 1.19

$$\frac{\sqrt{ia+ibx+1}\left(a^2-ia-b^2x^2-3ibx+2\right)}{2b^2\sqrt{-i(a+bx+i)}}+\frac{(-1)^{3/4}(1+2ia)\sqrt{-ib}\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{b^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] (Sqrt[1 + I*a + I*b*x]*(2 - I*a + a^2 - (3*I)*b*x - b^2*x^2))/(2*b^2*Sqrt[(
-I)*(I + a + b*x)]) + ((-1)^(3/4)*(1 + (2*I)*a)*Sqrt[(-I)*b]*ArcSinh[((1/2
+ I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(5/2)
```

Maple [B] time = 0.08, size = 350, normalized size = 3.2

$$\frac{-\frac{i}{2}x}{b}\sqrt{b^2x^2+2\,xab+a^2+1}-\frac{\frac{i}{2}a}{b^2}\sqrt{b^2x^2+2\,xab+a^2+1}-\frac{\frac{i}{2}}{b}\ln\left((b^2x+ab)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+2\,xab+a^2+1}\right)\frac{1}{\sqrt{b^2}}+\frac{ia}{b^2}\sqrt{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] -1/2*I/b*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*I/b^2*a*(b^2*x^2+2*a*b*x+a^2+1
)^(1/2)-1/2*I/b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(
b^2)^(1/2)+I/b^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a+1/b^2*((x-(I
-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)-1/b*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1
/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)*a+I/b*ln((I*b+
(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(
b^2)^(1/2)
```

Maxima [A] time = 1.51266, size = 131, normalized size = 1.19

$$-\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}x}{2\,b}-\frac{a\,\mathrm{arsinh}\,(bx+a)}{b^2}+\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}a}{2\,b^2}+\frac{i\,\mathrm{arsinh}\,(bx+a)}{2\,b^2}+\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b - a*arcsinh(b*x + a)/b^2 + 1/2
*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^2 + 1/2*I*arcsinh(b*x + a)/b^2 + s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2
```

Fricas [A] time = 2.29105, size = 200, normalized size = 1.82

$$\frac{3i\,a^2+(8\,a-4i)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+\sqrt{b^2x^2+2\,abx+a^2+1}(-4i\,bx+4i\,a+8)+4\,a}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*I*a^2 + (8*a - 4*I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-4*I*b*x + 4*I*a + 8) + 4*a)/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\sqrt{a^2+2abx+b^2x^2+1}}{ia+ibx+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(I*a + I*b*x + 1), x)
```

Giac [A] time = 1.13039, size = 105, normalized size = 0.95

$$-\frac{1}{2}\sqrt{(bx+a)^2+1}\left(\frac{ix}{b}-\frac{abi+2\,b}{b^3}\right)+\frac{(2\,a-i)\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{2\,b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt((b*x + a)^2 + 1)*(i*x/b - (a*b*i + 2*b)/b^3) + 1/2*(2*a - i)*log(
-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b*abs(b))
```

3.193 $\int e^{-i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=52

$$\frac{\sinh^{-1}(a+bx)}{b}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}$$

```
[Out] ((-I)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[a + b*x]/b
```

Rubi [A] time = 0.0337881, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5093, 50, 53, 619, 215}

$$\frac{\sinh^{-1}(a+bx)}{b}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-I)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[a + b*x]/b
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-i\tan^{-1}(a+bx)}\,dx=\int \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx$$
$$=-\frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b}+\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx$$
$$=-\frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b}+\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx$$
$$=-\frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx,x,2ab+2b^2x\right)}{2b^2}$$
$$=-\frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b}+\frac{\sinh^{-1}(a+bx)}{b}$$

Mathematica [A] time = 0.0193172, size = 28, normalized size = 0.54

$$\frac{\sinh^{-1}(a+bx)-i\sqrt{(a+bx)^2+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((-I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-I)*Sqrt[1 + (a + b*x)^2] + ArcSinh[a + b*x])/b
```

Maple [B] time = 0.054, size = 122, normalized size = 2.4

$$\frac{-i}{b}\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2i\left(x-\frac{i-a}{b}\right)b}+\ln\left(\left(ib+\left(x-\frac{i-a}{b}\right)b^2\right)\frac{1}{\sqrt{b^2}}+\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2i\left(x-\frac{i-a}{b}\right)b}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] -I/b*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)+ln((I*b+(x-(I-a)/b)*b^2)/(
b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)
```

Maxima [A] time = 1.49885, size = 47, normalized size = 0.9

$$\frac{\operatorname{arsinh}(bx+a)}{b}-\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(b*x + a)/b - I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

Fricas [A] time = 2.17001, size = 146, normalized size = 2.81

$$\frac{-i\,a-2i\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*a - 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*log(-b*x - a + sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a^2+2abx+b^2x^2+1}}{ia+ibx+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(I*a + I*b*x + 1), x)
```

Giac [A] time = 1.12331, size = 70, normalized size = 1.35

$$-\frac{\sqrt{(bx+a)^2+1}i}{b}-\frac{\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt((b*x + a)^2 + 1)*i/b - log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*
abs(b))/abs(b)
```

3.194 $\int \frac{e^{-i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=89

$$-i\sinh^{-1}(a+bx)-\frac{2\sqrt{a+i}\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}}$$

```
[Out] (-I)*ArcSinh[a + b*x] - (2*Sqrt[I + a]*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/Sqrt[I - a]
```

Rubi [A] time = 0.0651262, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 105, 53, 619, 215, 93, 208}

$$-i\sinh^{-1}(a+bx)-\frac{2\sqrt{a+i}\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a + b*x])*x),x]
```

```
[Out] (-I)*ArcSinh[a + b*x] - (2*Sqrt[I + a]*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/Sqrt[I - a]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dist[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b + d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-i\tan^{-1}(a+bx)}}{x}\,dx &= \int \frac{\sqrt{1-ia-ibx}}{x\sqrt{1+ia+ibx}}\,dx\\
&= -\left((-1+ia)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx\right)-(ib)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx\\
&= (2(1-ia))\,\mathrm{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)-(ib)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2a}}\\
&= -\frac{2\sqrt{i+a}\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i-a}}-\frac{i\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{2b}\\
&= -i\sinh^{-1}(a+bx)-\frac{2\sqrt{i+a}\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i-a}}\end{aligned}$$

Mathematica [A] time = 0.0785007, size = 132, normalized size = 1.48

$$\frac{2\sqrt[4]{-1}(-ib)^{3/2}\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{b^{3/2}}-2\sqrt{\frac{a+i}{a-i}}\tan^{-1}\left(\frac{\sqrt{-i(a+bx+i)}}{\sqrt{\frac{a+i}{a-i}}\sqrt{ia+ibx+1}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a + b*x])*x),x]
```

```
[Out] (2*(-1)^(1/4)*((-I)*b)^(3/2)*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a
+ b*x)])/Sqrt[(-I)*b]])/b^(3/2) - 2*Sqrt[(I + a)/(-I + a)]*ArcTan[Sqrt[(-I)
*(I + a + b*x)]/(Sqrt[(I + a)/(-I + a)]*Sqrt[1 + I*a + I*b*x])]
```

Maple [B] time = 0.116, size = 283, normalized size = 3.2

$$\frac{-i}{i-a}\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2i\left(x-\frac{i-a}{b}\right)b}+\frac{b}{i-a}\ln\left(\left(ib+\left(x-\frac{i-a}{b}\right)b^2\right)\frac{1}{\sqrt{b^2}}+\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2i\left(x-\frac{i-a}{b}\right)b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x)
```

```
[Out] -I/(I-a)*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)+1/(I-a)*b*ln((I*b+(x-(
I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)
^(1/2)+I/(I-a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+I/(I-a)*a*b*ln((b^2*x+a*b)/(b^
2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I/(I-a)*(a^2+1)^(1/2)*l
n((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.32046, size = 404, normalized size = 4.54

$$-\frac{1}{2}\sqrt{-\frac{4\,a+4i}{a-i}}\log\left(-bx+\frac{1}{2}\,(i\,a+1)\sqrt{-\frac{4\,a+4i}{a-i}}+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+\frac{1}{2}\sqrt{-\frac{4\,a+4i}{a-i}}\log\left(-bx+\frac{1}{2}\,(-i\,a-1)\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-(4*a + 4*I)/(a - I))*log(-b*x + 1/2*(I*a + 1)*sqrt(-(4*a + 4*I)/
(a - I)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 1/2*sqrt(-(4*a + 4*I)/(a -
I))*log(-b*x + 1/2*(-I*a - 1)*sqrt(-(4*a + 4*I)/(a - I)) + sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)) + I*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{x\,(ia+ibx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(x*(I*a + I*b*x + 1)), x)
```

Giac [A] time = 1.16925, size = 104, normalized size = 1.17

$$\frac{bi\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{|b|}+\frac{2\,(a+i)\arctan\left(-\frac{\left(x|b|-\sqrt{(bx+a)^2+1}\right)i}{\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] b*i*log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/abs(b) + 2*(a + i
)*arctan(-(x*abs(b) - sqrt((b*x + a)^2 + 1))*i/sqrt(a^2 + 1))/sqrt(a^2 + 1)
```

3.195 $\int \frac{e^{-i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1+ia)x}-\frac{2ib\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}\sqrt{a+i}}$$

```
[Out] -((Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 + I*a)*x)) - ((2*I)*b*A
rcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*
x])])/((I - a)^(3/2)*Sqrt[I + a])
```

Rubi [A] time = 0.0635947, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5095, 94, 93, 208}

$$-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1+ia)x}-\frac{2ib\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}\sqrt{a+i}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -((Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 + I*a)*x)) - ((2*I)*b*A
rcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*
x])])/((I - a)^(3/2)*Sqrt[I + a])
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-i\tan^{-1}(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{1-ia-ibx}}{x^2\sqrt{1+ia+ibx}}\,dx$$
$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{(1+ia)x} + \frac{b\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{i-a}$$
$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{(1+ia)x} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{i-a}$$
$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{(1+ia)x} - \frac{2ib\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}\sqrt{i+a}}$$

Mathematica [A] time = 0.0529437, size = 114, normalized size = 0.88

$$\frac{\frac{i\sqrt{a^2+2abx+b^2x^2+1}}{x} - \frac{2b\tan^{-1}\left(\frac{\sqrt{-i(a+bx+i)}}{\sqrt{\frac{a+i}{a-i}}\sqrt{ia+ibx+1}}\right)}{\sqrt{-1+ia}\sqrt{1+ia}}}{a-i}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a + b*x])*x^2),x]
```

```
[Out] ((I*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x - (2*b*ArcTan[Sqrt[(-I)*(I + a + b
*x)]/(Sqrt[(I + a)/(-I + a)]*Sqrt[1 + I*a + I*b*x])]))/(Sqrt[-1 + I*a]*Sqrt[
1 + I*a]))/(-I + a)
```

Maple [B] time = 0.145, size = 602, normalized size = 4.6

$$\frac{-ib}{(i-a)^2}\sqrt{\left(x-\frac{i-a}{b}\right)^2 b^2 + 2i\left(x-\frac{i-a}{b}\right)b} + \frac{b^2}{(i-a)^2}\ln\left(\left(ib+\left(x-\frac{i-a}{b}\right)b^2\right)\frac{1}{\sqrt{b^2}} + \sqrt{\left(x-\frac{i-a}{b}\right)^2 b^2 + 2i\left(x-\frac{i-a}{b}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x)
```

```
[Out] -I*b/(I-a)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)+b^2/(I-a)^2*ln((I*
b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))
/(b^2)^(1/2)-I/(I-a)/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+2*I/(I-a)*a*b/
(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+I/(I-a)*a^2*b^2/(a^2+1)*ln((b^2*x+a*b
)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I/(I-a)*a*b/(a^2+1
)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/
x)+I/(I-a)*b^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+I/(I-a)*b^2/(a^2+1)*
ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+I*b/(
I-a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+I*b^2/(I-a)^2*a*ln((b^2*x+a*b)/(b^2)^(
1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I*b/(I-a)^2*(a^2+1)^(1/2)*l
n((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(bx+a)^2+1}}{(i\,bx+i\,a+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x + a)^2 + 1)/((I*b*x + I*a + 1)*x^2), x)
```

Fricas [B] time = 2.24796, size = 545, normalized size = 4.19

$$-\frac{2\,(a-i)\sqrt{\frac{b^2}{a^4-2i\,a^3-2i\,a-1}}\,x\log\left(-\frac{b^2x-\sqrt{b^2x^2+2\,abx+a^2+1}\,b+\left(a^3-i\,a^2+a-i\right)\sqrt{\frac{b^2}{a^4-2i\,a^3-2i\,a-1}}}{b}\right)-2\,(a-i)\sqrt{\frac{b^2}{a^4-2i\,a^3-2i\,a-1}}\,x\log\left(-\frac{b^2x-\sqrt{b^2x^2+2}}{}\right.}{(2\,a-2i)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*(a - I)*sqrt(b^2/(a^4 - 2*I*a^3 - 2*I*a - 1))*x*log(-(b^2*x - sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*b + (a^3 - I*a^2 + a - I)*sqrt(b^2/(a^4 - 2*I*a^3
- 2*I*a - 1)))/b) - 2*(a - I)*sqrt(b^2/(a^4 - 2*I*a^3 - 2*I*a - 1))*x*log(-
(b^2*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b - (a^3 - I*a^2 + a - I)*sqrt(b
^2/(a^4 - 2*I*a^3 - 2*I*a - 1)))/b) - 2*I*b*x - 2*I*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1))/((2*a - 2*I)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{x^2\,(ia+ibx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(x**2*(I*a + I*b*x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.196 $\int \frac{e^{-i\tan^{-1}(a+bx)}}{x^3} dx$

Optimal. Leaf size=201

$$-\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{2\left(a^2+1\right)x^2}+\frac{(1-2ia)b^2\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{5/2}(a+i)^{3/2}}+\frac{(1-2ia)b\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(-a+i)^2(a+i)x}$$

```
[Out] ((1 - (2*I)*a)*b*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*(I - a)^2*
(I + a)*x) - ((1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(2*(1 + a^2)*x
^2) + ((1 - (2*I)*a)*b^2*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[
I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a)^(5/2)*(I + a)^(3/2))
```

Rubi [A] time = 0.117133, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5095, 96, 94, 93, 208}

$$-\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{2\left(a^2+1\right)x^2}+\frac{(1-2ia)b^2\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{5/2}(a+i)^{3/2}}+\frac{(1-2ia)b\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(-a+i)^2(a+i)x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a + b*x])*x^3),x]
```

```
[Out] ((1 - (2*I)*a)*b*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*(I - a)^2*
(I + a)*x) - ((1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(2*(1 + a^2)*x
^2) + ((1 - (2*I)*a)*b^2*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[
I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a)^(5/2)*(I + a)^(3/2))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-i\tan^{-1}(a+bx)}}{x^3}\,dx = \int \frac{\sqrt{1-ia-ibx}}{x^3\sqrt{1+ia+ibx}}\,dx$$
$$= -\frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2\left(1+a^2\right)x^2} - \frac{((i+2a)b)\int \frac{\sqrt{1-ia-ibx}}{x^2\sqrt{1+ia+ibx}}\,dx}{2\left(1+a^2\right)}$$
$$= \frac{(1-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2(i-a)^2(i+a)x} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2\left(1+a^2\right)x^2} + \frac{\left((i+2a)b^2\right)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+}}}{2(i-a)^2(i+a)}$$
$$= \frac{(1-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2(i-a)^2(i+a)x} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2\left(1+a^2\right)x^2} + \frac{\left((i+2a)b^2\right)\mathrm{Subst}\left(\int \frac{}{-1-ia}\right.}{(i-a)^2}$$
$$= \frac{(1-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2(i-a)^2(i+a)x} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2\left(1+a^2\right)x^2} + \frac{(1-2ia)b^2\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1}}{\sqrt{i-a}\sqrt{1}}\right.}{(i-a)^{5/2}(i+a)^{3/2}}$$

Mathematica [A] time = 0.111014, size = 149, normalized size = 0.74

$$\frac{\frac{i\left(a^2-abx-2ibx+1\right)\sqrt{a^2+2abx+b^2x^2+1}}{x^2}+\frac{2(2a+i)b^2\tan^{-1}\left(\frac{\sqrt{-i(a+bx+i)}}{\sqrt{\frac{a+i}{a-i}}\sqrt{ia+ibx+1}}\right)}{\sqrt{-1+ia}\sqrt{1+ia}}}{2(a-i)^2(a+i)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a + b*x])*x^3),x]
```

```
[Out] ((I*(1 + a^2 - (2*I)*b*x - a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x^2 +
(2*(I + 2*a)*b^2*ArcTan[Sqrt[(-I)*(I + a + b*x)]/(Sqrt[(I + a)/(-I + a)]*Sq
rt[1 + I*a + I*b*x])])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a]))/(2*(-I + a)^2*(I + a
))
```

Maple [B] time = 0.124, size = 1146, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x)
```

```
[Out] 2*I*b^2/(I-a)^2*a/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+I*b^3/(I-a)^2*a^2/(
a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2
)-1/2*I/(I-a)*a^3*b^3/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x
+a^2+1)^(1/2))/(b^2)^(1/2)-1/2*I/(I-a)/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^
(3/2)+1/2*I/(I-a)*a*b/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*I/(I-a)
*b^2/(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/x)-I*b^2/(I-a)^3*(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/
2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+I*b^3/(I-a)^3*a*ln((b^2*x+a*b)/(b^2)^(
1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*I/(I-a)*a^2*b^2/(a^2+1)
^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x
)-I*b^2/(I-a)^2*a/(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^
2+2*a*b*x+a^2+1)^(1/2))/x)+I*b^2/(I-a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+b^3/
(I-a)^3*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a
)/b)*b)^(1/2))/(b^2)^(1/2)-I*b^2/(I-a)^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)
*b)^(1/2)-I*b/(I-a)^2/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+1/2*I/(I-a)*b
^3/(a^2+1)*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2
)^(1/2)-I/(I-a)*a^2*b^2/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+I*b^3/(I-a)
^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+1/2*I/(I-a)*b^2/(a^2+1)*(b^2*x^2
+2*a*b*x+a^2+1)^(1/2)-1/2*I/(I-a)*a*b^3/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)*x-1/2*I/(I-a)*a*b^3/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*
b*x+a^2+1)^(1/2))/(b^2)^(1/2)+I*b^3/(I-a)^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1
/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(bx+a)^2+1}}{(i\,bx+i\,a+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x + a)^2 + 1)/((I*b*x + I*a + 1)*x^3), x)
```

Fricas [B] time = 2.36476, size = 1106, normalized size = 5.5

$$\frac{(-i\,a+2)b^2x^2+\sqrt{\frac{\left(4\,a^2+4i\,a-1\right)b^4}{a^8-2i\,a^7+2\,a^6-6i\,a^5-6i\,a^3-2\,a^2-2i\,a-1}}\left(a^3-i\,a^2+a-i\right)x^2\log\left(-\frac{(2\,a+i)b^3x-\sqrt{b^2x^2+2\,abx+a^2+1}(2\,a+i)b^2+\left(a^5-i\,a^4+2\,a^3\cdots\right.}{(2\,a+i)b^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] ((-I*a + 2)*b^2*x^2 + sqrt((4*a^2 + 4*I*a - 1)*b^4/(a^8 - 2*I*a^7 + 2*a^6 -
 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))*(a^3 - I*a^2 + a - I)*x^2*log(-((2
*a + I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a + I)*b^2 + (a^5 - I*
a^4 + 2*a^3 - 2*I*a^2 + a - I)*sqrt((4*a^2 + 4*I*a - 1)*b^4/(a^8 - 2*I*a^7
+ 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1)))/((2*a + I)*b^2)) - sqrt(
(4*a^2 + 4*I*a - 1)*b^4/(a^8 - 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2
- 2*I*a - 1))*(a^3 - I*a^2 + a - I)*x^2*log(-((2*a + I)*b^3*x - sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*(2*a + I)*b^2 - (a^5 - I*a^4 + 2*a^3 - 2*I*a^2 + a -
 I)*sqrt((4*a^2 + 4*I*a - 1)*b^4/(a^8 - 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3
```

```
 - 2*a^2 - 2*I*a - 1)))/((2*a + I)*b^2)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)*((-I*a + 2)*b*x + I*a^2 + I))/((2*a^3 - 2*I*a^2 + 2*a - 2*I)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2 + 1}}{x^3\,(ia + ibx + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(x**3*(I*a + I*b*x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.197 $\int \frac{e^{-i\tan^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=283

$$\frac{\left(-2a^2-9ia+4\right)b^2\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{6(1+ia)\left(a^2+1\right)^2x}+\frac{\left(2a+i\left(1-2a^2\right)\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{7/2}(a+i)^{5/2}}+\frac{(3-2ia)b\sqrt{-ia-}}{6(-a+}$$

```
[Out] -(Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(3*(1 + I*a)*x^3) + ((3 - (2
*I)*a)*b*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(6*(I - a)^2*(I + a)*
x^2) + ((4 - (9*I)*a - 2*a^2)*b^2*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*
x])/(6*(1 + I*a)*(1 + a^2)^2*x) + ((2*a + I*(1 - 2*a^2))*b^3*ArcTanh[(Sqrt[
I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a
)^(7/2)*(I + a)^(5/2))
```

Rubi [A] time = 0.183009, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5095, 99, 151, 12, 93, 208}

$$\frac{\left(-2a^2-9ia+4\right)b^2\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{6(1+ia)\left(a^2+1\right)^2x}+\frac{\left(-2ia^2+2a+i\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{7/2}(a+i)^{5/2}}+\frac{(3-2ia)b\sqrt{-ia-i}}{6(-a+i)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a + b*x])*x^4),x]
```

```
[Out] -(Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(3*(1 + I*a)*x^3) + ((3 - (2
*I)*a)*b*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(6*(I - a)^2*(I + a)*
x^2) + ((4 - (9*I)*a - 2*a^2)*b^2*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*
x])/(6*(1 + I*a)*(1 + a^2)^2*x) + ((I + 2*a - (2*I)*a^2)*b^3*ArcTanh[(Sqrt[
I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a
)^(7/2)*(I + a)^(5/2))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-i\tan^{-1}(a+bx)}}{x^4}\,dx = \int \frac{\sqrt{1-ia-ibx}}{x^4\sqrt{1+ia+ibx}}\,dx$$

$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1+ia)x^3} + \frac{\int \frac{-(3i+2a)b-2b^2x}{x^3\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{3(1+ia)}$$

$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1+ia)x^3} + \frac{(3-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2} - \frac{\int \frac{\left(4-9ia-2a^2\right)b^2-(3i+2a)b^3x}{x^2\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{6(1+ia)\left(1+a^2\right)}$$

$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1+ia)x^3} + \frac{(3-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2} + \frac{\left(4-9ia-2a^2\right)b^2\sqrt{1-ia-}}{6(1+ia)\left(1+a\right.}$$

$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1+ia)x^3} + \frac{(3-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2} + \frac{\left(4-9ia-2a^2\right)b^2\sqrt{1-ia-}}{6(1+ia)\left(1+a\right.}$$

$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1+ia)x^3} + \frac{(3-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2} + \frac{\left(4-9ia-2a^2\right)b^2\sqrt{1-ia-}}{6(1+ia)\left(1+a\right.}$$

$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1+ia)x^3} + \frac{(3-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2} + \frac{\left(4-9ia-2a^2\right)b^2\sqrt{1-ia-}}{6(1+ia)\left(1+a\right.}$$

Mathematica [A] time = 0.23213, size = 251, normalized size = 0.89

$$\frac{(1-4ia)bx(a+bx+i)\sqrt{a^2+2abx+b^2x^2+1} + 2(1+ia)(a+i)(a+bx+i)\sqrt{a^2+2abx+b^2x^2+1} + \frac{3i\left(2a^2+2ia-1\right)b^2x^2\left(\sqrt{-1+i}\right.}{}}{6\left(a^2+1\right)^2x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a + b*x])*x^4),x]
```

```
[Out] (2*(1 + I*a)*(I + a)*(I + a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (1 -
 (4*I)*a)*b*x*(I + a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + ((3*I)*(-1
+ (2*I)*a + 2*a^2)*b^2*x^2*(Sqrt[-1 + I*a]*Sqrt[1 + I*a]*Sqrt[1 + a^2 + 2*a
*b*x + b^2*x^2] + (2*I)*b*x*ArcTan[Sqrt[(-I)*(I + a + b*x)]/(Sqrt[(I + a)/(
-I + a)]*Sqrt[1 + I*a + I*b*x])]))/(Sqrt[-1 + I*a]*Sqrt[1 + I*a]*(-I + a)))
/(6*(1 + a^2)^2*x^3)
```

Maple [B] time = 0.129, size = 1738, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x)
```

```
[Out] I*b^3/(I-a)^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+b^4/(I-a)^4*ln((I*b+(x-(I-a)/b)
*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)-
I*b^3/(I-a)^4*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)-1/2*I*b^4/(I-a)^2
*a/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-1/2*I*b^4/(I-a)^2*a/(a^2+1)^2*
ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*I
*b^4/(I-a)^2/(a^2+1)*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(
1/2))/(b^2)^(1/2)+I*b^4/(I-a)^3*a^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2
*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*I/(I-a)*a*b/(a^2+1)^2/x^2*(b^2*x
^2+2*a*b*x+a^2+1)^(3/2)-1/2*I/(I-a)*a^2*b^2/(a^2+1)^3/x*(b^2*x^2+2*a*b*x+a^
2+1)^(3/2)+1/2*I/(I-a)*a^4*b^4/(a^2+1)^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^
2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*I/(I-a)*a^2*b^4/(a^2+1)^3*(b^2*x^2+
2*a*b*x+a^2+1)^(1/2)*x+1/2*I/(I-a)*a^2*b^4/(a^2+1)^3*ln((b^2*x+a*b)/(b^2)^(
1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/2*I/(I-a)*a^2*b^4/(a^2+1)
^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/
2*I*b^2/(I-a)^2*a/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*I*b^4/(I-a)
^2*a^3/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/
(b^2)^(1/2)-1/3*I/(I-a)/(a^2+1)/x^3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+1/2*I*b^3
/(I-a)^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*I*b^3/(I-a)^2/(a^2+1)^(1
/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-I
*b^3/(I-a)^4*(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a
*b*x+a^2+1)^(1/2))/x)+2*I*b^3/(I-a)^3*a/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)-I*b^3/(I-a)^3*a/(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x
^2+2*a*b*x+a^2+1)^(1/2))/x)+I*b^4/(I-a)^3/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)*x+I*b^4/(I-a)^3/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^
2+1)^(1/2))/(b^2)^(1/2)-I*b^2/(I-a)^3/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/
2)-1/2*I/(I-a)*a^3*b^3/(a^2+1)^(5/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b
^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-1/2*I/(I-a)*a*b^3/(a^2+1)^2*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2)+1/2*I/(I-a)*a*b^3/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1
)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+I*b^4/(I-a)^4*a*ln((b^2*x+a*b)/(b
^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*I*b^3/(I-a)^2*a^2/
(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(
1/2))/x)-1/2*I*b/(I-a)^2/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-I*b^3/(I
-a)^2*a^2/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+I/(I-a)*a^3*b^3/(a^2+1)^3
*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(bx+a)^2+1}}{(ibx+ia+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x + a)^2 + 1)/((I*b*x + I*a + 1)*x^4), x)
```

Fricas [B] time = 2.45153, size = 1766, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] ((2*I*a^2 - 9*a - 4*I)*b^3*x^3 - sqrt((4*a^4 + 8*I*a^3 - 8*a^2 - 4*I*a + 1)
*b^6/(a^12 - 2*I*a^11 + 4*a^10 - 10*I*a^9 + 5*a^8 - 20*I*a^7 - 20*I*a^5 - 5
*a^4 - 10*I*a^3 - 4*a^2 - 2*I*a - 1))*(3*a^5 - 3*I*a^4 + 6*a^3 - 6*I*a^2 +
3*a - 3*I)*x^3*log(-((2*a^2 + 2*I*a - 1)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*(2*a^2 + 2*I*a - 1)*b^3 + (a^7 - I*a^6 + 3*a^5 - 3*I*a^4 + 3*a^3 -
3*I*a^2 + a - I)*sqrt((4*a^4 + 8*I*a^3 - 8*a^2 - 4*I*a + 1)*b^6/(a^12 - 2*I
*a^11 + 4*a^10 - 10*I*a^9 + 5*a^8 - 20*I*a^7 - 20*I*a^5 - 5*a^4 - 10*I*a^3
- 4*a^2 - 2*I*a - 1)))/((2*a^2 + 2*I*a - 1)*b^3)) + sqrt((4*a^4 + 8*I*a^3 -
 8*a^2 - 4*I*a + 1)*b^6/(a^12 - 2*I*a^11 + 4*a^10 - 10*I*a^9 + 5*a^8 - 20*I
*a^7 - 20*I*a^5 - 5*a^4 - 10*I*a^3 - 4*a^2 - 2*I*a - 1))*(3*a^5 - 3*I*a^4 +
 6*a^3 - 6*I*a^2 + 3*a - 3*I)*x^3*log(-((2*a^2 + 2*I*a - 1)*b^4*x - sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*(2*a^2 + 2*I*a - 1)*b^3 - (a^7 - I*a^6 + 3*a^5 -
 3*I*a^4 + 3*a^3 - 3*I*a^2 + a - I)*sqrt((4*a^4 + 8*I*a^3 - 8*a^2 - 4*I*a +
 1)*b^6/(a^12 - 2*I*a^11 + 4*a^10 - 10*I*a^9 + 5*a^8 - 20*I*a^7 - 20*I*a^5
- 5*a^4 - 10*I*a^3 - 4*a^2 - 2*I*a - 1)))/((2*a^2 + 2*I*a - 1)*b^3)) + ((2*
I*a^2 - 9*a - 4*I)*b^2*x^2 + 2*I*a^4 + (-2*I*a^3 + 3*a^2 - 2*I*a + 3)*b*x +
 4*I*a^2 + 2*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((6*a^5 - 6*I*a^4 + 12*a
^3 - 12*I*a^2 + 6*a - 6*I)*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.198 $\int e^{-2i\tan^{-1}(a+bx)}x^4\,dx$

Optimal. Leaf size=99

$$\frac{2(1+ia)x^3}{3b^2}-\frac{i(-a+i)^2x^2}{b^3}-\frac{2(1+ia)^3x}{b^4}-\frac{2i(-a+i)^4\log(-a-bx+i)}{b^5}-\frac{ix^4}{2b}-\frac{x^5}{5}$$

```
[Out] (-2*(1 + I*a)^3*x)/b^4 - (I*(I - a)^2*x^2)/b^3 + (2*(1 + I*a)*x^3)/(3*b^2)
- ((I/2)*x^4)/b - x^5/5 - ((2*I)*(I - a)^4*Log[I - a - b*x])/b^5
```

Rubi [A] time = 0.0856157, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$\frac{2(1+ia)x^3}{3b^2}-\frac{i(-a+i)^2x^2}{b^3}-\frac{2(1+ia)^3x}{b^4}-\frac{2i(-a+i)^4\log(-a-bx+i)}{b^5}-\frac{ix^4}{2b}-\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] (-2*(1 + I*a)^3*x)/b^4 - (I*(I - a)^2*x^2)/b^3 + (2*(1 + I*a)*x^3)/(3*b^2)
- ((I/2)*x^4)/b - x^5/5 - ((2*I)*(I - a)^4*Log[I - a - b*x])/b^5
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{-2i\tan^{-1}(a+bx)}x^4\,dx&=\int\frac{x^4(1-ia-ibx)}{1+ia+ibx}\,dx\\&=\int\left(\frac{2(-1-ia)^3}{b^4}-\frac{2i(-i+a)^2x}{b^3}+\frac{2(1+ia)x^2}{b^2}-\frac{2ix^3}{b}-x^4-\frac{2i(-i+a)^4}{b^4(-i+a+bx)}\right)dx\\&=-\frac{2(1+ia)^3x}{b^4}-\frac{i(i-a)^2x^2}{b^3}+\frac{2(1+ia)x^3}{3b^2}-\frac{ix^4}{2b}-\frac{x^5}{5}-\frac{2i(i-a)^4\log(i-a-bx)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.0700133, size = 95, normalized size = 0.96

$$\frac{2(1+ia)x^3}{3b^2}-\frac{i(a-i)^2x^2}{b^3}-\frac{2(1+ia)^3x}{b^4}-\frac{2i(a-i)^4\log(-a-bx+i)}{b^5}-\frac{ix^4}{2b}-\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] (-2*(1 + I*a)^3*x)/b^4 - (I*(-I + a)^2*x^2)/b^3 + (2*(1 + I*a)*x^3)/(3*b^2)
 - ((I/2)*x^4)/b - x^5/5 - ((2*I)*(-I + a)^4*Log[I - a - b*x])/b^5
```

Maple [B] time = 0.043, size = 292, normalized size = 3.

$$-\frac{x^5}{5}-\frac{i\ln\left(b^2x^2+2\,xab+a^2+1\right)}{b^5}-\frac{6\,iax}{b^4}-\frac{ix^2a^2}{b^3}-\frac{i\ln\left(b^2x^2+2\,xab+a^2+1\right)a^4}{b^5}+\frac{2\,x^3}{3\,b^2}+\frac{\frac{2\,i}{3}x^3a}{b^2}-2\,\frac{ax^2}{b^3}-\frac{\frac{i}{2}x^4}{b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x)
```

```
[Out] -1/5*x^5-I/b^5*ln(b^2*x^2+2*a*b*x+a^2+1)-6*I/b^4*a*x-I/b^3*x^2*a^2-I/b^5*ln
(b^2*x^2+2*a*b*x+a^2+1)*a^4+2/3/b^2*x^3+2/3*I/b^2*x^3*a-2/b^3*x^2*a-1/2*I*x
^4/b+6/b^4*a^2*x-2/b^4*x+2/b^5*arctan(b*x+a)*a^4+6*I/b^5*ln(b^2*x^2+2*a*b*x
+a^2+1)*a^2-12/b^5*arctan(b*x+a)*a^2+2*I/b^4*a^3*x+8*I/b^5*arctan(b*x+a)*a-
4/b^5*ln(b^2*x^2+2*a*b*x+a^2+1)*a^3+2/b^5*arctan(b*x+a)+I/b^3*x^2-8*I/b^5*a
rctan(b*x+a)*a^3+4/b^5*ln(b^2*x^2+2*a*b*x+a^2+1)*a
```

Maxima [A] time = 1.0219, size = 138, normalized size = 1.39

$$-\frac{6\,b^4x^5+15i\,b^3x^4-20\,(i\,a+1)b^2x^3+\left(30i\,a^2+60\,a-30i\right)bx^2+\left(-60i\,a^3-180\,a^2+180i\,a+60\right)x}{30\,b^4}+\frac{\left(-2i\,a^4-8\,a^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/30*(6*b^4*x^5 + 15*I*b^3*x^4 - 20*(I*a + 1)*b^2*x^3 + (30*I*a^2 + 60*a -
 30*I)*b*x^2 + (-60*I*a^3 - 180*a^2 + 180*I*a + 60)*x)/b^4 + (-2*I*a^4 - 8*
a^3 + 12*I*a^2 + 8*a - 2*I)*log(I*b*x + I*a + 1)/b^5
```

Fricas [A] time = 2.02636, size = 286, normalized size = 2.89

$$-\frac{6\,b^5x^5+15i\,b^4x^4+20\,(-i\,a-1)b^3x^3-\left(-30i\,a^2-60\,a+30i\right)b^2x^2-\left(60i\,a^3+180\,a^2-180i\,a-60\right)bx-\left(-60i\,a^4-2\right.}{30\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/30*(6*b^5*x^5 + 15*I*b^4*x^4 + 20*(-I*a - 1)*b^3*x^3 - (-30*I*a^2 - 60*a
 + 30*I)*b^2*x^2 - (60*I*a^3 + 180*a^2 - 180*I*a - 60)*b*x - (-60*I*a^4 - 2
40*a^3 + 360*I*a^2 + 240*a - 60*I)*log((b*x + a - I)/b))/b^5
```

Sympy [B] time = 13.4273, size = 2315, normalized size = 23.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
```

```
[Out] -x**5/5 - x**4*(I*a**8 + 8*a**7 - 28*I*a**6 - 56*a**5 + 70*I*a**4 + 56*a**3
 - 28*I*a**2 - 8*a + I)/(2*a**8*b - 16*I*a**7*b - 56*a**6*b + 112*I*a**5*b
+ 140*a**4*b - 112*I*a**3*b - 56*a**2*b + 16*I*a*b + 2*b) + x**3*(2*I*a**17
 + 34*a**16 - 272*I*a**15 - 1360*a**14 + 4760*I*a**13 + 12376*a**12 - 24752
*I*a**11 - 38896*a**10 + 48620*I*a**9 + 48620*a**8 - 38896*I*a**7 - 24752*a
**6 + 12376*I*a**5 + 4760*a**4 - 1360*I*a**3 - 272*a**2 + 34*I*a + 2)/(3*a*
*16*b**2 - 48*I*a**15*b**2 - 360*a**14*b**2 + 1680*I*a**13*b**2 + 5460*a**1
2*b**2 - 13104*I*a**11*b**2 - 24024*a**10*b**2 + 34320*I*a**9*b**2 + 38610*
a**8*b**2 - 34320*I*a**7*b**2 - 24024*a**6*b**2 + 13104*I*a**5*b**2 + 5460*
a**4*b**2 - 1680*I*a**3*b**2 - 360*a**2*b**2 + 48*I*a*b**2 + 3*b**2) - x**2
*(I*a**26 + 26*a**25 - 325*I*a**24 - 2600*a**23 + 14950*I*a**22 + 65780*a**
21 - 230230*I*a**20 - 657800*a**19 + 1562275*I*a**18 + 3124550*a**17 - 5311
735*I*a**16 - 7726160*a**15 + 9657700*I*a**14 + 10400600*a**13 - 9657700*I*
a**12 - 7726160*a**11 + 5311735*I*a**10 + 3124550*a**9 - 1562275*I*a**8 - 6
57800*a**7 + 230230*I*a**6 + 65780*a**5 - 14950*I*a**4 - 2600*a**3 + 325*I*
a**2 + 26*a - I)/(a**24*b**3 - 24*I*a**23*b**3 - 276*a**22*b**3 + 2024*I*a*
*21*b**3 + 10626*a**20*b**3 - 42504*I*a**19*b**3 - 134596*a**18*b**3 + 3461
04*I*a**17*b**3 + 735471*a**16*b**3 - 1307504*I*a**15*b**3 - 1961256*a**14*
b**3 + 2496144*I*a**13*b**3 + 2704156*a**12*b**3 - 2496144*I*a**11*b**3 - 1
961256*a**10*b**3 + 1307504*I*a**9*b**3 + 735471*a**8*b**3 - 346104*I*a**7*
b**3 - 134596*a**6*b**3 + 42504*I*a**5*b**3 + 10626*a**4*b**3 - 2024*I*a**3
*b**3 - 276*a**2*b**3 + 24*I*a*b**3 + b**3) + x*(2*I*a**35 + 70*a**34 - 119
0*I*a**33 - 13090*a**32 + 104720*I*a**31 + 649264*a**30 - 3246320*I*a**29 -
 13449040*a**28 + 47071640*I*a**27 + 141214920*a**26 - 367158792*I*a**25 -
834451800*a**24 + 1668903600*I*a**23 + 2952675600*a**22 - 4639918800*I*a**2
1 - 6495886320*a**20 + 8119857900*I*a**19 + 9075135300*a**18 - 9075135300*I
*a**17 - 8119857900*a**16 + 6495886320*I*a**15 + 4639918800*a**14 - 2952675
600*I*a**13 - 1668903600*a**12 + 834451800*I*a**11 + 367158792*a**10 - 1412
14920*I*a**9 - 47071640*a**8 + 13449040*I*a**7 + 3246320*a**6 - 649264*I*a*
*5 - 104720*a**4 + 13090*I*a**3 + 1190*a**2 - 70*I*a - 2)/(a**32*b**4 - 32*
I*a**31*b**4 - 496*a**30*b**4 + 4960*I*a**29*b**4 + 35960*a**28*b**4 - 2013
76*I*a**27*b**4 - 906192*a**26*b**4 + 3365856*I*a**25*b**4 + 10518300*a**24
*b**4 - 28048800*I*a**23*b**4 - 64512240*a**22*b**4 + 129024480*I*a**21*b**
4 + 225792840*a**20*b**4 - 347373600*I*a**19*b**4 - 471435600*a**18*b**4 +
565722720*I*a**17*b**4 + 601080390*a**16*b**4 - 565722720*I*a**15*b**4 - 47
1435600*a**14*b**4 + 347373600*I*a**13*b**4 + 225792840*a**12*b**4 - 129024
480*I*a**11*b**4 - 64512240*a**10*b**4 + 28048800*I*a**9*b**4 + 10518300*a*
*8*b**4 - 3365856*I*a**7*b**4 - 906192*a**6*b**4 + 201376*I*a**5*b**4 + 359
60*a**4*b**4 - 4960*I*a**3*b**4 - 496*a**2*b**4 + 32*I*a*b**4 + b**4) + 2*(
-I*a**44 - 44*a**43 + 946*I*a**42 + 13244*a**41 - 135751*I*a**40 - 1086008*
a**39 + 7059052*I*a**38 + 38320568*a**37 - 177232627*I*a**36 - 708930508*a*
*35 + 2481256778*I*a**34 + 7669339132*a**33 - 21090682613*I*a**32 - 5191552
6432*a**31 + 114955808528*I*a**30 + 229911617056*a**29 - 416714805914*I*a**
28 - 686353797976*a**27 + 1029530696964*I*a**26 + 1408831480056*a**25 - 176
1039350070*I*a**24 - 2012616400080*a**23 + 2104098963720*I*a**22 + 20126164
00080*a**21 - 1761039350070*I*a**20 - 1408831480056*a**19 + 1029530696964*I
*a**18 + 686353797976*a**17 - 416714805914*I*a**16 - 229911617056*a**15 + 1
14955808528*I*a**14 + 51915526432*a**13 - 21090682613*I*a**12 - 7669339132*
a**11 + 2481256778*I*a**10 + 708930508*a**9 - 177232627*I*a**8 - 38320568*a
**7 + 7059052*I*a**6 + 1086008*a**5 - 135751*I*a**4 - 13244*a**3 + 946*I*a*
*2 + 44*a - I)*log(-a**41 + 41*I*a**40 + 820*a**39 - 10660*I*a**38 - 101270
*a**37 + 749398*I*a**36 + 4496388*a**35 - 22481940*I*a**34 - 95548245*a**33
 + 350343565*I*a**32 + 1121099408*a**31 - 3159461968*I*a**30 - 7898654920*a
**29 + 17620076360*I*a**28 + 35240152720*a**27 - 63432274896*I*a**26 - 1030
77446706*a**25 + 151584480450*I*a**24 + 202112640600*a**23 - 244662670200*I
```

```
*a**22 - 269128937220*a**21 + 269128937220*I*a**20 + 244662670200*a**19 - 2
02112640600*I*a**18 - 151584480450*a**17 + 103077446706*I*a**16 + 634322748
96*a**15 - 35240152720*I*a**14 - 17620076360*a**13 + 7898654920*I*a**12 + 3
159461968*a**11 - 1121099408*I*a**10 - 350343565*a**9 + 95548245*I*a**8 + 2
2481940*a**7 - 4496388*I*a**6 - 749398*a**5 + 101270*I*a**4 + 10660*a**3 -
820*I*a**2 - 41*a + x*(-a**40*b + 40*I*a**39*b + 780*a**38*b - 9880*I*a**37
*b - 91390*a**36*b + 658008*I*a**35*b + 3838380*a**34*b - 18643560*I*a**33*
b - 76904685*a**32*b + 273438880*I*a**31*b + 847660528*a**30*b - 2311801440
*I*a**29*b - 5586853480*a**28*b + 12033222880*I*a**27*b + 23206929840*a**26
*b - 40225345056*I*a**25*b - 62852101650*a**24*b + 88732378800*I*a**23*b +
113380261800*a**22*b - 131282408400*I*a**21*b - 137846528820*a**20*b + 1312
82408400*I*a**19*b + 113380261800*a**18*b - 88732378800*I*a**17*b - 6285210
1650*a**16*b + 40225345056*I*a**15*b + 23206929840*a**14*b - 12033222880*I*
a**13*b - 5586853480*a**12*b + 2311801440*I*a**11*b + 847660528*a**10*b - 2
73438880*I*a**9*b - 76904685*a**8*b + 18643560*I*a**7*b + 3838380*a**6*b -
658008*I*a**5*b - 91390*a**4*b + 9880*I*a**3*b + 780*a**2*b - 40*I*a*b - b)
 + I)/(b**5*(a**40 - 40*I*a**39 - 780*a**38 + 9880*I*a**37 + 91390*a**36 -
658008*I*a**35 - 3838380*a**34 + 18643560*I*a**33 + 76904685*a**32 - 273438
880*I*a**31 - 847660528*a**30 + 2311801440*I*a**29 + 5586853480*a**28 - 120
33222880*I*a**27 - 23206929840*a**26 + 40225345056*I*a**25 + 62852101650*a*
*24 - 88732378800*I*a**23 - 113380261800*a**22 + 131282408400*I*a**21 + 137
846528820*a**20 - 131282408400*I*a**19 - 113380261800*a**18 + 88732378800*I
*a**17 + 62852101650*a**16 - 40225345056*I*a**15 - 23206929840*a**14 + 1203
3222880*I*a**13 + 5586853480*a**12 - 2311801440*I*a**11 - 847660528*a**10 +
 273438880*I*a**9 + 76904685*a**8 - 18643560*I*a**7 - 3838380*a**6 + 658008
*I*a**5 + 91390*a**4 - 9880*I*a**3 - 780*a**2 + 40*I*a + 1))
```

Giac [B] time = 1.12598, size = 317, normalized size = 3.2

$$\frac{2\left(a^4i+4a^3-6a^2i-4a+i\right)\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b^5}+\frac{(bix+ai+1)^5\left(\frac{15(2ab-3bi)i}{(bix+ai+1)b}-\frac{20\left(3a^2b^2-10ab^2i-7b^2\right)i^2}{(bix+ai+1)^2b^2}+\frac{60\left(a^3b^3-6a^2b^3i-9\right.}{(bix+ai+1)}\right.}{30b^5i^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] 2*(a^4*i + 4*a^3 - 6*a^2*i - 4*a + i)*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))
/b^5 + 1/30*(b*i*x + a*i + 1)^5*(15*(2*a*b - 3*b*i)*i/((b*i*x + a*i + 1)*b)
 - 20*(3*a^2*b^2 - 10*a*b^2*i - 7*b^2)*i^2/((b*i*x + a*i + 1)^2*b^2) + 60*(
a^3*b^3 - 6*a^2*b^3*i - 9*a*b^3 + 4*b^3*i)*i^3/((b*i*x + a*i + 1)^3*b^3) -
30*(a^4*b^4 - 12*a^3*b^4*i - 30*a^2*b^4 + 28*a*b^4*i + 9*b^4)*i^4/((b*i*x +
 a*i + 1)^4*b^4) - 6)/(b^5*i^5)
```

3.199 $\int e^{-2i\tan^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=77

$$\frac{(1+ia)x^2}{b^2}-\frac{2i(-a+i)^2x}{b^3}-\frac{2(1+ia)^3\log(-a-bx+i)}{b^4}-\frac{2ix^3}{3b}-\frac{x^4}{4}$$

```
[Out] ((-2*I)*(I - a)^2*x)/b^3 + ((1 + I*a)*x^2)/b^2 - (((2*I)/3)*x^3)/b - x^4/4
- (2*(1 + I*a)^3*Log[I - a - b*x])/b^4
```

Rubi [A] time = 0.0592766, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$\frac{(1+ia)x^2}{b^2}-\frac{2i(-a+i)^2x}{b^3}-\frac{2(1+ia)^3\log(-a-bx+i)}{b^4}-\frac{2ix^3}{3b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-2*I)*(I - a)^2*x)/b^3 + ((1 + I*a)*x^2)/b^2 - (((2*I)/3)*x^3)/b - x^4/4
- (2*(1 + I*a)^3*Log[I - a - b*x])/b^4
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{-2i\tan^{-1}(a+bx)}x^3\,dx&=\int\frac{x^3(1-ia-ibx)}{1+ia+ibx}\,dx\\&=\int\left(-\frac{2i(-i+a)^2}{b^3}+\frac{2(1+ia)x}{b^2}-\frac{2ix^2}{b}-x^3+\frac{2(-1-ia)^3}{b^3(-i+a+bx)}\right)dx\\&=-\frac{2i(i-a)^2x}{b^3}+\frac{(1+ia)x^2}{b^2}-\frac{2ix^3}{3b}-\frac{x^4}{4}-\frac{2(1+ia)^3\log(i-a-bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.0583609, size = 77, normalized size = 1.

$$\frac{(1+ia)x^2}{b^2}-\frac{2i(-a+i)^2x}{b^3}-\frac{2(1+ia)^3\log(-a-bx+i)}{b^4}-\frac{2ix^3}{3b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-2*I)*(I - a)^2*x)/b^3 + ((1 + I*a)*x^2)/b^2 - (((2*I)/3)*x^3)/b - x^4/4
- (2*(1 + I*a)^3*Log[I - a - b*x])/b^4
```

Maple [B] time = 0.043, size = 211, normalized size = 2.7

$$-\frac{x^4}{4}-\frac{\frac{2i}{3}x^3}{b}+\frac{ix^2a}{b^2}-\frac{2\,ia^2x}{b^3}+\frac{x^2}{b^2}+\frac{2\,ix}{b^3}-4\,\frac{ax}{b^3}-2\,\frac{\arctan(bx+a)\,a^3}{b^4}+\frac{i\ln\left(b^2x^2+2\,xab+a^2+1\right)a^3}{b^4}+6\,\frac{\arctan(\ldots}{b\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x)
```

```
[Out] -1/4*x^4-2/3*I*x^3/b+I/b^2*x^2*a-2*I/b^3*a^2*x+1/b^2*x^2+2*I/b^3*x-4/b^3*a*
x-2/b^4*arctan(b*x+a)*a^3+I/b^4*ln(b^2*x^2+2*a*b*x+a^2+1)*a^3+6/b^4*arctan(
b*x+a)*a+6*I/b^4*arctan(b*x+a)*a^2-3*I/b^4*ln(b^2*x^2+2*a*b*x+a^2+1)*a+3/b^
4*ln(b^2*x^2+2*a*b*x+a^2+1)*a^2-2*I/b^4*arctan(b*x+a)-1/b^4*ln(b^2*x^2+2*a*
b*x+a^2+1)
```

Maxima [A] time = 1.05035, size = 100, normalized size = 1.3

$$-\frac{i\left(-3i\,b^3x^4+8\,b^2x^3-(12\,a-12i)bx^2+24\left(a^2-2i\,a-1\right)x\right)}{12\,b^3}+\frac{\left(2i\,a^3+6\,a^2-6i\,a-2\right)\log\left(i\,bx+i\,a+1\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/12*I*(-3*I*b^3*x^4 + 8*b^2*x^3 - (12*a - 12*I)*b*x^2 + 24*(a^2 - 2*I*a -
 1)*x)/b^3 + (2*I*a^3 + 6*a^2 - 6*I*a - 2)*log(I*b*x + I*a + 1)/b^4
```

Fricas [A] time = 2.12639, size = 203, normalized size = 2.64

$$-\frac{3\,b^4x^4+8i\,b^3x^3+12\,(-i\,a-1)b^2x^2-\left(-24i\,a^2-48\,a+24i\right)bx-\left(24i\,a^3+72\,a^2-72i\,a-24\right)\log\left(\frac{bx+a-i}{b}\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^4*x^4 + 8*I*b^3*x^3 + 12*(-I*a - 1)*b^2*x^2 - (-24*I*a^2 - 48*a
+ 24*I)*b*x - (24*I*a^3 + 72*a^2 - 72*I*a - 24)*log((b*x + a - I)/b))/b^4
```

Sympy [B] time = 6.41207, size = 1212, normalized size = 15.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
```

```
[Out] -x**4/4 - x**3*(2*I*a**6 + 12*a**5 - 30*I*a**4 - 40*a**3 + 30*I*a**2 + 12*a
 - 2*I)/(3*a**6*b - 18*I*a**5*b - 45*a**4*b + 60*I*a**3*b + 45*a**2*b - 18*
I*a*b - 3*b) + x**2*(I*a**13 + 13*a**12 - 78*I*a**11 - 286*a**10 + 715*I*a*
*9 + 1287*a**8 - 1716*I*a**7 - 1716*a**6 + 1287*I*a**5 + 715*a**4 - 286*I*a
**3 - 78*a**2 + 13*I*a + 1)/(a**12*b**2 - 12*I*a**11*b**2 - 66*a**10*b**2 +
 220*I*a**9*b**2 + 495*a**8*b**2 - 792*I*a**7*b**2 - 924*a**6*b**2 + 792*I*
a**5*b**2 + 495*a**4*b**2 - 220*I*a**3*b**2 - 66*a**2*b**2 + 12*I*a*b**2 +
b**2) - x*(2*I*a**20 + 40*a**19 - 380*I*a**18 - 2280*a**17 + 9690*I*a**16 +
 31008*a**15 - 77520*I*a**14 - 155040*a**13 + 251940*I*a**12 + 335920*a**11
 - 369512*I*a**10 - 335920*a**9 + 251940*I*a**8 + 155040*a**7 - 77520*I*a**
6 - 31008*a**5 + 9690*I*a**4 + 2280*a**3 - 380*I*a**2 - 40*a + 2*I)/(a**18*
b**3 - 18*I*a**17*b**3 - 153*a**16*b**3 + 816*I*a**15*b**3 + 3060*a**14*b**
3 - 8568*I*a**13*b**3 - 18564*a**12*b**3 + 31824*I*a**11*b**3 + 43758*a**10
*b**3 - 48620*I*a**9*b**3 - 43758*a**8*b**3 + 31824*I*a**7*b**3 + 18564*a**
6*b**3 - 8568*I*a**5*b**3 - 3060*a**4*b**3 + 816*I*a**3*b**3 + 153*a**2*b**
3 - 18*I*a*b**3 - b**3) + 2*(I*a**27 + 27*a**26 - 351*I*a**25 - 2925*a**24
+ 17550*I*a**23 + 80730*a**22 - 296010*I*a**21 - 888030*a**20 + 2220075*I*a
**19 + 4686825*a**18 - 8436285*I*a**17 - 13037895*a**16 + 17383860*I*a**15
+ 20058300*a**14 - 20058300*I*a**13 - 17383860*a**12 + 13037895*I*a**11 + 8
436285*a**10 - 4686825*I*a**9 - 2220075*a**8 + 888030*I*a**7 + 296010*a**6
- 80730*I*a**5 - 17550*a**4 + 2925*I*a**3 + 351*a**2 - 27*I*a - 1)*log(a**2
5 - 25*I*a**24 - 300*a**23 + 2300*I*a**22 + 12650*a**21 - 53130*I*a**20 - 1
77100*a**19 + 480700*I*a**18 + 1081575*a**17 - 2042975*I*a**16 - 3268760*a*
*15 + 4457400*I*a**14 + 5200300*a**13 - 5200300*I*a**12 - 4457400*a**11 + 3
268760*I*a**10 + 2042975*a**9 - 1081575*I*a**8 - 480700*a**7 + 177100*I*a**
6 + 53130*a**5 - 12650*I*a**4 - 2300*a**3 + 300*I*a**2 + 25*a + x*(a**24*b
- 24*I*a**23*b - 276*a**22*b + 2024*I*a**21*b + 10626*a**20*b - 42504*I*a**
19*b - 134596*a**18*b + 346104*I*a**17*b + 735471*a**16*b - 1307504*I*a**15
*b - 1961256*a**14*b + 2496144*I*a**13*b + 2704156*a**12*b - 2496144*I*a**1
1*b - 1961256*a**10*b + 1307504*I*a**9*b + 735471*a**8*b - 346104*I*a**7*b
- 134596*a**6*b + 42504*I*a**5*b + 10626*a**4*b - 2024*I*a**3*b - 276*a**2*
b + 24*I*a*b + b) - I)/(b**4*(a**24 - 24*I*a**23 - 276*a**22 + 2024*I*a**21
 + 10626*a**20 - 42504*I*a**19 - 134596*a**18 + 346104*I*a**17 + 735471*a**
16 - 1307504*I*a**15 - 1961256*a**14 + 2496144*I*a**13 + 2704156*a**12 - 24
96144*I*a**11 - 1961256*a**10 + 1307504*I*a**9 + 735471*a**8 - 346104*I*a**
7 - 134596*a**6 + 42504*I*a**5 + 10626*a**4 - 2024*I*a**3 - 276*a**2 + 24*I
*a + 1))
```

Giac [B] time = 1.09381, size = 234, normalized size = 3.04

$$-\frac{2\left(a^{3}i+3a^{2}-3ai-1\right)\log\left(\frac{1}{\sqrt{(bx+a)^{2}+1}|b|}\right)}{b^{4}}+\frac{(bix+ai+1)^{4}\left(\frac{4(3ab-5bi)i}{(bix+ai+1)b}-\frac{18\left(a^{2}b^{2}-4ab^{2}i-3b^{2}\right)i^{2}}{(bix+ai+1)^{2}b^{2}}+\frac{12\left(a^{3}b^{3}-9a^{2}b^{3}i-15ab^{3}+7b^{3}i\right)i^{3}}{(bix+ai+1)^{3}b^{3}}\right.}{12b^{4}i^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -2*(a^3*i + 3*a^2 - 3*a*i - 1)*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b^4 +
1/12*(b*i*x + a*i + 1)^4*(4*(3*a*b - 5*b*i)*i/((b*i*x + a*i + 1)*b) - 18*(a
^2*b^2 - 4*a*b^2*i - 3*b^2)*i^2/((b*i*x + a*i + 1)^2*b^2) + 12*(a^3*b^3 - 9
*a^2*b^3*i - 15*a*b^3 + 7*b^3*i)*i^3/((b*i*x + a*i + 1)^3*b^3) - 3)/(b^4*i^
4)
```

3.200 $\int e^{-2i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=59

$$\frac{2(1+ia)x}{b^2}-\frac{2i(-a+i)^2\log(-a-bx+i)}{b^3}-\frac{ix^2}{b}-\frac{x^3}{3}$$

```
[Out] (2*(1 + I*a)*x)/b^2 - (I*x^2)/b - x^3/3 - ((2*I)*(I - a)^2*Log[I - a - b*x]
)/b^3
```

Rubi [A] time = 0.047512, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$\frac{2(1+ia)x}{b^2}-\frac{2i(-a+i)^2\log(-a-bx+i)}{b^3}-\frac{ix^2}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] (2*(1 + I*a)*x)/b^2 - (I*x^2)/b - x^3/3 - ((2*I)*(I - a)^2*Log[I - a - b*x]
)/b^3
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int e^{-2i\tan^{-1}(a+bx)}x^2\,dx &= \int \frac{x^2(1-ia-ibx)}{1+ia+ibx}\,dx\\
&= \int \left(\frac{2i(-i+a)}{b^2}-\frac{2ix}{b}-x^2-\frac{2i(-i+a)^2}{b^2(-i+a+bx)}\right)dx\\
&= \frac{2(1+ia)x}{b^2}-\frac{ix^2}{b}-\frac{x^3}{3}-\frac{2i(i-a)^2\log(i-a-bx)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.0340041, size = 55, normalized size = 0.93

$$\frac{bx\left(6ia-b^2x^2-3ibx+6\right)-6i(a-i)^2\log(-a-bx+i)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] (b*x*(6 + (6*I)*a - (3*I)*b*x - b^2*x^2) - (6*I)*(-I + a)^2*Log[I - a - b*x
])/(3*b^3)
```

Maple [B] time = 0.043, size = 143, normalized size = 2.4

$$-\frac{x^3}{3}-\frac{ix^2}{b}+\frac{2\,iax}{b^2}+2\,\frac{x}{b^2}+2\,\frac{\arctan\left(bx+a\right)a^2}{b^3}-\frac{i\ln\left(b^2x^2+2\,xab+a^2+1\right)a^2}{b^3}-2\,\frac{\arctan\left(bx+a\right)}{b^3}-\frac{4\,i\arctan\left(bx+\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x)
```

```
[Out] -1/3*x^3-I*x^2/b+2*I/b^2*a*x+2/b^2*x+2/b^3*arctan(b*x+a)*a^2-I/b^3*ln(b^2*x
^2+2*a*b*x+a^2+1)*a^2-2/b^3*arctan(b*x+a)-4*I/b^3*arctan(b*x+a)*a+I/b^3*ln(
b^2*x^2+2*a*b*x+a^2+1)-2/b^3*ln(b^2*x^2+2*a*b*x+a^2+1)*a
```

Maxima [A] time = 1.02958, size = 70, normalized size = 1.19

$$-\frac{b^2x^3+3i\,bx^2+6\,(-i\,a-1)x}{3\,b^2}+\frac{\left(-2i\,a^2-4\,a+2i\right)\log\left(i\,bx+i\,a+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(b^2*x^3 + 3*I*b*x^2 + 6*(-I*a - 1)*x)/b^2 + (-2*I*a^2 - 4*a + 2*I)*lo
g(I*b*x + I*a + 1)/b^3
```

Fricas [A] time = 2.12139, size = 135, normalized size = 2.29

$$-\frac{b^3x^3+3i\,b^2x^2+6\,(-i\,a-1)bx-\left(-6i\,a^2-12\,a+6i\right)\log\left(\frac{bx+a-i}{b}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 + 3*I*b^2*x^2 + 6*(-I*a - 1)*b*x - (-6*I*a^2 - 12*a + 6*I)*lo
g((b*x + a - I)/b))/b^3
```

Sympy [B] time = 2.69192, size = 513, normalized size = 8.69

$$-\frac{x^3}{3}-\frac{x^2\left(ia^4+4a^3-6ia^2-4a+i\right)}{a^4b-4ia^3b-6a^2b+4iab+b}+\frac{x\left(2ia^9+18a^8-72ia^7-168a^6+252ia^5+252a^4-168ia^3-72a^2+18ia+2\right)}{a^8b^2-8ia^7b^2-28a^6b^2+56ia^5b^2+70a^4b^2-56ia^3b^2-28a^2b^2+8iab^2+b^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
```

```
[Out] -x**3/3 - x**2*(I*a**4 + 4*a**3 - 6*I*a**2 - 4*a + I)/(a**4*b - 4*I*a**3*b
- 6*a**2*b + 4*I*a*b + b) + x*(2*I*a**9 + 18*a**8 - 72*I*a**7 - 168*a**6 +
252*I*a**5 + 252*a**4 - 168*I*a**3 - 72*a**2 + 18*I*a + 2)/(a**8*b**2 - 8*I
*a**7*b**2 - 28*a**6*b**2 + 56*I*a**5*b**2 + 70*a**4*b**2 - 56*I*a**3*b**2
- 28*a**2*b**2 + 8*I*a*b**2 + b**2) + 2*(-I*a**14 - 14*a**13 + 91*I*a**12 +
 364*a**11 - 1001*I*a**10 - 2002*a**9 + 3003*I*a**8 + 3432*a**7 - 3003*I*a*
*6 - 2002*a**5 + 1001*I*a**4 + 364*a**3 - 91*I*a**2 - 14*a + I)*log(-a**13
+ 13*I*a**12 + 78*a**11 - 286*I*a**10 - 715*a**9 + 1287*I*a**8 + 1716*a**7
- 1716*I*a**6 - 1287*a**5 + 715*I*a**4 + 286*a**3 - 78*I*a**2 - 13*a + x*(-
a**12*b + 12*I*a**11*b + 66*a**10*b - 220*I*a**9*b - 495*a**8*b + 792*I*a**
7*b + 924*a**6*b - 792*I*a**5*b - 495*a**4*b + 220*I*a**3*b + 66*a**2*b - 1
2*I*a*b - b) + I)/(b**3*(a**12 - 12*I*a**11 - 66*a**10 + 220*I*a**9 + 495*a
**8 - 792*I*a**7 - 924*a**6 + 792*I*a**5 + 495*a**4 - 220*I*a**3 - 66*a**2
+ 12*I*a + 1))
```

Giac [B] time = 1.12451, size = 162, normalized size = 2.75

$$\frac{2\left(a^2 i + 2a - i\right)\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b^3} + \frac{(bix + ai + 1)^3\left(\frac{3(ab-2bi)i}{(bix+ai+1)b} - \frac{3\left(a^2b^2-6ab^2i-5b^2\right)i^2}{(bix+ai+1)^2b^2} - 1\right)}{3b^3i^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] 2*(a^2*i + 2*a - i)*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b^3 + 1/3*(b*i*x
+ a*i + 1)^3*(3*(a*b - 2*b*i)*i/((b*i*x + a*i + 1)*b) - 3*(a^2*b^2 - 6*a*b^
2*i - 5*b^2)*i^2/((b*i*x + a*i + 1)^2*b^2) - 1)/(b^3*i^3)
```

3.201 $\int e^{-2i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=40

$$\frac{2(1+ia)\log(-a-bx+i)}{b^2}-\frac{2ix}{b}-\frac{x^2}{2}$$

```
[Out] ((-2*I)*x)/b - x^2/2 + (2*(1 + I*a)*Log[I - a - b*x])/b^2
```

Rubi [A] time = 0.0317712, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5095, 77}

$$\frac{2(1+ia)\log(-a-bx+i)}{b^2}-\frac{2ix}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-2*I)*x)/b - x^2/2 + (2*(1 + I*a)*Log[I - a - b*x])/b^2
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{-2i\tan^{-1}(a+bx)}x\,dx &= \int \frac{x(1-ia-ibx)}{1+ia+ibx}\,dx\\ &= \int \left(-\frac{2i}{b}-x+\frac{2(1+ia)}{b(-i+a+bx)}\right)dx\\ &= -\frac{2ix}{b}-\frac{x^2}{2}+\frac{2(1+ia)\log(i-a-bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0203792, size = 40, normalized size = 1.

$$\frac{2(1+ia)\log(-a-bx+i)}{b^2}-\frac{2ix}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-2*I)*x)/b - x^2/2 + (2*(1 + I*a)*Log[I - a - b*x])/b^2
```

Maple [B] time = 0.043, size = 85, normalized size = 2.1

$$-\frac{x^2}{2}-\frac{2\,ix}{b}-2\,\frac{\arctan\left(bx+a\right)a}{b^2}+\frac{i\ln\left(b^2x^2+2\,xab+a^2+1\right)a}{b^2}+\frac{2\,i\arctan\left(bx+a\right)}{b^2}+\frac{\ln\left(b^2x^2+2\,xab+a^2+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x)
```

```
[Out] -1/2*x^2-2*I*x/b-2/b^2*arctan(b*x+a)*a+I/b^2*ln(b^2*x^2+2*a*b*x+a^2+1)*a+2*
I/b^2*arctan(b*x+a)+1/b^2*ln(b^2*x^2+2*a*b*x+a^2+1)
```

Maxima [A] time = 1.02399, size = 49, normalized size = 1.22

$$\frac{i\left(i\,bx^2-4\,x\right)}{2\,b}-\frac{2\left(-i\,a-1\right)\log\left(i\,bx+i\,a+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/2*I*(I*b*x^2 - 4*x)/b - 2*(-I*a - 1)*log(I*b*x + I*a + 1)/b^2
```

Fricas [A] time = 2.19443, size = 89, normalized size = 2.22

$$-\frac{b^2x^2+4i\,bx+4\left(-i\,a-1\right)\log\left(\frac{bx+a-i}{b}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 + 4*I*b*x + 4*(-I*a - 1)*log((b*x + a - I)/b))/b^2
```

Sympy [B] time = 1.02915, size = 148, normalized size = 3.7

$$-\frac{x^2}{2}-\frac{x\left(2ia^2+4a-2i\right)}{a^2b-2iab-b}+\frac{2\left(ia^5+5a^4-10ia^3-10a^2+5ia+1\right)\log\left(a^5-5ia^4-10a^3+10ia^2+5a+x\left(a^4b-4ia^3b\right.\right.}{b^2\left(a^4-4ia^3-6a^2+4ia+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
```

```
[Out] -x**2/2 - x*(2*I*a**2 + 4*a - 2*I)/(a**2*b - 2*I*a*b - b) + 2*(I*a**5 + 5*a
**4 - 10*I*a**3 - 10*a**2 + 5*I*a + 1)*log(a**5 - 5*I*a**4 - 10*a**3 + 10*I
*a**2 + 5*a + x*(a**4*b - 4*I*a**3*b - 6*a**2*b + 4*I*a*b + b) - I)/(b**2*(
```

```
a**4 - 4*I*a**3 - 6*a**2 + 4*I*a + 1))
```

Giac [B] time = 1.11572, size = 108, normalized size = 2.7

$$-\frac{i\left(\frac{4\,(a-i)\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b}-\frac{(bix+ai+1)^2\left(i-\frac{2\,(abi+3\,b)i}{(bix+ai+1)b}\right)}{bi^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/2*i*(4*(a - i)*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b - (b*i*x + a*i +
1)^2*(i - 2*(a*b*i + 3*b)*i/((b*i*x + a*i + 1)*b))/(b*i^2))/b
```

3.202 $\int e^{-2i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=23

$$-x-\frac{2i\log(-a-bx+i)}{b}$$

```
[Out] -x - ((2*I)*Log[I - a - b*x])/b
```

Rubi [A] time = 0.0125188, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5093, 43}

$$-x-\frac{2i\log(-a-bx+i)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-2*I)*ArcTan[a + b*x]),x]
```

```
[Out] -x - ((2*I)*Log[I - a - b*x])/b
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2i\tan^{-1}(a+bx)}\,dx &= \int \frac{1-ia-ibx}{1+ia+ibx}\,dx\\ &= \int \left(-1-\frac{2i}{-i+a+bx}\right)dx\\ &= -x-\frac{2i\log(i-a-bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0110591, size = 32, normalized size = 1.39

$$-\frac{i\log\left((a+bx)^2+1\right)}{b}+\frac{2\tan^{-1}(a+bx)}{b}-x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-2*I)*ArcTan[a + b*x]),x]
```

```
[Out] -x + (2*ArcTan[a + b*x])/b - (I*Log[1 + (a + b*x)^2])/b
```

Maple [A] time = 0.042, size = 40, normalized size = 1.7

$$-x-\frac{i\ln\left(b^2x^2+2\,xab+a^2+1\right)}{b}+2\,\frac{\arctan\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x)
```

```
[Out] -x-I/b*ln(b^2*x^2+2*a*b*x+a^2+1)+2/b*arctan(b*x+a)
```

Maxima [A] time = 0.986961, size = 26, normalized size = 1.13

$$-x-\frac{2i\,\log\left(i\,bx+i\,a+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -x - 2*I*log(I*b*x + I*a + 1)/b
```

Fricas [A] time = 2.09417, size = 50, normalized size = 2.17

$$-\frac{bx+2i\,\log\left(\frac{bx+a-i}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -(b*x + 2*I*log((b*x + a - I)/b))/b
```

Sympy [A] time = 0.378319, size = 15, normalized size = 0.65

$$-x-\frac{2i\log\left(a+bx-i\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
```

```
[Out] -x - 2*I*log(a + b*x - I)/b
```

Giac [A] time = 1.10408, size = 51, normalized size = 2.22

$$\frac{(bix+ai+1)i}{b}+\frac{2\,i\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] (b*i*x + a*i + 1)*i/b + 2*i*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b
```

3.203 $\int \frac{e^{-2i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=41

$$\frac{(a+i)\log(x)}{-a+i}-\frac{2\log(-a-bx+i)}{1+ia}$$

```
[Out] ((I + a)*Log[x])/(I - a) - (2*Log[I - a - b*x])/(1 + I*a)
```

Rubi [A] time = 0.0356451, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 72}

$$\frac{(a+i)\log(x)}{-a+i}-\frac{2\log(-a-bx+i)}{1+ia}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a + b*x])*x),x]
```

```
[Out] ((I + a)*Log[x])/(I - a) - (2*Log[I - a - b*x])/(1 + I*a)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2i\tan^{-1}(a+bx)}}{x}\,dx &= \int \frac{1-ia-ibx}{x(1+ia+ibx)}\,dx\\ &= \int \left(\frac{-i-a}{(-i+a)x}+\frac{2ib}{(-i+a)(-i+a+bx)}\right)dx\\ &= \frac{(i+a)\log(x)}{i-a}-\frac{2\log(i-a-bx)}{1+ia}\end{aligned}$$

Mathematica [A] time = 0.0187367, size = 34, normalized size = 0.83

$$\frac{2i\log(-a-bx+i)-(a+i)\log(x)}{a-i}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a + b*x])*x),x]
```

```
[Out] (-((I + a)*Log[x]) + (2*I)*Log[I - a - b*x])/(-I + a)
```

Maple [A] time = 0.063, size = 74, normalized size = 1.8

$$\frac{-i\ln\left(b^2x^2+2xab+a^2+1\right)}{i-a}+2\frac{\arctan\left(bx+a\right)}{i-a}-\frac{a^2\ln\left(x\right)}{\left(i-a\right)^2}-\frac{\ln\left(x\right)}{\left(i-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x)
```

```
[Out] -I/(I-a)*ln(b^2*x^2+2*a*b*x+a^2+1)+2/(I-a)*arctan(b*x+a)-1/(I-a)^2*ln(x)*a^
2-1/(I-a)^2*ln(x)
```

Maxima [A] time = 1.02132, size = 63, normalized size = 1.54

$$-\frac{2\left(-i\,a-1\right)\log\left(i\,bx+i\,a+1\right)}{a^2-2i\,a-1}-\frac{\left(a^2+1\right)\log\left(x\right)}{a^2-2i\,a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] -2*(-I*a - 1)*log(I*b*x + I*a + 1)/(a^2 - 2*I*a - 1) - (a^2 + 1)*log(x)/(a^
2 - 2*I*a - 1)
```

Fricas [A] time = 2.22017, size = 73, normalized size = 1.78

$$-\frac{\left(a+i\right)\log\left(x\right)-2i\,\log\left(\frac{bx+a-i}{b}\right)}{a-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x, algorithm="fricas")
```

```
[Out] -((a + I)*log(x) - 2*I*log((b*x + a - I)/b))/(a - I)
```

Sympy [B] time = 2.54326, size = 1538, normalized size = 37.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2)/x,x)
```

```
[Out] (-sqrt((-a**8 - 12*I*a**7 + 60*a**6 + 164*I*a**5 - 270*a**4 - 276*I*a**3 +
172*a**2 + 60*I*a - 9)*(-a**8 - 4*I*a**7 + 4*a**6 - 4*I*a**5 + 10*a**4 + 4*
I*a**3 + 4*a**2 + 4*I*a - 1))/(2*(a**8 + 4*I*a**7 - 4*a**6 + 4*I*a**5 - 10*
```

```
a**4 - 4*I*a**3 - 4*a**2 - 4*I*a + 1)) - 1/2)*log(x + (-sqrt((-a**8 - 12*I*
a**7 + 60*a**6 + 164*I*a**5 - 270*a**4 - 276*I*a**3 + 172*a**2 + 60*I*a - 9
)*(-a**8 - 4*I*a**7 + 4*a**6 - 4*I*a**5 + 10*a**4 + 4*I*a**3 + 4*a**2 + 4*I
*a - 1))/(2*(a**8 + 4*I*a**7 - 4*a**6 + 4*I*a**5 - 10*a**4 - 4*I*a**3 - 4*a
**2 - 4*I*a + 1)) - 1/2)*(a**11 + 9*I*a**10 - 31*a**9 - 47*I*a**8 + 10*a**7
 - 70*I*a**6 + 98*a**5 + 34*I*a**4 + 37*a**3 + 45*I*a**2 - 19*a - 3*I)/(a**
10*b + 14*I*a**9*b - 85*a**8*b - 296*I*a**7*b + 658*a**6*b + 980*I*a**5*b -
 994*a**4*b - 680*I*a**3*b + 301*a**2*b + 78*I*a*b - 9*b) + (a**25 + 29*I*a
**24 - 396*a**23 - 3388*I*a**22 + 20378*a**21 + 91602*I*a**20 - 319116*a**1
9 - 880764*I*a**18 + 1948887*a**17 + 3465467*I*a**16 - 4901848*a**15 - 5325
624*I*a**14 + 3970316*a**13 + 967708*I*a**12 + 2488392*a**11 + 4876008*I*a*
*10 - 5388609*a**9 - 4348701*I*a**8 + 2724068*a**7 + 1346548*I*a**6 - 52378
2*a**5 - 157646*I*a**4 + 35524*a**3 + 5652*I*a**2 - 567*a - 27*I)/(a**24*b
+ 32*I*a**23*b - 484*a**22*b - 4608*I*a**21*b + 31026*a**20*b + 157344*I*a*
*19*b - 624948*a**18*b - 1995456*I*a**17*b + 5216127*a**16*b + 11307584*I*a
**15*b - 20514376*a**14*b - 31338752*I*a**13*b + 40461564*a**12*b + 4421740
8*I*a**11*b - 40876296*a**10*b - 31876224*I*a**9*b + 20859663*a**8*b + 1136
1696*I*a**7*b - 5089492*a**6*b - 1842944*I*a**5*b + 526066*a**4*b + 113952*
I*a**3*b - 17604*a**2*b - 1728*I*a*b + 81*b)) + (sqrt((-a**8 - 12*I*a**7 +
60*a**6 + 164*I*a**5 - 270*a**4 - 276*I*a**3 + 172*a**2 + 60*I*a - 9)*(-a**
8 - 4*I*a**7 + 4*a**6 - 4*I*a**5 + 10*a**4 + 4*I*a**3 + 4*a**2 + 4*I*a - 1)
)/(2*(a**8 + 4*I*a**7 - 4*a**6 + 4*I*a**5 - 10*a**4 - 4*I*a**3 - 4*a**2 - 4
*I*a + 1)) - 1/2)*log(x + (sqrt((-a**8 - 12*I*a**7 + 60*a**6 + 164*I*a**5 -
 270*a**4 - 276*I*a**3 + 172*a**2 + 60*I*a - 9)*(-a**8 - 4*I*a**7 + 4*a**6
- 4*I*a**5 + 10*a**4 + 4*I*a**3 + 4*a**2 + 4*I*a - 1))/(2*(a**8 + 4*I*a**7
- 4*a**6 + 4*I*a**5 - 10*a**4 - 4*I*a**3 - 4*a**2 - 4*I*a + 1)) - 1/2)*(a**
11 + 9*I*a**10 - 31*a**9 - 47*I*a**8 + 10*a**7 - 70*I*a**6 + 98*a**5 + 34*I
*a**4 + 37*a**3 + 45*I*a**2 - 19*a - 3*I)/(a**10*b + 14*I*a**9*b - 85*a**8*
b - 296*I*a**7*b + 658*a**6*b + 980*I*a**5*b - 994*a**4*b - 680*I*a**3*b +
301*a**2*b + 78*I*a*b - 9*b) + (a**25 + 29*I*a**24 - 396*a**23 - 3388*I*a**
22 + 20378*a**21 + 91602*I*a**20 - 319116*a**19 - 880764*I*a**18 + 1948887*
a**17 + 3465467*I*a**16 - 4901848*a**15 - 5325624*I*a**14 + 3970316*a**13 +
 967708*I*a**12 + 2488392*a**11 + 4876008*I*a**10 - 5388609*a**9 - 4348701*
I*a**8 + 2724068*a**7 + 1346548*I*a**6 - 523782*a**5 - 157646*I*a**4 + 3552
4*a**3 + 5652*I*a**2 - 567*a - 27*I)/(a**24*b + 32*I*a**23*b - 484*a**22*b
- 4608*I*a**21*b + 31026*a**20*b + 157344*I*a**19*b - 624948*a**18*b - 1995
456*I*a**17*b + 5216127*a**16*b + 11307584*I*a**15*b - 20514376*a**14*b - 3
1338752*I*a**13*b + 40461564*a**12*b + 44217408*I*a**11*b - 40876296*a**10*
b - 31876224*I*a**9*b + 20859663*a**8*b + 11361696*I*a**7*b - 5089492*a**6*
b - 1842944*I*a**5*b + 526066*a**4*b + 113952*I*a**3*b - 17604*a**2*b - 172
8*I*a*b + 81*b))
```

Giac [B] time = 1.09915, size = 104, normalized size = 2.54

$$bi\left(\frac{(ai-1)\log\left(-\frac{ai^2}{bix+ai+1}+i-\frac{i}{bix+ai+1}\right)}{ab-bi}-\frac{i\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x, algorithm="giac")
```

```
[Out] b*i*((a*i - 1)*log(-a*i^2/(b*i*x + a*i + 1) + i - i/(b*i*x + a*i + 1))/(a*b
 - b*i) - i*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b)
```

3.204 $\int \frac{e^{-2i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=62

$$\frac{2ib\log(x)}{(-a+i)^2}-\frac{2ib\log(-a-bx+i)}{(-a+i)^2}-\frac{a+i}{(-a+i)x}$$

```
[Out] -((I + a)/((I - a)*x)) + ((2*I)*b*Log[x])/(I - a)^2 - ((2*I)*b*Log[I - a -
b*x])/(I - a)^2
```

Rubi [A] time = 0.0423323, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$\frac{2ib\log(x)}{(-a+i)^2}-\frac{2ib\log(-a-bx+i)}{(-a+i)^2}-\frac{a+i}{(-a+i)x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -((I + a)/((I - a)*x)) + ((2*I)*b*Log[x])/(I - a)^2 - ((2*I)*b*Log[I - a -
b*x])/(I - a)^2
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2i\tan^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{1-ia-ibx}{x^2(1+ia+ibx)}\,dx\\ &= \int \left(\frac{-i-a}{(-i+a)x^2}+\frac{2ib}{(-i+a)^2x}-\frac{2ib^2}{(-i+a)^2(-i+a+bx)}\right)dx\\ &= -\frac{i+a}{(i-a)x}+\frac{2ib\log(x)}{(i-a)^2}-\frac{2ib\log(i-a-bx)}{(i-a)^2}\end{aligned}$$

Mathematica [A] time = 0.0233313, size = 42, normalized size = 0.68

$$\frac{a^2-2ibx\log(-a-bx+i)+2ibx\log(x)+1}{(a-i)^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] (1 + a^2 + (2*I)*b*x*Log[x] - (2*I)*b*x*Log[I - a - b*x])/((-I + a)^2*x)
```

Maple [B] time = 0.052, size = 152, normalized size = 2.5

$$\frac{ib\ln\left(b^2x^2+2\,xab+a^2+1\right)a}{(i-a)^3}+\frac{b\ln\left(b^2x^2+2\,xab+a^2+1\right)}{(i-a)^3}-2\,\frac{b\arctan\left(bx+a\right)a}{(i-a)^3}+\frac{2\,ib\arctan\left(bx+a\right)}{(i-a)^3}+\frac{a^2}{x\left(i-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x)
```

```
[Out] I*b/(I-a)^3*ln(b^2*x^2+2*a*b*x+a^2+1)*a+b/(I-a)^3*ln(b^2*x^2+2*a*b*x+a^2+1)
-2*b/(I-a)^3*arctan(b*x+a)*a+2*I*b/(I-a)^3*arctan(b*x+a)+1/x/(I-a)^2*a^2+1/
x/(I-a)^2-2*I*b/(I-a)^3*ln(x)*a-2*b/(I-a)^3*ln(x)
```

Maxima [B] time = 1.01585, size = 153, normalized size = 2.47

$$-\frac{(2\,a-2i)b\log\left(i\,bx+i\,a+1\right)}{-i\,a^3-3\,a^2+3i\,a+1}+\frac{(2\,a-2i)b\log\left(x\right)}{-i\,a^3-3\,a^2+3i\,a+1}+\frac{a^3+\left(a^2+1\right)bx-i\,a^2+a-i}{\left(a^2-2i\,a-1\right)bx^2+\left(a^3-3i\,a^2-3\,a+i\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] -(2*a - 2*I)*b*log(I*b*x + I*a + 1)/(-I*a^3 - 3*a^2 + 3*I*a + 1) + (2*a - 2
*I)*b*log(x)/(-I*a^3 - 3*a^2 + 3*I*a + 1) + (a^3 + (a^2 + 1)*b*x - I*a^2 +
a - I)/((a^2 - 2*I*a - 1)*b*x^2 + (a^3 - 3*I*a^2 - 3*a + I)*x)
```

Fricas [A] time = 2.14245, size = 109, normalized size = 1.76

$$\frac{2i\,bx\log\left(x\right)-2i\,bx\log\left(\frac{bx+a-i}{b}\right)+a^2+1}{\left(a^2-2i\,a-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*I*b*x*log(x) - 2*I*b*x*log((b*x + a - I)/b) + a^2 + 1)/((a^2 - 2*I*a - 1
)*x)
```

Sympy [B] time = 8.05596, size = 3550, normalized size = 57.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2)/x**2,x)
```

```
[Out] -2*b*sqrt((-a**23 - 23*I*a**22 + 253*a**21 + 1771*I*a**20 - 8855*a**19 - 33
649*I*a**18 + 100947*a**17 + 245157*I*a**16 - 490314*a**15 - 817190*I*a**14
 + 1144066*a**13 + 1352078*I*a**12 - 1352078*a**11 - 1144066*I*a**10 + 8171
90*a**9 + 490314*I*a**8 - 245157*a**7 - 100947*I*a**6 + 33649*a**5 + 8855*I
*a**4 - 1771*a**3 - 253*I*a**2 + 23*a + I)/(a**27 + 19*I*a**26 - 167*a**25
- 893*I*a**24 + 3198*a**23 + 7866*I*a**22 - 12650*a**21 - 9614*I*a**20 - 10
373*a**19 - 43263*I*a**18 + 62491*a**17 + 37145*I*a**16 + 29716*a**15 + 891
48*I*a**14 - 89148*a**13 - 29716*I*a**12 - 37145*a**11 - 62491*I*a**10 + 43
263*a**9 + 10373*I*a**8 + 9614*a**7 + 12650*I*a**6 - 7866*a**5 - 3198*I*a**
4 + 893*a**3 + 167*I*a**2 - 19*a - I))*log(-2*b*sqrt((-a**23 - 23*I*a**22 +
 253*a**21 + 1771*I*a**20 - 8855*a**19 - 33649*I*a**18 + 100947*a**17 + 245
157*I*a**16 - 490314*a**15 - 817190*I*a**14 + 1144066*a**13 + 1352078*I*a**
12 - 1352078*a**11 - 1144066*I*a**10 + 817190*a**9 + 490314*I*a**8 - 245157
*a**7 - 100947*I*a**6 + 33649*a**5 + 8855*I*a**4 - 1771*a**3 - 253*I*a**2 +
 23*a + I)/(a**27 + 19*I*a**26 - 167*a**25 - 893*I*a**24 + 3198*a**23 + 786
6*I*a**22 - 12650*a**21 - 9614*I*a**20 - 10373*a**19 - 43263*I*a**18 + 6249
1*a**17 + 37145*I*a**16 + 29716*a**15 + 89148*I*a**14 - 89148*a**13 - 29716
*I*a**12 - 37145*a**11 - 62491*I*a**10 + 43263*a**9 + 10373*I*a**8 + 9614*a
**7 + 12650*I*a**6 - 7866*a**5 - 3198*I*a**4 + 893*a**3 + 167*I*a**2 - 19*a
 - I))*(I*a**33 - 27*a**32 - 348*I*a**31 + 2844*a**30 + 16500*I*a**29 - 720
36*a**28 - 244412*I*a**27 + 654588*a**26 + 1384344*I*a**25 - 2262000*a**24
- 2646540*I*a**23 + 1560780*a**22 - 1560780*I*a**21 + 5882940*a**20 + 90045
00*I*a**19 - 8364180*a**18 - 3421710*I*a**17 - 3421710*a**16 - 8364180*I*a*
*15 + 9004500*a**14 + 5882940*I*a**13 - 1560780*a**12 + 1560780*I*a**11 - 2
646540*a**10 - 2262000*I*a**9 + 1384344*a**8 + 654588*I*a**7 - 244412*a**6
- 72036*I*a**5 + 16500*a**4 + 2844*I*a**3 - 348*a**2 - 27*I*a + 1)/(4*a**30
*b**2 + 120*I*a**29*b**2 - 1740*a**28*b**2 - 16240*I*a**27*b**2 + 109620*a*
*26*b**2 + 570024*I*a**25*b**2 - 2375100*a**24*b**2 - 8143200*I*a**23*b**2
+ 23411700*a**22*b**2 + 57228600*I*a**21*b**2 - 120180060*a**20*b**2 - 2185
09200*I*a**19*b**2 + 345972900*a**18*b**2 + 479039400*I*a**17*b**2 - 581690
700*a**16*b**2 - 620470080*I*a**15*b**2 + 581690700*a**14*b**2 + 479039400*
I*a**13*b**2 - 345972900*a**12*b**2 - 218509200*I*a**11*b**2 + 120180060*a*
*10*b**2 + 57228600*I*a**9*b**2 - 23411700*a**8*b**2 - 8143200*I*a**7*b**2
+ 2375100*a**6*b**2 + 570024*I*a**5*b**2 - 109620*a**4*b**2 - 16240*I*a**3*
b**2 + 1740*a**2*b**2 + 120*I*a*b**2 - 4*b**2) + x + (a**50 + 48*I*a**49 -
1127*a**48 - 17248*I*a**47 + 193452*a**46 + 1695008*I*a**45 - 12076932*a**4
4 - 71916768*I*a**43 + 365077482*a**42 + 1603477568*I*a**41 - 6163366902*a*
*40 - 20918093728*I*a**39 + 63127818572*a**38 + 170333048928*I*a**37 - 4126
52088772*a**36 - 900331830048*I*a**35 + 1772528290407*a**34 + 3151161405168
*I*a**33 - 5054988087457*a**32 - 7297426411968*I*a**31 + 9425842448792*a**3
0 + 10772391370048*I*a**29 - 10649977831752*a**28 - 8643460269248*I*a**27 +
 4861946401452*a**26 + 4861946401452*a**24 + 8643460269248*I*a**23 - 106499
77831752*a**22 - 10772391370048*I*a**21 + 9425842448792*a**20 + 72974264119
68*I*a**19 - 5054988087457*a**18 - 3151161405168*I*a**17 + 1772528290407*a*
*16 + 900331830048*I*a**15 - 412652088772*a**14 - 170333048928*I*a**13 + 63
127818572*a**12 + 20918093728*I*a**11 - 6163366902*a**10 - 1603477568*I*a**
9 + 365077482*a**8 + 71916768*I*a**7 - 12076932*a**6 - 1695008*I*a**5 + 193
452*a**4 + 17248*I*a**3 - 1127*a**2 - 48*I*a + 1)/(2*a**49*b + 98*I*a**48*b
 - 2352*a**47*b - 36848*I*a**46*b + 423752*a**45*b + 3813768*I*a**44*b - 27
967632*a**43*b - 171801168*I*a**42*b + 901956132*a**41*b + 4108911268*I*a**
40*b - 16435645072*a**39*b - 58271832528*I*a**38*b + 184527469672*a**37*b +
 525193567528*I*a**36*b - 1350497745072*a**35*b - 3151161405168*I*a**34*b +
 6696217985982*a**33*b + 12998540796318*I*a**32*b - 23108516971232*a**31*b
- 37703369795168*I*a**30*b + 56555054692752*a**29*b + 78099837432848*I*a**2
8*b - 99399793096352*a**27*b - 116686713634848*I*a**26*b + 126410606437752*
```

a**25*b + 126410606437752*I*a**24*b - 116686713634848*a**23*b - 99399793096
352*I*a**22*b + 78099837432848*a**21*b + 56555054692752*I*a**20*b - 3770336
9795168*a**19*b - 23108516971232*I*a**18*b + 12998540796318*a**17*b + 66962
17985982*I*a**16*b - 3151161405168*a**15*b - 1350497745072*I*a**14*b + 5251
93567528*a**13*b + 184527469672*I*a**12*b - 58271832528*a**11*b - 164356450
72*I*a**10*b + 4108911268*a**9*b + 901956132*I*a**8*b - 171801168*a**7*b -
27967632*I*a**6*b + 3813768*a**5*b + 423752*I*a**4*b - 36848*a**3*b - 2352*
I*a**2*b + 98*a*b + 2*I*b)) + 2*b*sqrt((-a**23 - 23*I*a**22 + 253*a**21 + 1
771*I*a**20 - 8855*a**19 - 33649*I*a**18 + 100947*a**17 + 245157*I*a**16 -
490314*a**15 - 817190*I*a**14 + 1144066*a**13 + 1352078*I*a**12 - 1352078*a
11 - 1144066*I*a10 + 817190*a**9 + 490314*I*a**8 - 245157*a**7 - 100947
*I*a**6 + 33649*a**5 + 8855*I*a**4 - 1771*a**3 - 253*I*a**2 + 23*a + I)/(a*
*27 + 19*I*a**26 - 167*a**25 - 893*I*a**24 + 3198*a**23 + 7866*I*a**22 - 12
650*a**21 - 9614*I*a**20 - 10373*a**19 - 43263*I*a**18 + 62491*a**17 + 3714
5*I*a**16 + 29716*a**15 + 89148*I*a**14 - 89148*a**13 - 29716*I*a**12 - 371
45*a**11 - 62491*I*a**10 + 43263*a**9 + 10373*I*a**8 + 9614*a**7 + 12650*I*
a**6 - 7866*a**5 - 3198*I*a**4 + 893*a**3 + 167*I*a**2 - 19*a - I))*log(2*b
*sqrt((-a**23 - 23*I*a**22 + 253*a**21 + 1771*I*a**20 - 8855*a**19 - 33649*
I*a**18 + 100947*a**17 + 245157*I*a**16 - 490314*a**15 - 817190*I*a**14 + 1
144066*a**13 + 1352078*I*a**12 - 1352078*a**11 - 1144066*I*a**10 + 817190*a
9 + 490314*I*a8 - 245157*a**7 - 100947*I*a**6 + 33649*a**5 + 8855*I*a**
4 - 1771*a**3 - 253*I*a**2 + 23*a + I)/(a**27 + 19*I*a**26 - 167*a**25 - 89
3*I*a**24 + 3198*a**23 + 7866*I*a**22 - 12650*a**21 - 9614*I*a**20 - 10373*
a**19 - 43263*I*a**18 + 62491*a**17 + 37145*I*a**16 + 29716*a**15 + 89148*I
*a**14 - 89148*a**13 - 29716*I*a**12 - 37145*a**11 - 62491*I*a**10 + 43263*
a**9 + 10373*I*a**8 + 9614*a**7 + 12650*I*a**6 - 7866*a**5 - 3198*I*a**4 +
893*a**3 + 167*I*a**2 - 19*a - I))*(I*a**33 - 27*a**32 - 348*I*a**31 + 2844
*a**30 + 16500*I*a**29 - 72036*a**28 - 244412*I*a**27 + 654588*a**26 + 1384
344*I*a**25 - 2262000*a**24 - 2646540*I*a**23 + 1560780*a**22 - 1560780*I*a
21 + 5882940*a20 + 9004500*I*a**19 - 8364180*a**18 - 3421710*I*a**17 -
3421710*a**16 - 8364180*I*a**15 + 9004500*a**14 + 5882940*I*a**13 - 1560780
*a**12 + 1560780*I*a**11 - 2646540*a**10 - 2262000*I*a**9 + 1384344*a**8 +
654588*I*a**7 - 244412*a**6 - 72036*I*a**5 + 16500*a**4 + 2844*I*a**3 - 348
*a**2 - 27*I*a + 1)/(4*a**30*b**2 + 120*I*a**29*b**2 - 1740*a**28*b**2 - 16
240*I*a**27*b**2 + 109620*a**26*b**2 + 570024*I*a**25*b**2 - 2375100*a**24*
b**2 - 8143200*I*a**23*b**2 + 23411700*a**22*b**2 + 57228600*I*a**21*b**2 -
120180060*a**20*b**2 - 218509200*I*a**19*b**2 + 345972900*a**18*b**2 + 479
039400*I*a**17*b**2 - 581690700*a**16*b**2 - 620470080*I*a**15*b**2 + 58169
0700*a**14*b**2 + 479039400*I*a**13*b**2 - 345972900*a**12*b**2 - 218509200
*I*a**11*b**2 + 120180060*a**10*b**2 + 57228600*I*a**9*b**2 - 23411700*a**8
*b**2 - 8143200*I*a**7*b**2 + 2375100*a**6*b**2 + 570024*I*a**5*b**2 - 1096
20*a**4*b**2 - 16240*I*a**3*b**2 + 1740*a**2*b**2 + 120*I*a*b**2 - 4*b**2)
+ x + (a**50 + 48*I*a**49 - 1127*a**48 - 17248*I*a**47 + 193452*a**46 + 169
5008*I*a**45 - 12076932*a**44 - 71916768*I*a**43 + 365077482*a**42 + 160347
7568*I*a**41 - 6163366902*a**40 - 20918093728*I*a**39 + 63127818572*a**38 +
170333048928*I*a**37 - 412652088772*a**36 - 900331830048*I*a**35 + 1772528
290407*a**34 + 3151161405168*I*a**33 - 5054988087457*a**32 - 7297426411968*
I*a**31 + 9425842448792*a**30 + 10772391370048*I*a**29 - 10649977831752*a**
28 - 8643460269248*I*a**27 + 4861946401452*a**26 + 4861946401452*a**24 + 86
43460269248*I*a**23 - 10649977831752*a**22 - 10772391370048*I*a**21 + 94258
42448792*a**20 + 7297426411968*I*a**19 - 5054988087457*a**18 - 315116140516
8*I*a**17 + 1772528290407*a**16 + 900331830048*I*a**15 - 412652088772*a**14
- 170333048928*I*a**13 + 63127818572*a**12 + 20918093728*I*a**11 - 6163366
902*a**10 - 1603477568*I*a**9 + 365077482*a**8 + 71916768*I*a**7 - 12076932
*a**6 - 1695008*I*a**5 + 193452*a**4 + 17248*I*a**3 - 1127*a**2 - 48*I*a +
1)/(2*a**49*b + 98*I*a**48*b - 2352*a**47*b - 36848*I*a**46*b + 423752*a**4
5*b + 3813768*I*a**44*b - 27967632*a**43*b - 171801168*I*a**42*b + 90195613
2*a**41*b + 4108911268*I*a**40*b - 16435645072*a**39*b - 58271832528*I*a**3
8*b + 184527469672*a**37*b + 525193567528*I*a**36*b - 1350497745072*a**35*b

```
 - 3151161405168*I*a**34*b + 6696217985982*a**33*b + 12998540796318*I*a**32
*b - 23108516971232*a**31*b - 37703369795168*I*a**30*b + 56555054692752*a**
29*b + 78099837432848*I*a**28*b - 99399793096352*a**27*b - 116686713634848*
I*a**26*b + 126410606437752*a**25*b + 126410606437752*I*a**24*b - 116686713
634848*a**23*b - 99399793096352*I*a**22*b + 78099837432848*a**21*b + 565550
54692752*I*a**20*b - 37703369795168*a**19*b - 23108516971232*I*a**18*b + 12
998540796318*a**17*b + 6696217985982*I*a**16*b - 3151161405168*a**15*b - 13
50497745072*I*a**14*b + 525193567528*a**13*b + 184527469672*I*a**12*b - 582
71832528*a**11*b - 16435645072*I*a**10*b + 4108911268*a**9*b + 901956132*I*
a**8*b - 171801168*a**7*b - 27967632*I*a**6*b + 3813768*a**5*b + 423752*I*a
**4*b - 36848*a**3*b - 2352*I*a**2*b + 98*a*b + 2*I*b)) + (a**5 + 5*I*a**4
- 10*a**3 - 10*I*a**2 + 5*a + I)/(x*(a**5 + 3*I*a**4 - 2*a**3 + 2*I*a**2 -
3*a - I))
```

Giac [B] time = 1.11125, size = 143, normalized size = 2.31

$$-\frac{2\,b^2\log\left(-\frac{ai}{bix+ai+1}+\frac{i^2}{bix+ai+1}+1\right)}{a^2bi+2\,ab-bi}-\frac{ab+bi}{(a-i)^2\left(\frac{ai}{bix+ai+1}-\frac{i^2}{bix+ai+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x, algorithm="giac")
```

```
[Out] -2*b^2*log(-a*i/(b*i*x + a*i + 1) + i^2/(b*i*x + a*i + 1) + 1)/(a^2*b*i + 2
*a*b - b*i) - (a*b + b*i)/((a - i)^2*(a*i/(b*i*x + a*i + 1) - i^2/(b*i*x +
a*i + 1) - 1))
```

3.205 $\int \frac{e^{-2i\tan^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=81

$$-\frac{2b^2\log(x)}{(1+ia)^3}+\frac{2b^2\log(-a-bx+i)}{(1+ia)^3}-\frac{2ib}{(-a+i)^2x}-\frac{a+i}{2(-a+i)x^2}$$

```
[Out] -(I + a)/(2*(I - a)*x^2) - ((2*I)*b)/((I - a)^2*x) - (2*b^2*Log[x])/(1 + I*
a)^3 + (2*b^2*Log[I - a - b*x])/(1 + I*a)^3
```

Rubi [A] time = 0.0516425, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$-\frac{2b^2\log(x)}{(1+ia)^3}+\frac{2b^2\log(-a-bx+i)}{(1+ia)^3}-\frac{2ib}{(-a+i)^2x}-\frac{a+i}{2(-a+i)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a + b*x])*x^3),x]
```

```
[Out] -(I + a)/(2*(I - a)*x^2) - ((2*I)*b)/((I - a)^2*x) - (2*b^2*Log[x])/(1 + I*
a)^3 + (2*b^2*Log[I - a - b*x])/(1 + I*a)^3
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2i\tan^{-1}(a+bx)}}{x^3}\,dx &= \int \frac{1-ia-ibx}{x^3(1+ia+ibx)}\,dx\\
&= \int \left(\frac{-i-a}{(-i+a)x^3}+\frac{2ib}{(-i+a)^2x^2}-\frac{2ib^2}{(-i+a)^3x}+\frac{2ib^3}{(-i+a)^3(-i+a+bx)}\right)dx\\
&= -\frac{i+a}{2(i-a)x^2}-\frac{2ib}{(i-a)^2x}-\frac{2b^2\log(x)}{(1+ia)^3}+\frac{2b^2\log(i-a-bx)}{(1+ia)^3}
\end{aligned}$$

Mathematica [A] time = 0.0361532, size = 66, normalized size = 0.81

$$\frac{(a-i)\left(a^2-4ibx+1\right)+4ib^2x^2\log(-a-bx+i)-4ib^2x^2\log(x)}{2(a-i)^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a + b*x])*x^3),x]
```

```
[Out] ((-I + a)*(1 + a^2 - (4*I)*b*x) - (4*I)*b^2*x^2*Log[x] + (4*I)*b^2*x^2*Log[
I - a - b*x])/(2*(-I + a)^3*x^2)
```

Maple [B] time = 0.053, size = 246, normalized size = 3.

$$\frac{ib^2\ln\left(b^2x^2+2\,xab+a^2+1\right)a}{(i-a)^4}+\frac{b^2\ln\left(b^2x^2+2\,xab+a^2+1\right)}{(i-a)^4}-2\,\frac{b^2\arctan(bx+a)\,a}{(i-a)^4}+\frac{2\,ib^2\arctan(bx+a)}{(i-a)^4}-\frac{}{(i-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x)
```

```
[Out] I*b^2/(I-a)^4*ln(b^2*x^2+2*a*b*x+a^2+1)*a+b^2/(I-a)^4*ln(b^2*x^2+2*a*b*x+a^
2+1)-2*b^2/(I-a)^4*arctan(b*x+a)*a+2*I*b^2/(I-a)^4*arctan(b*x+a)-I/(I-a)^4/
x^2*a^3+1/2/(I-a)^4/x^2*a^4-I/(I-a)^4/x^2*a-1/2/(I-a)^4/x^2-2*I*b/(I-a)^4/x
*a^2+2*I*b/(I-a)^4/x-4*b/(I-a)^4/x*a-2*I*b^2/(I-a)^4*ln(x)*a-2*b^2/(I-a)^4*
ln(x)
```

Maxima [B] time = 1.03118, size = 220, normalized size = 2.72

$$-\frac{2\,(-i\,a-1)b^2\log(i\,bx+i\,a+1)}{a^4-4i\,a^3-6\,a^2+4i\,a+1}-\frac{2\,(i\,a+1)b^2\log(x)}{a^4-4i\,a^3-6\,a^2+4i\,a+1}+\frac{4\,(-i\,a-1)b^2x^2+a^4-2i\,a^3+\left(a^3-5i\,a^2-7\,a+3i\right)}{\left(2\,a^3-6i\,a^2-6\,a+2i\right)bx^3+\left(2\,a^4-8i\,a^3-12\,a^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x, algorithm="maxima")
```

```
[Out] -2*(-I*a - 1)*b^2*log(I*b*x + I*a + 1)/(a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1)
- 2*(I*a + 1)*b^2*log(x)/(a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1) + (4*(-I*a - 1
)*b^2*x^2 + a^4 - 2*I*a^3 + (a^3 - 5*I*a^2 - 7*a + 3*I)*b*x - 2*I*a - 1)/((
2*a^3 - 6*I*a^2 - 6*a + 2*I)*b*x^3 + (2*a^4 - 8*I*a^3 - 12*a^2 + 8*I*a + 2)
*x^2)
```

Fricas [A] time = 2.18259, size = 181, normalized size = 2.23

$$\frac{-4i\,b^2x^2\log(x)+4i\,b^2x^2\log\left(\frac{bx+a-i}{b}\right)+a^3-4\,(i\,a+1)bx-i\,a^2+a-i}{\left(2\,a^3-6i\,a^2-6\,a+2i\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x, algorithm="fricas")
```

```
[Out] (-4*I*b^2*x^2*log(x) + 4*I*b^2*x^2*log((b*x + a - I)/b) + a^3 - 4*(I*a + 1)
*b*x - I*a^2 + a - I)/((2*a^3 - 6*I*a^2 - 6*a + 2*I)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2)/x**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10766, size = 212, normalized size = 2.62

$$\frac{2\,b^3\log\left(-\frac{ai}{bix+ai+1}+\frac{i^2}{bix+ai+1}+1\right)}{a^3bi+3\,a^2b-3\,abi-b}-\frac{\frac{2\left(ab^3i-3\,b^3\right)i^2}{(bix+ai+1)b}+\frac{ab^2i-5\,b^2}{ai+1}}{2\,(a-i)^2\left(\frac{ai}{bix+ai+1}-\frac{i^2}{bix+ai+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x, algorithm="giac")
```

```
[Out] 2*b^3*log(-a*i/(b*i*x + a*i + 1) + i^2/(b*i*x + a*i + 1) + 1)/(a^3*b*i + 3*
a^2*b - 3*a*b*i - b) - 1/2*(2*(a*b^3*i - 3*b^3)*i^2/((b*i*x + a*i + 1)*b) +
 (a*b^2*i - 5*b^2)/(a*i + 1))/((a - i)^2*(a*i/(b*i*x + a*i + 1) - i^2/(b*i*
x + a*i + 1) - 1)^2)
```

3.206 $\int \frac{e^{-2 i \tan^{-1}(a+bx)}}{x^4} \, dx$

Optimal. Leaf size=102

$$\frac{2b^2}{(1+ia)^3x}+\frac{2ib^3\log(x)}{(-a+i)^4}-\frac{2ib^3\log(-a-bx+i)}{(-a+i)^4}-\frac{ib}{(-a+i)^2x^2}-\frac{a+i}{3(-a+i)x^3}$$

```
[Out] -(I + a)/(3*(I - a)*x^3) - (I*b)/((I - a)^2*x^2) + (2*b^2)/((1 + I*a)^3*x)
+ ((2*I)*b^3*Log[x])/(I - a)^4 - ((2*I)*b^3*Log[I - a - b*x])/(I - a)^4
```

Rubi [A] time = 0.0606169, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$\frac{2b^2}{(1+ia)^3x}+\frac{2ib^3\log(x)}{(-a+i)^4}-\frac{2ib^3\log(-a-bx+i)}{(-a+i)^4}-\frac{ib}{(-a+i)^2x^2}-\frac{a+i}{3(-a+i)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a + b*x])*x^4),x]
```

```
[Out] -(I + a)/(3*(I - a)*x^3) - (I*b)/((I - a)^2*x^2) + (2*b^2)/((1 + I*a)^3*x)
+ ((2*I)*b^3*Log[x])/(I - a)^4 - ((2*I)*b^3*Log[I - a - b*x])/(I - a)^4
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2i\tan^{-1}(a+bx)}}{x^4}\,dx &= \int \frac{1-ia-ibx}{x^4(1+ia+ibx)}\,dx\\ &= \int \left(\frac{-i-a}{(-i+a)x^4}+\frac{2ib}{(-i+a)^2x^3}-\frac{2ib^2}{(-i+a)^3x^2}+\frac{2ib^3}{(-i+a)^4x}-\frac{2ib^4}{(-i+a)^4(-i+a+bx)}\right)dx\\ &= -\frac{i+a}{3(i-a)x^3}-\frac{ib}{(i-a)^2x^2}+\frac{2b^2}{(1+ia)^3x}+\frac{2ib^3\log(x)}{(i-a)^4}-\frac{2ib^3\log(i-a-bx)}{(i-a)^4}\end{aligned}$$

Mathematica [A] time = 0.0462582, size = 91, normalized size = 0.89

$$\frac{(a-i)\left(a^3-ia^2-3iabx+a+6ib^2x^2-3bx-i\right)-6ib^3x^3\log(-a-bx+i)+6ib^3x^3\log(x)}{3(a-i)^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a + b*x])*x^4),x]
```

```
[Out] ((-I + a)*(-I + a - I*a^2 + a^3 - 3*b*x - (3*I)*a*b*x + (6*I)*b^2*x^2) + (6
*I)*b^3*x^3*Log[x] - (6*I)*b^3*x^3*Log[I - a - b*x])/(3*(-I + a)^4*x^3)
```

Maple [B] time = 0.054, size = 349, normalized size = 3.4

$$\frac{-2\,i b^3\ln(x)\,a}{(i-a)^5}+\frac{b^3\ln\left(b^2x^2+2\,xab+a^2+1\right)}{(i-a)^5}-2\,\frac{b^3\arctan(bx+a)\,a}{(i-a)^5}+\frac{\frac{2\,i}{3}a^2}{(i-a)^5\,x^3}+\frac{ia^4}{(i-a)^5\,x^3}+\frac{2\,ib^3\arctan(bx+a)}{(i-a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x)
```

```
[Out] -2*I*b^3/(I-a)^5*ln(x)*a+b^3/(I-a)^5*ln(b^2*x^2+2*a*b*x+a^2+1)-2*b^3/(I-a)^
5*arctan(b*x+a)*a+2/3*I/(I-a)^5/x^3*a^2+I/(I-a)^5/x^3*a^4+2*I*b^3/(I-a)^5*a
rctan(b*x+a)+3*b/(I-a)^5/x^2*a^2-b/(I-a)^5/x^2+I*b/(I-a)^5/x^2*a^3-1/3*I/(I
-a)^5/x^3-4*b^2/(I-a)^5/x*a+I*b^3/(I-a)^5*ln(b^2*x^2+2*a*b*x+a^2+1)*a-1/3/(
I-a)^5/x^3*a^5+2*I*b^2/(I-a)^5/x+2/3/(I-a)^5/x^3*a^3-3*I*b/(I-a)^5/x^2*a+1/
(I-a)^5/x^3*a-2*I*b^2/(I-a)^5/x*a^2-2*b^3/(I-a)^5*ln(x)
```

Maxima [B] time = 1.05809, size = 300, normalized size = 2.94

$$\frac{(2\,a-2i)b^3\log(i\,bx+i\,a+1)}{i\,a^5+5\,a^4-10i\,a^3-10\,a^2+5i\,a+1}-\frac{(2\,a-2i)b^3\log(x)}{i\,a^5+5\,a^4-10i\,a^3-10\,a^2+5i\,a+1}-\frac{(6\,a-6i)b^3x^3-i\,a^5+3\left(a^2-2i\,a-1\right)b^2x^2}{\left(3i\,a^4+12\,a^3-18i\,a^2-12\,a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x, algorithm="maxima")
```

```
[Out] (2*a - 2*I)*b^3*log(I*b*x + I*a + 1)/(I*a^5 + 5*a^4 - 10*I*a^3 - 10*a^2 + 5
*I*a + 1) - (2*a - 2*I)*b^3*log(x)/(I*a^5 + 5*a^4 - 10*I*a^3 - 10*a^2 + 5*I
*a + 1) - ((6*a - 6*I)*b^3*x^3 - I*a^5 + 3*(a^2 - 2*I*a - 1)*b^2*x^2 - 3*a^
4 + 2*I*a^3 - (I*a^4 + 5*a^3 - 9*I*a^2 - 7*a + 2*I)*b*x - 2*a^2 + 3*I*a + 1
)/((3*I*a^4 + 12*a^3 - 18*I*a^2 - 12*a + 3*I)*b*x^4 + (3*I*a^5 + 15*a^4 - 3
0*I*a^3 - 30*a^2 + 15*I*a + 3)*x^3)
```

Fricas [A] time = 2.12128, size = 248, normalized size = 2.43

$$\frac{6i\,b^3x^3\log(x)-6i\,b^3x^3\log\left(\frac{bx+a-i}{b}\right)-6\,(-i\,a-1)b^2x^2+a^4-2i\,a^3+\left(-3i\,a^2-6\,a+3i\right)bx-2i\,a-1}{\left(3\,a^4-12i\,a^3-18\,a^2+12i\,a+3\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x, algorithm="fricas")
```

```
[Out] (6*I*b^3*x^3*log(x) - 6*I*b^3*x^3*log((b*x + a - I)/b) - 6*(-I*a - 1)*b^2*x
^2 + a^4 - 2*I*a^3 + (-3*I*a^2 - 6*a + 3*I)*b*x - 2*I*a - 1)/((3*a^4 - 12*I
```

```
*a^3 - 18*a^2 + 12*I*a + 3)*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2)/x**4,x)

[Out] Timed out
```

Giac [B] time = 1.13172, size = 273, normalized size = 2.68

$$-\frac{2\,b^4\log\left(-\frac{ai}{bix+ai+1}+\frac{i^2}{bix+ai+1}+1\right)}{a^4bi+4\,a^3b-6\,a^2bi-4\,ab+bi}-\frac{\frac{3\left(ab^4i-8\,b^4\right)i^2}{(bix+ai+1)b}+\frac{ab^3i-10\,b^3}{ai+1}+\frac{3\left(a^2b^5+4\,ab^5i+5\,b^5\right)i^2}{(bix+ai+1)^2b^2}}{3\,(a-i)^3\left(\frac{ai}{bix+ai+1}-\frac{i^2}{bix+ai+1}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x, algorithm="giac")

[Out] -2*b^4*log(-a*i/(b*i*x + a*i + 1) + i^2/(b*i*x + a*i + 1) + 1)/(a^4*b*i + 4
*a^3*b - 6*a^2*b*i - 4*a*b + b*i) - 1/3*(3*(a*b^4*i - 8*b^4)*i^2/((b*i*x +
a*i + 1)*b) + (a*b^3*i - 10*b^3)/(a*i + 1) + 3*(a^2*b^5 + 4*a*b^5*i + 5*b^5
)*i^2/((b*i*x + a*i + 1)^2*b^2))/((a - i)^3*(a*i/(b*i*x + a*i + 1) - i^2/(b
*i*x + a*i + 1) - 1)^3)
```

3.207 $\int e^{-3i\tan^{-1}(a+bx)}x^4\,dx$

Optimal. Leaf size=324

$$\frac{i(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-2\left(-52ia^2-118a+61i\right)bx-112ia^3-422a^2+458ia+163\right)}{40b^5}+\frac{3\left(8ia^4+48a^3-88ia^2\right.}{}$$

```
[Out] ((2*I)*x^4*(1 - I*a - I*b*x)^(3/2))/(b*Sqrt[1 + I*a + I*b*x]) + (3*(19*I -
68*a - (88*I)*a^2 + 48*a^3 + (8*I)*a^4)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a
+ I*b*x])/(8*b^5) - (3*(17*I - 16*a)*x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I
*a + I*b*x])/(20*b^3) - (11*x^3*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*
x])/(5*b^2) + ((I/40)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(163 +
(458*I)*a - 422*a^2 - (112*I)*a^3 - 2*(61*I - 118*a - (52*I)*a^2)*b*x))/b^5
 - (3*(19 + (68*I)*a - 88*a^2 - (48*I)*a^3 + 8*a^4)*ArcSinh[a + b*x])/(8*b^
5)
```

Rubi [A] time = 0.281299, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5095, 97, 153, 147, 50, 53, 619, 215}

$$\frac{i(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-2\left(-52ia^2-118a+61i\right)bx-112ia^3-422a^2+458ia+163\right)}{40b^5}+\frac{3\left(8ia^4+48a^3-88ia^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((2*I)*x^4*(1 - I*a - I*b*x)^(3/2))/(b*Sqrt[1 + I*a + I*b*x]) + (3*(19*I -
68*a - (88*I)*a^2 + 48*a^3 + (8*I)*a^4)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a
+ I*b*x])/(8*b^5) - (3*(17*I - 16*a)*x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I
*a + I*b*x])/(20*b^3) - (11*x^3*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*
x])/(5*b^2) + ((I/40)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(163 +
(458*I)*a - 422*a^2 - (112*I)*a^3 - 2*(61*I - 118*a - (52*I)*a^2)*b*x))/b^5
 - (3*(19 + (68*I)*a - 88*a^2 - (48*I)*a^3 + 8*a^4)*ArcSinh[a + b*x])/(8*b^
5)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
```

```
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-3i\tan^{-1}(a+bx)}x^4\,dx = \int \frac{x^4(1-ia-ibx)^{3/2}}{(1+ia+ibx)^{3/2}}\,dx$$

$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{(2i)\int \frac{x^3\sqrt{1-ia-ibx}\left(4(1-ia)-\frac{11ibx}{2}\right)}{\sqrt{1+ia+ibx}}\,dx}{b}$$

$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{11x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} - \frac{(2i)\int \frac{x^2\sqrt{1-ia-ibx}\left(\frac{33}{2}(1+ia)(i+a)b+\frac{3}{2}(17+16\ldots\right.}{\sqrt{1+ia+ibx}}}{5b^3}$$

$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{3(17i-16a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} - \frac{11x^3(1-ia-ibx)^{3/2}\sqrt{1+i\ldots}}{5b^2}$$

$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{3(17i-16a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} - \frac{11x^3(1-ia-ibx)^{3/2}\sqrt{1+i\ldots}}{5b^2}$$

$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(19i-68a-88ia^2+48a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{3(17i-\ldots}{}$$

$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(19i-68a-88ia^2+48a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{3(17i-\ldots}{}$$

$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(19i-68a-88ia^2+48a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{3(17i-\ldots}{}$$

$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(19i-68a-88ia^2+48a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{3(17i-\ldots}{}$$

Mathematica [A] time = 0.477621, size = 299, normalized size = 0.92

$$\frac{\frac{a^3\left(60b^2x^2-2004ibx-905\right)-a^2\left(20b^3x^3+356ib^2x^2+2635bx+836i\right)+a^5(410+8ibx)+2a^4(265bx-638i)+8ia^6+a\left(8ib^5x^5+10b^4x^4+116ib^3x^3-515b^2x^2+1468ibx-1315\right)+8\ldots}{\sqrt{a^2+2abx+b^2x^2+1}}}{40b^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((448*I + (8*I)*a^6 + 285*b*x + (224*I)*b^2*x^2 + 95*b^3*x^3 - (56*I)*b^4*x
^4 - 30*b^5*x^5 + (8*I)*b^6*x^6 + a^5*(410 + (8*I)*b*x) + 2*a^4*(-638*I + 2
65*b*x) + a^3*(-905 - (2004*I)*b*x + 60*b^2*x^2) - a^2*(836*I + 2635*b*x +
(356*I)*b^2*x^2 + 20*b^3*x^3) + a*(-1315 + (1468*I)*b*x - 515*b^2*x^2 + (11
6*I)*b^3*x^3 + 10*b^4*x^4 + (8*I)*b^5*x^5))/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^
2] + (30*(-1)^(1/4)*(19*I - 68*a - (88*I)*a^2 + 48*a^3 + (8*I)*a^4)*Sqrt[b]
*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/Sqrt
[(-I)*b])/(40*b^5)
```

Maple [B] time = 0.138, size = 2058, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x)
```

```
[Out] 33/b^5*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a^3-6/b^5*((x-(I-a)/b)^2
*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a-6/b^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+
((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)-9/8/b^4*x*(b^2*x^2
+2*a*b*x+a^2+1)^(1/2)-9/8/b^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a
^2+1)^(1/2))/(b^2)^(1/2)-3/2*I/b^4*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-9/8*a/b^5*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/4
/b^4*x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-3/4/b^5*a*(b^2*x^2+2*a*b*x+a^2+1)^(3/2
)-3/b^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a
)/b)*b)^(1/2))/(b^2)^(1/2)*a^4+33/b^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+
((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)*a^2-3/2*I/b^4*a*(b
^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-I/b^4*a*x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+24*I/
b^7/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)*a^2-24*I/b^4*
ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^
(1/2))/(b^2)^(1/2)*a+18*I/b^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a
)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)*a^3+18*I/b^4*((x-(I-a)/b)^
2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x*a^3-24*I/b^4*((x-(I-a)/b)^2*b^2+2*I*(x-(I-
a)/b)*b)^(1/2)*x*a-2*I/b^7/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)
*b)^(5/2)*a^4+4*I/b^8/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(
5/2)*a^3-4*I/b^8/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)*
a+1/5*I/b^5*(b^2*x^2+2*a*b*x+a^2+1)^(5/2)+4*I/b^5*((x-(I-a)/b)^2*b^2+2*I*(x
-(I-a)/b)*b)^(3/2)+12/b^5*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)*a^3-1
6/b^5*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)*a-1/b^8/(x-I/b+a/b)^3*((x
-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)-6/b^4*((x-(I-a)/b)^2*b^2+2*I*(x-(I
-a)/b)*b)^(1/2)*x-3/b^5*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a^5-1/b
^8/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)*a^4+6/b^8/(x-I
/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)*a^2-3/b^4*((x-(I-a)/b
)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x*a^4+33/b^4*((x-(I-a)/b)^2*b^2+2*I*(x-(I-
a)/b)*b)^(1/2)*x*a^2-12/b^7/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b
)*b)^(5/2)*a^3+20/b^7/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(
5/2)*a+2*I/b^5*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)*a^4-I/b^5*a^2*(b
^2*x^2+2*a*b*x+a^2+1)^(3/2)-3/2*I/b^5*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+18*
I/b^5*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a^4-24*I/b^5*((x-(I-a)/b)
^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a^2-22*I/b^5*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a
)/b)*b)^(3/2)*a^2-6*I/b^7/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*
b)^(5/2)
```

Maxima [B] time = 1.65956, size = 1847, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^4/(b^7*x^2 + 2*a*b^6*x + a^2*b^5 -
2*I*b^6*x - 2*I*a*b^5 - b^5) + 4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^3/(b
^7*x^2 + 2*a*b^6*x + a^2*b^5 - 2*I*b^6*x - 2*I*a*b^5 - b^5) + 4*(b^2*x^2 +
2*a*b*x + a^2 + 1)^(3/2)*a^3/(2*I*b^6*x + 2*I*a*b^5 + 2*b^5) + 6*I*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*a^4/(I*b^6*x + I*a*b^5 + b^5) - 6*I*(b^2*x^2 + 2*
a*b*x + a^2 + 1)^(3/2)*a^2/(b^7*x^2 + 2*a*b^6*x + a^2*b^5 - 2*I*b^6*x - 2*I
*a*b^5 - b^5) - 12*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/(2*I*b^6*x + 2
*I*a*b^5 + 2*b^5) + 24*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3/(I*b^6*x + I*a
*b^5 + b^5) - 4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(b^7*x^2 + 2*a*b^6*x
+ a^2*b^5 - 2*I*b^6*x - 2*I*a*b^5 - b^5) - 12*(b^2*x^2 + 2*a*b*x + a^2 + 1)
^(3/2)*a/(2*I*b^6*x + 2*I*a*b^5 + 2*b^5) - 36*I*sqrt(b^2*x^2 + 2*a*b*x + a^
```

```
2 + 1)*a^2/(I*b^6*x + I*a*b^5 + b^5) + I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2
)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5 - 2*I*b^6*x - 2*I*a*b^5 - b^5) + 4*I*(b^2*
x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(2*I*b^6*x + 2*I*a*b^5 + 2*b^5) - 24*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*a/(I*b^6*x + I*a*b^5 + b^5) - 3*a^4*arcsinh(b*x
+ a)/b^5 + 6*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(I*b^6*x + I*a*b^5 + b^5)
- I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a*x/b^4 - 3*sqrt(-b^2*x^2 - 2*a*b*x
 - a^2 + 4*I*b*x + 4*I*a + 3)*a^2*x/b^4 + 18*I*a^3*arcsinh(b*x + a)/b^5 + I
*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/b^5 + 6*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*a^3/b^5 - 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*a^
3/b^5 - 3/4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x/b^4 - 3/2*I*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*a*x/b^4 + 6*I*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x
+ 4*I*a + 3)*a*x/b^4 + 3*a^2*arcsin(I*b*x + I*a + 2)/b^5 + 36*a^2*arcsinh(b
*x + a)/b^5 + 1/5*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2)/b^5 + 13/4*(b^2*x^2
 + 2*a*b*x + a^2 + 1)^(3/2)*a/b^5 - 39/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)*a^2/b^5 + 12*I*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*a^2/b
^5 - 9/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^4 + 3*sqrt(-b^2*x^2 - 2*a*b*
x - a^2 + 4*I*b*x + 4*I*a + 3)*x/b^4 - 6*I*a*arcsin(I*b*x + I*a + 2)/b^5 -
63/2*I*a*arcsinh(b*x + a)/b^5 - 2*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/b^5
 - 153/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^5 + 15*sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 4*I*b*x + 4*I*a + 3)*a/b^5 - 3*arcsin(I*b*x + I*a + 2)/b^5 - 81/
8*arcsinh(b*x + a)/b^5 + 6*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^5 - 6*I*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)/b^5
```

Fricas [A] time = 2.40236, size = 818, normalized size = 2.52

$$\frac{62i\,a^6 + 2687\,a^5 - 11575i\,a^4 - 20350\,a^3 + \left(62i\,a^5 + 2625\,a^4 - 8950i\,a^3 - 11400\,a^2 + 6340i\,a + 1280\right)bx + 17740i\,a^2 + (96}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (62*I*a^6 + 2687*a^5 - 11575*I*a^4 - 20350*a^3 + (62*I*a^5 + 2625*a^4 - 895
0*I*a^3 - 11400*a^2 + 6340*I*a + 1280)*b*x + 17740*I*a^2 + (960*a^5 - 6720*
I*a^4 - 16320*a^3 + (960*a^4 - 5760*I*a^3 - 10560*a^2 + 8160*I*a + 2280)*b*
x + 18720*I*a^2 + 10440*a - 2280*I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)) + (64*I*b^5*x^5 - 176*b^4*x^4 + (256*a - 272*I)*b^3*x^3 + 64*I*a
^5 - 8*(52*a^2 - 118*I*a - 61)*b^2*x^2 + 3344*a^4 - 13552*I*a^3 + (896*a^3
- 3376*I*a^2 - 3664*a + 1304*I)*b*x - 20792*a^2 + 14104*I*a + 3584)*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1) + 7620*a - 1280*I)/(320*b^6*x + (320*a - 320*I)*
b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14108, size = 536, normalized size = 1.65

$$\frac{1}{40}\sqrt{(bx+a)^2+1}\left(\left(2\left(\left(\frac{4\,ix}{b}-\frac{4\,ab^{17}i+15\,b^{17}}{b^{19}}\right)x+\frac{4\,a^2b^{16}i+35\,ab^{16}-32\,b^{16}i}{b^{19}}\right)x-\frac{8\,a^3b^{15}i+130\,a^2b^{15}-252\,ab^{15}i-}{b^{19}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/40*sqrt((b*x + a)^2 + 1)*((2*((4*i*x/b - (4*a*b^17*i + 15*b^17)/b^19)*x +
 (4*a^2*b^16*i + 35*a*b^16 - 32*b^16*i)/b^19)*x - (8*a^3*b^15*i + 130*a^2*b
^15 - 252*a*b^15*i - 125*b^15)/b^19)*x + (8*a^4*b^14*i + 250*a^3*b^14 - 804
*a^2*b^14*i - 835*a*b^14 + 288*b^14*i)/b^19) + 1/4*(8*a^4*abs(b) - 48*a^3*i
*abs(b) - 88*a^2*abs(b) + 68*a*i*abs(b) + 19*abs(b))*log(96*b^5)/b^6 + 1/8*
(8*a^4*i + 48*a^3 - 88*a^2*i - 68*a + 19*i)*log(3*(x*abs(b) - sqrt((b*x + a
)^2 + 1))^2*a*b*i + a^3*b*i + (x*abs(b) - sqrt((b*x + a)^2 + 1))^3*i*abs(b)
 + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*i*abs(b) + 2*(x*abs(b) - sqrt((
b*x + a)^2 + 1))^2*b + 2*a^2*b - a*b*i + 4*(x*abs(b) - sqrt((b*x + a)^2 + 1
))*a*abs(b) - (x*abs(b) - sqrt((b*x + a)^2 + 1))*i*abs(b))/(b^4*i*abs(b))
```

3.208 $\int e^{-3i\tan^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=249

$$-\frac{i(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-22ia^2+2(11+10ia)bx-54a+29i\right)}{8b^4}+\frac{3\left(-8ia^3-36a^2+44ia+17\right)\sqrt{-ia-ibx+1}\sqrt{}}{8b^4}$$

```
[Out] ((2*I)*x^3*(1 - I*a - I*b*x)^(3/2))/(b*Sqrt[1 + I*a + I*b*x]) + (3*(17 + (4
4*I)*a - 36*a^2 - (8*I)*a^3)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(
8*b^4) - (9*x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(4*b^2) - ((
I/8)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(29*I - 54*a - (22*I)*a^
2 + 2*(11 + (10*I)*a)*b*x))/b^4 + (3*(17*I - 44*a - (36*I)*a^2 + 8*a^3)*Arc
Sinh[a + b*x])/(8*b^4)
```

Rubi [A] time = 0.245515, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5095, 97, 153, 147, 50, 53, 619, 215}

$$-\frac{i(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-22ia^2+2(11+10ia)bx-54a+29i\right)}{8b^4}+\frac{3\left(-8ia^3-36a^2+44ia+17\right)\sqrt{-ia-ibx+1}\sqrt{}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((3*I)*ArcTan[a + b*x]),x]
```


```
[Out] ((2*I)*x^3*(1 - I*a - I*b*x)^(3/2))/(b*Sqrt[1 + I*a + I*b*x]) + (3*(17 + (4
4*I)*a - 36*a^2 - (8*I)*a^3)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(
8*b^4) - (9*x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(4*b^2) - ((
I/8)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(29*I - 54*a - (22*I)*a^
2 + 2*(11 + (10*I)*a)*b*x))/b^4 + (3*(17*I - 44*a - (36*I)*a^2 + 8*a^3)*Arc
Sinh[a + b*x])/(8*b^4)
```


Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
```

```
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-3i\tan^{-1}(a+bx)}x^3\,dx = \int \frac{x^3(1-ia-ibx)^{3/2}}{(1+ia+ibx)^{3/2}}\,dx$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{(2i)\int \frac{x^2\sqrt{1-ia-ibx}\left(3(1-ia)-\frac{9ibx}{2}\right)}{\sqrt{1+ia+ibx}}\,dx}{b}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{9x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{i\int \frac{x\sqrt{1-ia-ibx}\left(9i(1+a^2)b+\frac{3}{2}(11+10ia)b^2x\right)}{\sqrt{1+ia+ibx}}\,dx}{2b^3}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{9x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{i(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}\,(29i-5}{8b^4}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(17+44ia-36a^2-8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{9x^2(1-ia-ibx}{4}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(17+44ia-36a^2-8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{9x^2(1-ia-ibx}{4}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(17+44ia-36a^2-8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{9x^2(1-ia-ibx}{4}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(17+44ia-36a^2-8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{9x^2(1-ia-ibx}{4}$$

Mathematica [A] time = 0.3541, size = 244, normalized size = 0.98

$$\frac{a^2\left(-12b^2x^2+265ibx+4\right)+a^4(-76-2ibx)-5a^3(20bx-31i)-2ia^5+a\left(2ib^4x^4+4b^3x^3+53ib^2x^2+212bx+157i\right)+2}{8b^4\sqrt{a^2+2abx+b^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] (80 - (2*I)*a^5 - (51*I)*b*x + 40*b^2*x^2 - (17*I)*b^3*x^3 - 8*b^4*x^4 + (2
*I)*b^5*x^5 + a^4*(-76 - (2*I)*b*x) - 5*a^3*(-31*I + 20*b*x) + a^2*(4 + (26
5*I)*b*x - 12*b^2*x^2) + a*(157*I + 212*b*x + (53*I)*b^2*x^2 + 4*b^3*x^3 +
(2*I)*b^4*x^4))/(8*b^4*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) + (3*(-1)^(1/4)*(
17*I - 44*a - (36*I)*a^2 + 8*a^3)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]
*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*b^(9/2))
```

Maple [B] time = 0.102, size = 1529, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x)
```

```
[Out] -27/2*I/b^3*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-
(I-a)/b)*b)^(1/2))/(b^2)^(1/2)*a^2-3*I/b^7/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2
```

```
+2*I*(x-(I-a)/b)*b)^(5/2)*a^2+2*I/b^6/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*
(x-(I-a)/b)*b)^(5/2)*a^3-12*I/b^6/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(
I-a)/b)*b)^(5/2)*a-5/b^6/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b
)^(5/2)-9/b^4*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)*a^2+3/b^4*((x-(I-
a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a^4+3/b^3*ln((I*b+(x-(I-a)/b)*b^2)/(b^
2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)*a^3-33/2/
b^3*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)
*b)^(1/2))/(b^2)^(1/2)*a-33/2/b^4*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/
2)*a^2-27/2*I/b^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x*a^2+3/8*I/b
^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+3/8*I/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^
2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/b^7/(x-I/b+a/b)^3*((x-(I-a)/b)^2*
b^2+2*I*(x-(I-a)/b)*b)^(5/2)*a^3-3/b^7/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I
*(x-(I-a)/b)*b)^(5/2)*a+I/b^7/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)
/b)*b)^(5/2)+9/b^6/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2
)*a^2+3/b^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x*a^3-33/2/b^3*((x-
(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x*a-2*I/b^4*((x-(I-a)/b)^2*b^2+2*I*
(x-(I-a)/b)*b)^(3/2)*a^3+1/4*I/b^3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)*x+1/4*I/b^
4*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)*a+3/8*I/b^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x
-27/2*I/b^4*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a^3+11*I/b^4*((x-(I
-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)*a+6*I/b^3*((x-(I-a)/b)^2*b^2+2*I*(x-(
I-a)/b)*b)^(1/2)*x+6*I/b^4*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a+6*
I/b^3*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/
b)*b)^(1/2))/(b^2)^(1/2)+4/b^4*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)
```

Maxima [B] time = 1.658, size = 1322, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^3/(b^6*x^2 + 2*a*b^5*x + a^2*b^4 -
 2*I*b^5*x - 2*I*a*b^4 - b^4) - 3*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/(
b^6*x^2 + 2*a*b^5*x + a^2*b^4 - 2*I*b^5*x - 2*I*a*b^4 - b^4) - 3*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^(3/2)*a^2/(2*I*b^5*x + 2*I*a*b^4 + 2*b^4) - 6*I*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*a^3/(I*b^5*x + I*a*b^4 + b^4) + 3*I*(b^2*x^2 + 2
*a*b*x + a^2 + 1)^(3/2)*a/(b^6*x^2 + 2*a*b^5*x + a^2*b^4 - 2*I*b^5*x - 2*I*
a*b^4 - b^4) + 6*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(2*I*b^5*x + 2*I*a
*b^4 + 2*b^4) - 18*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/(I*b^5*x + I*a*b^4
 + b^4) + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(b^6*x^2 + 2*a*b^5*x + a^2*b^
4 - 2*I*b^5*x - 2*I*a*b^4 - b^4) + 3*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(2
*I*b^5*x + 2*I*a*b^4 + 2*b^4) + 18*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/(I
*b^5*x + I*a*b^4 + b^4) + 3*a^3*arcsinh(b*x + a)/b^4 + 6*sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)/(I*b^5*x + I*a*b^4 + b^4) + 1/4*I*(b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2)*x/b^3 + 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)
*a*x/b^3 - 27/2*I*a^2*arcsinh(b*x + a)/b^4 - 3/4*I*(b^2*x^2 + 2*a*b*x + a^2
 + 1)^(3/2)*a/b^4 - 9/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/b^4 + 3/2*sqr
t(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*a^2/b^4 + 3/8*I*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*x/b^3 - 3/2*I*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*
b*x + 4*I*a + 3)*x/b^3 - 3/2*a*arcsin(I*b*x + I*a + 2)/b^4 - 18*a*arcsinh(b
*x + a)/b^4 - (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/b^4 + 75/8*I*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*a/b^4 - 9/2*I*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x
 + 4*I*a + 3)*a/b^4 + 3/2*I*arcsin(I*b*x + I*a + 2)/b^4 + 63/8*I*arcsinh(b*
x + a)/b^4 + 9/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4 - 3*sqrt(-b^2*x^2 -
2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)/b^4
```

Fricas [A] time = 2.36326, size = 635, normalized size = 2.55

$$-15i\,a^5 - 495\,a^4 + 1664i\,a^3 + \left(-15i\,a^4 - 480\,a^3 + 1184i\,a^2 + 968\,a - 256i\right)bx + 2152\,a^2 - \left(192\,a^4 - 1056i\,a^3 + \left(192\,a^3 - 8\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (-15*I*a^5 - 495*a^4 + 1664*I*a^3 + (-15*I*a^4 - 480*a^3 + 1184*I*a^2 + 968
*a - 256*I)*b*x + 2152*a^2 - (192*a^4 - 1056*I*a^3 + (192*a^3 - 864*I*a^2 -
 1056*a + 408*I)*b*x - 1920*a^2 + 1464*I*a + 408)*log(-b*x - a + sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)) + (16*I*b^4*x^4 - 48*b^3*x^3 + (80*a - 88*I)*b^2*x
^2 - 16*I*a^4 - 624*a^3 - 8*(22*a^2 - 54*I*a - 29)*b*x + 1864*I*a^2 + 1896*
a - 640*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 1224*I*a - 256)/(64*b^5*x +
(64*a - 64*I)*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1616, size = 459, normalized size = 1.84

$$-\frac{1}{8}\sqrt{(bx+a)^2+1}\left(\left(2x\left(\frac{x}{bi} - \frac{ab^{11} - 4\,b^{11}i}{b^{13}i}\right) + \frac{2\,a^2b^{10} - 20\,ab^{10}i - 19\,b^{10}}{b^{13}i}\right)x - \frac{2\,a^3b^9 - 44\,a^2b^9i - 93\,ab^9 + 48\,b^9i}{b^{13}i}\right) - \frac{(8\,a^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt((b*x + a)^2 + 1)*((2*x*(x/(b*i) - (a*b^11 - 4*b^11*i)/(b^13*i)) +
 (2*a^2*b^10 - 20*a*b^10*i - 19*b^10)/(b^13*i))*x - (2*a^3*b^9 - 44*a^2*b^9
*i - 93*a*b^9 + 48*b^9*i)/(b^13*i)) - 1/8*(8*a^3 - 36*a^2*i - 44*a + 17*i)*
log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b - 2*(x*abs(b) - sqrt
((b*x + a)^2 + 1))^2*b*i - 2*a^2*b*i + (x*abs(b) - sqrt((b*x + a)^2 + 1))^3
*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) - 4*(x*abs(b) - s
qrt((b*x + a)^2 + 1))*a*i*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))
*abs(b))/(b^3*abs(b)) - 1/4*(8*a^3*abs(b) - 36*a^2*i*abs(b) - 44*a*abs(b) +
 17*i*abs(b))*log(96*b^4)/b^5
```

3.209 $\int e^{-3i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=229

$$-\frac{\left(-6ia^2-18a+11i\right)\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{6b^3}-\frac{\left(-6ia^2-18a+11i\right)\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{2b^3}+\frac{\left(-6a^2+18\right.}{}$$

```
[Out] (I*(I - a)^2*(1 - I*a - I*b*x)^(5/2))/(b^3*Sqrt[1 + I*a + I*b*x]) - ((11*I
- 18*a - (6*I)*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^3) -
((11*I - 18*a - (6*I)*a^2)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(
6*b^3) - ((I/3)*(1 - I*a - I*b*x)^(5/2)*Sqrt[1 + I*a + I*b*x])/b^3 + ((11 +
 (18*I)*a - 6*a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rubi [A] time = 0.168235, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 89, 80, 50, 53, 619, 215}

$$-\frac{\left(-6ia^2-18a+11i\right)\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{6b^3}-\frac{\left(-6ia^2-18a+11i\right)\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{2b^3}+\frac{\left(-6a^2+18\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((3*I)*ArcTan[a + b*x]),x]
```


```
[Out] (I*(I - a)^2*(1 - I*a - I*b*x)^(5/2))/(b^3*Sqrt[1 + I*a + I*b*x]) - ((11*I
- 18*a - (6*I)*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^3) -
((11*I - 18*a - (6*I)*a^2)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(
6*b^3) - ((I/3)*(1 - I*a - I*b*x)^(5/2)*Sqrt[1 + I*a + I*b*x])/b^3 + ((11 +
 (18*I)*a - 6*a^2)*ArcSinh[a + b*x])/(2*b^3)
```


Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-3i\tan^{-1}(a+bx)}x^2\,dx = \int \frac{x^2(1-ia-ibx)^{3/2}}{(1+ia+ibx)^{3/2}}\,dx$$

$$= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{i\int \frac{(1-ia-ibx)^{3/2}\left(-(i-a)(3+2ia)b-b^2x\right)}{\sqrt{1+ia+ibx}}\,dx}{b^3}$$

$$= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} - \frac{i(1-ia-ibx)^{5/2}\sqrt{1+ia+ibx}}{3b^3} + \frac{\left(11+18ia-6a^2\right)\int \frac{(1-ia-ibx)^{3/2}}{\sqrt{1+ia+ibx}}\,dx}{3b^2}$$

$$= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{\left(18a-i\left(11-6a^2\right)\right)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} - \frac{i(1-ia-ibx)^{5/2}\sqrt{}}{3b^3}$$

$$= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{\left(18a-i\left(11-6a^2\right)\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{\left(18a-i\left(11-6a^2\right)\right)}{}$$

$$= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{\left(18a-i\left(11-6a^2\right)\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{\left(18a-i\left(11-6a^2\right)\right)}{}$$

$$= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{\left(18a-i\left(11-6a^2\right)\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{\left(18a-i\left(11-6a^2\right)\right)}{}$$

$$= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{\left(18a-i\left(11-6a^2\right)\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{\left(18a-i\left(11-6a^2\right)\right)}{}$$

Mathematica [A] time = 0.295245, size = 198, normalized size = 0.86

$$\frac{a^3(51+2ibx)+a^2(69bx-50i)+2ia^4+a\left(2ib^3x^3+9b^2x^2-106ibx+51\right)+i\left(2b^4x^4+9ib^3x^3-26b^2x^2+33ibx-52\right)}{6b^3\sqrt{a^2+2abx+b^2x^2+1}}+$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((2*I)*a^4 + a^3*(51 + (2*I)*b*x) + a^2*(-50*I + 69*b*x) + a*(51 - (106*I)*
b*x + 9*b^2*x^2 + (2*I)*b^3*x^3) + I*(-52 + (33*I)*b*x - 26*b^2*x^2 + (9*I)
*b^3*x^3 + 2*b^4*x^4))/(6*b^3*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) + ((-1)^(1
/4)*(11 + (18*I)*a - 6*a^2)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[
(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(7/2)
```

Maple [B] time = 0.094, size = 1026, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x)
```

```
[Out] 4*I/b^5/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)+9*I/b^2*l
n((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(
1/2))/(b^2)^(1/2)*a+1/b^6/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*
b)^(5/2)-1/b^6/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)*a^
2+2*I/b^6/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)*a-2*I/b
^5/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)*a^2+9*I/b^2*((
x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x*a-11/3*I/b^3*((x-(I-a)/b)^2*b^2
+2*I*(x-(I-a)/b)*b)^(3/2)-3/b^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)
*a^3-3/b^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x*a^2-3/b^2*ln((I*b+
(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(
b^2)^(1/2)*a^2+9*I/b^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a^2+2*I/
b^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)*a^2+6/b^3*a*((x-(I-a)/b)^2*
b^2+2*I*(x-(I-a)/b)*b)^(3/2)+11/2/b^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)
^(1/2)*x+11/2/b^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a+11/2/b^2*ln
((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1
/2))/(b^2)^(1/2)-6/b^5*a/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b
)^(5/2)
```

Maxima [B] time = 1.56663, size = 842, normalized size = 3.68

$$\frac{i\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}a^2}{b^5x^2+2\,ab^4x+a^2b^3-2i\,b^4x-2i\,ab^3-b^3}+\frac{2\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}a}{b^5x^2+2\,ab^4x+a^2b^3-2i\,b^4x-2i\,ab^3-b^3}+\frac{2\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{2i\,b^4x+2i\,ab^3+2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/(b^5*x^2 + 2*a*b^4*x + a^2*b^3 -
2*I*b^4*x - 2*I*a*b^3 - b^3) + 2*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(b^5
*x^2 + 2*a*b^4*x + a^2*b^3 - 2*I*b^4*x - 2*I*a*b^3 - b^3) + 2*(b^2*x^2 + 2*
a*b*x + a^2 + 1)^(3/2)*a/(2*I*b^4*x + 2*I*a*b^3 + 2*b^3) + 6*I*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*a^2/(I*b^4*x + I*a*b^3 + b^3) - I*(b^2*x^2 + 2*a*b*x
+ a^2 + 1)^(3/2)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3 - 2*I*b^4*x - 2*I*a*b^3 - b
^3) - 2*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(2*I*b^4*x + 2*I*a*b^3 + 2*b^
3) + 12*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/(I*b^4*x + I*a*b^3 + b^3) - 3*a
```

```
^2*arcsinh(b*x + a)/b^3 - 6*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(I*b^4*x +
I*a*b^3 + b^3) - 1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*x
/b^2 + 9*I*a*arcsinh(b*x + a)/b^3 + 1/3*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/
2)/b^3 + 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^3 - 1/2*sqrt(-b^2*x^2 - 2*
a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*a/b^3 + 1/2*arcsin(I*b*x + I*a + 2)/b^3
+ 6*arcsinh(b*x + a)/b^3 - 3*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^3 + I*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)/b^3
```

Fricas [A] time = 2.43877, size = 485, normalized size = 2.12

$$\frac{7i\,a^4+166\,a^3+\left(7i\,a^3+159\,a^2-249i\,a-96\right)bx-408i\,a^2+\left(72\,a^3+12\left(6\,a^2-18i\,a-11\right)bx-288i\,a^2-348\,a+132i\right)\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (7*I*a^4 + 166*a^3 + (7*I*a^3 + 159*a^2 - 249*I*a - 96)*b*x - 408*I*a^2 + (
72*a^3 + 12*(6*a^2 - 18*I*a - 11)*b*x - 288*I*a^2 - 348*a + 132*I)*log(-b*x
 - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (8*I*b^3*x^3 - 28*b^2*x^2 + 8*I
*a^3 + (64*a - 76*I)*b*x + 212*a^2 - 412*I*a - 208)*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1) - 345*a + 96*I)/(24*b^4*x + (24*a - 24*I)*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1758, size = 386, normalized size = 1.69

$$\frac{1}{6}\sqrt{(bx+a)^2+1}\left(\left(\frac{2\,ix}{b}-\frac{2\,ab^6i+9\,b^6}{b^8}\right)x+\frac{2\,a^2b^5i+27\,ab^5-28\,b^5i}{b^8}\right)+\frac{\left(6\,a^2i+18\,a-11\,i\right)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt((b*x + a)^2 + 1)*((2*i*x/b - (2*a*b^6*i + 9*b^6)/b^8)*x + (2*a^2*b
^5*i + 27*a*b^5 - 28*b^5*i)/b^8) + 1/6*(6*a^2*i + 18*a - 11*i)*log(3*(x*abs
(b) - sqrt((b*x + a)^2 + 1))^2*a*b*i + a^3*b*i + (x*abs(b) - sqrt((b*x + a)
^2 + 1))^3*i*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*i*abs(b) + 2
*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b + 2*a^2*b - a*b*i + 4*(x*abs(b) - s
qrt((b*x + a)^2 + 1))*a*abs(b) - (x*abs(b) - sqrt((b*x + a)^2 + 1))*i*abs(b
))/(b^2*i*abs(b)) + 1/3*(6*a^2*abs(b) - 18*a*i*abs(b) - 11*abs(b))*log(8*b^
3)/b^4
```

3.210 $\int e^{-3i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=163

$$-\frac{(1+ia)(-ia-ibx+1)^{5/2}}{b^2\sqrt{ia+ibx+1}}-\frac{(3+2ia)\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{2b^2}-\frac{3(3+2ia)\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{2b^2}-\frac{3(-2}{}$$

```
[Out] -(((1 + I*a)*(1 - I*a - I*b*x)^(5/2))/(b^2*Sqrt[1 + I*a + I*b*x])) - (3*(3
+ (2*I)*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^2) - ((3 + (2*
I)*a)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(2*b^2) - (3*(3*I - 2*
a)*ArcSinh[a + b*x])/(2*b^2)
```

Rubi [A] time = 0.119072, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5095, 78, 50, 53, 619, 215}

$$-\frac{(1+ia)(-ia-ibx+1)^{5/2}}{b^2\sqrt{ia+ibx+1}}-\frac{(3+2ia)\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{2b^2}-\frac{3(3+2ia)\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{2b^2}-\frac{3(-2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] -(((1 + I*a)*(1 - I*a - I*b*x)^(5/2))/(b^2*Sqrt[1 + I*a + I*b*x])) - (3*(3
+ (2*I)*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^2) - ((3 + (2*
I)*a)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(2*b^2) - (3*(3*I - 2*
a)*ArcSinh[a + b*x])/(2*b^2)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
```

```
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int e^{-3i\tan^{-1}(a+bx)}x\,dx=\int\frac{x(1-ia-ibx)^{3/2}}{(1+ia+ibx)^{3/2}}\,dx$$

$$=-\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}}-\frac{(3i-2a)\int\frac{(1-ia-ibx)^{3/2}}{\sqrt{1+ia+ibx}}\,dx}{b}$$

$$=-\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}}-\frac{(3+2ia)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2}-\frac{(3(3i-2a))\int\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx}{2b}$$

$$=-\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}}-\frac{3(3+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2}-\frac{(3+2ia)(1-ia-ibx)^{3/2}\sqrt{1+}}{2b^2}$$

$$=-\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}}-\frac{3(3+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2}-\frac{(3+2ia)(1-ia-ibx)^{3/2}\sqrt{1+}}{2b^2}$$

$$=-\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}}-\frac{3(3+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2}-\frac{(3+2ia)(1-ia-ibx)^{3/2}\sqrt{1+}}{2b^2}$$

$$=-\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}}-\frac{3(3+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2}-\frac{(3+2ia)(1-ia-ibx)^{3/2}\sqrt{1+}}{2b^2}$$

Mathematica [A] time = 0.299525, size = 157, normalized size = 0.96

$$\frac{i\left(a^2(-bx+14i)-a^3+a\left(b^2x^2+20ibx-1\right)+b^3x^3+6ib^2x^2+9bx+14i\right)}{2b^2\sqrt{a^2+2abx+b^2x^2+1}}+\frac{3\sqrt[4]{-1}(2a-3i)\sqrt{-ib}\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+bx+}}{\sqrt{-ib}}\right.}{b^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((I/2)*(14*I - a^3 + 9*b*x + (6*I)*b^2*x^2 + b^3*x^3 + a^2*(14*I - b*x) + a
*(-1 + (20*I)*b*x + b^2*x^2)))/(b^2*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) + (3
*(-1)^(1/4)*(-3*I + 2*a)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I
)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(5/2)
```

Maple [B] time = 0.086, size = 676, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x)
```

```
[Out] 2*I/b^4*a/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)+3/b^4/(
x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)-3/b^2*((x-(I-a)/b)
^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)-I/b^5/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*
(x-(I-a)/b)*b)^(5/2)-2*I/b^2*a*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)-
9/2*I/b*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x+1/b^5*a/(x-I/b+a/b)^3
*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)-9/2*I/b^2*((x-(I-a)/b)^2*b^2+2
*I*(x-(I-a)/b)*b)^(1/2)*a-9/2*I/b*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-
(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)+3/b*a*((x-(I-a)/b)^2*b
^2+2*I*(x-(I-a)/b)*b)^(1/2)*x+3/b^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(
1/2)*a^2+3/b*a*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*
(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)
```

Maxima [B] time = 1.55039, size = 396, normalized size = 2.43

$$-\frac{i\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}a}{b^4x^2+2\,ab^3x+a^2b^2-2i\,b^3x-2i\,ab^2-b^2}-\frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{b^4x^2+2\,ab^3x+a^2b^2-2i\,b^3x-2i\,ab^2-b^2}-\frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{2i\,b^3x+2i\,ab^2+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(b^4*x^2 + 2*a*b^3*x + a^2*b^2 - 2
*I*b^3*x - 2*I*a*b^2 - b^2) - (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(b^4*x^2
+ 2*a*b^3*x + a^2*b^2 - 2*I*b^3*x - 2*I*a*b^2 - b^2) - (b^2*x^2 + 2*a*b*x +
 a^2 + 1)^(3/2)/(2*I*b^3*x + 2*I*a*b^2 + 2*b^2) - 6*I*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)*a/(I*b^3*x + I*a*b^2 + b^2) + 3*a*arcsinh(b*x + a)/b^2 - 6*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)/(I*b^3*x + I*a*b^2 + b^2) - 9/2*I*arcsinh(b*
x + a)/b^2 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2
```

Fricas [A] time = 2.34392, size = 367, normalized size = 2.25

$$\frac{-3i\,a^3+\left(-3i\,a^2-44\,a+32i\right)bx-47\,a^2-\left((24\,a-36i)bx+24\,a^2-60i\,a-36\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right.}{8\,b^3x+(8\,a-8i)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (-3*I*a^3 + (-3*I*a^2 - 44*a + 32*I)*b*x - 47*a^2 - ((24*a - 36*I)*b*x + 24
*a^2 - 60*I*a - 36)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)*(4*I*b^2*x^2 - 4*I*a^2 - 20*b*x - 60*a + 56*
I) + 76*I*a + 32)/(8*b^3*x + (8*a - 8*I)*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15398, size = 325, normalized size = 1.99

$$-\frac{1}{2}\sqrt{(bx+a)^2+1}\left(\frac{x}{bi}-\frac{ab^2-6b^2i}{b^4i}\right)-\frac{(2a-3i)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b-2\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2bi-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt((b*x + a)^2 + 1)*(x/(b*i) - (a*b^2 - 6*b^2*i)/(b^4*i)) - 1/2*(2*a
 - 3*i)*log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b - 2*(x*abs(b
) - sqrt((b*x + a)^2 + 1))^2*b*i - 2*a^2*b*i + (x*abs(b) - sqrt((b*x + a)^2
 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) - 4*(x*ab
s(b) - sqrt((b*x + a)^2 + 1))*a*i*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)
^2 + 1))*abs(b))/(b*abs(b)) - (2*a*abs(b) - 3*i*abs(b))*log(24*b^2)/b^3
```

3.211 $\int e^{-3i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=94

$$\frac{2i(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}+\frac{3i\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{b}-\frac{3\sinh^{-1}(a+bx)}{b}$$

```
[Out] ((2*I)*(1 - I*a - I*b*x)^(3/2))/(b*Sqrt[1 + I*a + I*b*x]) + ((3*I)*Sqrt[1 -
 I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b - (3*ArcSinh[a + b*x])/b
```

Rubi [A] time = 0.0440134, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5093, 47, 50, 53, 619, 215}

$$\frac{2i(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}+\frac{3i\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{b}-\frac{3\sinh^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((2*I)*(1 - I*a - I*b*x)^(3/2))/(b*Sqrt[1 + I*a + I*b*x]) + ((3*I)*Sqrt[1 -
 I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b - (3*ArcSinh[a + b*x])/b
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{-3i\tan^{-1}(a+bx)}\,dx &= \int \frac{(1-ia-ibx)^{3/2}}{(1+ia+ibx)^{3/2}}\,dx\\
&= \frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - 3\int \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx\\
&= \frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - 3\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx\\
&= \frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - 3\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx\\
&= \frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{2b^2}\\
&= \frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{3\sinh^{-1}(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0346367, size = 45, normalized size = 0.48

$$-\frac{3\sinh^{-1}(a+bx)}{b} + \frac{\sqrt{(a+bx)^2+1}\left(\frac{4}{a+bx-i}+i\right)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-3*I)*ArcTan[a + b*x]),x]
```

```
[Out] (Sqrt[1 + (a + b*x)^2]*(I + 4/(-I + a + b*x)))/b - (3*ArcSinh[a + b*x])/b
```

Maple [B] time = 0.062, size = 329, normalized size = 3.5

$$-\frac{1}{b^4}\left(\left(x-\frac{i-a}{b}\right)^2 b^2 + 2i\left(x-\frac{i-a}{b}\right)b\right)^{\frac{5}{2}}\left(x-\frac{i}{b}+\frac{a}{b}\right)^{-3} - \frac{2i}{b^3}\left(\left(x-\frac{i-a}{b}\right)^2 b^2 + 2i\left(x-\frac{i-a}{b}\right)b\right)^{\frac{5}{2}}\left(x-\frac{i}{b}+\frac{a}{b}\right)^{-2} + \frac{2i}{b}\Bigg(\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x)
```

```
[Out] -1/b^4/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)-2*I/b^3/(x
-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)+2*I/b*((x-(I-a)/b)^
2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)-3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2
```

```
)*x-3/b*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a-3*ln((I*b+(x-(I-a)/b)
*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)
```

Maxima [A] time = 1.48851, size = 139, normalized size = 1.48

$$\frac{i\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{b^3x^2+2\,ab^2x+a^2b-2i\,b^2x-2i\,ab-b}-\frac{3\,\operatorname{arsinh}\left(bx+a\right)}{b}+\frac{6i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{i\,b^2x+i\,ab+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(b^3*x^2 + 2*a*b^2*x + a^2*b - 2*I*b^
2*x - 2*I*a*b - b) - 3*arcsinh(b*x + a)/b + 6*I*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)/(I*b^2*x + I*a*b + b)
```

Fricas [A] time = 2.22626, size = 262, normalized size = 2.79

$$\frac{(i\,a+8)bx+i\,a^2+(6\,bx+6\,a-6i)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+\sqrt{b^2x^2+2\,abx+a^2+1}(2i\,bx+2i\,a+10}{2\,b^2x+(2\,a-2i)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((I*a + 8)*b*x + I*a^2 + (6*b*x + 6*a - 6*I)*log(-b*x - a + sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*I*b*x + 2*I*a +
10) + 9*a - 8*I)/(2*b^2*x + (2*a - 2*I)*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14208, size = 265, normalized size = 2.82

$$\frac{\sqrt{(bx+a)^2+1}i}{b}+\frac{\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2abi+a^3bi+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3i|b|+3\left(x|b|-\sqrt{(bx+a)^2+1}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt((b*x + a)^2 + 1)*i/b + log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b*
i + a^3*b*i + (x*abs(b) - sqrt((b*x + a)^2 + 1))^3*i*abs(b) + 3*(x*abs(b) -
 sqrt((b*x + a)^2 + 1))*a^2*i*abs(b) + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))
^2*b + 2*a^2*b - a*b*i + 4*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a*abs(b) - (x
*abs(b) - sqrt((b*x + a)^2 + 1))*i*abs(b))/abs(b) + 2*abs(b)*log(12*b)/b^2
```

3.212 $\int \frac{e^{-3i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=134

$$\frac{4\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}+i\sinh^{-1}(a+bx)-\frac{2(a+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}}$$

```
[Out] (4*Sqrt[1 - I*a - I*b*x])/((1 + I*a)*Sqrt[1 + I*a + I*b*x]) + I*ArcSinh[a +
 b*x] - (2*(I + a)^(3/2)*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[
I - a]*Sqrt[1 - I*a - I*b*x])])/(I - a)^(3/2)
```

Rubi [A] time = 0.0940627, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5095, 98, 157, 53, 619, 215, 93, 208}

$$\frac{4\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}+i\sinh^{-1}(a+bx)-\frac{2(a+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a + b*x])*x),x]
```

```
[Out] (4*Sqrt[1 - I*a - I*b*x])/((1 + I*a)*Sqrt[1 + I*a + I*b*x]) + I*ArcSinh[a +
 b*x] - (2*(I + a)^(3/2)*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[
I - a]*Sqrt[1 - I*a - I*b*x])])/(I - a)^(3/2)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
```

```
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3i\tan^{-1}(a+bx)}}{x}\,dx &= \int \frac{(1-ia-ibx)^{3/2}}{x(1+ia+ibx)^{3/2}}\,dx\\
&= \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}}+\frac{2\int \frac{-\frac{1}{2}i(i+a)^2b-\frac{1}{2}(1+ia)b^2x}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{(i-a)b}\\
&= \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}}-\frac{(i+a)^2\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{1+ia}+(ib)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx\\
&= \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}}-\frac{\left(2(i+a)^2\right)\operatorname{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx,x,\frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{1+ia}+(ib)\int \frac{}{\sqrt{(1-ia)(1+}}\\
&= \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}}-\frac{2(i+a)^{3/2}\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}}+\frac{i\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx,x,2ab+2b^2x\right)}{2b}\\
&= \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}}+i\sinh^{-1}(a+bx)-\frac{2(i+a)^{3/2}\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.54472, size = 172, normalized size = 1.28

$$
-\frac{4\sqrt{a^2+2abx+b^2x^2+1}}{(a-i)(a+bx-i)}+2\left(\frac{a+i}{a-i}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{-i(a+bx+i)}}{\sqrt{\frac{a+i}{a-i}}\sqrt{ia+ibx+1}}\right)+\frac{2\sqrt[4]{-1}\sqrt{b}\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{\sqrt{-ib}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a + b*x])*x),x]
```

```
[Out] (-4*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/((-I + a)*(-I + a + b*x)) + (2*(-1)^
(1/4)*Sqrt[b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(
-I)*b]])/Sqrt[(-I)*b] + 2*((I + a)/(-I + a))^(3/2)*ArcTan[Sqrt[(-I)*(I + a
+ b*x)]/(Sqrt[(I + a)/(-I + a)]*Sqrt[1 + I*a + I*b*x])]
```

Maple [B] time = 0.13, size = 1278, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x)
```

```
[Out] -1/(I-a)^2/b^2/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)+1/
(I-a)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)+3/2*I/(I-a)^2*b*ln((I*b
+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/
(b^2)^(1/2)-3/2*I/(I-a)^3*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3/2*I/(I-a)^2*(
(x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a+2*I/(I-a)*((x-(I-a)/b)^2*b^2+2
*I*(x-(I-a)/b)*b)^(3/2)-1/2/(I-a)^3*b*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)
^(1/2)*x-1/2/(I-a)^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a-1/2/(I-a
)^3*b*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/
b)*b)^(1/2))/(b^2)^(1/2)-1/(I-a)/b^3/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*(
x-(I-a)/b)*b)^(5/2)+3/2*I/(I-a)^2*b*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(
1/2)*x-3/2*I/(I-a)^3*a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)
^(1/2))/(b^2)^(1/2)-3/(I-a)*b*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x
-3/(I-a)*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a-3/(I-a)*b*ln((I*b+(x
-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^
2)^(1/2)-2*I/(I-a)/b^2/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^
(5/2)+1/3*I/(I-a)^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)-I/(I-a)^3*a
^3*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+
I/(I-a)^3*(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/x)*a^2-1/3*I/(I-a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*I/(I
-a)^3*a*b*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+I/(I-a)^3*(a^2+1)^(1/2)*ln((2*a^2
+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-I/(I-a)^3*(b^2
*x^2+2*a*b*x+a^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((b*x + a)^2 + 1)^(3/2)/((I*b*x + I*a + 1)^3*x), x)
```

Fricas [B] time = 2.50144, size = 980, normalized size = 7.31

$$\left((a-i)bx+a^2-2ia-1\right)\sqrt{-\frac{4a^3+12ia^2-12a-4i}{a^3-3ia^2-3a+i}}\log\left(-\frac{(2a+2i)bx-\sqrt{b^2x^2+2abx+a^2+1}(2a+2i)-\left(ia^2+2a-i\right)\sqrt{-\frac{4a^3+12ia^2-12a-4i}{a^3-3ia^2-3a+i}}}{2a+2i}\right)-\left((a-i)bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (((a - I)*b*x + a^2 - 2*I*a - 1)*sqrt(-(4*a^3 + 12*I*a^2 - 12*a - 4*I)/(a^3
 - 3*I*a^2 - 3*a + I))*log(-((2*a + 2*I)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)*(2*a + 2*I) - (I*a^2 + 2*a - I)*sqrt(-(4*a^3 + 12*I*a^2 - 12*a - 4*I)
/(a^3 - 3*I*a^2 - 3*a + I)))/(2*a + 2*I)) - ((a - I)*b*x + a^2 - 2*I*a - 1)
*sqrt(-(4*a^3 + 12*I*a^2 - 12*a - 4*I)/(a^3 - 3*I*a^2 - 3*a + I))*log(-((2*
a + 2*I)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a + 2*I) - (-I*a^2 - 2*
a + I)*sqrt(-(4*a^3 + 12*I*a^2 - 12*a - 4*I)/(a^3 - 3*I*a^2 - 3*a + I)))/(2
*a + 2*I)) - 8*b*x - (2*(I*a + 1)*b*x + 2*I*a^2 + 4*a - 2*I)*log(-b*x - a +
 sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 8*a - 8*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1) + 8*I)/((2*a - 2*I)*b*x + 2*a^2 - 4*I*a - 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.213 $\int \frac{e^{-3i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=178

$$-\frac{(-ia-ibx+1)^{3/2}}{(1+ia)x\sqrt{ia+ibx+1}}+\frac{6ib\sqrt{-ia-ibx+1}}{(-a+i)^2\sqrt{ia+ibx+1}}-\frac{6i\sqrt{a+i}b\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{5/2}}$$

```
[Out] ((6*I)*b*Sqrt[1 - I*a - I*b*x])/((I - a)^2*Sqrt[1 + I*a + I*b*x]) - (1 - I*
a - I*b*x)^(3/2)/((1 + I*a)*x*Sqrt[1 + I*a + I*b*x]) - ((6*I)*Sqrt[I + a]*b
*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*
b*x])])/(I - a)^(5/2)
```

Rubi [A] time = 0.0854949, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5095, 94, 93, 208}

$$-\frac{(-ia-ibx+1)^{3/2}}{(1+ia)x\sqrt{ia+ibx+1}}+\frac{6ib\sqrt{-ia-ibx+1}}{(-a+i)^2\sqrt{ia+ibx+1}}-\frac{6i\sqrt{a+i}b\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] ((6*I)*b*Sqrt[1 - I*a - I*b*x])/((I - a)^2*Sqrt[1 + I*a + I*b*x]) - (1 - I*
a - I*b*x)^(3/2)/((1 + I*a)*x*Sqrt[1 + I*a + I*b*x]) - ((6*I)*Sqrt[I + a]*b
*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*
b*x])])/(I - a)^(5/2)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3i\tan^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{(1-ia-ibx)^{3/2}}{x^2(1+ia+ibx)^{3/2}}\,dx \\
&= -\frac{(1-ia-ibx)^{3/2}}{(1+ia)x\sqrt{1+ia+ibx}} + \frac{(3b)\int \frac{\sqrt{1-ia-ibx}}{x(1+ia+ibx)^{3/2}}\,dx}{i-a} \\
&= \frac{6ib\sqrt{1-ia-ibx}}{(i-a)^2\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{3/2}}{(1+ia)x\sqrt{1+ia+ibx}} + \frac{(3(i+a)b)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{(i-a)^2} \\
&= \frac{6ib\sqrt{1-ia-ibx}}{(i-a)^2\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{3/2}}{(1+ia)x\sqrt{1+ia+ibx}} + \frac{(6(i+a)b)\,\mathrm{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ib}}{\sqrt{1-ia-ib}}\right.}{(i-a)^2} \\
&= \frac{6ib\sqrt{1-ia-ibx}}{(i-a)^2\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{3/2}}{(1+ia)x\sqrt{1+ia+ibx}} - \frac{6i\sqrt{i+a}\,b\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.171053, size = 135, normalized size = 0.76

$$
\frac{\frac{\sqrt{-i(a+bx+i)}\left(a^2+abx+5ibx+1\right)}{x\sqrt{ia+ibx+1}} - 6i\sqrt{\frac{a+i}{a-i}}\,b\tan^{-1}\left(\frac{\sqrt{-i(a+bx+i)}}{\sqrt{\frac{a+i}{a-i}}\sqrt{ia+ibx+1}}\right)}{(a-i)^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] ((Sqrt[(-I)*(I + a + b*x)]*(1 + a^2 + (5*I)*b*x + a*b*x))/(x*Sqrt[1 + I*a +
 I*b*x]) - (6*I)*Sqrt[(I + a)/(-I + a)]*b*ArcTan[Sqrt[(-I)*(I + a + b*x)]/(
Sqrt[(I + a)/(-I + a)]*Sqrt[1 + I*a + I*b*x])])/(-I + a)^2
```

Maple [B] time = 0.132, size = 1917, normalized size = 10.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x)
```

```
[Out] 2/(I-a)^3*b*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)-2*I/(I-a)^2/b/(x-I/
b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)-3*b^2/(I-a)^2*ln((I*b+
(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(
b^2)^(1/2)-2*I/(I-a)^3*a*b/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-3*I/(I-a)^
3*a^4*b^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))
/(b^2)^(1/2)+3*I/(I-a)^3*a*b/(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1
/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-I/(I-a)^3*b^2/(a^2+1)*(b^2*x^2+2*a*b*
x+a^2+1)^(3/2)*x-3/2*I/(I-a)^3*b^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-
3/2*I/(I-a)^3*b^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1
)^(1/2))/(b^2)^(1/2)-3*I/(I-a)^4*b^2*a^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^
```

```
2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+3*I/(I-a)^4*b*(a^2+1)^(1/2)*ln((2*a^2+2
+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)*a^2-3/2*I/(I-a)^
4*b^2*a*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-9/2*I/(I-a)^4*b^2*a*ln((b^2*x+a*b)/
(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-9/2*I/(I-a)^3*a^3*b/
(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-9/2*I/(I-a)^3*a*b/(a^2+1)*(b^2*x^2+2*
a*b*x+a^2+1)^(1/2)+3*I/(I-a)^3*a^3*b/(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a
^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-3/2*I/(I-a)^3*a^2*b^2/(a^2+1)
*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-9/2*I/(I-a)^3*a^2*b^2/(a^2+1)*ln((b^2*x+a*
b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/(I-a)^2/b^2/(x-
I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)-3/(I-a)^2*b^2*((x-(I
-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x-3/(I-a)^2*b*((x-(I-a)/b)^2*b^2+2*I*
(x-(I-a)/b)*b)^(1/2)*a+2*I/(I-a)^2*b*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^
(3/2)-2/(I-a)^3/b/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)
+I/(I-a)^4*b*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)-3/2/(I-a)^4*b^2*((
x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x-3/2/(I-a)^4*b*((x-(I-a)/b)^2*b^
2+2*I*(x-(I-a)/b)*b)^(1/2)*a-3/2/(I-a)^4*b^2*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)
^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)-I/(I-a)^4*b
*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-3*I/(I-a)^4*b*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+
3*I/(I-a)^3*b^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x+3*I/(I-a)^3*b
*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a+3*I/(I-a)^3*b^2*ln((I*b+(x-(
I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)
^(1/2)+I/(I-a)^3/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(5/2)+3*I/(I-a)^4*b*(a^2
+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
)/x)-9/2*I/(I-a)^4*b*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((b*x + a)^2 + 1)^(3/2)/((I*b*x + I*a + 1)^3*x^2), x)
```

Fricas [B] time = 2.37605, size = 1029, normalized size = 5.78

$$-\frac{2\,(i\,a-5)b^2x^2-\left(-2i\,a^2+8\,a-10i\right)bx-\left(\left(a^2-2i\,a-1\right)bx^2+\left(a^3-3i\,a^2-3\,a+i\right)x\right)\sqrt{\frac{(36\,a+36i)b^2}{a^5-5i\,a^4-10\,a^3+10i\,a^2+5\,a-i}}\log\Bigg(-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*(I*a - 5)*b^2*x^2 - (-2*I*a^2 + 8*a - 10*I)*b*x - ((a^2 - 2*I*a - 1)*b*
x^2 + (a^3 - 3*I*a^2 - 3*a + I)*x)*sqrt((36*a + 36*I)*b^2/(a^5 - 5*I*a^4 -
10*a^3 + 10*I*a^2 + 5*a - I))*log(-1/6*(6*b^2*x + (a^3 - 3*I*a^2 - 3*a + I)
*sqrt((36*a + 36*I)*b^2/(a^5 - 5*I*a^4 - 10*a^3 + 10*I*a^2 + 5*a - I)) - 6*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b)/b) + ((a^2 - 2*I*a - 1)*b*x^2 + (a^3 -
 3*I*a^2 - 3*a + I)*x)*sqrt((36*a + 36*I)*b^2/(a^5 - 5*I*a^4 - 10*a^3 + 10*
```

```
I*a^2 + 5*a - I))*log(-1/6*(6*b^2*x - (a^3 - 3*I*a^2 - 3*a + I)*sqrt((36*a
+ 36*I)*b^2/(a^5 - 5*I*a^4 - 10*a^3 + 10*I*a^2 + 5*a - I)) - 6*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*b)/b) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*(I*a - 5
)*b*x + 2*I*a^2 + 2*I))/(2*(a^2 - 2*I*a - 1)*b*x^2 + (2*a^3 - 6*I*a^2 - 6*a
 + 2*I)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] undef
```

3.214 $\int \frac{e^{-3i\tan^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=264

$$-\frac{(-ia-ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{ia+ibx+1}}-\frac{3(2a+3i)b^2\sqrt{-ia-ibx+1}}{(1+ia)^3(a+i)\sqrt{ia+ibx+1}}+\frac{3(3-2ia)b^2\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{7/2}\sqrt{a+i}}+\frac{(3-2ia)b(-\ldots}{2(-a+i)^2(a+\ldots}$$

```
[Out] (-3*(3*I + 2*a)*b^2*Sqrt[1 - I*a - I*b*x])/((1 + I*a)^3*(I + a)*Sqrt[1 + I*
a + I*b*x]) + ((3 - (2*I)*a)*b*(1 - I*a - I*b*x)^(3/2))/(2*(I - a)^2*(I + a
)*x*Sqrt[1 + I*a + I*b*x]) - (1 - I*a - I*b*x)^(5/2)/(2*(1 + a^2)*x^2*Sqrt[
1 + I*a + I*b*x]) + (3*(3 - (2*I)*a)*b^2*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a
+ I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a)^(7/2)*Sqrt[I + a])
```

Rubi [A] time = 0.160294, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5095, 96, 94, 93, 208}

$$-\frac{(-ia-ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{ia+ibx+1}}-\frac{3(2a+3i)b^2\sqrt{-ia-ibx+1}}{(1+ia)^3(a+i)\sqrt{ia+ibx+1}}+\frac{3(3-2ia)b^2\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{7/2}\sqrt{a+i}}+\frac{(3-2ia)b(-\ldots}{2(-a+i)^2(a+\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a + b*x])*x^3),x]
```

```
[Out] (-3*(3*I + 2*a)*b^2*Sqrt[1 - I*a - I*b*x])/((1 + I*a)^3*(I + a)*Sqrt[1 + I*
a + I*b*x]) + ((3 - (2*I)*a)*b*(1 - I*a - I*b*x)^(3/2))/(2*(I - a)^2*(I + a
)*x*Sqrt[1 + I*a + I*b*x]) - (1 - I*a - I*b*x)^(5/2)/(2*(1 + a^2)*x^2*Sqrt[
1 + I*a + I*b*x]) + (3*(3 - (2*I)*a)*b^2*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a
+ I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a)^(7/2)*Sqrt[I + a])
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3i\tan^{-1}(a+bx)}}{x^3}\,dx = \int \frac{(1-ia-ibx)^{3/2}}{x^3(1+ia+ibx)^{3/2}}\,dx$$

$$= -\frac{(1-ia-ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1+ia+ibx}} - \frac{((3i+2a)b)\int \frac{(1-ia-ibx)^{3/2}}{x^2(1+ia+ibx)^{3/2}}\,dx}{2\left(1+a^2\right)}$$

$$= \frac{(3-2ia)b(1-ia-ibx)^{3/2}}{2(i-a)^2(i+a)x\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1+ia+ibx}} + \frac{\left(3(3i+2a)b^2\right)\int \frac{\sqrt{1-ia-ibx}}{x(1+ia+ibx)^{3/2}}\,dx}{2(i-a)^2(i+a)}$$

$$= -\frac{3(3-2ia)b^2\sqrt{1-ia-ibx}}{(i-a)^3(i+a)\sqrt{1+ia+ibx}} + \frac{(3-2ia)b(1-ia-ibx)^{3/2}}{2(i-a)^2(i+a)x\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1+ia+ibx}} + \frac{(3(3\cdots}{\cdots}$$

$$= -\frac{3(3-2ia)b^2\sqrt{1-ia-ibx}}{(i-a)^3(i+a)\sqrt{1+ia+ibx}} + \frac{(3-2ia)b(1-ia-ibx)^{3/2}}{2(i-a)^2(i+a)x\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1+ia+ibx}} + \frac{(3(3\cdots}{\cdots}$$

$$= -\frac{3(3-2ia)b^2\sqrt{1-ia-ibx}}{(i-a)^3(i+a)\sqrt{1+ia+ibx}} + \frac{(3-2ia)b(1-ia-ibx)^{3/2}}{2(i-a)^2(i+a)x\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1+ia+ibx}} + \frac{3(3\cdots}{\cdots}$$

Mathematica [A] time = 0.222158, size = 184, normalized size = 0.7

$$\frac{\frac{\sqrt{-i(a+bx+i)}\left(a^3-ia^2-ab^2x^2-5iabx+a-14ib^2x^2-5bx-i\right)}{x^2\sqrt{ia+ibx+1}} + \frac{6i\sqrt{\frac{a+i}{a-i}}(2a+3i)b^2\tan^{-1}\left(\frac{\sqrt{-i(a+bx+i)}}{\sqrt{\frac{a+i}{a-i}}\sqrt{ia+ibx+1}}\right)}{a+i}}{2(a-i)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a + b*x])*x^3),x]
```

```
[Out] ((Sqrt[(-I)*(I + a + b*x)]*(-I + a - I*a^2 + a^3 - 5*b*x - (5*I)*a*b*x - (1
4*I)*b^2*x^2 - a*b^2*x^2))/(x^2*Sqrt[1 + I*a + I*b*x]) + ((6*I)*Sqrt[(I + a
)/(-I + a)]*(3*I + 2*a)*b^2*ArcTan[Sqrt[(-I)*(I + a + b*x)]/(Sqrt[(I + a)/(
-I + a)]*Sqrt[1 + I*a + I*b*x]])])/(I + a))/(2*(-I + a)^3)
```

Maple [B] time = 0.135, size = 3042, normalized size = 11.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x)
```

```
[Out] 9/2*I/(I-a)^4*b^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x-27/2*I/(I-a
)^4*b^3*a^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2
))/(b^2)^(1/2)-9*I/(I-a)^4*b^3*a^4/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*
x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*I/(I-a)^3*a*b/(a^2+1)^2/x*(b^2*x^
2+2*a*b*x+a^2+1)^(5/2)-3/4*I/(I-a)^3*a^3*b^3/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2)*x-9/4*I/(I-a)^3*a^3*b^3/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2
*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-3/2*I/(I-a)^3*a^5*b^3/(a^2+1)^2*ln((
b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/2*I/(I-
a)^3*a*b^3/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)*x-3/4*I/(I-a)^3*a*b^3/(a
^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-3/4*I/(I-a)^3*a*b^3/(a^2+1)^2*ln((b
^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-3/4*I/(I-a
)^3*b^3/(a^2+1)*a*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-9/4*I/(I-a)^3*b^3/(a^2+1)
*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-3/
2*I/(I-a)^3*b^3/(a^2+1)*a^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/(b^2)^(1/2)-9/2*I/(I-a)^4*b^3*a^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1
)^(1/2)*x-3/(I-a)^4/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/
2)+3/(I-a)^4*b^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)-2*I/(I-a)^5*b^
2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-6*I/(I-a)^5*b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)-3/(I-a)^5*b^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x-3/(I-a)^5*b^
2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a-3/(I-a)^5*b^3*ln((I*b+(x-(I
-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^
(1/2)+2*I/(I-a)^5*b^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)-1/(I-a)^3
/b/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)-3/(I-a)^3*b^3*
((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x-3/(I-a)^3*b^2*((x-(I-a)/b)^2*
b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a-2*I/(I-a)^3/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2
+2*I*(x-(I-a)/b)*b)^(5/2)+2*I/(I-a)^3*b^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b
)*b)^(3/2)-9/4*I/(I-a)^3*b^2/(a^2+1)*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-I/(I
-a)^3*a^2*b^2/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-3*b^3/(I-a)^3*ln((I*b
+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/
(b^2)^(1/2)+9/2*I/(I-a)^4*b^2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a
+9/2*I/(I-a)^4*b^3*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+
2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)+1/2*I/(I-a)^3/(a^2+1)/x^2*(b^2*x^2+2*
a*b*x+a^2+1)^(5/2)-1/2*I/(I-a)^3*b^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-
3/2*I/(I-a)^3*b^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3/2*I/(I-a)^3*b^2/(
a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2))/x)-9*I/(I-a)^5*b^2*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+6*I/(I-a)^5*b^2*(
a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2))/x)+3/2*I/(I-a)^3*b^2/(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)
*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)*a^2+9*I/(I-a)^4*b^2*a/(a^2+1)^(1/2)*ln((
2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-3*I/(I-a)
^4*b^3/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)*x+3*I/(I-a)^4*b/(a^2+1)/x*(b^2
*x^2+2*a*b*x+a^2+1)^(5/2)+6*I/(I-a)^5*b^2*(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b
+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)*a^2-9/2*I/(I-a)^4*b^3/(a
^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-9/2*I/(I-a)^4*b^3/(a^2+1)*ln((b^2*x+a
*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-6*I/(I-a)^4*b^2*
a/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-27/2*I/(I-a)^4*b^2*a^3/(a^2+1)*(b^2
*x^2+2*a*b*x+a^2+1)^(1/2)-27/2*I/(I-a)^4*b^2*a/(a^2+1)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2)+9*I/(I-a)^4*b^2*a^3/(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(
1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+3/2*I/(I-a)^3*a^2*b^2/(a^2+1)^(3/2)*
ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-3*I/(
I-a)^5*b^3*a*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-9*I/(I-a)^5*b^3*a*ln((b^2*x+a*
```

```
b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-6*I/(I-a)^5*b^3*a
^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-9/
4*I/(I-a)^3*a^4*b^2/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-9/4*I/(I-a)^3*a
^2*b^2/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3/2*I/(I-a)^3*a^4*b^2/(a^2+1
)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/
x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((b*x + a)^2 + 1)^(3/2)/((I*b*x + I*a + 1)^3*x^3), x)
```

Fricas [B] time = 2.48799, size = 1458, normalized size = 5.52

$$(i\,a-14)b^3x^3+\left(i\,a^2-13\,a+14i\right)b^2x^2-3\left(\left(a^3-3i\,a^2-3\,a+i\right)bx^3+\left(a^4-4i\,a^3-6\,a^2+4i\,a+1\right)x^2\right)\sqrt{\frac{(4\,a^2+12}{a^8-6i\,a^7-14\,a^6+14i\,a^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] ((I*a - 14)*b^3*x^3 + (I*a^2 - 13*a + 14*I)*b^2*x^2 - 3*((a^3 - 3*I*a^2 - 3
*a + I)*b*x^3 + (a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1)*x^2)*sqrt((4*a^2 + 12*I
*a - 9)*b^4/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a
- 1))*log(-((6*a + 9*I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a + 9*
I)*b^2 + 3*(a^5 - 3*I*a^4 - 2*a^3 - 2*I*a^2 - 3*a + I)*sqrt((4*a^2 + 12*I*a
 - 9)*b^4/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a -
1)))/((6*a + 9*I)*b^2)) + 3*((a^3 - 3*I*a^2 - 3*a + I)*b*x^3 + (a^4 - 4*I*a
^3 - 6*a^2 + 4*I*a + 1)*x^2)*sqrt((4*a^2 + 12*I*a - 9)*b^4/(a^8 - 6*I*a^7 -
 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a - 1))*log(-((6*a + 9*I)*b^3*
x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a + 9*I)*b^2 - 3*(a^5 - 3*I*a^4 -
2*a^3 - 2*I*a^2 - 3*a + I)*sqrt((4*a^2 + 12*I*a - 9)*b^4/(a^8 - 6*I*a^7 - 1
4*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a - 1)))/((6*a + 9*I)*b^2)) + ((
I*a - 14)*b^2*x^2 - I*a^3 - (5*a - 5*I)*b*x - a^2 - I*a - 1)*sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1))/((2*a^3 - 6*I*a^2 - 6*a + 2*I)*b*x^3 + (2*a^4 - 8*I*a^
3 - 12*a^2 + 8*I*a + 2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

3.215 $\int \frac{e^{-3i\tan^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=339

$$-\frac{\left(-2a^2-51ia+52\right)b^3\sqrt{-ia-ibx+1}}{6(-a+i)^4(a+i)\sqrt{ia+ibx+1}}+\frac{\left(-6ia^2+18a+11i\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{9/2}(a+i)^{3/2}}+\frac{(19-16ia)b^2\sqrt{-ia-ibx+}}{6(-a+i)^3(a+i)x\sqrt{ia+ibx}}$$

```
[Out] -((52 - (51*I)*a - 2*a^2)*b^3*Sqrt[1 - I*a - I*b*x])/(6*(I - a)^4*(I + a)*S
qrt[1 + I*a + I*b*x]) - ((I + a)*Sqrt[1 - I*a - I*b*x])/(3*(I - a)*x^3*Sqrt
[1 + I*a + I*b*x]) - (((7*I)/6)*b*Sqrt[1 - I*a - I*b*x])/((I - a)^2*x^2*Sqr
t[1 + I*a + I*b*x]) + ((19 - (16*I)*a)*b^2*Sqrt[1 - I*a - I*b*x])/(6*(I - a
)^3*(I + a)*x*Sqrt[1 + I*a + I*b*x]) + ((11*I + 18*a - (6*I)*a^2)*b^3*ArcTa
nh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])]
)/((I - a)^(9/2)*(I + a)^(3/2))
```

Rubi [A] time = 0.271053, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 98, 151, 152, 12, 93, 208}

$$-\frac{\left(-2a^2-51ia+52\right)b^3\sqrt{-ia-ibx+1}}{6(-a+i)^4(a+i)\sqrt{ia+ibx+1}}+\frac{\left(-6ia^2+18a+11i\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{9/2}(a+i)^{3/2}}+\frac{(19-16ia)b^2\sqrt{-ia-ibx+}}{6(-a+i)^3(a+i)x\sqrt{ia+ibx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a + b*x])*x^4),x]
```


```
[Out] -((52 - (51*I)*a - 2*a^2)*b^3*Sqrt[1 - I*a - I*b*x])/(6*(I - a)^4*(I + a)*S
qrt[1 + I*a + I*b*x]) - ((I + a)*Sqrt[1 - I*a - I*b*x])/(3*(I - a)*x^3*Sqrt
[1 + I*a + I*b*x]) - (((7*I)/6)*b*Sqrt[1 - I*a - I*b*x])/((I - a)^2*x^2*Sqr
t[1 + I*a + I*b*x]) + ((19 - (16*I)*a)*b^2*Sqrt[1 - I*a - I*b*x])/(6*(I - a
)^3*(I + a)*x*Sqrt[1 + I*a + I*b*x]) + ((11*I + 18*a - (6*I)*a^2)*b^3*ArcTa
nh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])]
)/((I - a)^(9/2)*(I + a)^(3/2))
```


Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3i\tan^{-1}(a+bx)}}{x^4}\,dx = \int \frac{(1-ia-ibx)^{3/2}}{x^4(1+ia+ibx)^{3/2}}\,dx$$

$$= -\frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{\int \frac{7(i+a)b+6b^2x}{x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}\,dx}{3(1+ia)}$$

$$= -\frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+ibx}} + \frac{\int \frac{-(19-35ia-16a^2)b^2+14(i+a)b^3x}{x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}\,dx}{6(1+ia)\left(1+a^2\right)}$$

$$= -\frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+ibx}} + \frac{(19i+16a)b^2\sqrt{1-ia-ibx}}{6(1+ia)^3(i+a)x\sqrt{1+ia+ibx}} - \frac{\int \frac{-3(i+a)}{}}{}$$

$$= -\frac{\left(52-51ia-2a^2\right)b^3\sqrt{1-ia-ibx}}{6(i-a)^4(i+a)\sqrt{1+ia+ibx}} - \frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+ibx}} + \frac{(19i}{6(1+}$$

$$= -\frac{\left(52-51ia-2a^2\right)b^3\sqrt{1-ia-ibx}}{6(i-a)^4(i+a)\sqrt{1+ia+ibx}} - \frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+ibx}} + \frac{(19i}{6(1+}$$

$$= -\frac{\left(52-51ia-2a^2\right)b^3\sqrt{1-ia-ibx}}{6(i-a)^4(i+a)\sqrt{1+ia+ibx}} - \frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+ibx}} + \frac{(19i}{6(1+}$$

$$= -\frac{\left(52-51ia-2a^2\right)b^3\sqrt{1-ia-ibx}}{6(i-a)^4(i+a)\sqrt{1+ia+ibx}} - \frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+ibx}} + \frac{(19i}{6(1+}$$

Mathematica [A] time = 0.387144, size = 268, normalized size = 0.79

$$\frac{\left(-6ia^2+18a+11i\right)b^2x^2\left(-i\sqrt{1+ia}\sqrt{-i(a+bx+i)}\left(a^2+abx+5ibx+1\right)-6\sqrt{-1+ia}\,bx\sqrt{ia+ibx+1}\tan^{-1}\left(\frac{\sqrt{-i(a+bx+}}{\sqrt{\frac{a+i}{a-i}}\sqrt{ia+ibx}}\right.\right.}{6(1+ia)^{5/2}\left(a^2+1\right)^2x^3\sqrt{ia+ibx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a + b*x])*x^4),x]
```

```
[Out] (-2*(1 - I*a)*(1 + I*a)^(7/2)*((-I)*(I + a + b*x))^(5/2) + (1 + I*a)^(5/2)*
(3*I + 4*a)*b*x*((-I)*(I + a + b*x))^(5/2) + (11*I + 18*a - (6*I)*a^2)*b^2*
x^2*((-I)*Sqrt[1 + I*a]*Sqrt[(-I)*(I + a + b*x)]*(1 + a^2 + (5*I)*b*x + a*b
*x) - 6*Sqrt[-1 + I*a]*b*x*Sqrt[1 + I*a + I*b*x]*ArcTan[Sqrt[(-I)*(I + a +
b*x)]/(Sqrt[(I + a)/(-I + a)]*Sqrt[1 + I*a + I*b*x])]))/(6*(1 + I*a)^(5/2)*
(1 + a^2)^2*x^3*Sqrt[1 + I*a + I*b*x])
```

Maple [B] time = 0.142, size = 4390, normalized size = 13.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x)
```

```
[Out] -9/2*I/(I-a)^4*b^3/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+9/2*I/(I-a)^4*b^3/
(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(
```

```
1/2))/x)+6*I/(I-a)^5*b^4*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x+6*I/
(I-a)^5*b^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a+9/2*I/(I-a)^4*b^3
*a^2/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/x)+6*I/(I-a)^5*b^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a
)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)-2*I/(I-a)^4*b/(x-I/b+a/b)^
2*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(5/2)-15*I/(I-a)^6*b^3*a^2*(b^2*x^2
+2*a*b*x+a^2+1)^(1/2)+10*I/(I-a)^6*b^3*(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*
(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+1/3*I/(I-a)^3/(a^2+1)/x^3*(
b^2*x^2+2*a*b*x+a^2+1)^(5/2)+9/2*I/(I-a)^4*b^3*a^4/(a^2+1)^(3/2)*ln((2*a^2+
2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-9*I/(I-a)^5*b^4
/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-9*I/(I-a)^5*b^4/(a^2+1)*ln((b^2*x+
a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+6*I/(I-a)^5*b^2
/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(5/2)-12*I/(I-a)^5*b^3*a/(a^2+1)*(b^2*x^
2+2*a*b*x+a^2+1)^(3/2)-27*I/(I-a)^5*b^3*a^3/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)-27*I/(I-a)^5*b^3*a/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+18*I/(I-a)^
5*b^3*a^3/(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/x)+18*I/(I-a)^5*b^3*a/(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(
a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-6*I/(I-a)^5*b^4/(a^2+1)*(b^2
*x^2+2*a*b*x+a^2+1)^(3/2)*x-I/(I-a)^3*b^4/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)*x-I/(I-a)^3*b^4/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/(b^2)^(1/2)+1/3*I/(I-a)^3*a^3*b^3/(a^2+1)^3*(b^2*x^2+2*a*b*
x+a^2+1)^(3/2)+3/4*I/(I-a)^3*a^5*b^3/(a^2+1)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)+3/4*I/(I-a)^3*a^3*b^3/(a^2+1)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-7/6*I/(I-a)
^3*a*b^3/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-9/4*I/(I-a)^3*a^3*b^3/(a^2
+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-5/2*I/(I-a)^3*a*b^3/(a^2+1)^2*(b^2*x^2+
2*a*b*x+a^2+1)^(1/2)-1/2*I/(I-a)^3*a^5*b^3/(a^2+1)^(5/2)*ln((2*a^2+2+2*x*a*
b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-10*I/(I-a)^6*b^4*a^3*ln
((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+10*I/(I
-a)^6*b^3*(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/x)*a^2-27/4*I/(I-a)^4*b^3*a^4/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^
2+1)^(1/2)-27/4*I/(I-a)^4*b^3*a^2/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3
*b^4/(I-a)^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x
-(I-a)/b)*b)^(1/2))/(b^2)^(1/2)+3/2*I/(I-a)^4*b/(a^2+1)/x^2*(b^2*x^2+2*a*b*
x+a^2+1)^(5/2)-3*I/(I-a)^4*b^3*a^2/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-
1/2*I/(I-a)^3*a^3*b^3/(a^2+1)^(5/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+3/2*I/(I-a)^3*a^3*b^3/(a^2+1)^(3/2)*ln((2*a^
2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+3/2*I/(I-a)^3
*a*b^3/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a
^2+1)^(1/2))/x)+2/3*I/(I-a)^3*b^2/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(5/2)
-2/3*I/(I-a)^3*b^4/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)*x-3/2*I/(I-a)^4*
b^3/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+4/(I-a)^5*b^3*((x-(I-a)/b)^2*b^2+
2*I*(x-(I-a)/b)*b)^(3/2)-1/(I-a)^4/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*(x-
(I-a)/b)*b)^(5/2)-9/2*I/(I-a)^4*b^4/(a^2+1)*a^3*ln((b^2*x+a*b)/(b^2)^(1/2)+
(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-9*I/(I-a)^5*b^4*a^2/(a^2+1)*(b^2
*x^2+2*a*b*x+a^2+1)^(1/2)*x-27*I/(I-a)^5*b^4*a^2/(a^2+1)*ln((b^2*x+a*b)/(b^
2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-18*I/(I-a)^5*b^4*a^4/(a
^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
-3/2*I/(I-a)^3*a^4*b^4/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/(b^2)^(1/2)+1/6*I/(I-a)^3*a^2*b^4/(a^2+1)^3*(b^2*x^2+2*a*b*
x+a^2+1)^(3/2)*x+2*I/(I-a)^4*b^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2
)-5/(I-a)^6*b^4*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*x-5/(I-a)^6*b^3
*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a-5/(I-a)^6*b^4*ln((I*b+(x-(I-
a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2))/(b^2)^(
1/2)+10/3*I/(I-a)^6*b^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(3/2)-10/3*I/
(I-a)^6*b^3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-10*I/(I-a)^6*b^3*(b^2*x^2+2*a*b*x
+a^2+1)^(1/2)+1/2*I/(I-a)^3*a^6*b^4/(a^2+1)^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b
^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/4*I/(I-a)^3*a^2*b^4/(a^2+1)^3*(b
^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-1/6*I/(I-a)^3*a*b/(a^2+1)^2/x^2*(b^2*x^2+2*a*
b*x+a^2+1)^(5/2)-1/6*I/(I-a)^3*a^2*b^2/(a^2+1)^3/x*(b^2*x^2+2*a*b*x+a^2+1)^
```

```
(5/2)+1/4*I/(I-a)^3*a^4*b^4/(a^2+1)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+3/4*I
/(I-a)^3*a^4*b^4/(a^2+1)^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+
1)^(1/2))/(b^2)^(1/2)+1/4*I/(I-a)^3*a^2*b^4/(a^2+1)^3*ln((b^2*x+a*b)/(b^2)^
(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-3/4*I/(I-a)^3*a^2*b^4/(a^2
+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-9/4*I/(I-a)^3*a^2*b^4/(a^2+1)^2*ln((b
^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+3/2*I/(I-a
)^4*b^2*a/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(5/2)-9/4*I/(I-a)^4*b^4*a^3/(
a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-27/4*I/(I-a)^4*b^4*a^3/(a^2+1)^2*l
n((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-9/2*I/
(I-a)^4*b^4*a^5/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1
)^(1/2))/(b^2)^(1/2)-3/2*I/(I-a)^4*b^4*a/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^
(3/2)*x-9/4*I/(I-a)^4*b^4*a/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-9/4*I
/(I-a)^4*b^4*a/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)
^(1/2))/(b^2)^(1/2)-9/4*I/(I-a)^4*b^4/(a^2+1)*a*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)*x-27/4*I/(I-a)^4*b^4/(a^2+1)*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b
*x+a^2+1)^(1/2))/(b^2)^(1/2)-4/(I-a)^5*b/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2
*I*(x-(I-a)/b)*b)^(5/2)-3/(I-a)^4*b^4*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)
^(1/2)*x-3/(I-a)^4*b^3*((x-(I-a)/b)^2*b^2+2*I*(x-(I-a)/b)*b)^(1/2)*a-27/4*I
/(I-a)^4*b^3/(a^2+1)*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+9/2*I/(I-a)^4*b^3/(a
^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2))/x)*a^2-5*I/(I-a)^6*b^4*a*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-15*I/(I-a)^6*b
^4*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(((b*x + a)^2 + 1)^(3/2)/((I*b*x + I*a + 1)^3*x^4), x)
```

Fricas [B] time = 2.67903, size = 2219, normalized size = 6.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] ((-2*I*a^2 + 51*a + 52*I)*b^4*x^4 + (-2*I*a^3 + 49*a^2 + I*a + 52)*b^3*x^3
+ sqrt((36*a^4 + 216*I*a^3 - 456*a^2 - 396*I*a + 121)*b^6/(a^12 - 6*I*a^11
- 12*a^10 + 2*I*a^9 - 27*a^8 + 36*I*a^7 + 36*I*a^5 + 27*a^4 + 2*I*a^3 + 12*
a^2 - 6*I*a - 1))*((3*a^5 - 9*I*a^4 - 6*a^3 - 6*I*a^2 - 9*a + 3*I)*b*x^4 +
(3*a^6 - 12*I*a^5 - 15*a^4 - 15*a^2 + 12*I*a + 3)*x^3)*log(-((6*a^2 + 18*I*
a - 11)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a^2 + 18*I*a - 11)*b^3
 + (a^7 - 3*I*a^6 - a^5 - 5*I*a^4 - 5*a^3 - I*a^2 - 3*a + I)*sqrt((36*a^4 +
 216*I*a^3 - 456*a^2 - 396*I*a + 121)*b^6/(a^12 - 6*I*a^11 - 12*a^10 + 2*I*
a^9 - 27*a^8 + 36*I*a^7 + 36*I*a^5 + 27*a^4 + 2*I*a^3 + 12*a^2 - 6*I*a - 1)
))/((6*a^2 + 18*I*a - 11)*b^3)) - sqrt((36*a^4 + 216*I*a^3 - 456*a^2 - 396*
I*a + 121)*b^6/(a^12 - 6*I*a^11 - 12*a^10 + 2*I*a^9 - 27*a^8 + 36*I*a^7 + 3
6*I*a^5 + 27*a^4 + 2*I*a^3 + 12*a^2 - 6*I*a - 1))*((3*a^5 - 9*I*a^4 - 6*a^3
```

```
 - 6*I*a^2 - 9*a + 3*I)*b*x^4 + (3*a^6 - 12*I*a^5 - 15*a^4 - 15*a^2 + 12*I*
a + 3)*x^3)*log(-((6*a^2 + 18*I*a - 11)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*(6*a^2 + 18*I*a - 11)*b^3 - (a^7 - 3*I*a^6 - a^5 - 5*I*a^4 - 5*a^3 -
 I*a^2 - 3*a + I)*sqrt((36*a^4 + 216*I*a^3 - 456*a^2 - 396*I*a + 121)*b^6/(
a^12 - 6*I*a^11 - 12*a^10 + 2*I*a^9 - 27*a^8 + 36*I*a^7 + 36*I*a^5 + 27*a^4
 + 2*I*a^3 + 12*a^2 - 6*I*a - 1)))/((6*a^2 + 18*I*a - 11)*b^3)) + ((-2*I*a^
2 + 51*a + 52*I)*b^3*x^3 - 2*I*a^5 + (16*a^2 + 3*I*a + 19)*b^2*x^2 - 2*a^4
- 4*I*a^3 - (7*a^3 - 7*I*a^2 + 7*a - 7*I)*b*x - 4*a^2 - 2*I*a - 2)*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1))/((6*a^5 - 18*I*a^4 - 12*a^3 - 12*I*a^2 - 18*a +
6*I)*b*x^4 + (6*a^6 - 24*I*a^5 - 30*a^4 - 30*a^2 + 24*I*a + 6)*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

3.216 $\int e^{\frac{1}{2}i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=494

$$-\frac{\left(-8ia^2+4a+3i\right)(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{8b^3}-\frac{\left(-8ia^2+4a+3i\right)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3}+\frac{\left(-8ia^2+4a\right.}{}$$

```
[Out] -((3*I + 4*a - (8*I)*a^2)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/
(8*b^3) - ((I + 8*a)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(5/4))/(12*b
^3) + (x*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(5/4))/(3*b^2) + ((3*I +
 4*a - (8*I)*a^2)*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I
*b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((3*I + 4*a - (8*I)*a^2)*ArcTan[1 + (Sqrt[2
]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((3*
I + 4*a - (8*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] -
(Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(16*Sqrt[2]*b^3
) + ((3*I + 4*a - (8*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I
*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(16*Sqr
t[2]*b^3)
```

Rubi [A] time = 0.405493, antiderivative size = 494, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5095, 90, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\left(-8ia^2+4a+3i\right)(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{8b^3}-\frac{\left(-8ia^2+4a+3i\right)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3}+\frac{\left(-8ia^2+4a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a + b*x])*x^2,x]
```


```
[Out] -((3*I + 4*a - (8*I)*a^2)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/
(8*b^3) - ((I + 8*a)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(5/4))/(12*b
^3) + (x*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(5/4))/(3*b^2) + ((3*I +
 4*a - (8*I)*a^2)*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I
*b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((3*I + 4*a - (8*I)*a^2)*ArcTan[1 + (Sqrt[2
]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((3*
I + 4*a - (8*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] -
(Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(16*Sqrt[2]*b^3
) + ((3*I + 4*a - (8*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I
*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(16*Sqr
t[2]*b^3)
```


Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
```

```
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{1}{2}i\tan^{-1}(a+bx)}x^2\,dx = \int \frac{x^2\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\,dx$$

$$= \frac{x(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{3b^2} + \frac{\int \frac{\sqrt[4]{1+ia+ibx}\left(-1-a^2-\frac{1}{2}(i+8a)bx\right)}{\sqrt[4]{1-ia-ibx}}\,dx}{3b^2}$$

$$= -\frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} + \frac{x(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{3b^2} - \frac{\left(3-4ia-8a^2\right)\int}{8b^2}$$

$$= -\frac{\left(3i+4a-8ia^2\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3} - \frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} + \frac{x(1-}{}$$

$$= -\frac{\left(3i+4a-8ia^2\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3} - \frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} + \frac{x(1-}{}$$

$$= -\frac{\left(3i+4a-8ia^2\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3} - \frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} + \frac{x(1-}{}$$

$$= -\frac{\left(3i+4a-8ia^2\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3} - \frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} + \frac{x(1-}{}$$

$$= -\frac{\left(3i+4a-8ia^2\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3} - \frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} + \frac{x(1-}{}$$

$$= -\frac{\left(3i+4a-8ia^2\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3} - \frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} + \frac{x(1-}{}$$

$$= -\frac{\left(3i+4a-8ia^2\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3} - \frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} + \frac{x(1-}{}$$

Mathematica [C] time = 0.0852281, size = 121, normalized size = 0.24

$$\frac{(-i(a+bx+i))^{3/4}\left(2i\sqrt[4]{2}\left(8a^2+4ia-3\right)\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{1}{2}i(a+bx+i)\right)-i\sqrt[4]{ia+ibx+1}\left(8a^2+a(4bx-7i)-4b^2x^2+5ib\right.\right.}{12b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (((-I)*(I + a + b*x))^(3/4)*((-I)*(1 + I*a + I*b*x)^(1/4)*(1 + 8*a^2 + (5*I
)*b*x - 4*b^2*x^2 + a*(-7*I + 4*b*x)) + (2*I)*2^(1/4)*(-3 + (4*I)*a + 8*a^2
)*Hypergeometric2F1[-1/4, 3/4, 7/4, (-I/2)*(I + a + b*x)]))/(12*b^3)
```

Maple [F] time = 0.312, size = 0, normalized size = 0.

$$\int \sqrt{(1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(x^2*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Fricas [A] time = 2.46128, size = 1432, normalized size = 2.9

$$3\,b^3\sqrt{\frac{64i\,a^4-64\,a^3-64i\,a^2+24\,a+9i}{b^6}}\log\left(\frac{i\,b^3\sqrt{\frac{64i\,a^4-64\,a^3-64i\,a^2+24\,a+9i}{b^6}}+\left(8\,a^2+4i\,a-3\right)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{8\,a^2+4i\,a-3}\right)-3\,b^3\sqrt{\frac{64i\,a^4-64\,a^3-64i\,a^2+24\,a+9i}{b^6}}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x, algorithm="frica
s")
```

```
[Out] 1/48*(3*b^3*sqrt((64*I*a^4 - 64*a^3 - 64*I*a^2 + 24*a + 9*I)/b^6)*log((I*b^
3*sqrt((64*I*a^4 - 64*a^3 - 64*I*a^2 + 24*a + 9*I)/b^6) + (8*a^2 + 4*I*a -
3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 + 4*I*a
- 3)) - 3*b^3*sqrt((64*I*a^4 - 64*a^3 - 64*I*a^2 + 24*a + 9*I)/b^6)*log((-I
*b^3*sqrt((64*I*a^4 - 64*a^3 - 64*I*a^2 + 24*a + 9*I)/b^6) + (8*a^2 + 4*I*a
 - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 + 4*I
*a - 3)) + 3*b^3*sqrt((-64*I*a^4 + 64*a^3 + 64*I*a^2 - 24*a - 9*I)/b^6)*log
((I*b^3*sqrt((-64*I*a^4 + 64*a^3 + 64*I*a^2 - 24*a - 9*I)/b^6) + (8*a^2 + 4
*I*a - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 +
```

```
 4*I*a - 3)) - 3*b^3*sqrt((-64*I*a^4 + 64*a^3 + 64*I*a^2 - 24*a - 9*I)/b^6)
*log((-I*b^3*sqrt((-64*I*a^4 + 64*a^3 + 64*I*a^2 - 24*a - 9*I)/b^6) + (8*a^
2 + 4*I*a - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*
a^2 + 4*I*a - 3)) + 2*(8*b^3*x^3 - 2*I*b^2*x^2 + 8*a^3 + (8*I*a - 1)*b*x +
34*I*a^2 - 37*a - 11*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a +
 I)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)*x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{\frac{i\,bx + i\,a + 1}{\sqrt{(bx + a)^2 + 1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x, algorithm="giac"
)
```

```
[Out] integrate(x^2*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

3.217 $\int e^{\frac{1}{2}i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=410

$$\frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2}+\frac{(1-4ia)(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{4b^2}+\frac{(1-4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right.}{8\sqrt{2}b^2}$$

```
[Out] ((1 - (4*I)*a)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/(4*b^2) + (
(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(5/4))/(2*b^2) - ((1 - (4*I)*a)*A
rcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(4*Sq
rt[2]*b^2) + ((1 - (4*I)*a)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1
 + I*a + I*b*x)^(1/4)])/(4*Sqrt[2]*b^2) + ((1 - (4*I)*a)*Log[1 + Sqrt[1 - I
*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 +
I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2) - ((1 - (4*I)*a)*Log[1 + Sqrt[1 - I*a
- I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a
 + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2)
```

Rubi [A] time = 0.307194, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {5095, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2}+\frac{(1-4ia)(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{4b^2}+\frac{(1-4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right.}{8\sqrt{2}b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a + b*x])*x,x]
```


```
[Out] ((1 - (4*I)*a)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/(4*b^2) + (
(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(5/4))/(2*b^2) - ((1 - (4*I)*a)*A
rcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(4*Sq
rt[2]*b^2) + ((1 - (4*I)*a)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1
 + I*a + I*b*x)^(1/4)])/(4*Sqrt[2]*b^2) + ((1 - (4*I)*a)*Log[1 + Sqrt[1 - I
*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 +
I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2) - ((1 - (4*I)*a)*Log[1 + Sqrt[1 - I*a
- I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a
 + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2)
```


Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{1}{2}i\tan^{-1}(a+bx)}x\,dx = \int \frac{x\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\,dx$$
$$= \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} - \frac{(i+4a)\int \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\,dx}{4b}$$
$$= \frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} - \frac{(i+4a)\int \frac{1}{\sqrt[4]{1-ia-ibx(}}}{8b}$$
$$= \frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} + \frac{(1-4ia)\,\mathrm{Subst}\left(\int\right.}{}$$
$$= \frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} + \frac{(1-4ia)\,\mathrm{Subst}\left(\int\right.}{2}$$
$$= \frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} - \frac{(1-4ia)\,\mathrm{Subst}\left(\int\right.}{4}$$
$$= \frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} + \frac{(1-4ia)\,\mathrm{Subst}\left(\int\right.}{}$$
$$= \frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} + \frac{(1-4ia)\log\left(1+\right.}{8}$$
$$= \frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} - \frac{(1-4ia)\tan^{-1}\left(1-\right.}{4\sqrt{2}b^2}$$

Mathematica [C] time = 0.0460453, size = 81, normalized size = 0.2

$$\frac{(-i(a+bx+i))^{3/4}\left(2\sqrt[4]{2}(1-4ia)\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{1}{2}i(a+bx+i)\right)+3(ia+ibx+1)^{5/4}\right)}{6b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a + b*x])*x,x]
```

```
[Out] (((-I)*(I + a + b*x))^(3/4)*(3*(1 + I*a + I*b*x)^(5/4) + 2*2^(1/4)*(1 - (4*
I)*a)*Hypergeometric2F1[-1/4, 3/4, 7/4, (-I/2)*(I + a + b*x)]))/(6*b^2)
```

Maple [F] time = 0.224, size = 0, normalized size = 0.

$$\int \sqrt{(1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\frac{i\,bx + i\,a + 1}{\sqrt{(bx+a)^2 + 1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x, algorithm="maxima"
)
```

```
[Out] integrate(x*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Fricas [A] time = 2.33364, size = 1015, normalized size = 2.48

$$-\frac{b^2\sqrt{\frac{16i\,a^2-8\,a-i}{b^4}}\log\left(\frac{i\,b^2\sqrt{\frac{16i\,a^2-8\,a-i}{b^4}}+(4\,a+i)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{4\,a+i}\right)-b^2\sqrt{\frac{16i\,a^2-8\,a-i}{b^4}}\log\left(\frac{-i\,b^2\sqrt{\frac{16i\,a^2-8\,a-i}{b^4}}+(4\,a+i)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{4\,a+i}\right)+b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x, algorithm="fricas"
)
```

```
[Out] -1/8*(b^2*sqrt((16*I*a^2 - 8*a - I)/b^4)*log((I*b^2*sqrt((16*I*a^2 - 8*a -
I)/b^4) + (4*a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))
)/(4*a + I)) - b^2*sqrt((16*I*a^2 - 8*a - I)/b^4)*log((-I*b^2*sqrt((16*I*a^
2 - 8*a - I)/b^4) + (4*a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x
 + a + I)))/(4*a + I)) + b^2*sqrt((-16*I*a^2 + 8*a + I)/b^4)*log((I*b^2*sqr
t((-16*I*a^2 + 8*a + I)/b^4) + (4*a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)/(b*x + a + I)))/(4*a + I)) - b^2*sqrt((-16*I*a^2 + 8*a + I)/b^4)*log
((-I*b^2*sqrt((-16*I*a^2 + 8*a + I)/b^4) + (4*a + I)*sqrt(I*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)/(b*x + a + I)))/(4*a + I)) - 2*(2*b^2*x^2 - 2*a^2 - I*b*
x - 5*I*a + 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)*x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\frac{ibx+ia+1}{\sqrt{(bx+a)^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x, algorithm="giac")
```

```
[Out] integrate(x*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

3.218 $\int e^{\frac{1}{2}i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=338

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}+\frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}-\frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}-\frac{i\tan^{-1}(}{}$$

```
[Out] (I*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/b - (I*ArcTan[1 - (Sqrt
[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) + (I*Arc
Tan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2
]*b) + ((I/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2
]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) - ((I/2)*L
og[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*
b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b)
```

Rubi [A] time = 0.195212, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5093, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}+\frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}-\frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}-\frac{i\tan^{-1}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a + b*x]),x]
```

```
[Out] (I*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/b - (I*ArcTan[1 - (Sqrt
[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) + (I*Arc
Tan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2
]*b) + ((I/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2
]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) - ((I/2)*L
og[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*
b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b)
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{\frac{1}{2}i\tan^{-1}(a+bx)}\,dx = \int \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\,dx$$
$$= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{1}{2}\int \frac{1}{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}\,dx$$
$$= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{(2i)\operatorname{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right)}{b}$$
$$= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{(2i)\operatorname{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b}$$
$$= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} - \frac{i\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} + \frac{i\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right.}{b}$$
$$= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{i\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b} + \frac{i\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x,\right.}{2b}$$
$$= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} - \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} - \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} + \frac{\sqrt{2}\sqrt[4]{1-ia}}{\sqrt[4]{1+ia+}}\right.}{2\sqrt{2}b}$$
$$= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} - \frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{i\log\left(1+\right.}{}$$

Mathematica [C] time = 0.015582, size = 45, normalized size = 0.13

$$-\frac{8ie^{\frac{5}{2}i\tan^{-1}(a+bx)}\,{}_2F_1\left(\frac{5}{4},2;\frac{9}{4};-e^{2i\tan^{-1}(a+bx)}\right)}{5b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a + b*x]),x]
```

```
[Out] (((-8*I)/5)*E^(((5*I)/2)*ArcTan[a + b*x])*Hypergeometric2F1[5/4, 2, 9/4, -E
^((2*I)*ArcTan[a + b*x])])/b
```

Maple [F] time = 0.194, size = 0, normalized size = 0.

$$\int \sqrt{(1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{i\,bx + i\,a + 1}{\sqrt{(bx+a)^2 + 1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Fricas [A] time = 2.3546, size = 636, normalized size = 1.88

$$\frac{b\sqrt{\frac{i}{b^2}}\log\left(i\,b\sqrt{\frac{i}{b^2}} + \sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) - b\sqrt{\frac{i}{b^2}}\log\left(-i\,b\sqrt{\frac{i}{b^2}} + \sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) + b\sqrt{-\frac{i}{b^2}}\log\left(i\,b\sqrt{-\frac{i}{b^2}} + \sqrt{\frac{i\sqrt{b^2x}}{}}\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*sqrt(I/b^2)*log(I*b*sqrt(I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)/(b*x + a + I))) - b*sqrt(I/b^2)*log(-I*b*sqrt(I/b^2) + sqrt(I*sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + b*sqrt(-I/b^2)*log(I*b*sqrt(-
I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(
-I/b^2)*log(-I*b*sqrt(-I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b
*x + a + I))) + (2*b*x + 2*a + 2*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)/(b*x + a + I)))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{i\,bx + i\,a + 1}{\sqrt{(bx+a)^2 + 1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

3.219 $\int \frac{e^{\frac{1}{2}i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=395

$$-\frac{\log\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+1\right)}{\sqrt{2}}-\frac{2\sqrt[4]{-a+i}\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{a+i}}-\sqrt{2}\tan$$

```
[Out] (-2*(I - a)^(1/4)*ArcTan[((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(
1/4)*(1 - I*(a + b*x))^(1/4))])/(I + a)^(1/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 + I*(a + b*x))^(1/4))/(1 - I*(a + b*x))^(1/4)] + Sqrt[2]*ArcTan[1 + (Sq
rt[2]*(1 + I*(a + b*x))^(1/4))/(1 - I*(a + b*x))^(1/4)] - (2*(I - a)^(1/4)*
ArcTanh[((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(1/4)*(1 - I*(a +
b*x))^(1/4))])/(I + a)^(1/4) - Log[1 - (Sqrt[2]*(1 + I*(a + b*x))^(1/4))/(1
 - I*(a + b*x))^(1/4) + Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]]/Sqrt[2
] + Log[1 + (Sqrt[2]*(1 + I*(a + b*x))^(1/4))/(1 - I*(a + b*x))^(1/4) + Sqr
t[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]]/Sqrt[2]
```

Rubi [A] time = 0.245591, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5094, 445, 481, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$-\frac{\log\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+1\right)}{\sqrt{2}}-\frac{2\sqrt[4]{-a+i}\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{a+i}}-\sqrt{2}\tan$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a + b*x])/x,x]
```


```
[Out] (-2*(I - a)^(1/4)*ArcTan[((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(
1/4)*(1 - I*(a + b*x))^(1/4))])/(I + a)^(1/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 + I*(a + b*x))^(1/4))/(1 - I*(a + b*x))^(1/4)] + Sqrt[2]*ArcTan[1 + (Sq
rt[2]*(1 + I*(a + b*x))^(1/4))/(1 - I*(a + b*x))^(1/4)] - (2*(I - a)^(1/4)*
ArcTanh[((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(1/4)*(1 - I*(a +
b*x))^(1/4))])/(I + a)^(1/4) - Log[1 - (Sqrt[2]*(1 + I*(a + b*x))^(1/4))/(1
 - I*(a + b*x))^(1/4) + Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]]/Sqrt[2
] + Log[1 + (Sqrt[2]*(1 + I*(a + b*x))^(1/4))/(1 - I*(a + b*x))^(1/4) + Sqr
t[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]]/Sqrt[2]
```


Rule 5094

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_))*(x_)^(m_), x_Symbol] :> Dist
[4/(I^m*n*b^(m + 1)*c^(m + 1)), Subst[Int[(x^(2/(I*n))*(1 - I*a*c - (1 + I*
a*c)*x^(2/(I*n)))^m)/(1 + x^(2/(I*n)))^(m + 2), x], x, (1 - I*c*(a + b*x))^
((I*n)/2)/(1 + I*c*(a + b*x))^((I*n)/2)], x] /; FreeQ[{a, b, c}, x] && ILtQ
[m, 0] && LtQ[-1, I*n, 1]
```

Rule 445

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2} i \tan^{-1}(a+bx)}}{x} dx = 8 \operatorname{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x^4}\right)\left(1-ia-\frac{1+ia}{x^4}\right)x^4} dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)$$
$$= 8 \operatorname{Subst}\left(\int \frac{x^4}{(1+x^4)\left(-1-ia+(1-ia)x^4\right)} dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)$$
$$= 4 \operatorname{Subst}\left(\int \frac{1}{1+x^4} dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) + (4(1+ia)) \operatorname{Subst}\left(\int \frac{1}{-1-ia+(1-ia)x^4} dx, x, \frac{\sqrt[4]{1+i}}{\sqrt[4]{1-i}}\right.$$
$$= 2 \operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) + 2 \operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) - \left(2\sqrt{i-a}\right)$$
$$= -\frac{2\sqrt[4]{i-a}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \frac{2\sqrt[4]{i-a}\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx, x, \frac{\sqrt[4]{}}{\sqrt[4]{}}\right.}{\sqrt{2}}$$
$$= -\frac{2\sqrt[4]{i-a}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \frac{2\sqrt[4]{i-a}\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \frac{\log\left(1-\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+\frac{\sqrt{1+i}}{\sqrt{1-i}}\right.}{\sqrt{2}}$$
$$= -\frac{2\sqrt[4]{i-a}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1+i(a}}{\sqrt[4]{1-i(a+}}\right.$$

Mathematica [C] time = 0.0874389, size = 124, normalized size = 0.31

$$\frac{2}{3}(-i(a+bx+i))^{3/4}\left(\frac{2(a-i)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)}{(a+i)(ia+ibx+1)^{3/4}} - \sqrt[4]{2}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{1}{2}i(a+bx+i)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a + b*x])/x,x]
```

```
[Out] (2*((-I)*(I + a + b*x))^(3/4)*(-(2^(1/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (
-I/2)*(I + a + b*x)]) + (2*(-I + a)*Hypergeometric2F1[3/4, 1, 7/4, (1 + a^2
 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)])/((I + a)*(1 + I*a + I*b*x)^(3
/4))))/3
```

Maple [F] time = 0.215, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{(1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))/x, x)
```

Fricas [A] time = 2.44152, size = 1141, normalized size = 2.89

$$\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)-\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)+\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] 1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/
(b*x + a + I))) - 1/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)/(b*x + a + I))) + 1/2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + sq
rt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 1/2*sqrt(-4*I)*log
(-1/2*sqrt(-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))
 - (-(a - I)/(a + I))^(1/4)*log(sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b
*x + a + I)) + (-(a - I)/(a + I))^(1/4)) - I*(-(a - I)/(a + I))^(1/4)*log(s
qrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + I*(-(a - I)/(a + I
))^(1/4)) + I*(-(a - I)/(a + I))^(1/4)*log(sqrt(I*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)/(b*x + a + I)) - I*(-(a - I)/(a + I))^(1/4)) + (-(a - I)/(a + I))^
(1/4)*log(sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (-(a -
I)/(a + I))^(1/4))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))/x, x)
```

3.220 $\int \frac{e^{\frac{1}{2}i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=205

$$-\frac{\sqrt[4]{1+i(a+bx)}(a+bx+i)}{(a+i)x\sqrt[4]{1-i(a+bx)}}+\frac{ib\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right)}{(-a+i)^{3/4}(a+i)^{5/4}}+\frac{ib\tanh^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right)}{(-a+i)^{3/4}(a+i)^{5/4}}$$

```
[Out] -(((I + a + b*x)*(1 + I*(a + b*x))^(1/4))/((I + a)*x*(1 - I*(a + b*x))^(1/4
))) + (I*b*ArcTan[((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(1/4)*(1
 - I*(a + b*x))^(1/4))])/((I - a)^(3/4)*(I + a)^(5/4)) + (I*b*ArcTanh[((I +
 a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))]
)/((I - a)^(3/4)*(I + a)^(5/4))
```

Rubi [A] time = 0.102695, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5094, 263, 288, 212, 208, 205}

$$-\frac{\sqrt[4]{1+i(a+bx)}(a+bx+i)}{(a+i)x\sqrt[4]{1-i(a+bx)}}+\frac{ib\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right)}{(-a+i)^{3/4}(a+i)^{5/4}}+\frac{ib\tanh^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right)}{(-a+i)^{3/4}(a+i)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] -(((I + a + b*x)*(1 + I*(a + b*x))^(1/4))/((I + a)*x*(1 - I*(a + b*x))^(1/4
))) + (I*b*ArcTan[((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(1/4)*(1
 - I*(a + b*x))^(1/4))])/((I - a)^(3/4)*(I + a)^(5/4)) + (I*b*ArcTanh[((I +
 a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))]
)/((I - a)^(3/4)*(I + a)^(5/4))
```

Rule 5094

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_))*(x_)^(m_), x_Symbol] :> Dist
[4/(I^m*n*b^(m + 1)*c^(m + 1)), Subst[Int[(x^(2/(I*n))*(1 - I*a*c - (1 + I*
a*c)*x^(2/(I*n)))^m)/(1 + x^(2/(I*n)))^(m + 2), x], x, (1 - I*c*(a + b*x))^
((I*n)/2)/(1 + I*c*(a + b*x))^((I*n)/2)], x] /; FreeQ[{a, b, c}, x] && ILtQ
[m, 0] && LtQ[-1, I*n, 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{2}i\tan^{-1}(a+bx)}}{x^2}\,dx &= (8ib)\,\mathrm{Subst}\left(\int \frac{1}{\left(1-ia-\frac{1+ia}{x^4}\right)^2 x^4}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)\\
&= (8ib)\,\mathrm{Subst}\left(\int \frac{x^4}{\left(-1-ia+(1-ia)x^4\right)^2}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)\\
&= -\frac{(i+a+bx)\sqrt[4]{1+i(a+bx)}}{(i+a)x\sqrt[4]{1-i(a+bx)}} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{-1-ia+(1-ia)x^4}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)}{i+a}\\
&= -\frac{(i+a+bx)\sqrt[4]{1+i(a+bx)}}{(i+a)x\sqrt[4]{1-i(a+bx)}} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{i-a}-\sqrt{i+a}x^2}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt{i-a}(1-ia)} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{i-a}+\sqrt{i+a}x^2}\,d\right.}{\sqrt{i-a}(1-}\\
&= -\frac{(i+a+bx)\sqrt[4]{1+i(a+bx)}}{(i+a)x\sqrt[4]{1-i(a+bx)}} + \frac{ib\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{(i-a)^{3/4}(i+a)^{5/4}} + \frac{ib\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{(i-a)^{3/4}(i+a)^{5/4}}
\end{aligned}$$

Mathematica [C] time = 0.0268317, size = 110, normalized size = 0.54

$$\frac{(-i(a+bx+i))^{3/4}\left(2ibx\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)+3(a+i)(a+bx-i)\right)}{3(a+i)^2x(ia+ibx+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] (((-I)*(I + a + b*x))^(3/4)*(3*(I + a)*(-I + a + b*x) + (2*I)*b*x*Hypergeom
etric2F1[3/4, 1, 7/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)])
)/(3*(I + a)^2*x*(1 + I*a + I*b*x)^(3/4))
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{(1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))/x^2, x)
```

Fricas [B] time = 2.37556, size = 1575, normalized size = 7.68

$$\left(-\frac{b^4}{16\,a^8+32i\,a^7+32\,a^6+96i\,a^5+96i\,a^3-32\,a^2+32i\,a-16}\right)^{\frac{1}{4}}(-i\,a+1)x\log\left(\frac{b\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}+2\left(-\frac{b^4}{16\,a^8+32i\,a^7+32\,a^6+96i\,a^5+96i\,a^3-32\,a^2+32i\,a-16}\right)^{\frac{1}{4}}(a}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] ((-b^4/(16*a^8 + 32*I*a^7 + 32*a^6 + 96*I*a^5 + 96*I*a^3 - 32*a^2 + 32*I*a
- 16))^(1/4)*(-I*a + 1)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(
b*x + a + I)) + 2*(-b^4/(16*a^8 + 32*I*a^7 + 32*a^6 + 96*I*a^5 + 96*I*a^3 -
 32*a^2 + 32*I*a - 16))^(1/4)*(a^2 + 1))/b) + (-b^4/(16*a^8 + 32*I*a^7 + 32
*a^6 + 96*I*a^5 + 96*I*a^3 - 32*a^2 + 32*I*a - 16))^(1/4)*(I*a - 1)*x*log((
b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - 2*(-b^4/(16*a^8
 + 32*I*a^7 + 32*a^6 + 96*I*a^5 + 96*I*a^3 - 32*a^2 + 32*I*a - 16))^(1/4)*(
a^2 + 1))/b) + (-b^4/(16*a^8 + 32*I*a^7 + 32*a^6 + 96*I*a^5 + 96*I*a^3 - 32
*a^2 + 32*I*a - 16))^(1/4)*(a + I)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)/(b*x + a + I)) + (-b^4/(16*a^8 + 32*I*a^7 + 32*a^6 + 96*I*a^5 + 9
6*I*a^3 - 32*a^2 + 32*I*a - 16))^(1/4)*(2*I*a^2 + 2*I))/b) - (-b^4/(16*a^8
+ 32*I*a^7 + 32*a^6 + 96*I*a^5 + 96*I*a^3 - 32*a^2 + 32*I*a - 16))^(1/4)*(a
 + I)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (-
b^4/(16*a^8 + 32*I*a^7 + 32*a^6 + 96*I*a^5 + 96*I*a^3 - 32*a^2 + 32*I*a - 1
6))^(1/4)*(-2*I*a^2 - 2*I))/b) - (b*x + a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)/(b*x + a + I)))/((a + I)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ibx+ia+1}{\sqrt{(bx+a)^2+1}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))/x^2, x)
```

3.221 $\int e^{\frac{3}{2}i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=494

$$-\frac{\left(-24ia^2+36a+17i\right)\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{24b^3}+\frac{\left(-24ia^2+36a+17i\right)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3}-\cdots$$

```
[Out] -((17*I + 36*a - (24*I)*a^2)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4
))/(24*b^3) - ((3*I + 8*a)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(7/4))
/(12*b^3) + (x*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(7/4))/(3*b^2) + (
(17*I + 36*a - (24*I)*a^2)*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1
+ I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((17*I + 36*a - (24*I)*a^2)*ArcTan
[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]
*b^3) + ((17*I + 36*a - (24*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 +
I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/
(16*Sqrt[2]*b^3) - ((17*I + 36*a - (24*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x
]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*
x)^(1/4)])/(16*Sqrt[2]*b^3)
```

Rubi [A] time = 0.382775, antiderivative size = 494, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5095, 90, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\left(-24ia^2+36a+17i\right)\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{24b^3}+\frac{\left(-24ia^2+36a+17i\right)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a + b*x])*x^2,x]
```


```
[Out] -((17*I + 36*a - (24*I)*a^2)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4
))/(24*b^3) - ((3*I + 8*a)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(7/4))
/(12*b^3) + (x*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(7/4))/(3*b^2) + (
(17*I + 36*a - (24*I)*a^2)*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1
+ I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((17*I + 36*a - (24*I)*a^2)*ArcTan
[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]
*b^3) + ((17*I + 36*a - (24*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 +
I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/
(16*Sqrt[2]*b^3) - ((17*I + 36*a - (24*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x
]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*
x)^(1/4)])/(16*Sqrt[2]*b^3)
```


Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
```

```
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{3}{2}i\tan^{-1}(a+bx)}x^2\,dx = \int \frac{x^2(1+ia+ibx)^{3/4}}{(1-ia-ibx)^{3/4}}\,dx$$

$$= \frac{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{3b^2} + \frac{\int \frac{(1+ia+ibx)^{3/4}\left(-1-a^2-\frac{1}{2}(3i+8a)bx\right)}{(1-ia-ibx)^{3/4}}\,dx}{3b^2}$$

$$= -\frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} + \frac{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{3b^2} - \frac{\left(17-36ia-24a^2\right.}{24}$$

$$= -\frac{\left(17i+36a-24ia^2\right)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} +$$

$$= -\frac{\left(17i+36a-24ia^2\right)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} +$$

$$= -\frac{\left(17i+36a-24ia^2\right)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} +$$

$$= -\frac{\left(17i+36a-24ia^2\right)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} +$$

$$= -\frac{\left(17i+36a-24ia^2\right)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} +$$

$$= -\frac{\left(17i+36a-24ia^2\right)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} +$$

$$= -\frac{\left(17i+36a-24ia^2\right)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} +$$

Mathematica [C] time = 0.0878406, size = 121, normalized size = 0.24

$$\frac{\sqrt[4]{-i(a+bx+i)}\left(2i2^{3/4}\left(24a^2+36ia-17\right)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};-\frac{1}{2}i(a+bx+i)\right)-i(ia+ibx+1)^{3/4}\left(8a^2+a(4bx-5i)-4b^2x\right.\right.}{12b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (((-I)*(I + a + b*x))^(1/4)*((-I)*(1 + I*a + I*b*x))^(3/4)*(3 + 8*a^2 + (7*I
)*b*x - 4*b^2*x^2 + a*(-5*I + 4*b*x)) + (2*I)*2^(3/4)*(-17 + (36*I)*a + 24*
a^2)*Hypergeometric2F1[-3/4, 1/4, 5/4, (-I/2)*(I + a + b*x)]))/(12*b^3)
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int \left((1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{i\,bx + i\,a + 1}{\sqrt{(bx+a)^2 + 1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(x^2*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

Fricas [A] time = 2.51913, size = 1571, normalized size = 3.18

$$\frac{3\,b^3\sqrt{\frac{576i\,a^4-1728\,a^3-2112i\,a^2+1224\,a+289i}{b^6}}\log\left(\frac{b^3\sqrt{\frac{576i\,a^4-1728\,a^3-2112i\,a^2+1224\,a+289i}{b^6}}+\left(24\,a^2+36i\,a-17\right)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{24\,a^2+36i\,a-17}\right)-3\,b^3\sqrt{\frac{576i\,a^4-1728}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x, algorithm="frica
s")
```

```
[Out] 1/48*(3*b^3*sqrt((576*I*a^4 - 1728*a^3 - 2112*I*a^2 + 1224*a + 289*I)/b^6)*
log((b^3*sqrt((576*I*a^4 - 1728*a^3 - 2112*I*a^2 + 1224*a + 289*I)/b^6) + (
24*a^2 + 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I
)))/(24*a^2 + 36*I*a - 17)) - 3*b^3*sqrt((576*I*a^4 - 1728*a^3 - 2112*I*a^2
 + 1224*a + 289*I)/b^6)*log(-(b^3*sqrt((576*I*a^4 - 1728*a^3 - 2112*I*a^2 +
 1224*a + 289*I)/b^6) - (24*a^2 + 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)/(b*x + a + I)))/(24*a^2 + 36*I*a - 17)) - 3*b^3*sqrt((-576*I*a
```

```
^4 + 1728*a^3 + 2112*I*a^2 - 1224*a - 289*I)/b^6)*log((b^3*sqrt((-576*I*a^4
 + 1728*a^3 + 2112*I*a^2 - 1224*a - 289*I)/b^6) + (24*a^2 + 36*I*a - 17)*sq
rt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(24*a^2 + 36*I*a - 1
7)) + 3*b^3*sqrt((-576*I*a^4 + 1728*a^3 + 2112*I*a^2 - 1224*a - 289*I)/b^6)
*log(-(b^3*sqrt((-576*I*a^4 + 1728*a^3 + 2112*I*a^2 - 1224*a - 289*I)/b^6)
- (24*a^2 + 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a
+ I)))/(24*a^2 + 36*I*a - 17)) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(8*I*b
^2*x^2 - 2*(4*I*a - 7)*b*x + 8*I*a^2 - 46*a - 23*I)*sqrt(I*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)/(b*x + a + I)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)*x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{i\,bx + i\,a + 1}{\sqrt{(bx+a)^2 + 1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x, algorithm="giac"
)
```

```
[Out] integrate(x^2*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

3.222 $\int e^{\frac{3}{2}i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=410

$$\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{2b^2}+\frac{(3-4ia)\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{4b^2}-\frac{3(3-4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right.}{8\sqrt{2}b^2}$$

```
[Out] ((3 - (4*I)*a)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/(4*b^2) + (
(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(7/4))/(2*b^2) - (3*(3 - (4*I)*a)
*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(4*
Sqrt[2]*b^2) + (3*(3 - (4*I)*a)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4)
)/(1 + I*a + I*b*x)^(1/4)])/(4*Sqrt[2]*b^2) - (3*(3 - (4*I)*a)*Log[1 + Sqrt
[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))
/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2) + (3*(3 - (4*I)*a)*Log[1 + Sqrt[
1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/
(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2)
```

Rubi [A] time = 0.292121, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {5095, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{2b^2}+\frac{(3-4ia)\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{4b^2}-\frac{3(3-4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right.}{8\sqrt{2}b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a + b*x])*x,x]
```

```
[Out] ((3 - (4*I)*a)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/(4*b^2) + (
(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(7/4))/(2*b^2) - (3*(3 - (4*I)*a)
*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(4*
Sqrt[2]*b^2) + (3*(3 - (4*I)*a)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4)
)/(1 + I*a + I*b*x)^(1/4)])/(4*Sqrt[2]*b^2) - (3*(3 - (4*I)*a)*Log[1 + Sqrt
[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))
/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2) + (3*(3 - (4*I)*a)*Log[1 + Sqrt[
1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/
(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{3}{2}i\tan^{-1}(a+bx)}x\,dx = \int \frac{x(1+ia+ibx)^{3/4}}{(1-ia-ibx)^{3/4}}\,dx$$

$$= \frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2} - \frac{(3i+4a)\int \frac{(1+ia+ibx)^{3/4}}{(1-ia-ibx)^{3/4}}\,dx}{4b}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2} - \frac{(3(3i+4a))\int \frac{1}{(1-ia-ibx)^{3/4}\cdots}}{8b}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2} + \frac{(3(3-4ia))\,\mathrm{Subst}\left(\int \cdots\right.}{2b\cdots}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2} + \frac{(3(3-4ia))\,\mathrm{Subst}\left(\int \frac{1}{1+\cdots}\right.}{2b^2}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2} + \frac{(3(3-4ia))\,\mathrm{Subst}\left(\int \frac{1-\cdots}{1+\cdots}\right.}{4b^2}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2} + \frac{(3(3-4ia))\,\mathrm{Subst}\left(\int \frac{\cdots}{1-\cdots}\right.}{8b\cdots}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2} - \frac{3(3-4ia)\log\left(1+\frac{\sqrt{1-ia\cdots}}{\sqrt{1+ia\cdots}}\right.}{8\sqrt{2}b^2\cdots}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2} - \frac{3(3-4ia)\tan^{-1}\left(1-\frac{\sqrt{2}\cdots}{\sqrt[4]{\cdots}}\right.}{4\sqrt{2}b^2}$$

Mathematica [C] time = 0.0419517, size = 79, normalized size = 0.19

$$\frac{\sqrt[4]{-i(a+bx+i)}\left(2\,2^{3/4}(3-4ia)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};-\frac{1}{2}i(a+bx+i)\right)+(ia+ibx+1)^{7/4}\right)}{2b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a + b*x])*x,x]
```

```
[Out] (((-I)*(I + a + b*x))^(1/4)*((1 + I*a + I*b*x)^(7/4) + 2*2^(3/4)*(3 - (4*I)
*a)*Hypergeometric2F1[-3/4, 1/4, 5/4, (-I/2)*(I + a + b*x)]))/(2*b^2)
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int \left((1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x, algorithm="maxima"
)
```

```
[Out] integrate(x*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

Fricas [A] time = 2.316, size = 1123, normalized size = 2.74

$$-\frac{b^2\sqrt{\frac{144i\,a^2-216\,a-81i}{b^4}}\log\left(\frac{b^2\sqrt{\frac{144i\,a^2-216\,a-81i}{b^4}}+(12\,a+9i)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{12\,a+9i}\right)-b^2\sqrt{\frac{144i\,a^2-216\,a-81i}{b^4}}\log\left(-\frac{b^2\sqrt{\frac{144i\,a^2-216\,a-81i}{b^4}}-(12\,a+9i)}{12\,a+9i}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x, algorithm="fricas"
)
```

```
[Out] -1/8*(b^2*sqrt((144*I*a^2 - 216*a - 81*I)/b^4)*log((b^2*sqrt((144*I*a^2 - 2
16*a - 81*I)/b^4) + (12*a + 9*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(
b*x + a + I)))/(12*a + 9*I)) - b^2*sqrt((144*I*a^2 - 216*a - 81*I)/b^4)*log
(-(b^2*sqrt((144*I*a^2 - 216*a - 81*I)/b^4) - (12*a + 9*I)*sqrt(I*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(12*a + 9*I)) - b^2*sqrt((-144*I*a
^2 + 216*a + 81*I)/b^4)*log((b^2*sqrt((-144*I*a^2 + 216*a + 81*I)/b^4) + (1
2*a + 9*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(12*a +
 9*I)) + b^2*sqrt((-144*I*a^2 + 216*a + 81*I)/b^4)*log(-(b^2*sqrt((-144*I*a
^2 + 216*a + 81*I)/b^4) - (12*a + 9*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)/(b*x + a + I)))/(12*a + 9*I)) - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2
*I*b*x - 2*I*a + 5)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))
)/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)*x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{ibx+ia+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x, algorithm="giac")
```

```
[Out] integrate(x*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

3.223 $\int e^{\frac{3}{2}i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=338

$$\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}-\frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}+\frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}-\frac{3i}{}$$

```
[Out] (I*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/b - ((3*I)*ArcTan[1 - (
Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) + ((
3*I)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])
/(Sqrt[2]*b) - (((3*I)/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*
x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b
) + (((3*I)/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[
2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b)
```

Rubi [A] time = 0.197481, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5093, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}-\frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}+\frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}-\frac{3i}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] (I*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/b - ((3*I)*ArcTan[1 - (
Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) + ((
3*I)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])
/(Sqrt[2]*b) - (((3*I)/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*
x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b
) + (((3*I)/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[
2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b)
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{3}{2}i\tan^{-1}(a+bx)}\,dx = \int \frac{(1+ia+ibx)^{3/4}}{(1-ia-ibx)^{3/4}}\,dx$$
$$= \frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{3}{2}\int \frac{1}{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx$$
$$= \frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{(6i)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right)}{b}$$
$$= \frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{(6i)\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b}$$
$$= \frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \right.}{b}$$
$$= \frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x}\right.}{2b}$$
$$= \frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} - \frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} - \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} + \frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} + \right.}{2\sqrt{2}b}$$
$$= \frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} - \frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} - \frac{3i\log}{}$$

Mathematica [C] time = 0.0171085, size = 45, normalized size = 0.13

$$-\frac{8ie^{\frac{7}{2}i\tan^{-1}(a+bx)}\,{}_2F_1\left(\frac{7}{4},2;\frac{11}{4};-e^{2i\tan^{-1}(a+bx)}\right)}{7b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] (((-8*I)/7)*E^(((7*I)/2)*ArcTan[a + b*x])*Hypergeometric2F1[7/4, 2, 11/4, -
E^((2*I)*ArcTan[a + b*x])])/b
```

Maple [F] time = 0.232, size = 0, normalized size = 0.

$$\int \left((1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{i\,bx + i\,a + 1}{\sqrt{(bx+a)^2 + 1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

Fricas [A] time = 2.40525, size = 693, normalized size = 2.05

$$\frac{b\sqrt{\frac{9i}{b^2}}\log\left(\frac{1}{3}b\sqrt{\frac{9i}{b^2}} + \sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) - b\sqrt{\frac{9i}{b^2}}\log\left(-\frac{1}{3}b\sqrt{\frac{9i}{b^2}} + \sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) - b\sqrt{-\frac{9i}{b^2}}\log\left(\frac{1}{3}b\sqrt{-\frac{9i}{b^2}} + \sqrt{\frac{i\sqrt{b^2x^2\ldots}}{b\ldots}}\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*sqrt(9*I/b^2)*log(1/3*b*sqrt(9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)/(b*x + a + I))) - b*sqrt(9*I/b^2)*log(-1/3*b*sqrt(9*I/b^2) + s
qrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(-9*I/b^2)*
log(1/3*b*sqrt(-9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x +
a + I))) + b*sqrt(-9*I/b^2)*log(-1/3*b*sqrt(-9*I/b^2) + sqrt(I*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{i\,bx + i\,a + 1}{\sqrt{(bx+a)^2 + 1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

3.224 $\int \frac{e^{\frac{3}{2}i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=427

$$\frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}+\frac{2(-a+i)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(a+i)^{3/4}}+\sqrt{2}$$

```
[Out] (2*(I - a)^(3/4)*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1
/4)*(1 - I*a - I*b*x)^(1/4))])/(I + a)^(3/4) + Sqrt[2]*ArcTan[1 - (Sqrt[2]*
(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqr
t[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)] - (2*(I - a)^(3/4)*A
rcTanh[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I*
b*x)^(1/4))])/(I + a)^(3/4) + Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a +
I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]
 - Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a
- I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.251418, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {5095, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 205, 208}

$$\frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}+\frac{2(-a+i)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(a+i)^{3/4}}+\sqrt{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a + b*x])/x,x]
```

```
[Out] (2*(I - a)^(3/4)*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1
/4)*(1 - I*a - I*b*x)^(1/4))])/(I + a)^(3/4) + Sqrt[2]*ArcTan[1 - (Sqrt[2]*
(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqr
t[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)] - (2*(I - a)^(3/4)*A
rcTanh[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I*
b*x)^(1/4))])/(I + a)^(3/4) + Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a +
I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]
 - Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a
- I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}i\tan^{-1}(a+bx)}}{x}\,dx = \int \frac{(1+ia+ibx)^{3/4}}{x(1-ia-ibx)^{3/4}}\,dx$$

$$= -\left((-1-ia)\int \frac{1}{x(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx\right)+(ib)\int \frac{1}{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx$$

$$= -\left(4\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx,x,\sqrt[4]{1-ia-ibx}\right)\right)+(4(1+ia))\,\mathrm{Subst}\left(\int \frac{x^2}{-1-ia-(-1+ia)x^4}\,dx,\right.$$

$$= -\left(4\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)\right)-\frac{(2(i-a))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{i-a}-\sqrt{i+a}x^2}\,dx,x,\frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{\sqrt{i+a}}$$

$$= \frac{2(i-a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}}-\frac{2(i-a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}}-2\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,\right.$$

$$= \frac{2(i-a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}}-\frac{2(i-a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}}+\frac{\mathrm{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,\right.}{\sqrt{2}}$$

$$= \frac{2(i-a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}}-\frac{2(i-a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}}+\frac{\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\right.}{\sqrt{2}}$$

$$= \frac{2(i-a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}}+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)-\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-i}}{\sqrt[4]{1+ia}}\right.$$

Mathematica [C] time = 0.0959046, size = 122, normalized size = 0.29

$$2\sqrt[4]{-i(a+bx+i)}\left(\frac{2(a-i)\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)}{(a+i)\sqrt[4]{ia+ibx+1}}-2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{1}{2}i(a+bx+i)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a + b*x])/x,x]
```

```
[Out] 2*((-I)*(I + a + b*x))^(1/4)*(-(2^(3/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (-
I/2)*(I + a + b*x)]) + (2*(-I + a)*Hypergeometric2F1[1/4, 1, 5/4, (1 + a^2
- I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)])/((I + a)*(1 + I*a + I*b*x)^(1/
4)))
```

Maple [F] time = 0.218, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left((1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ibx+ia+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)/x, x)
```

Fricas [B] time = 2.69826, size = 1840, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="fricas"
)
```

```
[Out] 1/2*sqrt(4*I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)/(b*x + a + I))) - 1/2*sqrt(4*I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 1/2*sqrt(-4*I)*log(1/2*I*sqrt(-4*I
) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 1/2*sqrt(-4*
I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x +
a + I))) - (-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^(1/4)*log
(((a^2 + 2*I*a - 1)*(-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^
(3/4) + (a^2 - 2*I*a - 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a
 + I)))/(a^2 - 2*I*a - 1)) + (-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3
```

```
*a - I))^(1/4)*log(-((a^2 + 2*I*a - 1)*(-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3
*I*a^2 - 3*a - I))^(3/4) - (a^2 - 2*I*a - 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)/(b*x + a + I)))/(a^2 - 2*I*a - 1)) + I*(-(a^3 - 3*I*a^2 - 3*a +
I)/(a^3 + 3*I*a^2 - 3*a - I))^(1/4)*log(((I*a^2 - 2*a - I)*(-(a^3 - 3*I*a^2
 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^(3/4) + (a^2 - 2*I*a - 1)*sqrt(I*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a^2 - 2*I*a - 1)) - I*(-(a^
3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^(1/4)*log(((-I*a^2 + 2*a
+ I)*(-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^(3/4) + (a^2 -
2*I*a - 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a^2 -
2*I*a - 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)/x, x)
```

3.225 $\int \frac{e^{\frac{3}{2}i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=211

$$-\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{(1-ia)x}-\frac{3ib\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{\sqrt[4]{-a+i}(a+i)^{7/4}}+\frac{3ib\tanh^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{\sqrt[4]{-a+i}(a+i)^{7/4}}$$

```
[Out] -(((1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/((1 - I*a)*x)) - ((3*I)
*b*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a -
 I*b*x)^(1/4))])/((I - a)^(1/4)*(I + a)^(7/4)) + ((3*I)*b*ArcTanh[((I + a)^
(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I*b*x)^(1/4))])/((
I - a)^(1/4)*(I + a)^(7/4))
```

Rubi [A] time = 0.107824, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5095, 94, 93, 298, 205, 208}

$$-\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{(1-ia)x}-\frac{3ib\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{\sqrt[4]{-a+i}(a+i)^{7/4}}+\frac{3ib\tanh^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{\sqrt[4]{-a+i}(a+i)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] -(((1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/((1 - I*a)*x)) - ((3*I)
*b*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a -
 I*b*x)^(1/4))])/((I - a)^(1/4)*(I + a)^(7/4)) + ((3*I)*b*ArcTanh[((I + a)^
(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I*b*x)^(1/4))])/((
I - a)^(1/4)*(I + a)^(7/4))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}i\tan^{-1}(a+bx)}}{x^2}\,dx = \int \frac{(1+ia+ibx)^{3/4}}{x^2(1-ia-ibx)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{(1-ia)x} - \frac{(3b)\int \frac{1}{x(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx}{2(i+a)}$$

$$= -\frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{(1-ia)x} - \frac{(6b)\,\mathrm{Subst}\left(\int \frac{x^2}{-1-ia-(-1+ia)x^4}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{i+a}$$

$$= -\frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{(1-ia)x} + \frac{(3ib)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{i-a}-\sqrt{i+a}x^2}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/2}} - \frac{(3ib)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{(1-ia)x} - \frac{3ib\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{\sqrt[4]{i-a}(i+a)^{7/4}} + \frac{3ib\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{\sqrt[4]{i-a}(i+a)^{7/4}}$$

Mathematica [C] time = 0.0221918, size = 106, normalized size = 0.5

$$\frac{\sqrt[4]{-i(a+bx+i)}\left(6ibx\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)+a^2+abx+ibx+1\right)}{(a+i)^2x\sqrt[4]{ia+ibx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] (((-I)*(I + a + b*x))^(1/4)*(1 + a^2 + I*b*x + a*b*x + (6*I)*b*x*Hypergeome
tric2F1[1/4, 1, 5/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]))
/((I + a)^2*x*(1 + I*a + I*b*x)^(1/4))
```

Maple [F] time = 0.24, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left((1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)/x^2, x)
```

Fricas [B] time = 2.72177, size = 1886, normalized size = 8.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="frica
s")
```

```
[Out] (3*(-b^4/(16*a^8 + 96*I*a^7 - 224*a^6 - 224*I*a^5 - 224*I*a^3 + 224*a^2 + 9
6*I*a - 16))^(1/4)*(-I*a + 1)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)/(b*x + a + I)) + (8*a^6 + 32*I*a^5 - 40*a^4 - 40*a^2 - 32*I*a + 8)*(
-b^4/(16*a^8 + 96*I*a^7 - 224*a^6 - 224*I*a^5 - 224*I*a^3 + 224*a^2 + 96*I*
a - 16))^(3/4))/b^3) + 3*(-b^4/(16*a^8 + 96*I*a^7 - 224*a^6 - 224*I*a^5 - 2
24*I*a^3 + 224*a^2 + 96*I*a - 16))^(1/4)*(I*a - 1)*x*log((b^3*sqrt(I*sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (8*a^6 + 32*I*a^5 - 40*a^4 - 4
0*a^2 - 32*I*a + 8)*(-b^4/(16*a^8 + 96*I*a^7 - 224*a^6 - 224*I*a^5 - 224*I*
a^3 + 224*a^2 + 96*I*a - 16))^(3/4))/b^3) - 3*(-b^4/(16*a^8 + 96*I*a^7 - 22
4*a^6 - 224*I*a^5 - 224*I*a^3 + 224*a^2 + 96*I*a - 16))^(1/4)*(a + I)*x*log
((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (8*I*a^6 -
32*a^5 - 40*I*a^4 - 40*I*a^2 + 32*a + 8*I)*(-b^4/(16*a^8 + 96*I*a^7 - 224*a
^6 - 224*I*a^5 - 224*I*a^3 + 224*a^2 + 96*I*a - 16))^(3/4))/b^3) + 3*(-b^4/
(16*a^8 + 96*I*a^7 - 224*a^6 - 224*I*a^5 - 224*I*a^3 + 224*a^2 + 96*I*a - 1
6))^(1/4)*(a + I)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x
+ a + I)) + (-8*I*a^6 + 32*a^5 + 40*I*a^4 + 40*I*a^2 - 32*a - 8*I)*(-b^4/(1
6*a^8 + 96*I*a^7 - 224*a^6 - 224*I*a^5 - 224*I*a^3 + 224*a^2 + 96*I*a - 16)
)^(3/4))/b^3) - I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*sqrt(I*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)/(b*x + a + I)))/((a + I)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="giac"
)
```

```
[Out] integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)/x^2, x)
```

3.226 $\int e^{-\frac{1}{2}i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=494

$$\frac{\left(-8ia^2-4a+3i\right)(ia+ibx+1)^{3/4}\sqrt[4]{-ia-ibx+1}}{8b^3}+\frac{\left(-8ia^2-4a+3i\right)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3}-\frac{\left(-8ia^2-4a+\right.}{}$$

```
[Out] ((3*I - 4*a - (8*I)*a^2)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/(
8*b^3) + ((I - 8*a)*(1 - I*a - I*b*x)^(5/4)*(1 + I*a + I*b*x)^(3/4))/(12*b^
3) + (x*(1 - I*a - I*b*x)^(5/4)*(1 + I*a + I*b*x)^(3/4))/(3*b^2) + ((3*I -
4*a - (8*I)*a^2)*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*
b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((3*I - 4*a - (8*I)*a^2)*ArcTan[1 + (Sqrt[2]
*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^3) + ((3*I
 - 4*a - (8*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (
Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(16*Sqrt[2]*b^3)
 - ((3*I - 4*a - (8*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*
b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(16*Sqrt
[2]*b^3)
```

Rubi [A] time = 0.401267, antiderivative size = 494, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5095, 90, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{\left(-8ia^2-4a+3i\right)(ia+ibx+1)^{3/4}\sqrt[4]{-ia-ibx+1}}{8b^3}+\frac{\left(-8ia^2-4a+3i\right)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3}-\frac{\left(-8ia^2-4a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((I/2)*ArcTan[a + b*x]),x]
```


```
[Out] ((3*I - 4*a - (8*I)*a^2)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/(
8*b^3) + ((I - 8*a)*(1 - I*a - I*b*x)^(5/4)*(1 + I*a + I*b*x)^(3/4))/(12*b^
3) + (x*(1 - I*a - I*b*x)^(5/4)*(1 + I*a + I*b*x)^(3/4))/(3*b^2) + ((3*I -
4*a - (8*I)*a^2)*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*
b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((3*I - 4*a - (8*I)*a^2)*ArcTan[1 + (Sqrt[2]
*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^3) + ((3*I
 - 4*a - (8*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (
Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(16*Sqrt[2]*b^3)
 - ((3*I - 4*a - (8*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*
b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(16*Sqrt
[2]*b^3)
```


Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
```

```
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}i\tan^{-1}(a+bx)}x^2\,dx = \int \frac{x^2\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\,dx$$

$$= \frac{x(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{3b^2} + \frac{\int \frac{\sqrt[4]{1-ia-ibx}\left(-1-a^2+\frac{1}{2}(i-8a)bx\right)}{\sqrt[4]{1+ia+ibx}}\,dx}{3b^2}$$

$$= \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} + \frac{x(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{3b^2} - \frac{\left(3+4ia-8a^2\right)\int}{8b^2}$$

$$= -\frac{\left(4a-i\left(3-8a^2\right)\right)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} + \frac{x(}{}$$

$$= -\frac{\left(4a-i\left(3-8a^2\right)\right)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} + \frac{x(}{}$$

$$= -\frac{\left(4a-i\left(3-8a^2\right)\right)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} + \frac{x(}{}$$

$$= -\frac{\left(4a-i\left(3-8a^2\right)\right)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} + \frac{x(}{}$$

$$= -\frac{\left(4a-i\left(3-8a^2\right)\right)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} + \frac{x(}{}$$

$$= -\frac{\left(4a-i\left(3-8a^2\right)\right)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} + \frac{x(}{}$$

$$= -\frac{\left(4a-i\left(3-8a^2\right)\right)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} + \frac{x(}{}$$

Mathematica [C] time = 0.0877648, size = 99, normalized size = 0.2

$$\frac{(-i(a+bx+i))^{5/4}\left(3\,2^{3/4}\left(8ia^2+4a-3i\right)\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};-\frac{1}{2}i(a+bx+i)\right)+5(ia+ibx+1)^{3/4}(-8a+4bx+i)\right)}{60b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^((I/2)*ArcTan[a + b*x]),x]
```

```
[Out] (((-I)*(I + a + b*x))^(5/4)*(5*(1 + I*a + I*b*x)^(3/4)*(I - 8*a + 4*b*x) +
3*2^(3/4)*(-3*I + 4*a + (8*I)*a^2)*Hypergeometric2F1[1/4, 5/4, 9/4, (-I/2)*
(I + a + b*x)]))/(60*b^3)
```

Maple [F] time = 0.23, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{(1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

```
[Out] int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\frac{ibx+ia+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(x^2/sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Fricas [A] time = 2.63519, size = 1445, normalized size = 2.93

$$3\,b^3\sqrt{\frac{64i\,a^4+64\,a^3-64i\,a^2-24\,a+9i}{b^6}}\log\left(\frac{b^3\sqrt{\frac{64i\,a^4+64\,a^3-64i\,a^2-24\,a+9i}{b^6}}+\left(8\,a^2-4i\,a-3\right)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{8\,a^2-4i\,a-3}\right)-3\,b^3\sqrt{\frac{64i\,a^4+64\,a^3-64i\,a^2-24\,a+9i}{b^6}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/48*(3*b^3*sqrt((64*I*a^4 + 64*a^3 - 64*I*a^2 - 24*a + 9*I)/b^6)*log((b^3*
sqrt((64*I*a^4 + 64*a^3 - 64*I*a^2 - 24*a + 9*I)/b^6) + (8*a^2 - 4*I*a - 3)
*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 - 4*I*a -
3)) - 3*b^3*sqrt((64*I*a^4 + 64*a^3 - 64*I*a^2 - 24*a + 9*I)/b^6)*log(-(b^3
*sqrt((64*I*a^4 + 64*a^3 - 64*I*a^2 - 24*a + 9*I)/b^6) - (8*a^2 - 4*I*a - 3
)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 - 4*I*a -
 3)) - 3*b^3*sqrt((-64*I*a^4 - 64*a^3 + 64*I*a^2 + 24*a - 9*I)/b^6)*log((b^
```

```
3*sqrt((-64*I*a^4 - 64*a^3 + 64*I*a^2 + 24*a - 9*I)/b^6) + (8*a^2 - 4*I*a -
 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 - 4*I*a
 - 3)) + 3*b^3*sqrt((-64*I*a^4 - 64*a^3 + 64*I*a^2 + 24*a - 9*I)/b^6)*log(-
(b^3*sqrt((-64*I*a^4 - 64*a^3 + 64*I*a^2 + 24*a - 9*I)/b^6) - (8*a^2 - 4*I*
a - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 - 4*
I*a - 3)) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-8*I*b^2*x^2 - 2*(-4*I*a -
 5)*b*x - 8*I*a^2 - 26*a + 11*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(
b*x + a + I)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(x^2/sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

3.227 $\int e^{-\frac{1}{2}i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=410

$$\frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2}+\frac{(1+4ia)(ia+ibx+1)^{3/4}\sqrt[4]{-ia-ibx+1}}{4b^2}+\frac{(1+4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right.}{8\sqrt{2}b^2}$$

```
[Out] ((1 + (4*I)*a)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/(4*b^2) + (
(1 - I*a - I*b*x)^(5/4)*(1 + I*a + I*b*x)^(3/4))/(2*b^2) + ((1 + (4*I)*a)*A
rcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(4*Sq
rt[2]*b^2) - ((1 + (4*I)*a)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1
 + I*a + I*b*x)^(1/4)])/(4*Sqrt[2]*b^2) + ((1 + (4*I)*a)*Log[1 + Sqrt[1 - I
*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 +
I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2) - ((1 + (4*I)*a)*Log[1 + Sqrt[1 - I*a
- I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a
 + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2)
```

Rubi [A] time = 0.307334, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {5095, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2}+\frac{(1+4ia)(ia+ibx+1)^{3/4}\sqrt[4]{-ia-ibx+1}}{4b^2}+\frac{(1+4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right.}{8\sqrt{2}b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((I/2)*ArcTan[a + b*x]),x]
```


```
[Out] ((1 + (4*I)*a)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/(4*b^2) + (
(1 - I*a - I*b*x)^(5/4)*(1 + I*a + I*b*x)^(3/4))/(2*b^2) + ((1 + (4*I)*a)*A
rcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(4*Sq
rt[2]*b^2) - ((1 + (4*I)*a)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1
 + I*a + I*b*x)^(1/4)])/(4*Sqrt[2]*b^2) + ((1 + (4*I)*a)*Log[1 + Sqrt[1 - I
*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 +
I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2) - ((1 + (4*I)*a)*Log[1 + Sqrt[1 - I*a
- I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a
 + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2)
```


Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}i\tan^{-1}(a+bx)}x\,dx = \int \frac{x\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\,dx$$

$$= \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} + \frac{(i-4a)\int \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\,dx}{4b}$$

$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} + \frac{(i-4a)\int \frac{}{(1-ia-ibx)}}{8b}$$

$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} - \frac{(1+4ia)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} - \frac{(1+4ia)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} - \frac{(1+4ia)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} - \frac{(1+4ia)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} + \frac{(1+4ia)\log\left(1+\right.}{8}$$

$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} + \frac{(1+4ia)\tan^{-1}\left(1\right.}{4\sqrt{2}b}$$

Mathematica [C] time = 0.0398892, size = 84, normalized size = 0.2

$$-\frac{i(-i(a+bx+i))^{5/4}\left(2^{3/4}(4a-i)\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};-\frac{1}{2}i(a+bx+i)\right)+5i(ia+ibx+1)^{3/4}\right)}{10b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^((I/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((-I/10)*((-I)*(I + a + b*x))^(5/4)*((5*I)*(1 + I*a + I*b*x)^(3/4) + 2^(3/4
)*(-I + 4*a)*Hypergeometric2F1[1/4, 5/4, 9/4, (-I/2)*(I + a + b*x)]))/b^2
```

Maple [F] time = 0.233, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{(1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

```
[Out] int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Fricas [A] time = 2.51555, size = 1027, normalized size = 2.5

$$-\frac{b^2\sqrt{\frac{16i\,a^2+8\,a-i}{b^4}}\log\left(\frac{b^2\sqrt{\frac{16i\,a^2+8\,a-i}{b^4}}+(4\,a-i)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{4\,a-i}\right)-b^2\sqrt{\frac{16i\,a^2+8\,a-i}{b^4}}\log\left(-\frac{b^2\sqrt{\frac{16i\,a^2+8\,a-i}{b^4}}-(4\,a-i)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{4\,a-i}\right)-b^2\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/8*(b^2*sqrt((16*I*a^2 + 8*a - I)/b^4)*log((b^2*sqrt((16*I*a^2 + 8*a - I)
/b^4) + (4*a - I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/
(4*a - I)) - b^2*sqrt((16*I*a^2 + 8*a - I)/b^4)*log(-(b^2*sqrt((16*I*a^2 +
8*a - I)/b^4) - (4*a - I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a
 + I)))/(4*a - I)) - b^2*sqrt((-16*I*a^2 - 8*a + I)/b^4)*log((b^2*sqrt((-16
*I*a^2 - 8*a + I)/b^4) + (4*a - I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
/(b*x + a + I)))/(4*a - I)) + b^2*sqrt((-16*I*a^2 - 8*a + I)/b^4)*log(-(b^2
*sqrt((-16*I*a^2 - 8*a + I)/b^4) - (4*a - I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)/(b*x + a + I)))/(4*a - I)) - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*(-2*I*b*x + 2*I*a + 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a +
 I)))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

3.228 $\int e^{-\frac{1}{2}i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=338

$$-\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}-\frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}+\frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}-\frac{i\tan^{-}}{}$$

```
[Out] ((-I)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/b - (I*ArcTan[1 - (S
qrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) + (I*
ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqr
t[2]*b) - ((I/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqr
t[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) + ((I/2
)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a -
 I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b)
```

Rubi [A] time = 0.197272, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5093, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}-\frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}+\frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}-\frac{i\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-I/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((-I)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/b - (I*ArcTan[1 - (S
qrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) + (I*
ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqr
t[2]*b) - ((I/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqr
t[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) + ((I/2
)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a -
 I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b)
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}i\tan^{-1}(a+bx)}\,dx = \int \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\,dx$$
$$= -\frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{1}{2}\int \frac{1}{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx$$
$$= -\frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{(2i)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right)}{b}$$
$$= -\frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{(2i)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b}$$
$$= -\frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{i\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} + \frac{i\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-}}{\sqrt[4]{1+ia+}}\right.}{b}$$
$$= -\frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b} + \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,\right.}{2b}$$
$$= -\frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} - \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} - \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} + \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} + \frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1+}}\right.}{2\sqrt{2}b}$$
$$= -\frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} - \frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} - \frac{i\log\left(1\right.}{}$$

Mathematica [C] time = 0.0165781, size = 45, normalized size = 0.13

$$-\frac{8ie^{\frac{3}{2}i\tan^{-1}(a+bx)}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-e^{2i\tan^{-1}(a+bx)}\right)}{3b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-I/2)*ArcTan[a + b*x]),x]
```

```
[Out] (((-8*I)/3)*E^(((3*I)/2)*ArcTan[a + b*x])*Hypergeometric2F1[3/4, 2, 7/4, -E
^((2*I)*ArcTan[a + b*x])])/b
```

Maple [F] time = 0.203, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Fricas [A] time = 2.54888, size = 649, normalized size = 1.92

$$\frac{b\sqrt{\frac{i}{b^2}}\log\left(b\sqrt{\frac{i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{\frac{i}{b^2}}\log\left(-b\sqrt{\frac{i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{-\frac{i}{b^2}}\log\left(b\sqrt{-\frac{i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2\,\ldots}}{bx+\ldots}}\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(b*sqrt(I/b^2)*log(b*sqrt(I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)/(b*x + a + I))) - b*sqrt(I/b^2)*log(-b*sqrt(I/b^2) + sqrt(I*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(-I/b^2)*log(b*sqrt(-I/b^2)
 + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + b*sqrt(-I/b^2
)*log(-b*sqrt(-I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a +
 I))) - 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)/(b*x + a + I)))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

3.229 $\int \frac{e^{-\frac{1}{2}i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=395

$$-\frac{\log\left(\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}+1\right)}{\sqrt{2}}-\frac{2\sqrt[4]{a+i}\tan^{-1}\left(\frac{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{-a+i}}-\sqrt{2}\tan$$

```
[Out] (-2*(I + a)^(1/4)*ArcTan[((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(
1/4)*(1 + I*(a + b*x))^(1/4))])/(I - a)^(1/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 - I*(a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)] + Sqrt[2]*ArcTan[1 + (Sq
rt[2]*(1 - I*(a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)] - (2*(I + a)^(1/4)*
ArcTanh[((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(1/4)*(1 + I*(a +
b*x))^(1/4))])/(I - a)^(1/4) - Log[1 + Sqrt[1 - I*(a + b*x)]/Sqrt[1 + I*(a
+ b*x)] - (Sqrt[2]*(1 - I*(a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)]/Sqrt[2
] + Log[1 + Sqrt[1 - I*(a + b*x)]/Sqrt[1 + I*(a + b*x)] + (Sqrt[2]*(1 - I*(
a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.205775, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {5094, 481, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$-\frac{\log\left(\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}+1\right)}{\sqrt{2}}-\frac{2\sqrt[4]{a+i}\tan^{-1}\left(\frac{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{-a+i}}-\sqrt{2}\tan$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((I/2)*ArcTan[a + b*x])*x),x]
```

```
[Out] (-2*(I + a)^(1/4)*ArcTan[((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(
1/4)*(1 + I*(a + b*x))^(1/4))])/(I - a)^(1/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 - I*(a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)] + Sqrt[2]*ArcTan[1 + (Sq
rt[2]*(1 - I*(a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)] - (2*(I + a)^(1/4)*
ArcTanh[((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(1/4)*(1 + I*(a +
b*x))^(1/4))])/(I - a)^(1/4) - Log[1 + Sqrt[1 - I*(a + b*x)]/Sqrt[1 + I*(a
+ b*x)] - (Sqrt[2]*(1 - I*(a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)]/Sqrt[2
] + Log[1 + Sqrt[1 - I*(a + b*x)]/Sqrt[1 + I*(a + b*x)] + (Sqrt[2]*(1 - I*(
a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)]/Sqrt[2]
```

Rule 5094

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_))*(x_)^(m_), x_Symbol] :> Dist
[4/(I^m*n*b^(m + 1)*c^(m + 1)), Subst[Int[(x^(2/(I*n))*(1 - I*a*c - (1 + I*
a*c)*x^(2/(I*n)))^m)/(1 + x^(2/(I*n)))^(m + 2), x], x, (1 - I*c*(a + b*x))^
((I*n)/2)/(1 + I*c*(a + b*x))^((I*n)/2)], x] /; FreeQ[{a, b, c}, x] && ILtQ
[m, 0] && LtQ[-1, I*n, 1]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
```

```
2*n - 1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}i\tan^{-1}(a+bx)}}{x}\,dx = -\left(8\,\text{Subst}\left(\int \frac{x^4}{(1+x^4)\left(1-ia-(1+ia)x^4\right)}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)\right)$$
$$= 4\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) - (4(1-ia))\,\text{Subst}\left(\int \frac{1}{1-ia+(-1-ia)x^4}\,dx, x, \ldots$$
$$= 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) + 2\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) - \left(2\sqrt{i\ldots}\right.$$
$$= -\frac{2\sqrt[4]{i+a}\tan^{-1}\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} - \frac{2\sqrt[4]{i+a}\tanh^{-1}\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, \ldots\right)}{\sqrt{2}} \ldots$$
$$= -\frac{2\sqrt[4]{i+a}\tan^{-1}\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} - \frac{2\sqrt[4]{i+a}\tanh^{-1}\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} - \frac{\log\left(1+\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}} - \ldots\right)}{\sqrt{2}} \ldots$$
$$= -\frac{2\sqrt[4]{i+a}\tan^{-1}\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} - \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\ldots}}{\sqrt[4]{1+\ldots}}\right.\ldots$$

Mathematica [C] time = 0.0325684, size = 126, normalized size = 0.32

$$\frac{2\sqrt[4]{-i(a+bx+i)}\left(2^{3/4}\sqrt[4]{ia+ibx+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{1}{2}i(a+bx+i)\right)-2\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)\right)}{\sqrt[4]{ia+ibx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((I/2)*ArcTan[a + b*x])*x),x]
```

```
[Out] (2*((-I)*(I + a + b*x))^(1/4)*(2^(3/4)*(1 + I*a + I*b*x)^(1/4)*Hypergeometr
ic2F1[1/4, 1/4, 5/4, (-I/2)*(I + a + b*x)] - 2*Hypergeometric2F1[1/4, 1, 5/
4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]))/(1 + I*a + I*b*x)
^(1/4)
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{(1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

```
[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\frac{ibx+ia+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="maxim
a")
```

```
[Out] integrate(1/(x*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))), x)
```

Fricas [A] time = 2.82535, size = 1296, normalized size = 3.28

$$-\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)+\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)+\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="frica
s")
```

```
[Out] -1/2*sqrt(4*I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)/(b*x + a + I))) + 1/2*sqrt(4*I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 1/2*sqrt(-4*I)*log(1/2*I*sqrt(-4*
I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 1/2*sqrt(-4
*I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x +
 a + I))) + (-(a + I)/(a - I))^(1/4)*log(((a - I)*(-(a + I)/(a - I))^(3/4)
+ (a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a + I))
 - (-(a + I)/(a - I))^(1/4)*log(-((a - I)*(-(a + I)/(a - I))^(3/4) - (a + I
)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a + I)) - I*(-(
a + I)/(a - I))^(1/4)*log(((I*a + 1)*(-(a + I)/(a - I))^(3/4) + (a + I)*sqr
t(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a + I)) + I*(-(a + I
)/(a - I))^(1/4)*log(((-I*a - 1)*(-(a + I)/(a - I))^(3/4) + (a + I)*sqrt(I*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a + I))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\frac{ibx+ia+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="giac"
)
```

```
[Out] integrate(1/(x*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))), x)
```

3.230 $\int \frac{e^{-\frac{1}{2}i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=210

$$-\frac{\sqrt[4]{1-i(a+bx)}(-a-bx+i)}{(-a+i)x\sqrt[4]{1+i(a+bx)}}-\frac{ib\tan^{-1}\left(\frac{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}\right)}{(-a+i)^{5/4}(a+i)^{3/4}}-\frac{ib\tanh^{-1}\left(\frac{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}\right)}{(-a+i)^{5/4}(a+i)^{3/4}}$$

```
[Out] -(((I - a - b*x)*(1 - I*(a + b*x))^(1/4))/((I - a)*x*(1 + I*(a + b*x))^(1/4
))) - (I*b*ArcTan[((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(1/4)*(1
 + I*(a + b*x))^(1/4))])/((I - a)^(5/4)*(I + a)^(3/4)) - (I*b*ArcTanh[((I -
 a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))]
)/((I - a)^(5/4)*(I + a)^(3/4))
```

Rubi [A] time = 0.101343, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5094, 288, 212, 208, 205}

$$-\frac{\sqrt[4]{1-i(a+bx)}(-a-bx+i)}{(-a+i)x\sqrt[4]{1+i(a+bx)}}-\frac{ib\tan^{-1}\left(\frac{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}\right)}{(-a+i)^{5/4}(a+i)^{3/4}}-\frac{ib\tanh^{-1}\left(\frac{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}\right)}{(-a+i)^{5/4}(a+i)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((I/2)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -(((I - a - b*x)*(1 - I*(a + b*x))^(1/4))/((I - a)*x*(1 + I*(a + b*x))^(1/4
))) - (I*b*ArcTan[((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(1/4)*(1
 + I*(a + b*x))^(1/4))])/((I - a)^(5/4)*(I + a)^(3/4)) - (I*b*ArcTanh[((I -
 a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))]
)/((I - a)^(5/4)*(I + a)^(3/4))
```

Rule 5094

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_))*(x_)^(m_), x_Symbol] :> Dist
[4/(I^m*n*b^(m + 1)*c^(m + 1)), Subst[Int[(x^(2/(I*n))*(1 - I*a*c - (1 + I*
a*c)*x^(2/(I*n)))^m)/(1 + x^(2/(I*n)))^(m + 2), x], x, (1 - I*c*(a + b*x))^
((I*n)/2)/(1 + I*c*(a + b*x))^((I*n)/2)], x] /; FreeQ[{a, b, c}, x] && ILtQ
[m, 0] && LtQ[-1, I*n, 1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\frac{1}{2}i\tan^{-1}(a+bx)}}{x^2}\,dx &= -\left((8ib)\,\text{Subst}\left(\int \frac{x^4}{\left(1-ia-(1+ia)x^4\right)^2}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)\right)\\
&= -\frac{(i-a-bx)\sqrt[4]{1-i(a+bx)}}{(i-a)x\sqrt[4]{1+i(a+bx)}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{1-ia+(-1-ia)x^4}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{i-a}\\
&= -\frac{(i-a-bx)\sqrt[4]{1-i(a+bx)}}{(i-a)x\sqrt[4]{1+i(a+bx)}} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{i+a}-\sqrt{i-a}x^2}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{(1+ia)\sqrt{i+a}} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{i+a}+\sqrt{i-a}\ldots}\right.}{(1+ia)\ldots}\\
&= -\frac{(i-a-bx)\sqrt[4]{1-i(a+bx)}}{(i-a)x\sqrt[4]{1+i(a+bx)}} - \frac{ib\tan^{-1}\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{(i-a)^{5/4}(i+a)^{3/4}} - \frac{ib\tanh^{-1}\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{(i-a)^{5/4}(i+a)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0259856, size = 107, normalized size = 0.51

$$
-\frac{\sqrt[4]{-i(a+bx+i)}\left(-2ibx\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)+a^2+abx+ibx+1\right)}{\left(a^2+1\right)x\sqrt[4]{ia+ibx+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((I/2)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -((((-I)*(I + a + b*x))^(1/4)*(1 + a^2 + I*b*x + a*b*x - (2*I)*b*x*Hypergeo
metric2F1[1/4, 1, 5/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]
))/((1 + a^2)*x*(1 + I*a + I*b*x)^(1/4)))
```

Maple [F] time = 0.225, size = 0, normalized size = 0.

$$
\int \frac{1}{x^2}\frac{1}{\sqrt{(1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\frac{ibx+ia+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(1/(x^2*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))), x)
```

Fricas [B] time = 2.69079, size = 1875, normalized size = 8.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] ((-b^4/(16*a^8 - 32*I*a^7 + 32*a^6 - 96*I*a^5 - 96*I*a^3 - 32*a^2 - 32*I*a
- 16))^(1/4)*(-I*a - 1)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
/(b*x + a + I)) + (8*a^6 - 16*I*a^5 + 8*a^4 - 32*I*a^3 - 8*a^2 - 16*I*a - 8
)*(-b^4/(16*a^8 - 32*I*a^7 + 32*a^6 - 96*I*a^5 - 96*I*a^3 - 32*a^2 - 32*I*a
 - 16))^(3/4))/b^3) + (-b^4/(16*a^8 - 32*I*a^7 + 32*a^6 - 96*I*a^5 - 96*I*a
^3 - 32*a^2 - 32*I*a - 16))^(1/4)*(I*a + 1)*x*log((b^3*sqrt(I*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (8*a^6 - 16*I*a^5 + 8*a^4 - 32*I*a^3
- 8*a^2 - 16*I*a - 8)*(-b^4/(16*a^8 - 32*I*a^7 + 32*a^6 - 96*I*a^5 - 96*I*a
^3 - 32*a^2 - 32*I*a - 16))^(3/4))/b^3) - (-b^4/(16*a^8 - 32*I*a^7 + 32*a^6
 - 96*I*a^5 - 96*I*a^3 - 32*a^2 - 32*I*a - 16))^(1/4)*(a - I)*x*log((b^3*sq
rt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (8*I*a^6 + 16*a^5 +
 8*I*a^4 + 32*a^3 - 8*I*a^2 + 16*a - 8*I)*(-b^4/(16*a^8 - 32*I*a^7 + 32*a^6
 - 96*I*a^5 - 96*I*a^3 - 32*a^2 - 32*I*a - 16))^(3/4))/b^3) + (-b^4/(16*a^8
 - 32*I*a^7 + 32*a^6 - 96*I*a^5 - 96*I*a^3 - 32*a^2 - 32*I*a - 16))^(1/4)*(
a - I)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) +
 (-8*I*a^6 - 16*a^5 - 8*I*a^4 - 32*a^3 + 8*I*a^2 - 16*a + 8*I)*(-b^4/(16*a^
8 - 32*I*a^7 + 32*a^6 - 96*I*a^5 - 96*I*a^3 - 32*a^2 - 32*I*a - 16))^(3/4))
/b^3) + I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)/(b*x + a + I)))/((a - I)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\frac{ibx+ia+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="gia
c")
```

```
[Out] integrate(1/(x^2*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))), x)
```

3.231 $\int e^{-\frac{3}{2}i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=494

$$\frac{\left(-24ia^2-36a+17i\right)\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{3/4}}{24b^3}-\frac{\left(-24ia^2-36a+17i\right)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3}+\frac{(-24i}{}$$

```
[Out] ((17*I - 36*a - (24*I)*a^2)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4)
)/(24*b^3) + ((3*I - 8*a)*(1 - I*a - I*b*x)^(7/4)*(1 + I*a + I*b*x)^(1/4))/
(12*b^3) + (x*(1 - I*a - I*b*x)^(7/4)*(1 + I*a + I*b*x)^(1/4))/(3*b^2) + ((
17*I - 36*a - (24*I)*a^2)*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 +
 I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((17*I - 36*a - (24*I)*a^2)*ArcTan[
1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*
b^3) - ((17*I - 36*a - (24*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I
*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(
16*Sqrt[2]*b^3) + ((17*I - 36*a - (24*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]
/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x
)^(1/4)])/(16*Sqrt[2]*b^3)
```

Rubi [A] time = 0.390344, antiderivative size = 494, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5095, 90, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{\left(-24ia^2-36a+17i\right)\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{3/4}}{24b^3}-\frac{\left(-24ia^2-36a+17i\right)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3}+\frac{(-24i}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((17*I - 36*a - (24*I)*a^2)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4)
)/(24*b^3) + ((3*I - 8*a)*(1 - I*a - I*b*x)^(7/4)*(1 + I*a + I*b*x)^(1/4))/
(12*b^3) + (x*(1 - I*a - I*b*x)^(7/4)*(1 + I*a + I*b*x)^(1/4))/(3*b^2) + ((
17*I - 36*a - (24*I)*a^2)*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 +
 I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((17*I - 36*a - (24*I)*a^2)*ArcTan[
1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*
b^3) - ((17*I - 36*a - (24*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I
*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(
16*Sqrt[2]*b^3) + ((17*I - 36*a - (24*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]
/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x
)^(1/4)])/(16*Sqrt[2]*b^3)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
```

```
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{-\frac{3}{2}i\tan^{-1}(a+bx)}x^2\,dx = \int \frac{x^2(1-ia-ibx)^{3/4}}{(1+ia+ibx)^{3/4}}\,dx$$

$$= \frac{x(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{3b^2} + \frac{\int \frac{(1-ia-ibx)^{3/4}\left(-1-a^2+\frac{1}{2}(3i-8a)bx\right)}{(1+ia+ibx)^{3/4}}\,dx}{3b^2}$$

$$= \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3} + \frac{x(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{3b^2} - \frac{\left(17+36ia-24a^2\right)\int}{24b^2}$$

$$= -\frac{\left(36a-i\left(17-24a^2\right)\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3} +$$

$$= -\frac{\left(36a-i\left(17-24a^2\right)\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3} +$$

$$= -\frac{\left(36a-i\left(17-24a^2\right)\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3} +$$

$$= -\frac{\left(36a-i\left(17-24a^2\right)\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3} +$$

$$= -\frac{\left(36a-i\left(17-24a^2\right)\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3} +$$

$$= -\frac{\left(36a-i\left(17-24a^2\right)\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3} +$$

$$= -\frac{\left(36a-i\left(17-24a^2\right)\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3} +$$

Mathematica [C] time = 0.0850889, size = 98, normalized size = 0.2

$$\frac{(-i(a+bx+i))^{7/4}\left(\sqrt[4]{2}\left(24ia^2+36a-17i\right){}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};-\frac{1}{2}i(a+bx+i)\right)+7\sqrt[4]{ia+ibx+1}(-8a+4bx+3i)\right)}{84b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] (((-I)*(I + a + b*x))^(7/4)*(7*(1 + I*a + I*b*x)^(1/4)*(3*I - 8*a + 4*b*x)
+ 2^(1/4)*(-17*I + 36*a + (24*I)*a^2)*Hypergeometric2F1[3/4, 7/4, 11/4, (-I
/2)*(I + a + b*x)]))/(84*b^3)
```

Maple [F] time = 0.245, size = 0, normalized size = 0.

$$\int x^2 \left((1 + i (bx + a)) \frac{1}{\sqrt{1 + (bx + a)^2}}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

```
[Out] int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\frac{i bx + i a + 1}{\sqrt{(bx+a)^2 + 1}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(x^2/((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

Fricas [A] time = 2.73873, size = 1566, normalized size = 3.17

$$3 b^3 \sqrt{\frac{576 i a^4 + 1728 a^3 - 2112 i a^2 - 1224 a + 289 i}{b^6}} \log\left(\frac{i b^3 \sqrt{\frac{576 i a^4 + 1728 a^3 - 2112 i a^2 - 1224 a + 289 i}{b^6}} + \left(24 a^2 - 36 i a - 17\right) \sqrt{\frac{i \sqrt{b^2 x^2 + 2 abx + a^2 + 1}}{bx + a + i}}}{24 a^2 - 36 i a - 17}\right) - 3 b^3 \sqrt{576 i a^4 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/48*(3*b^3*sqrt((576*I*a^4 + 1728*a^3 - 2112*I*a^2 - 1224*a + 289*I)/b^6)*
log((I*b^3*sqrt((576*I*a^4 + 1728*a^3 - 2112*I*a^2 - 1224*a + 289*I)/b^6) +
 (24*a^2 - 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a +
 I)))/(24*a^2 - 36*I*a - 17)) - 3*b^3*sqrt((576*I*a^4 + 1728*a^3 - 2112*I*a
^2 - 1224*a + 289*I)/b^6)*log((-I*b^3*sqrt((576*I*a^4 + 1728*a^3 - 2112*I*a
^2 - 1224*a + 289*I)/b^6) + (24*a^2 - 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)/(b*x + a + I)))/(24*a^2 - 36*I*a - 17)) + 3*b^3*sqrt((-576
```

```
*I*a^4 - 1728*a^3 + 2112*I*a^2 + 1224*a - 289*I)/b^6)*log((I*b^3*sqrt((-576
*I*a^4 - 1728*a^3 + 2112*I*a^2 + 1224*a - 289*I)/b^6) + (24*a^2 - 36*I*a -
17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(24*a^2 - 36*I
*a - 17)) - 3*b^3*sqrt((-576*I*a^4 - 1728*a^3 + 2112*I*a^2 + 1224*a - 289*I
)/b^6)*log((-I*b^3*sqrt((-576*I*a^4 - 1728*a^3 + 2112*I*a^2 + 1224*a - 289*
I)/b^6) + (24*a^2 - 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(
b*x + a + I)))/(24*a^2 - 36*I*a - 17)) - 2*(8*b^3*x^3 + 22*I*b^2*x^2 + 8*a^
3 - (40*I*a + 37)*b*x - 38*I*a^2 + 23*a - 23*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)/(b*x + a + I)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(x^2/((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

3.232 $\int e^{-\frac{3}{2}i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=410

$$\frac{\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{2b^2}+\frac{(3+4ia)\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{3/4}}{4b^2}-\frac{3(3+4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right.}{8\sqrt{2}b^2}$$

```
[Out] ((3 + (4*I)*a)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/(4*b^2) + (
(1 - I*a - I*b*x)^(7/4)*(1 + I*a + I*b*x)^(1/4))/(2*b^2) + (3*(3 + (4*I)*a)
*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(4*
Sqrt[2]*b^2) - (3*(3 + (4*I)*a)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4)
)/(1 + I*a + I*b*x)^(1/4)])/(4*Sqrt[2]*b^2) - (3*(3 + (4*I)*a)*Log[1 + Sqrt
[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))
/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2) + (3*(3 + (4*I)*a)*Log[1 + Sqrt[
1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/
(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2)
```

Rubi [A] time = 0.29704, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {5095, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{2b^2}+\frac{(3+4ia)\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{3/4}}{4b^2}-\frac{3(3+4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right.}{8\sqrt{2}b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((3 + (4*I)*a)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/(4*b^2) + (
(1 - I*a - I*b*x)^(7/4)*(1 + I*a + I*b*x)^(1/4))/(2*b^2) + (3*(3 + (4*I)*a)
*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(4*
Sqrt[2]*b^2) - (3*(3 + (4*I)*a)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4)
)/(1 + I*a + I*b*x)^(1/4)])/(4*Sqrt[2]*b^2) - (3*(3 + (4*I)*a)*Log[1 + Sqrt
[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))
/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2) + (3*(3 + (4*I)*a)*Log[1 + Sqrt[
1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/
(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2)
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{-\frac{3}{2}i\tan^{-1}(a+bx)}x\,dx = \int \frac{x(1-ia-ibx)^{3/4}}{(1+ia+ibx)^{3/4}}\,dx$$

$$= \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} + \frac{(3i-4a)\int \frac{(1-ia-ibx)^{3/4}}{(1+ia+ibx)^{3/4}}\,dx}{4b}$$

$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} + \frac{(3(3i-4a))\int \frac{}{\sqrt[4]{1-ia-}}}{8b}$$

$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} - \frac{(3(3+4ia))\,\mathrm{Subst}\Big(}{}$$

$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} - \frac{(3(3+4ia))\,\mathrm{Subst}\Big(}{}$$

$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} + \frac{(3(3+4ia))\,\mathrm{Subst}\Big(}{}$$

$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} - \frac{(3(3+4ia))\,\mathrm{Subst}\Big(}{}$$

$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} - \frac{3(3+4ia)\log\Big(1+}{8}$$

$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} + \frac{3(3+4ia)\tan^{-1}\Big(1}{4\sqrt{2}b^2}$$

Mathematica [C] time = 0.038326, size = 84, normalized size = 0.2

$$-\frac{i(-i(a+bx+i))^{7/4}\left(\sqrt[4]{2}(4a-3i)\,{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};-\frac{1}{2}i(a+bx+i)\right)+7i\sqrt[4]{ia+ibx+1}\right)}{14b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((-I/14)*((-I)*(I + a + b*x))^(7/4)*((7*I)*(1 + I*a + I*b*x)^(1/4) + 2^(1/4
)*(-3*I + 4*a)*Hypergeometric2F1[3/4, 7/4, 11/4, (-I/2)*(I + a + b*x)]))/b^
2
```

Maple [F] time = 0.233, size = 0, normalized size = 0.

$$\int x\left((1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

```
[Out] int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

Fricas [A] time = 2.64128, size = 1115, normalized size = 2.72

$$-\frac{b^2\sqrt{\frac{144i\,a^2+216\,a-81i}{b^4}}\log\left(\frac{i\,b^2\sqrt{\frac{144i\,a^2+216\,a-81i}{b^4}}+(12\,a-9i)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{12\,a-9i}\right)-b^2\sqrt{\frac{144i\,a^2+216\,a-81i}{b^4}}\log\left(\frac{-i\,b^2\sqrt{\frac{144i\,a^2+216\,a-81i}{b^4}}+(12\,a-9i)\sqrt{\cdots}}{12\,a-9i}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/8*(b^2*sqrt((144*I*a^2 + 216*a - 81*I)/b^4)*log((I*b^2*sqrt((144*I*a^2 +
 216*a - 81*I)/b^4) + (12*a - 9*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
/(b*x + a + I)))/(12*a - 9*I)) - b^2*sqrt((144*I*a^2 + 216*a - 81*I)/b^4)*l
og((-I*b^2*sqrt((144*I*a^2 + 216*a - 81*I)/b^4) + (12*a - 9*I)*sqrt(I*sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(12*a - 9*I)) + b^2*sqrt((-144
*I*a^2 - 216*a + 81*I)/b^4)*log((I*b^2*sqrt((-144*I*a^2 - 216*a + 81*I)/b^4
) + (12*a - 9*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(
12*a - 9*I)) - b^2*sqrt((-144*I*a^2 - 216*a + 81*I)/b^4)*log((-I*b^2*sqrt((
-144*I*a^2 - 216*a + 81*I)/b^4) + (12*a - 9*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)/(b*x + a + I)))/(12*a - 9*I)) + 2*(2*b^2*x^2 - 2*a^2 + 7*I*b*x
 + 3*I*a - 5)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

3.233 $\int e^{-\frac{3}{2}i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=338

$$-\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}+\frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}-\frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}-\frac{3i\,\mathrm{ta}\cdots}{\cdots}$$

```
[Out] ((-I)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/b - ((3*I)*ArcTan[1
- (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) +
 ((3*I)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4
)])/(Sqrt[2]*b) + (((3*I)/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I
*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2
]*b) - (((3*I)/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sq
rt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b)
```

Rubi [A] time = 0.19681, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5093, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}+\frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}-\frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}-\frac{3i\,\mathrm{ta}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((-3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((-I)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/b - ((3*I)*ArcTan[1
- (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) +
 ((3*I)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4
)])/(Sqrt[2]*b) + (((3*I)/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I
*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2
]*b) - (((3*I)/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sq
rt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b)
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^{-\frac{3}{2}i\tan^{-1}(a+bx)}\,dx = \int \frac{(1-ia-ibx)^{3/4}}{(1+ia+ibx)^{3/4}}\,dx$$
$$= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{3}{2}\int \frac{1}{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}\,dx$$
$$= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{(6i)\,\mathrm{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right)}{b}$$
$$= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{(6i)\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b}$$
$$= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} - \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \right.}{b}$$
$$= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+\right.}{2b}$$
$$= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} - \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} - \frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} + \frac{\sqrt{2}}{\right.}{2\sqrt{2}b}$$
$$= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} - \frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{3i\log}{}$$

Mathematica [C] time = 0.0165455, size = 43, normalized size = 0.13

$$-\frac{8ie^{\frac{1}{2}i\tan^{-1}(a+bx)}\,{}_2F_1\left(\frac{1}{4},2;\frac{5}{4};-e^{2i\tan^{-1}(a+bx)}\right)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((-3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((-8*I)*E^((I/2)*ArcTan[a + b*x])*Hypergeometric2F1[1/4, 2, 5/4, -E^((2*I)*
ArcTan[a + b*x])])/b
```

Maple [F] time = 0.211, size = 0, normalized size = 0.

$$\int \left((1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(-3/2), x)
```

Fricas [A] time = 2.44927, size = 679, normalized size = 2.01

$$\frac{b\sqrt{\frac{9i}{b^2}}\log\left(\frac{1}{3}i\,b\sqrt{\frac{9i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{\frac{9i}{b^2}}\log\left(-\frac{1}{3}i\,b\sqrt{\frac{9i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)+b\sqrt{-\frac{9i}{b^2}}\log\left(\frac{1}{3}i\,b\sqrt{-\frac{9i}{b^2}}+\sqrt{\frac{i}{}}\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(b*sqrt(9*I/b^2)*log(1/3*I*b*sqrt(9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(9*I/b^2)*log(-1/3*I*b*sqrt(9*I/b^2)
 + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + b*sqrt(-9*I/b
^2)*log(1/3*I*b*sqrt(-9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(
b*x + a + I))) - b*sqrt(-9*I/b^2)*log(-1/3*I*b*sqrt(-9*I/b^2) + sqrt(I*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - (2*b*x + 2*a + 2*I)*sqrt(I*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(-3/2), x)
```

3.234 $\int \frac{e^{-\frac{3}{2}i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=427

$$\frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}-\frac{2(a+i)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(-a+i)^{3/4}}-\sqrt{2}\,\mathrm{t}$$

```
[Out] (-2*(I + a)^(3/4)*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(
1/4)*(1 - I*a - I*b*x)^(1/4))])/(I - a)^(3/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sq
rt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)] - (2*(I + a)^(3/4)*
ArcTanh[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I
*b*x)^(1/4))])/(I - a)^(3/4) + Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a +
 I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2
] - Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a
 - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.23995, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {5095, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 208, 205}

$$\frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}-\frac{2(a+i)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(-a+i)^{3/4}}-\sqrt{2}\,\mathrm{t}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a + b*x])*x),x]
```


```
[Out] (-2*(I + a)^(3/4)*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(
1/4)*(1 - I*a - I*b*x)^(1/4))])/(I - a)^(3/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sq
rt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)] - (2*(I + a)^(3/4)*
ArcTanh[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I
*b*x)^(1/4))])/(I - a)^(3/4) + Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a +
 I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2
] - Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a
 - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]
```


Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\frac{3}{2}i\tan^{-1}(a+bx)}}{x}\,dx &= \int \frac{(1-ia-ibx)^{3/4}}{x(1+ia+ibx)^{3/4}}\,dx\\
&= -\left((-1+ia)\int \frac{1}{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}\,dx\right)-(ib)\int \frac{1}{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}\,dx\\
&= 4\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx,x,\sqrt[4]{1-ia-ibx}\right)+(4(1-ia))\,\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^4}\,dx,x\right.\\
&= 4\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)-\frac{(2(i+a))\,\text{Subst}\left(\int \frac{1}{\sqrt{i-a}-\sqrt{i+a}x^2}\,dx,x,\frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{\sqrt{i-a}}-\\
&= -\frac{2(i+a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}}-\frac{2(i+a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}}-2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,d\right.\\
&= -\frac{2(i+a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}}-\frac{2(i+a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}}+\frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\right.}{\sqrt{2}}\\
&= -\frac{2(i+a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}}-\frac{2(i+a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}}+\frac{\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\right.}{\sqrt{2}}\\
&= -\frac{2(i+a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}}-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)+\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1+}}\right.
\end{aligned}
$$

Mathematica [C] time = 0.0361035, size = 128, normalized size = 0.3

$$
\frac{2(-i(a+bx+i))^{3/4}\left(\sqrt[4]{2}(ia+ibx+1)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{1}{2}i(a+bx+i)\right)-2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)\right)}{3(ia+ibx+1)^{3/4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a + b*x])*x),x]
```

```
[Out] (2*((-I)*(I + a + b*x))^(3/4)*(2^(1/4)*(1 + I*a + I*b*x)^(3/4)*Hypergeometr
ic2F1[3/4, 3/4, 7/4, (-I/2)*(I + a + b*x)] - 2*Hypergeometric2F1[3/4, 1, 7/
4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]))/(3*(1 + I*a + I*b
*x)^(3/4))
```

Maple [F] time = 0.234, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left((1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

```
[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="maxim
a")
```

```
[Out] integrate(1/(x*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)), x)
```

Fricas [B] time = 2.85048, size = 1673, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="frica
s")
```

```
[Out] -1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
/(b*x + a + I))) + 1/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 1/2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + s
qrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 1/2*sqrt(-4*I)*lo
g(-1/2*sqrt(-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))
) + (-(a^3 + 3*I*a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4)*log(((a +
I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (a - I)*(-(a^3
 + 3*I*a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4))/(a + I)) - (-(a^3 +
 3*I*a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4)*log(((a + I)*sqrt(I*sq
```

```
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (a - I)*(-(a^3 + 3*I*a^2 -
 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4))/(a + I)) + I*(-(a^3 + 3*I*a^2 -
 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4)*log(((a + I)*sqrt(I*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (I*a + 1)*(-(a^3 + 3*I*a^2 - 3*a - I
)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4))/(a + I)) - I*(-(a^3 + 3*I*a^2 - 3*a - I
)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4)*log(((a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)/(b*x + a + I)) + (-I*a - 1)*(-(a^3 + 3*I*a^2 - 3*a - I)/(a^3
- 3*I*a^2 - 3*a + I))^(1/4))/(a + I))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="giac"
)
```

```
[Out] integrate(1/(x*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)), x)
```

3.235 $\int \frac{e^{-\frac{3}{2}i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=211

$$-\frac{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{(1+ia)x}-\frac{3ib\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(-a+i)^{7/4}\sqrt[4]{a+i}}-\frac{3ib\tanh^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(-a+i)^{7/4}\sqrt[4]{a+i}}$$

```
[Out] -(((1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/((1 + I*a)*x)) - ((3*I)
*b*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a -
 I*b*x)^(1/4))])/((I - a)^(7/4)*(I + a)^(1/4)) - ((3*I)*b*ArcTanh[((I + a)^
(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I*b*x)^(1/4))])/((
I - a)^(7/4)*(I + a)^(1/4))
```

Rubi [A] time = 0.118121, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5095, 94, 93, 212, 208, 205}

$$-\frac{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{(1+ia)x}-\frac{3ib\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(-a+i)^{7/4}\sqrt[4]{a+i}}-\frac{3ib\tanh^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(-a+i)^{7/4}\sqrt[4]{a+i}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -(((1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/((1 + I*a)*x)) - ((3*I)
*b*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a -
 I*b*x)^(1/4))])/((I - a)^(7/4)*(I + a)^(1/4)) - ((3*I)*b*ArcTanh[((I + a)^
(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I*b*x)^(1/4))])/((
I - a)^(7/4)*(I + a)^(1/4))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\frac{3}{2}i\tan^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{(1-ia-ibx)^{3/4}}{x^2(1+ia+ibx)^{3/4}}\,dx\\
&= -\frac{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{(1+ia)x} + \frac{(3b)\int \frac{1}{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}\,dx}{2(i-a)}\\
&= -\frac{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{(1+ia)x} + \frac{(6b)\,\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^4}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{i-a}\\
&= -\frac{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{(1+ia)x} - \frac{(3ib)\,\text{Subst}\left(\int \frac{1}{\sqrt{i-a}-\sqrt{i+a}x^2}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/2}} - \frac{(3ib)\,\text{Subst}\Big(\int}{}\\
&= -\frac{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{(1+ia)x} - \frac{3ib\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{7/4}\sqrt[4]{i+a}} - \frac{3ib\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{7/4}\sqrt[4]{i+a}}
\end{aligned}$$

Mathematica [C] time = 0.0258798, size = 107, normalized size = 0.51

$$-\frac{(-i(a+bx+i))^{3/4}\left(-2ibx\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)+a^2+abx+ibx+1\right)}{\left(a^2+1\right)x(ia+ibx+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -((((-I)*(I + a + b*x))^(3/4)*(1 + a^2 + I*b*x + a*b*x - (2*I)*b*x*Hypergeo
metric2F1[3/4, 1, 7/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]
))/((1 + a^2)*x*(1 + I*a + I*b*x)^(3/4)))
```

Maple [F] time = 0.232, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left((1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\frac{ibx+ia+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(1/(x^2*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)), x)
```

Fricas [B] time = 2.62222, size = 1667, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] (3*(-b^4/(16*a^8 - 96*I*a^7 - 224*a^6 + 224*I*a^5 + 224*I*a^3 + 224*a^2 - 9
6*I*a - 16))^(1/4)*(-I*a - 1)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)/(b*x + a + I)) + 2*(-b^4/(16*a^8 - 96*I*a^7 - 224*a^6 + 224*I*a^5 + 22
4*I*a^3 + 224*a^2 - 96*I*a - 16))^(1/4)*(a^2 - 2*I*a - 1))/b) + 3*(-b^4/(16
*a^8 - 96*I*a^7 - 224*a^6 + 224*I*a^5 + 224*I*a^3 + 224*a^2 - 96*I*a - 16))
^(1/4)*(I*a + 1)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a
 + I)) - 2*(-b^4/(16*a^8 - 96*I*a^7 - 224*a^6 + 224*I*a^5 + 224*I*a^3 + 224
*a^2 - 96*I*a - 16))^(1/4)*(a^2 - 2*I*a - 1))/b) + 3*(-b^4/(16*a^8 - 96*I*a
^7 - 224*a^6 + 224*I*a^5 + 224*I*a^3 + 224*a^2 - 96*I*a - 16))^(1/4)*(a - I
)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (-b^4/
(16*a^8 - 96*I*a^7 - 224*a^6 + 224*I*a^5 + 224*I*a^3 + 224*a^2 - 96*I*a - 1
6))^(1/4)*(2*I*a^2 + 4*a - 2*I))/b) - 3*(-b^4/(16*a^8 - 96*I*a^7 - 224*a^6
+ 224*I*a^5 + 224*I*a^3 + 224*a^2 - 96*I*a - 16))^(1/4)*(a - I)*x*log((b*sq
rt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (-b^4/(16*a^8 - 96*
I*a^7 - 224*a^6 + 224*I*a^5 + 224*I*a^3 + 224*a^2 - 96*I*a - 16))^(1/4)*(-2
*I*a^2 - 4*a + 2*I))/b) + (b*x + a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)/(b*x + a + I)))/((a - I)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="gia
c")
```

```
[Out] integrate(1/(x^2*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)), x)
```

3.236 $\int e^{n\tan^{-1}(a+bx)}x^m\,dx$

Optimal. Leaf size=140

$$\frac{x^{m+1}(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\left(1-\frac{bx}{-a+i}\right)^{\frac{in}{2}}\left(1+\frac{bx}{a+i}\right)^{-\frac{in}{2}}F_1\left(m+1;-\frac{in}{2},\frac{in}{2};m+2;-\frac{bx}{a+i},\frac{bx}{i-a}\right)}{m+1}$$

```
[Out] (x^(1 + m)*(1 - I*a - I*b*x)^((I/2)*n)*(1 - (b*x)/(I - a))^((I/2)*n)*Appell
F1[1 + m, (-I/2)*n, (I/2)*n, 2 + m, -((b*x)/(I + a)), (b*x)/(I - a)])/((1 +
 m)*(1 + I*a + I*b*x)^((I/2)*n)*(1 + (b*x)/(I + a))^((I/2)*n))
```

Rubi [A] time = 0.0744353, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5095, 135, 133}

$$\frac{x^{m+1}(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\left(1-\frac{bx}{-a+i}\right)^{\frac{in}{2}}\left(1+\frac{bx}{a+i}\right)^{-\frac{in}{2}}F_1\left(m+1;-\frac{in}{2},\frac{in}{2};m+2;-\frac{bx}{a+i},\frac{bx}{i-a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a + b*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(1 - I*a - I*b*x)^((I/2)*n)*(1 - (b*x)/(I - a))^((I/2)*n)*Appell
F1[1 + m, (-I/2)*n, (I/2)*n, 2 + m, -((b*x)/(I + a)), (b*x)/(I - a)])/((1 +
 m)*(1 + I*a + I*b*x)^((I/2)*n)*(1 + (b*x)/(I + a))^((I/2)*n))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int e^{n\tan^{-1}(a+bx)}x^m\,dx = \int x^m(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx$$
$$= \left((1-ia-ibx)^{\frac{in}{2}}\left(1-\frac{ibx}{1-ia}\right)^{-\frac{in}{2}}\right)\int x^m(1+ia+ibx)^{-\frac{in}{2}}\left(1-\frac{ibx}{1-ia}\right)^{\frac{in}{2}}\,dx$$
$$= \left((1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\left(1-\frac{ibx}{1-ia}\right)^{-\frac{in}{2}}\left(1+\frac{ibx}{1+ia}\right)^{\frac{in}{2}}\right)\int x^m\left(1-\frac{ibx}{1-ia}\right)^{\frac{in}{2}}\left(1+\frac{ibx}{1+i}\right.$$
$$= \frac{x^{1+m}(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\left(1-\frac{bx}{i-a}\right)^{\frac{in}{2}}\left(1+\frac{bx}{i+a}\right)^{-\frac{in}{2}}F_1\left(1+m;-\frac{in}{2},\frac{in}{2};2+m;-\frac{bx}{i+a},\frac{bx}{i-a}\right.}{1+m}$$

Mathematica [F] time = 0.807305, size = 0, normalized size = 0.

$$\int e^{n\tan^{-1}(a+bx)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTan[a + b*x])*x^m,x]

[Out] Integrate[E^(n*ArcTan[a + b*x])*x^m, x]
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int e^{n\arctan(bx+a)}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(b*x+a))*x^m,x)

[Out] int(exp(n*arctan(b*x+a))*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m e^{(n\arctan(bx+a))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^m,x, algorithm="maxima")

[Out] integrate(x^m*e^(n*arctan(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m e^{(n\arctan(bx+a))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*e^(n*arctan(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(b*x+a))*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m e^{(n \arctan(bx+a))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*e^(n*arctan(b*x + a)), x)
```

3.237 $\int e^{n \tan^{-1}(a+bx)} x^3 \, dx$

Optimal. Leaf size=260

$$\frac{2^{-2-\frac{in}{2}}\left(36a^2n+24a^3-12a\left(2-n^2\right)-n\left(8-n^2\right)\right)(-ia-ibx+1)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(-ia-ibx+1)\right)}{3b^4(-n+2i)}-\frac{(-ia-\ldots}{}$$

```
[Out] (x^2*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n))/(4*b^
2) - ((1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n)*(6 -
18*a^2 - 10*a*n - n^2 + 2*b*(6*a + n)*x))/(24*b^4) + (2^(-2 - (I/2)*n)*(24*
a^3 + 36*a^2*n - 12*a*(2 - n^2) - n*(8 - n^2))*(1 - I*a - I*b*x)^(1 + (I/2)
*n)*Hypergeometric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/
2])/(3*b^4*(2*I - n))
```

Rubi [A] time = 0.185642, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5095, 100, 147, 69}

$$\frac{2^{-2-\frac{in}{2}}\left(36a^2n+24a^3-12a\left(2-n^2\right)-n\left(8-n^2\right)\right)(-ia-ibx+1)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(-ia-ibx+1)\right)}{3b^4(-n+2i)}-\frac{(-ia-\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a + b*x])*x^3,x]
```

```
[Out] (x^2*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n))/(4*b^
2) - ((1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n)*(6 -
18*a^2 - 10*a*n - n^2 + 2*b*(6*a + n)*x))/(24*b^4) + (2^(-2 - (I/2)*n)*(24*
a^3 + 36*a^2*n - 12*a*(2 - n^2) - n*(8 - n^2))*(1 - I*a - I*b*x)^(1 + (I/2)
*n)*Hypergeometric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/
2])/(3*b^4*(2*I - n))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
```

```
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tan^{-1}(a+bx)}x^3\,dx = \int x^3(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx$$

$$= \frac{x^2(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{4b^2} + \frac{\int x(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\left(-2\left(1+a^2\right)-b(6a+n\right.}{4b^2}$$

$$= \frac{x^2(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{4b^2} - \frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}\left(6-18a^2-10an-n^2\right.}{24b^4}$$

$$= \frac{x^2(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{4b^2} - \frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}\left(6-18a^2-10an-n^2\right.}{24b^4}$$

Mathematica [A] time = 0.300176, size = 272, normalized size = 1.05

$$\frac{(-i(a+bx+i))^{1+\frac{in}{2}}\left(b^2(-n+2i)x^2(ia+ibx+1)^{1-\frac{in}{2}}-2^{3-\frac{in}{2}}(6a+n)\,{}_2F_1\left(\frac{in}{2}-2,\frac{in}{2}+1;\frac{in}{2}+2;-\frac{1}{2}i(a+bx+i)\right)+(1+ia)\right.}{4b^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTan[a + b*x])*x^3,x]
```

```
[Out] (((-I)*(I + a + b*x))^(1 + (I/2)*n)*(b^2*(2*I - n)*x^2*(1 + I*a + I*b*x)^(1
 - (I/2)*n) - 2^(3 - (I/2)*n)*(6*a + n)*Hypergeometric2F1[-2 + (I/2)*n, 1 +
 (I/2)*n, 2 + (I/2)*n, (-I/2)*(I + a + b*x)] + 2^(3 - (I/2)*n)*(1 + I*a)*(-
I + 5*a + n)*Hypergeometric2F1[-1 + (I/2)*n, 1 + (I/2)*n, 2 + (I/2)*n, (-I/
2)*(I + a + b*x)] + 2^(1 - (I/2)*n)*(-I + a)^2*(-2*I + 4*a + n)*Hypergeomet
ric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (-I/2)*(I + a + b*x)]))/(4*b^4*(2
*I - n))
```

Maple [F] time = 0.197, size = 0, normalized size = 0.

$$\int e^{n\arctan(bx+a)}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(b*x+a))*x^3,x)
```

```
[Out] int(exp(n*arctan(b*x+a))*x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 e^{(n \arctan(bx+a))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*e^(n*arctan(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3 e^{(n \arctan(bx+a))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*e^(n*arctan(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 e^{n \operatorname{atan}(a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(b*x+a))*x**3,x)
```

```
[Out] Integral(x**3*exp(n*atan(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 e^{(n \arctan(bx+a))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*e^(n*arctan(b*x + a)), x)
```

3.238 $\int e^{n\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=220

$$\frac{2^{-\frac{in}{2}}\left(-6a^2-6an-n^2+2\right)(-ia-ibx+1)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(-ia-ibx+1)\right)}{3b^3(-n+2i)}-\frac{(4a+n)(-ia-ibx+1)^{1+\frac{in}{2}}(ia+}{6b^3}$$

```
[Out] -((4*a + n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n)
)/(6*b^3) + (x*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)
*n))/(3*b^2) + ((2 - 6*a^2 - 6*a*n - n^2)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*H
ypergeometric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/2])/(
3*2^((I/2)*n)*b^3*(2*I - n))
```

Rubi [A] time = 0.139089, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5095, 90, 80, 69}

$$\frac{2^{-\frac{in}{2}}\left(-6a^2-6an-n^2+2\right)(-ia-ibx+1)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(-ia-ibx+1)\right)}{3b^3(-n+2i)}-\frac{(4a+n)(-ia-ibx+1)^{1+\frac{in}{2}}(ia+}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a + b*x])*x^2,x]
```

```
[Out] -((4*a + n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n)
)/(6*b^3) + (x*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)
*n))/(3*b^2) + ((2 - 6*a^2 - 6*a*n - n^2)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*H
ypergeometric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/2])/(
3*2^((I/2)*n)*b^3*(2*I - n))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tan^{-1}(a+bx)}x^2\,dx = \int x^2(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx$$

$$= \frac{x(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{3b^2} + \frac{\int (1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\left(-1-a^2-b(4a+n)x\right.}{3b^2}$$

$$= -\frac{(4a+n)(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{6b^3} + \frac{x(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{3b^2} - \frac{\left(2-6a^2\right.}{}$$

$$= -\frac{(4a+n)(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{6b^3} + \frac{x(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{3b^2} + \frac{2^{-\frac{in}{2}}\left(2-\right.}{}$$

Mathematica [A] time = 0.135236, size = 160, normalized size = 0.73

$$\frac{(-i(a+bx+i))^{1+\frac{in}{2}}\left(\frac{2^{1-\frac{in}{2}}\left(6a^2+6an+n^2-2\right){}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;-\frac{1}{2}i(a+bx+i)\right)}{n-2i}-(4a+n)(ia+ibx+1)^{1-\frac{in}{2}}+2bx(ia+ibx+1)^{1-\frac{in}{2}}\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (((-I)*(I + a + b*x))^(1 + (I/2)*n)*(-((4*a + n)*(1 + I*a + I*b*x)^(1 - (I/
2)*n)) + 2*b*x*(1 + I*a + I*b*x)^(1 - (I/2)*n) + (2^(1 - (I/2)*n)*(-2 + 6*a
^2 + 6*a*n + n^2)*Hypergeometric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (-I/
2)*(I + a + b*x)])/(-2*I + n)))/(6*b^3)
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int e^{n\arctan(bx+a)}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(b*x+a))*x^2,x)
```

```
[Out] int(exp(n*arctan(b*x+a))*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 e^{(n\arctan(bx+a))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*e^(n*arctan(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2e^{(n\arctan(bx+a))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*e^(n*arctan(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2e^{n\operatorname{atan}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(b*x+a))*x**2,x)
```

```
[Out] Integral(x**2*exp(n*atan(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2e^{(n\arctan(bx+a))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*e^(n*arctan(b*x + a)), x)
```

3.239 $\int e^{n\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=147

$$\frac{2^{-\frac{in}{2}}(2a+n)(-ia-ibx+1)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(-ia-ibx+1)\right)}{b^2(-n+2i)}+\frac{(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{2b^2}$$

```
[Out] ((1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n))/(2*b^2) +
 ((2*a + n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*Hypergeometric2F1[1 + (I/2)*n,
(I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/2])/(2^((I/2)*n)*b^2*(2*I - n))
```

Rubi [A] time = 0.0672028, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5095, 80, 69}

$$\frac{2^{-\frac{in}{2}}(2a+n)(-ia-ibx+1)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(-ia-ibx+1)\right)}{b^2(-n+2i)}+\frac{(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a + b*x])*x,x]
```

```
[Out] ((1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n))/(2*b^2) +
 ((2*a + n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*Hypergeometric2F1[1 + (I/2)*n,
(I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/2])/(2^((I/2)*n)*b^2*(2*I - n))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tan^{-1}(a+bx)}x\,dx = \int x(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx$$
$$= \frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{2b^2} - \frac{(2a+n)\int(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx}{2b}$$
$$= \frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{2b^2} + \frac{2^{-\frac{in}{2}}(2a+n)(1-ia-ibx)^{1+\frac{in}{2}}\,{}_2F_1\left(1+\frac{in}{2},\frac{in}{2};2+\frac{in}{2};\frac{1}{2}(1-ia-\right.}{b^2(2i-n)}$$

Mathematica [A] time = 0.114278, size = 128, normalized size = 0.87

$$\frac{i(-i(a+bx+i))^{1+\frac{in}{2}}\left(\frac{2^{1-\frac{in}{2}}(2a+n)\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;-\frac{1}{2}i(a+bx+i)\right)}{-2-in}+(a+bx-i)(ia+ibx+1)^{-\frac{in}{2}}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a + b*x])*x,x]
```

```
[Out] ((I/2)*((-I)*(I + a + b*x))^(1 + (I/2)*n)*((-I + a + b*x)/(1 + I*a + I*b*x)
^((I/2)*n) + (2^(1 - (I/2)*n)*(2*a + n)*Hypergeometric2F1[1 + (I/2)*n, (I/2
)*n, 2 + (I/2)*n, (-I/2)*(I + a + b*x)])/(-2 - I*n)))/b^2
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int e^{n\arctan(bx+a)}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(b*x+a))*x,x)
```

```
[Out] int(exp(n*arctan(b*x+a))*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{(n\arctan(bx+a))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x,x, algorithm="maxima")
```

```
[Out] integrate(x*e^(n*arctan(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(xe^{(n\arctan(bx+a))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x,x, algorithm="fricas")
```

```
[Out] integral(x*e^(n*arctan(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{n\operatorname{atan}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(b*x+a))*x,x)
```

```
[Out] Integral(x*exp(n*atan(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{(n\arctan(bx+a))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x,x, algorithm="giac")
```

```
[Out] integrate(x*e^(n*arctan(b*x + a)), x)
```

3.240 $\int e^{n\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=91

$$-\frac{2^{1-\frac{in}{2}}(-ia-ibx+1)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(-ia-ibx+1)\right)}{b(-n+2i)}$$

```
[Out] -((2^(1 - (I/2)*n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*Hypergeometric2F1[1 + (I
/2)*n, (I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/2])/(b*(2*I - n)))
```

Rubi [A] time = 0.013713, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5093, 69}

$$-\frac{2^{1-\frac{in}{2}}(-ia-ibx+1)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(-ia-ibx+1)\right)}{b(-n+2i)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a + b*x]),x]
```

```
[Out] -((2^(1 - (I/2)*n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*Hypergeometric2F1[1 + (I
/2)*n, (I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/2])/(b*(2*I - n)))
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{n\tan^{-1}(a+bx)}\,dx &= \int (1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx\\ &= -\frac{2^{1-\frac{in}{2}}(1-ia-ibx)^{1+\frac{in}{2}}\,{}_2F_1\left(1+\frac{in}{2},\frac{in}{2};2+\frac{in}{2};\frac{1}{2}(1-ia-ibx)\right)}{b(2i-n)}\end{aligned}$$

Mathematica [A] time = 0.0283222, size = 60, normalized size = 0.66

$$\frac{4e^{(n+2i)\tan^{-1}(a+bx)}\,{}_2F_1\left(2,1-\frac{in}{2};2-\frac{in}{2};-e^{2i\tan^{-1}(a+bx)}\right)}{b(n+2i)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTan[a + b*x]),x]
```

```
[Out] (4*E^((2*I + n)*ArcTan[a + b*x])*Hypergeometric2F1[2, 1 - (I/2)*n, 2 - (I/2
)*n, -E^((2*I)*ArcTan[a + b*x])])/(b*(2*I + n))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int e^{n \arctan(bx+a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(b*x+a)),x)
```

```
[Out] int(exp(n*arctan(b*x+a)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(n \arctan(bx+a))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(e^{(n \arctan(bx+a))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a)),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n \operatorname{atan}(a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(b*x+a)),x)
```

```
[Out] Integral(exp(n*atan(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(n\arctan(bx+a))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(b*x + a)), x)
```

3.241 $\int \frac{e^{n\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=191

$$\frac{2i(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\,{}_2F_1\left(1,\frac{in}{2};\frac{in}{2}+1;\frac{(i-a)(-ia-ibx+1)}{(a+i)(ia+ibx+1)}\right)}{n}-\frac{i2^{1-\frac{in}{2}}(-ia-ibx+1)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;\frac{1}{2}(-ia-\right.}{n}$$

```
[Out] ((2*I)*(1 - I*a - I*b*x)^((I/2)*n)*Hypergeometric2F1[1, (I/2)*n, 1 + (I/2)*
n, ((I - a)*(1 - I*a - I*b*x))/((I + a)*(1 + I*a + I*b*x))])/(n*(1 + I*a +
I*b*x)^((I/2)*n)) - (I*2^(1 - (I/2)*n)*(1 - I*a - I*b*x)^((I/2)*n)*Hypergeo
metric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I*a - I*b*x)/2])/n
```

Rubi [A] time = 0.0742903, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5095, 105, 69, 131}

$$\frac{2i(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\,{}_2F_1\left(1,\frac{in}{2};\frac{in}{2}+1;\frac{(i-a)(-ia-ibx+1)}{(a+i)(ia+ibx+1)}\right)}{n}-\frac{i2^{1-\frac{in}{2}}(-ia-ibx+1)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;\frac{1}{2}(-ia-\right.}{n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a + b*x])/x,x]
```

```
[Out] ((2*I)*(1 - I*a - I*b*x)^((I/2)*n)*Hypergeometric2F1[1, (I/2)*n, 1 + (I/2)*
n, ((I - a)*(1 - I*a - I*b*x))/((I + a)*(1 + I*a + I*b*x))])/(n*(1 + I*a +
I*b*x)^((I/2)*n)) - (I*2^(1 - (I/2)*n)*(1 - I*a - I*b*x)^((I/2)*n)*Hypergeo
metric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I*a - I*b*x)/2])/n
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
```

```
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(a+bx)}}{x}\,dx = \int \frac{(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}}{x}\,dx$$
$$= -\left((-1+ia)\int \frac{(1-ia-ibx)^{-1+\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}}{x}\,dx\right) - (ib)\int (1-ia-ibx)^{-1+\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,d$$
$$= \frac{2i(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,{}_2F_1\left(1,\frac{in}{2};1+\frac{in}{2};\frac{(i-a)(1-ia-ibx)}{(i+a)(1+ia+ibx)}\right)}{n} - \frac{i2^{1-\frac{in}{2}}(1-ia-ibx)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\right.}{n}$$

Mathematica [A] time = 0.0344468, size = 170, normalized size = 0.89

$$\frac{2i(ia+ibx+1)^{-\frac{in}{2}}(-i(a+bx+i))^{\frac{in}{2}}\left({}_2F_1\left(1,\frac{in}{2};\frac{in}{2}+1;\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)-2^{-\frac{in}{2}}(ia+ibx+1)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;-\frac{1}{2}i(a+bx+\right.\right.}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a + b*x])/x,x]
```

```
[Out] ((2*I)*((-I)*(I + a + b*x))^((I/2)*n)*(Hypergeometric2F1[1, (I/2)*n, 1 + (I
/2)*n, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)] - ((1 + I*a + I
*b*x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (-I/2)*(I
+ a + b*x)])/2^((I/2)*n)))/(n*(1 + I*a + I*b*x)^((I/2)*n))
```

Maple [F] time = 0.198, size = 0, normalized size = 0.

$$\int \frac{e^{n\arctan(bx+a)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(b*x+a))/x,x)
```

```
[Out] int(exp(n*arctan(b*x+a))/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(bx+a))}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(b*x + a))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(n\arctan(bx+a))}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(b*x + a))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{atan}{(a+bx)}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(b*x+a))/x,x)
```

```
[Out] Integral(exp(n*atan(a + b*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(bx+a))}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(b*x + a))/x, x)
```

3.242 $\int \frac{e^{n\tan^{-1}(a+bx)}}{x^2} dx$

Optimal. Leaf size=128

$$-\frac{4b(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{-1-\frac{in}{2}}\,{}_2F_1\left(2,\frac{in}{2}+1;\frac{in}{2}+2;\frac{(i-a)(-ia-ibx+1)}{(a+i)(ia+ibx+1)}\right)}{(a+i)^2(-n+2i)}$$

```
[Out] (-4*b*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(-1 - (I/2)*n)*Hype
rgeometric2F1[2, 1 + (I/2)*n, 2 + (I/2)*n, ((I - a)*(1 - I*a - I*b*x))/((I
+ a)*(1 + I*a + I*b*x))])/((I + a)^2*(2*I - n))
```

Rubi [A] time = 0.0428307, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5095, 131}

$$-\frac{4b(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{-1-\frac{in}{2}}\,{}_2F_1\left(2,\frac{in}{2}+1;\frac{in}{2}+2;\frac{(i-a)(-ia-ibx+1)}{(a+i)(ia+ibx+1)}\right)}{(a+i)^2(-n+2i)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a + b*x])/x^2,x]
```

```
[Out] (-4*b*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(-1 - (I/2)*n)*Hype
rgeometric2F1[2, 1 + (I/2)*n, 2 + (I/2)*n, ((I - a)*(1 - I*a - I*b*x))/((I
+ a)*(1 + I*a + I*b*x))])/((I + a)^2*(2*I - n))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tan^{-1}(a+bx)}}{x^2} dx &= \int \frac{(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}}{x^2} dx\\ &= -\frac{4b(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{-1-\frac{in}{2}}\,{}_2F_1\left(2,1+\frac{in}{2};2+\frac{in}{2};\frac{(i-a)(1-ia-ibx)}{(i+a)(1+ia+ibx)}\right)}{(i+a)^2(2i-n)}\end{aligned}$$

Mathematica [A] time = 0.022431, size = 125, normalized size = 0.98

$$-\frac{4ib(ia+ibx+1)^{-\frac{in}{2}}(-i(a+bx+i))^{1+\frac{in}{2}}\,{}_2F_1\left(2,\frac{in}{2}+1;\frac{in}{2}+2;\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)}{(a+i)^2(n-2i)(a+bx-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a + b*x])/x^2,x]
```

```
[Out] ((-4*I)*b*((-I)*(I + a + b*x))^(1 + (I/2)*n)*Hypergeometric2F1[2, 1 + (I/2)
*n, 2 + (I/2)*n, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)])/((I
+ a)^2*(-2*I + n)*(1 + I*a + I*b*x)^((I/2)*n)*(-I + a + b*x))
```

Maple [F] time = 0.193, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{n \arctan(bx+a)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(b*x+a))/x^2,x)
```

```
[Out] int(exp(n*arctan(b*x+a))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n \arctan(bx+a))}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(b*x + a))/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^{(n \arctan(bx+a))}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x^2,x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(b*x + a))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{atan}(a+bx)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(b*x+a))/x**2,x)
```

```
[Out] Integral(exp(n*atan(a + b*x))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n \arctan(bx+a))}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(b*x + a))/x^2, x)
```

3.243 $\int \frac{e^{n\tan^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=207

$$-\frac{(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{2\left(a^2+1\right)x^2}-\frac{2b^2(2a-n)(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{-1-\frac{in}{2}}\,{}_2F_1\left(2,\frac{in}{2}+1;\frac{in}{2}+2;\frac{(i-a)(-ia}{(a+i)(ia}\right.}{(-a+i)(a+i)^3(-n+2i)}$$

```
[Out] -((1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n))/(2*(1 +
a^2)*x^2) - (2*b^2*(2*a - n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b
*x)^(-1 - (I/2)*n)*Hypergeometric2F1[2, 1 + (I/2)*n, 2 + (I/2)*n, ((I - a)*
(1 - I*a - I*b*x))/((I + a)*(1 + I*a + I*b*x))])/((I - a)*(I + a)^3*(2*I -
n))
```

Rubi [A] time = 0.108783, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5095, 96, 131}

$$-\frac{(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{2\left(a^2+1\right)x^2}-\frac{2b^2(2a-n)(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{-1-\frac{in}{2}}\,{}_2F_1\left(2,\frac{in}{2}+1;\frac{in}{2}+2;\frac{(i-a)(-ia}{(a+i)(ia}\right.}{(-a+i)(a+i)^3(-n+2i)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a + b*x])/x^3,x]
```


```
[Out] -((1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n))/(2*(1 +
a^2)*x^2) - (2*b^2*(2*a - n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b
*x)^(-1 - (I/2)*n)*Hypergeometric2F1[2, 1 + (I/2)*n, 2 + (I/2)*n, ((I - a)*
(1 - I*a - I*b*x))/((I + a)*(1 + I*a + I*b*x))])/((I - a)*(I + a)^3*(2*I -
n))
```


Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(a+bx)}}{x^3}\,dx = \int \frac{(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}}{x^3}\,dx$$
$$= -\frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{2\left(1+a^2\right)x^2} - \frac{(b(2a-n))\int \frac{(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}}{x^2}\,dx}{2\left(1+a^2\right)}$$
$$= -\frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{2\left(1+a^2\right)x^2} + \frac{2b^2(2a-n)(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{-1-\frac{in}{2}}\,{}_2F_1\left(2,1+\frac{in}{2}\right.}{(i+a)^2\left(1+a^2\right)(2i-n)}$$

Mathematica [A] time = 0.0644245, size = 173, normalized size = 0.84

$$-\frac{i(ia+ibx+1)^{-\frac{in}{2}}(-i(a+bx+i))^{1+\frac{in}{2}}\left(4b^2x^2(n-2a)\,{}_2F_1\left(2,\frac{in}{2}+1;\frac{in}{2}+2;\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)+(a+i)^2(n-2i)(a+bx-i)^2\right)}{2(a-i)(a+i)^3(n-2i)x^2(a+bx-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a + b*x])/x^3,x]
```

```
[Out] ((-I/2)*((-I)*(I + a + b*x))^(1 + (I/2)*n)*((I + a)^2*(-2*I + n)*(-I + a +
b*x)^2 + 4*b^2*(-2*a + n)*x^2*Hypergeometric2F1[2, 1 + (I/2)*n, 2 + (I/2)*n
, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]))/((-I + a)*(I + a)^
3*(-2*I + n)*x^2*(1 + I*a + I*b*x)^((I/2)*n)*(-I + a + b*x))
```

Maple [F] time = 0.205, size = 0, normalized size = 0.

$$\int \frac{e^{n\arctan(bx+a)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(b*x+a))/x^3,x)
```

```
[Out] int(exp(n*arctan(b*x+a))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(bx+a))}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(b*x + a))/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(n \arctan(bx+a))}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x^3,x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(b*x + a))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{atan}(a+bx)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(b*x+a))/x**3,x)
```

```
[Out] Integral(exp(n*atan(a + b*x))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n \arctan(bx+a))}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x^3,x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(b*x + a))/x^3, x)
```

3.244 $\int e^{\tan^{-1}(ax)} \left(c + a^2cx^2\right)^p \, dx$

Optimal. Leaf size=102

$$\frac{i2^{p+\left(1-\frac{i}{2}\right)}(1-iax)^{p+\left(1+\frac{i}{2}\right)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(\frac{i}{2}-p,p+\left(1+\frac{i}{2}\right);p+\left(2+\frac{i}{2}\right);\frac{1}{2}(1-iax)\right)}{a(2p+(2+i))}$$

```
[Out] (I*2^((1 - I/2) + p)*(1 - I*a*x)^((1 + I/2) + p)*(c + a^2*c*x^2)^p*Hypergeo
metric2F1[I/2 - p, (1 + I/2) + p, (2 + I/2) + p, (1 - I*a*x)/2])/(a*((2 + I
) + 2*p)*(1 + a^2*x^2)^p)
```

Rubi [A] time = 0.0751491, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5076, 5073, 69}

$$\frac{i2^{p+\left(1-\frac{i}{2}\right)}(1-iax)^{p+\left(1+\frac{i}{2}\right)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(\frac{i}{2}-p,p+\left(1+\frac{i}{2}\right);p+\left(2+\frac{i}{2}\right);\frac{1}{2}(1-iax)\right)}{a(2p+(2+i))}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]*(c + a^2*c*x^2)^p,x]
```

```
[Out] (I*2^((1 - I/2) + p)*(1 - I*a*x)^((1 + I/2) + p)*(c + a^2*c*x^2)^p*Hypergeo
metric2F1[I/2 - p, (1 + I/2) + p, (2 + I/2) + p, (1 - I*a*x)/2])/(a*((2 + I
) + 2*p)*(1 + a^2*x^2)^p)
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx = \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{\tan^{-1}(ax)}\left(1+a^2x^2\right)^p\,dx$$
$$= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int (1-iax)^{\frac{i}{2}+p}(1+iax)^{-\frac{i}{2}+p}\,dx$$
$$= \frac{i2^{\left(1-\frac{i}{2}\right)+p}(1-iax)^{\left(1+\frac{i}{2}\right)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\,{}_2F_1\left(\frac{i}{2}-p,\left(1+\frac{i}{2}\right)+p;\left(2+\frac{i}{2}\right)+p,\right.}{a((2+i)+2p)}$$

Mathematica [A] time = 0.0281641, size = 102, normalized size = 1.

$$\frac{i2^{p-\frac{i}{2}}(1-iax)^{p+\left(1+\frac{i}{2}\right)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(\frac{i}{2}-p,p+\left(1+\frac{i}{2}\right);p+\left(2+\frac{i}{2}\right);\frac{1}{2}(1-iax)\right)}{a\left(p+\left(1+\frac{i}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]*(c + a^2*c*x^2)^p,x]
```

```
[Out] (I*2^(-I/2 + p)*(1 - I*a*x)^((1 + I/2) + p)*(c + a^2*c*x^2)^p*Hypergeometri
c2F1[I/2 - p, (1 + I/2) + p, (2 + I/2) + p, (1 - I*a*x)/2])/(a*((1 + I/2) +
 p)*(1 + a^2*x^2)^p)
```

Maple [F] time = 0.322, size = 0, normalized size = 0.

$$\int e^{\arctan(ax)}\left(a^2cx^2+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

```
[Out] int(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^p e^{(\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^p*e^(arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^2+c\right)^p e^{(\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^p*e^(arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\left(a^2x^2+1\right)\right)^p e^{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))*(a**2*c*x**2+c)**p,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**p*exp(atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^p e^{(\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^p*e^(arctan(a*x)), x)
```

3.245 $\int e^{\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx$

Optimal. Leaf size=63

$$\frac{\left(\frac{1}{37}+\frac{6i}{37}\right)2^{3-\frac{i}{2}}c^2(1-iax)^{3+\frac{i}{2}}\,{}_2F_1\left(-2+\frac{i}{2},3+\frac{i}{2};4+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] ((1/37 + (6*I)/37)*2^(3 - I/2)*c^2*(1 - I*a*x)^(3 + I/2)*Hypergeometric2F1[
-2 + I/2, 3 + I/2, 4 + I/2, (1 - I*a*x)/2])/a
```

Rubi [A] time = 0.0372467, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5073, 69}

$$\frac{\left(\frac{1}{37}+\frac{6i}{37}\right)2^{3-\frac{i}{2}}c^2(1-iax)^{3+\frac{i}{2}}\,{}_2F_1\left(-2+\frac{i}{2},3+\frac{i}{2};4+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]*(c + a^2*c*x^2)^2,x]
```

```
[Out] ((1/37 + (6*I)/37)*2^(3 - I/2)*c^2*(1 - I*a*x)^(3 + I/2)*Hypergeometric2F1[
-2 + I/2, 3 + I/2, 4 + I/2, (1 - I*a*x)/2])/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx &= c^2\int (1-iax)^{2+\frac{i}{2}}(1+iax)^{2-\frac{i}{2}}\,dx\\ &= \frac{\left(\frac{1}{37}+\frac{6i}{37}\right)2^{3-\frac{i}{2}}c^2(1-iax)^{3+\frac{i}{2}}\,{}_2F_1\left(-2+\frac{i}{2},3+\frac{i}{2};4+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0139906, size = 63, normalized size = 1.

$$\frac{\left(\frac{1}{37}+\frac{6i}{37}\right)2^{3-\frac{i}{2}}c^2(1-iax)^{3+\frac{i}{2}}\,{}_2F_1\left(-2+\frac{i}{2},3+\frac{i}{2};4+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]*(c + a^2*c*x^2)^2,x]
```

```
[Out] ((1/37 + (6*I)/37)*2^(3 - I/2)*c^2*(1 - I*a*x)^(3 + I/2)*Hypergeometric2F1[
-2 + I/2, 3 + I/2, 4 + I/2, (1 - I*a*x)/2])/a
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int \mathrm{e}^{\arctan(ax)} \left(a^2 c x^2 + c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 c x^2 + c\right)^2 e^{(\arctan(ax))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*e^(arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4 c^2 x^4 + 2\, a^2 c^2 x^2 + c^2\right) e^{(\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2 \left(\int 2a^2 x^2 e^{\operatorname{atan}(ax)}\, dx + \int a^4 x^4 e^{\operatorname{atan}(ax)}\, dx + \int e^{\operatorname{atan}(ax)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))*(a**2*c*x**2+c)**2,x)
```

```
[Out] c**2*(Integral(2*a**2*x**2*exp(atan(a*x)), x) + Integral(a**4*x**4*exp(atan
(a*x)), x) + Integral(exp(atan(a*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^2 e^{(\arctan(ax))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*e^(arctan(a*x)), x)
```

3.246 $\int e^{\tan^{-1}(ax)}\left(c+a^2cx^2\right)\,dx$

Optimal. Leaf size=61

$$\frac{\left(\frac{1}{17}+\frac{4i}{17}\right)2^{2-\frac{i}{2}}c(1-iax)^{2+\frac{i}{2}}\,{}_2F_1\left(-1+\frac{i}{2},2+\frac{i}{2};3+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] ((1/17 + (4*I)/17)*2^(2 - I/2)*c*(1 - I*a*x)^(2 + I/2)*Hypergeometric2F1[-1
 + I/2, 2 + I/2, 3 + I/2, (1 - I*a*x)/2])/a
```

Rubi [A] time = 0.0261251, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5073, 69}

$$\frac{\left(\frac{1}{17}+\frac{4i}{17}\right)2^{2-\frac{i}{2}}c(1-iax)^{2+\frac{i}{2}}\,{}_2F_1\left(-1+\frac{i}{2},2+\frac{i}{2};3+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]*(c + a^2*c*x^2),x]
```

```
[Out] ((1/17 + (4*I)/17)*2^(2 - I/2)*c*(1 - I*a*x)^(2 + I/2)*Hypergeometric2F1[-1
 + I/2, 2 + I/2, 3 + I/2, (1 - I*a*x)/2])/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{\tan^{-1}(ax)}\left(c+a^2cx^2\right)\,dx &= c\int (1-iax)^{1+\frac{i}{2}}(1+iax)^{1-\frac{i}{2}}\,dx\\ &= \frac{\left(\frac{1}{17}+\frac{4i}{17}\right)2^{2-\frac{i}{2}}c(1-iax)^{2+\frac{i}{2}}\,{}_2F_1\left(-1+\frac{i}{2},2+\frac{i}{2};3+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0114601, size = 61, normalized size = 1.

$$\frac{\left(\frac{1}{17}+\frac{4i}{17}\right)2^{2-\frac{i}{2}}c(1-iax)^{2+\frac{i}{2}}\,{}_2F_1\left(-1+\frac{i}{2},2+\frac{i}{2};3+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]*(c + a^2*c*x^2),x]
```

```
[Out] ((1/17 + (4*I)/17)*2^(2 - I/2)*c*(1 - I*a*x)^(2 + I/2)*Hypergeometric2F1[-1
 + I/2, 2 + I/2, 3 + I/2, (1 - I*a*x)/2])/a
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \mathrm{e}^{\arctan(ax)} \left(a^2cx^2 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))*(a^2*c*x^2+c),x)
```

```
[Out] int(exp(arctan(a*x))*(a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)e^{(\arctan(ax))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2cx^2 + c\right)e^{(\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int a^2x^2e^{\operatorname{atan}(ax)}\, dx + \int e^{\operatorname{atan}(ax)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))*(a**2*c*x**2+c),x)
```

```
[Out] c*(Integral(a**2*x**2*exp(atan(a*x)), x) + Integral(exp(atan(a*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)e^{(\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

3.247 $\int e^{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=60

$$\frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-\frac{i}{2}}(1-iax)^{1+\frac{i}{2}}\,{}_2F_1\left(\frac{i}{2},1+\frac{i}{2};2+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] ((1/5 + (2*I)/5)*2^(1 - I/2)*(1 - I*a*x)^(1 + I/2)*Hypergeometric2F1[I/2, 1
 + I/2, 2 + I/2, (1 - I*a*x)/2])/a
```

Rubi [A] time = 0.0113063, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5061, 69}

$$\frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-\frac{i}{2}}(1-iax)^{1+\frac{i}{2}}\,{}_2F_1\left(\frac{i}{2},1+\frac{i}{2};2+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x],x]
```

```
[Out] ((1/5 + (2*I)/5)*2^(1 - I/2)*(1 - I*a*x)^(1 + I/2)*Hypergeometric2F1[I/2, 1
 + I/2, 2 + I/2, (1 - I*a*x)/2])/a
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{\tan^{-1}(ax)}\,dx &= \int (1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}\,dx\\ &= \frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-\frac{i}{2}}(1-iax)^{1+\frac{i}{2}}\,{}_2F_1\left(\frac{i}{2},1+\frac{i}{2};2+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0154955, size = 45, normalized size = 0.75

$$\frac{\left(\frac{4}{5}-\frac{8i}{5}\right)e^{(1+2i)\tan^{-1}(ax)}\,{}_2F_1\left(1-\frac{i}{2},2;2-\frac{i}{2};-e^{2i\tan^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTan[a*x],x]
```

```
[Out] ((4/5 - (8*I)/5)*E^((1 + 2*I)*ArcTan[a*x])*Hypergeometric2F1[1 - I/2, 2, 2
- I/2, -E^((2*I)*ArcTan[a*x])])/a
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \mathrm{e}^{\arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x)),x)
```

```
[Out] int(exp(arctan(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(\arctan(ax))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{(\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral(e^(arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\operatorname{atan}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x)),x)
```

```
[Out] Integral(exp(atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x)),x, algorithm="giac")
```

```
[Out] integrate(e^(arctan(a*x)), x)
```

3.248 $\int \frac{e^{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=13

$$\frac{e^{\tan^{-1}(ax)}}{ac}$$

```
[Out] E^ArcTan[a*x]/(a*c)
```

Rubi [A] time = 0.0258877, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5071}

$$\frac{e^{\tan^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2),x]

[Out] E^ArcTan[a*x]/(a*c)
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx = \frac{e^{\tan^{-1}(ax)}}{ac}$$

Mathematica [C] time = 0.0072651, size = 35, normalized size = 2.69

$$\frac{(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}}{ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2),x]

[Out] (1 - I*a*x)^(I/2)/(a*c*(1 + I*a*x)^(I/2))
```

Maple [A] time = 0.036, size = 13, normalized size = 1.

$$\frac{\mathrm{e}^{\arctan(ax)}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c),x)
```

```
[Out] exp(arctan(a*x))/a/c
```

Maxima [A] time = 1.49114, size = 16, normalized size = 1.23

$$\frac{e^{(\arctan(ax))}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] e^(arctan(a*x))/(a*c)
```

Fricas [A] time = 2.04406, size = 31, normalized size = 2.38

$$\frac{e^{(\arctan(ax))}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] e^(arctan(a*x))/(a*c)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.10974, size = 16, normalized size = 1.23

$$\frac{e^{(\arctan(ax))}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] e^(arctan(a*x))/(a*c)
```

3.249 $\int \frac{e^{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=50

$$\frac{(2ax+1)e^{\tan^{-1}(ax)}}{5ac^2\left(a^2x^2+1\right)}+\frac{2e^{\tan^{-1}(ax)}}{5ac^2}$$

```
[Out] (2*E^ArcTan[a*x])/(5*a*c^2) + (E^ArcTan[a*x]*(1 + 2*a*x))/(5*a*c^2*(1 + a^2
*x^2))
```

Rubi [A] time = 0.0504912, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5070, 5071}

$$\frac{(2ax+1)e^{\tan^{-1}(ax)}}{5ac^2\left(a^2x^2+1\right)}+\frac{2e^{\tan^{-1}(ax)}}{5ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^2,x]
```

```
[Out] (2*E^ArcTan[a*x])/(5*a*c^2) + (E^ArcTan[a*x]*(1 + 2*a*x))/(5*a*c^2*(1 + a^2
*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx &= \frac{e^{\tan^{-1}(ax)}(1+2ax)}{5ac^2\left(1+a^2x^2\right)}+\frac{2\int \frac{e^{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{5c}\\ &= \frac{2e^{\tan^{-1}(ax)}}{5ac^2}+\frac{e^{\tan^{-1}(ax)}(1+2ax)}{5ac^2\left(1+a^2x^2\right)}\end{aligned}$$

Mathematica [C] time = 0.0240477, size = 60, normalized size = 1.2

$$\frac{(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}\left(2a^2x^2+2ax+3\right)}{5c^2\left(a^3x^2+a\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((1 - I*a*x)^(I/2)*(3 + 2*a*x + 2*a^2*x^2))/(5*c^2*(1 + I*a*x)^(I/2)*(a + a
^3*x^2))
```

Maple [A] time = 0.036, size = 39, normalized size = 0.8

$$\frac{\mathrm{e}^{\arctan(ax)}\left(2\,a^2x^2+2\,ax+3\right)}{\left(5\,a^2x^2+5\right)ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/5*exp(arctan(a*x))*(2*a^2*x^2+2*a*x+3)/(a^2*x^2+1)/a/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arctan(ax))}}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

Fricas [A] time = 2.08994, size = 90, normalized size = 1.8

$$\frac{\left(2\,a^2x^2+2\,ax+3\right)e^{(\arctan(ax))}}{5\left(a^3c^2x^2+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/5*(2*a^2*x^2 + 2*a*x + 3)*e^(arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arctan(ax))}}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

3.250 $\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3} dx$

Optimal. Leaf size=83

$$\frac{12(2ax+1)e^{\tan^{-1}(ax)}}{85ac^3\left(a^2x^2+1\right)}+\frac{(4ax+1)e^{\tan^{-1}(ax)}}{17ac^3\left(a^2x^2+1\right)^2}+\frac{24e^{\tan^{-1}(ax)}}{85ac^3}$$

```
[Out] (24*E^ArcTan[a*x])/(85*a*c^3) + (E^ArcTan[a*x]*(1 + 4*a*x))/(17*a*c^3*(1 +
a^2*x^2)^2) + (12*E^ArcTan[a*x]*(1 + 2*a*x))/(85*a*c^3*(1 + a^2*x^2))
```

Rubi [A] time = 0.0777701, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5070, 5071}

$$\frac{12(2ax+1)e^{\tan^{-1}(ax)}}{85ac^3\left(a^2x^2+1\right)}+\frac{(4ax+1)e^{\tan^{-1}(ax)}}{17ac^3\left(a^2x^2+1\right)^2}+\frac{24e^{\tan^{-1}(ax)}}{85ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^3,x]
```

```
[Out] (24*E^ArcTan[a*x])/(85*a*c^3) + (E^ArcTan[a*x]*(1 + 4*a*x))/(17*a*c^3*(1 +
a^2*x^2)^2) + (12*E^ArcTan[a*x]*(1 + 2*a*x))/(85*a*c^3*(1 + a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3} dx &= \frac{e^{\tan^{-1}(ax)}(1+4ax)}{17ac^3\left(1+a^2x^2\right)^2}+\frac{12\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2} dx}{17c}\\
&= \frac{e^{\tan^{-1}(ax)}(1+4ax)}{17ac^3\left(1+a^2x^2\right)^2}+\frac{12e^{\tan^{-1}(ax)}(1+2ax)}{85ac^3\left(1+a^2x^2\right)}+\frac{24\int \frac{e^{\tan^{-1}(ax)}}{c+a^2cx^2} dx}{85c^2}\\
&= \frac{24e^{\tan^{-1}(ax)}}{85ac^3}+\frac{e^{\tan^{-1}(ax)}(1+4ax)}{17ac^3\left(1+a^2x^2\right)^2}+\frac{12e^{\tan^{-1}(ax)}(1+2ax)}{85ac^3\left(1+a^2x^2\right)}
\end{aligned}$$

Mathematica [C] time = 0.248842, size = 114, normalized size = 1.37

$$\frac{\frac{5(4ax+1)e^{\tan^{-1}(ax)}}{(a^2x^2+1)^2}+\frac{24(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}(ax+(1-i))}{ax-i}+(12-24i)(1-iax)^{-1+\frac{i}{2}}(1+iax)^{-1-\frac{i}{2}}}{85ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^3,x]
```

```
[Out] ((12 - 24*I)/((1 - I*a*x)^(1 - I/2)*(1 + I*a*x)^(1 + I/2)) + (24*(1 - I*a*x
)^(I/2)*((1 - I) + a*x))/((1 + I*a*x)^(I/2)*(-I + a*x)) + (5*E^ArcTan[a*x]*
(1 + 4*a*x))/(1 + a^2*x^2)^2)/(85*a*c^3)
```

Maple [A] time = 0.036, size = 55, normalized size = 0.7

$$\frac{e^{\arctan(ax)}\left(24\,a^4x^4+24\,a^3x^3+60\,a^2x^2+44\,ax+41\right)}{85\,\left(a^2x^2+1\right)^2ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/85*exp(arctan(a*x))*(24*a^4*x^4+24*a^3*x^3+60*a^2*x^2+44*a*x+41)/(a^2*x^2
+1)^2/a/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arctan(ax))}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

Fricas [A] time = 2.02264, size = 153, normalized size = 1.84

$$\frac{\left(24\,a^4x^4+24\,a^3x^3+60\,a^2x^2+44\,ax+41\right)e^{(\arctan(ax))}}{85\,\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/85*(24*a^4*x^4 + 24*a^3*x^3 + 60*a^2*x^2 + 44*a*x + 41)*e^(arctan(a*x))/(
a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arctan(ax))}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

3.251 $\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=116

$$\frac{72(2ax+1)e^{\tan^{-1}(ax)}}{629ac^4\left(a^2x^2+1\right)}+\frac{30(4ax+1)e^{\tan^{-1}(ax)}}{629ac^4\left(a^2x^2+1\right)^2}+\frac{(6ax+1)e^{\tan^{-1}(ax)}}{37ac^4\left(a^2x^2+1\right)^3}+\frac{144e^{\tan^{-1}(ax)}}{629ac^4}$$

```
[Out] (144*E^ArcTan[a*x])/(629*a*c^4) + (E^ArcTan[a*x]*(1 + 6*a*x))/(37*a*c^4*(1
+ a^2*x^2)^3) + (30*E^ArcTan[a*x]*(1 + 4*a*x))/(629*a*c^4*(1 + a^2*x^2)^2)
+ (72*E^ArcTan[a*x]*(1 + 2*a*x))/(629*a*c^4*(1 + a^2*x^2))
```

Rubi [A] time = 0.109743, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5070, 5071}

$$\frac{72(2ax+1)e^{\tan^{-1}(ax)}}{629ac^4\left(a^2x^2+1\right)}+\frac{30(4ax+1)e^{\tan^{-1}(ax)}}{629ac^4\left(a^2x^2+1\right)^2}+\frac{(6ax+1)e^{\tan^{-1}(ax)}}{37ac^4\left(a^2x^2+1\right)^3}+\frac{144e^{\tan^{-1}(ax)}}{629ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^4,x]
```

```
[Out] (144*E^ArcTan[a*x])/(629*a*c^4) + (E^ArcTan[a*x]*(1 + 6*a*x))/(37*a*c^4*(1
+ a^2*x^2)^3) + (30*E^ArcTan[a*x]*(1 + 4*a*x))/(629*a*c^4*(1 + a^2*x^2)^2)
+ (72*E^ArcTan[a*x]*(1 + 2*a*x))/(629*a*c^4*(1 + a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4}\,dx = \frac{e^{\tan^{-1}(ax)}(1+6ax)}{37ac^4\left(1+a^2x^2\right)^3} + \frac{30\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx}{37c}$$

$$= \frac{e^{\tan^{-1}(ax)}(1+6ax)}{37ac^4\left(1+a^2x^2\right)^3} + \frac{30e^{\tan^{-1}(ax)}(1+4ax)}{629ac^4\left(1+a^2x^2\right)^2} + \frac{360\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{629c^2}$$

$$= \frac{e^{\tan^{-1}(ax)}(1+6ax)}{37ac^4\left(1+a^2x^2\right)^3} + \frac{30e^{\tan^{-1}(ax)}(1+4ax)}{629ac^4\left(1+a^2x^2\right)^2} + \frac{72e^{\tan^{-1}(ax)}(1+2ax)}{629ac^4\left(1+a^2x^2\right)} + \frac{144\int \frac{e^{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{629c^3}$$

$$= \frac{144e^{\tan^{-1}(ax)}}{629ac^4} + \frac{e^{\tan^{-1}(ax)}(1+6ax)}{37ac^4\left(1+a^2x^2\right)^3} + \frac{30e^{\tan^{-1}(ax)}(1+4ax)}{629ac^4\left(1+a^2x^2\right)^2} + \frac{72e^{\tan^{-1}(ax)}(1+2ax)}{629ac^4\left(1+a^2x^2\right)}$$

Mathematica [C] time = 0.263634, size = 123, normalized size = 1.06

$$\frac{17c(6ax+1)e^{\tan^{-1}(ax)} + 6\left(a^2cx^2+c\right)\left(5(4ax+1)e^{\tan^{-1}(ax)} + 12(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}(ax-i)(ax+i)\left(2a^2x^2+2ax+3\right)\right.}{629ac^2\left(a^2cx^2+c\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^4,x]
```

```
[Out] (17*c*E^ArcTan[a*x]*(1 + 6*a*x) + 6*(c + a^2*c*x^2)*(5*E^ArcTan[a*x]*(1 + 4
*a*x) + (12*(1 - I*a*x)^(I/2)*(-I + a*x)*(I + a*x)*(3 + 2*a*x + 2*a^2*x^2))
/(1 + I*a*x)^(I/2)))/(629*a*c^2*(c + a^2*c*x^2)^3)
```

Maple [A] time = 0.038, size = 71, normalized size = 0.6

$$\frac{e^{\arctan(ax)}\left(144\,a^6x^6+144\,a^5x^5+504\,a^4x^4+408\,a^3x^3+606\,a^2x^2+366\,ax+263\right)}{629\,\left(a^2x^2+1\right)^3ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^4,x)
```

```
[Out] 1/629*exp(arctan(a*x))*(144*a^6*x^6+144*a^5*x^5+504*a^4*x^4+408*a^3*x^3+606
*a^2*x^2+366*a*x+263)/(a^2*x^2+1)^3/a/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arctan(ax))}}{\left(a^2cx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

Fricas [A] time = 1.96869, size = 220, normalized size = 1.9

$$\frac{\left(144\,a^{6}x^{6}+144\,a^{5}x^{5}+504\,a^{4}x^{4}+408\,a^{3}x^{3}+606\,a^{2}x^{2}+366\,ax+263\right)e^{\left(\arctan\left(ax\right)\right)}}{629\left(a^{7}c^{4}x^{6}+3\,a^{5}c^{4}x^{4}+3\,a^{3}c^{4}x^{2}+ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/629*(144*a^6*x^6 + 144*a^5*x^5 + 504*a^4*x^4 + 408*a^3*x^3 + 606*a^2*x^2
+ 366*a*x + 263)*e^(arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x
^2 + a*c^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**4,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\arctan\left(ax\right)\right)}}{\left(a^{2}cx^{2}+c\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

3.252 $\int \frac{e^{\tan^{-1}(ax)}}{(c+a^2cx^2)^5}\,dx$

Optimal. Leaf size=149

$$\frac{4032(2ax+1)e^{\tan^{-1}(ax)}}{40885ac^5(a^2x^2+1)}+\frac{336(4ax+1)e^{\tan^{-1}(ax)}}{8177ac^5(a^2x^2+1)^2}+\frac{56(6ax+1)e^{\tan^{-1}(ax)}}{2405ac^5(a^2x^2+1)^3}+\frac{(8ax+1)e^{\tan^{-1}(ax)}}{65ac^5(a^2x^2+1)^4}+\frac{8064e^{\tan^{-1}(ax)}}{40885ac^5}$$

```
[Out] (8064*E^ArcTan[a*x])/(40885*a*c^5) + (E^ArcTan[a*x]*(1 + 8*a*x))/(65*a*c^5*
(1 + a^2*x^2)^4) + (56*E^ArcTan[a*x]*(1 + 6*a*x))/(2405*a*c^5*(1 + a^2*x^2)
^3) + (336*E^ArcTan[a*x]*(1 + 4*a*x))/(8177*a*c^5*(1 + a^2*x^2)^2) + (4032*
E^ArcTan[a*x]*(1 + 2*a*x))/(40885*a*c^5*(1 + a^2*x^2))
```

Rubi [A] time = 0.140465, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5070, 5071}

$$\frac{4032(2ax+1)e^{\tan^{-1}(ax)}}{40885ac^5(a^2x^2+1)}+\frac{336(4ax+1)e^{\tan^{-1}(ax)}}{8177ac^5(a^2x^2+1)^2}+\frac{56(6ax+1)e^{\tan^{-1}(ax)}}{2405ac^5(a^2x^2+1)^3}+\frac{(8ax+1)e^{\tan^{-1}(ax)}}{65ac^5(a^2x^2+1)^4}+\frac{8064e^{\tan^{-1}(ax)}}{40885ac^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^5,x]
```


```
[Out] (8064*E^ArcTan[a*x])/(40885*a*c^5) + (E^ArcTan[a*x]*(1 + 8*a*x))/(65*a*c^5*
(1 + a^2*x^2)^4) + (56*E^ArcTan[a*x]*(1 + 6*a*x))/(2405*a*c^5*(1 + a^2*x^2)
^3) + (336*E^ArcTan[a*x]*(1 + 4*a*x))/(8177*a*c^5*(1 + a^2*x^2)^2) + (4032*
E^ArcTan[a*x]*(1 + 2*a*x))/(40885*a*c^5*(1 + a^2*x^2))
```


Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(ax)}}{(c+a^2cx^2)^5}\,dx = \frac{e^{\tan^{-1}(ax)}(1+8ax)}{65ac^5(1+a^2x^2)^4} + \frac{56\int \frac{e^{\tan^{-1}(ax)}}{(c+a^2cx^2)^4}\,dx}{65c}$$

$$= \frac{e^{\tan^{-1}(ax)}(1+8ax)}{65ac^5(1+a^2x^2)^4} + \frac{56e^{\tan^{-1}(ax)}(1+6ax)}{2405ac^5(1+a^2x^2)^3} + \frac{336\int \frac{e^{\tan^{-1}(ax)}}{(c+a^2cx^2)^3}\,dx}{481c^2}$$

$$= \frac{e^{\tan^{-1}(ax)}(1+8ax)}{65ac^5(1+a^2x^2)^4} + \frac{56e^{\tan^{-1}(ax)}(1+6ax)}{2405ac^5(1+a^2x^2)^3} + \frac{336e^{\tan^{-1}(ax)}(1+4ax)}{8177ac^5(1+a^2x^2)^2} + \frac{4032\int \frac{e^{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx}{8177c^3}$$

$$= \frac{e^{\tan^{-1}(ax)}(1+8ax)}{65ac^5(1+a^2x^2)^4} + \frac{56e^{\tan^{-1}(ax)}(1+6ax)}{2405ac^5(1+a^2x^2)^3} + \frac{336e^{\tan^{-1}(ax)}(1+4ax)}{8177ac^5(1+a^2x^2)^2} + \frac{4032e^{\tan^{-1}(ax)}(1+2ax)}{40885ac^5(1+a^2x^2)} + \frac{80\ldots}{}$$

$$= \frac{8064e^{\tan^{-1}(ax)}}{40885ac^5} + \frac{e^{\tan^{-1}(ax)}(1+8ax)}{65ac^5(1+a^2x^2)^4} + \frac{56e^{\tan^{-1}(ax)}(1+6ax)}{2405ac^5(1+a^2x^2)^3} + \frac{336e^{\tan^{-1}(ax)}(1+4ax)}{8177ac^5(1+a^2x^2)^2} + \frac{4032e^{\tan^{-1}(a\ldots}}{40885ac^5(\ldots}$$

Mathematica [C] time = 0.257846, size = 153, normalized size = 1.03

$$\frac{629(8ax+1)e^{\tan^{-1}(ax)} + \frac{56(a^2cx^2+c)\left(17c(6ax+1)e^{\tan^{-1}(ax)}+6(a^2cx^2+c)\left(5(4ax+1)e^{\tan^{-1}(ax)}+12(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}(ax-i)(ax+i)(2a^2x^2+2ax+3)\right)\right)}{c^2}}{40885ac(a^2cx^2+c)^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^5,x]
```

```
[Out] (629*E^ArcTan[a*x]*(1 + 8*a*x) + (56*(c + a^2*c*x^2)*(17*c*E^ArcTan[a*x]*(1
 + 6*a*x) + 6*(c + a^2*c*x^2)*(5*E^ArcTan[a*x]*(1 + 4*a*x) + (12*(1 - I*a*x
)^(I/2)*(-I + a*x)*(I + a*x)*(3 + 2*a*x + 2*a^2*x^2))/(1 + I*a*x)^(I/2))))/
c^2)/(40885*a*c*(c + a^2*c*x^2)^4)
```

Maple [A] time = 0.037, size = 87, normalized size = 0.6

$$\frac{e^{\arctan(ax)}\left(8064a^8x^8+8064a^7x^7+36288a^6x^6+30912a^5x^5+62160a^4x^4+43344a^3x^3+48664a^2x^2+25528ax+153\ldots\right.}{40885\left(a^2x^2+1\right)^4c^5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^5,x)
```

```
[Out] 1/40885*exp(arctan(a*x))*(8064*a^8*x^8+8064*a^7*x^7+36288*a^6*x^6+30912*a^5
*x^5+62160*a^4*x^4+43344*a^3*x^3+48664*a^2*x^2+25528*a*x+15357)/(a^2*x^2+1)
^4/c^5/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^5, x)
```

Fricas [A] time = 1.97205, size = 304, normalized size = 2.04

$$\frac{\left(8064\,a^{8}x^{8}+8064\,a^{7}x^{7}+36288\,a^{6}x^{6}+30912\,a^{5}x^{5}+62160\,a^{4}x^{4}+43344\,a^{3}x^{3}+48664\,a^{2}x^{2}+25528\,ax+15357\right)e^{(ar}}{40885\left(a^{9}c^{5}x^{8}+4\,a^{7}c^{5}x^{6}+6\,a^{5}c^{5}x^{4}+4\,a^{3}c^{5}x^{2}+ac^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] 1/40885*(8064*a^8*x^8 + 8064*a^7*x^7 + 36288*a^6*x^6 + 30912*a^5*x^5 + 6216
0*a^4*x^4 + 43344*a^3*x^3 + 48664*a^2*x^2 + 25528*a*x + 15357)*e^(arctan(a*
x))/(a^9*c^5*x^8 + 4*a^7*c^5*x^6 + 6*a^5*c^5*x^4 + 4*a^3*c^5*x^2 + a*c^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arctan(ax))}}{\left(a^{2}cx^{2}+c\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^5,x, algorithm="giac")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^5, x)
```

3.253 $\int e^{\tan^{-1}(ax)} \left(c + a^2cx^2\right)^{3/2} dx$

Optimal. Leaf size=98

$$\frac{\left(\frac{1}{13}+\frac{5i}{13}\right)2^{\frac{3}{2}-\frac{i}{2}}c(1-iax)^{\frac{5}{2}+\frac{i}{2}}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2};\frac{7}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[Out] ((1/13 + (5*I)/13)*2^(3/2 - I/2)*c*(1 - I*a*x)^(5/2 + I/2)*Sqrt[c + a^2*c*x
^2]*Hypergeometric2F1[-3/2 + I/2, 5/2 + I/2, 7/2 + I/2, (1 - I*a*x)/2])/(a*
Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.0749484, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5076, 5073, 69}

$$\frac{\left(\frac{1}{13}+\frac{5i}{13}\right)2^{\frac{3}{2}-\frac{i}{2}}c(1-iax)^{\frac{5}{2}+\frac{i}{2}}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2};\frac{7}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1/13 + (5*I)/13)*2^(3/2 - I/2)*c*(1 - I*a*x)^(5/2 + I/2)*Sqrt[c + a^2*c*x
^2]*Hypergeometric2F1[-3/2 + I/2, 5/2 + I/2, 7/2 + I/2, (1 - I*a*x)/2])/(a*
Sqrt[1 + a^2*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{\tan^{-1}(ax)}\left(1+a^2x^2\right)^{3/2}\,dx}{\sqrt{1+a^2x^2}}$$
$$=\frac{\left(c\sqrt{c+a^2cx^2}\right)\int(1-iax)^{\frac{3}{2}+\frac{i}{2}}(1+iax)^{\frac{3}{2}-\frac{i}{2}}\,dx}{\sqrt{1+a^2x^2}}$$
$$=\frac{\left(\frac{1}{13}+\frac{5i}{13}\right)2^{\frac{3}{2}-\frac{i}{2}}c(1-iax)^{\frac{5}{2}+\frac{i}{2}}\sqrt{c+a^2cx^2}\,{}_2F_1\left(-\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2};\frac{7}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.0232194, size = 98, normalized size = 1.

$$\frac{\left(\frac{1}{13}+\frac{5i}{13}\right)2^{\frac{3}{2}-\frac{i}{2}}c(1-iax)^{\frac{5}{2}+\frac{i}{2}}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2};\frac{7}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1/13 + (5*I)/13)*2^(3/2 - I/2)*c*(1 - I*a*x)^(5/2 + I/2)*Sqrt[c + a^2*c*x
^2]*Hypergeometric2F1[-3/2 + I/2, 5/2 + I/2, 7/2 + I/2, (1 - I*a*x)/2])/(a*
Sqrt[1 + a^2*x^2])
```

Maple [F] time = 0.282, size = 0, normalized size = 0.

$$\int e^{\arctan(ax)}\left(a^2cx^2+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*e^(arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*e^(arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))*(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.254 $\int e^{\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal. Leaf size=97

$$\frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{3}{2}+\frac{i}{2}}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2};\frac{5}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[Out] ((1/5 + (3*I)/5)*2^(1/2 - I/2)*(1 - I*a*x)^(3/2 + I/2)*Sqrt[c + a^2*c*x^2]*
Hypergeometric2F1[-1/2 + I/2, 3/2 + I/2, 5/2 + I/2, (1 - I*a*x)/2])/(a*Sqrt
[1 + a^2*x^2])
```

Rubi [A] time = 0.0695813, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5076, 5073, 69}

$$\frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{3}{2}+\frac{i}{2}}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2};\frac{5}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1/5 + (3*I)/5)*2^(1/2 - I/2)*(1 - I*a*x)^(3/2 + I/2)*Sqrt[c + a^2*c*x^2]*
Hypergeometric2F1[-1/2 + I/2, 3/2 + I/2, 5/2 + I/2, (1 - I*a*x)/2])/(a*Sqrt
[1 + a^2*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx = \frac{\sqrt{c+a^2cx^2}\int e^{\tan^{-1}(ax)}\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}$$
$$= \frac{\sqrt{c+a^2cx^2}\int (1-iax)^{\frac{1}{2}+\frac{i}{2}}(1+iax)^{\frac{1}{2}-\frac{i}{2}}\,dx}{\sqrt{1+a^2x^2}}$$
$$= \frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{3}{2}+\frac{i}{2}}\sqrt{c+a^2cx^2}\,{}_2F_1\left(-\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2};\frac{5}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.0181764, size = 97, normalized size = 1.

$$\frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{3}{2}+\frac{i}{2}}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2};\frac{5}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1/5 + (3*I)/5)*2^(1/2 - I/2)*(1 - I*a*x)^(3/2 + I/2)*Sqrt[c + a^2*c*x^2]*
Hypergeometric2F1[-1/2 + I/2, 3/2 + I/2, 5/2 + I/2, (1 - I*a*x)/2])/(a*Sqrt
[1 + a^2*x^2])
```

Maple [F] time = 0.28, size = 0, normalized size = 0.

$$\int e^{\arctan(ax)}\sqrt{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2+c}e^{(\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^2cx^2+c}e^{(\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\left(a^2x^2+1\right)}e^{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*exp(atan(a*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.255 $\int \frac{e^{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=93

$$\frac{(1+i)2^{-\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{1}{2}+\frac{i}{2}}\sqrt{a^2x^2+1}\,{}_2F_1\left(\frac{1}{2}+\frac{i}{2},\frac{1}{2}+\frac{i}{2};\frac{3}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] ((1 + I)*(1 - I*a*x)^(1/2 + I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2 +
I/2, 1/2 + I/2, 3/2 + I/2, (1 - I*a*x)/2])/(2^(1/2 + I/2)*a*Sqrt[c + a^2*c*
x^2])
```

Rubi [A] time = 0.0709474, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5076, 5073, 69}

$$\frac{(1+i)2^{-\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{1}{2}+\frac{i}{2}}\sqrt{a^2x^2+1}\,{}_2F_1\left(\frac{1}{2}+\frac{i}{2},\frac{1}{2}+\frac{i}{2};\frac{3}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1 + I)*(1 - I*a*x)^(1/2 + I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2 +
I/2, 1/2 + I/2, 3/2 + I/2, (1 - I*a*x)/2])/(2^(1/2 + I/2)*a*Sqrt[c + a^2*c*
x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}+\frac{i}{2}}(1+iax)^{-\frac{1}{2}-\frac{i}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{(1+i)2^{-\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{1}{2}+\frac{i}{2}}\sqrt{1+a^2x^2}\,{}_2F_1\left(\frac{1}{2}+\frac{i}{2},\frac{1}{2}+\frac{i}{2};\frac{3}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0201034, size = 93, normalized size = 1.

$$\frac{(1+i)2^{-\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{1}{2}+\frac{i}{2}}\sqrt{a^2x^2+1}\,{}_2F_1\left(\frac{1}{2}+\frac{i}{2},\frac{1}{2}+\frac{i}{2};\frac{3}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1 + I)*(1 - I*a*x)^(1/2 + I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2 +
I/2, 1/2 + I/2, 3/2 + I/2, (1 - I*a*x)/2])/(2^(1/2 + I/2)*a*Sqrt[c + a^2*c*
x^2])
```

Maple [F] time = 0.29, size = 0, normalized size = 0.

$$\int e^{\arctan(ax)}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(\arctan(ax))}}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(e^(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\operatorname{atan}(ax)}}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.256 $\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{(ax+1)e^{\tan^{-1}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

```
[Out] (E^ArcTan[a*x]*(1 + a*x))/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0361105, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5069}

$$\frac{(ax+1)e^{\tan^{-1}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^(3/2),x]

[Out] (E^ArcTan[a*x]*(1 + a*x))/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{e^{\tan^{-1}(ax)}(1+ax)}{2ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0148304, size = 35, normalized size = 1.

$$\frac{(ax+1)e^{\tan^{-1}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^(3/2),x]

[Out] (E^ArcTan[a*x]*(1 + a*x))/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.038, size = 37, normalized size = 1.1

$$\frac{\left(a^2x^2+1\right)(ax+1)\,e^{\arctan(ax)}}{2\,a}\left(a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(a^2*x^2+1)*(a*x+1)*exp(arctan(a*x))/a/(a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A] time = 1.87468, size = 99, normalized size = 2.83

$$\frac{\sqrt{a^2cx^2+c}(ax+1)e^{(\arctan(ax))}}{2\left(a^3c^2x^2+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a^2*c*x^2 + c)*(a*x + 1)*e^(arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\operatorname{atan}(ax)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(exp(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

3.257 $\int \frac{e^{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{3(ax+1)e^{\tan^{-1}(ax)}}{10ac^2\sqrt{a^2cx^2+c}}+\frac{(3ax+1)e^{\tan^{-1}(ax)}}{10ac\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] (E^ArcTan[a*x]*(1 + 3*a*x))/(10*a*c*(c + a^2*c*x^2)^(3/2)) + (3*E^ArcTan[a*
x]*(1 + a*x))/(10*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0730196, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5069}

$$\frac{3(ax+1)e^{\tan^{-1}(ax)}}{10ac^2\sqrt{a^2cx^2+c}}+\frac{(3ax+1)e^{\tan^{-1}(ax)}}{10ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^ArcTan[a*x]*(1 + 3*a*x))/(10*a*c*(c + a^2*c*x^2)^(3/2)) + (3*E^ArcTan[a*
x]*(1 + a*x))/(10*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx &= \frac{e^{\tan^{-1}(ax)}(1+3ax)}{10ac\left(c+a^2cx^2\right)^{3/2}}+\frac{3\int \frac{e^{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx}{5c}\\ &= \frac{e^{\tan^{-1}(ax)}(1+3ax)}{10ac\left(c+a^2cx^2\right)^{3/2}}+\frac{3e^{\tan^{-1}(ax)}(1+ax)}{10ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0340716, size = 60, normalized size = 0.83

$$\frac{\left(3a^3x^3+3a^2x^2+6ax+4\right)e^{\tan^{-1}(ax)}}{10c^2\left(a^3x^2+a\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^ArcTan[a*x]*(4 + 6*a*x + 3*a^2*x^2 + 3*a^3*x^3))/(10*c^2*(a + a^3*x^2)*S
qrt[c + a^2*c*x^2])
```

Maple [A] time = 0.036, size = 54, normalized size = 0.8

$$\frac{\left(a^2x^2+1\right)\left(3\,a^3x^3+3\,a^2x^2+6\,ax+4\right)\mathrm{e}^{\arctan(ax)}}{10\,a}\left(a^2cx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/10*(a^2*x^2+1)*(3*a^3*x^3+3*a^2*x^2+6*a*x+4)*exp(arctan(a*x))/a/(a^2*c*x^
2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 2.08118, size = 157, normalized size = 2.18

$$\frac{\left(3\,a^3x^3+3\,a^2x^2+6\,ax+4\right)\sqrt{a^2cx^2+c}e^{(\arctan(ax))}}{10\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/10*(3*a^3*x^3 + 3*a^2*x^2 + 6*a*x + 4)*sqrt(a^2*c*x^2 + c)*e^(arctan(a*x)
)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

3.258 $\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=108

$$\frac{3(ax+1)e^{\tan^{-1}(ax)}}{13ac^3\sqrt{a^2cx^2+c}}+\frac{(3ax+1)e^{\tan^{-1}(ax)}}{13ac^2\left(a^2cx^2+c\right)^{3/2}}+\frac{(5ax+1)e^{\tan^{-1}(ax)}}{26ac\left(a^2cx^2+c\right)^{5/2}}$$

```
[Out] (E^ArcTan[a*x]*(1 + 5*a*x))/(26*a*c*(c + a^2*c*x^2)^(5/2)) + (E^ArcTan[a*x]
*(1 + 3*a*x))/(13*a*c^2*(c + a^2*c*x^2)^(3/2)) + (3*E^ArcTan[a*x]*(1 + a*x)
)/(13*a*c^3*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.113277, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5069}

$$\frac{3(ax+1)e^{\tan^{-1}(ax)}}{13ac^3\sqrt{a^2cx^2+c}}+\frac{(3ax+1)e^{\tan^{-1}(ax)}}{13ac^2\left(a^2cx^2+c\right)^{3/2}}+\frac{(5ax+1)e^{\tan^{-1}(ax)}}{26ac\left(a^2cx^2+c\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (E^ArcTan[a*x]*(1 + 5*a*x))/(26*a*c*(c + a^2*c*x^2)^(5/2)) + (E^ArcTan[a*x]
*(1 + 3*a*x))/(13*a*c^2*(c + a^2*c*x^2)^(3/2)) + (3*E^ArcTan[a*x]*(1 + a*x)
)/(13*a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx = \frac{e^{\tan^{-1}(ax)}(1+5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}} + \frac{10\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{13c}$$

$$= \frac{e^{\tan^{-1}(ax)}(1+5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}} + \frac{e^{\tan^{-1}(ax)}(1+3ax)}{13ac^2\left(c+a^2cx^2\right)^{3/2}} + \frac{6\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{13c^2}$$

$$= \frac{e^{\tan^{-1}(ax)}(1+5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}} + \frac{e^{\tan^{-1}(ax)}(1+3ax)}{13ac^2\left(c+a^2cx^2\right)^{3/2}} + \frac{3e^{\tan^{-1}(ax)}(1+ax)}{13ac^3\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0353404, size = 79, normalized size = 0.73

$$\frac{\left(6a^5x^5+6a^4x^4+18a^3x^3+14a^2x^2+17ax+9\right)e^{\tan^{-1}(ax)}}{26ac^3\left(a^2x^2+1\right)^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (E^ArcTan[a*x]*(9 + 17*a*x + 14*a^2*x^2 + 18*a^3*x^3 + 6*a^4*x^4 + 6*a^5*x^
5))/(26*a*c^3*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.036, size = 70, normalized size = 0.7

$$\frac{\left(a^2x^2+1\right)\left(6\,a^5x^5+6\,a^4x^4+18\,a^3x^3+14\,a^2x^2+17\,ax+9\right)\mathrm{e}^{\arctan(ax)}}{26\,a}\left(a^2cx^2+c\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/26*(a^2*x^2+1)*(6*a^5*x^5+6*a^4*x^4+18*a^3*x^3+14*a^2*x^2+17*a*x+9)*exp(a
rctan(a*x))/a/(a^2*c*x^2+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

Fricas [A] time = 2.01852, size = 215, normalized size = 1.99

$$\frac{\left(6\,a^{5}x^{5}+6\,a^{4}x^{4}+18\,a^{3}x^{3}+14\,a^{2}x^{2}+17\,ax+9\right)\sqrt{a^{2}cx^{2}+c}e^{(\arctan(ax))}}{26\left(a^{7}c^{4}x^{6}+3\,a^{5}c^{4}x^{4}+3\,a^{3}c^{4}x^{2}+ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/26*(6*a^5*x^5 + 6*a^4*x^4 + 18*a^3*x^3 + 14*a^2*x^2 + 17*a*x + 9)*sqrt(a^
2*c*x^2 + c)*e^(arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2 +
 a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arctan(ax))}}{\left(a^{2}cx^{2}+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

3.259 $\int e^{2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{i2^{p-i}(1-iax)^{p+(1+i)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(i-p,p+(1+i);p+(2+i);\frac{1}{2}(1-iax)\right)}{a(p+(1+i))}$$

```
[Out] (I*2^(-I + p)*(1 - I*a*x)^((1 + I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F1
[I - p, (1 + I) + p, (2 + I) + p, (1 - I*a*x)/2])/(a*((1 + I) + p)*(1 + a^2
*x^2)^p)
```

Rubi [A] time = 0.0703924, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5076, 5073, 69}

$$\frac{i2^{p-i}(1-iax)^{p+(1+i)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(i-p,p+(1+i);p+(2+i);\frac{1}{2}(1-iax)\right)}{a(p+(1+i))}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

```
[Out] (I*2^(-I + p)*(1 - I*a*x)^((1 + I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F1
[I - p, (1 + I) + p, (2 + I) + p, (1 - I*a*x)/2])/(a*((1 + I) + p)*(1 + a^2
*x^2)^p)
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx=\left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{2\tan^{-1}(ax)}\left(1+a^2x^2\right)^p\,dx$$
$$=\left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int(1-iax)^{i+p}(1+iax)^{-i+p}\,dx$$
$$=\frac{i2^{-i+p}(1-iax)^{(1+i)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\,{}_2F_1\left(i-p,(1+i)+p;(2+i)+p;\frac{1}{2}(1-\right.}{a((1+i)+p)}$$

Mathematica [A] time = 0.0237536, size = 90, normalized size = 1.

$$\frac{i2^{p-i}(1-iax)^{p+(1+i)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(i-p,p+(1+i);p+(2+i);\frac{1}{2}(1-iax)\right)}{a(p+(1+i))}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

```
[Out] (I*2^(-I + p)*(1 - I*a*x)^((1 + I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F1
[I - p, (1 + I) + p, (2 + I) + p, (1 - I*a*x)/2])/(a*((1 + I) + p)*(1 + a^2
*x^2)^p)
```

Maple [F] time = 0.279, size = 0, normalized size = 0.

$$\int e^{2\arctan(ax)}\left(a^2cx^2+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

```
[Out] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2cx^2+c\right)^p e^{(2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^p*e^(2*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^2+c\right)^p e^{(2\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^p*e^(2*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\left(a^2x^2+1\right)\right)^p e^{2\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))*(a**2*c*x**2+c)**p,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**p*exp(2*atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^p e^{(2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^p*e^(2*arctan(a*x)), x)
```

3.260 $\int e^{2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{1-i}c^2(1-iax)^{3+i}\,{}_2F_1\left(-2+i,3+i;4+i;\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] ((1/5 + (3*I)/5)*2^(1 - I)*c^2*(1 - I*a*x)^(3 + I)*Hypergeometric2F1[-2 + I
, 3 + I, 4 + I, (1 - I*a*x)/2])/a
```

Rubi [A] time = 0.0389405, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5073, 69}

$$\frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{1-i}c^2(1-iax)^{3+i}\,{}_2F_1\left(-2+i,3+i;4+i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^2,x]
```

```
[Out] ((1/5 + (3*I)/5)*2^(1 - I)*c^2*(1 - I*a*x)^(3 + I)*Hypergeometric2F1[-2 + I
, 3 + I, 4 + I, (1 - I*a*x)/2])/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx &= c^2\int(1-iax)^{2+i}(1+iax)^{2-i}\,dx\\ &= \frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{1-i}c^2(1-iax)^{3+i}\,{}_2F_1\left(-2+i,3+i;4+i;\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0131317, size = 53, normalized size = 1.

$$\frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{1-i}c^2(1-iax)^{3+i}\,{}_2F_1\left(-2+i,3+i;4+i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^2,x]
```

```
[Out] ((1/5 + (3*I)/5)*2^(1 - I)*c^2*(1 - I*a*x)^(3 + I)*Hypergeometric2F1[-2 + I
, 3 + I, 4 + I, (1 - I*a*x)/2])/a
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int e^{2\arctan(ax)}\left(a^2cx^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2 e^{(2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*e^(2*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)e^{(2\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(2*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int 2a^2x^2e^{2\operatorname{atan}(ax)}\,dx+\int a^4x^4e^{2\operatorname{atan}(ax)}\,dx+\int e^{2\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))*(a**2*c*x**2+c)**2,x)
```

```
[Out] c**2*(Integral(2*a**2*x**2*exp(2*atan(a*x)), x) + Integral(a**4*x**4*exp(2*
atan(a*x)), x) + Integral(exp(2*atan(a*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2 e^{(2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*e^(2*arctan(a*x)), x)
```

3.261 $\int e^{2\tan^{-1}(ax)}\left(c+a^2cx^2\right)dx$

Optimal. Leaf size=51

$$\frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-i}c(1-iax)^{2+i}{}_2F_1\left(-1+i,2+i;3+i;\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] ((1/5 + (2*I)/5)*2^(1 - I)*c*(1 - I*a*x)^(2 + I)*Hypergeometric2F1[-1 + I,
2 + I, 3 + I, (1 - I*a*x)/2])/a
```

Rubi [A] time = 0.0256805, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5073, 69}

$$\frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-i}c(1-iax)^{2+i}{}_2F_1\left(-1+i,2+i;3+i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])*(c + a^2*c*x^2),x]
```

```
[Out] ((1/5 + (2*I)/5)*2^(1 - I)*c*(1 - I*a*x)^(2 + I)*Hypergeometric2F1[-1 + I,
2 + I, 3 + I, (1 - I*a*x)/2])/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{2\tan^{-1}(ax)}\left(c+a^2cx^2\right)dx &= c\int (1-iax)^{1+i}(1+iax)^{1-i}\,dx\\ &= \frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-i}c(1-iax)^{2+i}{}_2F_1\left(-1+i,2+i;3+i;\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0122826, size = 51, normalized size = 1.

$$\frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-i}c(1-iax)^{2+i}{}_2F_1\left(-1+i,2+i;3+i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])*(c + a^2*c*x^2),x]
```

```
[Out] ((1/5 + (2*I)/5)*2^(1 - I)*c*(1 - I*a*x)^(2 + I)*Hypergeometric2F1[-1 + I,
2 + I, 3 + I, (1 - I*a*x)/2])/a
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int e^{2\arctan(ax)}\left(a^2cx^2+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))*(a^2*c*x^2+c),x)
```

```
[Out] int(exp(2*arctan(a*x))*(a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)e^{(2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^2+c\right)e^{(2\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int a^2x^2e^{2\operatorname{atan}(ax)}\,dx+\int e^{2\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))*(a**2*c*x**2+c),x)
```

```
[Out] c*(Integral(a**2*x**2*exp(2*atan(a*x)), x) + Integral(exp(2*atan(a*x)), x))
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)e^{(2 \arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

3.262 $\int e^{2\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=46

$$\frac{(1+i)2^{-1-i}(1-iax)^{1+i}\,{}_2F_1\left(i,1+i;2+i;\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] ((1 + I)*(1 - I*a*x)^(1 + I)*Hypergeometric2F1[I, 1 + I, 2 + I, (1 - I*a*x)
/2])/(2^(1 + I)*a)
```

Rubi [A] time = 0.0121184, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5061, 69}

$$\frac{(1+i)2^{-1-i}(1-iax)^{1+i}\,{}_2F_1\left(i,1+i;2+i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x]),x]
```

```
[Out] ((1 + I)*(1 - I*a*x)^(1 + I)*Hypergeometric2F1[I, 1 + I, 2 + I, (1 - I*a*x)
/2])/(2^(1 + I)*a)
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{2\tan^{-1}(ax)}\,dx &= \int (1-iax)^i(1+iax)^{-i}\,dx\\ &= \frac{(1+i)2^{-1-i}(1-iax)^{1+i}\,{}_2F_1\left(i,1+i;2+i;\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0192597, size = 37, normalized size = 0.8

$$\frac{(1-i)e^{(2+2i)\tan^{-1}(ax)}\,{}_2F_1\left(1-i,2;2-i;-e^{2i\tan^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTan[a*x]),x]
```

```
[Out] ((1 - I)*E^((2 + 2*I)*ArcTan[a*x])*Hypergeometric2F1[1 - I, 2, 2 - I, -E^((
2*I)*ArcTan[a*x])])/a
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int e^{2 \arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x)),x)
```

```
[Out] int(exp(2*arctan(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(2 \arctan(ax))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(e^(2*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{(2 \arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral(e^(2*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{2 \operatorname{atan}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x)),x)
```

```
[Out] Integral(exp(2*atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(2 \arctan(ax))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x)),x, algorithm="giac")
```

```
[Out] integrate(e^(2*arctan(a*x)), x)
```

3.263 $\int \frac{e^{2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{e^{2\tan^{-1}(ax)}}{2ac}$$

```
[Out] E^(2*ArcTan[a*x])/(2*a*c)
```

Rubi [A] time = 0.0281226, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5071}

$$\frac{e^{2\tan^{-1}(ax)}}{2ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2),x]

[Out] E^(2*ArcTan[a*x])/(2*a*c)
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx = \frac{e^{2\tan^{-1}(ax)}}{2ac}$$

Mathematica [C] time = 0.006442, size = 34, normalized size = 1.89

$$\frac{(1-iax)^i(1+iax)^{-i}}{2ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2),x]

[Out] (1 - I*a*x)^I/(2*a*c*(1 + I*a*x)^I)
```

Maple [A] time = 0.034, size = 16, normalized size = 0.9

$$\frac{e^{2\arctan(ax)}}{2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c),x)
```

```
[Out] 1/2*exp(2*arctan(a*x))/a/c
```

Maxima [A] time = 1.5258, size = 20, normalized size = 1.11

$$\frac{e^{(2\arctan(ax))}}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*e^(2*arctan(a*x))/(a*c)
```

Fricas [A] time = 2.00773, size = 39, normalized size = 2.17

$$\frac{e^{(2\arctan(ax))}}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*e^(2*arctan(a*x))/(a*c)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.08264, size = 20, normalized size = 1.11

$$\frac{e^{(2\arctan(ax))}}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*e^(2*arctan(a*x))/(a*c)
```

3.264 $\int \frac{e^{2\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=53

$$\frac{(ax+1)e^{2\tan^{-1}(ax)}}{4ac^2(a^2x^2+1)}+\frac{e^{2\tan^{-1}(ax)}}{8ac^2}$$

```
[Out] E^(2*ArcTan[a*x])/(8*a*c^2) + (E^(2*ArcTan[a*x])*(1 + a*x))/(4*a*c^2*(1 + a
^2*x^2))
```

Rubi [A] time = 0.057281, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$\frac{(ax+1)e^{2\tan^{-1}(ax)}}{4ac^2(a^2x^2+1)}+\frac{e^{2\tan^{-1}(ax)}}{8ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] E^(2*ArcTan[a*x])/(8*a*c^2) + (E^(2*ArcTan[a*x])*(1 + a*x))/(4*a*c^2*(1 + a
^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx &= \frac{e^{2\tan^{-1}(ax)}(1+ax)}{4ac^2(1+a^2x^2)}+\frac{\int \frac{e^{2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{4c}\\ &= \frac{e^{2\tan^{-1}(ax)}}{8ac^2}+\frac{e^{2\tan^{-1}(ax)}(1+ax)}{4ac^2(1+a^2x^2)}\end{aligned}$$

Mathematica [C] time = 0.0184911, size = 55, normalized size = 1.04

$$\frac{(1-iax)^i(1+iax)^{-i}(a^2x^2+2ax+3)}{8c^2(a^3x^2+a)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((1 - I*a*x)^I*(3 + 2*a*x + a^2*x^2))/(8*c^2*(1 + I*a*x)^I*(a + a^3*x^2))
```

Maple [A] time = 0.037, size = 40, normalized size = 0.8

$$\frac{e^{2\arctan(ax)}\left(a^2x^2+2ax+3\right)}{\left(8a^2x^2+8\right)ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x)
```

```
[Out] 1/8*exp(2*arctan(a*x))*(a^2*x^2+2*a*x+3)/(a^2*x^2+1)/a/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(2\arctan(ax))}}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

Fricas [A] time = 1.92883, size = 90, normalized size = 1.7

$$\frac{\left(a^2x^2+2ax+3\right)e^{(2\arctan(ax))}}{8\left(a^3c^2x^2+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(a^2*x^2 + 2*a*x + 3)*e^(2*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(2\arctan(ax))}}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

3.265 $\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=88

$$\frac{3(ax+1)e^{2\tan^{-1}(ax)}}{20ac^3\left(a^2x^2+1\right)}+\frac{(2ax+1)e^{2\tan^{-1}(ax)}}{10ac^3\left(a^2x^2+1\right)^2}+\frac{3e^{2\tan^{-1}(ax)}}{40ac^3}$$

```
[Out] (3*E^(2*ArcTan[a*x]))/(40*a*c^3) + (E^(2*ArcTan[a*x])*(1 + 2*a*x))/(10*a*c^
3*(1 + a^2*x^2)^2) + (3*E^(2*ArcTan[a*x])*(1 + a*x))/(20*a*c^3*(1 + a^2*x^2
))
```

Rubi [A] time = 0.0894982, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$\frac{3(ax+1)e^{2\tan^{-1}(ax)}}{20ac^3\left(a^2x^2+1\right)}+\frac{(2ax+1)e^{2\tan^{-1}(ax)}}{10ac^3\left(a^2x^2+1\right)^2}+\frac{3e^{2\tan^{-1}(ax)}}{40ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (3*E^(2*ArcTan[a*x]))/(40*a*c^3) + (E^(2*ArcTan[a*x])*(1 + 2*a*x))/(10*a*c^
3*(1 + a^2*x^2)^2) + (3*E^(2*ArcTan[a*x])*(1 + a*x))/(20*a*c^3*(1 + a^2*x^2
))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx &= \frac{e^{2\tan^{-1}(ax)}(1+2ax)}{10ac^3\left(1+a^2x^2\right)^2}+\frac{3\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{5c}\\
&= \frac{e^{2\tan^{-1}(ax)}(1+2ax)}{10ac^3\left(1+a^2x^2\right)^2}+\frac{3e^{2\tan^{-1}(ax)}(1+ax)}{20ac^3\left(1+a^2x^2\right)}+\frac{3\int \frac{e^{2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{20c^2}\\
&= \frac{3e^{2\tan^{-1}(ax)}}{40ac^3}+\frac{e^{2\tan^{-1}(ax)}(1+2ax)}{10ac^3\left(1+a^2x^2\right)^2}+\frac{3e^{2\tan^{-1}(ax)}(1+ax)}{20ac^3\left(1+a^2x^2\right)}
\end{aligned}$$

Mathematica [C] time = 0.11687, size = 86, normalized size = 0.98

$$\frac{4(2ax+1)e^{2\tan^{-1}(ax)}+3(1-iax)^i(1+iax)^{-i}\left(a^2x^2+1\right)\left(a^2x^2+2ax+3\right)}{40ac^3\left(a^2x^2+1\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (4*E^(2*ArcTan[a*x])*(1 + 2*a*x) + (3*(1 - I*a*x)^I*(1 + a^2*x^2)*(3 + 2*a*
x + a^2*x^2))/(1 + I*a*x)^I)/(40*a*c^3*(1 + a^2*x^2)^2)
```

Maple [A] time = 0.037, size = 57, normalized size = 0.7

$$\frac{e^{2\arctan(ax)}\left(3a^4x^4+6a^3x^3+12a^2x^2+14ax+13\right)}{40\left(a^2x^2+1\right)^2ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/40*exp(2*arctan(a*x))*(3*a^4*x^4+6*a^3*x^3+12*a^2*x^2+14*a*x+13)/(a^2*x^2
+1)^2/a/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{e^{(2\arctan(ax))}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

Fricas [A] time = 2.12437, size = 153, normalized size = 1.74

$$\frac{\left(3a^4x^4+6a^3x^3+12a^2x^2+14ax+13\right)e^{(2\arctan(ax))}}{40\left(a^5c^3x^4+2a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/40*(3*a^4*x^4 + 6*a^3*x^3 + 12*a^2*x^2 + 14*a*x + 13)*e^(2*arctan(a*x))/(
a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(2\arctan(ax))}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

3.266 $\int \frac{e^{2\tan^{-1}(ax)}}{(c+a^2cx^2)^4}\,dx$

Optimal. Leaf size=123

$$\frac{9(ax+1)e^{2\tan^{-1}(ax)}}{80ac^4(a^2x^2+1)}+\frac{3(2ax+1)e^{2\tan^{-1}(ax)}}{40ac^4(a^2x^2+1)^2}+\frac{(3ax+1)e^{2\tan^{-1}(ax)}}{20ac^4(a^2x^2+1)^3}+\frac{9e^{2\tan^{-1}(ax)}}{160ac^4}$$

```
[Out] (9*E^(2*ArcTan[a*x]))/(160*a*c^4) + (E^(2*ArcTan[a*x])*(1 + 3*a*x))/(20*a*c
^4*(1 + a^2*x^2)^3) + (3*E^(2*ArcTan[a*x])*(1 + 2*a*x))/(40*a*c^4*(1 + a^2*
x^2)^2) + (9*E^(2*ArcTan[a*x])*(1 + a*x))/(80*a*c^4*(1 + a^2*x^2))
```

Rubi [A] time = 0.119764, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$\frac{9(ax+1)e^{2\tan^{-1}(ax)}}{80ac^4(a^2x^2+1)}+\frac{3(2ax+1)e^{2\tan^{-1}(ax)}}{40ac^4(a^2x^2+1)^2}+\frac{(3ax+1)e^{2\tan^{-1}(ax)}}{20ac^4(a^2x^2+1)^3}+\frac{9e^{2\tan^{-1}(ax)}}{160ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^4,x]
```

```
[Out] (9*E^(2*ArcTan[a*x]))/(160*a*c^4) + (E^(2*ArcTan[a*x])*(1 + 3*a*x))/(20*a*c
^4*(1 + a^2*x^2)^3) + (3*E^(2*ArcTan[a*x])*(1 + 2*a*x))/(40*a*c^4*(1 + a^2*
x^2)^2) + (9*E^(2*ArcTan[a*x])*(1 + a*x))/(80*a*c^4*(1 + a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4}\,dx = \frac{e^{2\tan^{-1}(ax)}(1+3ax)}{20ac^4\left(1+a^2x^2\right)^3} + \frac{3\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx}{4c}$$

$$= \frac{e^{2\tan^{-1}(ax)}(1+3ax)}{20ac^4\left(1+a^2x^2\right)^3} + \frac{3e^{2\tan^{-1}(ax)}(1+2ax)}{40ac^4\left(1+a^2x^2\right)^2} + \frac{9\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{20c^2}$$

$$= \frac{e^{2\tan^{-1}(ax)}(1+3ax)}{20ac^4\left(1+a^2x^2\right)^3} + \frac{3e^{2\tan^{-1}(ax)}(1+2ax)}{40ac^4\left(1+a^2x^2\right)^2} + \frac{9e^{2\tan^{-1}(ax)}(1+ax)}{80ac^4\left(1+a^2x^2\right)} + \frac{9\int \frac{e^{2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{80c^3}$$

$$= \frac{9e^{2\tan^{-1}(ax)}}{160ac^4} + \frac{e^{2\tan^{-1}(ax)}(1+3ax)}{20ac^4\left(1+a^2x^2\right)^3} + \frac{3e^{2\tan^{-1}(ax)}(1+2ax)}{40ac^4\left(1+a^2x^2\right)^2} + \frac{9e^{2\tan^{-1}(ax)}(1+ax)}{80ac^4\left(1+a^2x^2\right)}$$

Mathematica [C] time = 0.24509, size = 122, normalized size = 0.99

$$\frac{8c(3ax+1)e^{2\tan^{-1}(ax)} + 3\left(a^2cx^2+c\right)\left(4(2ax+1)e^{2\tan^{-1}(ax)} + 3(1-iax)^i(1+iax)^{-i}(ax-i)(ax+i)\left(a^2x^2+2ax+3\right)\right)}{160ac^2\left(a^2cx^2+c\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^4,x]
```

```
[Out] (8*c*E^(2*ArcTan[a*x])*(1 + 3*a*x) + 3*(c + a^2*c*x^2)*(4*E^(2*ArcTan[a*x])
*(1 + 2*a*x) + (3*(1 - I*a*x)^I*(-I + a*x)*(I + a*x)*(3 + 2*a*x + a^2*x^2))
/(1 + I*a*x)^I))/(160*a*c^2*(c + a^2*c*x^2)^3)
```

Maple [A] time = 0.039, size = 73, normalized size = 0.6

$$\frac{e^{2\arctan(ax)}\left(9a^6x^6+18a^5x^5+45a^4x^4+60a^3x^3+75a^2x^2+66ax+47\right)}{160\left(a^2x^2+1\right)^3ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x)
```

```
[Out] 1/160*exp(2*arctan(a*x))*(9*a^6*x^6+18*a^5*x^5+45*a^4*x^4+60*a^3*x^3+75*a^2
*x^2+66*a*x+47)/(a^2*x^2+1)^3/a/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(2\arctan(ax))}}{\left(a^2cx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

Fricas [A] time = 2.02926, size = 212, normalized size = 1.72

$$\frac{\left(9 a^{6} x^{6} + 18 a^{5} x^{5} + 45 a^{4} x^{4} + 60 a^{3} x^{3} + 75 a^{2} x^{2} + 66 a x + 47\right) e^{\left(2 \arctan\left(a x\right)\right)}}{160 \left(a^{7} c^{4} x^{6} + 3 a^{5} c^{4} x^{4} + 3 a^{3} c^{4} x^{2} + a c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/160*(9*a^6*x^6 + 18*a^5*x^5 + 45*a^4*x^4 + 60*a^3*x^3 + 75*a^2*x^2 + 66*a
*x + 47)*e^(2*arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2 + a
*c^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**4,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(2 \arctan\left(a x\right)\right)}}{\left(a^{2} c x^{2} + c\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

3.267 $\int e^{2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\frac{2}{29}+\frac{5i}{29}\right)2^{\frac{5}{2}-i}c(1-iax)^{\frac{5}{2}+i}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{3}{2}+i,\frac{5}{2}+i;\frac{7}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[Out] ((2/29 + (5*I)/29)*2^(5/2 - I)*c*(1 - I*a*x)^(5/2 + I)*Sqrt[c + a^2*c*x^2]*
Hypergeometric2F1[-3/2 + I, 5/2 + I, 7/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 + a
^2*x^2])
```

Rubi [A] time = 0.08045, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$\frac{\left(\frac{2}{29}+\frac{5i}{29}\right)2^{\frac{5}{2}-i}c(1-iax)^{\frac{5}{2}+i}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{3}{2}+i,\frac{5}{2}+i;\frac{7}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2),x]
```


```
[Out] ((2/29 + (5*I)/29)*2^(5/2 - I)*c*(1 - I*a*x)^(5/2 + I)*Sqrt[c + a^2*c*x^2]*
Hypergeometric2F1[-3/2 + I, 5/2 + I, 7/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 + a
^2*x^2])
```


Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}dx=\frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{2\tan^{-1}(ax)}\left(1+a^2x^2\right)^{3/2}dx}{\sqrt{1+a^2x^2}}$$
$$=\frac{\left(c\sqrt{c+a^2cx^2}\right)\int (1-iax)^{\frac{3}{2}+i}(1+iax)^{\frac{3}{2}-i}dx}{\sqrt{1+a^2x^2}}$$
$$=\frac{\left(\frac{2}{29}+\frac{5i}{29}\right)2^{\frac{5}{2}-i}c(1-iax)^{\frac{5}{2}+i}\sqrt{c+a^2cx^2}\,{}_2F_1\left(-\frac{3}{2}+i,\frac{5}{2}+i;\frac{7}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.0210065, size = 88, normalized size = 1.

$$\frac{\left(\frac{2}{29}+\frac{5i}{29}\right)2^{\frac{5}{2}-i}c(1-iax)^{\frac{5}{2}+i}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{3}{2}+i,\frac{5}{2}+i;\frac{7}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((2/29 + (5*I)/29)*2^(5/2 - I)*c*(1 - I*a*x)^(5/2 + I)*Sqrt[c + a^2*c*x^2]*
Hypergeometric2F1[-3/2 + I, 5/2 + I, 7/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 + a
^2*x^2])
```

Maple [F] time = 0.279, size = 0, normalized size = 0.

$$\int e^{2\arctan(ax)}\left(a^2cx^2+c\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(2\arctan(ax))}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*e^(2*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(2\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*e^(2*arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))*(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.268 $\int e^{2\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal. Leaf size=87

$$\frac{\left(\frac{2}{13}+\frac{3i}{13}\right)2^{\frac{3}{2}-i}(1-iax)^{\frac{3}{2}+i}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{1}{2}+i,\frac{3}{2}+i;\frac{5}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[Out] ((2/13 + (3*I)/13)*2^(3/2 - I)*(1 - I*a*x)^(3/2 + I)*Sqrt[c + a^2*c*x^2]*Hy
pergeometric2F1[-1/2 + I, 3/2 + I, 5/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 + a^2
*x^2])
```

Rubi [A] time = 0.0744987, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$\frac{\left(\frac{2}{13}+\frac{3i}{13}\right)2^{\frac{3}{2}-i}(1-iax)^{\frac{3}{2}+i}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{1}{2}+i,\frac{3}{2}+i;\frac{5}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((2/13 + (3*I)/13)*2^(3/2 - I)*(1 - I*a*x)^(3/2 + I)*Sqrt[c + a^2*c*x^2]*Hy
pergeometric2F1[-1/2 + I, 3/2 + I, 5/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 + a^2
*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{2\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx = \frac{\sqrt{c+a^2cx^2}\int e^{2\tan^{-1}(ax)}\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}$$
$$= \frac{\sqrt{c+a^2cx^2}\int(1-iax)^{\frac{1}{2}+i}(1+iax)^{\frac{1}{2}-i}\,dx}{\sqrt{1+a^2x^2}}$$
$$= \frac{\left(\frac{2}{13}+\frac{3i}{13}\right)2^{\frac{3}{2}-i}(1-iax)^{\frac{3}{2}+i}\sqrt{c+a^2cx^2}\,{}_2F_1\left(-\frac{1}{2}+i,\frac{3}{2}+i;\frac{5}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.0178089, size = 87, normalized size = 1.

$$\frac{\left(\frac{2}{13}+\frac{3i}{13}\right)2^{\frac{3}{2}-i}(1-iax)^{\frac{3}{2}+i}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{1}{2}+i,\frac{3}{2}+i;\frac{5}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((2/13 + (3*I)/13)*2^(3/2 - I)*(1 - I*a*x)^(3/2 + I)*Sqrt[c + a^2*c*x^2]*Hy
pergeometric2F1[-1/2 + I, 3/2 + I, 5/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 + a^2
*x^2])
```

Maple [F] time = 0.276, size = 0, normalized size = 0.

$$\int e^{2\arctan(ax)}\sqrt{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2+c}\,e^{(2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^2cx^2+c}\,e^{(2\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\left(a^2x^2+1\right)}e^{2\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*exp(2*atan(a*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.269 $\int \frac{e^{2\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\left(\frac{2}{5}+\frac{i}{5}\right)2^{\frac{1}{2}-i}(1-iax)^{\frac{1}{2}+i}\sqrt{a^2x^2+1}\,{}_2F_1\left(\frac{1}{2}+i,\frac{1}{2}+i;\frac{3}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] ((2/5 + I/5)*2^(1/2 - I)*(1 - I*a*x)^(1/2 + I)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[1/2 + I, 1/2 + I, 3/2 + I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0759546, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$\frac{\left(\frac{2}{5}+\frac{i}{5}\right)2^{\frac{1}{2}-i}(1-iax)^{\frac{1}{2}+i}\sqrt{a^2x^2+1}\,{}_2F_1\left(\frac{1}{2}+i,\frac{1}{2}+i;\frac{3}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((2/5 + I/5)*2^(1/2 - I)*(1 - I*a*x)^(1/2 + I)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[1/2 + I, 1/2 + I, 3/2 + I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{2\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{2\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}+i}(1+iax)^{-\frac{1}{2}-i}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\left(\frac{2}{5}+\frac{i}{5}\right)2^{\frac{1}{2}-i}(1-iax)^{\frac{1}{2}+i}\sqrt{1+a^2x^2}\,{}_2F_1\left(\frac{1}{2}+i,\frac{1}{2}+i;\frac{3}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0183728, size = 87, normalized size = 1.

$$\frac{\left(\frac{2}{5}+\frac{i}{5}\right)2^{\frac{1}{2}-i}(1-iax)^{\frac{1}{2}+i}\sqrt{a^2x^2+1}\,{}_2F_1\left(\frac{1}{2}+i,\frac{1}{2}+i;\frac{3}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((2/5 + I/5)*2^(1/2 - I)*(1 - I*a*x)^(1/2 + I)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[1/2 + I, 1/2 + I, 3/2 + I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2])
```

Maple [F] time = 0.28, size = 0, normalized size = 0.

$$\int e^{2\arctan(ax)}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(2\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(2\arctan(ax))}}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(e^(2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{2\operatorname{atan}(ax)}}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(2*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(2\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.270 $\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{(ax+2)e^{2\tan^{-1}(ax)}}{5ac\sqrt{a^2cx^2+c}}$$

```
[Out] (E^(2*ArcTan[a*x])*(2 + a*x))/(5*a*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0397945, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5069}

$$\frac{(ax+2)e^{2\tan^{-1}(ax)}}{5ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^(2*ArcTan[a*x])*(2 + a*x))/(5*a*c*Sqrt[c + a^2*c*x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx=\frac{e^{2\tan^{-1}(ax)}(2+ax)}{5ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0160972, size = 37, normalized size = 1.

$$\frac{(ax+2)e^{2\tan^{-1}(ax)}}{5ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^(2*ArcTan[a*x])*(2 + a*x))/(5*a*c*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.039, size = 39, normalized size = 1.1

$$\frac{\left(a^2x^2+1\right)(ax+2)\,e^{2\arctan(ax)}}{5\,a}\left(a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/5*(a^2*x^2+1)*(a*x+2)*exp(2*arctan(a*x))/a/(a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A] time = 2.02885, size = 101, normalized size = 2.73

$$\frac{\sqrt{a^2cx^2+c}(ax+2)e^{(2\arctan(ax))}}{5\left(a^3c^2x^2+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(a^2*c*x^2 + c)*(a*x + 2)*e^(2*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{2\operatorname{atan}(ax)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(exp(2*atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")

[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

3.271 $\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{6(ax+2)e^{2\tan^{-1}(ax)}}{65ac^2\sqrt{a^2cx^2+c}}+\frac{(3ax+2)e^{2\tan^{-1}(ax)}}{13ac\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] (E^(2*ArcTan[a*x])*(2 + 3*a*x))/(13*a*c*(c + a^2*c*x^2)^(3/2)) + (6*E^(2*Ar
cTan[a*x])*(2 + a*x))/(65*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0811724, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5070, 5069}

$$\frac{6(ax+2)e^{2\tan^{-1}(ax)}}{65ac^2\sqrt{a^2cx^2+c}}+\frac{(3ax+2)e^{2\tan^{-1}(ax)}}{13ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(2*ArcTan[a*x])*(2 + 3*a*x))/(13*a*c*(c + a^2*c*x^2)^(3/2)) + (6*E^(2*Ar
cTan[a*x])*(2 + a*x))/(65*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx &= \frac{e^{2\tan^{-1}(ax)}(2+3ax)}{13ac\left(c+a^2cx^2\right)^{3/2}}+\frac{6\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{13c}\\
&= \frac{e^{2\tan^{-1}(ax)}(2+3ax)}{13ac\left(c+a^2cx^2\right)^{3/2}}+\frac{6e^{2\tan^{-1}(ax)}(2+ax)}{65ac^2\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0340057, size = 62, normalized size = 0.82

$$\frac{\left(6a^3x^3+12a^2x^2+21ax+22\right)e^{2\tan^{-1}(ax)}}{65c^2\left(a^3x^2+a\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(2*ArcTan[a*x])*(22 + 21*a*x + 12*a^2*x^2 + 6*a^3*x^3))/(65*c^2*(a + a^3
*x^2)*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.04, size = 56, normalized size = 0.7

$$\frac{\left(a^{2} x^{2}+1\right)\left(6 a^{3} x^{3}+12 a^{2} x^{2}+21 a x+22\right) \mathrm{e}^{2 \arctan(a x)}}{65 a}\left(a^{2} c x^{2}+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/65*(a^2*x^2+1)*(6*a^3*x^3+12*a^2*x^2+21*a*x+22)*exp(2*arctan(a*x))/a/(a^2
*c*x^2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(2 \arctan(a x))}}{\left(a^{2} c x^{2}+c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 2.01751, size = 163, normalized size = 2.14

$$\frac{\left(6 a^{3} x^{3}+12 a^{2} x^{2}+21 a x+22\right) \sqrt{a^{2} c x^{2}+c}\, e^{(2 \arctan(a x))}}{65\left(a^{5} c^{3} x^{4}+2 a^{3} c^{3} x^{2}+a c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/65*(6*a^3*x^3 + 12*a^2*x^2 + 21*a*x + 22)*sqrt(a^2*c*x^2 + c)*e^(2*arctan
(a*x))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(2\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

3.272 $\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=114

$$\frac{24(ax+2)e^{2\tan^{-1}(ax)}}{377ac^3\sqrt{a^2cx^2+c}}+\frac{20(3ax+2)e^{2\tan^{-1}(ax)}}{377ac^2\left(a^2cx^2+c\right)^{3/2}}+\frac{(5ax+2)e^{2\tan^{-1}(ax)}}{29ac\left(a^2cx^2+c\right)^{5/2}}$$

```
[Out] (E^(2*ArcTan[a*x])*(2 + 5*a*x))/(29*a*c*(c + a^2*c*x^2)^(5/2)) + (20*E^(2*A
rcTan[a*x])*(2 + 3*a*x))/(377*a*c^2*(c + a^2*c*x^2)^(3/2)) + (24*E^(2*ArcTa
n[a*x])*(2 + a*x))/(377*a*c^3*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.12535, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5070, 5069}

$$\frac{24(ax+2)e^{2\tan^{-1}(ax)}}{377ac^3\sqrt{a^2cx^2+c}}+\frac{20(3ax+2)e^{2\tan^{-1}(ax)}}{377ac^2\left(a^2cx^2+c\right)^{3/2}}+\frac{(5ax+2)e^{2\tan^{-1}(ax)}}{29ac\left(a^2cx^2+c\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (E^(2*ArcTan[a*x])*(2 + 5*a*x))/(29*a*c*(c + a^2*c*x^2)^(5/2)) + (20*E^(2*A
rcTan[a*x])*(2 + 3*a*x))/(377*a*c^2*(c + a^2*c*x^2)^(3/2)) + (24*E^(2*ArcTa
n[a*x])*(2 + a*x))/(377*a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx = \frac{e^{2\tan^{-1}(ax)}(2+5ax)}{29ac\left(c+a^2cx^2\right)^{5/2}} + \frac{20\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{29c}$$

$$= \frac{e^{2\tan^{-1}(ax)}(2+5ax)}{29ac\left(c+a^2cx^2\right)^{5/2}} + \frac{20e^{2\tan^{-1}(ax)}(2+3ax)}{377ac^2\left(c+a^2cx^2\right)^{3/2}} + \frac{120\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{377c^2}$$

$$= \frac{e^{2\tan^{-1}(ax)}(2+5ax)}{29ac\left(c+a^2cx^2\right)^{5/2}} + \frac{20e^{2\tan^{-1}(ax)}(2+3ax)}{377ac^2\left(c+a^2cx^2\right)^{3/2}} + \frac{24e^{2\tan^{-1}(ax)}(2+ax)}{377ac^3\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0378807, size = 81, normalized size = 0.71

$$\frac{\left(24a^5x^5+48a^4x^4+108a^3x^3+136a^2x^2+149ax+114\right)e^{2\tan^{-1}(ax)}}{377ac^3\left(a^2x^2+1\right)^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (E^(2*ArcTan[a*x])*(114 + 149*a*x + 136*a^2*x^2 + 108*a^3*x^3 + 48*a^4*x^4
+ 24*a^5*x^5))/(377*a*c^3*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.037, size = 72, normalized size = 0.6

$$\frac{\left(a^2x^2+1\right)\left(24\,a^5x^5+48\,a^4x^4+108\,a^3x^3+136\,a^2x^2+149\,ax+114\right)e^{2\arctan(ax)}}{377\,a}\left(a^2cx^2+c\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/377*(a^2*x^2+1)*(24*a^5*x^5+48*a^4*x^4+108*a^3*x^3+136*a^2*x^2+149*a*x+11
4)*exp(2*arctan(a*x))/a/(a^2*c*x^2+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(2\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

Fricas [A] time = 1.86729, size = 228, normalized size = 2.

$$\frac{\left(24\,a^{5}x^{5}+48\,a^{4}x^{4}+108\,a^{3}x^{3}+136\,a^{2}x^{2}+149\,ax+114\right)\sqrt{a^{2}cx^{2}+c}e^{(2\,\arctan(ax))}}{377\left(a^{7}c^{4}x^{6}+3\,a^{5}c^{4}x^{4}+3\,a^{3}c^{4}x^{2}+ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/377*(24*a^5*x^5 + 48*a^4*x^4 + 108*a^3*x^3 + 136*a^2*x^2 + 149*a*x + 114)
*sqrt(a^2*c*x^2 + c)*e^(2*arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3
*c^4*x^2 + a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(2\,\arctan(ax))}}{\left(a^{2}cx^{2}+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

3.273 $\int e^{-\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx$

Optimal. Leaf size=101

$$\frac{2^{p+\left(1+\frac{i}{2}\right)}(1-iax)^{p+\left(1-\frac{i}{2}\right)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p{}_2F_1\left(-p-\frac{i}{2},p+\left(1-\frac{i}{2}\right);p+\left(2-\frac{i}{2}\right);\frac{1}{2}(1-iax)\right)}{a(-2ip-(1+2i))}$$

```
[Out] (2^((1 + I/2) + p)*(1 - I*a*x)^((1 - I/2) + p)*(c + a^2*c*x^2)^p*Hypergeome
tric2F1[-I/2 - p, (1 - I/2) + p, (2 - I/2) + p, (1 - I*a*x)/2])/(a*((-1 - 2
*I) - (2*I)*p)*(1 + a^2*x^2)^p)
```

Rubi [A] time = 0.0732523, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5076, 5073, 69}

$$\frac{2^{p+\left(1+\frac{i}{2}\right)}(1-iax)^{p+\left(1-\frac{i}{2}\right)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p{}_2F_1\left(-p-\frac{i}{2},p+\left(1-\frac{i}{2}\right);p+\left(2-\frac{i}{2}\right);\frac{1}{2}(1-iax)\right)}{a(-2ip-(1+2i))}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^p/E^ArcTan[a*x],x]
```

```
[Out] (2^((1 + I/2) + p)*(1 - I*a*x)^((1 - I/2) + p)*(c + a^2*c*x^2)^p*Hypergeome
tric2F1[-I/2 - p, (1 - I/2) + p, (2 - I/2) + p, (1 - I*a*x)/2])/(a*((-1 - 2
*I) - (2*I)*p)*(1 + a^2*x^2)^p)
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{-\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx=\left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{-\tan^{-1}(ax)}\left(1+a^2x^2\right)^p\,dx$$
$$=\left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int(1-iax)^{-\frac{i}{2}+p}(1+iax)^{\frac{i}{2}+p}\,dx$$
$$=\frac{2^{\left(1+\frac{i}{2}\right)+p}(1-iax)^{\left(1-\frac{i}{2}\right)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\,{}_2F_1\left(-\frac{i}{2}-p,\left(1-\frac{i}{2}\right)+p;\left(2-\frac{i}{2}\right)+p;\ldots\right)}{a((-1-2i)-2ip)}$$

Mathematica [A] time = 0.0256839, size = 102, normalized size = 1.01

$$\frac{i2^{p+\frac{i}{2}}(1-iax)^{p+\left(1-\frac{i}{2}\right)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(-p-\frac{i}{2},p+\left(1-\frac{i}{2}\right);p+\left(2-\frac{i}{2}\right);\frac{1}{2}(1-iax)\right)}{a\left(p+\left(1-\frac{i}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^p/E^ArcTan[a*x],x]
```

```
[Out] (I*2^(I/2 + p)*(1 - I*a*x)^((1 - I/2) + p)*(c + a^2*c*x^2)^p*Hypergeometric
2F1[-I/2 - p, (1 - I/2) + p, (2 - I/2) + p, (1 - I*a*x)/2])/(a*((1 - I/2) +
 p)*(1 + a^2*x^2)^p)
```

Maple [F] time = 0.323, size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2+c\right)^p}{e^{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^p/exp(arctan(a*x)),x)
```

```
[Out] int((a^2*c*x^2+c)^p/exp(arctan(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2cx^2+c\right)^p e^{(-\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^p*e^(-arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^2+c\right)^p e^{(-\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^p*e^(-arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**p/exp(atan(a*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^p e^{(-\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(arctan(a*x)),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^p*e^(-arctan(a*x)), x)
```

3.274 $\int e^{-\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx$

Optimal. Leaf size=63

$$-\frac{\left(\frac{1}{37}-\frac{6i}{37}\right)2^{3+\frac{i}{2}}c^2(1-iax)^{3-\frac{i}{2}}\,{}_2F_1\left(-2-\frac{i}{2},3-\frac{i}{2};4-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] ((-1/37 + (6*I)/37)*2^(3 + I/2)*c^2*(1 - I*a*x)^(3 - I/2)*Hypergeometric2F1
[-2 - I/2, 3 - I/2, 4 - I/2, (1 - I*a*x)/2])/a
```

Rubi [A] time = 0.0366298, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5073, 69}

$$-\frac{\left(\frac{1}{37}-\frac{6i}{37}\right)2^{3+\frac{i}{2}}c^2(1-iax)^{3-\frac{i}{2}}\,{}_2F_1\left(-2-\frac{i}{2},3-\frac{i}{2};4-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2/E^ArcTan[a*x],x]
```

```
[Out] ((-1/37 + (6*I)/37)*2^(3 + I/2)*c^2*(1 - I*a*x)^(3 - I/2)*Hypergeometric2F1
[-2 - I/2, 3 - I/2, 4 - I/2, (1 - I*a*x)/2])/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{-\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx &= c^2\int(1-iax)^{2-\frac{i}{2}}(1+iax)^{2+\frac{i}{2}}\,dx\\ &= -\frac{\left(\frac{1}{37}-\frac{6i}{37}\right)2^{3+\frac{i}{2}}c^2(1-iax)^{3-\frac{i}{2}}\,{}_2F_1\left(-2-\frac{i}{2},3-\frac{i}{2};4-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0138363, size = 63, normalized size = 1.

$$-\frac{\left(\frac{1}{37}-\frac{6i}{37}\right)2^{3+\frac{i}{2}}c^2(1-iax)^{3-\frac{i}{2}}\,{}_2F_1\left(-2-\frac{i}{2},3-\frac{i}{2};4-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2/E^ArcTan[a*x],x]
```

```
[Out] ((-1/37 + (6*I)/37)*2^(3 + I/2)*c^2*(1 - I*a*x)^(3 - I/2)*Hypergeometric2F1
[-2 - I/2, 3 - I/2, 4 - I/2, (1 - I*a*x)/2])/a
```

Maple [F] time = 0.19, size = 0, normalized size = 0.

$$\int \frac{\left(a^2 c x^2 + c\right)^2}{\mathrm{e}^{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/exp(arctan(a*x)),x)
```

```
[Out] int((a^2*c*x^2+c)^2/exp(arctan(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 c x^2 + c\right)^2 e^{(-\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/exp(arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*e^(-arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^4 c^2 x^4 + 2\,a^2 c^2 x^2 + c^2\right) e^{(-\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/exp(arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(-arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/exp(atan(a*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^2 e^{(-\arctan(ax))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/exp(arctan(a*x)),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*e^(-arctan(a*x)), x)
```

3.275 $\int e^{-\tan^{-1}(ax)}\left(c+a^2cx^2\right)\,dx$

Optimal. Leaf size=61

$$-\frac{\left(\frac{1}{17}-\frac{4i}{17}\right)2^{2+\frac{i}{2}}c(1-iax)^{2-\frac{i}{2}}\,{}_2F_1\left(-1-\frac{i}{2},2-\frac{i}{2};3-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] ((-1/17 + (4*I)/17)*2^(2 + I/2)*c*(1 - I*a*x)^(2 - I/2)*Hypergeometric2F1[-
1 - I/2, 2 - I/2, 3 - I/2, (1 - I*a*x)/2])/a
```

Rubi [A] time = 0.0259134, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5073, 69}

$$-\frac{\left(\frac{1}{17}-\frac{4i}{17}\right)2^{2+\frac{i}{2}}c(1-iax)^{2-\frac{i}{2}}\,{}_2F_1\left(-1-\frac{i}{2},2-\frac{i}{2};3-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)/E^ArcTan[a*x],x]
```

```
[Out] ((-1/17 + (4*I)/17)*2^(2 + I/2)*c*(1 - I*a*x)^(2 - I/2)*Hypergeometric2F1[-
1 - I/2, 2 - I/2, 3 - I/2, (1 - I*a*x)/2])/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{-\tan^{-1}(ax)}\left(c+a^2cx^2\right)\,dx &= c\int(1-iax)^{1-\frac{i}{2}}(1+iax)^{1+\frac{i}{2}}\,dx\\ &= -\frac{\left(\frac{1}{17}-\frac{4i}{17}\right)2^{2+\frac{i}{2}}c(1-iax)^{2-\frac{i}{2}}\,{}_2F_1\left(-1-\frac{i}{2},2-\frac{i}{2};3-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0125183, size = 61, normalized size = 1.

$$-\frac{\left(\frac{1}{17}-\frac{4i}{17}\right)2^{2+\frac{i}{2}}c(1-iax)^{2-\frac{i}{2}}\,{}_2F_1\left(-1-\frac{i}{2},2-\frac{i}{2};3-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)/E^ArcTan[a*x],x]
```

```
[Out] ((-1/17 + (4*I)/17)*2^(2 + I/2)*c*(1 - I*a*x)^(2 - I/2)*Hypergeometric2F1[-
1 - I/2, 2 - I/2, 3 - I/2, (1 - I*a*x)/2])/a
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{a^2cx^2+c}{\mathrm{e}^{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/exp(arctan(a*x)),x)
```

```
[Out] int((a^2*c*x^2+c)/exp(arctan(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)e^{(-\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/exp(arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2cx^2+c\right)e^{(-\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/exp(arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/exp(atan(a*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)e^{(-\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/exp(arctan(a*x)),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

3.276 $\int e^{-\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=60

$$-\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+\frac{i}{2}}(1-iax)^{1-\frac{i}{2}}\,{}_2F_1\left(-\frac{i}{2},1-\frac{i}{2};2-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] ((-1/5 + (2*I)/5)*2^(1 + I/2)*(1 - I*a*x)^(1 - I/2)*Hypergeometric2F1[-I/2,
 1 - I/2, 2 - I/2, (1 - I*a*x)/2])/a
```

Rubi [A] time = 0.0113338, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5061, 69}

$$-\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+\frac{i}{2}}(1-iax)^{1-\frac{i}{2}}\,{}_2F_1\left(-\frac{i}{2},1-\frac{i}{2};2-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-ArcTan[a*x]),x]
```

```
[Out] ((-1/5 + (2*I)/5)*2^(1 + I/2)*(1 - I*a*x)^(1 - I/2)*Hypergeometric2F1[-I/2,
 1 - I/2, 2 - I/2, (1 - I*a*x)/2])/a
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] && !IntegerQ[(I*n - 1)/2]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{-\tan^{-1}(ax)}\,dx &= \int (1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}\,dx\\ &= -\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+\frac{i}{2}}(1-iax)^{1-\frac{i}{2}}\,{}_2F_1\left(-\frac{i}{2},1-\frac{i}{2};2-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0209074, size = 45, normalized size = 0.75

$$-\frac{\left(\frac{4}{5}+\frac{8i}{5}\right)e^{(-1+2i)\tan^{-1}(ax)}\,{}_2F_1\left(1+\frac{i}{2},2;2+\frac{i}{2};-e^{2i\tan^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-ArcTan[a*x]),x]
```

```
[Out] ((-4/5 - (8*I)/5)*Hypergeometric2F1[1 + I/2, 2, 2 + I/2, -E^((2*I)*ArcTan[a
*x])])/(a*E^((1 - 2*I)*ArcTan[a*x]))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \mathrm{e}^{-\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-arctan(a*x)),x)
```

```
[Out] int(exp(-arctan(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(-\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(e^(-arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(e^{(-\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral(e^(-arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{-\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-atan(a*x)),x)
```

```
[Out] Integral(exp(-atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(-\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-arctan(a*x)),x, algorithm="giac")
```

```
[Out] integrate(e^(-arctan(a*x)), x)
```

3.277 $\int \frac{e^{-\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=16

$$-\frac{e^{-\tan^{-1}(ax)}}{ac}$$

```
[Out] -(1/(a*c*E^ArcTan[a*x]))
```

Rubi [A] time = 0.0267153, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5071}

$$-\frac{e^{-\tan^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)),x]

[Out] -(1/(a*c*E^ArcTan[a*x]))
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{-\tan^{-1}(ax)}}{c+a^2cx^2}\,dx=-\frac{e^{-\tan^{-1}(ax)}}{ac}$$

Mathematica [C] time = 0.0084977, size = 36, normalized size = 2.25

$$-\frac{(1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}}{ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)),x]

[Out] -((1 + I*a*x)^(I/2)/(a*c*(1 - I*a*x)^(I/2)))
```

Maple [A] time = 0.038, size = 16, normalized size = 1.

$$-\frac{1}{ace^{\arctan(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c),x)

[Out] -1/a/c/exp(arctan(a*x))
```

Maxima [A] time = 1.04651, size = 31, normalized size = 1.94

$$-\frac{2 e^{(-\arctan(ax))}}{a^3cx^2 + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")

[Out] -2*e^(-arctan(a*x))/(a^3*c*x^2 + a*c)
```

Fricas [A] time = 1.96871, size = 34, normalized size = 2.12

$$-\frac{e^{(-\arctan(ax))}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")

[Out] -e^(-arctan(a*x))/(a*c)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c),x)

[Out] Exception raised: TypeError
```

Giac [A] time = 1.09327, size = 20, normalized size = 1.25

$$-\frac{e^{(-\arctan(ax))}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")

[Out] -e^(-arctan(a*x))/(a*c)
```

3.278 $\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2} \, dx$

Optimal. Leaf size=54

$$-\frac{(1-2ax)e^{-\tan^{-1}(ax)}}{5ac^2\left(a^2x^2+1\right)}-\frac{2e^{-\tan^{-1}(ax)}}{5ac^2}$$

```
[Out] -2/(5*a*c^2*E^ArcTan[a*x]) - (1 - 2*a*x)/(5*a*c^2*E^ArcTan[a*x]*(1 + a^2*x^
2))
```

Rubi [A] time = 0.0539602, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$-\frac{(1-2ax)e^{-\tan^{-1}(ax)}}{5ac^2\left(a^2x^2+1\right)}-\frac{2e^{-\tan^{-1}(ax)}}{5ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^2),x]
```

```
[Out] -2/(5*a*c^2*E^ArcTan[a*x]) - (1 - 2*a*x)/(5*a*c^2*E^ArcTan[a*x]*(1 + a^2*x^
2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2} \, dx &= -\frac{e^{-\tan^{-1}(ax)}(1-2ax)}{5ac^2\left(1+a^2x^2\right)}+\frac{2\int \frac{e^{-\tan^{-1}(ax)}}{c+a^2cx^2} \, dx}{5c}\\ &= -\frac{2e^{-\tan^{-1}(ax)}}{5ac^2}-\frac{e^{-\tan^{-1}(ax)}(1-2ax)}{5ac^2\left(1+a^2x^2\right)}\end{aligned}$$

Mathematica [C] time = 0.0222546, size = 60, normalized size = 1.11

$$-\frac{(1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}\left(2a^2x^2-2ax+3\right)}{5c^2\left(a^3x^2+a\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^2),x]
```

```
[Out] -((1 + I*a*x)^(I/2)*(3 - 2*a*x + 2*a^2*x^2))/(5*c^2*(1 - I*a*x)^(I/2)*(a +
a^3*x^2))
```

Maple [A] time = 0.036, size = 41, normalized size = 0.8

$$-\frac{2\,a^2x^2-2\,ax+3}{\left(5\,a^2x^2+5\right)c^2\mathrm{e}^{\arctan(ax)}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^2,x)
```

```
[Out] -1/5*(2*a^2*x^2-2*a*x+3)/(a^2*x^2+1)/c^2/exp(arctan(a*x))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-\arctan(ax))}}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

Fricas [A] time = 1.79043, size = 93, normalized size = 1.72

$$-\frac{\left(2\,a^2x^2-2\,ax+3\right)e^{(-\arctan(ax))}}{5\left(a^3c^2x^2+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/5*(2*a^2*x^2 - 2*a*x + 3)*e^(-arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

3.279 $\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3} dx$

Optimal. Leaf size=89

$$-\frac{(1-4ax)e^{-\tan^{-1}(ax)}}{17ac^3\left(a^2x^2+1\right)^2}-\frac{12(1-2ax)e^{-\tan^{-1}(ax)}}{85ac^3\left(a^2x^2+1\right)}-\frac{24e^{-\tan^{-1}(ax)}}{85ac^3}$$

```
[Out] -24/(85*a*c^3*E^ArcTan[a*x]) - (1 - 4*a*x)/(17*a*c^3*E^ArcTan[a*x]*(1 + a^2
*x^2)^2) - (12*(1 - 2*a*x))/(85*a*c^3*E^ArcTan[a*x]*(1 + a^2*x^2))
```

Rubi [A] time = 0.0825961, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$-\frac{(1-4ax)e^{-\tan^{-1}(ax)}}{17ac^3\left(a^2x^2+1\right)^2}-\frac{12(1-2ax)e^{-\tan^{-1}(ax)}}{85ac^3\left(a^2x^2+1\right)}-\frac{24e^{-\tan^{-1}(ax)}}{85ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^3),x]
```

```
[Out] -24/(85*a*c^3*E^ArcTan[a*x]) - (1 - 4*a*x)/(17*a*c^3*E^ArcTan[a*x]*(1 + a^2
*x^2)^2) - (12*(1 - 2*a*x))/(85*a*c^3*E^ArcTan[a*x]*(1 + a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3} dx &= -\frac{e^{-\tan^{-1}(ax)}(1-4ax)}{17ac^3\left(1+a^2x^2\right)^2}+\frac{12\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2} dx}{17c}\\
&= -\frac{e^{-\tan^{-1}(ax)}(1-4ax)}{17ac^3\left(1+a^2x^2\right)^2}-\frac{12e^{-\tan^{-1}(ax)}(1-2ax)}{85ac^3\left(1+a^2x^2\right)}+\frac{24\int \frac{e^{-\tan^{-1}(ax)}}{c+a^2cx^2} dx}{85c^2}\\
&= -\frac{24e^{-\tan^{-1}(ax)}}{85ac^3}-\frac{e^{-\tan^{-1}(ax)}(1-4ax)}{17ac^3\left(1+a^2x^2\right)^2}-\frac{12e^{-\tan^{-1}(ax)}(1-2ax)}{85ac^3\left(1+a^2x^2\right)}
\end{aligned}$$

Mathematica [C] time = 0.1476, size = 91, normalized size = 1.02

$$\frac{5(4ax-1)e^{-\tan^{-1}(ax)}-12(1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}\left(a^2x^2+1\right)\left(2a^2x^2-2ax+3\right)}{85ac^3\left(a^2x^2+1\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^3),x]
```

```
[Out] ((5*(-1 + 4*a*x))/E^ArcTan[a*x] - (12*(1 + I*a*x)^(I/2)*(1 + a^2*x^2)*(3 -
2*a*x + 2*a^2*x^2))/(1 - I*a*x)^(I/2))/(85*a*c^3*(1 + a^2*x^2)^2)
```

Maple [A] time = 0.036, size = 57, normalized size = 0.6

$$-\frac{24\,a^4x^4-24\,a^3x^3+60\,a^2x^2-44\,ax+41}{85\,c^3\left(a^2x^2+1\right)^2\mathrm{e}^{\arctan(ax)}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^3,x)
```

```
[Out] -1/85*(24*a^4*x^4-24*a^3*x^3+60*a^2*x^2-44*a*x+41)/(a^2*x^2+1)^2/c^3/exp(ar
ctan(a*x))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-\arctan(ax))}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

Fricas [A] time = 2.0356, size = 155, normalized size = 1.74

$$-\frac{\left(24\,a^4x^4-24\,a^3x^3+60\,a^2x^2-44\,ax+41\right)e^{(-\arctan(ax))}}{85\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/85*(24*a^4*x^4 - 24*a^3*x^3 + 60*a^2*x^2 - 44*a*x + 41)*e^(-arctan(a*x))
/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-\arctan(ax))}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

3.280 $\int \frac{e^{-\tan^{-1}(ax)}}{(c+a^2cx^2)^4}\,dx$

Optimal. Leaf size=124

$$-\frac{(1-6ax)e^{-\tan^{-1}(ax)}}{37ac^4\left(a^2x^2+1\right)^3}-\frac{72(1-2ax)e^{-\tan^{-1}(ax)}}{629ac^4\left(a^2x^2+1\right)}-\frac{30(1-4ax)e^{-\tan^{-1}(ax)}}{629ac^4\left(a^2x^2+1\right)^2}-\frac{144e^{-\tan^{-1}(ax)}}{629ac^4}$$

```
[Out] -144/(629*a*c^4*E^ArcTan[a*x]) - (1 - 6*a*x)/(37*a*c^4*E^ArcTan[a*x]*(1 + a
^2*x^2)^3) - (30*(1 - 4*a*x))/(629*a*c^4*E^ArcTan[a*x]*(1 + a^2*x^2)^2) - (
72*(1 - 2*a*x))/(629*a*c^4*E^ArcTan[a*x]*(1 + a^2*x^2))
```

Rubi [A] time = 0.119277, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$-\frac{(1-6ax)e^{-\tan^{-1}(ax)}}{37ac^4\left(a^2x^2+1\right)^3}-\frac{72(1-2ax)e^{-\tan^{-1}(ax)}}{629ac^4\left(a^2x^2+1\right)}-\frac{30(1-4ax)e^{-\tan^{-1}(ax)}}{629ac^4\left(a^2x^2+1\right)^2}-\frac{144e^{-\tan^{-1}(ax)}}{629ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^4),x]
```

```
[Out] -144/(629*a*c^4*E^ArcTan[a*x]) - (1 - 6*a*x)/(37*a*c^4*E^ArcTan[a*x]*(1 + a
^2*x^2)^3) - (30*(1 - 4*a*x))/(629*a*c^4*E^ArcTan[a*x]*(1 + a^2*x^2)^2) - (
72*(1 - 2*a*x))/(629*a*c^4*E^ArcTan[a*x]*(1 + a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4}\,dx = -\frac{e^{-\tan^{-1}(ax)}(1-6ax)}{37ac^4\left(1+a^2x^2\right)^3} + \frac{30\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx}{37c}$$

$$= -\frac{e^{-\tan^{-1}(ax)}(1-6ax)}{37ac^4\left(1+a^2x^2\right)^3} - \frac{30e^{-\tan^{-1}(ax)}(1-4ax)}{629ac^4\left(1+a^2x^2\right)^2} + \frac{360\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{629c^2}$$

$$= -\frac{e^{-\tan^{-1}(ax)}(1-6ax)}{37ac^4\left(1+a^2x^2\right)^3} - \frac{30e^{-\tan^{-1}(ax)}(1-4ax)}{629ac^4\left(1+a^2x^2\right)^2} - \frac{72e^{-\tan^{-1}(ax)}(1-2ax)}{629ac^4\left(1+a^2x^2\right)} + \frac{144\int \frac{e^{-\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{629c^3}$$

$$= -\frac{144e^{-\tan^{-1}(ax)}}{629ac^4} - \frac{e^{-\tan^{-1}(ax)}(1-6ax)}{37ac^4\left(1+a^2x^2\right)^3} - \frac{30e^{-\tan^{-1}(ax)}(1-4ax)}{629ac^4\left(1+a^2x^2\right)^2} - \frac{72e^{-\tan^{-1}(ax)}(1-2ax)}{629ac^4\left(1+a^2x^2\right)}$$

Mathematica [C] time = 0.346536, size = 127, normalized size = 1.02

$$\frac{17c(6ax-1)e^{-\tan^{-1}(ax)} - 6\left(a^2cx^2+c\right)\left(5(1-4ax)e^{-\tan^{-1}(ax)} + 12(1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}(ax-i)(ax+i)\left(2a^2x^2-2ax+3\right)\right)}{629ac^2\left(a^2cx^2+c\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^4),x]
```

```
[Out] ((17*c*(-1 + 6*a*x))/E^ArcTan[a*x] - 6*(c + a^2*c*x^2)*((5*(1 - 4*a*x))/E^A
rcTan[a*x] + (12*(1 + I*a*x)^(I/2)*(-I + a*x)*(I + a*x)*(3 - 2*a*x + 2*a^2*
x^2))/(1 - I*a*x)^(I/2)))/(629*a*c^2*(c + a^2*c*x^2)^3)
```

Maple [A] time = 0.039, size = 73, normalized size = 0.6

$$-\frac{144\,a^6x^6 - 144\,a^5x^5 + 504\,a^4x^4 - 408\,a^3x^3 + 606\,a^2x^2 - 366\,ax + 263}{629\,c^4\left(a^2x^2+1\right)^3 \mathrm{e}^{\arctan(ax)}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^4,x)
```

```
[Out] -1/629*(144*a^6*x^6-144*a^5*x^5+504*a^4*x^4-408*a^3*x^3+606*a^2*x^2-366*a*x
+263)/(a^2*x^2+1)^3/c^4/exp(arctan(a*x))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-\arctan(ax))}}{\left(a^2cx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

Fricas [A] time = 1.98611, size = 223, normalized size = 1.8

$$-\frac{\left(144\,a^{6}x^{6}-144\,a^{5}x^{5}+504\,a^{4}x^{4}-408\,a^{3}x^{3}+606\,a^{2}x^{2}-366\,ax+263\right)e^{(-\arctan(ax))}}{629\left(a^{7}c^{4}x^{6}+3\,a^{5}c^{4}x^{4}+3\,a^{3}c^{4}x^{2}+ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/629*(144*a^6*x^6 - 144*a^5*x^5 + 504*a^4*x^4 - 408*a^3*x^3 + 606*a^2*x^2
 - 366*a*x + 263)*e^(-arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4
*x^2 + a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-\arctan(ax))}}{\left(a^{2}cx^{2}+c\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

3.281 $\int e^{-\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=98

$$-\frac{\left(\frac{1}{13}-\frac{5i}{13}\right)2^{\frac{3}{2}+\frac{i}{2}}c(1-iax)^{\frac{5}{2}-\frac{i}{2}}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2};\frac{7}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[Out] ((-1/13 + (5*I)/13)*2^(3/2 + I/2)*c*(1 - I*a*x)^(5/2 - I/2)*Sqrt[c + a^2*c*
x^2]*Hypergeometric2F1[-3/2 - I/2, 5/2 - I/2, 7/2 - I/2, (1 - I*a*x)/2])/(a
*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.0764833, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$-\frac{\left(\frac{1}{13}-\frac{5i}{13}\right)2^{\frac{3}{2}+\frac{i}{2}}c(1-iax)^{\frac{5}{2}-\frac{i}{2}}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2};\frac{7}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/E^ArcTan[a*x],x]
```

```
[Out] ((-1/13 + (5*I)/13)*2^(3/2 + I/2)*c*(1 - I*a*x)^(5/2 - I/2)*Sqrt[c + a^2*c*
x^2]*Hypergeometric2F1[-3/2 - I/2, 5/2 - I/2, 7/2 - I/2, (1 - I*a*x)/2])/(a
*Sqrt[1 + a^2*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{-\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}dx = \frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{-\tan^{-1}(ax)}\left(1+a^2x^2\right)^{3/2}dx}{\sqrt{1+a^2x^2}}$$
$$= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int (1-iax)^{\frac{3}{2}-\frac{i}{2}}(1+iax)^{\frac{3}{2}+\frac{i}{2}}dx}{\sqrt{1+a^2x^2}}$$
$$= -\frac{\left(\frac{1}{13}-\frac{5i}{13}\right)2^{\frac{3}{2}+\frac{i}{2}}c(1-iax)^{\frac{5}{2}-\frac{i}{2}}\sqrt{c+a^2cx^2}\,{}_2F_1\left(-\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2};\frac{7}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.0222047, size = 98, normalized size = 1.

$$-\frac{\left(\frac{1}{13}-\frac{5i}{13}\right)2^{\frac{3}{2}+\frac{i}{2}}c(1-iax)^{\frac{5}{2}-\frac{i}{2}}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2};\frac{7}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/E^ArcTan[a*x],x]
```

```
[Out] ((-1/13 + (5*I)/13)*2^(3/2 + I/2)*c*(1 - I*a*x)^(5/2 - I/2)*Sqrt[c + a^2*c*
x^2]*Hypergeometric2F1[-3/2 - I/2, 5/2 - I/2, 7/2 - I/2, (1 - I*a*x)/2])/(a
*Sqrt[1 + a^2*x^2])
```

Maple [F] time = 0.301, size = 0, normalized size = 0.

$$\int \frac{1}{e^{\arctan(ax)}}\left(a^2cx^2+c\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(-\arctan(ax))}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*e^(-arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(-\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*e^(-arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/exp(atan(a*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.282 $\int e^{-\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal. Leaf size=97

$$-\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{3}{2}-\frac{i}{2}}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2};\frac{5}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[Out] ((-1/5 + (3*I)/5)*2^(1/2 + I/2)*(1 - I*a*x)^(3/2 - I/2)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-1/2 - I/2, 3/2 - I/2, 5/2 - I/2, (1 - I*a*x)/2])/(a*Sqr
t[1 + a^2*x^2])
```

Rubi [A] time = 0.0705675, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$-\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{3}{2}-\frac{i}{2}}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2};\frac{5}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]/E^ArcTan[a*x],x]
```

```
[Out] ((-1/5 + (3*I)/5)*2^(1/2 + I/2)*(1 - I*a*x)^(3/2 - I/2)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-1/2 - I/2, 3/2 - I/2, 5/2 - I/2, (1 - I*a*x)/2])/(a*Sqr
t[1 + a^2*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{-\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx = \frac{\sqrt{c+a^2cx^2}\int e^{-\tan^{-1}(ax)}\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}$$
$$= \frac{\sqrt{c+a^2cx^2}\int (1-iax)^{\frac{1}{2}-\frac{i}{2}}(1+iax)^{\frac{1}{2}+\frac{i}{2}}\,dx}{\sqrt{1+a^2x^2}}$$
$$= -\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{3}{2}-\frac{i}{2}}\sqrt{c+a^2cx^2}\,{}_2F_1\left(-\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2};\frac{5}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.0180481, size = 97, normalized size = 1.

$$-\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{3}{2}-\frac{i}{2}}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2};\frac{5}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/E^ArcTan[a*x],x]
```

```
[Out] ((-1/5 + (3*I)/5)*2^(1/2 + I/2)*(1 - I*a*x)^(3/2 - I/2)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-1/2 - I/2, 3/2 - I/2, 5/2 - I/2, (1 - I*a*x)/2])/(a*Sqr
t[1 + a^2*x^2])
```

Maple [F] time = 0.288, size = 0, normalized size = 0.

$$\int \frac{1}{e^{\arctan(ax)}}\sqrt{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2+c}\,e^{(-\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a^2cx^2+c}\,e^{(-\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/exp(atan(a*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.283 $\int \frac{e^{-\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=93

$$-\frac{(1-i)2^{-\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{1}{2}-\frac{i}{2}}\sqrt{a^2x^2+1}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},\frac{1}{2}-\frac{i}{2};\frac{3}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] ((-1 + I)*(1 - I*a*x)^(1/2 - I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2 -
 I/2, 1/2 - I/2, 3/2 - I/2, (1 - I*a*x)/2])/(2^(1/2 - I/2)*a*Sqrt[c + a^2*c
*x^2])
```

Rubi [A] time = 0.0762926, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$-\frac{(1-i)2^{-\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{1}{2}-\frac{i}{2}}\sqrt{a^2x^2+1}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},\frac{1}{2}-\frac{i}{2};\frac{3}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-1 + I)*(1 - I*a*x)^(1/2 - I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2 -
 I/2, 1/2 - I/2, 3/2 - I/2, (1 - I*a*x)/2])/(2^(1/2 - I/2)*a*Sqrt[c + a^2*c
*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{-\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{-\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}-\frac{i}{2}}(1+iax)^{-\frac{1}{2}+\frac{i}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{(1-i)2^{-\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{1}{2}-\frac{i}{2}}\sqrt{1+a^2x^2}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},\frac{1}{2}-\frac{i}{2};\frac{3}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0213591, size = 93, normalized size = 1.

$$-\frac{(1-i)2^{-\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{1}{2}-\frac{i}{2}}\sqrt{a^2x^2+1}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},\frac{1}{2}-\frac{i}{2};\frac{3}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-1 + I)*(1 - I*a*x)^(1/2 - I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2 -
 I/2, 1/2 - I/2, 3/2 - I/2, (1 - I*a*x)/2])/(2^(1/2 - I/2)*a*Sqrt[c + a^2*c
*x^2])
```

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int \frac{1}{e^{\arctan(ax)}}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(-arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^{(-\arctan(ax))}}{\sqrt{a^2cx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(e^(-arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.284 $\int \frac{e^{-\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{(1-ax)e^{-\tan^{-1}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

```
[Out] -(1 - a*x)/(2*a*c*E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.038571, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5069}

$$-\frac{(1-ax)e^{-\tan^{-1}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] -(1 - a*x)/(2*a*c*E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{-\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=-\frac{e^{-\tan^{-1}(ax)}(1-ax)}{2ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0202819, size = 37, normalized size = 0.97

$$\frac{(ax-1)e^{-\tan^{-1}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (-1 + a*x)/(2*a*c*E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.035, size = 39, normalized size = 1.

$$\frac{(a^2x^2+1)(ax-1)}{2\,a\mathrm{e}^{\arctan(ax)}}(a^2cx^2+c)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(a^2*x^2+1)*(a*x-1)/a/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A] time = 2.03495, size = 100, normalized size = 2.63

$$\frac{\sqrt{a^2cx^2+c}(ax-1)e^{(-\arctan(ax))}}{2\left(a^3c^2x^2+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a^2*c*x^2 + c)*(a*x - 1)*e^(-arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

3.285 $\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{3(1-ax)e^{-\tan^{-1}(ax)}}{10ac^2\sqrt{a^2cx^2+c}}-\frac{(1-3ax)e^{-\tan^{-1}(ax)}}{10ac\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] -(1 - 3*a*x)/(10*a*c*E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)) - (3*(1 - a*x))/(
10*a*c^2*E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0798143, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5070, 5069}

$$-\frac{3(1-ax)e^{-\tan^{-1}(ax)}}{10ac^2\sqrt{a^2cx^2+c}}-\frac{(1-3ax)e^{-\tan^{-1}(ax)}}{10ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(1 - 3*a*x)/(10*a*c*E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)) - (3*(1 - a*x))/(
10*a*c^2*E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2])
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx &= -\frac{e^{-\tan^{-1}(ax)}(1-3ax)}{10ac\left(c+a^2cx^2\right)^{3/2}}+\frac{3\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{5c}\\ &= -\frac{e^{-\tan^{-1}(ax)}(1-3ax)}{10ac\left(c+a^2cx^2\right)^{3/2}}-\frac{3e^{-\tan^{-1}(ax)}(1-ax)}{10ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0419894, size = 62, normalized size = 0.81

$$\frac{\left(3a^3x^3-3a^2x^2+6ax-4\right)e^{-\tan^{-1}(ax)}}{10c^2\left(a^3x^2+a\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] (-4 + 6*a*x - 3*a^2*x^2 + 3*a^3*x^3)/(10*c^2*E^ArcTan[a*x]*(a + a^3*x^2)*Sq
rt[c + a^2*c*x^2])
```

Maple [A] time = 0.036, size = 56, normalized size = 0.7

$$\frac{(a^2x^2+1)(3a^3x^3-3a^2x^2+6ax-4)}{10\,a\mathrm{e}^{\arctan(ax)}}\left(a^2cx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/10*(a^2*x^2+1)*(3*a^3*x^3-3*a^2*x^2+6*a*x-4)/a/exp(arctan(a*x))/(a^2*c*x^
2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 2.02203, size = 158, normalized size = 2.05

$$\frac{\left(3\,a^3x^3-3\,a^2x^2+6\,ax-4\right)\sqrt{a^2cx^2+c}e^{(-\arctan(ax))}}{10\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/10*(3*a^3*x^3 - 3*a^2*x^2 + 6*a*x - 4)*sqrt(a^2*c*x^2 + c)*e^(-arctan(a*x
))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

3.286 $\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{3(1-ax)e^{-\tan^{-1}(ax)}}{13ac^3\sqrt{a^2cx^2+c}}-\frac{(1-3ax)e^{-\tan^{-1}(ax)}}{13ac^2\left(a^2cx^2+c\right)^{3/2}}-\frac{(1-5ax)e^{-\tan^{-1}(ax)}}{26ac\left(a^2cx^2+c\right)^{5/2}}$$

```
[Out] -(1 - 5*a*x)/(26*a*c*E^ArcTan[a*x]*(c + a^2*c*x^2)^(5/2)) - (1 - 3*a*x)/(13
*a*c^2*E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)) - (3*(1 - a*x))/(13*a*c^3*E^Arc
Tan[a*x]*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.123632, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5070, 5069}

$$-\frac{3(1-ax)e^{-\tan^{-1}(ax)}}{13ac^3\sqrt{a^2cx^2+c}}-\frac{(1-3ax)e^{-\tan^{-1}(ax)}}{13ac^2\left(a^2cx^2+c\right)^{3/2}}-\frac{(1-5ax)e^{-\tan^{-1}(ax)}}{26ac\left(a^2cx^2+c\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(7/2)),x]
```

```
[Out] -(1 - 5*a*x)/(26*a*c*E^ArcTan[a*x]*(c + a^2*c*x^2)^(5/2)) - (1 - 3*a*x)/(13
*a*c^2*E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)) - (3*(1 - a*x))/(13*a*c^3*E^Arc
Tan[a*x]*Sqrt[c + a^2*c*x^2])
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx = -\frac{e^{-\tan^{-1}(ax)}(1-5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}} + \frac{10\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{13c}$$

$$= -\frac{e^{-\tan^{-1}(ax)}(1-5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}} - \frac{e^{-\tan^{-1}(ax)}(1-3ax)}{13ac^2\left(c+a^2cx^2\right)^{3/2}} + \frac{6\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{13c^2}$$

$$= -\frac{e^{-\tan^{-1}(ax)}(1-5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}} - \frac{e^{-\tan^{-1}(ax)}(1-3ax)}{13ac^2\left(c+a^2cx^2\right)^{3/2}} - \frac{3e^{-\tan^{-1}(ax)}(1-ax)}{13ac^3\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.045607, size = 81, normalized size = 0.7

$$\frac{\left(6a^5x^5-6a^4x^4+18a^3x^3-14a^2x^2+17ax-9\right)e^{-\tan^{-1}(ax)}}{26ac^3\left(a^2x^2+1\right)^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(7/2)),x]
```

```
[Out] (-9 + 17*a*x - 14*a^2*x^2 + 18*a^3*x^3 - 6*a^4*x^4 + 6*a^5*x^5)/(26*a*c^3*E
^ArcTan[a*x]*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.039, size = 72, normalized size = 0.6

$$\frac{\left(a^2x^2+1\right)\left(6\,a^5x^5-6\,a^4x^4+18\,a^3x^3-14\,a^2x^2+17\,ax-9\right)}{26\,ae^{\arctan(ax)}}\left(a^2cx^2+c\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/26*(a^2*x^2+1)*(6*a^5*x^5-6*a^4*x^4+18*a^3*x^3-14*a^2*x^2+17*a*x-9)/a/exp
(arctan(a*x))/(a^2*c*x^2+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

Fricas [A] time = 1.9506, size = 216, normalized size = 1.88

$$\frac{\left(6\,a^5x^5-6\,a^4x^4+18\,a^3x^3-14\,a^2x^2+17\,ax-9\right)\sqrt{a^2cx^2+c}e^{(-\arctan(ax))}}{26\left(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/26*(6*a^5*x^5 - 6*a^4*x^4 + 18*a^3*x^3 - 14*a^2*x^2 + 17*a*x - 9)*sqrt(a^
2*c*x^2 + c)*e^(-arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2
+ a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

3.287 $\int e^{-2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{i2^{p+i}(1-iax)^{p+(1-i)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(-p-i,p+(1-i);p+(2-i);\frac{1}{2}(1-iax)\right)}{a(p+(1-i))}$$

```
[Out] (I*2^(I + p)*(1 - I*a*x)^((1 - I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F1[
-I - p, (1 - I) + p, (2 - I) + p, (1 - I*a*x)/2])/(a*((1 - I) + p)*(1 + a^2
*x^2)^p)
```

Rubi [A] time = 0.0675013, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5076, 5073, 69}

$$\frac{i2^{p+i}(1-iax)^{p+(1-i)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(-p-i,p+(1-i);p+(2-i);\frac{1}{2}(1-iax)\right)}{a(p+(1-i))}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^p/E^(2*ArcTan[a*x]),x]
```

```
[Out] (I*2^(I + p)*(1 - I*a*x)^((1 - I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F1[
-I - p, (1 - I) + p, (2 - I) + p, (1 - I*a*x)/2])/(a*((1 - I) + p)*(1 + a^2
*x^2)^p)
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{-2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx = \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{-2\tan^{-1}(ax)}\left(1+a^2x^2\right)^p\,dx$$
$$= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int (1-iax)^{-i+p}(1+iax)^{i+p}\,dx$$
$$= \frac{i2^{i+p}(1-iax)^{(1-i)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\,{}_2F_1\left(-i-p,(1-i)+p;(2-i)+p;\frac{1}{2}(1-ia$$
$$a((1-i)+p)$$

Mathematica [A] time = 0.0222456, size = 90, normalized size = 1.

$$\frac{i2^{p+i}(1-iax)^{p+(1-i)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(-p-i,p+(1-i);p+(2-i);\frac{1}{2}(1-iax)\right)}{a(p+(1-i))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^p/E^(2*ArcTan[a*x]),x]
```

```
[Out] (I*2^(I + p)*(1 - I*a*x)^((1 - I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F1[
-I - p, (1 - I) + p, (2 - I) + p, (1 - I*a*x)/2])/(a*((1 - I) + p)*(1 + a^2
*x^2)^p)
```

Maple [F] time = 0.326, size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^p}{e^{2\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x)
```

```
[Out] int((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^p e^{(-2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^p*e^(-2*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^2+c\right)^p e^{(-2\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^p*e^(-2*arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**p/exp(2*atan(a*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^p e^{(-2\ \arctan(ax))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^p*e^(-2*arctan(a*x)), x)
```

3.288 $\int e^{-2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx$

Optimal. Leaf size=53

$$-\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{1+i}c^2(1-iax)^{3-i}{}_2F_1\left(-2-i,3-i;4-i;\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] ((-1/5 + (3*I)/5)*2^(1 + I)*c^2*(1 - I*a*x)^(3 - I)*Hypergeometric2F1[-2 -
I, 3 - I, 4 - I, (1 - I*a*x)/2])/a
```

Rubi [A] time = 0.0368077, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5073, 69}

$$-\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{1+i}c^2(1-iax)^{3-i}{}_2F_1\left(-2-i,3-i;4-i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-1/5 + (3*I)/5)*2^(1 + I)*c^2*(1 - I*a*x)^(3 - I)*Hypergeometric2F1[-2 -
I, 3 - I, 4 - I, (1 - I*a*x)/2])/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{-2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx&=c^2\int(1-iax)^{2-i}(1+iax)^{2+i}\,dx\\&=-\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{1+i}c^2(1-iax)^{3-i}{}_2F_1\left(-2-i,3-i;4-i;\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0140956, size = 53, normalized size = 1.

$$-\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{1+i}c^2(1-iax)^{3-i}{}_2F_1\left(-2-i,3-i;4-i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-1/5 + (3*I)/5)*2^(1 + I)*c^2*(1 - I*a*x)^(3 - I)*Hypergeometric2F1[-2 -
I, 3 - I, 4 - I, (1 - I*a*x)/2])/a
```

Maple [F] time = 0.192, size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{2}}{\mathrm{e}^{2 \arctan \left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x)
```

```
[Out] int((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{2} c x^{2} + c\right)^{2} e^{\left(-2 \, \arctan\left(a x\right)\right)}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*e^(-2*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left({\left(a^{4} c^{2} x^{4} + 2 \, a^{2} c^{2} x^{2} + c^{2}\right)} e^{\left(-2 \, \arctan\left(a x\right)\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(-2*arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/exp(2*atan(a*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2 e^{(-2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*e^(-2*arctan(a*x)), x)
```

3.289 $\int e^{-2\tan^{-1}(ax)}\left(c+a^2cx^2\right)dx$

Optimal. Leaf size=51

$$-\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+i}c(1-iax)^{2-i}\,{}_2F_1\left(-1-i,2-i;3-i;\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] ((-1/5 + (2*I)/5)*2^(1 + I)*c*(1 - I*a*x)^(2 - I)*Hypergeometric2F1[-1 - I,
 2 - I, 3 - I, (1 - I*a*x)/2])/a
```

Rubi [A] time = 0.0258661, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5073, 69}

$$-\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+i}c(1-iax)^{2-i}\,{}_2F_1\left(-1-i,2-i;3-i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-1/5 + (2*I)/5)*2^(1 + I)*c*(1 - I*a*x)^(2 - I)*Hypergeometric2F1[-1 - I,
 2 - I, 3 - I, (1 - I*a*x)/2])/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{-2\tan^{-1}(ax)}\left(c+a^2cx^2\right)dx &= c\int (1-iax)^{1-i}(1+iax)^{1+i}\,dx\\ &= -\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+i}c(1-iax)^{2-i}\,{}_2F_1\left(-1-i,2-i;3-i;\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0118533, size = 51, normalized size = 1.

$$-\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+i}c(1-iax)^{2-i}\,{}_2F_1\left(-1-i,2-i;3-i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-1/5 + (2*I)/5)*2^(1 + I)*c*(1 - I*a*x)^(2 - I)*Hypergeometric2F1[-1 - I,
 2 - I, 3 - I, (1 - I*a*x)/2])/a
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{e^{2\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/exp(2*arctan(a*x)),x)
```

```
[Out] int((a^2*c*x^2+c)/exp(2*arctan(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)e^{(-2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/exp(2*arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2cx^2 + c\right)e^{(-2\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/exp(2*arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/exp(2*atan(a*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)e^{(-2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/exp(2*arctan(a*x)),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

3.290 $\int e^{-2\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=46

$$-\frac{(1-i)2^{-1+i}(1-iax)^{1-i}\,{}_2F_1\left(-i,1-i;2-i;\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] ((-1 + I)*(1 - I*a*x)^(1 - I)*Hypergeometric2F1[-I, 1 - I, 2 - I, (1 - I*a*
x)/2])/(2^(1 - I)*a)
```

Rubi [A] time = 0.0108576, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5061, 69}

$$-\frac{(1-i)2^{-1+i}(1-iax)^{1-i}\,{}_2F_1\left(-i,1-i;2-i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-2*ArcTan[a*x]),x]
```

```
[Out] ((-1 + I)*(1 - I*a*x)^(1 - I)*Hypergeometric2F1[-I, 1 - I, 2 - I, (1 - I*a*
x)/2])/(2^(1 - I)*a)
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{-2\tan^{-1}(ax)}\,dx &= \int (1-iax)^{-i}(1+iax)^{i}\,dx\\ &= -\frac{(1-i)2^{-1+i}(1-iax)^{1-i}\,{}_2F_1\left(-i,1-i;2-i;\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0209654, size = 37, normalized size = 0.8

$$-\frac{(1+i)e^{(-2+2i)\tan^{-1}(ax)}\,{}_2F_1\left(1+i,2;2+i;-e^{2i\tan^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-2*ArcTan[a*x]),x]
```

```
[Out] ((-1 - I)*Hypergeometric2F1[1 + I, 2, 2 + I, -E^((2*I)*ArcTan[a*x])])/(a*E^
((2 - 2*I)*ArcTan[a*x]))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \mathrm{e}^{-2\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*arctan(a*x)),x)
```

```
[Out] int(exp(-2*arctan(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(-2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2*arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(e^(-2*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(e^{(-2\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2*arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral(e^(-2*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{-2\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2*atan(a*x)),x)
```

```
[Out] Integral(exp(-2*atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(-2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2*arctan(a*x)),x, algorithm="giac")
```

```
[Out] integrate(e^(-2*arctan(a*x)), x)
```

3.291 $\int \frac{e^{-2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=18

$$-\frac{e^{-2\tan^{-1}(ax)}}{2ac}$$

```
[Out] -1/(2*a*c*E^(2*ArcTan[a*x]))
```

Rubi [A] time = 0.028431, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5071}

$$-\frac{e^{-2\tan^{-1}(ax)}}{2ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)),x]
```

```
[Out] -1/(2*a*c*E^(2*ArcTan[a*x]))
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{-2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{-2\tan^{-1}(ax)}}{2ac}$$

Mathematica [C] time = 0.0081919, size = 34, normalized size = 1.89

$$-\frac{(1-iax)^{-i}(1+iax)^{i}}{2ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)),x]
```

```
[Out] -(1 + I*a*x)^I/(2*a*c*(1 - I*a*x)^I)
```

Maple [A] time = 0.036, size = 18, normalized size = 1.

$$-\frac{1}{2\,ac{e}^{2\,\arctan(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c),x)
```

```
[Out] -1/2/a/c/exp(2*arctan(a*x))
```

Maxima [A] time = 1.06396, size = 31, normalized size = 1.72

$$-\frac{e^{(-2\arctan(ax))}}{a^3cx^2 + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -e^(-2*arctan(a*x))/(a^3*c*x^2 + a*c)
```

Fricas [A] time = 1.95275, size = 42, normalized size = 2.33

$$-\frac{e^{(-2\arctan(ax))}}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*e^(-2*arctan(a*x))/(a*c)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.11948, size = 20, normalized size = 1.11

$$-\frac{e^{(-2\arctan(ax))}}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*e^(-2*arctan(a*x))/(a*c)
```

3.292 $\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=54

$$-\frac{(1-ax)e^{-2\tan^{-1}(ax)}}{4ac^2\left(a^2x^2+1\right)}-\frac{e^{-2\tan^{-1}(ax)}}{8ac^2}$$

```
[Out] -1/(8*a*c^2*E^(2*ArcTan[a*x])) - (1 - a*x)/(4*a*c^2*E^(2*ArcTan[a*x])*(1 +
a^2*x^2))
```

Rubi [A] time = 0.057639, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$-\frac{(1-ax)e^{-2\tan^{-1}(ax)}}{4ac^2\left(a^2x^2+1\right)}-\frac{e^{-2\tan^{-1}(ax)}}{8ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^2),x]
```

```
[Out] -1/(8*a*c^2*E^(2*ArcTan[a*x])) - (1 - a*x)/(4*a*c^2*E^(2*ArcTan[a*x])*(1 +
a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx &= -\frac{e^{-2\tan^{-1}(ax)}(1-ax)}{4ac^2\left(1+a^2x^2\right)}+\frac{\int \frac{e^{-2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{4c}\\ &= -\frac{e^{-2\tan^{-1}(ax)}}{8ac^2}-\frac{e^{-2\tan^{-1}(ax)}(1-ax)}{4ac^2\left(1+a^2x^2\right)}\end{aligned}$$

Mathematica [C] time = 0.0214295, size = 55, normalized size = 1.02

$$-\frac{(1-iax)^{-i}(1+iax)^{i}\left(a^2x^2-2ax+3\right)}{8c^2\left(a^3x^2+a\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^2),x]
```

```
[Out] -((1 + I*a*x)^I*(3 - 2*a*x + a^2*x^2))/(8*c^2*(1 - I*a*x)^I*(a + a^3*x^2))
```

Maple [A] time = 0.038, size = 42, normalized size = 0.8

$$-\frac{a^2x^2 - 2\,ax + 3}{\left(8\,a^2x^2 + 8\right)c^2\mathrm{e}^{2\,\arctan(ax)}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x)
```

```
[Out] -1/8*(a^2*x^2-2*a*x+3)/(a^2*x^2+1)/c^2/exp(2*arctan(a*x))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-2\,\arctan(ax))}}{\left(a^2cx^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

Fricas [A] time = 2.03854, size = 93, normalized size = 1.72

$$-\frac{\left(a^2x^2 - 2\,ax + 3\right)e^{(-2\,\arctan(ax))}}{8\left(a^3c^2x^2 + ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(a^2*x^2 - 2*a*x + 3)*e^(-2*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-2\arctan(ax))}}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

3.293 $\int \frac{e^{-2\tan^{-1}(ax)}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=89

$$-\frac{(1-2ax)e^{-2\tan^{-1}(ax)}}{10ac^3\left(a^2x^2+1\right)^2}-\frac{3(1-ax)e^{-2\tan^{-1}(ax)}}{20ac^3\left(a^2x^2+1\right)}-\frac{3e^{-2\tan^{-1}(ax)}}{40ac^3}$$

```
[Out] -3/(40*a*c^3*E^(2*ArcTan[a*x])) - (1 - 2*a*x)/(10*a*c^3*E^(2*ArcTan[a*x])*(
1 + a^2*x^2)^2) - (3*(1 - a*x))/(20*a*c^3*E^(2*ArcTan[a*x])*(1 + a^2*x^2))
```

Rubi [A] time = 0.0875743, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$-\frac{(1-2ax)e^{-2\tan^{-1}(ax)}}{10ac^3\left(a^2x^2+1\right)^2}-\frac{3(1-ax)e^{-2\tan^{-1}(ax)}}{20ac^3\left(a^2x^2+1\right)}-\frac{3e^{-2\tan^{-1}(ax)}}{40ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^3),x]
```

```
[Out] -3/(40*a*c^3*E^(2*ArcTan[a*x])) - (1 - 2*a*x)/(10*a*c^3*E^(2*ArcTan[a*x])*(
1 + a^2*x^2)^2) - (3*(1 - a*x))/(20*a*c^3*E^(2*ArcTan[a*x])*(1 + a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx &= -\frac{e^{-2\tan^{-1}(ax)}(1-2ax)}{10ac^3\left(1+a^2x^2\right)^2}+\frac{3\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{5c}\\
&= -\frac{e^{-2\tan^{-1}(ax)}(1-2ax)}{10ac^3\left(1+a^2x^2\right)^2}-\frac{3e^{-2\tan^{-1}(ax)}(1-ax)}{20ac^3\left(1+a^2x^2\right)}+\frac{3\int \frac{e^{-2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{20c^2}\\
&= -\frac{3e^{-2\tan^{-1}(ax)}}{40ac^3}-\frac{e^{-2\tan^{-1}(ax)}(1-2ax)}{10ac^3\left(1+a^2x^2\right)^2}-\frac{3e^{-2\tan^{-1}(ax)}(1-ax)}{20ac^3\left(1+a^2x^2\right)}
\end{aligned}$$

Mathematica [C] time = 0.14501, size = 85, normalized size = 0.96

$$\frac{(8ax-4)e^{-2\tan^{-1}(ax)}-3(1-iax)^{-i}(1+iax)^{i}\left(a^2x^2+1\right)\left(a^2x^2-2ax+3\right)}{40ac^3\left(a^2x^2+1\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^3),x]
```

```
[Out] ((-4 + 8*a*x)/E^(2*ArcTan[a*x]) - (3*(1 + I*a*x)^I*(1 + a^2*x^2)*(3 - 2*a*x
 + a^2*x^2))/(1 - I*a*x)^I)/(40*a*c^3*(1 + a^2*x^2)^2)
```

Maple [A] time = 0.038, size = 59, normalized size = 0.7

$$-\frac{3\,a^4x^4-6\,a^3x^3+12\,a^2x^2-14\,ax+13}{40\,c^3\left(a^2x^2+1\right)^2\mathrm{e}^{2\arctan(ax)}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x)
```

```
[Out] -1/40*(3*a^4*x^4-6*a^3*x^3+12*a^2*x^2-14*a*x+13)/(a^2*x^2+1)^2/c^3/exp(2*ar
ctan(a*x))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{e^{(-2\arctan(ax))}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

Fricas [A] time = 2.0834, size = 155, normalized size = 1.74

$$-\frac{\left(3\,a^4x^4-6\,a^3x^3+12\,a^2x^2-14\,ax+13\right)e^{(-2\arctan(ax))}}{40\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(3*a^4*x^4 - 6*a^3*x^3 + 12*a^2*x^2 - 14*a*x + 13)*e^(-2*arctan(a*x))
/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**3,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-2\arctan(ax))}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="giac")

[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

3.294 $\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=124

$$-\frac{(1-3ax)e^{-2\tan^{-1}(ax)}}{20ac^4\left(a^2x^2+1\right)^3}-\frac{9(1-ax)e^{-2\tan^{-1}(ax)}}{80ac^4\left(a^2x^2+1\right)}-\frac{3(1-2ax)e^{-2\tan^{-1}(ax)}}{40ac^4\left(a^2x^2+1\right)^2}-\frac{9e^{-2\tan^{-1}(ax)}}{160ac^4}$$

```
[Out] -9/(160*a*c^4*E^(2*ArcTan[a*x])) - (1 - 3*a*x)/(20*a*c^4*E^(2*ArcTan[a*x])*
(1 + a^2*x^2)^3) - (3*(1 - 2*a*x))/(40*a*c^4*E^(2*ArcTan[a*x])*(1 + a^2*x^2
)^2) - (9*(1 - a*x))/(80*a*c^4*E^(2*ArcTan[a*x])*(1 + a^2*x^2))
```

Rubi [A] time = 0.120785, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$-\frac{(1-3ax)e^{-2\tan^{-1}(ax)}}{20ac^4\left(a^2x^2+1\right)^3}-\frac{9(1-ax)e^{-2\tan^{-1}(ax)}}{80ac^4\left(a^2x^2+1\right)}-\frac{3(1-2ax)e^{-2\tan^{-1}(ax)}}{40ac^4\left(a^2x^2+1\right)^2}-\frac{9e^{-2\tan^{-1}(ax)}}{160ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^4),x]
```

```
[Out] -9/(160*a*c^4*E^(2*ArcTan[a*x])) - (1 - 3*a*x)/(20*a*c^4*E^(2*ArcTan[a*x])*
(1 + a^2*x^2)^3) - (3*(1 - 2*a*x))/(40*a*c^4*E^(2*ArcTan[a*x])*(1 + a^2*x^2
)^2) - (9*(1 - a*x))/(80*a*c^4*E^(2*ArcTan[a*x])*(1 + a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4}\,dx = -\frac{e^{-2\tan^{-1}(ax)}(1-3ax)}{20ac^4\left(1+a^2x^2\right)^3} + \frac{3\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx}{4c}$$
$$= -\frac{e^{-2\tan^{-1}(ax)}(1-3ax)}{20ac^4\left(1+a^2x^2\right)^3} - \frac{3e^{-2\tan^{-1}(ax)}(1-2ax)}{40ac^4\left(1+a^2x^2\right)^2} + \frac{9\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{20c^2}$$
$$= -\frac{e^{-2\tan^{-1}(ax)}(1-3ax)}{20ac^4\left(1+a^2x^2\right)^3} - \frac{3e^{-2\tan^{-1}(ax)}(1-2ax)}{40ac^4\left(1+a^2x^2\right)^2} - \frac{9e^{-2\tan^{-1}(ax)}(1-ax)}{80ac^4\left(1+a^2x^2\right)} + \frac{9\int \frac{e^{-2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{80c^3}$$
$$= -\frac{9e^{-2\tan^{-1}(ax)}}{160ac^4} - \frac{e^{-2\tan^{-1}(ax)}(1-3ax)}{20ac^4\left(1+a^2x^2\right)^3} - \frac{3e^{-2\tan^{-1}(ax)}(1-2ax)}{40ac^4\left(1+a^2x^2\right)^2} - \frac{9e^{-2\tan^{-1}(ax)}(1-ax)}{80ac^4\left(1+a^2x^2\right)}$$

Mathematica [C] time = 0.32242, size = 121, normalized size = 0.98

$$\frac{8c(3ax-1)e^{-2\tan^{-1}(ax)} - 3\left(a^2cx^2+c\right)\left((4-8ax)e^{-2\tan^{-1}(ax)} + 3(1-iax)^{-i}(1+iax)^{i}(ax-i)(ax+i)\left(a^2x^2-2ax+3\right)\right)}{160ac^2\left(a^2cx^2+c\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^4),x]
```

```
[Out] ((8*c*(-1 + 3*a*x))/E^(2*ArcTan[a*x]) - 3*(c + a^2*c*x^2)*((4 - 8*a*x)/E^(2
*ArcTan[a*x]) + (3*(1 + I*a*x)^I*(-I + a*x)*(I + a*x)*(3 - 2*a*x + a^2*x^2)
)/(1 - I*a*x)^I))/(160*a*c^2*(c + a^2*c*x^2)^3)
```

Maple [A] time = 0.039, size = 75, normalized size = 0.6

$$-\frac{9\,a^6x^6 - 18\,a^5x^5 + 45\,a^4x^4 - 60\,a^3x^3 + 75\,a^2x^2 - 66\,ax + 47}{160\,c^4\left(a^2x^2+1\right)^3 \mathrm{e}^{2\arctan(ax)}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x)
```

```
[Out] -1/160*(9*a^6*x^6-18*a^5*x^5+45*a^4*x^4-60*a^3*x^3+75*a^2*x^2-66*a*x+47)/(a
^2*x^2+1)^3/c^4/exp(2*arctan(a*x))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-2\arctan(ax))}}{\left(a^2cx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

Fricas [A] time = 1.99839, size = 215, normalized size = 1.73

$$-\frac{\left(9\,a^6x^6-18\,a^5x^5+45\,a^4x^4-60\,a^3x^3+75\,a^2x^2-66\,ax+47\right)e^{\left(-2\,\arctan(ax)\right)}}{160\,\left(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/160*(9*a^6*x^6 - 18*a^5*x^5 + 45*a^4*x^4 - 60*a^3*x^3 + 75*a^2*x^2 - 66*
a*x + 47)*e^(-2*arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2 +
 a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(-2\,\arctan(ax)\right)}}{\left(a^2cx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

3.295 $\int e^{-2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=88

$$-\frac{\left(\frac{2}{29}-\frac{5i}{29}\right)2^{\frac{5}{2}+i}c(1-iax)^{\frac{5}{2}-i}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{3}{2}-i,\frac{5}{2}-i;\frac{7}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[Out] ((-2/29 + (5*I)/29)*2^(5/2 + I)*c*(1 - I*a*x)^(5/2 - I)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-3/2 - I, 5/2 - I, 7/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1 +
a^2*x^2])
```

Rubi [A] time = 0.0792291, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$-\frac{\left(\frac{2}{29}-\frac{5i}{29}\right)2^{\frac{5}{2}+i}c(1-iax)^{\frac{5}{2}-i}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{3}{2}-i,\frac{5}{2}-i;\frac{7}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-2/29 + (5*I)/29)*2^(5/2 + I)*c*(1 - I*a*x)^(5/2 - I)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-3/2 - I, 5/2 - I, 7/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1 +
a^2*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{-2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}dx = \frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{-2\tan^{-1}(ax)}\left(1+a^2x^2\right)^{3/2}dx}{\sqrt{1+a^2x^2}}$$
$$= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int(1-iax)^{\frac{3}{2}-i}(1+iax)^{\frac{3}{2}+i}dx}{\sqrt{1+a^2x^2}}$$
$$= -\frac{\left(\frac{2}{29}-\frac{5i}{29}\right)2^{\frac{5}{2}+i}c(1-iax)^{\frac{5}{2}-i}\sqrt{c+a^2cx^2}\,{}_2F_1\left(-\frac{3}{2}-i,\frac{5}{2}-i;\frac{7}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.023851, size = 88, normalized size = 1.

$$-\frac{\left(\frac{2}{29}-\frac{5i}{29}\right)2^{\frac{5}{2}+i}c(1-iax)^{\frac{5}{2}-i}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{3}{2}-i,\frac{5}{2}-i;\frac{7}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-2/29 + (5*I)/29)*2^(5/2 + I)*c*(1 - I*a*x)^(5/2 - I)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-3/2 - I, 5/2 - I, 7/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1 +
a^2*x^2])
```

Maple [F] time = 0.3, size = 0, normalized size = 0.

$$\int\frac{1}{e^{2\arctan(ax)}}\left(a^2cx^2+c\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(-2\arctan(ax))}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*e^(-2*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(-2\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*e^(-2*arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/exp(2*atan(a*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.296 $\int e^{-2\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal. Leaf size=87

$$-\frac{\left(\frac{2}{13}-\frac{3i}{13}\right)2^{\frac{3}{2}+i}(1-iax)^{\frac{3}{2}-i}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{1}{2}-i,\frac{3}{2}-i;\frac{5}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[Out] ((-2/13 + (3*I)/13)*2^(3/2 + I)*(1 - I*a*x)^(3/2 - I)*Sqrt[c + a^2*c*x^2]*H
ypergeometric2F1[-1/2 - I, 3/2 - I, 5/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1 + a^
2*x^2])
```

Rubi [A] time = 0.0718009, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$-\frac{\left(\frac{2}{13}-\frac{3i}{13}\right)2^{\frac{3}{2}+i}(1-iax)^{\frac{3}{2}-i}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{1}{2}-i,\frac{3}{2}-i;\frac{5}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-2/13 + (3*I)/13)*2^(3/2 + I)*(1 - I*a*x)^(3/2 - I)*Sqrt[c + a^2*c*x^2]*H
ypergeometric2F1[-1/2 - I, 3/2 - I, 5/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1 + a^
2*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{-2\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx = \frac{\sqrt{c+a^2cx^2}\int e^{-2\tan^{-1}(ax)}\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}$$
$$= \frac{\sqrt{c+a^2cx^2}\int(1-iax)^{\frac{1}{2}-i}(1+iax)^{\frac{1}{2}+i}\,dx}{\sqrt{1+a^2x^2}}$$
$$= -\frac{\left(\frac{2}{13}-\frac{3i}{13}\right)2^{\frac{3}{2}+i}(1-iax)^{\frac{3}{2}-i}\sqrt{c+a^2cx^2}\,{}_2F_1\left(-\frac{1}{2}-i,\frac{3}{2}-i;\frac{5}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.0178719, size = 87, normalized size = 1.

$$-\frac{\left(\frac{2}{13}-\frac{3i}{13}\right)2^{\frac{3}{2}+i}(1-iax)^{\frac{3}{2}-i}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{1}{2}-i,\frac{3}{2}-i;\frac{5}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-2/13 + (3*I)/13)*2^(3/2 + I)*(1 - I*a*x)^(3/2 - I)*Sqrt[c + a^2*c*x^2]*H
ypergeometric2F1[-1/2 - I, 3/2 - I, 5/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1 + a^
2*x^2])
```

Maple [F] time = 0.283, size = 0, normalized size = 0.

$$\int \frac{1}{e^{2\arctan(ax)}}\sqrt{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2+c}e^{(-2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^2cx^2+c}e^{(-2\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/exp(2*atan(a*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.297 $\int \frac{e^{-2\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=87

$$-\frac{\left(\frac{2}{5}-\frac{i}{5}\right)2^{\frac{1}{2}+i}(1-iax)^{\frac{1}{2}-i}\sqrt{a^2x^2+1}\,{}_2F_1\left(\frac{1}{2}-i,\frac{1}{2}-i;\frac{3}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] ((-2/5 + I/5)*2^(1/2 + I)*(1 - I*a*x)^(1/2 - I)*Sqrt[1 + a^2*x^2]*Hypergeom
etric2F1[1/2 - I, 1/2 - I, 3/2 - I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0746697, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$-\frac{\left(\frac{2}{5}-\frac{i}{5}\right)2^{\frac{1}{2}+i}(1-iax)^{\frac{1}{2}-i}\sqrt{a^2x^2+1}\,{}_2F_1\left(\frac{1}{2}-i,\frac{1}{2}-i;\frac{3}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-2/5 + I/5)*2^(1/2 + I)*(1 - I*a*x)^(1/2 - I)*Sqrt[1 + a^2*x^2]*Hypergeom
etric2F1[1/2 - I, 1/2 - I, 3/2 - I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{-2\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{-2\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}-i}(1+iax)^{-\frac{1}{2}+i}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{\left(\frac{2}{5}-\frac{i}{5}\right)2^{\frac{1}{2}+i}(1-iax)^{\frac{1}{2}-i}\sqrt{1+a^2x^2}\,{}_2F_1\left(\frac{1}{2}-i,\frac{1}{2}-i;\frac{3}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0208826, size = 87, normalized size = 1.

$$-\frac{\left(\frac{2}{5}-\frac{i}{5}\right)2^{\frac{1}{2}+i}(1-iax)^{\frac{1}{2}-i}\sqrt{a^2x^2+1}\,{}_2F_1\left(\frac{1}{2}-i,\frac{1}{2}-i;\frac{3}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-2/5 + I/5)*2^(1/2 + I)*(1 - I*a*x)^(1/2 - I)*Sqrt[1 + a^2*x^2]*Hypergeom
etric2F1[1/2 - I, 1/2 - I, 3/2 - I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2])
```

Maple [F] time = 0.285, size = 0, normalized size = 0.

$$\int \frac{1}{e^{2\arctan(ax)}}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-2\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(-2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(-2\arctan(ax))}}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(e^(-2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-2 \arctan(ax))}}{\sqrt{a^2cx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.298 $\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{(2-ax)e^{-2\tan^{-1}(ax)}}{5ac\sqrt{a^2cx^2+c}}$$

```
[Out] -(2 - a*x)/(5*a*c*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0392175, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5069}

$$-\frac{(2-ax)e^{-2\tan^{-1}(ax)}}{5ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] -(2 - a*x)/(5*a*c*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx=-\frac{e^{-2\tan^{-1}(ax)}(2-ax)}{5ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0213199, size = 37, normalized size = 0.97

$$\frac{(ax-2)e^{-2\tan^{-1}(ax)}}{5ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (-2 + a*x)/(5*a*c*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.036, size = 41, normalized size = 1.1

$$\frac{\left(a^2x^2+1\right)(ax-2)}{5\,ae^{2\,\arctan(ax)}}\left(a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/5*(a^2*x^2+1)*(a*x-2)/a/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-2\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A] time = 1.94746, size = 103, normalized size = 2.71

$$\frac{\sqrt{a^2cx^2+c}(ax-2)e^{(-2\arctan(ax))}}{5\left(a^3c^2x^2+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(a^2*c*x^2 + c)*(a*x - 2)*e^(-2*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-2\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

3.299 $\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{6(2-ax)e^{-2\tan^{-1}(ax)}}{65ac^2\sqrt{a^2cx^2+c}}-\frac{(2-3ax)e^{-2\tan^{-1}(ax)}}{13ac\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] -(2 - 3*a*x)/(13*a*c*E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)) - (6*(2 - a*x
))/(65*a*c^2*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.080294, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5070, 5069}

$$-\frac{6(2-ax)e^{-2\tan^{-1}(ax)}}{65ac^2\sqrt{a^2cx^2+c}}-\frac{(2-3ax)e^{-2\tan^{-1}(ax)}}{13ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(2 - 3*a*x)/(13*a*c*E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)) - (6*(2 - a*x
))/(65*a*c^2*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx &= -\frac{e^{-2\tan^{-1}(ax)}(2-3ax)}{13ac\left(c+a^2cx^2\right)^{3/2}}+\frac{6\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{13c}\\ &= -\frac{e^{-2\tan^{-1}(ax)}(2-3ax)}{13ac\left(c+a^2cx^2\right)^{3/2}}-\frac{6e^{-2\tan^{-1}(ax)}(2-ax)}{65ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0458528, size = 62, normalized size = 0.81

$$\frac{\left(6a^3x^3-12a^2x^2+21ax-22\right)e^{-2\tan^{-1}(ax)}}{65c^2\left(a^3x^2+a\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] (-22 + 21*a*x - 12*a^2*x^2 + 6*a^3*x^3)/(65*c^2*E^(2*ArcTan[a*x])*(a + a^3*
x^2)*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.037, size = 58, normalized size = 0.8

$$\frac{\left(a^2x^2+1\right)\left(6\,a^3x^3-12\,a^2x^2+21\,ax-22\right)}{65\,a\mathrm{e}^{2\arctan(ax)}}\left(a^2cx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/65*(a^2*x^2+1)*(6*a^3*x^3-12*a^2*x^2+21*a*x-22)/a/exp(2*arctan(a*x))/(a^2
*c*x^2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-2\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 1.85976, size = 165, normalized size = 2.14

$$\frac{\left(6\,a^3x^3-12\,a^2x^2+21\,ax-22\right)\sqrt{a^2cx^2+c}e^{(-2\arctan(ax))}}{65\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/65*(6*a^3*x^3 - 12*a^2*x^2 + 21*a*x - 22)*sqrt(a^2*c*x^2 + c)*e^(-2*arcta
n(a*x))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-2\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

3.300 $\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{24(2-ax)e^{-2\tan^{-1}(ax)}}{377ac^3\sqrt{a^2cx^2+c}}-\frac{20(2-3ax)e^{-2\tan^{-1}(ax)}}{377ac^2\left(a^2cx^2+c\right)^{3/2}}-\frac{(2-5ax)e^{-2\tan^{-1}(ax)}}{29ac\left(a^2cx^2+c\right)^{5/2}}$$

```
[Out] -(2 - 5*a*x)/(29*a*c*E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(5/2)) - (20*(2 - 3*
a*x))/(377*a*c^2*E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)) - (24*(2 - a*x))/
(377*a*c^3*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.124918, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5070, 5069}

$$-\frac{24(2-ax)e^{-2\tan^{-1}(ax)}}{377ac^3\sqrt{a^2cx^2+c}}-\frac{20(2-3ax)e^{-2\tan^{-1}(ax)}}{377ac^2\left(a^2cx^2+c\right)^{3/2}}-\frac{(2-5ax)e^{-2\tan^{-1}(ax)}}{29ac\left(a^2cx^2+c\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(7/2)),x]
```

```
[Out] -(2 - 5*a*x)/(29*a*c*E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(5/2)) - (20*(2 - 3*
a*x))/(377*a*c^2*E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)) - (24*(2 - a*x))/
(377*a*c^3*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx = -\frac{e^{-2\tan^{-1}(ax)}(2-5ax)}{29ac\left(c+a^2cx^2\right)^{5/2}} + \frac{20\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{29c}$$

$$= -\frac{e^{-2\tan^{-1}(ax)}(2-5ax)}{29ac\left(c+a^2cx^2\right)^{5/2}} - \frac{20e^{-2\tan^{-1}(ax)}(2-3ax)}{377ac^2\left(c+a^2cx^2\right)^{3/2}} + \frac{120\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{377c^2}$$

$$= -\frac{e^{-2\tan^{-1}(ax)}(2-5ax)}{29ac\left(c+a^2cx^2\right)^{5/2}} - \frac{20e^{-2\tan^{-1}(ax)}(2-3ax)}{377ac^2\left(c+a^2cx^2\right)^{3/2}} - \frac{24e^{-2\tan^{-1}(ax)}(2-ax)}{377ac^3\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0490135, size = 81, normalized size = 0.7

$$\frac{\left(24a^5x^5-48a^4x^4+108a^3x^3-136a^2x^2+149ax-114\right)e^{-2\tan^{-1}(ax)}}{377ac^3\left(a^2x^2+1\right)^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(7/2)),x]
```

```
[Out] (-114 + 149*a*x - 136*a^2*x^2 + 108*a^3*x^3 - 48*a^4*x^4 + 24*a^5*x^5)/(377
*a*c^3*E^(2*ArcTan[a*x])*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.038, size = 74, normalized size = 0.6

$$\frac{\left(a^2x^2+1\right)\left(24\,a^5x^5-48\,a^4x^4+108\,a^3x^3-136\,a^2x^2+149\,ax-114\right)}{377\,a e^{2\arctan(ax)}}\left(a^2cx^2+c\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/377*(a^2*x^2+1)*(24*a^5*x^5-48*a^4*x^4+108*a^3*x^3-136*a^2*x^2+149*a*x-11
4)/a/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-2\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

Fricas [A] time = 1.97393, size = 230, normalized size = 2.

$$\frac{\left(24\,a^{5}x^{5}-48\,a^{4}x^{4}+108\,a^{3}x^{3}-136\,a^{2}x^{2}+149\,ax-114\right)\sqrt{a^{2}cx^{2}+c}e^{(-2\,\arctan(ax))}}{377\left(a^{7}c^{4}x^{6}+3\,a^{5}c^{4}x^{4}+3\,a^{3}c^{4}x^{2}+ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/377*(24*a^5*x^5 - 48*a^4*x^4 + 108*a^3*x^3 - 136*a^2*x^2 + 149*a*x - 114)
*sqrt(a^2*c*x^2 + c)*e^(-2*arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^
3*c^4*x^2 + a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-2\,\arctan(ax))}}{\left(a^{2}cx^{2}+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

3.301 $\int \frac{e^{5i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=50

$$\frac{4i}{a(1-iax)}-\frac{2i}{a(1-iax)^2}+\frac{i\log(ax+i)}{a}$$

```
[Out] (-2*I)/(a*(1 - I*a*x)^2) + (4*I)/(a*(1 - I*a*x)) + (I*Log[I + a*x])/a
```

Rubi [A] time = 0.0427698, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5073, 43}

$$\frac{4i}{a(1-iax)}-\frac{2i}{a(1-iax)^2}+\frac{i\log(ax+i)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-2*I)/(a*(1 - I*a*x)^2) + (4*I)/(a*(1 - I*a*x)) + (I*Log[I + a*x])/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{5i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx &= \int \frac{(1+iax)^2}{(1-iax)^3}\,dx\\ &= \int \left(\frac{4}{(1-iax)^3}-\frac{4}{(1-iax)^2}+\frac{1}{1-iax}\right)dx\\ &= -\frac{2i}{a(1-iax)^2}+\frac{4i}{a(1-iax)}+\frac{i\log(i+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0222498, size = 42, normalized size = 0.84

$$\frac{i\left(4iax+(ax+i)^2\log(ax+i)-2\right)}{a(ax+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((5*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (I*(-2 + (4*I)*a*x + (I + a*x)^2*Log[I + a*x]))/(a*(I + a*x)^2)
```

Maple [A] time = 0.049, size = 45, normalized size = 0.9

$$\frac{1}{(ax+i)^2}\left(-4x-\frac{2i}{a}\right)+\frac{\frac{i}{2}\ln\left(a^2x^2+1\right)}{a}+\frac{\arctan(ax)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^5/(a^2*x^2+1)^3,x)
```

```
[Out] (-4*x-2*I/a)/(a*x+I)^2+1/2*I/a*ln(a^2*x^2+1)+arctan(a*x)/a
```

Maxima [A] time = 1.48925, size = 85, normalized size = 1.7

$$-\frac{32\,a^3x^3-48i\,a^2x^2-16i}{8\left(a^5x^4+2\,a^3x^2+a\right)}+\frac{\arctan(ax)}{a}+\frac{i\log\left(a^2x^2+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(32*a^3*x^3 - 48*I*a^2*x^2 - 16*I)/(a^5*x^4 + 2*a^3*x^2 + a) + arctan(
a*x)/a + 1/2*I*log(a^2*x^2 + 1)/a
```

Fricas [A] time = 1.85058, size = 115, normalized size = 2.3

$$-\frac{4\,ax-\left(i\,a^2x^2-2\,ax-i\right)\log\left(\frac{ax+i}{a}\right)+2i}{a^3x^2+2i\,a^2x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -(4*a*x - (I*a^2*x^2 - 2*a*x - I)*log((a*x + I)/a) + 2*I)/(a^3*x^2 + 2*I*a^
2*x - a)
```

Sympy [A] time = 0.589308, size = 41, normalized size = 0.82

$$-\frac{4a^4x+2ia^3}{a^6x^2+2ia^5x-a^4}+\frac{i\log(ax+i)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**5/(a**2*x**2+1)**3,x)
```

```
[Out] -(4*a**4*x + 2*I*a**3)/(a**6*x**2 + 2*I*a**5*x - a**4) + I*log(a*x + I)/a
```

Giac [A] time = 1.11795, size = 41, normalized size = 0.82

$$\frac{i\log\left(ax+i\right)}{a}-\frac{2\left(2\,ax+i\right)}{\left(ax+i\right)^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] i*log(a*x + i)/a - 2*(2*a*x + i)/((a*x + i)^2*a)
```

3.302 $\int \frac{e^{4i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}}+\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}}+\frac{\sinh^{-1}(ax)}{a}$$

```
[Out] ((2*I)*Sqrt[1 + I*a*x])/(a*Sqrt[1 - I*a*x]) - (((2*I)/3)*(1 + I*a*x)^(3/2))
/(a*(1 - I*a*x)^(3/2)) + ArcSinh[a*x]/a
```

Rubi [A] time = 0.0388457, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5073, 47, 41, 215}

$$-\frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}}+\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}}+\frac{\sinh^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ((2*I)*Sqrt[1 + I*a*x])/(a*Sqrt[1 - I*a*x]) - (((2*I)/3)*(1 + I*a*x)^(3/2))
/(a*(1 - I*a*x)^(3/2)) + ArcSinh[a*x]/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx &= \int \frac{(1+iax)^{3/2}}{(1-iax)^{5/2}}\,dx\\ &= -\frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}} - \int \frac{\sqrt{1+iax}}{(1-iax)^{3/2}}\,dx\\ &= \frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}} + \int \frac{1}{\sqrt{1-iax}\sqrt{1+iax}}\,dx\\ &= \frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}} + \int \frac{1}{\sqrt{1+a^2x^2}}\,dx\\ &= \frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}} + \frac{\sinh^{-1}(ax)}{a}\end{aligned}$$

Mathematica [C] time = 0.0119682, size = 48, normalized size = 0.66

$$-\frac{4i\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{1}{2}(1-iax)\right)}{3a(1-iax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (((-4*I)/3)*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - I*a*x)/2])/(a*
(1 - I*a*x)^(3/2))
```

Maple [A] time = 0.067, size = 113, normalized size = 1.6

$$\frac{7x}{3}\left(a^2x^2+1\right)^{-\frac{3}{2}} - \frac{7x}{3}\frac{1}{\sqrt{a^2x^2+1}} - \frac{a^2x^3}{3}\left(a^2x^2+1\right)^{-\frac{3}{2}} + \ln\left(a^2x\frac{1}{\sqrt{a^2}} + \sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2}} + 4\,iax^2\left(a^2x^2+1\right)^{-\frac{3}{2}} + \frac{\frac{4\,i}{3}}{a}\,(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x)
```

```
[Out] 7/3*x/(a^2*x^2+1)^(3/2)-7/3*x/(a^2*x^2+1)^(1/2)-1/3*a^2*x^3/(a^2*x^2+1)^(3/
2)+ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)+4*I*a*x^2/(a^2*x^2+1
)^(3/2)+4/3*I/a/(a^2*x^2+1)^(3/2)
```

Maxima [B] time = 0.995436, size = 163, normalized size = 2.23

$$-\frac{1}{3}a^4x\left(\frac{3\,x^2}{\left(a^2x^2+1\right)^{\frac{3}{2}}a^2} + \frac{2}{\left(a^2x^2+1\right)^{\frac{3}{2}}a^4}\right) + \frac{4i\,ax^2}{\left(a^2x^2+1\right)^{\frac{3}{2}}} - \frac{5\,x}{3\sqrt{a^2x^2+1}} + \frac{\operatorname{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}} + \frac{7\,x}{3\left(a^2x^2+1\right)^{\frac{3}{2}}} + \frac{4i}{3\left(a^2x^2+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a^4*x*(3*x^2/((a^2*x^2 + 1)^(3/2)*a^2) + 2/((a^2*x^2 + 1)^(3/2)*a^4))
+ 4*I*a*x^2/(a^2*x^2 + 1)^(3/2) - 5/3*x/sqrt(a^2*x^2 + 1) + arcsinh(a^2*x/s
```

```
qrt(a^2))/sqrt(a^2) + 7/3*x/(a^2*x^2 + 1)^(3/2) + 4/3*I/((a^2*x^2 + 1)^(3/2
)*a)
```

Fricas [A] time = 1.88621, size = 204, normalized size = 2.79

$$-\frac{8\,a^2x^2 + 16i\,ax + \left(3\,a^2x^2 + 6i\,ax - 3\right)\log\left(-ax + \sqrt{a^2x^2 + 1}\right) + \sqrt{a^2x^2 + 1}(8\,ax + 4i) - 8}{3\,a^3x^2 + 6i\,a^2x - 3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -(8*a^2*x^2 + 16*I*a*x + (3*a^2*x^2 + 6*I*a*x - 3)*log(-a*x + sqrt(a^2*x^2
+ 1)) + sqrt(a^2*x^2 + 1)*(8*a*x + 4*I) - 8)/(3*a^3*x^2 + 6*I*a^2*x - 3*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(iax + 1)^4}{\left(a^2x^2 + 1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**(5/2),x)
```

```
[Out] Integral((I*a*x + 1)**4/(a**2*x**2 + 1)**(5/2), x)
```

Giac [A] time = 1.13991, size = 32, normalized size = 0.44

$$-\frac{\log\left(-x|a| + \sqrt{a^2x^2 + 1}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] -log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a)
```

3.303 $\int \frac{e^{3i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=30

$$\frac{2}{a(ax+i)}-\frac{i\log(ax+i)}{a}$$

```
[Out] 2/(a*(I + a*x)) - (I*Log[I + a*x])/a
```

Rubi [A] time = 0.0382867, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5073, 43}

$$\frac{2}{a(ax+i)}-\frac{i\log(ax+i)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] 2/(a*(I + a*x)) - (I*Log[I + a*x])/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx &= \int \frac{1+iax}{(1-iax)^2}\,dx\\
&= \int \left(-\frac{2}{(i+ax)^2}-\frac{i}{i+ax}\right)dx\\
&= \frac{2}{a(i+ax)}-\frac{i\log(i+ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0141234, size = 30, normalized size = 1.

$$\frac{2}{a(ax+i)}-\frac{i\log(ax+i)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] 2/(a*(I + a*x)) - (I*Log[I + a*x])/a
```

Maple [A] time = 0.044, size = 40, normalized size = 1.3

$$2\frac{1}{a(ax+i)} - \frac{\frac{i}{2}\ln\left(a^2x^2+1\right)}{a} - \frac{\arctan(ax)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^2,x)
```

```
[Out] 2/a/(a*x+I)-1/2*I/a*ln(a^2*x^2+1)-arctan(a*x)/a
```

Maxima [A] time = 1.47707, size = 59, normalized size = 1.97

$$\frac{4\,ax-4i}{2\left(a^3x^2+a\right)} - \frac{\arctan(ax)}{a} - \frac{i\log\left(a^2x^2+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*x - 4*I)/(a^3*x^2 + a) - arctan(a*x)/a - 1/2*I*log(a^2*x^2 + 1)/a
```

Fricas [A] time = 1.77536, size = 69, normalized size = 2.3

$$\frac{(-i\,ax+1)\log\left(\frac{ax+i}{a}\right)+2}{a^2x+i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] ((-I*a*x + 1)*log((a*x + I)/a) + 2)/(a^2*x + I*a)
```

Sympy [A] time = 0.458285, size = 22, normalized size = 0.73

$$\frac{2a}{a^3x+ia^2} - \frac{i\log(ax+i)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**2,x)
```

```
[Out] 2*a/(a**3*x + I*a**2) - I*log(a*x + I)/a
```

Giac [A] time = 1.1164, size = 34, normalized size = 1.13

$$-\frac{i\log\left(ax+i\right)}{a}+\frac{2}{\left(ax+i\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] -i*log(a*x + i)/a + 2/((a*x + i)*a)
```

3.304 $\int \frac{e^{2i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=41

$$-\frac{\sinh^{-1}(ax)}{a}-\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}}$$

```
[Out] ((-2*I)*Sqrt[1 + I*a*x])/(a*Sqrt[1 - I*a*x]) - ArcSinh[a*x]/a
```

Rubi [A] time = 0.0346615, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5073, 47, 41, 215}

$$-\frac{\sinh^{-1}(ax)}{a}-\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + I*a*x])/(a*Sqrt[1 - I*a*x]) - ArcSinh[a*x]/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{e^{2i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx = \int \frac{\sqrt{1+iax}}{(1-iax)^{3/2}}\,dx$$
$$= -\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \int \frac{1}{\sqrt{1-iax}\sqrt{1+iax}}\,dx$$
$$= -\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \int \frac{1}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \frac{\sinh^{-1}(ax)}{a}$$

Mathematica [A] time = 0.0296359, size = 52, normalized size = 1.27

$$-\frac{2i\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}+\sin^{-1}\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ((-2*I)*(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x] + ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]])
)/a
```

Maple [A] time = 0.064, size = 63, normalized size = 1.5

$$2\frac{x}{\sqrt{a^2x^2+1}} - \ln\left(a^2x\frac{1}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2}} - \frac{2i}{a}\frac{1}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x)
```

```
[Out] 2*x/(a^2*x^2+1)^(1/2)-ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)-2
*I/a/(a^2*x^2+1)^(1/2)
```

Maxima [A] time = 0.988022, size = 66, normalized size = 1.61

$$\frac{2x}{\sqrt{a^2x^2+1}} - \frac{\operatorname{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}} - \frac{2i}{\sqrt{a^2x^2+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*x/sqrt(a^2*x^2 + 1) - arcsinh(a^2*x/sqrt(a^2))/sqrt(a^2) - 2*I/(sqrt(a^2*
x^2 + 1)*a)
```

Fricas [A] time = 1.76655, size = 126, normalized size = 3.07

$$\frac{2\,ax + (ax + i)\log\left(-ax + \sqrt{a^2x^2 + 1}\right) + 2\,\sqrt{a^2x^2 + 1} + 2i}{a^2x + i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (2*a*x + (a*x + I)*log(-a*x + sqrt(a^2*x^2 + 1)) + 2*sqrt(a^2*x^2 + 1) + 2*
I)/(a^2*x + I*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(iax + 1)^2}{\left(a^2x^2 + 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((I*a*x + 1)**2/(a**2*x**2 + 1)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.305 $\int \frac{e^{i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{i\log(ax+i)}{a}$$

```
[Out] (I*Log[I + a*x])/a
```

Rubi [A] time = 0.02944, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5073, 31}

$$\frac{i\log(ax+i)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (I*Log[I + a*x])/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx &= \int \frac{1}{1-iax}\,dx\\ &= \frac{i\log(i+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0052172, size = 15, normalized size = 1.

$$\frac{i\log(ax+i)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (I*Log[I + a*x])/a
```

Maple [A] time = 0.033, size = 26, normalized size = 1.7

$$\frac{\frac{i}{2}\ln\left(a^2x^2+1\right)}{a}+\frac{\arctan\left(ax\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1),x)
```

```
[Out] 1/2*I/a*ln(a^2*x^2+1)+arctan(a*x)/a
```

Maxima [B] time = 1.49732, size = 32, normalized size = 2.13

$$\frac{\arctan\left(ax\right)}{a}+\frac{i\,\log\left(a^2x^2+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] arctan(a*x)/a + 1/2*I*log(a^2*x^2 + 1)/a
```

Fricas [A] time = 1.78238, size = 30, normalized size = 2.

$$\frac{i\,\log\left(\frac{ax+i}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] I*log((a*x + I)/a)/a
```

Sympy [A] time = 0.089788, size = 12, normalized size = 0.8

$$\frac{i\log\left(a^2x+ia\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1),x)
```

```
[Out] I*log(a**2*x + I*a)/a
```

Giac [A] time = 1.09913, size = 18, normalized size = 1.2

$$\frac{i\log\left(-aix+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1),x, algorithm="giac")
```

```
[Out] i*log(-a*i*x + 1)/a
```

3.306 $\int \frac{e^{-i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=16

$$-\frac{i\log(-ax+i)}{a}$$

```
[Out] ((-I)*Log[I - a*x])/a
```

Rubi [A] time = 0.0315238, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5073, 31}

$$-\frac{i\log(-ax+i)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] ((-I)*Log[I - a*x])/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx &= \int \frac{1}{1+iax}\,dx\\ &= -\frac{i\log(i-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0049329, size = 16, normalized size = 1.

$$-\frac{i\log(-ax+i)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] ((-I)*Log[I - a*x])/a
```

Maple [A] time = 0.033, size = 26, normalized size = 1.6

$$\frac{-\frac{i}{2}\ln\left(a^2x^2+1\right)}{a}+\frac{\arctan\left(ax\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x),x)
```

```
[Out] -1/2*I/a*ln(a^2*x^2+1)+arctan(a*x)/a
```

Maxima [A] time = 0.973474, size = 16, normalized size = 1.

$$-\frac{i\log\left(i\,ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x),x, algorithm="maxima")
```

```
[Out] -I*log(I*a*x + 1)/a
```

Fricas [A] time = 1.70131, size = 31, normalized size = 1.94

$$-\frac{i\log\left(\frac{ax-i}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x),x, algorithm="fricas")
```

```
[Out] -I*log((a*x - I)/a)/a
```

Sympy [A] time = 0.070349, size = 12, normalized size = 0.75

$$-\frac{i\log\left(iax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x),x)
```

```
[Out] -I*log(I*a*x + 1)/a
```

Giac [A] time = 1.11537, size = 18, normalized size = 1.12

$$-\frac{i\log\left(aix+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x),x, algorithm="giac")

[Out] -i*log(a*i*x + 1)/a
```

3.307 $\int \frac{e^{-2i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=41

$$-\frac{\sinh^{-1}(ax)}{a}+\frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}}$$

```
[Out] ((2*I)*Sqrt[1 - I*a*x])/(a*Sqrt[1 + I*a*x]) - ArcSinh[a*x]/a
```

Rubi [A] time = 0.0352676, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5073, 47, 41, 215}

$$-\frac{\sinh^{-1}(ax)}{a}+\frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] ((2*I)*Sqrt[1 - I*a*x])/(a*Sqrt[1 + I*a*x]) - ArcSinh[a*x]/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{e^{-2i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx = \int \frac{\sqrt{1-iax}}{(1+iax)^{3/2}}\,dx$$
$$= \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} - \int \frac{1}{\sqrt{1-iax}\sqrt{1+iax}}\,dx$$
$$= \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} - \int \frac{1}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} - \frac{\sinh^{-1}(ax)}{a}$$

Mathematica [A] time = 0.0577804, size = 56, normalized size = 1.37

$$\frac{2\left(\sqrt{a^2x^2+1}+(-1-iax)\sin^{-1}\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{a(ax-i)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (2*(Sqrt[1 + a^2*x^2] + (-1 - I*a*x)*ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]))/(a*(
-I + a*x))
```

Maple [B] time = 0.068, size = 143, normalized size = 3.5

$$\frac{-i}{a^3}\left(a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}\left(x-\frac{i}{a}\right)^{-2}+\frac{i}{a}\sqrt{a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)}-\ln\left(\left(ia+a^2\left(x-\frac{i}{a}\right)\right)\frac{1}{\sqrt{a^2}}+\sqrt{a^2\left(x-\frac{i}{a}\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x)
```

```
[Out] -I/a^3/(x-I/a)^2*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(3/2)+I/a*(a^2*(x-I/a)^2+2*I
*a*(x-I/a))^(1/2)-ln((I*a+a^2*(x-I/a))/(a^2)^(1/2)+(a^2*(x-I/a)^2+2*I*a*(x-
I/a))^(1/2))/(a^2)^(1/2)
```

Maxima [A] time = 1.46689, size = 45, normalized size = 1.1

$$-\frac{\operatorname{arsinh}(ax)}{a}+\frac{2i\sqrt{a^2x^2+1}}{i\,a^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(a*x)/a + 2*I*sqrt(a^2*x^2 + 1)/(I*a^2*x + a)
```

Fricas [A] time = 1.95559, size = 126, normalized size = 3.07

$$\frac{2\,ax + (ax - i)\log\left(-ax + \sqrt{a^2x^2 + 1}\right) + 2\,\sqrt{a^2x^2 + 1} - 2i}{a^2x - i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*a*x + (a*x - I)*log(-a*x + sqrt(a^2*x^2 + 1)) + 2*sqrt(a^2*x^2 + 1) - 2*
I)/(a^2*x - I*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2x^2 + 1}}{(iax + 1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2*x**2 + 1)/(I*a*x + 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] undef
```

3.308 $\int \frac{e^{-3i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{i\log(-ax+i)}{a}-\frac{2}{a(-ax+i)}$$

```
[Out] -2/(a*(I - a*x)) + (I*Log[I - a*x])/a
```

Rubi [A] time = 0.0380563, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5073, 43}

$$\frac{i\log(-ax+i)}{a}-\frac{2}{a(-ax+i)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -2/(a*(I - a*x)) + (I*Log[I - a*x])/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx &= \int \frac{1-iax}{(1+iax)^2}\,dx\\ &= \int \left(-\frac{2}{(-i+ax)^2}+\frac{i}{-i+ax}\right)dx\\ &= -\frac{2}{a(i-ax)}+\frac{i\log(i-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0150184, size = 32, normalized size = 1.

$$\frac{i\log(-ax+i)}{a}-\frac{2}{a(-ax+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -2/(a*(I - a*x)) + (I*Log[I - a*x])/a
```

Maple [A] time = 0.043, size = 41, normalized size = 1.3

$$\frac{\frac{i}{2}\ln\left(a^2x^2+1\right)}{a}-\frac{\arctan\left(ax\right)}{a}-2\frac{1}{a\left(-ax+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1),x)
```

```
[Out] 1/2*I/a*ln(a^2*x^2+1)-arctan(a*x)/a-2/a/(-a*x+I)
```

Maxima [A] time = 0.979072, size = 55, normalized size = 1.72

$$-\frac{4\left(-i\,ax-1\right)}{2i\,a^3x^2+4\,a^2x-2i\,a}+\frac{i\,\log\left(i\,ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -4*(-I*a*x - 1)/(2*I*a^3*x^2 + 4*a^2*x - 2*I*a) + I*log(I*a*x + 1)/a
```

Fricas [A] time = 1.72624, size = 68, normalized size = 2.12

$$\frac{\left(i\,ax+1\right)\log\left(\frac{ax-i}{a}\right)+2}{a^2x-i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] ((I*a*x + 1)*log((a*x - I)/a) + 2)/(a^2*x - I*a)
```

Sympy [A] time = 0.38787, size = 24, normalized size = 0.75

$$\frac{2a}{a^3x-ia^2}+\frac{i\log\left(iax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1),x)
```

```
[Out] 2*a/(a**3*x - I*a**2) + I*log(I*a*x + 1)/a
```

Giac [A] time = 1.09108, size = 38, normalized size = 1.19

$$\frac{i\log\left(ax-i\right)}{a}+\frac{2}{\left(ax-i\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1),x, algorithm="giac")
```

```
[Out] i*log(a*x - i)/a + 2/((a*x - i)*a)
```

3.309 $\int \frac{e^{-4i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=73

$$\frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}}-\frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}}+\frac{\sinh^{-1}(ax)}{a}$$

```
[Out] (((2*I)/3)*(1 - I*a*x)^(3/2))/(a*(1 + I*a*x)^(3/2)) - ((2*I)*Sqrt[1 - I*a*x
])/(a*Sqrt[1 + I*a*x]) + ArcSinh[a*x]/a
```

Rubi [A] time = 0.0376806, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5073, 47, 41, 215}

$$\frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}}-\frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}}+\frac{\sinh^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((4*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (((2*I)/3)*(1 - I*a*x)^(3/2))/(a*(1 + I*a*x)^(3/2)) - ((2*I)*Sqrt[1 - I*a*x
])/(a*Sqrt[1 + I*a*x]) + ArcSinh[a*x]/a
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-4i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx &= \int \frac{(1-iax)^{3/2}}{(1+iax)^{5/2}}\,dx \\
&= \frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}} - \int \frac{\sqrt{1-iax}}{(1+iax)^{3/2}}\,dx \\
&= \frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}} - \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} + \int \frac{1}{\sqrt{1-iax}\sqrt{1+iax}}\,dx \\
&= \frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}} - \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} + \int \frac{1}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}} - \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} + \frac{\sinh^{-1}(ax)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0715867, size = 82, normalized size = 1.12

$$
\frac{2i\left(\frac{2\sqrt{1+iax}(2a^2x^2+iax+1)}{\sqrt{1-iax}(ax-i)^2}+3\sin^{-1}\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{3a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((4*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (((2*I)/3)*((2*Sqrt[1 + I*a*x]*(1 + I*a*x + 2*a^2*x^2))/(Sqrt[1 - I*a*x]*(-
I + a*x)^2) + 3*ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]))/a
```

Maple [B] time = 0.069, size = 262, normalized size = 3.6

$$
\frac{\frac{i}{3}}{a^5}\left(a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}\left(x-\frac{i}{a}\right)^{-4}+\frac{1}{3\,a^4}\left(a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}\left(x-\frac{i}{a}\right)^{-3}+\frac{\frac{2\,i}{3}}{a^3}\left(a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/3*I/a^5/(x-I/a)^4*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(5/2)+1/3/a^4/(x-I/a)^3*(
a^2*(x-I/a)^2+2*I*a*(x-I/a))^(5/2)+2/3*I/a^3/(x-I/a)^2*(a^2*(x-I/a)^2+2*I*a
*(x-I/a))^(5/2)-2/3*I/a*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(3/2)+(a^2*(x-I/a)^2+
2*I*a*(x-I/a))^(1/2)*x+ln((I*a+a^2*(x-I/a))/(a^2)^(1/2)+(a^2*(x-I/a)^2+2*I*
a*(x-I/a))^(1/2))/(a^2)^(1/2)
```

Maxima [B] time = 1.54856, size = 146, normalized size = 2.

$$
\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}}{-3i\,a^4x^3-9\,a^3x^2+9i\,a^2x+3\,a}+\frac{\operatorname{arsinh}(ax)}{a}-\frac{2i\,\sqrt{a^2x^2+1}}{3\,a^3x^2-6i\,a^2x-3\,a}-\frac{7i\,\sqrt{a^2x^2+1}}{3i\,a^2x+3\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] I*(a^2*x^2 + 1)^(3/2)/(-3*I*a^4*x^3 - 9*a^3*x^2 + 9*I*a^2*x + 3*a) + arcsin
h(a*x)/a - 2*I*sqrt(a^2*x^2 + 1)/(3*a^3*x^2 - 6*I*a^2*x - 3*a) - 7*I*sqrt(a
^2*x^2 + 1)/(3*I*a^2*x + 3*a)
```

Fricas [A] time = 1.9496, size = 204, normalized size = 2.79

$$-\frac{8\,a^2x^2 - 16i\,ax + \left(3\,a^2x^2 - 6i\,ax - 3\right)\log\left(-ax + \sqrt{a^2x^2+1}\right) + \sqrt{a^2x^2+1}(8\,ax - 4i) - 8}{3\,a^3x^2 - 6i\,a^2x - 3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(8*a^2*x^2 - 16*I*a*x + (3*a^2*x^2 - 6*I*a*x - 3)*log(-a*x + sqrt(a^2*x^2
+ 1)) + sqrt(a^2*x^2 + 1)*(8*a*x - 4*I) - 8)/(3*a^3*x^2 - 6*I*a^2*x - 3*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{(iax+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**4*(a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((a**2*x**2 + 1)**(3/2)/(I*a*x + 1)**4, x)
```

Giac [A] time = 1.15249, size = 32, normalized size = 0.44

$$-\frac{\log\left(-x|a| + \sqrt{a^2x^2+1}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a)
```

3.310 $\int \frac{e^{5i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=131

$$\frac{4i\sqrt{a^2x^2+1}}{a(1-iax)\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}}{a(1-iax)^2\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] ((-2*I)*Sqrt[1 + a^2*x^2])/(a*(1 - I*a*x)^2*Sqrt[c + a^2*c*x^2]) + ((4*I)*S
qrt[1 + a^2*x^2])/(a*(1 - I*a*x)*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2
]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0831411, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5076, 5073, 43}

$$\frac{4i\sqrt{a^2x^2+1}}{a(1-iax)\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}}{a(1-iax)^2\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + a^2*x^2])/(a*(1 - I*a*x)^2*Sqrt[c + a^2*c*x^2]) + ((4*I)*S
qrt[1 + a^2*x^2])/(a*(1 - I*a*x)*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2
]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{5i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{5i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{(1+iax)^2}{(1-iax)^3}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \left(\frac{4}{(1-iax)^3}-\frac{4}{(1-iax)^2}+\frac{1}{1-iax}\right)dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2i\sqrt{1+a^2x^2}}{a(1-iax)^2\sqrt{c+a^2cx^2}}+\frac{4i\sqrt{1+a^2x^2}}{a(1-iax)\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\log(i+ax)}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0333269, size = 69, normalized size = 0.53

$$\frac{i\sqrt{a^2x^2+1}\left(4iax+(ax+i)^2\log(ax+i)-2\right)}{a(ax+i)^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((5*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (I*Sqrt[1 + a^2*x^2]*(-2 + (4*I)*a*x + (I + a*x)^2*Log[I + a*x]))/(a*(I + a
*x)^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.151, size = 84, normalized size = 0.6

$$\frac{i\ln(ax+i)x^2a^2-2\ln(ax+i)xa-i\ln(ax+i)-4ax-2i}{ac(ax+i)^2}\sqrt{c\left(a^2x^2+1\right)}\frac{1}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(I*ln(a*x+I)*x^2*a^2-2*ln(a*x+I)*
x*a-I*ln(a*x+I)-4*a*x-2*I)/c/a/(a*x+I)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)^5}{\sqrt{a^2cx^2+c}\left(a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((I*a*x + 1)^5/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^(5/2)), x)
```

Fricas [B] time = 2.2175, size = 815, normalized size = 6.22

$$\frac{-4i\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}ax^2+\left(ia^4cx^4-2a^3cx^3-2acx-ic\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{\left(ia^6x^2-2a^5x-2ia^4\right)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+\left(ia^9cx^4-2a^8cx^3+ia^7c\right.}{8a^3x^3+8ia^2x^2+8ax+8i}\right.}{2a^4cx^4+4ia^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] (-4*I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a*x^2 + (I*a^4*c*x^4 - 2*a^3*c*
x^3 - 2*a*c*x - I*c)*sqrt(1/(a^2*c))*log(((I*a^6*x^2 - 2*a^5*x - 2*I*a^4)*s
qrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1) + (I*a^9*c*x^4 - 2*a^8*c*x^3 + I*a^7*c
*x^2 - 2*a^6*c*x)*sqrt(1/(a^2*c)))/(8*a^3*x^3 + 8*I*a^2*x^2 + 8*a*x + 8*I))
 + (-I*a^4*c*x^4 + 2*a^3*c*x^3 + 2*a*c*x + I*c)*sqrt(1/(a^2*c))*log(((I*a^6
*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1) + (-I*a^9*c
*x^4 + 2*a^8*c*x^3 - I*a^7*c*x^2 + 2*a^6*c*x)*sqrt(1/(a^2*c)))/(8*a^3*x^3 +
 8*I*a^2*x^2 + 8*a*x + 8*I)))/(2*a^4*c*x^4 + 4*I*a^3*c*x^3 + 4*I*a*c*x - 2*
c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(iax+1)^5}{\sqrt{c\left(a^2x^2+1\right)}\left(a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**5/(a**2*x**2+1)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((I*a*x + 1)**5/(sqrt(c*(a**2*x**2 + 1))*(a**2*x**2 + 1)**(5/2)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(iax+1)^5}{\sqrt{a^2cx^2+c}\left(a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*x + 1)^5/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^(5/2)), x)
```

3.311 $\int \frac{e^{4i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=96

$$-\frac{2ic(1+iax)^3}{3a\left(a^2cx^2+c\right)^{3/2}}+\frac{2i(1+iax)}{a\sqrt{a^2cx^2+c}}+\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}$$

```
[Out] (((-2*I)/3)*c*(1 + I*a*x)^3)/(a*(c + a^2*c*x^2)^(3/2)) + ((2*I)*(1 + I*a*x)
)/(a*Sqrt[c + a^2*c*x^2]) + ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a*S
qrt[c])
```

Rubi [A] time = 0.0845581, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5075, 669, 653, 217, 206}

$$-\frac{2ic(1+iax)^3}{3a\left(a^2cx^2+c\right)^{3/2}}+\frac{2i(1+iax)}{a\sqrt{a^2cx^2+c}}+\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (((-2*I)/3)*c*(1 + I*a*x)^3)/(a*(c + a^2*c*x^2)^(3/2)) + ((2*I)*(1 + I*a*x)
)/(a*Sqrt[c + a^2*c*x^2]) + ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a*S
qrt[c])
```

Rule 5075

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[1/c^((I*n)/2), Int[(c + d*x^2)^(p + (I*n)/2)/(1 + I*a*x)^(I*n), x], x] /
; FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0]) &
& ILtQ[(I*n)/2, 0]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] && !IntegerQ[p] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx &= c^2\int \frac{(1+iax)^4}{\left(c+a^2cx^2\right)^{5/2}}\,dx \\
&= -\frac{2ic(1+iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}} - c\int \frac{(1+iax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx \\
&= -\frac{2ic(1+iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}} + \frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} + \int \frac{1}{\sqrt{c+a^2cx^2}}\,dx \\
&= -\frac{2ic(1+iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}} + \frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} + \text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{x}{\sqrt{c+a^2cx^2}}\right) \\
&= -\frac{2ic(1+iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}} + \frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} + \frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}
\end{aligned}$$

Mathematica [C] time = 0.0216242, size = 71, normalized size = 0.74

$$-\frac{4i\sqrt{2a^2x^2+2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{1}{2}(1-iax)\right)}{3a(1-iax)^{3/2}\sqrt{a^2cx^2+c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (((-4*I)/3)*Sqrt[2 + 2*a^2*x^2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - I*
a*x)/2])/(a*(1 - I*a*x)^(3/2)*Sqrt[c + a^2*c*x^2])
```

Maple [B] time = 0.229, size = 800, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] ln(x*a^2*c/(a^2*c)^(1/2)+(a^2*c*x^2+c)^(1/2))/(a^2*c)^(1/2)-2/a^3*(I*(-a^2)
^(1/2)+a)/c/(x-(-a^2)^(1/2)/a^2)*((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(
1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2)+2/a^3*(I*(-a^2)^(1/2)-a)/c/(x+(-a^2)^(1/2)
/a^2)*((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^
(1/2)+2/3*I/a^3/c/(x-(-a^2)^(1/2)/a^2)^2*((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*
(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2)+2/3/a^2/c/(-a^2)^(1/2)/(x-(-a^2)^(
1/2)/a^2)^2*((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/
a^2))^(1/2)+2/3*I/a^3/c/(x-(-a^2)^(1/2)/a^2)*((x-(-a^2)^(1/2)/a^2)^2*a^2*c+
2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2)*(-a^2)^(1/2)+2/3/a^2/c/(x-(-a^
2)^(1/2)/a^2)*((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2
```

```
)/a^2))^(1/2)+2/3*I/a^3/c/(x+(-a^2)^(1/2)/a^2)^2*((x+(-a^2)^(1/2)/a^2)^2*a^
2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)-2/3/a^2/c/(-a^2)^(1/2)/(x+
(-a^2)^(1/2)/a^2)^2*((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2
)^(1/2)/a^2))^(1/2)-2/3*I/a^3/c/(x+(-a^2)^(1/2)/a^2)*((x+(-a^2)^(1/2)/a^2)^
2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)*(-a^2)^(1/2)+2/3/a^2/c
/(x+(-a^2)^(1/2)/a^2)*((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a
^2)^(1/2)/a^2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)^4}{\sqrt{a^2cx^2+c}\left(a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((I*a*x + 1)^4/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^2), x)
```

Fricas [B] time = 2.22531, size = 417, normalized size = 4.34

$$\frac{\left(3\,a^3cx^2+6i\,a^2cx-3\,ac\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx+\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-\left(3\,a^3cx^2+6i\,a^2cx-3\,ac\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx-\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2}}\right.}{x}\cdots}{6\,a^3cx^2+12i\,a^2cx-6\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] ((3*a^3*c*x^2 + 6*I*a^2*c*x - 3*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x + sqrt(
a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*c)))/x) - (3*a^3*c*x^2 + 6*I*a^2*c*x - 3*a
*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x - sqrt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*
c)))/x) - sqrt(a^2*c*x^2 + c)*(16*a*x + 8*I))/(6*a^3*c*x^2 + 12*I*a^2*c*x -
 6*a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(iax+1)^4}{\sqrt{c\left(a^2x^2+1\right)}\left(a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((I*a*x + 1)**4/(sqrt(c*(a**2*x**2 + 1))*(a**2*x**2 + 1)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.312 $\int \frac{e^{3i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=84

$$\frac{2\sqrt{a^2x^2+1}}{a(ax+i)\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] (2*Sqrt[1 + a^2*x^2])/(a*(I + a*x)*Sqrt[c + a^2*c*x^2]) - (I*Sqrt[1 + a^2*x
^2]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0767305, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5076, 5073, 43}

$$\frac{2\sqrt{a^2x^2+1}}{a(ax+i)\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2])/(a*(I + a*x)*Sqrt[c + a^2*c*x^2]) - (I*Sqrt[1 + a^2*x
^2]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{3i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{3i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{1+iax}{(1-iax)^2}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \left(-\frac{2}{(i+ax)^2}-\frac{i}{i+ax}\right)dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{2\sqrt{1+a^2x^2}}{a(i+ax)\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\log(i+ax)}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0262165, size = 55, normalized size = 0.65

$$\frac{\sqrt{a^2x^2+1}\left(\frac{2}{ax+i}-i\log(ax+i)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]

[Out] (Sqrt[1 + a^2*x^2]*(2/(I + a*x) - I*Log[I + a*x]))/(a*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.143, size = 61, normalized size = 0.7

$$\frac{-i\ln(ax+i)\,xa+\ln(ax+i)+2}{ac\,(ax+i)}\sqrt{c\left(a^2x^2+1\right)}\frac{1}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x)

[Out] (-I*ln(a*x+I)*x*a+ln(a*x+I)+2)/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/c/a/
(a*x+I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)^3}{\sqrt{a^2cx^2+c}\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")

[Out] integrate((I*a*x + 1)^3/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^(3/2)), x)
```

Fricas [B] time = 2.26181, size = 798, normalized size = 9.5

$$\frac{\left(-i\,a^3cx^3+a^2cx^2-i\,acx+c\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{\left(i\,a^6x^2-2\,a^5x-2i\,a^4\right)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+\left(i\,a^9cx^4-2\,a^8cx^3+i\,a^7cx^2-2\,a^6cx\right)\sqrt{\frac{1}{a^2c}}}{8\,a^3x^3+8i\,a^2x^2+8\,ax+8i}\right)+\left(i\,a^3cx^3-a^2c\right.}{2\,a^3cx^3+2i\,a^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] ((-I*a^3*c*x^3 + a^2*c*x^2 - I*a*c*x + c)*sqrt(1/(a^2*c))*log(((I*a^6*x^2 -
 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1) + (I*a^9*c*x^4 -
2*a^8*c*x^3 + I*a^7*c*x^2 - 2*a^6*c*x)*sqrt(1/(a^2*c)))/(8*a^3*x^3 + 8*I*a^
2*x^2 + 8*a*x + 8*I)) + (I*a^3*c*x^3 - a^2*c*x^2 + I*a*c*x - c)*sqrt(1/(a^2
*c))*log(((I*a^6*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2
+ 1) + (-I*a^9*c*x^4 + 2*a^8*c*x^3 - I*a^7*c*x^2 + 2*a^6*c*x)*sqrt(1/(a^2*c
)))/(8*a^3*x^3 + 8*I*a^2*x^2 + 8*a*x + 8*I)) + 4*I*sqrt(a^2*c*x^2 + c)*sqrt
(a^2*x^2 + 1)*x)/(2*a^3*c*x^3 + 2*I*a^2*c*x^2 + 2*a*c*x + 2*I*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(iax+1)^3}{\sqrt{c\left(a^2x^2+1\right)}\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((I*a*x + 1)**3/(sqrt(c*(a**2*x**2 + 1))*(a**2*x**2 + 1)**(3/2)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)^3}{\sqrt{a^2cx^2+c}\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*x + 1)^3/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^(3/2)), x)
```

3.313 $\int \frac{e^{2i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=63

$$-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}-\frac{2i(1+iax)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] ((-2*I)*(1 + I*a*x))/(a*Sqrt[c + a^2*c*x^2]) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c
 + a^2*c*x^2]]/(a*Sqrt[c])
```

Rubi [A] time = 0.0602542, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {5075, 653, 217, 206}

$$-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}-\frac{2i(1+iax)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((-2*I)*(1 + I*a*x))/(a*Sqrt[c + a^2*c*x^2]) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c
 + a^2*c*x^2]]/(a*Sqrt[c])
```

Rule 5075

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[1/c^((I*n)/2), Int[(c + d*x^2)^(p + (I*n)/2)/(1 + I*a*x)^(I*n), x], x] /
; FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0]) &
& ILtQ[(I*n)/2, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] && !IntegerQ[p] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx &= c\int \frac{(1+iax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx \\
&= -\frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} - \int \frac{1}{\sqrt{c+a^2cx^2}}\,dx \\
&= -\frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} - \text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{x}{\sqrt{c+a^2cx^2}}\right) \\
&= -\frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.0272302, size = 91, normalized size = 1.44

$$
-\frac{2i\sqrt{a^2x^2+1}\left(\sqrt{1+iax}+\sqrt{1-iax}\sin^{-1}\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-iax}\sqrt{a^2cx^2+c}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + a^2*x^2]*(Sqrt[1 + I*a*x] + Sqrt[1 - I*a*x]*ArcSin[Sqrt[1
- I*a*x]/Sqrt[2]]))/(a*Sqrt[1 - I*a*x]*Sqrt[c + a^2*c*x^2])
```

Maple [B] time = 0.172, size = 204, normalized size = 3.2

$$
-\ln\left(a^2cx\frac{1}{\sqrt{a^2c}}+\sqrt{a^2cx^2+c}\right)\frac{1}{\sqrt{a^2c}}-\frac{1}{a^3c}\left(i\sqrt{-a^2}-a\right)\sqrt{\left(x+\frac{1}{a^2}\sqrt{-a^2}\right)^2a^2c-2c\sqrt{-a^2}\left(x+\frac{\sqrt{-a^2}}{a^2}\right)}\left(x+\frac{1}{a^2}\sqrt{-a^2}\right)^{-1}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -ln(x*a^2*c/(a^2*c)^(1/2)+(a^2*c*x^2+c)^(1/2))/(a^2*c)^(1/2)-1/a^3*(I*(-a^2
)^(1/2)-a)/c/(x+(-a^2)^(1/2)/a^2)*((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^
(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)+1/a^3*(I*(-a^2)^(1/2)+a)/c/(x-(-a^2)^(1/2
)/a^2)*((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{(iax+1)^2}{\sqrt{a^2cx^2+c}\left(a^2x^2+1\right)}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*x + 1)^2/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)), x)
```

Fricas [B] time = 2.00894, size = 332, normalized size = 5.27

$$-\frac{\left(a^2cx+i\,ac\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx+\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-\left(a^2cx+i\,ac\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx-\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-4\sqrt{a^2cx^2+c}}{2\,a^2cx+2i\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -((a^2*c*x + I*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x + sqrt(a^2*c*x^2 + c)*a^
2*c*sqrt(1/(a^2*c)))/x) - (a^2*c*x + I*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x
- sqrt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*c)))/x) - 4*sqrt(a^2*c*x^2 + c))/(2
*a^2*c*x + 2*I*a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(iax+1)^2}{\sqrt{c\left(a^2x^2+1\right)}\left(a^2x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((I*a*x + 1)**2/(sqrt(c*(a**2*x**2 + 1))*(a**2*x**2 + 1)), x)
```

Giac [A] time = 1.14073, size = 97, normalized size = 1.54

$$\frac{\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{a\sqrt{c}}-\frac{4}{\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)i-\sqrt{c}\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/(a*sqrt(c)) - 4/(((sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 + c))*i - sqrt(c))*a)
```

3.314 $\int \frac{e^{i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=42

$$\frac{i\sqrt{a^2x^2+1}\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] (I*Sqrt[1 + a^2*x^2]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0648809, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5076, 5073, 31}

$$\frac{i\sqrt{a^2x^2+1}\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (I*Sqrt[1 + a^2*x^2]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int \frac{1}{1-iax}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{i\sqrt{1+a^2x^2}\log(i+ax)}{a\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0129689, size = 42, normalized size = 1.

$$\frac{i\sqrt{a^2x^2+1}\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (I*Sqrt[1 + a^2*x^2]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.125, size = 53, normalized size = 1.3

$$\frac{i\ln\left(a^2x^2+1\right)+2\arctan(ax)}{2ac}\sqrt{c\left(a^2x^2+1\right)}\frac{1}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(I*ln(a^2*x^2+1)+2*arctan(a*x))
/c/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.0996, size = 575, normalized size = 13.69

$$\frac{1}{2}i\sqrt{\frac{1}{a^2c}}\log\left(\frac{\left(ia^6x^2-2a^5x-2ia^4\right)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+\left(ia^9cx^4-2a^8cx^3+ia^7cx^2-2a^6cx\right)\sqrt{\frac{1}{a^2c}}}{8a^3x^3+8ia^2x^2+8ax+8i}\right)-\frac{1}{2}i\sqrt{\frac{1}{a^2c}}\log\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*I*sqrt(1/(a^2*c))*log(((I*a^6*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 +
 c)*sqrt(a^2*x^2 + 1) + (I*a^9*c*x^4 - 2*a^8*c*x^3 + I*a^7*c*x^2 - 2*a^6*c*
x)*sqrt(1/(a^2*c)))/(8*a^3*x^3 + 8*I*a^2*x^2 + 8*a*x + 8*I)) - 1/2*I*sqrt(1
/(a^2*c))*log(((I*a^6*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2
*x^2 + 1) + (-I*a^9*c*x^4 + 2*a^8*c*x^3 - I*a^7*c*x^2 + 2*a^6*c*x)*sqrt(1/(
```

```
a^2*c)))/(8*a^3*x^3 + 8*I*a^2*x^2 + 8*a*x + 8*I))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{iax+1}{\sqrt{c\left(a^2x^2+1\right)}\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((I*a*x + 1)/(sqrt(c*(a**2*x**2 + 1))*sqrt(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{i\,ax+1}{\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((I*a*x + 1)/(sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)), x)
```

3.315 $\int \frac{e^{-i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=43

$$-\frac{i\sqrt{a^2x^2+1}\log(-ax+i)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] ((-I)*Sqrt[1 + a^2*x^2]*Log[I - a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0658911, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5076, 5073, 31}

$$-\frac{i\sqrt{a^2x^2+1}\log(-ax+i)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + a^2*x^2]*Log[I - a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{-i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\int \frac{1}{1+iax}\,dx}{\sqrt{c+a^2cx^2}}\\ &= -\frac{i\sqrt{1+a^2x^2}\log(i-ax)}{a\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0157528, size = 43, normalized size = 1.

$$-\frac{i\sqrt{a^2x^2+1}\log(-ax+i)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + a^2*x^2]*Log[I - a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.109, size = 42, normalized size = 1.

$$\frac{-i\ln(1+iax)}{ac}\sqrt{c\left(a^2x^2+1\right)}\frac{1}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -I/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/c*ln(1+I*a*x)/a
```

Maxima [A] time = 0.998187, size = 20, normalized size = 0.47

$$-\frac{i\log(i\,ax+1)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] -I*log(I*a*x + 1)/(a*sqrt(c))
```

Fricas [B] time = 2.25501, size = 578, normalized size = 13.44

$$\frac{1}{2}i\sqrt{\frac{1}{a^2c}}\log\left(\frac{\left(-i\,a^6x^2-2\,a^5x+2i\,a^4\right)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+\left(i\,a^9cx^4+2\,a^8cx^3+i\,a^7cx^2+2\,a^6cx\right)\sqrt{\frac{1}{a^2c}}}{8\,a^3x^3-8i\,a^2x^2+8\,ax-8i}\right)-\frac{1}{2}i\sqrt{\frac{1}{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/2*I*sqrt(1/(a^2*c))*log(((-I*a^6*x^2 - 2*a^5*x + 2*I*a^4)*sqrt(a^2*c*x^2
+ c)*sqrt(a^2*x^2 + 1) + (I*a^9*c*x^4 + 2*a^8*c*x^3 + I*a^7*c*x^2 + 2*a^6*c
*x)*sqrt(1/(a^2*c)))/(8*a^3*x^3 - 8*I*a^2*x^2 + 8*a*x - 8*I)) - 1/2*I*sqrt(
1/(a^2*c))*log(((-I*a^6*x^2 - 2*a^5*x + 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a
^2*x^2 + 1) + (-I*a^9*c*x^4 - 2*a^8*c*x^3 - I*a^7*c*x^2 - 2*a^6*c*x)*sqrt(1
```

```
/(a^2*c)))/(8*a^3*x^3 - 8*I*a^2*x^2 + 8*a*x - 8*I))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2x^2+1}}{\sqrt{c\left(a^2x^2+1\right)}\,(iax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2*x**2 + 1)/(sqrt(c*(a**2*x**2 + 1))*(I*a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2x^2+1}}{\sqrt{a^2cx^2+c}(i\,ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)/(sqrt(a^2*c*x^2 + c)*(I*a*x + 1)), x)
```

3.316 $\int \frac{e^{-2i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=63

$$-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}+\frac{2i(1-iax)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] ((2*I)*(1 - I*a*x))/(a*Sqrt[c + a^2*c*x^2]) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c
+ a^2*c*x^2]]/(a*Sqrt[c])
```

Rubi [A] time = 0.0611335, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {5074, 653, 217, 206}

$$-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}+\frac{2i(1-iax)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((2*I)*(1 - I*a*x))/(a*Sqrt[c + a^2*c*x^2]) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c
+ a^2*c*x^2]]/(a*Sqrt[c])
```

Rule 5074

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[c^((I*n)/2), Int[(c + d*x^2)^(p - (I*n)/2)*(1 - I*a*x)^(I*n), x], x] /;
FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &&
IGtQ[(I*n)/2, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx &= c\int \frac{(1-iax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} - \int \frac{1}{\sqrt{c+a^2cx^2}}\,dx\\
&= \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} - \text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{x}{\sqrt{c+a^2cx^2}}\right)\\
&= \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.0577683, size = 117, normalized size = 1.86

$$
\frac{2\sqrt{a^2x^2+1}\left((1-iax)\sqrt{1+iax}-i\sqrt{1-iax}(ax-i)\sin^{-1}\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-iax}(ax-i)\sqrt{a^2cx^2+c}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*((1 - I*a*x)*Sqrt[1 + I*a*x] - I*Sqrt[1 - I*a*x]*(-I +
 a*x)*ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]))/(a*Sqrt[1 - I*a*x]*(-I + a*x)*Sqrt[
c + a^2*c*x^2])
```

Maple [A] time = 0.201, size = 87, normalized size = 1.4

$$
-\ln\left(a^2cx\frac{1}{\sqrt{a^2c}}+\sqrt{a^2cx^2+c}\right)\frac{1}{\sqrt{a^2c}}+2\,\frac{1}{a^2c}\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2\,iac\left(x-\frac{i}{a}\right)}\left(x-\frac{i}{a}\right)^{-1}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -ln(x*a^2*c/(a^2*c)^(1/2)+(a^2*c*x^2+c)^(1/2))/(a^2*c)^(1/2)+2/a^2/c/(x-I/a
)*((x-I/a)^2*a^2*c+2*I*a*c*(x-I/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.99296, size = 332, normalized size = 5.27

$$-\frac{\left(a^2cx - i\,ac\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx+\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right) - \left(a^2cx - i\,ac\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx-\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right) - 4\sqrt{a^2cx^2+c}}{2\,a^2cx - 2i\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] -((a^2*c*x - I*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x + sqrt(a^2*c*x^2 + c)*a^
2*c*sqrt(1/(a^2*c)))/x) - (a^2*c*x - I*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x
- sqrt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*c)))/x) - 4*sqrt(a^2*c*x^2 + c))/(2
*a^2*c*x - 2*I*a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2x^2+1}{\sqrt{c\left(a^2x^2+1\right)}\,(iax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a**2*x**2 + 1)/(sqrt(c*(a**2*x**2 + 1))*(I*a*x + 1)**2), x)
```

Giac [A] time = 1.32336, size = 101, normalized size = 1.6

$$-i^2\left(\frac{\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{a\sqrt{c}} + \frac{4}{\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)i+\sqrt{c}\right)a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] -i^2*(log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/(a*sqrt(c)) + 4/(((sqr
t(a^2*c)*x - sqrt(a^2*c*x^2 + c))*i + sqrt(c))*a))
```

3.317 $\int \frac{e^{-3i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{i\sqrt{a^2x^2+1}\log(-ax+i)}{a\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{a(-ax+i)\sqrt{a^2cx^2+c}}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*(I - a*x)*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*
x^2]*Log[I - a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0755422, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5076, 5073, 43}

$$\frac{i\sqrt{a^2x^2+1}\log(-ax+i)}{a\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{a(-ax+i)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*(I - a*x)*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*
x^2]*Log[I - a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{-3i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{-3i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{1-iax}{(1+iax)^2}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \left(-\frac{2}{(-i+ax)^2}+\frac{i}{-i+ax}\right)dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2\sqrt{1+a^2x^2}}{a(i-ax)\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\log(i-ax)}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0275034, size = 60, normalized size = 0.7

$$\frac{\sqrt{a^2x^2+1}\left(\frac{i\log(-ax+i)}{a}-\frac{2}{a(-ax+i)}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(-2/(a*(I - a*x)) + (I*Log[I - a*x])/a))/Sqrt[c + a^2*c*
x^2]
```

Maple [A] time = 0.142, size = 66, normalized size = 0.8

$$\frac{-i\ln(-ax+i)\,xa-\ln(-ax+i)-2}{ac\,(-ax+i)}\sqrt{c\left(a^2x^2+1\right)}\frac{1}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-I*ln(-a*x+I)*x*a-ln(-a*x+I)-2)/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/c/
a/(-a*x+I)
```

Maxima [A] time = 0.998757, size = 47, normalized size = 0.55

$$\frac{i\log(i\,ax+1)}{a\sqrt{c}}+\frac{2}{a^2\sqrt{c}x-i\,a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] I*log(I*a*x + 1)/(a*sqrt(c)) + 2/(a^2*sqrt(c)*x - I*a*sqrt(c))
```

Fricas [B] time = 2.25482, size = 801, normalized size = 9.31

$$\frac{\left(-i\,a^3cx^3-a^2cx^2-i\,acx-c\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{\left(-i\,a^6x^2-2\,a^5x+2i\,a^4\right)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+\left(i\,a^9cx^4+2\,a^8cx^3+i\,a^7cx^2+2\,a^6cx\right)\sqrt{\frac{1}{a^2c}}}{8\,a^3x^3-8i\,a^2x^2+8\,ax-8i}\right)+\left(i\,a^3cx^3+a^2cx^2\cdots\right.}{2\,a^3cx^3-2i\,a^2cx^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] ((-I*a^3*c*x^3 - a^2*c*x^2 - I*a*c*x - c)*sqrt(1/(a^2*c))*log(((-I*a^6*x^2
- 2*a^5*x + 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1) + (I*a^9*c*x^4 +
 2*a^8*c*x^3 + I*a^7*c*x^2 + 2*a^6*c*x)*sqrt(1/(a^2*c)))/(8*a^3*x^3 - 8*I*a
^2*x^2 + 8*a*x - 8*I)) + (I*a^3*c*x^3 + a^2*c*x^2 + I*a*c*x + c)*sqrt(1/(a^
2*c))*log(((-I*a^6*x^2 - 2*a^5*x + 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^
2 + 1) + (-I*a^9*c*x^4 - 2*a^8*c*x^3 - I*a^7*c*x^2 - 2*a^6*c*x)*sqrt(1/(a^2
*c)))/(8*a^3*x^3 - 8*I*a^2*x^2 + 8*a*x - 8*I)) - 4*I*sqrt(a^2*c*x^2 + c)*sq
rt(a^2*x^2 + 1)*x)/(2*a^3*c*x^3 - 2*I*a^2*c*x^2 + 2*a*c*x - 2*I*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}(i\,ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/(sqrt(a^2*c*x^2 + c)*(I*a*x + 1)^3), x)
```

3.318 $\int \frac{e^{-4i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=96

$$\frac{2ic(1-iax)^3}{3a\left(a^2cx^2+c\right)^{3/2}}-\frac{2i(1-iax)}{a\sqrt{a^2cx^2+c}}+\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}$$

```
[Out] (((2*I)/3)*c*(1 - I*a*x)^3)/(a*(c + a^2*c*x^2)^(3/2)) - ((2*I)*(1 - I*a*x))
/(a*Sqrt[c + a^2*c*x^2]) + ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a*Sq
rt[c])
```

Rubi [A] time = 0.0775118, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5074, 669, 653, 217, 206}

$$\frac{2ic(1-iax)^3}{3a\left(a^2cx^2+c\right)^{3/2}}-\frac{2i(1-iax)}{a\sqrt{a^2cx^2+c}}+\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((4*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (((2*I)/3)*c*(1 - I*a*x)^3)/(a*(c + a^2*c*x^2)^(3/2)) - ((2*I)*(1 - I*a*x))
/(a*Sqrt[c + a^2*c*x^2]) + ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a*Sq
rt[c])
```

Rule 5074

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[c^((I*n)/2), Int[(c + d*x^2)^(p - (I*n)/2)*(1 - I*a*x)^(I*n), x], x] /;
FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &&
IGtQ[(I*n)/2, 0]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-4i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx &= c^2\int \frac{(1-iax)^4}{\left(c+a^2cx^2\right)^{5/2}}\,dx\\
&= \frac{2ic(1-iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}} - c\int \frac{(1-iax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{2ic(1-iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}} - \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} + \int \frac{1}{\sqrt{c+a^2cx^2}}\,dx\\
&= \frac{2ic(1-iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}} - \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} + \text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{x}{\sqrt{c+a^2cx^2}}\right)\\
&= \frac{2ic(1-iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}} - \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} + \frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.0796571, size = 132, normalized size = 1.38

$$\frac{2\sqrt{a^2x^2+1}\left(2i\sqrt{1+iax}\left(2a^2x^2+iax+1\right)+3i\sqrt{1-iax}(ax-i)^2\sin^{-1}\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{3a\sqrt{1-iax}(ax-i)^2\sqrt{a^2cx^2+c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((4*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*((2*I)*Sqrt[1 + I*a*x]*(1 + I*a*x + 2*a^2*x^2) + (3*I)
*Sqrt[1 - I*a*x]*(-I + a*x)^2*ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]))/(3*a*Sqrt[1
 - I*a*x]*(-I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.172, size = 136, normalized size = 1.4

$$\ln\left(a^2cx\frac{1}{\sqrt{a^2c}}+\sqrt{a^2cx^2+c}\right)\frac{1}{\sqrt{a^2c}}-\frac{8}{3\,a^2c}\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2\,iac\left(x-\frac{i}{a}\right)}\left(x-\frac{i}{a}\right)^{-1}-\frac{\frac{4\,i}{3}}{a^3c}\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2\,iac\left(x-\frac{i}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] ln(x*a^2*c/(a^2*c)^(1/2)+(a^2*c*x^2+c)^(1/2))/(a^2*c)^(1/2)-8/3/a^2/c/(x-I/
a)*((x-I/a)^2*a^2*c+2*I*a*c*(x-I/a))^(1/2)-4/3*I/a^3/c/(x-I/a)^2*((x-I/a)^2
*a^2*c+2*I*a*c*(x-I/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.16369, size = 417, normalized size = 4.34

$$\frac{\left(3\,a^3cx^2-6i\,a^2cx-3\,ac\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx+\sqrt{a^2cx^2+c}\,a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-\left(3\,a^3cx^2-6i\,a^2cx-3\,ac\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx-\sqrt{a^2cx^2+c}\,a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)}{6\,a^3cx^2-12i\,a^2cx-6\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] ((3*a^3*c*x^2 - 6*I*a^2*c*x - 3*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x + sqrt(
a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*c)))/x) - (3*a^3*c*x^2 - 6*I*a^2*c*x - 3*a
*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x - sqrt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*
c)))/x) - sqrt(a^2*c*x^2 + c)*(16*a*x - 8*I))/(6*a^3*c*x^2 - 12*I*a^2*c*x -
 6*a*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**4*(a**2*x**2+1)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError
```

3.319 $\int \frac{e^{5i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=35

$$-\frac{i}{2a(ax+i)^2}-\frac{2}{3a(ax+i)^3}$$

```
[Out] -2/(3*a*(I + a*x)^3) - (I/2)/(a*(I + a*x)^2)
```

Rubi [A] time = 0.0384182, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5073, 43}

$$-\frac{i}{2a(ax+i)^2}-\frac{2}{3a(ax+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] -2/(3*a*(I + a*x)^3) - (I/2)/(a*(I + a*x)^2)
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{5i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx &= \int \frac{1+iax}{(1-iax)^4}\,dx\\
&= \int \left(\frac{2}{(i+ax)^4}+\frac{i}{(i+ax)^3}\right)dx\\
&= -\frac{2}{3a(i+ax)^3}-\frac{i}{2a(i+ax)^2}
\end{aligned}$$

Mathematica [A] time = 0.0160125, size = 24, normalized size = 0.69

$$-\frac{1+3iax}{6a(ax+i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((5*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] -(1 + (3*I)*a*x)/(6*a*(I + a*x)^3)
```

Maple [A] time = 0.049, size = 20, normalized size = 0.6

$$\frac{1}{(ax+i)^{3}}\left(-\frac{i}{2}x-\frac{1}{6a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^5/(a^2*x^2+1)^4,x)
```

```
[Out] (-1/2*I*x-1/6/a)/(a*x+I)^3
```

Maxima [B] time = 1.51073, size = 80, normalized size = 2.29

$$\frac{-24i\,a^{4}x^{4}-80\,a^{3}x^{3}+96i\,a^{2}x^{2}+48\,ax-8i}{48\left(a^{7}x^{6}+3\,a^{5}x^{4}+3\,a^{3}x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(-24*I*a^4*x^4 - 80*a^3*x^3 + 96*I*a^2*x^2 + 48*a*x - 8*I)/(a^7*x^6 +
3*a^5*x^4 + 3*a^3*x^2 + a)
```

Fricas [A] time = 1.91439, size = 84, normalized size = 2.4

$$\frac{-3i\,ax-1}{6\,a^{4}x^{3}+18i\,a^{3}x^{2}-18\,a^{2}x-6i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^4,x, algorithm="fricas")
```

```
[Out] (-3*I*a*x - 1)/(6*a^4*x^3 + 18*I*a^3*x^2 - 18*a^2*x - 6*I*a)
```

Sympy [A] time = 0.647313, size = 44, normalized size = 1.26

$$-\frac{3ia^{7}x+a^{6}}{6a^{10}x^{3}+18ia^{9}x^{2}-18a^{8}x-6ia^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**5/(a**2*x**2+1)**4,x)
```

```
[Out] -(3*I*a**7*x + a**6)/(6*a**10*x**3 + 18*I*a**9*x**2 - 18*a**8*x - 6*I*a**7)
```

Giac [A] time = 1.11667, size = 26, normalized size = 0.74

$$-\frac{3\,aix+1}{6\,(ax+i)^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*a*i*x + 1)/((a*x + i)^3*a)
```

3.320 $\int \frac{e^{4i\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{i(1+iax)^{3/2}}{15a(1-iax)^{3/2}}-\frac{i(1+iax)^{3/2}}{5a(1-iax)^{5/2}}$$

```
[Out] ((-I/5)*(1 + I*a*x)^(3/2))/(a*(1 - I*a*x)^(5/2)) - ((I/15)*(1 + I*a*x)^(3/2
))/(a*(1 - I*a*x)^(3/2))
```

Rubi [A] time = 0.0385951, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5073, 45, 37}

$$-\frac{i(1+iax)^{3/2}}{15a(1-iax)^{3/2}}-\frac{i(1+iax)^{3/2}}{5a(1-iax)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] ((-I/5)*(1 + I*a*x)^(3/2))/(a*(1 - I*a*x)^(5/2)) - ((I/15)*(1 + I*a*x)^(3/2
))/(a*(1 - I*a*x)^(3/2))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{4i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx = \int \frac{\sqrt{1+iax}}{(1-iax)^{7/2}}\,dx$$
$$= -\frac{i(1+iax)^{3/2}}{5a(1-iax)^{5/2}} + \frac{1}{5}\int \frac{\sqrt{1+iax}}{(1-iax)^{5/2}}\,dx$$
$$= -\frac{i(1+iax)^{3/2}}{5a(1-iax)^{5/2}} - \frac{i(1+iax)^{3/2}}{15a(1-iax)^{3/2}}$$

Mathematica [A] time = 0.0153257, size = 47, normalized size = 0.7

$$\frac{(1+iax)^{3/2}(ax+4i)}{15a\sqrt{1-iax}(ax+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] ((1 + I*a*x)^(3/2)*(4*I + a*x))/(15*a*Sqrt[1 - I*a*x]*(I + a*x)^2)
```

Maple [B] time = 0.067, size = 269, normalized size = 4.

$$\frac{x}{5}\left(a^2x^2+1\right)^{-\frac{5}{2}} + \frac{4x}{15}\left(a^2x^2+1\right)^{-\frac{3}{2}} + \frac{8x}{15}\frac{1}{\sqrt{a^2x^2+1}} + a^4\left(-\frac{x^3}{2a^2}\left(a^2x^2+1\right)^{-\frac{5}{2}} + \frac{3}{2a^2}\left(-\frac{x}{4a^2}\left(a^2x^2+1\right)^{-\frac{5}{2}} + \frac{1}{4a^2}\left(\frac{x}{5}\left(a^2x^2\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x)
```

```
[Out] 1/5*x/(a^2*x^2+1)^(5/2)+4/15*x/(a^2*x^2+1)^(3/2)+8/15*x/(a^2*x^2+1)^(1/2)+a
^4*(-1/2*x^3/a^2/(a^2*x^2+1)^(5/2)+3/2/a^2*(-1/4*x/a^2/(a^2*x^2+1)^(5/2)+1/
4/a^2*(1/5*x/(a^2*x^2+1)^(5/2)+4/15*x/(a^2*x^2+1)^(3/2)+8/15*x/(a^2*x^2+1)^
(1/2))))-4*I*a^3*(-1/3*x^2/a^2/(a^2*x^2+1)^(5/2)-2/15/a^4/(a^2*x^2+1)^(5/2)
)-6*a^2*(-1/4*x/a^2/(a^2*x^2+1)^(5/2)+1/4/a^2*(1/5*x/(a^2*x^2+1)^(5/2)+4/15
*x/(a^2*x^2+1)^(3/2)+8/15*x/(a^2*x^2+1)^(1/2)))-4/5*I/a/(a^2*x^2+1)^(5/2)
```

Maxima [B] time = 0.99578, size = 128, normalized size = 1.91

$$-\frac{a^2x^3}{2\left(a^2x^2+1\right)^{\frac{5}{2}}} - \frac{x}{15\sqrt{a^2x^2+1}} + \frac{4i\,ax^2}{3\left(a^2x^2+1\right)^{\frac{5}{2}}} - \frac{x}{30\left(a^2x^2+1\right)^{\frac{3}{2}}} + \frac{11\,x}{10\left(a^2x^2+1\right)^{\frac{5}{2}}} - \frac{4i}{15\left(a^2x^2+1\right)^{\frac{5}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*x^3/(a^2*x^2 + 1)^(5/2) - 1/15*x/sqrt(a^2*x^2 + 1) + 4/3*I*a*x^2/(
a^2*x^2 + 1)^(5/2) - 1/30*x/(a^2*x^2 + 1)^(3/2) + 11/10*x/(a^2*x^2 + 1)^(5/
2) - 4/15*I/((a^2*x^2 + 1)^(5/2)*a)
```

Fricas [A] time = 1.86898, size = 176, normalized size = 2.63

$$-\frac{a^3x^3+3i\,a^2x^2-3\,ax+\left(a^2x^2+3i\,ax+4\right)\sqrt{a^2x^2+1}-i}{15\,a^4x^3+45i\,a^3x^2-45\,a^2x-15i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x, algorithm="fricas")
```

```
[Out] -(a^3*x^3 + 3*I*a^2*x^2 - 3*a*x + (a^2*x^2 + 3*I*a*x + 4)*sqrt(a^2*x^2 + 1)
 - I)/(15*a^4*x^3 + 45*I*a^3*x^2 - 45*a^2*x - 15*I*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(iax+1)^4}{\left(a^2x^2+1\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**(7/2),x)
```

```
[Out] Integral((I*a*x + 1)**4/(a**2*x**2 + 1)**(7/2), x)
```

Giac [B] time = 1.16208, size = 150, normalized size = 2.24

$$-\frac{2\left(5\,a^3i\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)-15\,ai\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^3-4\,a^4+25\,a^2\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^2-15\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^4\right)}{15\left(ai+\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(5*a^3*i*(sqrt(a^2 + 1/x^2) - 1/x) - 15*a*i*(sqrt(a^2 + 1/x^2) - 1/x)
^3 - 4*a^4 + 25*a^2*(sqrt(a^2 + 1/x^2) - 1/x)^2 - 15*(sqrt(a^2 + 1/x^2) - 1
/x)^4)/(a*i + sqrt(a^2 + 1/x^2) - 1/x)^5
```

3.321 $\int \frac{e^{3i\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{i}{2a(1-iax)^2}$$

```
[Out] (-I/2)/(a*(1 - I*a*x)^2)
```

Rubi [A] time = 0.0320956, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5073, 32}

$$-\frac{i}{2a(1-iax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] (-I/2)/(a*(1 - I*a*x)^2)
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3i\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx &= \int \frac{1}{(1-iax)^3}\,dx\\ &= -\frac{i}{2a(1-iax)^2}\end{aligned}$$

Mathematica [A] time = 0.0175488, size = 18, normalized size = 0.95

$$\frac{i}{2a(ax+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] (I/2)/(a*(I + a*x)^2)
```

Maple [A] time = 0.047, size = 15, normalized size = 0.8

$$\frac{\frac{i}{2}}{a\,(ax+i)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^3,x)
```

```
[Out] 1/2*I/a/(a*x+I)^2
```

Maxima [B] time = 1.48044, size = 47, normalized size = 2.47

$$\frac{4i\,a^2x^2 + 8\,ax - 4i}{8\left(a^5x^4 + 2\,a^3x^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*I*a^2*x^2 + 8*a*x - 4*I)/(a^5*x^4 + 2*a^3*x^2 + a)
```

Fricas [A] time = 1.78899, size = 45, normalized size = 2.37

$$\frac{i}{2\,a^3x^2 + 4i\,a^2x - 2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] I/(2*a^3*x^2 + 4*I*a^2*x - 2*a)
```

Sympy [A] time = 0.589506, size = 26, normalized size = 1.37

$$\frac{ia^6}{2a^9x^2 + 4ia^8x - 2a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**3,x)
```

```
[Out] I*a**6/(2*a**9*x**2 + 4*I*a**8*x - 2*a**7)
```

Giac [A] time = 1.09756, size = 18, normalized size = 0.95

$$\frac{i}{2\,(ax+i)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/2*i/((a*x + i)^2*a)
```

3.322 $\int \frac{e^{2i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{i\sqrt{1+iax}}{3a\sqrt{1-iax}}-\frac{i\sqrt{1+iax}}{3a(1-iax)^{3/2}}$$

```
[Out] ((-I/3)*Sqrt[1 + I*a*x])/(a*(1 - I*a*x)^(3/2)) - ((I/3)*Sqrt[1 + I*a*x])/(a
*Sqrt[1 - I*a*x])
```

Rubi [A] time = 0.0389113, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5073, 45, 37}

$$-\frac{i\sqrt{1+iax}}{3a\sqrt{1-iax}}-\frac{i\sqrt{1+iax}}{3a(1-iax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] ((-I/3)*Sqrt[1 + I*a*x])/(a*(1 - I*a*x)^(3/2)) - ((I/3)*Sqrt[1 + I*a*x])/(a
*Sqrt[1 - I*a*x])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{2i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx = \int \frac{1}{(1-iax)^{5/2}\sqrt{1+iax}}\,dx$$
$$= -\frac{i\sqrt{1+iax}}{3a(1-iax)^{3/2}} + \frac{1}{3}\int \frac{1}{(1-iax)^{3/2}\sqrt{1+iax}}\,dx$$
$$= -\frac{i\sqrt{1+iax}}{3a(1-iax)^{3/2}} - \frac{i\sqrt{1+iax}}{3a\sqrt{1-iax}}$$

Mathematica [A] time = 0.0125056, size = 48, normalized size = 0.72

$$\frac{(2-iax)\sqrt{1+iax}}{3a\sqrt{1-iax}(ax+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] ((2 - I*a*x)*Sqrt[1 + I*a*x])/(3*a*Sqrt[1 - I*a*x]*(I + a*x))
```

Maple [B] time = 0.062, size = 104, normalized size = 1.6

$$-a^2\left(-\frac{x}{2a^2}\left(a^2x^2+1\right)^{-\frac{3}{2}} + \frac{1}{2a^2}\left(\frac{x}{3}\left(a^2x^2+1\right)^{-\frac{3}{2}} + \frac{2x}{3}\frac{1}{\sqrt{a^2x^2+1}}\right)\right) - \frac{\frac{2i}{3}}{a}\left(a^2x^2+1\right)^{-\frac{3}{2}} + \frac{x}{3}\left(a^2x^2+1\right)^{-\frac{3}{2}} + \frac{2x}{3}\frac{1}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x)
```

```
[Out] -a^2*(-1/2*x/a^2/(a^2*x^2+1)^(3/2)+1/2/a^2*(1/3*x/(a^2*x^2+1)^(3/2)+2/3*x/(
a^2*x^2+1)^(1/2)))-2/3*I/a/(a^2*x^2+1)^(3/2)+1/3*x/(a^2*x^2+1)^(3/2)+2/3*x/
(a^2*x^2+1)^(1/2)
```

Maxima [A] time = 0.967046, size = 61, normalized size = 0.91

$$\frac{x}{3\sqrt{a^2x^2+1}} + \frac{2x}{3\left(a^2x^2+1\right)^{\frac{3}{2}}} - \frac{2i}{3\left(a^2x^2+1\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x/sqrt(a^2*x^2 + 1) + 2/3*x/(a^2*x^2 + 1)^(3/2) - 2/3*I/((a^2*x^2 + 1)^
(3/2)*a)
```

Fricas [A] time = 1.94631, size = 117, normalized size = 1.75

$$\frac{a^2x^2+2i\,ax+\sqrt{a^2x^2+1}(ax+2i)-1}{3\,a^3x^2+6i\,a^2x-3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] (a^2*x^2 + 2*I*a*x + sqrt(a^2*x^2 + 1)*(a*x + 2*I) - 1)/(3*a^3*x^2 + 6*I*a^
2*x - 3*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(iax+1)^2}{\left(a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)**(5/2),x)
```

```
[Out] Integral((I*a*x + 1)**2/(a**2*x**2 + 1)**(5/2), x)
```

Giac [A] time = 1.14777, size = 89, normalized size = 1.33

$$\frac{2\left(3\,ai\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)-2\,a^2+3\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^2\right)}{3\left(ai+\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*a*i*(sqrt(a^2 + 1/x^2) - 1/x) - 2*a^2 + 3*(sqrt(a^2 + 1/x^2) - 1/x)^
2)/(a*i + sqrt(a^2 + 1/x^2) - 1/x)^3
```

3.323 $\int \frac{e^{i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\frac{\tan^{-1}(ax)}{2a}+\frac{1}{2a(ax+i)}$$

```
[Out] 1/(2*a*(I + a*x)) + ArcTan[a*x]/(2*a)
```

Rubi [A] time = 0.0422823, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5073, 44, 203}

$$\frac{\tan^{-1}(ax)}{2a}+\frac{1}{2a(ax+i)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] 1/(2*a*(I + a*x)) + ArcTan[a*x]/(2*a)
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx &= \int \frac{1}{(1-iax)^2(1+iax)}\,dx\\
&= \int \left(-\frac{1}{2(i+ax)^2}+\frac{1}{2\left(1+a^2x^2\right)}\right)dx\\
&= \frac{1}{2a(i+ax)}+\frac{1}{2}\int \frac{1}{1+a^2x^2}\,dx\\
&= \frac{1}{2a(i+ax)}+\frac{\tan^{-1}(ax)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.0155222, size = 21, normalized size = 0.75

$$\frac{\tan^{-1}(ax)+\frac{1}{ax+i}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] ((I + a*x)^(-1) + ArcTan[a*x])/(2*a)
```

Maple [A] time = 0.037, size = 38, normalized size = 1.4

$$\frac{2\,a^2x-2\,ia}{4\,a^2\left(a^2x^2+1\right)}+\frac{\arctan\left(ax\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^2,x)
```

```
[Out] 1/4*(2*a^2*x-2*I*a)/a^2/(a^2*x^2+1)+1/2*arctan(a*x)/a
```

Maxima [A] time = 1.45607, size = 38, normalized size = 1.36

$$\frac{ax-i}{2\left(a^3x^2+a\right)}+\frac{\arctan\left(ax\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*x - I)/(a^3*x^2 + a) + 1/2*arctan(a*x)/a
```

Fricas [B] time = 1.93926, size = 116, normalized size = 4.14

$$\frac{(i\,ax-1)\log\left(\frac{ax+i}{a}\right)+(-i\,ax+1)\log\left(\frac{ax-i}{a}\right)+2}{4\left(a^2x+i\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((I*a*x - 1)*log((a*x + I)/a) + (-I*a*x + 1)*log((a*x - I)/a) + 2)/(a^2
*x + I*a)
```

Sympy [B] time = 0.474389, size = 39, normalized size = 1.39

$$ia\left(-\frac{ia}{2a^4x+2ia^3}+\frac{-\frac{\log\left(x-\frac{i}{a}\right)}{4}+\frac{\log\left(x+\frac{i}{a}\right)}{4}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**2,x)
```

```
[Out] I*a*(-I*a/(2*a**4*x + 2*I*a**3) + (-log(x - I/a)/4 + log(x + I/a)/4)/a**2)
```

Giac [A] time = 1.10985, size = 55, normalized size = 1.96

$$\frac{i\log\left(ax+i\right)}{4a}+\frac{\log\left(-aix-1\right)}{4ai}+\frac{1}{2\left(ax+i\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/4*i*log(a*x + i)/a + 1/4*log(-a*i*x - 1)/(a*i) + 1/2/((a*x + i)*a)
```

3.324 $\int \frac{e^{-i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\tan^{-1}(ax)}{2a}-\frac{1}{2a(-ax+i)}$$

```
[Out] -1/(2*a*(I - a*x)) + ArcTan[a*x]/(2*a)
```

Rubi [A] time = 0.0412167, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5073, 44, 203}

$$\frac{\tan^{-1}(ax)}{2a}-\frac{1}{2a(-ax+i)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] -1/(2*a*(I - a*x)) + ArcTan[a*x]/(2*a)
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx &= \int \frac{1}{(1-iax)(1+iax)^2}\,dx\\
&= \int \left(-\frac{1}{2(-i+ax)^2}+\frac{1}{2\left(1+a^2x^2\right)}\right)dx\\
&= -\frac{1}{2a(i-ax)}+\frac{1}{2}\int \frac{1}{1+a^2x^2}\,dx\\
&= -\frac{1}{2a(i-ax)}+\frac{\tan^{-1}(ax)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.0158269, size = 21, normalized size = 0.72

$$\frac{\tan^{-1}(ax)+\frac{1}{ax-i}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] ((-I + a*x)^(-1) + ArcTan[a*x])/(2*a)
```

Maple [A] time = 0.053, size = 25, normalized size = 0.9

$$-\frac{1}{2\,a\,(-ax+i)}+\frac{\arctan(ax)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)/(a^2*x^2+1),x)
```

```
[Out] -1/2/a/(-a*x+I)+1/2*arctan(a*x)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)/(a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 1.8327, size = 116, normalized size = 4.

$$\frac{(i\,ax+1)\log\left(\frac{ax+i}{a}\right)+(-i\,ax-1)\log\left(\frac{ax-i}{a}\right)+2}{4\left(a^2x-i\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)/(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*((I*a*x + 1)*log((a*x + I)/a) + (-I*a*x - 1)*log((a*x - I)/a) + 2)/(a^2
*x - I*a)
```

Sympy [B] time = 0.40411, size = 36, normalized size = 1.24

$$\frac{a}{2a^3x-2ia^2}+\frac{-\frac{i\log\left(x-\frac{i}{a}\right)}{4}+\frac{i\log\left(x+\frac{i}{a}\right)}{4}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)/(a**2*x**2+1),x)
```

```
[Out] a/(2*a**3*x - 2*I*a**2) + (-I*log(x - I/a)/4 + I*log(x + I/a)/4)/a
```

Giac [A] time = 1.1191, size = 59, normalized size = 2.03

$$-\frac{i\log\left(ax-i\right)}{4\,a}-\frac{\log\left(aix-1\right)}{4\,ai}+\frac{1}{2\,(ax-i)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)/(a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*i*log(a*x - i)/a - 1/4*log(a*i*x - 1)/(a*i) + 1/2/((a*x - i)*a)
```

3.325 $\int \frac{e^{-2i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{i\sqrt{1-iax}}{3a\sqrt{1+iax}}+\frac{i\sqrt{1-iax}}{3a(1+iax)^{3/2}}$$

```
[Out] ((I/3)*Sqrt[1 - I*a*x])/(a*(1 + I*a*x)^(3/2)) + ((I/3)*Sqrt[1 - I*a*x])/(a*
Sqrt[1 + I*a*x])
```

Rubi [A] time = 0.0403993, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5073, 45, 37}

$$\frac{i\sqrt{1-iax}}{3a\sqrt{1+iax}}+\frac{i\sqrt{1-iax}}{3a(1+iax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] ((I/3)*Sqrt[1 - I*a*x])/(a*(1 + I*a*x)^(3/2)) + ((I/3)*Sqrt[1 - I*a*x])/(a*
Sqrt[1 + I*a*x])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{-2i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\sqrt{1-iax}(1+iax)^{5/2}}\,dx$$
$$= \frac{i\sqrt{1-iax}}{3a(1+iax)^{3/2}} + \frac{1}{3}\int \frac{1}{\sqrt{1-iax}(1+iax)^{3/2}}\,dx$$
$$= \frac{i\sqrt{1-iax}}{3a(1+iax)^{3/2}} + \frac{i\sqrt{1-iax}}{3a\sqrt{1+iax}}$$

Mathematica [A] time = 0.0142586, size = 48, normalized size = 0.72

$$\frac{\sqrt{1-iax}(2+iax)}{3a\sqrt{1+iax}(ax-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - I*a*x]*(2 + I*a*x))/(3*a*Sqrt[1 + I*a*x]*(-I + a*x))
```

Maple [A] time = 0.067, size = 93, normalized size = 1.4

$$-\frac{1}{a^2}\left(\frac{\frac{i}{3}}{a}\sqrt{a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)}\left(x-\frac{i}{a}\right)^{-2}-\frac{1}{3}\sqrt{a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)}\left(x-\frac{i}{a}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/a^2*(1/3*I/a/(x-I/a)^2*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2)-1/3/(x-I/a)*(
a^2*(x-I/a)^2+2*I*a*(x-I/a))^(1/2))
```

Maxima [A] time = 1.47495, size = 80, normalized size = 1.19

$$-\frac{i\sqrt{a^2x^2+1}}{3\,a^3x^2-6i\,a^2x-3\,a}+\frac{i\sqrt{a^2x^2+1}}{3i\,a^2x+3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*sqrt(a^2*x^2 + 1)/(3*a^3*x^2 - 6*I*a^2*x - 3*a) + I*sqrt(a^2*x^2 + 1)/(3
*I*a^2*x + 3*a)
```

Fricas [A] time = 1.89626, size = 117, normalized size = 1.75

$$\frac{a^2x^2-2i\,ax+\sqrt{a^2x^2+1}(ax-2i)-1}{3\,a^3x^2-6i\,a^2x-3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a^2*x^2 - 2*I*a*x + sqrt(a^2*x^2 + 1)*(a*x - 2*I) - 1)/(3*a^3*x^2 - 6*I*a^
2*x - 3*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2x^2+1}\,(iax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a**2*x**2 + 1)*(I*a*x + 1)**2), x)
```

Giac [A] time = 1.21691, size = 93, normalized size = 1.39

$$-\frac{2\left(3\,ai\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)+2\,a^2-3\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^2\right)i^2}{3\left(ai-\sqrt{a^2+\frac{1}{x^2}}+\frac{1}{x}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*a*i*(sqrt(a^2 + 1/x^2) - 1/x) + 2*a^2 - 3*(sqrt(a^2 + 1/x^2) - 1/x)
^2)*i^2/(a*i - sqrt(a^2 + 1/x^2) + 1/x)^3
```

3.326 $\int \frac{e^{-3i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{i}{2a(1+iax)^2}$$

```
[Out] (I/2)/(a*(1 + I*a*x)^2)
```

Rubi [A] time = 0.0323292, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5073, 32}

$$\frac{i}{2a(1+iax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] (I/2)/(a*(1 + I*a*x)^2)
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx &= \int \frac{1}{(1+iax)^3}\,dx\\ &= \frac{i}{2a(1+iax)^2}\end{aligned}$$

Mathematica [A] time = 0.0177464, size = 18, normalized size = 0.95

$$-\frac{i}{2a(ax-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] (-I/2)/(a*(-I + a*x)^2)
```

Maple [A] time = 0.033, size = 16, normalized size = 0.8

$$\frac{\frac{i}{2}}{a\left(1+iax\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3,x)
```

```
[Out] 1/2*I/a/(1+I*a*x)^2
```

Maxima [A] time = 1.00784, size = 18, normalized size = 0.95

$$\frac{i}{2\left(i\,ax+1\right)^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*I/((I*a*x + 1)^2*a)
```

Fricas [A] time = 1.67099, size = 46, normalized size = 2.42

$$-\frac{i}{2\,a^{3}x^{2}-4i\,a^{2}x-2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3,x, algorithm="fricas")
```

```
[Out] -I/(2*a^3*x^2 - 4*I*a^2*x - 2*a)
```

Sympy [B] time = 0.395081, size = 27, normalized size = 1.42

$$-\frac{ia^{3}}{2a^{6}x^{2}-4ia^{5}x-2a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3,x)
```

```
[Out] -I*a**3/(2*a**6*x**2 - 4*I*a**5*x - 2*a**4)
```

Giac [A] time = 1.09596, size = 19, normalized size = 1.

$$\frac{i}{2\left(aix+1\right)^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3,x, algorithm="giac")
```

```
[Out] 1/2*i/((a*i*x + 1)^2*a)
```

3.327 $\int \frac{e^{-4i\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{i(1-iax)^{3/2}}{15a(1+iax)^{3/2}}+\frac{i(1-iax)^{3/2}}{5a(1+iax)^{5/2}}$$

```
[Out] ((I/5)*(1 - I*a*x)^(3/2))/(a*(1 + I*a*x)^(5/2)) + ((I/15)*(1 - I*a*x)^(3/2)
)/(a*(1 + I*a*x)^(3/2))
```

Rubi [A] time = 0.0387899, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5073, 45, 37}

$$\frac{i(1-iax)^{3/2}}{15a(1+iax)^{3/2}}+\frac{i(1-iax)^{3/2}}{5a(1+iax)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((4*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] ((I/5)*(1 - I*a*x)^(3/2))/(a*(1 + I*a*x)^(5/2)) + ((I/15)*(1 - I*a*x)^(3/2)
)/(a*(1 + I*a*x)^(3/2))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{-4i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx = \int \frac{\sqrt{1-iax}}{(1+iax)^{7/2}}\,dx$$
$$= \frac{i(1-iax)^{3/2}}{5a(1+iax)^{5/2}} + \frac{1}{5}\int \frac{\sqrt{1-iax}}{(1+iax)^{5/2}}\,dx$$
$$= \frac{i(1-iax)^{3/2}}{5a(1+iax)^{5/2}} + \frac{i(1-iax)^{3/2}}{15a(1+iax)^{3/2}}$$

Mathematica [A] time = 0.0142884, size = 47, normalized size = 0.7

$$\frac{(1-iax)^{3/2}(ax-4i)}{15a\sqrt{1+iax}(ax-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((4*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] ((1 - I*a*x)^(3/2)*(-4*I + a*x))/(15*a*Sqrt[1 + I*a*x]*(-I + a*x)^2)
```

Maple [A] time = 0.062, size = 92, normalized size = 1.4

$$\frac{1}{a^4}\left(\frac{\frac{i}{5}}{a}\left(a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}\left(x-\frac{i}{a}\right)^{-4}-\frac{1}{15}\left(a^2\left(x-\frac{i}{a}\right)^2+2\,ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}\left(x-\frac{i}{a}\right)^{-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/a^4*(1/5*I/a/(x-I/a)^4*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(3/2)-1/15/(x-I/a)^3
*(a^2*(x-I/a)^2+2*I*a*(x-I/a))^(3/2))
```

Maxima [B] time = 0.978653, size = 135, normalized size = 2.01

$$\frac{2i\,\sqrt{a^2x^2+1}}{-5i\,a^4x^3-15\,a^3x^2+15i\,a^2x+5\,a}+\frac{i\,\sqrt{a^2x^2+1}}{15\,a^3x^2-30i\,a^2x-15\,a}-\frac{i\,\sqrt{a^2x^2+1}}{15i\,a^2x+15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*I*sqrt(a^2*x^2 + 1)/(-5*I*a^4*x^3 - 15*a^3*x^2 + 15*I*a^2*x + 5*a) + I*sq
rt(a^2*x^2 + 1)/(15*a^3*x^2 - 30*I*a^2*x - 15*a) - I*sqrt(a^2*x^2 + 1)/(15*
I*a^2*x + 15*a)
```

Fricas [A] time = 1.92368, size = 176, normalized size = 2.63

$$-\frac{a^3x^3-3i\,a^2x^2-3\,ax+\left(a^2x^2-3i\,ax+4\right)\sqrt{a^2x^2+1}+i}{15\,a^4x^3-45i\,a^3x^2-45\,a^2x+15i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(a^3*x^3 - 3*I*a^2*x^2 - 3*a*x + (a^2*x^2 - 3*I*a*x + 4)*sqrt(a^2*x^2 + 1)
 + I)/(15*a^4*x^3 - 45*I*a^3*x^2 - 45*a^2*x + 15*I*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2x^2+1}}{(iax+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**4*(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2*x**2 + 1)/(I*a*x + 1)**4, x)
```

Giac [B] time = 1.18679, size = 150, normalized size = 2.24

$$-\frac{2\left(5\,a^3 i\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)-15\,ai\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^3+4\,a^4-25\,a^2\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^2+15\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^4\right)}{15\left(ai-\sqrt{a^2+\frac{1}{x^2}}+\frac{1}{x}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(5*a^3*i*(sqrt(a^2 + 1/x^2) - 1/x) - 15*a*i*(sqrt(a^2 + 1/x^2) - 1/x)
^3 + 4*a^4 - 25*a^2*(sqrt(a^2 + 1/x^2) - 1/x)^2 + 15*(sqrt(a^2 + 1/x^2) - 1
/x)^4)/(a*i - sqrt(a^2 + 1/x^2) + 1/x)^5
```

3.328 $\int \frac{e^{5i\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{i\sqrt{a^2x^2+1}}{2ac(ax+i)^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{3ac(ax+i)^3\sqrt{a^2cx^2+c}}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*c*(I + a*x)^3*Sqrt[c + a^2*c*x^2]) - ((I/2)*Sqr
t[1 + a^2*x^2])/(a*c*(I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0799853, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5076, 5073, 43}

$$-\frac{i\sqrt{a^2x^2+1}}{2ac(ax+i)^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{3ac(ax+i)^3\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*c*(I + a*x)^3*Sqrt[c + a^2*c*x^2]) - ((I/2)*Sqr
t[1 + a^2*x^2])/(a*c*(I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{5 i \tan^{-1}(a x)}}{\left(c+a^2 c x^2\right)^{3/2}} d x &= \frac{\sqrt{1+a^2 x^2} \int \frac{e^{5 i \tan^{-1}(a x)}}{\left(1+a^2 x^2\right)^{3/2}} d x}{c \sqrt{c+a^2 c x^2}} \\
&= \frac{\sqrt{1+a^2 x^2} \int \frac{1+i a x}{(1-i a x)^4} d x}{c \sqrt{c+a^2 c x^2}} \\
&= \frac{\sqrt{1+a^2 x^2} \int \left(\frac{2}{(i+a x)^4}+\frac{i}{(i+a x)^3}\right) d x}{c \sqrt{c+a^2 c x^2}} \\
&= -\frac{2 \sqrt{1+a^2 x^2}}{3 a c(i+a x)^3 \sqrt{c+a^2 c x^2}}-\frac{i \sqrt{1+a^2 x^2}}{2 a c(i+a x)^2 \sqrt{c+a^2 c x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0290565, size = 56, normalized size = 0.59

$$
-\frac{i(3 a x-i) \sqrt{a^2 x^2+1}}{6 a c(a x+i)^3 \sqrt{a^2 c x^2+c}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((5*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((-I/6)*(-I + 3*a*x)*Sqrt[1 + a^2*x^2])/(a*c*(I + a*x)^3*Sqrt[c + a^2*c*x^2
])
```

Maple [A] time = 0.15, size = 48, normalized size = 0.5

$$
-\frac{3 i a x+1}{6 a c^2 (a x+i)^3} \sqrt{c \left(a^2 x^2+1\right)} \frac{1}{\sqrt{a^2 x^2+1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/6/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(3*I*a*x+1)/c^2/a/(a*x+I)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{(i a x+1)^5}{\left(a^2 c x^2+c\right)^{\frac{3}{2}} \left(a^2 x^2+1\right)^{\frac{5}{2}}} d x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((I*a*x + 1)^5/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)^(5/2)), x)
```

Fricas [A] time = 2.04973, size = 220, normalized size = 2.32

$$\frac{\sqrt{a^2cx^2+c}\left(i\,a^2x^3-3\,ax^2-6i\,x\right)\sqrt{a^2x^2+1}}{6\,a^5c^2x^5+18i\,a^4c^2x^4-12\,a^3c^2x^3+12i\,a^2c^2x^2-18\,ac^2x-6i\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*(I*a^2*x^3 - 3*a*x^2 - 6*I*x)*sqrt(a^2*x^2 + 1)/(6*a^5*
c^2*x^5 + 18*I*a^4*c^2*x^4 - 12*a^3*c^2*x^3 + 12*I*a^2*c^2*x^2 - 18*a*c^2*x
 - 6*I*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**5/(a**2*x**2+1)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)^5}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*x + 1)^5/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)^(5/2)), x)
```

3.329 $\int \frac{e^{4 i \tan^{-1}(a x)}}{\left(c+a^2 c x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=69

$$\frac{ic(1+iax)^5}{15a\left(a^2cx^2+c\right)^{5/2}}-\frac{ic(1+iax)^4}{3a\left(a^2cx^2+c\right)^{5/2}}$$

```
[Out] ((-I/3)*c*(1 + I*a*x)^4)/(a*(c + a^2*c*x^2)^(5/2)) + ((I/15)*c*(1 + I*a*x)^
5)/(a*(c + a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.0652056, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5075, 659, 651}

$$\frac{ic(1+iax)^5}{15a\left(a^2cx^2+c\right)^{5/2}}-\frac{ic(1+iax)^4}{3a\left(a^2cx^2+c\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((-I/3)*c*(1 + I*a*x)^4)/(a*(c + a^2*c*x^2)^(5/2)) + ((I/15)*c*(1 + I*a*x)^
5)/(a*(c + a^2*c*x^2)^(5/2))
```

Rule 5075

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[1/c^((I*n)/2), Int[(c + d*x^2)^(p + (I*n)/2)/(1 + I*a*x)^(I*n), x], x] /
; FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0]) &
& ILtQ[(I*n)/2, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{4i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = c^2\int \frac{(1+iax)^4}{\left(c+a^2cx^2\right)^{7/2}}\,dx$$
$$= -\frac{ic(1+iax)^4}{3a\left(c+a^2cx^2\right)^{5/2}} - \frac{1}{3}c^2\int \frac{(1+iax)^5}{\left(c+a^2cx^2\right)^{7/2}}\,dx$$
$$= -\frac{ic(1+iax)^4}{3a\left(c+a^2cx^2\right)^{5/2}} + \frac{ic(1+iax)^5}{15a\left(c+a^2cx^2\right)^{5/2}}$$

Mathematica [A] time = 0.0336332, size = 77, normalized size = 1.12

$$\frac{(1+iax)^{3/2}(ax+4i)\sqrt{a^2x^2+1}}{15ac\sqrt{1-iax}(ax+i)^2\sqrt{a^2cx^2+c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 + I*a*x)^(3/2)*(4*I + a*x)*Sqrt[1 + a^2*x^2])/(15*a*c*Sqrt[1 - I*a*x]*(
I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Maple [B] time = 0.082, size = 940, normalized size = 13.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] x/c/(a^2*c*x^2+c)^(1/2)-2/a*(I*(-a^2)^(1/2)-a)/(-a^2)^(1/2)*(1/3/c/(-a^2)^(
1/2)/(x+(-a^2)^(1/2)/a^2)/((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x
+(-a^2)^(1/2)/a^2))^(1/2)+1/3/c^2/(-a^2)^(1/2)*(2*(x+(-a^2)^(1/2)/a^2)*a^2*
c-2*c*(-a^2)^(1/2))/((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2
)^(1/2)/a^2))^(1/2))+2/a^3*(I*(-a^2)^(1/2)-a)*(1/5/c/(-a^2)^(1/2)/(x+(-a^2)
^(1/2)/a^2)^2/((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2
)/a^2))^(1/2)+3/5*a^2/(-a^2)^(1/2)*(1/3/c/(-a^2)^(1/2)/(x+(-a^2)^(1/2)/a^2)
/((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)
+1/3/c^2/(-a^2)^(1/2)*(2*(x+(-a^2)^(1/2)/a^2)*a^2*c-2*c*(-a^2)^(1/2))/((x+(
-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)))-2/a
*(I*(-a^2)^(1/2)+a)/(-a^2)^(1/2)*(-1/3/c/(-a^2)^(1/2)/(x-(-a^2)^(1/2)/a^2)/
((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2)-
1/3/c^2/(-a^2)^(1/2)*(2*(x-(-a^2)^(1/2)/a^2)*a^2*c+2*c*(-a^2)^(1/2))/((x-(-
a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2))-2/a^3
*(I*(-a^2)^(1/2)+a)*(-1/5/c/(-a^2)^(1/2)/(x-(-a^2)^(1/2)/a^2)^2/((x-(-a^2)^
(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2)-3/5*a^2/(-a
^2)^(1/2)*(-1/3/c/(-a^2)^(1/2)/(x-(-a^2)^(1/2)/a^2)/((x-(-a^2)^(1/2)/a^2)^2
*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2)-1/3/c^2/(-a^2)^(1/2)*(2
*(x-(-a^2)^(1/2)/a^2)*a^2*c+2*c*(-a^2)^(1/2))/((x-(-a^2)^(1/2)/a^2)^2*a^2*c
+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)^4}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((I*a*x + 1)^4/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)^2), x)
```

Fricas [A] time = 2.83624, size = 149, normalized size = 2.16

$$-\frac{\sqrt{a^2cx^2+c}\left(a^2x^2+3i\,ax+4\right)}{15\,a^4c^2x^3+45i\,a^3c^2x^2-45\,a^2c^2x-15i\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] -sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 3*I*a*x + 4)/(15*a^4*c^2*x^3 + 45*I*a^3*c^2
*x^2 - 45*a^2*c^2*x - 15*I*a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(iax+1)^4}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\left(a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((I*a*x + 1)**4/((c*(a**2*x**2 + 1))**(3/2)*(a**2*x**2 + 1)**2), x)
```

Giac [B] time = 1.13021, size = 182, normalized size = 2.64

$$-\frac{2\left(5\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^2ci-15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^3\sqrt{c}+c^2i+5\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)c^{\frac{3}{2}}\right)}{15\left(\sqrt{c}i+\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^5ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] -2/15*(5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))^2*c*i - 15*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 + c))^3*sqrt(c) + c^2*i + 5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
+ c))*c^(3/2))/((sqrt(c)*i + sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))^5*a*c)
```

3.330 $\int \frac{e^{3i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{i\sqrt{a^2x^2+1}}{2ac(1-iax)^2\sqrt{a^2cx^2+c}}$$

```
[Out] ((-I/2)*Sqrt[1 + a^2*x^2])/(a*c*(1 - I*a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0688765, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5076, 5073, 32}

$$-\frac{i\sqrt{a^2x^2+1}}{2ac(1-iax)^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((-I/2)*Sqrt[1 + a^2*x^2])/(a*c*(1 - I*a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{3i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1-iax)^3}\,dx}{c\sqrt{c+a^2cx^2}}\\ &= -\frac{i\sqrt{1+a^2x^2}}{2ac(1-iax)^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0355115, size = 57, normalized size = 1.16

$$\frac{i\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{2ac^2(ax-i)(ax+i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((I/2)*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(a*c^2*(-I + a*x)*(I + a*x)^3
)
```

Maple [A] time = 0.056, size = 44, normalized size = 0.9

$$-\frac{(ax+i)\,(1+iax)^3}{2\,a}\left(a^2x^2+1\right)^{-\frac{3}{2}}\left(a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*(a*x+I)/a*(1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.16015, size = 150, normalized size = 3.06

$$\frac{\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}\left(i\,ax^2-2\,x\right)}{2\,a^4c^2x^4+4i\,a^3c^2x^3+4i\,ac^2x-2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*(I*a*x^2 - 2*x)/(2*a^4*c^2*x^4 + 4*I*
a^3*c^2*x^3 + 4*I*a*c^2*x - 2*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(iax+1)^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((I*a*x + 1)**3/((c*(a**2*x**2 + 1))**(3/2)*(a**2*x**2 + 1)**(3/2))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*x + 1)^3/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)^(3/2)), x)
```

3.331 $\int \frac{e^{2i\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{x}{3c\sqrt{a^2cx^2+c}}-\frac{2i(1+iax)}{3a\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] (((-2*I)/3)*(1 + I*a*x))/(a*(c + a^2*c*x^2)^(3/2)) + x/(3*c*Sqrt[c + a^2*c*
x^2])
```

Rubi [A] time = 0.0549454, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5075, 653, 191}

$$\frac{x}{3c\sqrt{a^2cx^2+c}}-\frac{2i(1+iax)}{3a\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (((-2*I)/3)*(1 + I*a*x))/(a*(c + a^2*c*x^2)^(3/2)) + x/(3*c*Sqrt[c + a^2*c*
x^2])
```

Rule 5075

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[1/c^((I*n)/2), Int[(c + d*x^2)^(p + (I*n)/2)/(1 + I*a*x)^(I*n), x], x] /
; FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &
& ILtQ[(I*n)/2, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= c\int \frac{(1+iax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx\\ &= -\frac{2i(1+iax)}{3a\left(c+a^2cx^2\right)^{3/2}}+\frac{1}{3}\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx\\ &= -\frac{2i(1+iax)}{3a\left(c+a^2cx^2\right)^{3/2}}+\frac{x}{3c\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0267966, size = 78, normalized size = 1.44

$$\frac{(2-iax)\sqrt{1+iax}\sqrt{a^2x^2+1}}{3ac\sqrt{1-iax}(ax+i)\sqrt{a^2cx^2+c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((2 - I*a*x)*Sqrt[1 + I*a*x]*Sqrt[1 + a^2*x^2])/(3*a*c*Sqrt[1 - I*a*x]*(I +
 a*x)*Sqrt[c + a^2*c*x^2])
```

Maple [B] time = 0.074, size = 398, normalized size = 7.4

$$-\frac{x}{c}\frac{1}{\sqrt{a^2cx^2+c}}+\frac{1}{a}\left(i\sqrt{-a^2}+a\right)\left(-\frac{1}{3c}\frac{1}{\sqrt{-a^2}}\left(x-\frac{1}{a^2}\sqrt{-a^2}\right)^{-1}\frac{1}{\sqrt{\left(x-\frac{1}{a^2}\sqrt{-a^2}\right)^2a^2c+2c\sqrt{-a^2}\left(x-\frac{\sqrt{-a^2}}{a^2}\right)}}-\frac{1}{3c^2}\left(2\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -x/c/(a^2*c*x^2+c)^(1/2)+1/a*(I*(-a^2)^(1/2)+a)/(-a^2)^(1/2)*(-1/3/c/(-a^2)
^(1/2)/(x-(-a^2)^(1/2)/a^2)/((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*
(x-(-a^2)^(1/2)/a^2))^(1/2)-1/3/c^2/(-a^2)^(1/2)*(2*(x-(-a^2)^(1/2)/a^2)*a^
2*c+2*c*(-a^2)^(1/2))/((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a
^2)^(1/2)/a^2))^(1/2))+1/a*(I*(-a^2)^(1/2)-a)/(-a^2)^(1/2)*(1/3/c/(-a^2)^(1
/2)/(x+(-a^2)^(1/2)/a^2)/((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+
(-a^2)^(1/2)/a^2))^(1/2)+1/3/c^2/(-a^2)^(1/2)*(2*(x+(-a^2)^(1/2)/a^2)*a^2*c
-2*c*(-a^2)^(1/2))/((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)
^(1/2)/a^2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(i\,ax+1)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*x + 1)^2/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)), x)
```

Fricas [A] time = 2.37992, size = 101, normalized size = 1.87

$$\frac{\sqrt{a^2cx^2+c}(ax+2i)}{3\,a^3c^2x^2+6i\,a^2c^2x-3\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(a^2*c*x^2 + c)*(a*x + 2*I)/(3*a^3*c^2*x^2 + 6*I*a^2*c^2*x - 3*a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(iax+1)^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\left(a^2x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((I*a*x + 1)**2/((c*(a**2*x**2 + 1))**(3/2)*(a**2*x**2 + 1)), x)
```

Giac [A] time = 1.14482, size = 103, normalized size = 1.91

$$-\frac{2\sqrt{a^2c}\left(\sqrt{c}i+3\sqrt{a^2c}x-3\sqrt{a^2cx^2+c}\right)}{3\left(\sqrt{c}i+\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^3a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(a^2*c)*(sqrt(c)*i + 3*sqrt(a^2*c)*x - 3*sqrt(a^2*c*x^2 + c))/((sq
rt(c)*i + sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))^3*a^2*c)
```

3.332 $\int \frac{e^{i\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{a^2x^2+1}\tan^{-1}(ax)}{2ac\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}}{2ac(ax+i)\sqrt{a^2cx^2+c}}$$

```
[Out] Sqrt[1 + a^2*x^2]/(2*a*c*(I + a*x)*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2
]*ArcTan[a*x])/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0813627, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {5076, 5073, 44, 203}

$$\frac{\sqrt{a^2x^2+1}\tan^{-1}(ax)}{2ac\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}}{2ac(ax+i)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Sqrt[1 + a^2*x^2]/(2*a*c*(I + a*x)*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2
]*ArcTan[a*x])/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1-iax)^2(1+iax)}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \left(-\frac{1}{2(i+ax)^2}+\frac{1}{2\left(1+a^2x^2\right)}\right)dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}}{2ac(i+ax)\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\int \frac{1}{1+a^2x^2}\,dx}{2c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}}{2ac(i+ax)\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\tan^{-1}(ax)}{2ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0307032, size = 51, normalized size = 0.58

$$\frac{\sqrt{a^2x^2+1}\left(\tan^{-1}(ax)+\frac{1}{ax+i}\right)}{2ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*((I + a*x)^(-1) + ArcTan[a*x]))/(2*a*c*Sqrt[c + a^2*c*x^
2])
```

Maple [A] time = 0.138, size = 58, normalized size = 0.7

$$-\frac{-\arctan(ax)x^2a^2-ax+i-\arctan(ax)}{2\,ac^2}\sqrt{c\left(a^2x^2+1\right)}\left(a^2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2/(a^2*x^2+1)^(3/2)*(c*(a^2*x^2+1))^(1/2)/a*(-arctan(a*x)*x^2*a^2-a*x+I-
arctan(a*x))/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.44164, size = 698, normalized size = 7.93

$$\frac{\left(-i\,a^3c^2x^3+a^2c^2x^2-i\,ac^2x+c^2\right)\sqrt{\frac{1}{a^2c^3}}\log\left(\frac{8\,\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}a^6x+\left(4i\,a^{10}c^2x^4-4i\,a^6c^2\right)\sqrt{\frac{1}{a^2c^3}}}{2\left(a^4x^4+2\,a^2x^2+1\right)}\right)+\left(i\,a^3c^2x^3-a^2c^2x^2+i\,ac^2x-\right.}{2\left(4\,a^3c^2x^3+4i\,a^2c^2x^2+4\,ac^2x+4i\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*((-I*a^3*c^2*x^3 + a^2*c^2*x^2 - I*a*c^2*x + c^2)*sqrt(1/(a^2*c^3))*log
(1/2*(8*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^6*x + (4*I*a^10*c^2*x^4 - 4
*I*a^6*c^2)*sqrt(1/(a^2*c^3)))/(a^4*x^4 + 2*a^2*x^2 + 1)) + (I*a^3*c^2*x^3
- a^2*c^2*x^2 + I*a*c^2*x - c^2)*sqrt(1/(a^2*c^3))*log(1/2*(8*sqrt(a^2*c*x^
2 + c)*sqrt(a^2*x^2 + 1)*a^6*x + (-4*I*a^10*c^2*x^4 + 4*I*a^6*c^2)*sqrt(1/(
a^2*c^3)))/(a^4*x^4 + 2*a^2*x^2 + 1)) + 4*I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^
2 + 1)*x)/(4*a^3*c^2*x^3 + 4*I*a^2*c^2*x^2 + 4*a*c^2*x + 4*I*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{iax+1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((I*a*x + 1)/((c*(a**2*x**2 + 1))**(3/2)*sqrt(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{i\,ax+1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((I*a*x + 1)/((a^2*c*x^2 + c)^(3/2)*sqrt(a^2*x^2 + 1)), x)
```

3.333 $\int \frac{e^{-i\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{a^2x^2+1}\tan^{-1}(ax)}{2ac\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}}{2ac(-ax+i)\sqrt{a^2cx^2+c}}$$

```
[Out] -Sqrt[1 + a^2*x^2]/(2*a*c*(I - a*x)*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^
2]*ArcTan[a*x])/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0806776, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {5076, 5073, 44, 203}

$$\frac{\sqrt{a^2x^2+1}\tan^{-1}(ax)}{2ac\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}}{2ac(-ax+i)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(2*a*c*(I - a*x)*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^
2]*ArcTan[a*x])/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{-i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1-iax)(1+iax)^2}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \left(-\frac{1}{2(-i+ax)^2}+\frac{1}{2\left(1+a^2x^2\right)}\right)dx}{c\sqrt{c+a^2cx^2}}$$
$$= -\frac{\sqrt{1+a^2x^2}}{2ac(i-ax)\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\int \frac{1}{1+a^2x^2}\,dx}{2c\sqrt{c+a^2cx^2}}$$
$$= -\frac{\sqrt{1+a^2x^2}}{2ac(i-ax)\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\tan^{-1}(ax)}{2ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0339056, size = 60, normalized size = 0.67

$$\frac{\sqrt{a^2x^2+1}\left(\frac{\tan^{-1}(ax)}{2a}-\frac{1}{2a(-ax+i)}\right)}{c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(-1/(2*a*(I - a*x)) + ArcTan[a*x]/(2*a)))/(c*Sqrt[c + a^
2*c*x^2])
```

Maple [A] time = 0.154, size = 86, normalized size = 1.

$$\frac{i\ln(-ax+i)\,xa - i\ln(ax+i)\,xa + \ln(-ax+i) - \ln(ax+i) - 2}{4\,ac^2\,(-ax+i)}\sqrt{c\left(a^2x^2+1\right)}\frac{1}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(I*ln(-a*x+I)*x*a-I*ln(a*x+I)*x
*a+ln(-a*x+I)-ln(a*x+I)-2)/c^2/a/(-a*x+I)
```

Maxima [A] time = 1.02318, size = 70, normalized size = 0.79

$$\frac{\sqrt{c}}{2\,a^2c^2x - 2i\,ac^2} - \frac{i\,\log(ax-i)}{4\,ac^{\frac{3}{2}}} + \frac{i\,\log(i\,ax-1)}{4\,ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] sqrt(c)/(2*a^2*c^2*x - 2*I*a*c^2) - 1/4*I*log(a*x - I)/(a*c^(3/2)) + 1/4*I*
log(I*a*x - 1)/(a*c^(3/2))
```

Fricas [B] time = 2.47163, size = 698, normalized size = 7.84

$$\frac{\left(-i\,a^{3}c^{2}x^{3}-a^{2}c^{2}x^{2}-i\,ac^{2}x-c^{2}\right)\sqrt{\frac{1}{a^{2}c^{3}}}\log\left(\frac{8\,\sqrt{a^{2}cx^{2}+c}\sqrt{a^{2}x^{2}+1}a^{6}x+\left(4i\,a^{10}c^{2}x^{4}-4i\,a^{6}c^{2}\right)\sqrt{\frac{1}{a^{2}c^{3}}}}{2\left(a^{4}x^{4}+2\,a^{2}x^{2}+1\right)}\right)+\left(i\,a^{3}c^{2}x^{3}+a^{2}c^{2}x^{2}+i\,ac^{2}x+c^{2}\right)}{2\left(4\,a^{3}c^{2}x^{3}-4i\,a^{2}c^{2}x^{2}+4\,ac^{2}x-4i\,c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/2*((-I*a^3*c^2*x^3 - a^2*c^2*x^2 - I*a*c^2*x - c^2)*sqrt(1/(a^2*c^3))*log
(1/2*(8*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^6*x + (4*I*a^10*c^2*x^4 - 4
*I*a^6*c^2)*sqrt(1/(a^2*c^3)))/(a^4*x^4 + 2*a^2*x^2 + 1)) + (I*a^3*c^2*x^3
+ a^2*c^2*x^2 + I*a*c^2*x + c^2)*sqrt(1/(a^2*c^3))*log(1/2*(8*sqrt(a^2*c*x^
2 + c)*sqrt(a^2*x^2 + 1)*a^6*x + (-4*I*a^10*c^2*x^4 + 4*I*a^6*c^2)*sqrt(1/(
a^2*c^3)))/(a^4*x^4 + 2*a^2*x^2 + 1)) - 4*I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^
2 + 1)*x)/(4*a^3*c^2*x^3 - 4*I*a^2*c^2*x^2 + 4*a*c^2*x - 4*I*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^{2}x^{2}+1}}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}(iax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a**2*x**2 + 1)/((c*(a**2*x**2 + 1))**(3/2)*(I*a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^{2}x^{2}+1}}{\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}(i\,ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)/((a^2*c*x^2 + c)^(3/2)*(I*a*x + 1)), x)
```

3.334 $\int \frac{e^{-2i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{x}{3c\sqrt{a^2cx^2+c}}+\frac{2i(1-iax)}{3a\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] (((2*I)/3)*(1 - I*a*x))/(a*(c + a^2*c*x^2)^(3/2)) + x/(3*c*Sqrt[c + a^2*c*x
^2])
```

Rubi [A] time = 0.0548632, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5074, 653, 191}

$$\frac{x}{3c\sqrt{a^2cx^2+c}}+\frac{2i(1-iax)}{3a\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (((2*I)/3)*(1 - I*a*x))/(a*(c + a^2*c*x^2)^(3/2)) + x/(3*c*Sqrt[c + a^2*c*x
^2])
```

Rule 5074

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[c^((I*n)/2), Int[(c + d*x^2)^(p - (I*n)/2)*(1 - I*a*x)^(I*n), x], x] /;
FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &&
IGtQ[(I*n)/2, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= c\int \frac{(1-iax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx\\ &= \frac{2i(1-iax)}{3a\left(c+a^2cx^2\right)^{3/2}}+\frac{1}{3}\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx\\ &= \frac{2i(1-iax)}{3a\left(c+a^2cx^2\right)^{3/2}}+\frac{x}{3c\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0333535, size = 78, normalized size = 1.44

$$\frac{\sqrt{1-iax}(2+iax)\sqrt{a^2x^2+1}}{3ac\sqrt{1+iax}(ax-i)\sqrt{a^2cx^2+c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - I*a*x]*(2 + I*a*x)*Sqrt[1 + a^2*x^2])/(3*a*c*Sqrt[1 + I*a*x]*(-I
+ a*x)*Sqrt[c + a^2*c*x^2])
```

Maple [B] time = 0.07, size = 137, normalized size = 2.5

$$-\frac{x}{c}\frac{1}{\sqrt{a^2cx^2+c}}-\frac{2i}{a}\left(\frac{\frac{i}{3}}{ac}\left(x-\frac{i}{a}\right)^{-1}\frac{1}{\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}+\frac{\frac{i}{3}}{ac^2}\left(2\left(x-\frac{i}{a}\right)a^2c+2iac\right)\frac{1}{\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -x/c/(a^2*c*x^2+c)^(1/2)-2*I/a*(1/3*I/a/c/(x-I/a)/((x-I/a)^2*a^2*c+2*I*a*c*
(x-I/a))^(1/2)+1/3*I/a/c^2*(2*(x-I/a)*a^2*c+2*I*a*c)/((x-I/a)^2*a^2*c+2*I*a
*c*(x-I/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.38089, size = 101, normalized size = 1.87

$$\frac{\sqrt{a^2cx^2+c}(ax-2i)}{3a^3c^2x^2-6ia^2c^2x-3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c*x^2 + c)*(a*x - 2*I)/(3*a^3*c^2*x^2 - 6*I*a^2*c^2*x - 3*a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2x^2+1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}(iax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a**2*x**2 + 1)/((c*(a**2*x**2 + 1))**(3/2)*(I*a*x + 1)**2), x)
```

Giac [A] time = 1.22282, size = 105, normalized size = 1.94

$$\frac{2\sqrt{a^2c}\left(\sqrt{c}i-3\sqrt{a^2c}x+3\sqrt{a^2cx^2+c}\right)i^2}{3\left(\sqrt{c}i-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right)^3a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] 2/3*sqrt(a^2*c)*(sqrt(c)*i - 3*sqrt(a^2*c)*x + 3*sqrt(a^2*c*x^2 + c))*i^2/(
(sqrt(c)*i - sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c))^3*a^2*c)
```

3.335 $\int \frac{e^{-3i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=49

$$\frac{i\sqrt{a^2x^2+1}}{2ac(1+iax)^2\sqrt{a^2cx^2+c}}$$

```
[Out] ((I/2)*Sqrt[1 + a^2*x^2])/(a*c*(1 + I*a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0720498, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5076, 5073, 32}

$$\frac{i\sqrt{a^2x^2+1}}{2ac(1+iax)^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((I/2)*Sqrt[1 + a^2*x^2])/(a*c*(1 + I*a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{-3i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1+iax)^3}\,dx}{c\sqrt{c+a^2cx^2}}\\ &= \frac{i\sqrt{1+a^2x^2}}{2ac(1+iax)^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0405942, size = 57, normalized size = 1.16

$$-\frac{i\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{2ac^2(ax-i)^3(ax+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((-I/2)*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(a*c^2*(-I + a*x)^3*(I + a*x
))
```

Maple [A] time = 0.041, size = 45, normalized size = 0.9

$$\frac{-ax+i}{2\,a\,(1+iax)^3}\left(a^2x^2+1\right)^{\frac{3}{2}}\left(a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(-a*x+I)/a/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2)
```

Maxima [A] time = 1.02254, size = 39, normalized size = 0.8

$$\frac{1}{2i\,a^3c^{\frac{3}{2}}x^2+4\,a^2c^{\frac{3}{2}}x-2i\,ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] 1/(2*I*a^3*c^(3/2)*x^2 + 4*a^2*c^(3/2)*x - 2*I*a*c^(3/2))
```

Fricas [A] time = 2.23682, size = 151, normalized size = 3.08

$$\frac{\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}\left(-i\,ax^2-2\,x\right)}{2\,a^4c^2x^4-4i\,a^3c^2x^3-4i\,ac^2x-2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*(-I*a*x^2 - 2*x)/(2*a^4*c^2*x^4 - 4*I
*a^3*c^2*x^3 - 4*I*a*c^2*x - 2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}(i\,ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/((a^2*c*x^2 + c)^(3/2)*(I*a*x + 1)^3), x)
```

3.336 $\int \frac{e^{-4i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{ic(1-iax)^4}{3a\left(a^2cx^2+c\right)^{5/2}}-\frac{ic(1-iax)^5}{15a\left(a^2cx^2+c\right)^{5/2}}$$

```
[Out] ((I/3)*c*(1 - I*a*x)^4)/(a*(c + a^2*c*x^2)^(5/2)) - ((I/15)*c*(1 - I*a*x)^5
)/(a*(c + a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.0661462, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5074, 659, 651}

$$\frac{ic(1-iax)^4}{3a\left(a^2cx^2+c\right)^{5/2}}-\frac{ic(1-iax)^5}{15a\left(a^2cx^2+c\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((4*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((I/3)*c*(1 - I*a*x)^4)/(a*(c + a^2*c*x^2)^(5/2)) - ((I/15)*c*(1 - I*a*x)^5
)/(a*(c + a^2*c*x^2)^(5/2))
```

Rule 5074

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[c^((I*n)/2), Int[(c + d*x^2)^(p - (I*n)/2)*(1 - I*a*x)^(I*n), x], x] /;
FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &&
IGtQ[(I*n)/2, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{-4i\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = c^2\int \frac{(1-iax)^4}{(c+a^2cx^2)^{7/2}}\,dx$$
$$= \frac{ic(1-iax)^4}{3a(c+a^2cx^2)^{5/2}} - \frac{1}{3}c^2\int \frac{(1-iax)^5}{(c+a^2cx^2)^{7/2}}\,dx$$
$$= \frac{ic(1-iax)^4}{3a(c+a^2cx^2)^{5/2}} - \frac{ic(1-iax)^5}{15a(c+a^2cx^2)^{5/2}}$$

Mathematica [A] time = 0.0392665, size = 77, normalized size = 1.12

$$\frac{(1-iax)^{3/2}(ax-4i)\sqrt{a^2x^2+1}}{15ac\sqrt{1+iax}(ax-i)^2\sqrt{a^2cx^2+c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((4*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((1 - I*a*x)^(3/2)*(-4*I + a*x)*Sqrt[1 + a^2*x^2])/(15*a*c*Sqrt[1 + I*a*x]*
(-I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Maple [B] time = 0.072, size = 307, normalized size = 4.5

$$\frac{x}{c}\frac{1}{\sqrt{a^2cx^2+c}} - 4\frac{1}{a^2}\left(\frac{i/5}{ac}\left(x-\frac{i}{a}\right)^{-2}\frac{1}{\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2\,iac\left(x-\frac{i}{a}\right)}} + 3/5\,ia\left(\frac{i/3}{ac}\left(x-\frac{i}{a}\right)^{-1}\frac{1}{\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2\,iac\left(x-\frac{i}{a}\right)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] x/c/(a^2*c*x^2+c)^(1/2)-4/a^2*(1/5*I/a/c/(x-I/a)^2/((x-I/a)^2*a^2*c+2*I*a*c
*(x-I/a))^(1/2)+3/5*I*a*(1/3*I/a/c/(x-I/a)/((x-I/a)^2*a^2*c+2*I*a*c*(x-I/a)
)^(1/2)+1/3*I/a/c^2*(2*(x-I/a)*a^2*c+2*I*a*c)/((x-I/a)^2*a^2*c+2*I*a*c*(x-I
/a))^(1/2))+4*I/a*(1/3*I/a/c/(x-I/a)/((x-I/a)^2*a^2*c+2*I*a*c*(x-I/a))^(1/
2)+1/3*I/a/c^2*(2*(x-I/a)*a^2*c+2*I*a*c)/((x-I/a)^2*a^2*c+2*I*a*c*(x-I/a))^
(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.75508, size = 149, normalized size = 2.16

$$-\frac{\sqrt{a^2cx^2+c}\left(a^2x^2-3i\,ax+4\right)}{15\,a^4c^2x^3-45i\,a^3c^2x^2-45\,a^2c^2x+15i\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] -sqrt(a^2*c*x^2 + c)*(a^2*x^2 - 3*I*a*x + 4)/(15*a^4*c^2*x^3 - 45*I*a^3*c^2
*x^2 - 45*a^2*c^2*x + 15*I*a*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**4*(a**2*x**2+1)**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15478, size = 181, normalized size = 2.62

$$-\frac{2\left(5\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^2ci+15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^3\sqrt{c}+c^2i-5\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)c^{\frac{3}{2}}\right)}{15\left(\sqrt{c}i-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right)^5ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] -2/15*(5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))^2*c*i + 15*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 + c))^3*sqrt(c) + c^2*i - 5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
+ c))*c^(3/2))/((sqrt(c)*i - sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c))^5*a*c)
```

3.337 $\int e^{n\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx$

Optimal. Leaf size=86

$$-\frac{c^2 2^{3-\frac{in}{2}}(1-iax)^{3+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}-2,\frac{in}{2}+3;\frac{in}{2}+4;\frac{1}{2}(1-iax)\right)}{a(-n+6i)}$$

```
[Out] -((2^(3 - (I/2)*n)*c^2*(1 - I*a*x)^(3 + (I/2)*n)*Hypergeometric2F1[-2 + (I/
2)*n, 3 + (I/2)*n, 4 + (I/2)*n, (1 - I*a*x)/2])/(a*(6*I - n)))
```

Rubi [A] time = 0.052661, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5073, 69}

$$-\frac{c^2 2^{3-\frac{in}{2}}(1-iax)^{3+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}-2,\frac{in}{2}+3;\frac{in}{2}+4;\frac{1}{2}(1-iax)\right)}{a(-n+6i)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^2,x]
```

```
[Out] -((2^(3 - (I/2)*n)*c^2*(1 - I*a*x)^(3 + (I/2)*n)*Hypergeometric2F1[-2 + (I/
2)*n, 3 + (I/2)*n, 4 + (I/2)*n, (1 - I*a*x)/2])/(a*(6*I - n)))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{n\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx &= c^2\int(1-iax)^{2+\frac{in}{2}}(1+iax)^{2-\frac{in}{2}}\,dx\\ &= -\frac{2^{3-\frac{in}{2}}c^2(1-iax)^{3+\frac{in}{2}}\,{}_2F_1\left(-2+\frac{in}{2},3+\frac{in}{2};4+\frac{in}{2};\frac{1}{2}(1-iax)\right)}{a(6i-n)}\end{aligned}$$

Mathematica [A] time = 0.0230905, size = 90, normalized size = 1.05

$$\frac{ic^2 2^{2-\frac{in}{2}}(1-iax)^{3+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}-2,\frac{in}{2}+3;\frac{in}{2}+4;\frac{1}{2}(1-iax)\right)}{a\left(3+\frac{in}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^2,x]
```

```
[Out] (I*2^(2 - (I/2)*n)*c^2*(1 - I*a*x)^(3 + (I/2)*n)*Hypergeometric2F1[-2 + (I/
2)*n, 3 + (I/2)*n, 4 + (I/2)*n, (1 - I*a*x)/2])/(a*(3 + (I/2)*n))
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int e^{n \arctan(ax)} \left(a^2 c x^2 + c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 c x^2 + c\right)^2 e^{(n \arctan(ax))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*e^(n*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4 c^2 x^4 + 2\, a^2 c^2 x^2 + c^2\right) e^{(n \arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(n*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2 \left(\int 2a^2 x^2 e^{n \operatorname{atan}(ax)}\, dx + \int a^4 x^4 e^{n \operatorname{atan}(ax)}\, dx + \int e^{n \operatorname{atan}(ax)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**2,x)
```

```
[Out] c**2*(Integral(2*a**2*x**2*exp(n*atan(a*x)), x) + Integral(a**4*x**4*exp(n*
atan(a*x)), x) + Integral(exp(n*atan(a*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^2 e^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*e^(n*arctan(a*x)), x)
```

3.338 $\int e^{n\tan^{-1}(ax)}\left(c+a^2cx^2\right)\,dx$

Optimal. Leaf size=84

$$-\frac{c2^{2-\frac{in}{2}}(1-iax)^{2+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}-1,\frac{in}{2}+2;\frac{in}{2}+3;\frac{1}{2}(1-iax)\right)}{a(-n+4i)}$$

```
[Out] -((2^(2 - (I/2)*n)*c*(1 - I*a*x)^(2 + (I/2)*n)*Hypergeometric2F1[-1 + (I/2)
*n, 2 + (I/2)*n, 3 + (I/2)*n, (1 - I*a*x)/2])/(a*(4*I - n)))
```

Rubi [A] time = 0.0390547, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5073, 69}

$$-\frac{c2^{2-\frac{in}{2}}(1-iax)^{2+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}-1,\frac{in}{2}+2;\frac{in}{2}+3;\frac{1}{2}(1-iax)\right)}{a(-n+4i)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2),x]
```

```
[Out] -((2^(2 - (I/2)*n)*c*(1 - I*a*x)^(2 + (I/2)*n)*Hypergeometric2F1[-1 + (I/2)
*n, 2 + (I/2)*n, 3 + (I/2)*n, (1 - I*a*x)/2])/(a*(4*I - n)))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{n\tan^{-1}(ax)}\left(c+a^2cx^2\right)\,dx &= c\int(1-iax)^{1+\frac{in}{2}}(1+iax)^{1-\frac{in}{2}}\,dx\\ &= -\frac{2^{2-\frac{in}{2}}c(1-iax)^{2+\frac{in}{2}}\,{}_2F_1\left(-1+\frac{in}{2},2+\frac{in}{2};3+\frac{in}{2};\frac{1}{2}(1-iax)\right)}{a(4i-n)}\end{aligned}$$

Mathematica [A] time = 0.0155312, size = 88, normalized size = 1.05

$$\frac{ic2^{1-\frac{in}{2}}(1-iax)^{2+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}-1,\frac{in}{2}+2;\frac{in}{2}+3;\frac{1}{2}(1-iax)\right)}{a\left(2+\frac{in}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2),x]
```

```
[Out] (I*2^(1 - (I/2)*n)*c*(1 - I*a*x)^(2 + (I/2)*n)*Hypergeometric2F1[-1 + (I/2)
*n, 2 + (I/2)*n, 3 + (I/2)*n, (1 - I*a*x)/2])/(a*(2 + (I/2)*n))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \mathrm{e}^{n \arctan(ax)} \left(a^2 c x^2 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 c x^2 + c\right) e^{(n \arctan(ax))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2 c x^2 + c\right) e^{(n \arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c \left(\int a^2 x^2 e^{n \operatorname{atan}(ax)}\, dx + \int e^{n \operatorname{atan}(ax)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c),x)
```

```
[Out] c*(Integral(a**2*x**2*exp(n*atan(a*x)), x) + Integral(exp(n*atan(a*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)e^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

3.339 $\int e^{n\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=81

$$-\frac{2^{1-\frac{in}{2}}(1-iax)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(1-iax)\right)}{a(-n+2i)}$$

```
[Out] -((2^(1 - (I/2)*n)*(1 - I*a*x)^(1 + (I/2)*n)*Hypergeometric2F1[1 + (I/2)*n,
 (I/2)*n, 2 + (I/2)*n, (1 - I*a*x)/2])/(a*(2*I - n)))
```

Rubi [A] time = 0.0156714, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5061, 69}

$$-\frac{2^{1-\frac{in}{2}}(1-iax)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(1-iax)\right)}{a(-n+2i)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x]),x]
```

```
[Out] -((2^(1 - (I/2)*n)*(1 - I*a*x)^(1 + (I/2)*n)*Hypergeometric2F1[1 + (I/2)*n,
 (I/2)*n, 2 + (I/2)*n, (1 - I*a*x)/2])/(a*(2*I - n)))
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{n\tan^{-1}(ax)}\,dx &= \int (1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\,dx\\ &= -\frac{2^{1-\frac{in}{2}}(1-iax)^{1+\frac{in}{2}}\,{}_2F_1\left(1+\frac{in}{2},\frac{in}{2};2+\frac{in}{2};\frac{1}{2}(1-iax)\right)}{a(2i-n)}\end{aligned}$$

Mathematica [A] time = 0.0177068, size = 56, normalized size = 0.69

$$\frac{4e^{(n+2i)\tan^{-1}(ax)}\,{}_2F_1\left(2,1-\frac{in}{2};2-\frac{in}{2};-e^{2i\tan^{-1}(ax)}\right)}{a(n+2i)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTan[a*x]),x]
```

```
[Out] (4*E^((2*I + n)*ArcTan[a*x])*Hypergeometric2F1[2, 1 - (I/2)*n, 2 - (I/2)*n,
 -E^((2*I)*ArcTan[a*x])])/(a*(2*I + n))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int e^{n\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x)),x)
```

```
[Out] int(exp(n*arctan(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(e^{(n\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x)),x)
```

```
[Out] Integral(exp(n*atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x)),x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(a*x)), x)
```

3.340 $\int \frac{e^{n\tan^{-1}(ax)}x^3}{c+a^2cx^2}\,dx$

Optimal. Leaf size=131

$$\frac{i\left(n^2-2\right)e^{n\tan^{-1}(ax)}\,{}_2F_1\left(1,-\frac{in}{2};1-\frac{in}{2};-e^{2i\tan^{-1}(ax)}\right)}{a^4cn}+\frac{\left(-in^2+n+2i\right)e^{n\tan^{-1}(ax)}}{2a^4cn}+\frac{x^2e^{n\tan^{-1}(ax)}}{2a^2c}-\frac{nxe^{n\tan^{-1}(ax)}}{2a^3c}$$

```
[Out] (E^(n*ArcTan[a*x])*(2*I + n - I*n^2))/(2*a^4*c*n) - (E^(n*ArcTan[a*x])*n*x)
/(2*a^3*c) + (E^(n*ArcTan[a*x])*x^2)/(2*a^2*c) + (I*E^(n*ArcTan[a*x])*(-2 +
 n^2)*Hypergeometric2F1[1, (-I/2)*n, 1 - (I/2)*n, -E^((2*I)*ArcTan[a*x])])/
(a^4*c*n)
```

Rubi [A] time = 0.226768, antiderivative size = 206, normalized size of antiderivative = 1.57, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5082, 100, 143, 69}

$$\frac{2^{-1-\frac{in}{2}}\left(2-n^2\right)(1-iax)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2}+1;\frac{in}{2}+2;\frac{1}{2}(1-iax)\right)}{a^4c(2+in)}+\frac{i(1+iax)^{-\frac{in}{2}}\left(ian^2x-n^2-in+2\right)(1-iax)^{\frac{in}{2}}}{2a^4cn}+$$

Warning: Unable to verify antiderivative.

```
[In] Int[(E^(n*ArcTan[a*x])*x^3)/(c + a^2*c*x^2),x]
```

```
[Out] (x^2*(1 - I*a*x)^((I/2)*n))/(2*a^2*c*(1 + I*a*x)^((I/2)*n)) + ((I/2)*(1 - I
*a*x)^((I/2)*n)*(2 - I*n - n^2 + I*a*n^2*x))/(a^4*c*n*(1 + I*a*x)^((I/2)*n)
) + (2^(-1 - (I/2)*n)*(2 - n^2)*(1 - I*a*x)^(1 + (I/2)*n)*Hypergeometric2F1
[1 + (I/2)*n, 1 + (I/2)*n, 2 + (I/2)*n, (1 - I*a*x)/2])/(a^4*c*(2 + I*n))
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}x^3}{c+a^2cx^2}\,dx = \frac{\int x^3(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,dx}{c}$$

$$= \frac{x^2(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2a^2c} + \frac{\int x(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}(-2-anx)\,dx}{2a^2c}$$

$$= \frac{x^2(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2a^2c} + \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2-in-n^2+ian^2x\right)}{2a^4cn} - \frac{\left(i\left(2-n^2\right)\right)\int(1-iax)^{\frac{in}{2}}}{2a^3c}$$

$$= \frac{x^2(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2a^2c} + \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2-in-n^2+ian^2x\right)}{2a^4cn} + \frac{2^{-1-\frac{in}{2}}\left(2-n^2\right)(1-iax)}{}$$

Mathematica [A] time = 0.113165, size = 141, normalized size = 1.08

$$\frac{(1-iax)^{\frac{in}{2}}\left(\frac{\left(a^2nx^2+n^2(-(ax+i))+n+2i\right)(1+iax)^{-\frac{in}{2}}}{n}+\frac{2^{-\frac{in}{2}}\left(n^2-2\right)(ax+i)\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2}+1;\frac{in}{2}+2;\frac{1}{2}(1-iax)\right)}{n-2i}\right)}{2a^4c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^3)/(c + a^2*c*x^2),x]
```

```
[Out] ((1 - I*a*x)^((I/2)*n)*((2*I + n + a^2*n*x^2 - n^2*(I + a*x))/(n*(1 + I*a*x
)^((I/2)*n)) + ((-2 + n^2)*(I + a*x)*Hypergeometric2F1[1 + (I/2)*n, 1 + (I/
2)*n, 2 + (I/2)*n, (1 - I*a*x)/2])/(2^((I/2)*n)*(-2*I + n))))/(2*a^4*c)
```

Maple [F] time = 0.266, size = 0, normalized size = 0.

$$\int \frac{e^{n\arctan(ax)}x^3}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3e^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^3*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3 e^{(n \arctan(ax))}}{a^2 c x^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^3*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^3 e^{n \operatorname{atan}(ax)}}{a^2 x^2 + 1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**3*exp(n*atan(a*x))/(a**2*x**2 + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 e^{(n \arctan(ax))}}{a^2 c x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^3*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

3.341 $\int \frac{e^{n\tan^{-1}(ax)}x^2}{c+a^2cx^2}\,dx$

Optimal. Leaf size=164

$$\frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;\frac{1}{2}(1-iax)\right)}{a^3c}+\frac{x(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2c}-\frac{(1+in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^3cn}$$

```
[Out] -(((1 + I*n)*(1 - I*a*x)^((I/2)*n))/(a^3*c*n*(1 + I*a*x)^((I/2)*n))) + (x*(
1 - I*a*x)^((I/2)*n))/(a^2*c*(1 + I*a*x)^((I/2)*n)) + (I*2^(1 - (I/2)*n)*(1
 - I*a*x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I
*a*x)/2])/(a^3*c)
```

Rubi [A] time = 0.131597, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5082, 90, 79, 69}

$$\frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;\frac{1}{2}(1-iax)\right)}{a^3c}+\frac{x(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2c}-\frac{(1+in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^3cn}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^2)/(c + a^2*c*x^2),x]
```

```
[Out] -(((1 + I*n)*(1 - I*a*x)^((I/2)*n))/(a^3*c*n*(1 + I*a*x)^((I/2)*n))) + (x*(
1 - I*a*x)^((I/2)*n))/(a^2*c*(1 + I*a*x)^((I/2)*n)) + (I*2^(1 - (I/2)*n)*(1
 - I*a*x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I
*a*x)/2])/(a^3*c)
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}x^2}{c+a^2cx^2}\,dx = \frac{\int x^2(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,dx}{c}$$
$$= \frac{x(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2c} + \frac{\int (1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}(-1-anx)\,dx}{a^2c}$$
$$= -\frac{(1+in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^3cn} + \frac{x(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2c} + \frac{(in)\int (1-iax)^{-1+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\,dx}{a^2c}$$
$$= -\frac{(1+in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^3cn} + \frac{x(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2c} + \frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};1+\frac{in}{2};\right.}{a^3c}$$

Mathematica [A] time = 0.120062, size = 121, normalized size = 0.74

$$\frac{(1-iax)^{\frac{in}{2}}(2+2iax)^{-\frac{in}{2}}\left(2in(1+iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;\frac{1}{2}(1-iax)\right)+2^{\frac{in}{2}}(-1+n(ax-i))\right)}{a^3cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^2)/(c + a^2*c*x^2),x]
```

```
[Out] ((1 - I*a*x)^((I/2)*n)*(2^((I/2)*n)*(-1 + n*(-I + a*x)) + (2*I)*n*(1 + I*a*
x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I*a*x)/2
]))/(a^3*c*n*(2 + (2*I)*a*x)^((I/2)*n))
```

Maple [F] time = 0.254, size = 0, normalized size = 0.

$$\int \frac{e^{n\arctan(ax)}x^2}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2e^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^2*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2e^{(n\arctan(ax))}}{a^2cx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^2*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2e^{n\,\mathrm{atan}\,(ax)}}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**2*exp(n*atan(a*x))/(a**2*x**2 + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2e^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^2*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

3.342 $\int \frac{e^{n\tan^{-1}(ax)}x}{c+a^2cx^2}\,dx$

Optimal. Leaf size=122

$$\frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2cn}-\frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;\frac{1}{2}(1-iax)\right)}{a^2cn}$$

```
[Out] (I*(1 - I*a*x)^((I/2)*n))/(a^2*c*n*(1 + I*a*x)^((I/2)*n)) - (I*2^(1 - (I/2)
*n)*(1 - I*a*x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n,
(1 - I*a*x)/2])/(a^2*c*n)
```

Rubi [A] time = 0.0779773, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5082, 79, 69}

$$\frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2cn}-\frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;\frac{1}{2}(1-iax)\right)}{a^2cn}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x)/(c + a^2*c*x^2),x]
```

```
[Out] (I*(1 - I*a*x)^((I/2)*n))/(a^2*c*n*(1 + I*a*x)^((I/2)*n)) - (I*2^(1 - (I/2)
*n)*(1 - I*a*x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n,
(1 - I*a*x)/2])/(a^2*c*n)
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}x}{c+a^2cx^2}\,dx = \frac{\int x(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,dx}{c}$$
$$= \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2cn} - \frac{i\int (1-iax)^{-1+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\,dx}{ac}$$
$$= \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2cn} - \frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};1+\frac{in}{2};\frac{1}{2}(1-iax)\right)}{a^2cn}$$

Mathematica [A] time = 0.0597226, size = 109, normalized size = 0.89

$$\frac{i(1-iax)^{\frac{in}{2}}(2+2iax)^{-\frac{in}{2}}\left(2^{\frac{in}{2}}-2(1+iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;\frac{1}{2}(1-iax)\right)\right)}{a^2cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x)/(c + a^2*c*x^2),x]
```

```
[Out] (I*(1 - I*a*x)^((I/2)*n)*(2^((I/2)*n) - 2*(1 + I*a*x)^((I/2)*n)*Hypergeomet
ric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I*a*x)/2]))/(a^2*c*n*(2 + (2*I)*
a*x)^((I/2)*n))
```

Maple [F] time = 0.318, size = 0, normalized size = 0.

$$\int \frac{e^{n\arctan(ax)}x}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{xe^{(n\arctan(ax))}}{a^2cx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{xe^{n\operatorname{atan}(ax)}}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x*exp(n*atan(a*x))/(a**2*x**2 + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

3.343 $\int \frac{e^{n\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{e^{n\tan^{-1}(ax)}}{acn}$$

[Out] E^(n*ArcTan[a*x])/(a*c*n)

Rubi [A] time = 0.0303301, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5071}

$$\frac{e^{n\tan^{-1}(ax)}}{acn}$$

Antiderivative was successfully verified.

[In] Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2),x]

[Out] E^(n*ArcTan[a*x])/(a*c*n)

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{c+a^2cx^2}\,dx = \frac{e^{n\tan^{-1}(ax)}}{acn}$$

Mathematica [C] time = 0.0065214, size = 42, normalized size = 2.33

$$\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{acn}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2),x]

[Out] (1 - I*a*x)^((I/2)*n)/(a*c*n*(1 + I*a*x)^((I/2)*n))

Maple [A] time = 0.04, size = 18, normalized size = 1.

$$\frac{e^{n\arctan(ax)}}{can}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c),x)
```

```
[Out] exp(n*arctan(a*x))/a/c/n
```

Maxima [A] time = 1.52385, size = 23, normalized size = 1.28

$$\frac{e^{(n\arctan(ax))}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] e^(n*arctan(a*x))/(a*c*n)
```

Fricas [A] time = 2.23902, size = 36, normalized size = 2.

$$\frac{e^{(n\arctan(ax))}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] e^(n*arctan(a*x))/(a*c*n)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.085, size = 23, normalized size = 1.28

$$\frac{e^{(n\arctan(ax))}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] e^(n*arctan(a*x))/(a*c*n)
```

3.344 $\int \frac{e^{n\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)}\,dx$

Optimal. Leaf size=65

$$\frac{ie^{n\tan^{-1}(ax)}}{cn}-\frac{2ie^{n\tan^{-1}(ax)}\,{}_2F_1\left(1,-\frac{in}{2};1-\frac{in}{2};e^{2i\tan^{-1}(ax)}\right)}{cn}$$

```
[Out] (I*E^(n*ArcTan[a*x]))/(c*n) - ((2*I)*E^(n*ArcTan[a*x])*Hypergeometric2F1[1,
 (-I/2)*n, 1 - (I/2)*n, E^((2*I)*ArcTan[a*x])])/(c*n)
```

Rubi [B] time = 0.0984313, antiderivative size = 132, normalized size of antiderivative = 2.03, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5082, 96, 131}

$$\frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn}-\frac{2(1-iax)^{1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,{}_2F_1\left(1,\frac{in}{2}+1;\frac{in}{2}+2;\frac{1-iax}{iax+1}\right)}{c(2+in)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTan[a*x])/(x*(c + a^2*c*x^2)),x]
```

```
[Out] (I*(1 - I*a*x)^((I/2)*n))/(c*n*(1 + I*a*x)^((I/2)*n)) - (2*(1 - I*a*x)^(1 +
 (I/2)*n)*(1 + I*a*x)^(-1 - (I/2)*n)*Hypergeometric2F1[1, 1 + (I/2)*n, 2 +
(I/2)*n, (1 - I*a*x)/(1 + I*a*x)])/(c*(2 + I*n))
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)}dx = \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x}dx}{c}$$
$$= \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn} + \frac{\int \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x}dx}{c}$$
$$= \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn} - \frac{2(1-iax)^{1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,{}_2F_1\left(1,1+\frac{in}{2};2+\frac{in}{2};\frac{1-iax}{1+iax}\right)}{c(2+in)}$$

Mathematica [A] time = 0.0363659, size = 120, normalized size = 1.85

$$\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2(n-ianx)\,{}_2F_1\left(1,\frac{in}{2}+1;\frac{in}{2}+2;\frac{ax+i}{i-ax}\right)+(2+in)(ax-i)\right)}{cn(n-2i)(ax-i)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTan[a*x])/(x*(c + a^2*c*x^2)),x]
```

```
[Out] ((1 - I*a*x)^((I/2)*n)*((2 + I*n)*(-I + a*x) + 2*(n - I*a*n*x)*Hypergeometr
ic2F1[1, 1 + (I/2)*n, 2 + (I/2)*n, (I + a*x)/(I - a*x)]))/(c*n*(-2*I + n)*(
1 + I*a*x)^((I/2)*n)*(-I + a*x))
```

Maple [F] time = 0.323, size = 0, normalized size = 0.

$$\int \frac{e^{n\arctan(ax)}}{x\left(a^2cx^2+c\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^{(n\arctan(ax))}}{a^2cx^3+cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(a*x))/(a^2*c*x^3 + c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^{n\,\mathrm{atan}\,(ax)}}{a^2x^3+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/x/(a**2*c*x**2+c),x)
```

```
[Out] Integral(exp(n*atan(a*x))/(a**2*x**3 + x), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x), x)
```

3.345 $\int \frac{e^{n\tan^{-1}(ax)}}{x^2(c+a^2cx^2)}\,dx$

Optimal. Leaf size=90

$$-\frac{2ia\,{}_2F_1\left(1,-\frac{in}{2};1-\frac{in}{2};\frac{2i}{ax+i}-1\right)e^{n\tan^{-1}(ax)}}{c}+\frac{ia(n+i)e^{n\tan^{-1}(ax)}}{cn}-\frac{e^{n\tan^{-1}(ax)}}{cx}$$

```
[Out] (I*a*E^(n*ArcTan[a*x])*(I + n))/(c*n) - E^(n*ArcTan[a*x])/(c*x) - ((2*I)*a*
E^(n*ArcTan[a*x])*Hypergeometric2F1[1, (-I/2)*n, 1 - (I/2)*n, -1 + (2*I)/(I
 + a*x)])/c
```

Rubi [A] time = 0.137301, antiderivative size = 180, normalized size of antiderivative = 2., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5082, 129, 155, 12, 131}

$$-\frac{2an(1-iax)^{1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,{}_2F_1\left(1,\frac{in}{2}+1;\frac{in}{2}+2;\frac{1-iax}{iax+1}\right)}{c(2+in)}-\frac{a(1-in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn}-\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cx}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTan[a*x])/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] -((a*(1 - I*n)*(1 - I*a*x)^((I/2)*n))/(c*n*(1 + I*a*x)^((I/2)*n))) - (1 - I
*a*x)^((I/2)*n)/(c*x*(1 + I*a*x)^((I/2)*n)) - (2*a*n*(1 - I*a*x)^(1 + (I/2)
*n)*(1 + I*a*x)^(-1 - (I/2)*n)*Hypergeometric2F1[1, 1 + (I/2)*n, 2 + (I/2)*
n, (1 - I*a*x)/(1 + I*a*x)])/(c*(2 + I*n))
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
```

```
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\tan^{-1}(ax)}}{x^2\left(c+a^2cx^2\right)}\,dx &= \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x^2}\,dx}{c}\\
&= -\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cx}-\frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\left(-an+a^2x\right)}{x}\,dx}{c}\\
&= -\frac{a(1-in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn}-\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cx}+\frac{\int \frac{a^2n^2(1-iax)^{\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x}\,dx}{acn}\\
&= -\frac{a(1-in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn}-\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cx}+\frac{(an)\int \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x}\,dx}{c}\\
&= -\frac{a(1-in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn}-\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cx}-\frac{2an(1-iax)^{1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,{}_2F_1\left(1,\right.}{c(2+in)}
\end{aligned}$$

Mathematica [A] time = 0.0458572, size = 142, normalized size = 1.58

$$\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2an^2x(1-iax)\,{}_2F_1\left(1,\frac{in}{2}+1;\frac{in}{2}+2;\frac{ax+i}{i-ax}\right)+(n-2i)(1+iax)(n(ax+i)+iax)\right)}{cn(n-2i)x(ax-i)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTan[a*x])/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] ((1 - I*a*x)^((I/2)*n)*((-2*I + n)*(1 + I*a*x)*(I*a*x + n*(I + a*x)) + 2*a*
n^2*x*(1 - I*a*x)*Hypergeometric2F1[1, 1 + (I/2)*n, 2 + (I/2)*n, (I + a*x)/
(I - a*x)]))/(c*n*(-2*I + n)*x*(1 + I*a*x)^((I/2)*n)*(-I + a*x))
```

Maple [F] time = 0.326, size = 0, normalized size = 0.

$$\int \frac{e^{n\arctan(ax)}}{x^2\left(a^2cx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(n\arctan(ax))}}{a^2cx^4+cx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(a*x))/(a^2*c*x^4 + c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^{n\operatorname{atan}(ax)}}{a^2x^4+x^2}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/x**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(exp(n*atan(a*x))/(a**2*x**4 + x**2), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x^2), x)
```

3.346 $\int \frac{e^{n\tan^{-1}(ax)}}{x^3(c+a^2cx^2)}\,dx$

Optimal. Leaf size=126

$$-\frac{ia^2\left(n^2-2\right)e^{n\tan^{-1}(ax)}\,{}_2F_1\left(1,-\frac{in}{2};1-\frac{in}{2};e^{2i\tan^{-1}(ax)}\right)}{cn}+\frac{ia^2\left(n^2+in-2\right)e^{n\tan^{-1}(ax)}}{2cn}-\frac{e^{n\tan^{-1}(ax)}}{2cx^2}-\frac{ane^{n\tan^{-1}(ax)}}{2cx}$$

```
[Out] ((I/2)*a^2*E^(n*ArcTan[a*x])*(-2 + I*n + n^2))/(c*n) - E^(n*ArcTan[a*x])/(2
*c*x^2) - (a*E^(n*ArcTan[a*x])*n)/(2*c*x) - (I*a^2*E^(n*ArcTan[a*x])*(-2 +
n^2)*Hypergeometric2F1[1, (-I/2)*n, 1 - (I/2)*n, E^((2*I)*ArcTan[a*x])])/(c
*n)
```

Rubi [A] time = 0.182806, antiderivative size = 242, normalized size of antiderivative = 1.92, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5082, 129, 151, 155, 12, 131}

$$\frac{a^2\left(2-n^2\right)(1-iax)^{1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,{}_2F_1\left(1,\frac{in}{2}+1;\frac{in}{2}+2;\frac{1-iax}{iax+1}\right)}{c(2+in)}-\frac{a^2\left(-in^2+n+2i\right)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cn}-\frac{(1-iax)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTan[a*x])/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] -(a^2*(2*I + n - I*n^2)*(1 - I*a*x)^((I/2)*n))/(2*c*n*(1 + I*a*x)^((I/2)*n)
) - (1 - I*a*x)^((I/2)*n)/(2*c*x^2*(1 + I*a*x)^((I/2)*n)) - (a*n*(1 - I*a*x
)^((I/2)*n))/(2*c*x*(1 + I*a*x)^((I/2)*n)) + (a^2*(2 - n^2)*(1 - I*a*x)^(1
+ (I/2)*n)*(1 + I*a*x)^(-1 - (I/2)*n)*Hypergeometric2F1[1, 1 + (I/2)*n, 2 +
 (I/2)*n, (1 - I*a*x)/(1 + I*a*x)])/(c*(2 + I*n))
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{x^3\left(c+a^2cx^2\right)}\,dx = \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x^3}\,dx}{c}$$

$$= -\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx^2} - \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\left(-an+2a^2x\right)}{x^2}\,dx}{2c}$$

$$= -\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx^2} - \frac{an(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx} + \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\left(-a^2\left(2-n^2\right)-a^3nx\right)}{x}\,d}{2c}$$

$$= -\frac{a^2\left(2i+n-in^2\right)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cn} - \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx^2} - \frac{an(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx}$$

$$= -\frac{a^2\left(2i+n-in^2\right)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cn} - \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx^2} - \frac{an(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx}$$

$$= -\frac{a^2\left(2i+n-in^2\right)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cn} - \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx^2} - \frac{an(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx}$$

Mathematica [A] time = 0.0683832, size = 174, normalized size = 1.38

$$\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2a^2n\left(n^2-2\right)x^2(1-iax)\,{}_2F_1\left(1,\frac{in}{2}+1;\frac{in}{2}+2;\frac{ax+i}{i-ax}\right)+i(n-2i)(ax-i)\left(in\left(a^2x^2+1\right)-2a^2x^2+\right.\right.}{2cn(n-2i)x^2(ax-i)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTan[a*x])/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] ((1 - I*a*x)^((I/2)*n)*(I*(-2*I + n)*(-I + a*x)*(-2*a^2*x^2 + a*n^2*x*(I +
a*x) + I*n*(1 + a^2*x^2)) + 2*a^2*n*(-2 + n^2)*x^2*(1 - I*a*x)*Hypergeometr
ic2F1[1, 1 + (I/2)*n, 2 + (I/2)*n, (I + a*x)/(I - a*x)]))/(2*c*n*(-2*I + n)
*x^2*(1 + I*a*x)^((I/2)*n)*(-I + a*x))
```

Maple [F] time = 0.346, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{n\arctan(ax)}}{x^3\left(a^2cx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^{(n\arctan(ax))}}{a^2cx^5+cx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(a*x))/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^{n\operatorname{atan}(ax)}}{a^2x^5+x^3}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/x**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(exp(n*atan(a*x))/(a**2*x**5 + x**3), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x^3), x)
```

3.347 $\int \frac{e^{n\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=181

$$\frac{360(2ax+n)e^{n\tan^{-1}(ax)}}{ac^4\left(n^2+4\right)\left(n^2+16\right)\left(n^2+36\right)\left(a^2x^2+1\right)}+\frac{30(4ax+n)e^{n\tan^{-1}(ax)}}{ac^4\left(n^2+16\right)\left(n^2+36\right)\left(a^2x^2+1\right)^2}+\frac{(6ax+n)e^{n\tan^{-1}(ax)}}{ac^4\left(n^2+36\right)\left(a^2x^2+1\right)^3}+\frac{}{ac^4n\left(n^2\right.}$$

```
[Out] (720*E^(n*ArcTan[a*x]))/(a*c^4*n*(4 + n^2)*(16 + n^2)*(36 + n^2)) + (E^(n*A
rcTan[a*x])*(n + 6*a*x))/(a*c^4*(36 + n^2)*(1 + a^2*x^2)^3) + (30*E^(n*ArcT
an[a*x])*(n + 4*a*x))/(a*c^4*(16 + n^2)*(36 + n^2)*(1 + a^2*x^2)^2) + (360*
E^(n*ArcTan[a*x])*(n + 2*a*x))/(a*c^4*(4 + n^2)*(16 + n^2)*(36 + n^2)*(1 +
a^2*x^2))
```

Rubi [A] time = 0.175117, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$\frac{360(2ax+n)e^{n\tan^{-1}(ax)}}{ac^4\left(n^2+4\right)\left(n^2+16\right)\left(n^2+36\right)\left(a^2x^2+1\right)}+\frac{30(4ax+n)e^{n\tan^{-1}(ax)}}{ac^4\left(n^2+16\right)\left(n^2+36\right)\left(a^2x^2+1\right)^2}+\frac{(6ax+n)e^{n\tan^{-1}(ax)}}{ac^4\left(n^2+36\right)\left(a^2x^2+1\right)^3}+\frac{}{ac^4n\left(n^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^4,x]
```

```
[Out] (720*E^(n*ArcTan[a*x]))/(a*c^4*n*(4 + n^2)*(16 + n^2)*(36 + n^2)) + (E^(n*A
rcTan[a*x])*(n + 6*a*x))/(a*c^4*(36 + n^2)*(1 + a^2*x^2)^3) + (30*E^(n*ArcT
an[a*x])*(n + 4*a*x))/(a*c^4*(16 + n^2)*(36 + n^2)*(1 + a^2*x^2)^2) + (360*
E^(n*ArcTan[a*x])*(n + 2*a*x))/(a*c^4*(4 + n^2)*(16 + n^2)*(36 + n^2)*(1 +
a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4}\,dx = \frac{e^{n\tan^{-1}(ax)}(n+6ax)}{ac^4\left(36+n^2\right)\left(1+a^2x^2\right)^3} + \frac{30\int \frac{e^{n\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx}{c\left(36+n^2\right)}$$

$$= \frac{e^{n\tan^{-1}(ax)}(n+6ax)}{ac^4\left(36+n^2\right)\left(1+a^2x^2\right)^3} + \frac{30e^{n\tan^{-1}(ax)}(n+4ax)}{ac^4\left(16+n^2\right)\left(36+n^2\right)\left(1+a^2x^2\right)^2} + \frac{360\int \frac{e^{n\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{c^2\left(16+n^2\right)\left(36+n^2\right)}$$

$$= \frac{e^{n\tan^{-1}(ax)}(n+6ax)}{ac^4\left(36+n^2\right)\left(1+a^2x^2\right)^3} + \frac{30e^{n\tan^{-1}(ax)}(n+4ax)}{ac^4\left(16+n^2\right)\left(36+n^2\right)\left(1+a^2x^2\right)^2} + \frac{360e^{n\tan^{-1}(ax)}(n+2}{ac^4\left(4+n^2\right)\left(16+n^2\right)\left(36+n\right.}$$

$$= \frac{720e^{n\tan^{-1}(ax)}}{ac^4n\left(4+n^2\right)\left(16+n^2\right)\left(36+n^2\right)} + \frac{e^{n\tan^{-1}(ax)}(n+6ax)}{ac^4\left(36+n^2\right)\left(1+a^2x^2\right)^3} + \frac{30e^{n\tan^{-1}(ax)}(n+4ax)}{ac^4\left(16+n^2\right)\left(36+n^2\right)\left(1+a^2\right.}$$

Mathematica [C] time = 0.440163, size = 165, normalized size = 0.91

$$\frac{(6ax+n)e^{n\tan^{-1}(ax)} + \frac{30\left(a^2cx^2+c\right)\left(12(ax-i)(ax+i)(1-iax)^{\frac{in}{2}}\left(2a^2x^2+2anx+n^2+2\right)(1+iax)^{-\frac{in}{2}}+n(n-2i)(n+2i)(4ax+n)e^{n\tan^{-1}(ax)}\right)}{cn\left(n^4+20n^2+64\right)}}{ac\left(n^2+36\right)\left(a^2cx^2+c\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^4,x]
```

```
[Out] (E^(n*ArcTan[a*x])*(n + 6*a*x) + (30*(c + a^2*c*x^2)*(E^(n*ArcTan[a*x])*n*(
-2*I + n)*(2*I + n)*(n + 4*a*x) + (12*(1 - I*a*x)^((I/2)*n)*(-I + a*x)*(I +
 a*x)*(2 + n^2 + 2*a*n*x + 2*a^2*x^2))/(1 + I*a*x)^((I/2)*n)))/(c*n*(64 + 2
0*n^2 + n^4)))/(a*c*(36 + n^2)*(c + a^2*c*x^2)^3)
```

Maple [A] time = 0.04, size = 166, normalized size = 0.9

$$\frac{\left(720\,a^6x^6+720\,a^5x^5n+360\,a^4n^2x^4+120\,a^3n^3x^3+2160\,a^4x^4+30\,a^2n^4x^2+1920\,a^3x^3n+6\,an^5x+840\,a^2n^2x^2+n^6\right.}{\left(a^2x^2+1\right)^3c^4an\left(n^6+56\,n^4+784\,n^2+2304\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^4,x)
```

```
[Out] (720*a^6*x^6+720*a^5*n*x^5+360*a^4*n^2*x^4+120*a^3*n^3*x^3+2160*a^4*x^4+30*
a^2*n^4*x^2+1920*a^3*n*x^3+6*a*n^5*x+840*a^2*n^2*x^2+n^6+240*a*n^3*x+2160*a
^2*x^2+50*n^4+1584*a*n*x+544*n^2+720)*exp(n*arctan(a*x))/(a^2*x^2+1)^3/c^4/
a/n/(n^6+56*n^4+784*n^2+2304)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

Fricas [A] time = 2.00418, size = 666, normalized size = 3.68

$$\frac{\left(720\,a^6x^6 + 720\,a^5nx^5 + n^6 + 360\left(a^4n^2 + 6\,a^4\right)x^4 + 50\,n^4 + 120\left(a^3n^3 + 16\,a^3n\right)x^3 + 30\left(a^2n^4 + 28\,a^2n^2\right.\right.}{ac^4n^7 + 56\,ac^4n^5 + 784\,ac^4n^3 + \left(a^7c^4n^7 + 56\,a^7c^4n^5 + 784\,a^7c^4n^3 + 2304\,a^7c^4n\right)x^6 + 2304\,ac^4n + 3\left(a^5c^4n^7 + 56\,a^5c^4n^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] (720*a^6*x^6 + 720*a^5*n*x^5 + n^6 + 360*(a^4*n^2 + 6*a^4)*x^4 + 50*n^4 + 1
20*(a^3*n^3 + 16*a^3*n)*x^3 + 30*(a^2*n^4 + 28*a^2*n^2 + 72*a^2)*x^2 + 544*
n^2 + 6*(a*n^5 + 40*a*n^3 + 264*a*n)*x + 720)*e^(n*arctan(a*x))/(a*c^4*n^7
+ 56*a*c^4*n^5 + 784*a*c^4*n^3 + (a^7*c^4*n^7 + 56*a^7*c^4*n^5 + 784*a^7*c^
4*n^3 + 2304*a^7*c^4*n)*x^6 + 2304*a*c^4*n + 3*(a^5*c^4*n^7 + 56*a^5*c^4*n^
5 + 784*a^5*c^4*n^3 + 2304*a^5*c^4*n)*x^4 + 3*(a^3*c^4*n^7 + 56*a^3*c^4*n^5
 + 784*a^3*c^4*n^3 + 2304*a^3*c^4*n)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{\left(a^2cx^2 + c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

3.348 $\int e^{n\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=121

$$-\frac{c2^{\frac{5}{2}-\frac{in}{2}}\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(5+in)}\,{}_2F_1\left(\frac{1}{2}(in-3),\frac{1}{2}(in+5);\frac{1}{2}(in+7);\frac{1}{2}(1-iax)\right)}{a(-n+5i)\sqrt{a^2x^2+1}}$$

```
[Out] -((2^(5/2 - (I/2)*n)*c*(1 - I*a*x)^((5 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hyperg
eometric2F1[(-3 + I*n)/2, (5 + I*n)/2, (7 + I*n)/2, (1 - I*a*x)/2])/(a*(5*I
 - n)*Sqrt[1 + a^2*x^2]))
```

Rubi [A] time = 0.103858, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$-\frac{c2^{\frac{5}{2}-\frac{in}{2}}\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(5+in)}\,{}_2F_1\left(\frac{1}{2}(in-3),\frac{1}{2}(in+5);\frac{1}{2}(in+7);\frac{1}{2}(1-iax)\right)}{a(-n+5i)\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -((2^(5/2 - (I/2)*n)*c*(1 - I*a*x)^((5 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hyperg
eometric2F1[(-3 + I*n)/2, (5 + I*n)/2, (7 + I*n)/2, (1 - I*a*x)/2])/(a*(5*I
 - n)*Sqrt[1 + a^2*x^2]))
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx = \frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{n\tan^{-1}(ax)}\left(1+a^2x^2\right)^{3/2}\,dx}{\sqrt{1+a^2x^2}}$$
$$= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int(1-iax)^{\frac{3}{2}+\frac{in}{2}}(1+iax)^{\frac{3}{2}-\frac{in}{2}}\,dx}{\sqrt{1+a^2x^2}}$$
$$= -\frac{2^{\frac{5}{2}-\frac{in}{2}}c(1-iax)^{\frac{1}{2}(5+in)}\sqrt{c+a^2cx^2}\,{}_2F_1\left(\frac{1}{2}(-3+in),\frac{1}{2}(5+in);\frac{1}{2}(7+in);\frac{1}{2}(1-iax)\right)}{a(5i-n)\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.104526, size = 118, normalized size = 0.98

$$\frac{c2^{\frac{5}{2}-\frac{in}{2}}\sqrt{a^2cx^2+c}(1-iax)^{\frac{5}{2}+\frac{in}{2}}\,{}_2F_1\left(\frac{1}{2}(in+5),\frac{1}{2}i(n+3i);\frac{1}{2}(in+7);\frac{1}{2}(1-iax)\right)}{a(n-5i)\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (2^(5/2 - (I/2)*n)*c*(1 - I*a*x)^(5/2 + (I/2)*n)*Sqrt[c + a^2*c*x^2]*Hyperg
eometric2F1[(5 + I*n)/2, (I/2)*(3*I + n), (7 + I*n)/2, (1 - I*a*x)/2])/(a*(
-5*I + n)*Sqrt[1 + a^2*x^2])
```

Maple [F] time = 0.285, size = 0, normalized size = 0.

$$\int e^{n\arctan(ax)}\left(a^2cx^2+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*e^(n*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(n\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*e^(n*arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.349 $\int e^{n\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal. Leaf size=120

$$-\frac{2^{\frac{3}{2}-\frac{in}{2}}\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(3+in)}\,{}_2F_1\left(\frac{1}{2}(in-1),\frac{1}{2}(in+3);\frac{1}{2}(in+5);\frac{1}{2}(1-iax)\right)}{a(-n+3i)\sqrt{a^2x^2+1}}$$

```
[Out] -((2^(3/2 - (I/2)*n)*(1 - I*a*x)^((3 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hypergeo
metric2F1[(-1 + I*n)/2, (3 + I*n)/2, (5 + I*n)/2, (1 - I*a*x)/2])/(a*(3*I -
 n)*Sqrt[1 + a^2*x^2]))
```

Rubi [A] time = 0.0966217, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$-\frac{2^{\frac{3}{2}-\frac{in}{2}}\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(3+in)}\,{}_2F_1\left(\frac{1}{2}(in-1),\frac{1}{2}(in+3);\frac{1}{2}(in+5);\frac{1}{2}(1-iax)\right)}{a(-n+3i)\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -((2^(3/2 - (I/2)*n)*(1 - I*a*x)^((3 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hypergeo
metric2F1[(-1 + I*n)/2, (3 + I*n)/2, (5 + I*n)/2, (1 - I*a*x)/2])/(a*(3*I -
 n)*Sqrt[1 + a^2*x^2]))
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx = \frac{\sqrt{c+a^2cx^2}\int e^{n\tan^{-1}(ax)}\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}$$
$$= \frac{\sqrt{c+a^2cx^2}\int(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{1+a^2x^2}}$$
$$= -\frac{2^{\frac{3}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}(3+in)}\sqrt{c+a^2cx^2}\,{}_2F_1\left(\frac{1}{2}(-1+in),\frac{1}{2}(3+in);\frac{1}{2}(5+in);\frac{1}{2}(1-iax)\right)}{a(3i-n)\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.052151, size = 117, normalized size = 0.98

$$\frac{2^{\frac{3}{2}-\frac{in}{2}}\sqrt{a^2cx^2+c}(1-iax)^{\frac{3}{2}+\frac{in}{2}}\,{}_2F_1\left(\frac{1}{2}(in+3),\frac{1}{2}i(n+i);\frac{1}{2}(in+5);\frac{1}{2}(1-iax)\right)}{a(n-3i)\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(3/2 - (I/2)*n)*(1 - I*a*x)^(3/2 + (I/2)*n)*Sqrt[c + a^2*c*x^2]*Hypergeo
metric2F1[(3 + I*n)/2, (I/2)*(I + n), (5 + I*n)/2, (1 - I*a*x)/2])/(a*(-3*I
 + n)*Sqrt[1 + a^2*x^2])
```

Maple [F] time = 0.284, size = 0, normalized size = 0.

$$\int e^{n\arctan(ax)}\sqrt{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2+c}\,e^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^2cx^2+c}\,e^{(n\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\left(a^2x^2+1\right)}e^{n\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*exp(n*atan(a*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.350 $\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=120

$$-\frac{2^{\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}\,{}_2F_1\left(\frac{1}{2}(in+1),\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1}{2}(1-iax)\right)}{a(-n+i)\sqrt{a^2cx^2+c}}$$

```
[Out] -((2^(1/2 - (I/2)*n)*(1 - I*a*x)^((1 + I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[(1 + I*n)/2, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/2])/(a*(I - n)*S
qrt[c + a^2*c*x^2]))
```

Rubi [A] time = 0.0975418, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$-\frac{2^{\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}\,{}_2F_1\left(\frac{1}{2}(in+1),\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1}{2}(1-iax)\right)}{a(-n+i)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -((2^(1/2 - (I/2)*n)*(1 - I*a*x)^((1 + I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[(1 + I*n)/2, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/2])/(a*(I - n)*S
qrt[c + a^2*c*x^2]))
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2^{\frac{1}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,{}_2F_1\left(\frac{1}{2}(1+in),\frac{1}{2}(1+in);\frac{1}{2}(3+in);\frac{1}{2}(1-iax)\right)}{a(i-n)\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.035477, size = 117, normalized size = 0.98

$$\frac{2^{\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+\frac{1}{2},\frac{in}{2}+\frac{1}{2};\frac{in}{2}+\frac{3}{2};\frac{1}{2}-\frac{iax}{2}\right)}{a(n-i)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(1/2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[1/2 + (I/2)*n, 1/2 + (I/2)*n, 3/2 + (I/2)*n, 1/2 - (I/2)*a*x])/(a*(
-I + n)*Sqrt[c + a^2*c*x^2])
```

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int e^{n\arctan(ax)}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{atan}(ax)}}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.351 $\int e^{n\tan^{-1}(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=283

$$\frac{c2^{\frac{3}{2}-\frac{in}{2}}\left(5-n^2\right)\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(5+in)}\,{}_2F_1\left(\frac{1}{2}(in-3),\frac{1}{2}(in+5);\frac{1}{2}(in+7);\frac{1}{2}(1-iax)\right)}{15a^3(-n+5i)\sqrt{a^2x^2+1}}-\frac{cn\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(5+in)}(}{30a^3\sqrt{a^2x^2+1}}$$

```
[Out] -(c*n*(1 - I*a*x)^((5 + I*n)/2)*(1 + I*a*x)^((5 - I*n)/2)*Sqrt[c + a^2*c*x^
2])/(30*a^3*Sqrt[1 + a^2*x^2]) + (c*x*(1 - I*a*x)^((5 + I*n)/2)*(1 + I*a*x)
^((5 - I*n)/2)*Sqrt[c + a^2*c*x^2])/(6*a^2*Sqrt[1 + a^2*x^2]) + (2^(3/2 - (
I/2)*n)*c*(5 - n^2)*(1 - I*a*x)^((5 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hypergeom
etric2F1[(-3 + I*n)/2, (5 + I*n)/2, (7 + I*n)/2, (1 - I*a*x)/2])/(15*a^3*(5
*I - n)*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.310511, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5085, 5082, 90, 80, 69}

$$\frac{c2^{\frac{3}{2}-\frac{in}{2}}\left(5-n^2\right)\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(5+in)}\,{}_2F_1\left(\frac{1}{2}(in-3),\frac{1}{2}(in+5);\frac{1}{2}(in+7);\frac{1}{2}(1-iax)\right)}{15a^3(-n+5i)\sqrt{a^2x^2+1}}-\frac{cn\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(5+in)}(}{30a^3\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*x^2*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -(c*n*(1 - I*a*x)^((5 + I*n)/2)*(1 + I*a*x)^((5 - I*n)/2)*Sqrt[c + a^2*c*x^
2])/(30*a^3*Sqrt[1 + a^2*x^2]) + (c*x*(1 - I*a*x)^((5 + I*n)/2)*(1 + I*a*x)
^((5 - I*n)/2)*Sqrt[c + a^2*c*x^2])/(6*a^2*Sqrt[1 + a^2*x^2]) + (2^(3/2 - (
I/2)*n)*c*(5 - n^2)*(1 - I*a*x)^((5 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hypergeom
etric2F1[(-3 + I*n)/2, (5 + I*n)/2, (7 + I*n)/2, (1 - I*a*x)/2])/(15*a^3*(5
*I - n)*Sqrt[1 + a^2*x^2])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tan^{-1}(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx=\frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{n\tan^{-1}(ax)}x^2\left(1+a^2x^2\right)^{3/2}\,dx}{\sqrt{1+a^2x^2}}$$
$$=\frac{\left(c\sqrt{c+a^2cx^2}\right)\int x^2(1-iax)^{\frac{3}{2}+\frac{in}{2}}(1+iax)^{\frac{3}{2}-\frac{in}{2}}\,dx}{\sqrt{1+a^2x^2}}$$
$$=\frac{cx(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{c+a^2cx^2}}{6a^2\sqrt{1+a^2x^2}}+\frac{\left(c\sqrt{c+a^2cx^2}\right)\int(1-iax)^{\frac{3}{2}+\frac{in}{2}}(1+ia}{6a^2\sqrt{1+a^2x^2}}$$
$$=-\frac{cn(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{c+a^2cx^2}}{30a^3\sqrt{1+a^2x^2}}+\frac{cx(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{}}{6a^2\sqrt{1+a^2x^2}}$$
$$=-\frac{cn(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{c+a^2cx^2}}{30a^3\sqrt{1+a^2x^2}}+\frac{cx(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{}}{6a^2\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.246871, size = 217, normalized size = 0.77

$$\frac{c2^{-1-\frac{in}{2}}(ax+i)^2\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2^{\frac{in}{2}}(n-5i)\sqrt{1+iax}(ax-i)^2(5ax-n)-4\sqrt{2}\left(n^2-5\right)(1+iax)^{\frac{in}{2}}\right.}{15a^3(n-5i)\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])*x^2*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (2^(-1 - (I/2)*n)*c*(1 - I*a*x)^(1/2 + (I/2)*n)*(I + a*x)^2*Sqrt[c + a^2*c*
x^2]*(2^((I/2)*n)*(-5*I + n)*Sqrt[1 + I*a*x]*(-I + a*x)^2*(-n + 5*a*x) - 4*
Sqrt[2]*(-5 + n^2)*(1 + I*a*x)^((I/2)*n)*Hypergeometric2F1[(5 + I*n)/2, (I/
2)*(3*I + n), (7 + I*n)/2, (1 - I*a*x)/2]))/(15*a^3*(-5*I + n)*(1 + I*a*x)^
((I/2)*n)*Sqrt[1 + a^2*x^2])
```

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int e^{n\arctan(ax)}x^2\left(a^2cx^2+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}}x^2e^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^2*e^(n*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^4+cx^2\right)\sqrt{a^2cx^2+c}e^{(n\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^4 + c*x^2)*sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**2*(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.352 $\int e^{n\tan^{-1}(ax)}x^2\sqrt{c+a^2cx^2}\,dx$

Optimal. Leaf size=280

$$\frac{2^{-\frac{1}{2}-\frac{in}{2}}\left(3-n^2\right)\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(3+in)}\,{}_2F_1\left(\frac{1}{2}(in-1),\frac{1}{2}(in+3);\frac{1}{2}(in+5);\frac{1}{2}(1-iax)\right)}{3a^3(-n+3i)\sqrt{a^2x^2+1}}-\frac{n\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(3+i}}{12a^3\sqrt{a^2x^2-}}$$

```
[Out] -(n*(1 - I*a*x)^((3 + I*n)/2)*(1 + I*a*x)^((3 - I*n)/2)*Sqrt[c + a^2*c*x^2]
)/(12*a^3*Sqrt[1 + a^2*x^2]) + (x*(1 - I*a*x)^((3 + I*n)/2)*(1 + I*a*x)^((3
 - I*n)/2)*Sqrt[c + a^2*c*x^2])/(4*a^2*Sqrt[1 + a^2*x^2]) + (2^(-1/2 - (I/2
)*n)*(3 - n^2)*(1 - I*a*x)^((3 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hypergeometric
2F1[(-1 + I*n)/2, (3 + I*n)/2, (5 + I*n)/2, (1 - I*a*x)/2])/(3*a^3*(3*I - n
)*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.288978, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5085, 5082, 90, 80, 69}

$$\frac{2^{-\frac{1}{2}-\frac{in}{2}}\left(3-n^2\right)\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(3+in)}\,{}_2F_1\left(\frac{1}{2}(in-1),\frac{1}{2}(in+3);\frac{1}{2}(in+5);\frac{1}{2}(1-iax)\right)}{3a^3(-n+3i)\sqrt{a^2x^2+1}}-\frac{n\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(3+i}}{12a^3\sqrt{a^2x^2-}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*x^2*Sqrt[c + a^2*c*x^2],x]
```


```
[Out] -(n*(1 - I*a*x)^((3 + I*n)/2)*(1 + I*a*x)^((3 - I*n)/2)*Sqrt[c + a^2*c*x^2]
)/(12*a^3*Sqrt[1 + a^2*x^2]) + (x*(1 - I*a*x)^((3 + I*n)/2)*(1 + I*a*x)^((3
 - I*n)/2)*Sqrt[c + a^2*c*x^2])/(4*a^2*Sqrt[1 + a^2*x^2]) + (2^(-1/2 - (I/2
)*n)*(3 - n^2)*(1 - I*a*x)^((3 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hypergeometric
2F1[(-1 + I*n)/2, (3 + I*n)/2, (5 + I*n)/2, (1 - I*a*x)/2])/(3*a^3*(3*I - n
)*Sqrt[1 + a^2*x^2])
```


Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tan^{-1}(ax)}x^2\sqrt{c+a^2cx^2}\,dx = \frac{\sqrt{c+a^2cx^2}\int e^{n\tan^{-1}(ax)}x^2\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\int x^2(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{x(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{4a^2\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\int(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{\frac{1}{2}-\frac{in}{2}}(-1}{4a^2\sqrt{1+a^2x^2}}$$

$$= -\frac{n(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{12a^3\sqrt{1+a^2x^2}} + \frac{x(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{4a^2\sqrt{1+a^2x^2}}$$

$$= -\frac{n(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{12a^3\sqrt{1+a^2x^2}} + \frac{x(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{4a^2\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.19168, size = 214, normalized size = 0.76

$$\frac{2^{-2-\frac{in}{2}}(ax+i)\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2^{\frac{in}{2}}(n-3i)\sqrt{1+iax}(ax-i)(3ax-n)-2i\sqrt{2}\left(n^2-3\right)(1+iax)^{\frac{in}{2}}\,{}_2F_1\left(\right.\right.}{3a^3(n-3i)\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])*x^2*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(-2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*(I + a*x)*Sqrt[c + a^2*c*x^2]
*(2^((I/2)*n)*(-3*I + n)*Sqrt[1 + I*a*x]*(-I + a*x)*(-n + 3*a*x) - (2*I)*Sq
rt[2]*(-3 + n^2)*(1 + I*a*x)^((I/2)*n)*Hypergeometric2F1[(3 + I*n)/2, (I/2)
*(I + n), (5 + I*n)/2, (1 - I*a*x)/2]))/(3*a^3*(-3*I + n)*(1 + I*a*x)^((I/2
)*n)*Sqrt[1 + a^2*x^2])
```

Maple [F] time = 0.302, size = 0, normalized size = 0.

$$\int e^{n\arctan(ax)}x^2\sqrt{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x)

[Out] int(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2 + c}x^2e^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a^2*c*x^2 + c)*x^2*e^(n*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^2cx^2 + c}x^2e^{(n\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*c*x^2 + c)*x^2*e^(n*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{c\left(a^2x^2 + 1\right)}e^{n\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**2*(a**2*c*x**2+c)**(1/2),x)

[Out] Integral(x**2*sqrt(c*(a**2*x**2 + 1))*exp(n*atan(a*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.353 $\int \frac{e^{n\tan^{-1}(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=322

$$\frac{2^{-\frac{1}{2}-\frac{in}{2}}n\left(5-n^2\right)\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(3+in)}\,{}_2F_1\left(\frac{1}{2}(in+1),\frac{1}{2}(in+3);\frac{1}{2}(in+5);\frac{1}{2}(1-iax)\right)}{3a^4\left(4n-i\left(3-n^2\right)\right)\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(a(}{6a^4(1}$$

```
[Out] (x^2*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])
/(3*a^2*Sqrt[c + a^2*c*x^2]) - ((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 -
 I*n)/2)*(4 - I*n - n^2 + a*(1 + I*n)*n*x)*Sqrt[1 + a^2*x^2])/(6*a^4*(1 + I
*n)*Sqrt[c + a^2*c*x^2]) + (2^(-1/2 - (I/2)*n)*n*(5 - n^2)*(1 - I*a*x)^((3
+ I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[(1 + I*n)/2, (3 + I*n)/2, (5
+ I*n)/2, (1 - I*a*x)/2])/(3*a^4*(4*n - I*(3 - n^2))*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.356904, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5085, 5082, 100, 146, 69}

$$\frac{2^{-\frac{1}{2}-\frac{in}{2}}n\left(5-n^2\right)\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(3+in)}\,{}_2F_1\left(\frac{1}{2}(in+1),\frac{1}{2}(in+3);\frac{1}{2}(in+5);\frac{1}{2}(1-iax)\right)}{3a^4\left(4n-i\left(3-n^2\right)\right)\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(a(}{6a^4(1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^3)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (x^2*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])
/(3*a^2*Sqrt[c + a^2*c*x^2]) - ((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 -
 I*n)/2)*(4 - I*n - n^2 + a*(1 + I*n)*n*x)*Sqrt[1 + a^2*x^2])/(6*a^4*(1 + I
*n)*Sqrt[c + a^2*c*x^2]) + (2^(-1/2 - (I/2)*n)*n*(5 - n^2)*(1 - I*a*x)^((3
+ I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[(1 + I*n)/2, (3 + I*n)/2, (5
+ I*n)/2, (1 - I*a*x)/2])/(3*a^4*(4*n - I*(3 - n^2))*Sqrt[c + a^2*c*x^2])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
```

```
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}x^3}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int x^3(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{x^2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{3a^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\int x(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}(-2-an}{3a^2\sqrt{c+a^2cx^2}}$$

$$= \frac{x^2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{3a^2\sqrt{c+a^2cx^2}} - \frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\left(4-in-n^2+a(1+\right.}{6a^4(1+in)\sqrt{c+a^2cx^2}}$$

$$= \frac{x^2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{3a^2\sqrt{c+a^2cx^2}} - \frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\left(4-in-n^2+a(1+\right.}{6a^4(1+in)\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.291598, size = 248, normalized size = 0.77

$$\frac{2^{-\frac{3}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2^{\frac{1}{2}+\frac{in}{2}}(n-3i)\sqrt{1+iax}\left(n\left(2a^2x^2+iax+1\right)-2i\left(a^2x^2-2\right)+n^2(-(ax+i))\right)\right.}{3a^4\left(n^2-4in-3\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^3)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(-3/2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*(2^(1/2 +
 (I/2)*n)*(-3*I + n)*Sqrt[1 + I*a*x]*(-(n^2*(I + a*x)) - (2*I)*(-2 + a^2*x^
2) + n*(1 + I*a*x + 2*a^2*x^2)) + 2*n*(-5 + n^2)*(1 + I*a*x)^((I/2)*n)*(I +
 a*x)*Hypergeometric2F1[1/2 + (I/2)*n, 3/2 + (I/2)*n, 5/2 + (I/2)*n, 1/2 -
```

```
(I/2)*a*x]))/(3*a^4*(-3 - (4*I)*n + n^2)*(1 + I*a*x)^((I/2)*n)*Sqrt[c + a^2
*c*x^2])
```

Maple [F] time = 0.29, size = 0, normalized size = 0.

$$\int e^{n\arctan(ax)} x^3 \frac{1}{\sqrt{a^2 c x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 e^{(n\arctan(ax))}}{\sqrt{a^2 c x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3 e^{(n\arctan(ax))}}{\sqrt{a^2 c x^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 e^{n \operatorname{atan}(ax)}}{\sqrt{c\left(a^2 x^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.354 $\int \frac{e^{n\tan^{-1}(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=291

$$-\frac{i2^{\frac{1}{2}-\frac{in}{2}}\left(1-n^2\right)\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}{}_2F_1\left(\frac{1}{2}(in-1),\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1}{2}(1-iax)\right)}{a^3\left(n^2+1\right)\sqrt{a^2cx^2+c}}+\frac{x\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1}{2a^2\sqrt{a^2cx^2+c}}$$

```
[Out] -((1 + I*n)*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^
2*x^2])/(2*a^3*(I + n)*Sqrt[c + a^2*c*x^2]) + (x*(1 - I*a*x)^((1 + I*n)/2)*
(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])/(2*a^2*Sqrt[c + a^2*c*x^2]) -
(I*2^(1/2 - (I/2)*n)*(1 - n^2)*(1 - I*a*x)^((1 + I*n)/2)*Sqrt[1 + a^2*x^2]*
Hypergeometric2F1[(-1 + I*n)/2, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/2])/(
a^3*(1 + n^2)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.342169, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5085, 5082, 90, 79, 69}

$$-\frac{i2^{\frac{1}{2}-\frac{in}{2}}\left(1-n^2\right)\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}{}_2F_1\left(\frac{1}{2}(in-1),\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1}{2}(1-iax)\right)}{a^3\left(n^2+1\right)\sqrt{a^2cx^2+c}}+\frac{x\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1}{2a^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -((1 + I*n)*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^
2*x^2])/(2*a^3*(I + n)*Sqrt[c + a^2*c*x^2]) + (x*(1 - I*a*x)^((1 + I*n)/2)*
(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])/(2*a^2*Sqrt[c + a^2*c*x^2]) -
(I*2^(1/2 - (I/2)*n)*(1 - n^2)*(1 - I*a*x)^((1 + I*n)/2)*Sqrt[1 + a^2*x^2]*
Hypergeometric2F1[(-1 + I*n)/2, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/2])/(
a^3*(1 + n^2)*Sqrt[c + a^2*c*x^2])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}x^2}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int x^2(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{x(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\int(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}(-1-anx)}{2a^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{(1+in)(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^3(i+n)\sqrt{c+a^2cx^2}} + \frac{x(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^2\sqrt{c+a^2cx^2}} -$$

$$= -\frac{(1+in)(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^3(i+n)\sqrt{c+a^2cx^2}} + \frac{x(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^2\sqrt{c+a^2cx^2}} -$$

Mathematica [A] time = 0.167357, size = 206, normalized size = 0.71

$$\frac{2^{-1-\frac{in}{2}}\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2i\sqrt{2}\left(n^2-1\right)(1+iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{1}{2}(in+1),\frac{1}{2}i(n+i);\frac{1}{2}(in+3);\frac{1}{2}(1-iax)\right)+2\right.}{a^3\left(n^2+1\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(-1 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*(2^((I/2)*n
)*(-I + n)*Sqrt[1 + I*a*x]*(-1 + I*a*x + n*(-I + a*x)) + (2*I)*Sqrt[2]*(-1
+ n^2)*(1 + I*a*x)^((I/2)*n)*Hypergeometric2F1[(1 + I*n)/2, (I/2)*(I + n),
(3 + I*n)/2, (1 - I*a*x)/2]))/(a^3*(1 + n^2)*(1 + I*a*x)^((I/2)*n)*Sqrt[c +
 a^2*c*x^2])
```

Maple [F] time = 0.294, size = 0, normalized size = 0.

$$\int e^{n \arctan(ax)} x^2 \frac{1}{\sqrt{a^2 c x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2 e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 e^{n \operatorname{atan}(ax)}}{\sqrt{c \left(a^2 x^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.355 $\int \frac{e^{n\tan^{-1}(ax)}x}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=202

$$\frac{\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{a^2(1-in)\sqrt{a^2cx^2+c}}-\frac{i2^{\frac{3}{2}-\frac{in}{2}}n\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}{}_2F_1\left(\frac{1}{2}(in-1),\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1}{2}(1-iax\right.}{a^2\left(n^2+1\right)\sqrt{a^2cx^2+c}}$$

```
[Out] ((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])/(a^
2*(1 - I*n)*Sqrt[c + a^2*c*x^2]) - (I*2^(3/2 - (I/2)*n)*n*(1 - I*a*x)^((1 +
 I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[(-1 + I*n)/2, (1 + I*n)/2, (3
+ I*n)/2, (1 - I*a*x)/2])/(a^2*(1 + n^2)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.186635, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5085, 5082, 79, 69}

$$\frac{\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{a^2(1-in)\sqrt{a^2cx^2+c}}-\frac{i2^{\frac{3}{2}-\frac{in}{2}}n\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}{}_2F_1\left(\frac{1}{2}(in-1),\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1}{2}(1-iax\right.}{a^2\left(n^2+1\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])/(a^
2*(1 - I*n)*Sqrt[c + a^2*c*x^2]) - (I*2^(3/2 - (I/2)*n)*n*(1 - I*a*x)^((1 +
 I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[(-1 + I*n)/2, (1 + I*n)/2, (3
+ I*n)/2, (1 - I*a*x)/2])/(a^2*(1 + n^2)*Sqrt[c + a^2*c*x^2])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\tan^{-1}(ax)}x}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}x}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int x(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{a^2(1-in)\sqrt{c+a^2cx^2}} - \frac{\left(n\sqrt{1+a^2x^2}\right)\int(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}i(i+n)}\,dx}{a(1-in)\sqrt{c+a^2cx^2}}\\
&= \frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{a^2(1-in)\sqrt{c+a^2cx^2}} - \frac{i2^{\frac{3}{2}-\frac{in}{2}}n(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,{}_2F_1\left(\frac{1}{2}(-1+in),\frac{1}{2}(1\right.}{a^2\left(1+n^2\right)\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.137188, size = 187, normalized size = 0.93

$$\frac{i2^{-\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2^{\frac{1}{2}+\frac{in}{2}}(n-i)\sqrt{1+iax}-4n(1+iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{1}{2}(in+1),\frac{1}{2}i(n+i);\frac{1}{2}(in+3);\frac{1}{2}\right.\right.}{a^2\left(n^2+1\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (I*2^(-1/2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*(2^(1/2
 + (I/2)*n)*(-I + n)*Sqrt[1 + I*a*x] - 4*n*(1 + I*a*x)^((I/2)*n)*Hypergeome
tric2F1[(1 + I*n)/2, (I/2)*(I + n), (3 + I*n)/2, (1 - I*a*x)/2]))/(a^2*(1 +
 n^2)*(1 + I*a*x)^((I/2)*n)*Sqrt[c + a^2*c*x^2])
```

Maple [F] time = 0.289, size = 0, normalized size = 0.

$$\int e^{n\arctan(ax)}x\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{xe^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{xe^{n\,\mathrm{atan}\,(ax)}}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{xe^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.356 $\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=120

$$-\frac{2^{\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}\,{}_2F_1\left(\frac{1}{2}(in+1),\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1}{2}(1-iax)\right)}{a(-n+i)\sqrt{a^2cx^2+c}}$$

```
[Out] -((2^(1/2 - (I/2)*n)*(1 - I*a*x)^((1 + I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[(1 + I*n)/2, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/2])/(a*(I - n)*S
qrt[c + a^2*c*x^2]))
```

Rubi [A] time = 0.0833224, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$-\frac{2^{\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}\,{}_2F_1\left(\frac{1}{2}(in+1),\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1}{2}(1-iax)\right)}{a(-n+i)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -((2^(1/2 - (I/2)*n)*(1 - I*a*x)^((1 + I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[(1 + I*n)/2, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/2])/(a*(I - n)*S
qrt[c + a^2*c*x^2]))
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2^{\frac{1}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,{}_2F_1\left(\frac{1}{2}(1+in),\frac{1}{2}(1+in);\frac{1}{2}(3+in);\frac{1}{2}(1-iax)\right)}{a(i-n)\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0249861, size = 117, normalized size = 0.98

$$\frac{2^{\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+\frac{1}{2},\frac{in}{2}+\frac{1}{2};\frac{in}{2}+\frac{3}{2};\frac{1}{2}-\frac{iax}{2}\right)}{a(n-i)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(1/2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[1/2 + (I/2)*n, 1/2 + (I/2)*n, 3/2 + (I/2)*n, 1/2 - (I/2)*a*x])/(a*(
-I + n)*Sqrt[c + a^2*c*x^2])
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int e^{n\arctan(ax)}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{atan}(ax)}}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.357 $\int \frac{e^{n\tan^{-1}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=121

$$-\frac{2\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\,{}_2F_1\left(1,\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1-iax}{iax+1}\right)}{(1+in)\sqrt{a^2cx^2+c}}$$

```
[Out] (-2*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((-1 - I*n)/2)*Sqrt[1 + a^2*x^2]*
Hypergeometric2F1[1, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/(1 + I*a*x)])/((
1 + I*n)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.203202, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5085, 5082, 131}

$$-\frac{2\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\,{}_2F_1\left(1,\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1-iax}{iax+1}\right)}{(1+in)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-2*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((-1 - I*n)/2)*Sqrt[1 + a^2*x^2]*
Hypergeometric2F1[1, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/(1 + I*a*x)])/((
1 + I*n)*Sqrt[c + a^2*c*x^2])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}}{x\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}}{x}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\sqrt{1+a^2x^2}\,{}_2F_1\left(1,\frac{1}{2}(1+in);\frac{1}{2}(3+in);\frac{1-iax}{1+iax}\right)}{(1+in)\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.043392, size = 120, normalized size = 0.99

$$\frac{2\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,{}_2F_1\left(1,\frac{in}{2}+\frac{1}{2};\frac{in}{2}+\frac{3}{2};\frac{ax+i}{i-ax}\right)}{(-1-in)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (2*(1 - I*a*x)^(1/2 + (I/2)*n)*(1 + I*a*x)^(-1/2 - (I/2)*n)*Sqrt[1 + a^2*x^
2]*Hypergeometric2F1[1, 1/2 + (I/2)*n, 3/2 + (I/2)*n, (I + a*x)/(I - a*x)])
/((-1 - I*n)*Sqrt[c + a^2*c*x^2])
```

Maple [F] time = 0.287, size = 0, normalized size = 0.

$$\int \frac{e^{n\arctan(ax)}}{x}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}e^{(n\arctan(ax))}}{a^2cx^3+cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x))/(a^2*c*x^3 + c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n\,\mathrm{atan}\,(ax)}}{x\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/x/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(n*atan(a*x))/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x), x)
```

3.358 $\int \frac{e^{n\tan^{-1}(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=196

$$-\frac{2an\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\,{}_2F_1\left(1,\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1-iax}{iax+1}\right)}{(1+in)\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)}{x\sqrt{a^2cx^2+c}}$$

```
[Out] -(((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])/(
x*Sqrt[c + a^2*c*x^2])) - (2*a*n*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((-1
 - I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1, (1 + I*n)/2, (3 + I*n)/2,
 (1 - I*a*x)/(1 + I*a*x)])/((1 + I*n)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.232639, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5085, 5082, 96, 131}

$$-\frac{2an\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\,{}_2F_1\left(1,\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1-iax}{iax+1}\right)}{(1+in)\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)}{x\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -(((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])/(
x*Sqrt[c + a^2*c*x^2])) - (2*a*n*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((-1
 - I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1, (1 + I*n)/2, (3 + I*n)/2,
 (1 - I*a*x)/(1 + I*a*x)])/((1 + I*n)*Sqrt[c + a^2*c*x^2])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}}{x^2\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}}{x^2}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{x\sqrt{c+a^2cx^2}} + \frac{\left(an\sqrt{1+a^2x^2}\right)\int \frac{(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}}{x}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{x\sqrt{c+a^2cx^2}} - \frac{2an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\sqrt{1+a^2x^2}\,{}_2F_1\left(1,\frac{1}{2}\right.}{(1+in)\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0650814, size = 142, normalized size = 0.72

$$\frac{\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\left(2anx\,{}_2F_1\left(1,\frac{in}{2}+\frac{1}{2};\frac{in}{2}+\frac{3}{2};\frac{ax+i}{i-ax}\right)-(n-i)(ax-i)\right)}{(-1-in)x\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((1 - I*a*x)^(1/2 + (I/2)*n)*(1 + I*a*x)^(-1/2 - (I/2)*n)*Sqrt[1 + a^2*x^2]
*(-((-I + n)*(-I + a*x)) + 2*a*n*x*Hypergeometric2F1[1, 1/2 + (I/2)*n, 3/2
+ (I/2)*n, (I + a*x)/(I - a*x)]))/((-1 - I*n)*x*Sqrt[c + a^2*c*x^2])
```

Maple [F] time = 0.289, size = 0, normalized size = 0.

$$\int \frac{e^{n\arctan(ax)}}{x^2}\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,e^{(n\arctan(ax))}}{a^2cx^4+cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x))/(a^2*c*x^4 + c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n\,\mathrm{atan}\,(ax)}}{x^2\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/x**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(n*atan(a*x))/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

3.359 $\int \frac{e^{n\tan^{-1}(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=281

$$\frac{a^2(1-n^2)\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}{}_2F_1\left(1,\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1-iax}{iax+1}\right)}{(1+in)\sqrt{a^2cx^2+c}}-\frac{an\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+}{2x\sqrt{a^2cx^2+c}}$$

```
[Out] -((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])/(2
*x^2*Sqrt[c + a^2*c*x^2]) - (a*n*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1
- I*n)/2)*Sqrt[1 + a^2*x^2])/(2*x*Sqrt[c + a^2*c*x^2]) + (a^2*(1 - n^2)*(1
- I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((-1 - I*n)/2)*Sqrt[1 + a^2*x^2]*Hyperge
ometric2F1[1, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/(1 + I*a*x)])/((1 + I*n
)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.25253, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5085, 5082, 129, 151, 12, 131}

$$\frac{a^2(1-n^2)\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}{}_2F_1\left(1,\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1-iax}{iax+1}\right)}{(1+in)\sqrt{a^2cx^2+c}}-\frac{an\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+}{2x\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])/(2
*x^2*Sqrt[c + a^2*c*x^2]) - (a*n*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1
- I*n)/2)*Sqrt[1 + a^2*x^2])/(2*x*Sqrt[c + a^2*c*x^2]) + (a^2*(1 - n^2)*(1
- I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((-1 - I*n)/2)*Sqrt[1 + a^2*x^2]*Hyperge
ometric2F1[1, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/(1 + I*a*x)])/((1 + I*n
)*Sqrt[c + a^2*c*x^2])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
```

```
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}}{x^3\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int \frac{(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}}{x^3}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\int \frac{(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\left(-an+a^2x\right)}{x^2}\,dx}{2\sqrt{c+a^2cx^2}}$$

$$= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x^2\sqrt{c+a^2cx^2}} - \frac{an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+\cdots}}{\cdots}$$

$$= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x^2\sqrt{c+a^2cx^2}} - \frac{an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x\sqrt{c+a^2cx^2}} - \frac{\left(a^2\left(\cdots\right.\right.}{\cdots}$$

$$= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x^2\sqrt{c+a^2cx^2}} - \frac{an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x\sqrt{c+a^2cx^2}} + \frac{a^2\left(1\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.0766287, size = 159, normalized size = 0.57

$$\frac{i\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\left(2a^2\left(n^2-1\right)x^2\,{}_2F_1\left(1,\frac{in}{2}+\frac{1}{2};\frac{in}{2}+\frac{3}{2};\frac{ax+i}{i-ax}\right)-(n-i)(ax-i)(anx+1)\right)}{2(n-i)x^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((I/2)*(1 - I*a*x)^(1/2 + (I/2)*n)*(1 + I*a*x)^(-1/2 - (I/2)*n)*Sqrt[1 + a^
2*x^2]*(-((-I + n)*(-I + a*x)*(1 + a*n*x)) + 2*a^2*(-1 + n^2)*x^2*Hypergeom
etric2F1[1, 1/2 + (I/2)*n, 3/2 + (I/2)*n, (I + a*x)/(I - a*x)]))/((-I + n)*
x^2*Sqrt[c + a^2*c*x^2])
```

Maple [F] time = 0.293, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{n \arctan(ax)}}{x^3} \frac{1}{\sqrt{a^2cx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2cx^2 + c}x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2 + c}e^{(n \arctan(ax))}}{a^2cx^5 + cx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x))/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{atan}(ax)}}{x^3\sqrt{c\left(a^2x^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/x**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(n*atan(a*x))/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2cx^2 + c}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

3.360 $\int e^{n\tan^{-1}(ax)}\sqrt[3]{c+a^2cx^2}\,dx$

Optimal. Leaf size=120

$$-\frac{3\,2^{\frac{4}{3}-\frac{in}{2}}\sqrt[3]{a^2cx^2+c}(1-iax)^{\frac{1}{6}(8+3in)}\,{}_2F_1\left(\frac{1}{6}(3in-2),\frac{1}{6}(3in+8);\frac{1}{6}(3in+14);\frac{1}{2}(1-iax)\right)}{a(-3n+8i)\sqrt[3]{a^2x^2+1}}$$

```
[Out] (-3*2^(4/3 - (I/2)*n)*(1 - I*a*x)^((8 + (3*I)*n)/6)*(c + a^2*c*x^2)^(1/3)*H
ypergeometric2F1[(-2 + (3*I)*n)/6, (8 + (3*I)*n)/6, (14 + (3*I)*n)/6, (1 -
I*a*x)/2])/(a*(8*I - 3*n)*(1 + a^2*x^2)^(1/3))
```

Rubi [A] time = 0.106687, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$-\frac{3\,2^{\frac{4}{3}-\frac{in}{2}}\sqrt[3]{a^2cx^2+c}(1-iax)^{\frac{1}{6}(8+3in)}\,{}_2F_1\left(\frac{1}{6}(3in-2),\frac{1}{6}(3in+8);\frac{1}{6}(3in+14);\frac{1}{2}(1-iax)\right)}{a(-3n+8i)\sqrt[3]{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^(1/3),x]
```

```
[Out] (-3*2^(4/3 - (I/2)*n)*(1 - I*a*x)^((8 + (3*I)*n)/6)*(c + a^2*c*x^2)^(1/3)*H
ypergeometric2F1[(-2 + (3*I)*n)/6, (8 + (3*I)*n)/6, (14 + (3*I)*n)/6, (1 -
I*a*x)/2])/(a*(8*I - 3*n)*(1 + a^2*x^2)^(1/3))
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tan^{-1}(ax)}\sqrt[3]{c+a^2cx^2}\,dx=\frac{\sqrt[3]{c+a^2cx^2}\int e^{n\tan^{-1}(ax)}\sqrt[3]{1+a^2x^2}\,dx}{\sqrt[3]{1+a^2x^2}}$$
$$=\frac{\sqrt[3]{c+a^2cx^2}\int(1-iax)^{\frac{1}{3}+\frac{in}{2}}(1+iax)^{\frac{1}{3}-\frac{in}{2}}\,dx}{\sqrt[3]{1+a^2x^2}}$$
$$=-\frac{3\,2^{\frac{4}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(8+3in)}\sqrt[3]{c+a^2cx^2}\,{}_2F_1\left(\frac{1}{6}(-2+3in),\frac{1}{6}(8+3in);\frac{1}{6}(14+3in);\frac{1}{2}(1-iax\right.}{a(8i-3n)\sqrt[3]{1+a^2x^2}}$$

Mathematica [A] time = 0.0629626, size = 120, normalized size = 1.

$$\frac{3\,2^{\frac{4}{3}-\frac{in}{2}}\sqrt[3]{a^2cx^2+c}(1-iax)^{\frac{4}{3}+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}-\frac{1}{3},\frac{in}{2}+\frac{4}{3};\frac{in}{2}+\frac{7}{3};\frac{1}{2}-\frac{iax}{2}\right)}{a(3n-8i)\sqrt[3]{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^(1/3),x]
```

```
[Out] (3*2^(4/3 - (I/2)*n)*(1 - I*a*x)^(4/3 + (I/2)*n)*(c + a^2*c*x^2)^(1/3)*Hype
rgeometric2F1[-1/3 + (I/2)*n, 4/3 + (I/2)*n, 7/3 + (I/2)*n, 1/2 - (I/2)*a*x
])/(a*(-8*I + 3*n)*(1 + a^2*x^2)^(1/3))
```

Maple [F] time = 0.255, size = 0, normalized size = 0.

$$\int e^{n\arctan(ax)}\sqrt[3]{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x)
```

```
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2cx^2+c\right)^{\frac{1}{3}}e^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(1/3)*e^(n*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^2+c\right)^{\frac{1}{3}}e^{(n\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(1/3)*e^(n*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{c\left(a^2x^2+1\right)}e^{n\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**(1/3),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(1/3)*exp(n*atan(a*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.361 $\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt[3]{c+a^2cx^2}}\,dx$

Optimal. Leaf size=120

$$-\frac{3\,2^{\frac{2}{3}-\frac{in}{2}}\sqrt[3]{a^2x^2+1}(1-iax)^{\frac{1}{6}(4+3in)}\,{}_2F_1\left(\frac{1}{6}(3in+2),\frac{1}{6}(3in+4);\frac{1}{6}(3in+10);\frac{1}{2}(1-iax)\right)}{a(-3n+4i)\sqrt[3]{a^2cx^2+c}}$$

```
[Out] (-3*2^(2/3 - (I/2)*n)*(1 - I*a*x)^((4 + (3*I)*n)/6)*(1 + a^2*x^2)^(1/3)*Hyp
ergeometric2F1[(2 + (3*I)*n)/6, (4 + (3*I)*n)/6, (10 + (3*I)*n)/6, (1 - I*a
*x)/2])/(a*(4*I - 3*n)*(c + a^2*c*x^2)^(1/3))
```

Rubi [A] time = 0.108826, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$-\frac{3\,2^{\frac{2}{3}-\frac{in}{2}}\sqrt[3]{a^2x^2+1}(1-iax)^{\frac{1}{6}(4+3in)}\,{}_2F_1\left(\frac{1}{6}(3in+2),\frac{1}{6}(3in+4);\frac{1}{6}(3in+10);\frac{1}{2}(1-iax)\right)}{a(-3n+4i)\sqrt[3]{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(1/3),x]
```

```
[Out] (-3*2^(2/3 - (I/2)*n)*(1 - I*a*x)^((4 + (3*I)*n)/6)*(1 + a^2*x^2)^(1/3)*Hyp
ergeometric2F1[(2 + (3*I)*n)/6, (4 + (3*I)*n)/6, (10 + (3*I)*n)/6, (1 - I*a
*x)/2])/(a*(4*I - 3*n)*(c + a^2*c*x^2)^(1/3))
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt[3]{c+a^2cx^2}}\,dx = \frac{\sqrt[3]{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt[3]{1+a^2x^2}}\,dx}{\sqrt[3]{c+a^2cx^2}}$$
$$= \frac{\sqrt[3]{1+a^2x^2}\int (1-iax)^{-\frac{1}{3}+\frac{in}{2}}(1+iax)^{-\frac{1}{3}-\frac{in}{2}}\,dx}{\sqrt[3]{c+a^2cx^2}}$$
$$= -\frac{3\,2^{\frac{2}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(4+3in)}\sqrt[3]{1+a^2x^2}\,{}_2F_1\left(\frac{1}{6}(2+3in),\frac{1}{6}(4+3in);\frac{1}{6}(10+3in);\frac{1}{2}(1-iax)\right)}{a(4i-3n)\sqrt[3]{c+a^2cx^2}}$$

Mathematica [A] time = 0.0461303, size = 120, normalized size = 1.

$$\frac{3\,2^{\frac{2}{3}-\frac{in}{2}}\sqrt[3]{a^2x^2+1}(1-iax)^{\frac{2}{3}+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+\frac{1}{3},\frac{in}{2}+\frac{2}{3};\frac{in}{2}+\frac{5}{3};\frac{1}{2}-\frac{iax}{2}\right)}{a(3n-4i)\sqrt[3]{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(1/3),x]
```

```
[Out] (3*2^(2/3 - (I/2)*n)*(1 - I*a*x)^(2/3 + (I/2)*n)*(1 + a^2*x^2)^(1/3)*Hyperg
eometric2F1[1/3 + (I/2)*n, 2/3 + (I/2)*n, 5/3 + (I/2)*n, 1/2 - (I/2)*a*x])/
(a*(-4*I + 3*n)*(c + a^2*c*x^2)^(1/3))
```

Maple [F] time = 0.271, size = 0, normalized size = 0.

$$\int e^{n\arctan(ax)}\frac{1}{\sqrt[3]{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x)
```

```
[Out] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n\operatorname{atan}(ax)}}{\sqrt[3]{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(1/3),x)
```

```
[Out] Integral(exp(n*atan(a*x))/(c*(a**2*x**2 + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(1/3), x)
```

3.362 $\int \frac{e^{n\tan^{-1}(ax)}}{(c+a^2cx^2)^{2/3}}\,dx$

Optimal. Leaf size=120

$$-\frac{3\,2^{\frac{1}{3}-\frac{in}{2}}\left(a^2x^2+1\right)^{2/3}(1-iax)^{\frac{1}{6}(2+3in)}\,{}_2F_1\left(\frac{1}{6}(3in+2),\frac{1}{6}(3in+4);\frac{1}{6}(3in+8);\frac{1}{2}(1-iax)\right)}{a(-3n+2i)\left(a^2cx^2+c\right)^{2/3}}$$

```
[Out] (-3*2^(1/3 - (I/2)*n)*(1 - I*a*x)^((2 + (3*I)*n)/6)*(1 + a^2*x^2)^(2/3)*Hyp
ergeometric2F1[(2 + (3*I)*n)/6, (4 + (3*I)*n)/6, (8 + (3*I)*n)/6, (1 - I*a*
x)/2])/(a*(2*I - 3*n)*(c + a^2*c*x^2)^(2/3))
```

Rubi [A] time = 0.110613, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$-\frac{3\,2^{\frac{1}{3}-\frac{in}{2}}\left(a^2x^2+1\right)^{2/3}(1-iax)^{\frac{1}{6}(2+3in)}\,{}_2F_1\left(\frac{1}{6}(3in+2),\frac{1}{6}(3in+4);\frac{1}{6}(3in+8);\frac{1}{2}(1-iax)\right)}{a(-3n+2i)\left(a^2cx^2+c\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(2/3),x]
```

```
[Out] (-3*2^(1/3 - (I/2)*n)*(1 - I*a*x)^((2 + (3*I)*n)/6)*(1 + a^2*x^2)^(2/3)*Hyp
ergeometric2F1[(2 + (3*I)*n)/6, (4 + (3*I)*n)/6, (8 + (3*I)*n)/6, (1 - I*a*
x)/2])/(a*(2*I - 3*n)*(c + a^2*c*x^2)^(2/3))
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{2/3}}\,dx = \frac{\left(1+a^2x^2\right)^{2/3}\int \frac{e^{n\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{2/3}}\,dx}{\left(c+a^2cx^2\right)^{2/3}}$$

$$= \frac{\left(1+a^2x^2\right)^{2/3}\int (1-iax)^{-\frac{2}{3}+\frac{in}{2}}(1+iax)^{-\frac{2}{3}-\frac{in}{2}}\,dx}{\left(c+a^2cx^2\right)^{2/3}}$$

$$= -\frac{3\,2^{\frac{1}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(2+3in)}\left(1+a^2x^2\right)^{2/3}{}_2F_1\left(\frac{1}{6}(2+3in),\frac{1}{6}(4+3in);\frac{1}{6}(8+3in);\frac{1}{2}(1-iax)\right)}{a(2i-3n)\left(c+a^2cx^2\right)^{2/3}}$$

Mathematica [A] time = 0.0433382, size = 120, normalized size = 1.

$$\frac{3\,2^{\frac{1}{3}-\frac{in}{2}}\left(a^2x^2+1\right)^{2/3}(1-iax)^{\frac{1}{3}+\frac{in}{2}}{}_2F_1\left(\frac{in}{2}+\frac{1}{3},\frac{in}{2}+\frac{2}{3};\frac{in}{2}+\frac{4}{3};\frac{1}{2}-\frac{iax}{2}\right)}{a(3n-2i)\left(a^2cx^2+c\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(2/3),x]
```

```
[Out] (3*2^(1/3 - (I/2)*n)*(1 - I*a*x)^(1/3 + (I/2)*n)*(1 + a^2*x^2)^(2/3)*Hyperg
eometric2F1[1/3 + (I/2)*n, 2/3 + (I/2)*n, 4/3 + (I/2)*n, 1/2 - (I/2)*a*x])/
(a*(-2*I + 3*n)*(c + a^2*c*x^2)^(2/3))
```

Maple [F] time = 0.341, size = 0, normalized size = 0.

$$\int e^{n\arctan(ax)}\left(a^2cx^2+c\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x)
```

```
[Out] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n\operatorname{atan}(ax)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(2/3),x)
```

```
[Out] Integral(exp(n*atan(a*x))/(c*(a**2*x**2 + 1))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(2/3), x)
```

3.363 $\int \frac{e^{n\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=123

$$\frac{3\,2^{-\frac{1}{3}-\frac{in}{2}}\sqrt[3]{a^2x^2+1}(1-iax)^{\frac{1}{6}(-2+3in)}\,{}_2F_1\left(\frac{1}{6}(3in-2),\frac{1}{6}(3in+8);\frac{1}{6}(3in+4);\frac{1}{2}(1-iax)\right)}{ac(3n+2i)\sqrt[3]{a^2cx^2+c}}$$

```
[Out] (3*2^(-1/3 - (I/2)*n)*(1 - I*a*x)^((-2 + (3*I)*n)/6)*(1 + a^2*x^2)^(1/3)*Hy
pergeometric2F1[(-2 + (3*I)*n)/6, (8 + (3*I)*n)/6, (4 + (3*I)*n)/6, (1 - I*
a*x)/2])/(a*c*(2*I + 3*n)*(c + a^2*c*x^2)^(1/3))
```

Rubi [A] time = 0.119131, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5076, 5073, 69}

$$\frac{3\,2^{-\frac{1}{3}-\frac{in}{2}}\sqrt[3]{a^2x^2+1}(1-iax)^{\frac{1}{6}(-2+3in)}\,{}_2F_1\left(\frac{1}{6}(3in-2),\frac{1}{6}(3in+8);\frac{1}{6}(3in+4);\frac{1}{2}(1-iax)\right)}{ac(3n+2i)\sqrt[3]{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(4/3),x]
```

```
[Out] (3*2^(-1/3 - (I/2)*n)*(1 - I*a*x)^((-2 + (3*I)*n)/6)*(1 + a^2*x^2)^(1/3)*Hy
pergeometric2F1[(-2 + (3*I)*n)/6, (8 + (3*I)*n)/6, (4 + (3*I)*n)/6, (1 - I*
a*x)/2])/(a*c*(2*I + 3*n)*(c + a^2*c*x^2)^(1/3))
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{4/3}}\,dx = \frac{\sqrt[3]{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{4/3}}\,dx}{c\sqrt[3]{c+a^2cx^2}}$$
$$= \frac{\sqrt[3]{1+a^2x^2}\int (1-iax)^{-\frac{4}{3}+\frac{in}{2}}(1+iax)^{-\frac{4}{3}-\frac{in}{2}}\,dx}{c\sqrt[3]{c+a^2cx^2}}$$
$$= \frac{3\,2^{-\frac{1}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(-2+3in)}\sqrt[3]{1+a^2x^2}\,{}_2F_1\left(\frac{1}{6}(-2+3in),\frac{1}{6}(8+3in);\frac{1}{6}(4+3in);\frac{1}{2}(1-iax)\right)}{ac(2i+3n)\sqrt[3]{c+a^2cx^2}}$$

Mathematica [A] time = 0.0533024, size = 123, normalized size = 1.

$$\frac{3\,2^{-\frac{1}{3}-\frac{in}{2}}\sqrt[3]{a^2x^2+1}(1-iax)^{-\frac{1}{3}+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}-\frac{1}{3},\frac{in}{2}+\frac{4}{3};\frac{in}{2}+\frac{2}{3};\frac{1}{2}-\frac{iax}{2}\right)}{ac(3n+2i)\sqrt[3]{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(4/3),x]
```

```
[Out] (3*2^(-1/3 - (I/2)*n)*(1 - I*a*x)^(-1/3 + (I/2)*n)*(1 + a^2*x^2)^(1/3)*Hype
rgeometric2F1[-1/3 + (I/2)*n, 4/3 + (I/2)*n, 2/3 + (I/2)*n, 1/2 - (I/2)*a*x
])/(a*c*(2*I + 3*n)*(c + a^2*c*x^2)^(1/3))
```

Maple [F] time = 0.266, size = 0, normalized size = 0.

$$\int e^{n\arctan(ax)}\left(a^2cx^2+c\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x)
```

```
[Out] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{2}cx^{2}+c\right)^{\frac{2}{3}}e^{(n\arctan(ax))}}{a^{4}c^{2}x^{4}+2\,a^{2}c^{2}x^{2}+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(2/3)*e^(n*arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2*x
^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{e^{n\operatorname{atan}(ax)}}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(4/3),x)
```

```
[Out] Integral(exp(n*atan(a*x))/(c*(a**2*x**2 + 1))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{e^{(n\arctan(ax))}}{\left(a^{2}cx^{2}+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(4/3), x)
```

3.364 $\int e^{n\tan^{-1}(ax)}x^m\left(c+a^2cx^2\right)dx$

Optimal. Leaf size=49

$$\frac{cx^{m+1}F_1\left(m+1;-\frac{in}{2}-1,\frac{in}{2}-1;m+2;iax,-iax\right)}{m+1}$$

```
[Out] (c*x^(1 + m)*AppellF1[1 + m, -1 - (I/2)*n, -1 + (I/2)*n, 2 + m, I*a*x, (-I)
*a*x])/(1 + m)
```

Rubi [A] time = 0.0681657, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5082, 133}

$$\frac{cx^{m+1}F_1\left(m+1;-\frac{in}{2}-1,\frac{in}{2}-1;m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*x^m*(c + a^2*c*x^2),x]
```

```
[Out] (c*x^(1 + m)*AppellF1[1 + m, -1 - (I/2)*n, -1 + (I/2)*n, 2 + m, I*a*x, (-I)
*a*x])/(1 + m)
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int e^{n\tan^{-1}(ax)}x^m\left(c+a^2cx^2\right)dx=c\int x^m(1-iax)^{1+\frac{in}{2}}(1+iax)^{1-\frac{in}{2}}\,dx$$
$$=\frac{cx^{1+m}F_1\left(1+m;-1-\frac{in}{2},-1+\frac{in}{2};2+m;iax,-iax\right)}{1+m}$$

Mathematica [F] time = 0.459059, size = 0, normalized size = 0.

$$\int e^{n\tan^{-1}(ax)}x^m\left(c+a^2cx^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTan[a*x])*x^m*(c + a^2*c*x^2),x]

[Out] Integrate[E^(n*ArcTan[a*x])*x^m*(c + a^2*c*x^2), x]
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int e^{n \arctan(ax)} x^m \left(a^2 c x^2 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x)

[Out] int(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 c x^2 + c\right) x^m e^{(n \arctan(ax))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x, algorithm="maxima")

[Out] integrate((a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2 c x^2 + c\right) x^m e^{(n \arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x, algorithm="fricas")

[Out] integral((a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**m*(a**2*c*x**2+c),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)x^m e^{(n \arctan(ax))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x)), x)
```

3.365 $\int \frac{e^{n \tan^{-1}(ax)} x^m}{c+a^2 c x^2} \, dx$

Optimal. Leaf size=51

$$\frac{x^{m+1} F_1\left(m+1;1-\frac{in}{2},\frac{in}{2}+1;m+2;iax,-iax\right)}{c(m+1)}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1 - (I/2)*n, 1 + (I/2)*n, 2 + m, I*a*x, (-I)*a*x
])/(c*(1 + m))
```

Rubi [A] time = 0.0923885, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5082, 133}

$$\frac{x^{m+1} F_1\left(m+1;1-\frac{in}{2},\frac{in}{2}+1;m+2;iax,-iax\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1 - (I/2)*n, 1 + (I/2)*n, 2 + m, I*a*x, (-I)*a*x
])/(c*(1 + m))
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n \tan^{-1}(ax)} x^m}{c+a^2 c x^2} \, dx &= \frac{\int x^m (1-iax)^{-1+\frac{in}{2}} (1+iax)^{-1-\frac{in}{2}} \, dx}{c}\\ &= \frac{x^{1+m} F_1\left(1+m;1-\frac{in}{2},1+\frac{in}{2};2+m;iax,-iax\right)}{c(1+m)}\end{aligned}$$

Mathematica [A] time = 0.152072, size = 96, normalized size = 1.88

$$\frac{x^m \left(1-e^{2i\tan^{-1}(ax)}\right)^{-m} \left(1+e^{2i\tan^{-1}(ax)}\right)^m e^{n\tan^{-1}(ax)} F_1\left(-\frac{in}{2};m,-m;1-\frac{in}{2};-e^{2i\tan^{-1}(ax)},e^{2i\tan^{-1}(ax)}\right)}{acn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2),x]
```

```
[Out] (E^(n*ArcTan[a*x])*(1 + E^((2*I)*ArcTan[a*x]))^m*x^m*AppellF1[(-I/2)*n, m,
-m, 1 - (I/2)*n, -E^((2*I)*ArcTan[a*x]), E^((2*I)*ArcTan[a*x])])/(a*c*(1 -
E^((2*I)*ArcTan[a*x]))^m*n)
```

Maple [F] time = 0.553, size = 0, normalized size = 0.

$$\int \frac{e^{n \arctan(ax)} x^m}{a^2 c x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m e^{(n \arctan(ax))}}{a^2 c x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m e^{(n \arctan(ax))}}{a^2 c x^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m e^{n \operatorname{atan}(ax)}}{a^2 x^2 + 1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**m*exp(n*atan(a*x))/(a**2*x**2 + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m e^{(n \arctan(ax))}}{a^2cx^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

3.366 $\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=51

$$\frac{x^{m+1}F_1\left(m+1;2-\frac{in}{2},\frac{in}{2}+2;m+2;iax,-iax\right)}{c^2(m+1)}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, 2 - (I/2)*n, 2 + (I/2)*n, 2 + m, I*a*x, (-I)*a*x
])/(c^2*(1 + m))
```

Rubi [A] time = 0.0897596, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5082, 133}

$$\frac{x^{m+1}F_1\left(m+1;2-\frac{in}{2},\frac{in}{2}+2;m+2;iax,-iax\right)}{c^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 2 - (I/2)*n, 2 + (I/2)*n, 2 + m, I*a*x, (-I)*a*x
])/(c^2*(1 + m))
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^2}\,dx &= \frac{\int x^m(1-iax)^{-2+\frac{in}{2}}(1+iax)^{-2-\frac{in}{2}}\,dx}{c^2}\\ &= \frac{x^{1+m}F_1\left(1+m;2-\frac{in}{2},2+\frac{in}{2};2+m;iax,-iax\right)}{c^2(1+m)}\end{aligned}$$

Mathematica [F] time = 0.444131, size = 0, normalized size = 0.

$$\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^2,x]
```

```
[Out] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^2, x]
```

Maple [F] time = 0.48, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{n \arctan(ax)} x^{m}}{\left(a^{2} c \,x^{2}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m} e^{\left(n \arctan\left(a x\right)\right)}}{{\left(a^{2} c x^{2} + c\right)}^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{x^{m} e^{\left(n \arctan\left(a x\right)\right)}}{a^{4} c^{2} x^{4} + 2 \, a^{2} c^{2} x^{2} + c^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*e^(n*arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m e^{(n \arctan(ax))}}{\left(a^2cx^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

3.367 $\int \frac{e^{n\tan^{-1}(ax)}x^m}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=51

$$\frac{x^{m+1}F_1\left(m+1;3-\frac{in}{2},\frac{in}{2}+3;m+2;iax,-iax\right)}{c^3(m+1)}$$

```
[Out] (x^(1 + m)*AppellF1[1 + m, 3 - (I/2)*n, 3 + (I/2)*n, 2 + m, I*a*x, (-I)*a*x
])/(c^3*(1 + m))
```

Rubi [A] time = 0.0891178, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5082, 133}

$$\frac{x^{m+1}F_1\left(m+1;3-\frac{in}{2},\frac{in}{2}+3;m+2;iax,-iax\right)}{c^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 3 - (I/2)*n, 3 + (I/2)*n, 2 + m, I*a*x, (-I)*a*x
])/(c^3*(1 + m))
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^3}\,dx &= \frac{\int x^m(1-iax)^{-3+\frac{in}{2}}(1+iax)^{-3-\frac{in}{2}}\,dx}{c^3}\\ &= \frac{x^{1+m}F_1\left(1+m;3-\frac{in}{2},3+\frac{in}{2};2+m;iax,-iax\right)}{c^3(1+m)}\end{aligned}$$

Mathematica [F] time = 0.678413, size = 0, normalized size = 0.

$$\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^3,x]
```

```
[Out] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^3, x]
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{e^{n\arctan(ax)}x^m}{(a^2cx^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m e^{(n\arctan(ax))}}{a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*e^(n*arctan(a*x))/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m e^{(n \arctan(ax))}}{\left(a^2 c x^2 + c\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

3.368 $\int \frac{e^{n\tan^{-1}(ax)}x^m}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{a^2x^2+1}x^{m+1}F_1\left(m+1;\frac{1}{2}(1-in),\frac{1}{2}(in+1);m+2;iax,-iax\right)}{(m+1)\sqrt{a^2cx^2+c}}$$

```
[Out] (x^(1 + m)*Sqrt[1 + a^2*x^2]*AppellF1[1 + m, (1 - I*n)/2, (1 + I*n)/2, 2 +
m, I*a*x, (-I)*a*x])/((1 + m)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.194392, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5085, 5082, 133}

$$\frac{\sqrt{a^2x^2+1}x^{m+1}F_1\left(m+1;\frac{1}{2}(1-in),\frac{1}{2}(in+1);m+2;iax,-iax\right)}{(m+1)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + a^2*x^2]*AppellF1[1 + m, (1 - I*n)/2, (1 + I*n)/2, 2 +
m, I*a*x, (-I)*a*x])/((1 + m)*Sqrt[c + a^2*c*x^2])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}x^m}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}x^m}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int x^m(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{x^{1+m}\sqrt{1+a^2x^2}F_1\left(1+m;\frac{1}{2}(1-in),\frac{1}{2}(1+in);2+m;iax,-iax\right)}{(1+m)\sqrt{c+a^2cx^2}}$$

Mathematica [F] time = 0.330356, size = 0, normalized size = 0.

$$\int \frac{e^{n\tan^{-1}(ax)}x^m}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(E^(n*ArcTan[a*x])*x^m)/Sqrt[c + a^2*c*x^2], x]
```

Maple [F] time = 0.309, size = 0, normalized size = 0.

$$\int e^{n\arctan(ax)}x^m\frac{1}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

[Out] integral(x^m*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**(1/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")

[Out] integrate(x^m*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.369 $\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{a^2x^2+1}x^{m+1}F_1\left(m+1;\frac{1}{2}(3-in),\frac{1}{2}(in+3);m+2;iax,-iax\right)}{c(m+1)\sqrt{a^2cx^2+c}}$$

```
[Out] (x^(1 + m)*Sqrt[1 + a^2*x^2]*AppellF1[1 + m, (3 - I*n)/2, (3 + I*n)/2, 2 +
m, I*a*x, (-I)*a*x])/(c*(1 + m)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.211071, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5085, 5082, 133}

$$\frac{\sqrt{a^2x^2+1}x^{m+1}F_1\left(m+1;\frac{1}{2}(3-in),\frac{1}{2}(in+3);m+2;iax,-iax\right)}{c(m+1)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + a^2*x^2]*AppellF1[1 + m, (3 - I*n)/2, (3 + I*n)/2, 2 +
m, I*a*x, (-I)*a*x])/(c*(1 + m)*Sqrt[c + a^2*c*x^2])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(1+a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int x^m(1-iax)^{-\frac{3}{2}+\frac{in}{2}}(1+iax)^{-\frac{3}{2}-\frac{in}{2}}\,dx}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{x^{1+m}\sqrt{1+a^2x^2}F_1\left(1+m;\frac{1}{2}(3-in),\frac{1}{2}(3+in);2+m;iax,-iax\right)}{c(1+m)\sqrt{c+a^2cx^2}}$$

Mathematica [F] time = 0.432636, size = 0, normalized size = 0.

$$\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(3/2), x]
```

Maple [F] time = 0.309, size = 0, normalized size = 0.

$$\int \mathrm{e}^{n\arctan(ax)}x^m\left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^m e^{(n\arctan(ax))}}{a^4c^2x^4+2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2
*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m e^{(n \arctan(ax))}}{\left(a^2cx^2 + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

3.370 $\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{a^2x^2+1}\,x^{m+1}F_1\left(m+1;\frac{1}{2}(5-in),\frac{1}{2}(in+5);m+2;iax,-iax\right)}{c^2(m+1)\sqrt{a^2cx^2+c}}$$

```
[Out] (x^(1 + m)*Sqrt[1 + a^2*x^2]*AppellF1[1 + m, (5 - I*n)/2, (5 + I*n)/2, 2 +
m, I*a*x, (-I)*a*x])/(c^2*(1 + m)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.214208, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5085, 5082, 133}

$$\frac{\sqrt{a^2x^2+1}\,x^{m+1}F_1\left(m+1;\frac{1}{2}(5-in),\frac{1}{2}(in+5);m+2;iax,-iax\right)}{c^2(m+1)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + a^2*x^2]*AppellF1[1 + m, (5 - I*n)/2, (5 + I*n)/2, 2 +
m, I*a*x, (-I)*a*x])/(c^2*(1 + m)*Sqrt[c + a^2*c*x^2])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(1+a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int x^m(1-iax)^{-\frac{5}{2}+\frac{in}{2}}(1+iax)^{-\frac{5}{2}-\frac{in}{2}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{x^{1+m}\sqrt{1+a^2x^2}F_1\left(1+m;\frac{1}{2}(5-in),\frac{1}{2}(5+in);2+m;iax,-iax\right)}{c^2(1+m)\sqrt{c+a^2cx^2}}$$

Mathematica [F] time = 0.570156, size = 0, normalized size = 0.

$$\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(5/2), x]
```

Maple [F] time = 0.317, size = 0, normalized size = 0.

$$\int e^{n\arctan(ax)}x^m\left(a^2cx^2+c\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^m e^{(n\arctan(ax))}}{a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x))/(a^6*c^3*x^6 + 3*a^4*c^3
*x^4 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m e^{(n \arctan(ax))}}{\left(a^2cx^2 + c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

3.371 $\int e^{n\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx$

Optimal. Leaf size=115

$$\frac{2^{-\frac{in}{2}+p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p(1-iax)^{\frac{in}{2}+p+1}\,{}_2F_1\left(\frac{in}{2}-p,\frac{in}{2}+p+1;\frac{in}{2}+p+2;\frac{1}{2}(1-iax)\right)}{a(n-2i(p+1))}$$

```
[Out] (2^(1 - (I/2)*n + p)*(1 - I*a*x)^(1 + (I/2)*n + p)*(c + a^2*c*x^2)^p*Hyperg
eometric2F1[(I/2)*n - p, 1 + (I/2)*n + p, 2 + (I/2)*n + p, (1 - I*a*x)/2])/
(a*(n - (2*I)*(1 + p))*(1 + a^2*x^2)^p)
```

Rubi [A] time = 0.0867331, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5076, 5073, 69}

$$\frac{2^{-\frac{in}{2}+p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p(1-iax)^{\frac{in}{2}+p+1}\,{}_2F_1\left(\frac{in}{2}-p,\frac{in}{2}+p+1;\frac{in}{2}+p+2;\frac{1}{2}(1-iax)\right)}{a(n-2i(p+1))}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

```
[Out] (2^(1 - (I/2)*n + p)*(1 - I*a*x)^(1 + (I/2)*n + p)*(c + a^2*c*x^2)^p*Hyperg
eometric2F1[(I/2)*n - p, 1 + (I/2)*n + p, 2 + (I/2)*n + p, (1 - I*a*x)/2])/
(a*(n - (2*I)*(1 + p))*(1 + a^2*x^2)^p)
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int e^{n\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx = \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{n\tan^{-1}(ax)}\left(1+a^2x^2\right)^p\,dx$$

$$= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int (1-iax)^{\frac{in}{2}+p}(1+iax)^{-\frac{in}{2}+p}\,dx$$

$$= \frac{2^{1-\frac{in}{2}+p}(1-iax)^{1+\frac{in}{2}+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\,{}_2F_1\left(\frac{in}{2}-p,1+\frac{in}{2}+p;2+\frac{in}{2}+p;\frac{1}{2}(1-\right.}{a(n-2i(1+p))}$$

Mathematica [A] time = 0.0320095, size = 115, normalized size = 1.

$$\frac{2^{-\frac{in}{2}+p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p(1-iax)^{\frac{in}{2}+p+1}\,{}_2F_1\left(\frac{in}{2}-p,\frac{in}{2}+p+1;\frac{in}{2}+p+2;\frac{1}{2}(1-iax)\right)}{a(n-2i(p+1))}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

```
[Out] (2^(1 - (I/2)*n + p)*(1 - I*a*x)^(1 + (I/2)*n + p)*(c + a^2*c*x^2)^p*Hyperg
eometric2F1[(I/2)*n - p, 1 + (I/2)*n + p, 2 + (I/2)*n + p, (1 - I*a*x)/2])/
(a*(n - (2*I)*(1 + p))*(1 + a^2*x^2)^p)
```

Maple [F] time = 0.31, size = 0, normalized size = 0.

$$\int e^{n\arctan(ax)}\left(a^2cx^2+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

```
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^p e^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^p*e^(n*arctan(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^2+c\right)^p e^{(n\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^p*e^(n*arctan(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\left(a^2x^2+1\right)\right)^p e^{n\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**p,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**p*exp(n*atan(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^p e^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^p*e^(n*arctan(a*x)), x)
```

3.372 $\int e^{-2ip\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx$

Optimal. Leaf size=53

$$\frac{i(1-iax)^{2p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p}{a(2p+1)}$$

```
[Out] (I*(1 - I*a*x)^(1 + 2*p)*(c + a^2*c*x^2)^p)/(a*(1 + 2*p)*(1 + a^2*x^2)^p)
```

Rubi [A] time = 0.0649463, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5076, 5073, 32}

$$\frac{i(1-iax)^{2p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^p/E^((2*I)*p*ArcTan[a*x]),x]
```

```
[Out] (I*(1 - I*a*x)^(1 + 2*p)*(c + a^2*c*x^2)^p)/(a*(1 + 2*p)*(1 + a^2*x^2)^p)
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{-2ip\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx&=\left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{-2ip\tan^{-1}(ax)}\left(1+a^2x^2\right)^p\,dx\\&=\left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int(1-iax)^{2p}\,dx\\&=\frac{i(1-iax)^{1+2p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p}{a(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.0283349, size = 39, normalized size = 0.74

$$\frac{(ax+i)\left(a^2cx^2+c\right)^p e^{-2ip\tan^{-1}(ax)}}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^p/E^((2*I)*p*ArcTan[a*x]),x]
```

```
[Out] ((I + a*x)*(c + a^2*c*x^2)^p)/(E^((2*I)*p*ArcTan[a*x])*(a + 2*a*p))
```

Maple [A] time = 0.046, size = 41, normalized size = 0.8

$$\frac{(ax+i)\left(a^2cx^2+c\right)^p}{a\left(1+2p\right)\mathrm{e}^{2ip\arctan(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x)),x)
```

```
[Out] (a*x+I)/a/(1+2*p)*(a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x))
```

Maxima [A] time = 1.0347, size = 103, normalized size = 1.94

$$\frac{(ac^px+ic^p)\left(a^2x^2+1\right)^p\cos\left(2\,p\arctan\left(ax\right)\right)-(i\,ac^px-c^p)\left(a^2x^2+1\right)^p\sin\left(2\,p\arctan\left(ax\right)\right)}{2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x)),x, algorithm="maxima")
```

```
[Out] ((a*c^p*x + I*c^p)*(a^2*x^2 + 1)^p*cos(2*p*arctan(a*x)) - (I*a*c^p*x - c^p)
*(a^2*x^2 + 1)^p*sin(2*p*arctan(a*x)))/(2*a*p + a)
```

Fricas [A] time = 2.28301, size = 89, normalized size = 1.68

$$\frac{(ax+i)\left(a^2cx^2+c\right)^p\left(-\frac{ax+i}{ax-i}\right)^p}{2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x)),x, algorithm="fricas")
```

```
[Out] (a*x + I)*(a^2*c*x^2 + c)^p*(-(a*x + I)/(a*x - I))^p/(2*a*p + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**p/exp(2*I*p*atan(a*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1315, size = 74, normalized size = 1.4

$$\frac{axe^{\left(-\pi ip+2\,p\log(ax+i)+p\log(c)\right)}+ie^{\left(-\pi ip+2\,p\log(ax+i)+p\log(c)\right)}}{2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x)),x, algorithm="giac")
```

```
[Out] (a*x*e^(-pi*i*p + 2*p*log(a*x + i) + p*log(c)) + i*e^(-pi*i*p + 2*p*log(a*x
 + i) + p*log(c)))/(2*a*p + a)
```

3.373 $\int e^{2ip\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx$

Optimal. Leaf size=53

$$-\frac{i(1+iax)^{2p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p}{a(2p+1)}$$

```
[Out] ((-I)*(1 + I*a*x)^(1 + 2*p)*(c + a^2*c*x^2)^p)/(a*(1 + 2*p)*(1 + a^2*x^2)^p
)
```

Rubi [A] time = 0.0610917, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5076, 5073, 32}

$$-\frac{i(1+iax)^{2p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*p*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

```
[Out] ((-I)*(1 + I*a*x)^(1 + 2*p)*(c + a^2*c*x^2)^p)/(a*(1 + 2*p)*(1 + a^2*x^2)^p
)
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{2ip\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{2ip\tan^{-1}(ax)}\left(1+a^2x^2\right)^p\,dx\\ &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int (1+iax)^{2p}\,dx\\ &= -\frac{i(1+iax)^{1+2p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p}{a(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.0248986, size = 39, normalized size = 0.74

$$\frac{(ax-i)\left(a^2cx^2+c\right)^p e^{2ip\tan^{-1}(ax)}}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*p*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

```
[Out] (E^((2*I)*p*ArcTan[a*x])*(-I + a*x)*(c + a^2*c*x^2)^p)/(a + 2*a*p)
```

Maple [A] time = 0.046, size = 41, normalized size = 0.8

$$-\frac{(-ax+i)\,\mathrm{e}^{2ip\arctan(ax)}\left(a^2cx^2+c\right)^p}{a\left(1+2p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

```
[Out] -(-a*x+I)/a/(1+2*p)*exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^p e^{\left(2ip\arctan(ax)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^p*e^(2*I*p*arctan(a*x)), x)
```

Fricas [A] time = 2.21179, size = 92, normalized size = 1.74

$$\frac{(ax-i)\left(a^2cx^2+c\right)^p}{\left(2\,ap+a\right)\left(-\frac{ax+i}{ax-i}\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] (a*x - I)*(a^2*c*x^2 + c)^p/((2*a*p + a)*(-(a*x + I)/(a*x - I))^p)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*I*p*atan(a*x))*(a**2*c*x**2+c)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14425, size = 78, normalized size = 1.47

$$\frac{axe^{\left(\pi ip+2\,p\log(ax-i)+p\log(c)\right)}-ie^{\left(\pi ip+2\,p\log(ax-i)+p\log(c)\right)}}{2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] (a*x*e^(pi*i*p + 2*p*log(a*x - i) + p*log(c)) - i*e^(pi*i*p + 2*p*log(a*x -
 i) + p*log(c)))/(2*a*p + a)
```

3.374 $\int e^{in\tan^{-1}(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx$

Optimal. Leaf size=60

$$\frac{i(1-ianx)\left(a^2cx^2+c\right)^{-\frac{n^2}{2}}e^{in\tan^{-1}(ax)}}{a^3cn\left(1-n^2\right)}$$

```
[Out] (I*E^(I*n*ArcTan[a*x])*(1 - I*a*n*x))/(a^3*c*n*(1 - n^2)*(c + a^2*c*x^2)^(n
^2/2))
```

Rubi [A] time = 0.111918, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {5079}

$$\frac{i(1-ianx)\left(a^2cx^2+c\right)^{-\frac{n^2}{2}}e^{in\tan^{-1}(ax)}}{a^3cn\left(1-n^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])*x^2*(c + a^2*c*x^2)^(-1 - n^2/2),x]
```

```
[Out] (I*E^(I*n*ArcTan[a*x])*(1 - I*a*n*x))/(a^3*c*n*(1 - n^2)*(c + a^2*c*x^2)^(n
^2/2))
```

Rule 5079

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> -Simp[((1 - a*n*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*d*n*(n^
2 + 1)), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && EqQ[n^2 - 2*(p +
1), 0] &&  !IntegerQ[I*n]
```

Rubi steps

$$\int e^{in\tan^{-1}(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=\frac{ie^{in\tan^{-1}(ax)}(1-ianx)\left(c+a^2cx^2\right)^{-\frac{n^2}{2}}}{a^3cn\left(1-n^2\right)}$$

Mathematica [A] time = 0.024067, size = 55, normalized size = 0.92

$$-\frac{(anx+i)\left(a^2cx^2+c\right)^{-\frac{n^2}{2}}e^{in\tan^{-1}(ax)}}{a^3cn\left(n^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*n*ArcTan[a*x])*x^2*(c + a^2*c*x^2)^(-1 - n^2/2),x]
```

```
[Out] -((E^(I*n*ArcTan[a*x])*(I + a*n*x))/(a^3*c*n*(-1 + n^2)*(c + a^2*c*x^2)^(n^
2/2)))
```

Maple [A] time = 0.06, size = 62, normalized size = 1.

$$\frac{(-ax+i)\,(ax+i)\,(nax+i)\,\mathrm{e}^{in\arctan(ax)}}{na^3\,(n^2-1)}\left(a^2cx^2+c\right)^{-1-\frac{n^2}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x)
```

```
[Out] (-a*x+I)*(a*x+I)*(n*a*x+I)*exp(I*n*arctan(a*x))*(a^2*c*x^2+c)^(-1-1/2*n^2)/
n/a^3/(n^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{-\frac{1}{2}\,n^2-1}x^2e^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm=
"maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(-1/2*n^2 - 1)*x^2*e^(I*n*arctan(a*x)), x)
```

Fricas [A] time = 2.24294, size = 162, normalized size = 2.7

$$-\frac{\left(a^3nx^3+i\,a^2x^2+anx+i\right)\left(a^2cx^2+c\right)^{-\frac{1}{2}\,n^2-1}}{\left(a^3n^3-a^3n\right)\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm=
"fricas")
```

```
[Out] -(a^3*n*x^3 + I*a^2*x^2 + a*n*x + I)*(a^2*c*x^2 + c)^(-1/2*n^2 - 1)/((a^3*n
^3 - a^3*n)*(-(a*x + I)/(a*x - I))^(1/2*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))*x**2*(a**2*c*x**2+c)**(-1-1/2*n**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14228, size = 487, normalized size = 8.12

$$-\frac{a^3nx^3e^{\left(\frac{1}{2}\pi in-\frac{1}{2}n^2\log(ax+i)-\frac{1}{2}n^2\log(ax-i)-\frac{1}{2}n^2\log(c)-\frac{1}{2}n\log(ax+i)+\frac{1}{2}n\log(ax-i)-\log(ax+i)-\log(ax-i)-\log(c)\right)}+a^2ix^2e^{\left(\frac{1}{2}\pi in-\frac{1}{2}n^2\log(ax+i)-\frac{1}{2}\cdots\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm=
"giac")
```

```
[Out] -(a^3*n*x^3*e^(1/2*pi*i*n - 1/2*n^2*log(a*x + i) - 1/2*n^2*log(a*x - i) - 1
/2*n^2*log(c) - 1/2*n*log(a*x + i) + 1/2*n*log(a*x - i) - log(a*x + i) - lo
g(a*x - i) - log(c)) + a^2*i*x^2*e^(1/2*pi*i*n - 1/2*n^2*log(a*x + i) - 1/2
*n^2*log(a*x - i) - 1/2*n^2*log(c) - 1/2*n*log(a*x + i) + 1/2*n*log(a*x - i
) - log(a*x + i) - log(a*x - i) - log(c)) + a*n*x*e^(1/2*pi*i*n - 1/2*n^2*l
og(a*x + i) - 1/2*n^2*log(a*x - i) - 1/2*n^2*log(c) - 1/2*n*log(a*x + i) +
1/2*n*log(a*x - i) - log(a*x + i) - log(a*x - i) - log(c)) + i*e^(1/2*pi*i*
n - 1/2*n^2*log(a*x + i) - 1/2*n^2*log(a*x - i) - 1/2*n^2*log(c) - 1/2*n*lo
g(a*x + i) + 1/2*n*log(a*x - i) - log(a*x + i) - log(a*x - i) - log(c)))/(a
^3*n^3 - a^3*n)
```

3.375 $\int \frac{e^{6i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{19}}\,dx$

Optimal. Leaf size=38

$$-\frac{6ax+i}{210a^3c^{19}(1-iax)^{21}(1+iax)^{15}}$$

```
[Out] -(I + 6*a*x)/(210*a^3*c^19*(1 - I*a*x)^21*(1 + I*a*x)^15)
```

Rubi [A] time = 0.0785909, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5082, 81}

$$-\frac{6ax+i}{210a^3c^{19}(1-iax)^{21}(1+iax)^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^((6*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^19,x]

[Out] -(I + 6*a*x)/(210*a^3*c^19*(1 - I*a*x)^21*(1 + I*a*x)^15)
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{6i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{19}}\,dx &= \frac{\int \frac{x^2}{(1-iax)^{22}(1+iax)^{16}}\,dx}{c^{19}}\\ &= -\frac{i+6ax}{210a^3c^{19}(1-iax)^{21}(1+iax)^{15}}\end{aligned}$$

Mathematica [A] time = 1.05516, size = 36, normalized size = 0.95

$$\frac{6ax+i}{210a^3c^{19}(ax-i)^{15}(ax+i)^{21}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^((6*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^19,x]
```

```
[Out] (I + 6*a*x)/(210*a^3*c^19*(-I + a*x)^15*(I + a*x)^21)
```

Maple [A] time = 0.342, size = 34, normalized size = 0.9

$$\frac{1}{c^{19}\left(ax+i\right)^{21}\left(ax-i\right)^{15}}\left(\frac{x}{35\,a^{2}}+\frac{\frac{i}{210}}{a^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x)
```

```
[Out] 1/c^19*(1/35*x/a^2+1/210*I/a^3)/(a*x+I)^21/(a*x-I)^15
```

Maxima [B] time = 2.0134, size = 394, normalized size = 10.37

$$\frac{20775821962641408\cdots}{727153768692449280\left(a^{45}c^{19}x^{42}+21\,a^{43}c^{19}x^{40}+210\,a^{41}c^{19}x^{38}+1330\,a^{39}c^{19}x^{36}+5985\,a^{37}c^{19}x^{34}+20349\,a^{35}c^{19}x^{32}+5\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x, algorithm="maxi
ma")
```

```
[Out] 1/727153768692449280*(20775821962641408*a^7*x^7 - 121192294782074880*I*a^6*
x^6 - 290861507476979712*a^5*x^5 + 363576884346224640*I*a^4*x^4 + 242384589
564149760*a^3*x^3 - 72715376869244928*I*a^2*x^2 - 3462636993773568*I)/(a^45
*c^19*x^42 + 21*a^43*c^19*x^40 + 210*a^41*c^19*x^38 + 1330*a^39*c^19*x^36 +
 5985*a^37*c^19*x^34 + 20349*a^35*c^19*x^32 + 54264*a^33*c^19*x^30 + 116280
*a^31*c^19*x^28 + 203490*a^29*c^19*x^26 + 293930*a^27*c^19*x^24 + 352716*a^
25*c^19*x^22 + 352716*a^23*c^19*x^20 + 293930*a^21*c^19*x^18 + 203490*a^19*
c^19*x^16 + 116280*a^17*c^19*x^14 + 54264*a^15*c^19*x^12 + 20349*a^13*c^19*
x^10 + 5985*a^11*c^19*x^8 + 1330*a^9*c^19*x^6 + 210*a^7*c^19*x^4 + 21*a^5*c
^19*x^2 + a^3*c^19)
```

Fricas [B] time = 10.8487, size = 1139, normalized size = 29.97

$$\frac{\cdots}{210\,a^{39}c^{19}x^{36}+1260i\,a^{38}c^{19}x^{35}+14700i\,a^{36}c^{19}x^{33}-22050\,a^{35}c^{19}x^{32}+70560i\,a^{34}c^{19}x^{31}-188160\,a^{33}c^{19}x^{30}+151200i\,a^{3}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x, algorithm="fric
as")
```

```
[Out] (6*a*x + I)/(210*a^39*c^19*x^36 + 1260*I*a^38*c^19*x^35 + 14700*I*a^36*c^19
*x^33 - 22050*a^35*c^19*x^32 + 70560*I*a^34*c^19*x^31 - 188160*a^33*c^19*x^
30 + 151200*I*a^32*c^19*x^29 - 819000*a^31*c^19*x^28 - 58800*I*a^30*c^19*x^
27 - 2257920*a^29*c^19*x^26 - 1375920*I*a^28*c^19*x^25 - 4204200*a^27*c^19*
x^24 - 4586400*I*a^26*c^19*x^23 - 5241600*a^25*c^19*x^22 - 9129120*I*a^24*c
```

```
^19*x^21 - 3783780*a^23*c^19*x^20 - 12612600*I*a^22*c^19*x^19 - 12612600*I*
a^20*c^19*x^17 + 3783780*a^19*c^19*x^16 - 9129120*I*a^18*c^19*x^15 + 524160
0*a^17*c^19*x^14 - 4586400*I*a^16*c^19*x^13 + 4204200*a^15*c^19*x^12 - 1375
920*I*a^14*c^19*x^11 + 2257920*a^13*c^19*x^10 - 58800*I*a^12*c^19*x^9 + 819
000*a^11*c^19*x^8 + 151200*I*a^10*c^19*x^7 + 188160*a^9*c^19*x^6 + 70560*I*
a^8*c^19*x^5 + 22050*a^7*c^19*x^4 + 14700*I*a^6*c^19*x^3 + 1260*I*a^4*c^19*
x - 210*a^3*c^19)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**6/(a**2*x**2+1)**3*x**2/(a**2*c*x**2+c)**19,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13747, size = 429, normalized size = 11.29

$$-\frac{358229025\,a^{14}x^{14} - 5340869100\,a^{13}ix^{13} - 37114698075\,a^{12}x^{12} + 159416118225\,a^{11}ix^{11} + 473088806190\,a^{10}x^{10} - 10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x, algorithm="giac
")
```

```
[Out] -1/901943132160*(358229025*a^14*x^14 - 5340869100*a^13*i*x^13 - 37114698075
*a^12*x^12 + 159416118225*a^11*i*x^11 + 473088806190*a^10*x^10 - 1026819468
675*a^9*i*x^9 - 1682288472150*a^8*x^8 + 2115551402250*a^7*i*x^7 + 205443504
6125*a^6*x^6 - 1535397250002*a^5*i*x^5 - 870854759775*a^4*x^4 + 36430753320
5*a^3*i*x^3 + 106553746740*a^2*x^2 - 19571887695*a*i*x - 1710785408)/((a*x
- i)^15*a^3*c^19) + 1/901943132160*(358229025*a^20*x^20 + 7555375800*a^19*i
*x^19 - 75901131600*a^18*x^18 - 483051354975*a^17*i*x^17 + 2184946607340*a^
16*x^16 + 7469205450840*a^15*i*x^15 - 20031221295000*a^14*x^14 - 4317700403
7300*a^13*i*x^13 + 76013078916950*a^12*x^12 + 110448380006328*a^11*i*x^11 -
 133277726128008*a^10*x^10 - 133908931763530*a^9*i*x^9 + 111933156213900*a^
8*x^8 + 77492989590120*a^7*i*x^7 - 44041557267624*a^6*x^6 - 20244576347604*
a^5*i*x^5 + 7349182966545*a^4*x^4 + 2026362494800*a^3*i*x^3 - 396520754280*
a^2*x^2 - 48177926223*a*i*x + 2584181888)/((a*x + i)^21*a^3*c^19)
```

3.376 $\int \frac{e^{4i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^9}\,dx$

Optimal. Leaf size=38

$$-\frac{4ax+i}{60a^3c^9(1-iax)^{10}(1+iax)^6}$$

```
[Out] -(I + 4*a*x)/(60*a^3*c^9*(1 - I*a*x)^10*(1 + I*a*x)^6)
```

Rubi [A] time = 0.0775599, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5082, 81}

$$-\frac{4ax+i}{60a^3c^9(1-iax)^{10}(1+iax)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(E^((4*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^9,x]

[Out] -(I + 4*a*x)/(60*a^3*c^9*(1 - I*a*x)^10*(1 + I*a*x)^6)
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^9}\,dx &= \frac{\int \frac{x^2}{(1-iax)^{11}(1+iax)^7}\,dx}{c^9}\\
&= -\frac{i+4ax}{60a^3c^9(1-iax)^{10}(1+iax)^6}
\end{aligned}$$

Mathematica [A] time = 0.19339, size = 36, normalized size = 0.95

$$-\frac{4ax+i}{60a^3c^9(ax-i)^6(ax+i)^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^((4*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^9,x]
```

```
[Out] -(I + 4*a*x)/(60*a^3*c^9*(-I + a*x)^6*(I + a*x)^10)
```

Maple [A] time = 0.13, size = 35, normalized size = 0.9

$$-\frac{1}{c^{9}(ax+i)^{10}(ax-i)^{6}}\left(\frac{\frac{i}{60}}{a^{3}}+\frac{x}{15\,a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x)
```

```
[Out] -1/c^9*(1/60*I/a^3+1/15*x/a^2)/(a*x+I)^10/(a*x-I)^6
```

Maxima [B] time = 1.58723, size = 209, normalized size = 5.5

$$-\frac{5505024\,a^{5}x^{5}-20643840i\,a^{4}x^{4}-27525120\,a^{3}x^{3}+13762560i\,a^{2}x^{2}+1376256i}{82575360\left(a^{23}c^{9}x^{20}+10\,a^{21}c^{9}x^{18}+45\,a^{19}c^{9}x^{16}+120\,a^{17}c^{9}x^{14}+210\,a^{15}c^{9}x^{12}+252\,a^{13}c^{9}x^{10}+210\,a^{11}c^{9}x^{8}+120\,a^{9}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x, algorithm="maxim
a")
```

```
[Out] -1/82575360*(5505024*a^5*x^5 - 20643840*I*a^4*x^4 - 27525120*a^3*x^3 + 1376
2560*I*a^2*x^2 + 1376256*I)/(a^23*c^9*x^20 + 10*a^21*c^9*x^18 + 45*a^19*c^9
*x^16 + 120*a^17*c^9*x^14 + 210*a^15*c^9*x^12 + 252*a^13*c^9*x^10 + 210*a^1
1*c^9*x^8 + 120*a^9*c^9*x^6 + 45*a^7*c^9*x^4 + 10*a^5*c^9*x^2 + a^3*c^9)
```

Fricas [B] time = 2.49146, size = 429, normalized size = 11.29

$$-\frac{4\,ax+i}{60\,a^{19}c^{9}x^{16}+240i\,a^{18}c^{9}x^{15}+1200i\,a^{16}c^{9}x^{13}-1200\,a^{15}c^{9}x^{12}+2160i\,a^{14}c^{9}x^{11}-3840\,a^{13}c^{9}x^{10}+1200i\,a^{12}c^{9}x^{9}-540}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x, algorithm="frica
s")
```

```
[Out] -(4*a*x + I)/(60*a^19*c^9*x^16 + 240*I*a^18*c^9*x^15 + 1200*I*a^16*c^9*x^13
 - 1200*a^15*c^9*x^12 + 2160*I*a^14*c^9*x^11 - 3840*a^13*c^9*x^10 + 1200*I*
a^12*c^9*x^9 - 5400*a^11*c^9*x^8 - 1200*I*a^10*c^9*x^7 - 3840*a^9*c^9*x^6 -
 2160*I*a^8*c^9*x^5 - 1200*a^7*c^9*x^4 - 1200*I*a^6*c^9*x^3 - 240*I*a^4*c^9
*x + 60*a^3*c^9)
```

Sympy [B] time = 111.484, size = 202, normalized size = 5.32

$$-\frac{a^{4}\left(4a^{1}\right.}{60a^{143}c^{9}x^{16}+240ia^{142}c^{9}x^{15}+1200ia^{140}c^{9}x^{13}-1200a^{139}c^{9}x^{12}+2160ia^{138}c^{9}x^{11}-3840a^{137}c^{9}x^{10}+1200ia^{136}c^{9}x^{9}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x**2/(a**2*c*x**2+c)**9,x)
```

```
[Out] -a**4*(4*a**121*x + I*a**120)/(60*a**143*c**9*x**16 + 240*I*a**142*c**9*x**
15 + 1200*I*a**140*c**9*x**13 - 1200*a**139*c**9*x**12 + 2160*I*a**138*c**9
*x**11 - 3840*a**137*c**9*x**10 + 1200*I*a**136*c**9*x**9 - 5400*a**135*c**
9*x**8 - 1200*I*a**134*c**9*x**7 - 3840*a**133*c**9*x**6 - 2160*I*a**132*c*
*9*x**5 - 1200*a**131*c**9*x**4 - 1200*I*a**130*c**9*x**3 - 240*I*a**128*c*
*9*x + 60*a**127*c**9)
```

Giac [B] time = 1.14383, size = 204, normalized size = 5.37

$$-\frac{2145\,a^5x^5-12540\,a^4ix^4-30030\,a^3x^3+37080\,a^2ix^2+23841\,ax-6476\,i}{983040\,(ax-i)^6a^3c^9}+\frac{2145\,a^9x^9+21780\,a^8ix^8-99660\,a^7x^7-27}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x, algorithm="giac"
)
```

```
[Out] -1/983040*(2145*a^5*x^5 - 12540*a^4*i*x^4 - 30030*a^3*x^3 + 37080*a^2*i*x^2
 + 23841*a*x - 6476*i)/((a*x - i)^6*a^3*c^9) + 1/983040*(2145*a^9*x^9 + 217
80*a^8*i*x^8 - 99660*a^7*x^7 - 270480*a^6*i*x^6 + 481446*a^5*x^5 + 584920*a
^4*i*x^4 - 486220*a^3*x^3 - 265680*a^2*i*x^2 + 84065*a*x + 9908*i)/((a*x +
i)^10*a^3*c^9)
```

3.377 $\int \frac{e^{2i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=38

$$-\frac{2ax+i}{6a^3c^3(1-iax)^3(1+iax)}$$

```
[Out] -(I + 2*a*x)/(6*a^3*c^3*(1 - I*a*x)^3*(1 + I*a*x))
```

Rubi [A] time = 0.0767827, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5082, 81}

$$-\frac{2ax+i}{6a^3c^3(1-iax)^3(1+iax)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^((2*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] -(I + 2*a*x)/(6*a^3*c^3*(1 - I*a*x)^3*(1 + I*a*x))
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^3}\,dx &= \frac{\int \frac{x^2}{(1-iax)^4(1+iax)^2}\,dx}{c^3}\\ &= -\frac{i+2ax}{6a^3c^3(1-iax)^3(1+iax)}\end{aligned}$$

Mathematica [A] time = 0.0309703, size = 36, normalized size = 0.95

$$\frac{2ax+i}{6a^3c^3(ax-i)(ax+i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^((2*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (I + 2*a*x)/(6*a^3*c^3*(-I + a*x)*(I + a*x)^3)
```

Maple [A] time = 0.051, size = 34, normalized size = 0.9

$$\frac{1}{c^3 (ax+i)^3 (ax-i)} \left(\frac{x}{3a^2} + \frac{\frac{i}{6}}{a^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/c^3*(1/3*x/a^2+1/6*I/a^3)/(a*x+I)^3/(a*x-I)
```

Maxima [B] time = 1.50358, size = 84, normalized size = 2.21

$$\frac{16\,a^3x^3 - 24i\,a^2x^2 - 8i}{48\left(a^9c^3x^6 + 3\,a^7c^3x^4 + 3\,a^5c^3x^2 + a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] 1/48*(16*a^3*x^3 - 24*I*a^2*x^2 - 8*I)/(a^9*c^3*x^6 + 3*a^7*c^3*x^4 + 3*a^5
*c^3*x^2 + a^3*c^3)
```

Fricas [A] time = 2.02252, size = 104, normalized size = 2.74

$$\frac{2\,ax + i}{6\,a^7c^3x^4 + 12i\,a^6c^3x^3 + 12i\,a^4c^3x - 6\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] (2*a*x + I)/(6*a^7*c^3*x^4 + 12*I*a^6*c^3*x^3 + 12*I*a^4*c^3*x - 6*a^3*c^3)
```

Sympy [A] time = 0.859227, size = 60, normalized size = 1.58

$$\frac{a^2\left(2a^9x + ia^8\right)}{6a^{17}c^3x^4 + 12ia^{16}c^3x^3 + 12ia^{14}c^3x - 6a^{13}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)*x**2/(a**2*c*x**2+c)**3,x)
```

```
[Out] a**2*(2*a**9*x + I*a**8)/(6*a**17*c**3*x**4 + 12*I*a**16*c**3*x**3 + 12*I*a
**14*c**3*x - 6*a**13*c**3)
```

Giac [A] time = 1.15881, size = 65, normalized size = 1.71

$$-\frac{1}{16\,(ax-i)a^3c^3}+\frac{3\,a^2x^2+12\,aix-5}{48\,(ax+i)^3a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/16/((a*x - i)*a^3*c^3) + 1/48*(3*a^2*x^2 + 12*a*i*x - 5)/((a*x + i)^3*a^
3*c^3)
```

3.378 $\int \frac{e^{-2i\tan^{-1}(ax)}x^2}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=38

$$\frac{-2ax+i}{6a^3c^3(1-iax)(1+iax)^3}$$

```
[Out] (I - 2*a*x)/(6*a^3*c^3*(1 - I*a*x)*(1 + I*a*x)^3)
```

Rubi [A] time = 0.0774534, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5082, 81}

$$\frac{-2ax+i}{6a^3c^3(1-iax)(1+iax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^((2*I)*ArcTan[a*x])*(c + a^2*c*x^2)^3),x]

[Out] (I - 2*a*x)/(6*a^3*c^3*(1 - I*a*x)*(1 + I*a*x)^3)
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2i\tan^{-1}(ax)}x^2}{(c+a^2cx^2)^3}\,dx &= \frac{\int \frac{x^2}{(1-iax)^2(1+iax)^4}\,dx}{c^3}\\ &= \frac{i-2ax}{6a^3c^3(1-iax)(1+iax)^3}\end{aligned}$$

Mathematica [A] time = 0.032676, size = 36, normalized size = 0.95

$$\frac{2ax-i}{6a^3c^3(ax-i)^3(ax+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^((2*I)*ArcTan[a*x])*(c + a^2*c*x^2)^3),x]
```

```
[Out] (-I + 2*a*x)/(6*a^3*c^3*(-I + a*x)^3*(I + a*x))
```

Maple [A] time = 0.053, size = 62, normalized size = 1.6

$$\frac{1}{c^3}\left(\frac{-\frac{i}{8}}{a^3(-ax+i)^2}-\frac{1}{12\,a^3(-ax+i)^3}-\frac{1}{16\,a^3(-ax+i)}-\frac{1}{16\,a^3(ax+i)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/c^3*(-1/8*I/a^3/(-a*x+I)^2-1/12/a^3/(-a*x+I)^3-1/16/a^3/(-a*x+I)-1/16/a^3
/(a*x+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.13267, size = 104, normalized size = 2.74

$$\frac{2\,ax-i}{6\,a^7c^3x^4-12i\,a^6c^3x^3-12i\,a^4c^3x-6\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] (2*a*x - I)/(6*a^7*c^3*x^4 - 12*I*a^6*c^3*x^3 - 12*I*a^4*c^3*x - 6*a^3*c^3)
```

Sympy [A] time = 0.8945, size = 56, normalized size = 1.47

$$\frac{2a^9x-ia^8}{6a^{15}c^3x^4-12ia^{14}c^3x^3-12ia^{12}c^3x-6a^{11}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*a*x)**2*(a**2*x**2+1)/(a**2*c*x**2+c)**3,x)
```

```
[Out] (2*a**9*x - I*a**8)/(6*a**15*c**3*x**4 - 12*I*a**14*c**3*x**3 - 12*I*a**12*
c**3*x - 6*a**11*c**3)
```

Giac [B] time = 1.1234, size = 119, normalized size = 3.13

$$\frac{1}{32\,a^3c^3\left(i-\frac{2\,i}{aix+1}\right)}-\frac{\frac{6\,a^3c^6i^3}{(aix+1)^2}-\frac{3\,a^3c^6i}{aix+1}-\frac{4\,a^3c^6i^3}{(aix+1)^3}}{48\,a^6c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/32/(a^3*c^3*(i - 2*i/(a*i*x + 1))) - 1/48*(6*a^3*c^6*i^3/(a*i*x + 1)^2 -
3*a^3*c^6*i/(a*i*x + 1) - 4*a^3*c^6*i^3/(a*i*x + 1)^3)/(a^6*c^9)
```

3.379 $\int \frac{e^{-4i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^9}\,dx$

Optimal. Leaf size=38

$$\frac{-4ax+i}{60a^3c^9(1-iax)^6(1+iax)^{10}}$$

```
[Out] (I - 4*a*x)/(60*a^3*c^9*(1 - I*a*x)^6*(1 + I*a*x)^10)
```

Rubi [A] time = 0.0775632, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5082, 81}

$$\frac{-4ax+i}{60a^3c^9(1-iax)^6(1+iax)^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^((4*I)*ArcTan[a*x])*(c + a^2*c*x^2)^9),x]
```

```
[Out] (I - 4*a*x)/(60*a^3*c^9*(1 - I*a*x)^6*(1 + I*a*x)^10)
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-4i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^9}\,dx &= \frac{\int \frac{x^2}{(1-iax)^7(1+iax)^{11}}\,dx}{c^9}\\ &= \frac{i-4ax}{60a^3c^9(1-iax)^6(1+iax)^{10}}\end{aligned}$$

Mathematica [A] time = 0.194454, size = 36, normalized size = 0.95

$$\frac{-4ax+i}{60a^3c^9(ax-i)^{10}(ax+i)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^((4*I)*ArcTan[a*x])*(c + a^2*c*x^2)^9),x]
```

```
[Out] (I - 4*a*x)/(60*a^3*c^9*(-I + a*x)^10*(I + a*x)^6)
```

Maple [B] time = 0.086, size = 218, normalized size = 5.7

$$\frac{1}{c^9}\left(\frac{\frac{21\,i}{8192}}{a^3\,(-ax+i)^4}+\frac{\frac{i}{1280}}{a^3\,(-ax+i)^{10}}-\frac{\frac{i}{1024}}{a^3\,(-ax+i)^8}-\frac{\frac{7\,i}{6144}}{a^3\,(-ax+i)^6}-\frac{\frac{165\,i}{65536}}{a^3\,(-ax+i)^2}+\frac{1}{768\,a^3\,(-ax+i)^9}-\frac{21}{10240\,a^3\,(-ax+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x)
```

```
[Out] 1/c^9*(21/8192*I/a^3/(-a*x+I)^4+1/1280*I/a^3/(-a*x+I)^10-1/1024*I/a^3/(-a*x
+I)^8-7/6144*I/a^3/(-a*x+I)^6-165/65536*I/a^3/(-a*x+I)^2+1/768/a^3/(-a*x+I)
^9-21/10240/a^3/(-a*x+I)^5+11/4096/a^3/(-a*x+I)^3-143/65536/a^3/(-a*x+I)+13
/16384*I/a^3/(a*x+I)^4-1/12288*I/a^3/(a*x+I)^6-121/65536*I/a^3/(a*x+I)^2-7/
20480/a^3/(a*x+I)^5+11/8192/a^3/(a*x+I)^3-143/65536/a^3/(a*x+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.56486, size = 429, normalized size = 11.29

$$-\frac{4\,ax-i}{60\,a^{19}c^9x^{16}-240i\,a^{18}c^9x^{15}-1200i\,a^{16}c^9x^{13}-1200\,a^{15}c^9x^{12}-2160i\,a^{14}c^9x^{11}-3840\,a^{13}c^9x^{10}-1200i\,a^{12}c^9x^9-5400\,a^{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x, algorithm="frica
s")
```

```
[Out] -(4*a*x - I)/(60*a^19*c^9*x^16 - 240*I*a^18*c^9*x^15 - 1200*I*a^16*c^9*x^13
 - 1200*a^15*c^9*x^12 - 2160*I*a^14*c^9*x^11 - 3840*a^13*c^9*x^10 - 1200*I*
a^12*c^9*x^9 - 5400*a^11*c^9*x^8 + 1200*I*a^10*c^9*x^7 - 3840*a^9*c^9*x^6 +
 2160*I*a^8*c^9*x^5 - 1200*a^7*c^9*x^4 + 1200*I*a^6*c^9*x^3 + 240*I*a^4*c^9
*x + 60*a^3*c^9)
```

Sympy [B] time = 110.205, size = 199, normalized size = 5.24

$$-\frac{4a^{121}x-}{60a^{139}c^9x^{16}-240ia^{138}c^9x^{15}-1200ia^{136}c^9x^{13}-1200a^{135}c^9x^{12}-2160ia^{134}c^9x^{11}-3840a^{133}c^9x^{10}-1200ia^{132}c^9x^9-540}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*a*x)**4*(a**2*x**2+1)**2/(a**2*c*x**2+c)**9,x)
```

```
[Out] -(4*a**121*x - I*a**120)/(60*a**139*c**9*x**16 - 240*I*a**138*c**9*x**15 -
1200*I*a**136*c**9*x**13 - 1200*a**135*c**9*x**12 - 2160*I*a**134*c**9*x**1
1 - 3840*a**133*c**9*x**10 - 1200*I*a**132*c**9*x**9 - 5400*a**131*c**9*x**
8 + 1200*I*a**130*c**9*x**7 - 3840*a**129*c**9*x**6 + 2160*I*a**128*c**9*x*
*5 - 1200*a**127*c**9*x**4 + 1200*I*a**126*c**9*x**3 + 240*I*a**124*c**9*x
+ 60*a**123*c**9)
```

Giac [B] time = 1.11882, size = 204, normalized size = 5.37

$$-\frac{2145\,a^5x^5+12540\,a^4ix^4-30030\,a^3x^3-37080\,a^2ix^2+23841\,ax+6476\,i}{983040\,(ax+i)^6a^3c^9}+\frac{2145\,a^9x^9-21780\,a^8ix^8-99660\,a^7x^7+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x, algorithm="giac"
)
```

```
[Out] -1/983040*(2145*a^5*x^5 + 12540*a^4*i*x^4 - 30030*a^3*x^3 - 37080*a^2*i*x^2
 + 23841*a*x + 6476*i)/((a*x + i)^6*a^3*c^9) + 1/983040*(2145*a^9*x^9 - 217
80*a^8*i*x^8 - 99660*a^7*x^7 + 270480*a^6*i*x^6 + 481446*a^5*x^5 - 584920*a
^4*i*x^4 - 486220*a^3*x^3 + 265680*a^2*i*x^2 + 84065*a*x - 9908*i)/((a*x -
i)^10*a^3*c^9)
```

3.380 $\int \frac{e^{5i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{27/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{(5ax+i)\sqrt{a^2x^2+1}}{120a^3c^{13}(1-iax)^{15}(1+iax)^{10}\sqrt{a^2cx^2+c}}$$

```
[Out] -((I + 5*a*x)*Sqrt[1 + a^2*x^2])/(120*a^3*c^13*(1 - I*a*x)^15*(1 + I*a*x)^1
0*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.201334, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5085, 5082, 81}

$$-\frac{(5ax+i)\sqrt{a^2x^2+1}}{120a^3c^{13}(1-iax)^{15}(1+iax)^{10}\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^((5*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(27/2),x]
```

```
[Out] -((I + 5*a*x)*Sqrt[1 + a^2*x^2])/(120*a^3*c^13*(1 - I*a*x)^15*(1 + I*a*x)^1
0*Sqrt[c + a^2*c*x^2])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\int \frac{e^{5i\tan^{-1}(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{5i\tan^{-1}(ax)}x^2}{(1+a^2x^2)^{27/2}}\,dx}{c^{13}\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)^{16}(1+iax)^{11}}\,dx}{c^{13}\sqrt{c+a^2cx^2}}$$
$$= -\frac{(i+5ax)\sqrt{1+a^2x^2}}{120a^3c^{13}(1-iax)^{15}(1+iax)^{10}\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.481836, size = 63, normalized size = 0.97

$$\frac{(1-5iax)\sqrt{a^2x^2+1}}{120a^3c^{13}(ax-i)^{10}(ax+i)^{15}\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^((5*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(27/2),x]
```

```
[Out] ((1 - (5*I)*a*x)*Sqrt[1 + a^2*x^2])/(120*a^3*c^13*(-I + a*x)^10*(I + a*x)^1
5*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.098, size = 58, normalized size = 0.9

$$\frac{(-ax+i)(ax+i)(i+5ax)(1+iax)^5}{120a^3}\left(a^2x^2+1\right)^{-\frac{5}{2}}\left(a^2cx^2+c\right)^{-\frac{27}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x)
```

```
[Out] 1/120*(a*x+I)*(-a*x+I)*(I+5*a*x)*(1+I*a*x)^5/a^3/(a^2*x^2+1)^(5/2)/(a^2*c*x
^2+c)^(27/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 4.18018, size = 1369, normalized size = 21.06

$$\frac{\left(i\,a^{22}x^{25}-5\,a^{21}x^{24}\right.}{120\,a^{27}c^{14}x^{27}+600i\,a^{26}c^{14}x^{26}+120\,a^{25}c^{14}x^{25}+5400i\,a^{24}c^{14}x^{24}-6000\,a^{23}c^{14}x^{23}+19920i\,a^{22}c^{14}x^{22}-39600\,a^{21}c^{14}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x, algorit
hm="fricas")
```

```
[Out] (I*a^22*x^25 - 5*a^21*x^24 - 40*a^19*x^22 - 50*I*a^18*x^21 - 126*a^17*x^20
- 280*I*a^16*x^19 - 160*a^15*x^18 - 765*I*a^14*x^17 + 105*a^13*x^16 - 1248*
I*a^12*x^15 + 720*a^11*x^14 - 1260*I*a^10*x^13 + 1260*a^9*x^12 - 720*I*a^8*
x^11 + 1248*a^7*x^10 - 105*I*a^6*x^9 + 765*a^5*x^8 + 160*I*a^4*x^7 + 280*a^
3*x^6 + 126*I*a^2*x^5 + 50*a*x^4 + 40*I*x^3)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x
^2 + 1)/(120*a^27*c^14*x^27 + 600*I*a^26*c^14*x^26 + 120*a^25*c^14*x^25 + 5
400*I*a^24*c^14*x^24 - 6000*a^23*c^14*x^23 + 19920*I*a^22*c^14*x^22 - 39600
*a^21*c^14*x^21 + 34320*I*a^20*c^14*x^20 - 125400*a^19*c^14*x^19 + 6600*I*a
^18*c^14*x^18 - 241560*a^17*c^14*x^17 - 99000*I*a^16*c^14*x^16 - 300960*a^1
5*c^14*x^15 - 237600*I*a^14*c^14*x^14 - 237600*a^13*c^14*x^13 - 300960*I*a^
12*c^14*x^12 - 99000*a^11*c^14*x^11 - 241560*I*a^10*c^14*x^10 + 6600*a^9*c^
14*x^9 - 125400*I*a^8*c^14*x^8 + 34320*a^7*c^14*x^7 - 39600*I*a^6*c^14*x^6
+ 19920*a^5*c^14*x^5 - 6000*I*a^4*c^14*x^4 + 5400*a^3*c^14*x^3 + 120*I*a^2*
c^14*x^2 + 600*a*c^14*x + 120*I*c^14)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**5/(a**2*x**2+1)**(5/2)*x**2/(a**2*c*x**2+c)**(27/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)^5x^2}{\left(a^2cx^2+c\right)^{\frac{27}{2}}\left(a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x, algorit
hm="giac")
```

```
[Out] integrate((I*a*x + 1)^5*x^2/((a^2*c*x^2 + c)^(27/2)*(a^2*x^2 + 1)^(5/2)), x
)
```

3.381 $\int \frac{e^{3i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{11/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{(3ax+i)\sqrt{a^2x^2+1}}{24a^3c^5(1-iax)^6(1+iax)^3\sqrt{a^2cx^2+c}}$$

```
[Out] -((I + 3*a*x)*Sqrt[1 + a^2*x^2])/(24*a^3*c^5*(1 - I*a*x)^6*(1 + I*a*x)^3*Sq
rt[c + a^2*c*x^2])
```

Rubi [A] time = 0.203903, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5085, 5082, 81}

$$-\frac{(3ax+i)\sqrt{a^2x^2+1}}{24a^3c^5(1-iax)^6(1+iax)^3\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^((3*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(11/2),x]
```

```
[Out] -((I + 3*a*x)*Sqrt[1 + a^2*x^2])/(24*a^3*c^5*(1 - I*a*x)^6*(1 + I*a*x)^3*Sq
rt[c + a^2*c*x^2])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\int \frac{e^{3i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{11/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{3i\tan^{-1}(ax)}x^2}{\left(1+a^2x^2\right)^{11/2}}\,dx}{c^5\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)^7(1+iax)^4}\,dx}{c^5\sqrt{c+a^2cx^2}}$$
$$= -\frac{(i+3ax)\sqrt{1+a^2x^2}}{24a^3c^5(1-iax)^6(1+iax)^3\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0921947, size = 65, normalized size = 1.

$$\frac{i(3ax+i)\sqrt{a^2x^2+1}}{24a^3c^5(ax-i)^3(ax+i)^6\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^((3*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(11/2),x]
```

```
[Out] ((I/24)*(I + 3*a*x)*Sqrt[1 + a^2*x^2])/(a^3*c^5*(-I + a*x)^3*(I + a*x)^6*Sq
rt[c + a^2*c*x^2])
```

Maple [A] time = 0.07, size = 58, normalized size = 0.9

$$\frac{(-ax+i)\,(ax+i)\,(i+3\,ax)\,(1+iax)^3}{24\,a^3}\left(a^2x^2+1\right)^{-\frac{3}{2}}\left(a^2cx^2+c\right)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x)
```

```
[Out] 1/24*(a*x+I)*(-a*x+I)*(I+3*a*x)*(1+I*a*x)^3/a^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^
2+c)^(11/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.39638, size = 436, normalized size = 6.71

$$\frac{\left(i\,a^6x^9-3\,a^5x^8-8\,a^3x^6-6i\,a^2x^5-6\,ax^4-8i\,x^3\right)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}}{24\,a^{11}c^6x^{11}+72i\,a^{10}c^6x^{10}+24\,a^9c^6x^9+264i\,a^8c^6x^8-144\,a^7c^6x^7+336i\,a^6c^6x^6-336\,a^5c^6x^5+144i\,a^4c^6x^4-264\,a^3c^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x, algorit
hm="fricas")
```

```
[Out] (I*a^6*x^9 - 3*a^5*x^8 - 8*a^3*x^6 - 6*I*a^2*x^5 - 6*a*x^4 - 8*I*x^3)*sqrt(
a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/(24*a^11*c^6*x^11 + 72*I*a^10*c^6*x^10 + 2
4*a^9*c^6*x^9 + 264*I*a^8*c^6*x^8 - 144*a^7*c^6*x^7 + 336*I*a^6*c^6*x^6 - 3
36*a^5*c^6*x^5 + 144*I*a^4*c^6*x^4 - 264*a^3*c^6*x^3 - 24*I*a^2*c^6*x^2 - 7
2*a*c^6*x - 24*I*c^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x**2/(a**2*c*x**2+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax+1)^3x^2}{\left(a^2cx^2+c\right)^{\frac{11}{2}}\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x, algorit
hm="giac")
```

```
[Out] integrate((I*a*x + 1)^3*x^2/((a^2*c*x^2 + c)^(11/2)*(a^2*x^2 + 1)^(3/2)), x
)
```

3.382 $\int \frac{e^{i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{\sqrt{a^2x^2+1}}{2a^3c(ax+i)\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\log(-ax+i)}{4a^3c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\log(ax+i)}{4a^3c\sqrt{a^2cx^2+c}}$$

```
[Out] -Sqrt[1 + a^2*x^2]/(2*a^3*c*(I + a*x)*Sqrt[c + a^2*c*x^2]) + ((I/4)*Sqrt[1
+ a^2*x^2]*Log[I - a*x])/(a^3*c*Sqrt[c + a^2*c*x^2]) + (((3*I)/4)*Sqrt[1 +
a^2*x^2]*Log[I + a*x])/(a^3*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.218828, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5085, 5082, 88}

$$-\frac{\sqrt{a^2x^2+1}}{2a^3c(ax+i)\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\log(-ax+i)}{4a^3c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\log(ax+i)}{4a^3c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(I*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(2*a^3*c*(I + a*x)*Sqrt[c + a^2*c*x^2]) + ((I/4)*Sqrt[1
+ a^2*x^2]*Log[I - a*x])/(a^3*c*Sqrt[c + a^2*c*x^2]) + (((3*I)/4)*Sqrt[1 +
a^2*x^2]*Log[I + a*x])/(a^3*c*Sqrt[c + a^2*c*x^2])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{i\tan^{-1}(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{i\tan^{-1}(ax)}x^2}{(1+a^2x^2)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)^2(1+iax)}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \left(\frac{i}{4a^2(-i+ax)}+\frac{1}{2a^2(i+ax)^2}+\frac{3i}{4a^2(i+ax)}\right)dx}{c\sqrt{c+a^2cx^2}}$$
$$= -\frac{\sqrt{1+a^2x^2}}{2a^3c(i+ax)\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\log(i-ax)}{4a^3c\sqrt{c+a^2cx^2}}+\frac{3i\sqrt{1+a^2x^2}\log(i+ax)}{4a^3c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.051473, size = 74, normalized size = 0.52

$$\frac{\sqrt{a^2x^2+1}\left(-\frac{2}{ax+i}+i\log(-ax+i)+3i\log(ax+i)\right)}{4a^3c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(I*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(-2/(I + a*x) + I*Log[I - a*x] + (3*I)*Log[I + a*x]))/(4
*a^3*c*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.161, size = 87, normalized size = 0.6

$$\frac{3\,i\ln(ax+i)\,xa+i\ln(-ax+i)\,xa-3\,\ln(ax+i)-\ln(-ax+i)-2}{4\,c^2a^3\,(ax+i)}\sqrt{c\left(a^2x^2+1\right)}\frac{1}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(3*I*ln(a*x+I)*x*a+I*ln(-a*x+I)
*x*a-3*ln(a*x+I)-ln(-a*x+I)-2)/c^2/a^3/(a*x+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] 1/2*((-3*I*a^5*c^2*x^3 + 3*a^4*c^2*x^2 - 3*I*a^3*c^2*x + 3*a^2*c^2)*sqrt(1/
(a^6*c^3))*log((I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6
*c^3)) + I*a^2*x^3 + I*x)/(a^3*x^3 + I*a^2*x^2 + a*x + I)) + (3*I*a^5*c^2*x
^3 - 3*a^4*c^2*x^2 + 3*I*a^3*c^2*x - 3*a^2*c^2)*sqrt(1/(a^6*c^3))*log((-I*s
qrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) + I*a^2*x^3
+ I*x)/(a^3*x^3 + I*a^2*x^2 + a*x + I)) + (I*a^5*c^2*x^3 - a^4*c^2*x^2 + I*
a^3*c^2*x - a^2*c^2)*sqrt(1/(a^6*c^3))*log((I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*
x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) - I*a^2*x^3 - I*x)/(a^3*x^3 - I*a^2*x^2
+ a*x - I)) + (-I*a^5*c^2*x^3 + a^4*c^2*x^2 - I*a^3*c^2*x + a^2*c^2)*sqrt(1
/(a^6*c^3))*log((-I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a
^6*c^3)) - I*a^2*x^3 - I*x)/(a^3*x^3 - I*a^2*x^2 + a*x - I)) + (4*I*a^5*c^2
*x^3 - 4*a^4*c^2*x^2 + 4*I*a^3*c^2*x - 4*a^2*c^2)*sqrt(1/(a^6*c^3))*log((sq
rt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) + a^2*x^3 + x
)/(a^2*x^2 + 1)) + (-4*I*a^5*c^2*x^3 + 4*a^4*c^2*x^2 - 4*I*a^3*c^2*x + 4*a^
2*c^2)*sqrt(1/(a^6*c^3))*log(-(sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*
x*sqrt(1/(a^6*c^3)) - a^2*x^3 - x)/(a^2*x^2 + 1)) - 4*I*sqrt(a^2*c*x^2 + c)
*sqrt(a^2*x^2 + 1)*x + 2*(4*a^5*c^2*x^3 + 4*I*a^4*c^2*x^2 + 4*a^3*c^2*x + 4
*I*a^2*c^2)*integral(1/2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*(2*I*a*x + 1
)/(a^6*c^2*x^4 + 2*a^4*c^2*x^2 + a^2*c^2), x))/(4*a^5*c^2*x^3 + 4*I*a^4*c^2
*x^2 + 4*a^3*c^2*x + 4*I*a^2*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (iax + 1)}{\left(c\left(a^2x^2 + 1\right)\right)^{\frac{3}{2}} \sqrt{a^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*(I*a*x + 1)/((c*(a**2*x**2 + 1))**(3/2)*sqrt(a**2*x**2 + 1)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,ax + 1)x^2}{\left(a^2cx^2 + c\right)^{\frac{3}{2}} \sqrt{a^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((I*a*x + 1)*x^2/((a^2*c*x^2 + c)^(3/2)*sqrt(a^2*x^2 + 1)), x)
```

3.383 $\int \frac{e^{-i\tan^{-1}(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{\sqrt{a^2x^2+1}}{2a^3c(-ax+i)\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\log(-ax+i)}{4a^3c\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\log(ax+i)}{4a^3c\sqrt{a^2cx^2+c}}$$

```
[Out] Sqrt[1 + a^2*x^2]/(2*a^3*c*(I - a*x)*Sqrt[c + a^2*c*x^2]) - (((3*I)/4)*Sqrt
[1 + a^2*x^2]*Log[I - a*x])/(a^3*c*Sqrt[c + a^2*c*x^2]) - ((I/4)*Sqrt[1 + a
^2*x^2]*Log[I + a*x])/(a^3*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.223591, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5085, 5082, 88}

$$\frac{\sqrt{a^2x^2+1}}{2a^3c(-ax+i)\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\log(-ax+i)}{4a^3c\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\log(ax+i)}{4a^3c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^(I*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] Sqrt[1 + a^2*x^2]/(2*a^3*c*(I - a*x)*Sqrt[c + a^2*c*x^2]) - (((3*I)/4)*Sqrt
[1 + a^2*x^2]*Log[I - a*x])/(a^3*c*Sqrt[c + a^2*c*x^2]) - ((I/4)*Sqrt[1 + a
^2*x^2]*Log[I + a*x])/(a^3*c*Sqrt[c + a^2*c*x^2])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{-i\tan^{-1}(ax)}x^2}{\left(1+a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)(1+iax)^2}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \left(\frac{1}{2a^2(-i+ax)^2}-\frac{3i}{4a^2(-i+ax)}-\frac{i}{4a^2(i+ax)}\right)dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}}{2a^3c(i-ax)\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1+a^2x^2}\log(i-ax)}{4a^3c\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\log(i+ax)}{4a^3c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.056661, size = 75, normalized size = 0.52

$$\frac{\sqrt{a^2x^2+1}\left(\frac{2}{-ax+i}-3i\log(-ax+i)-i\log(ax+i)\right)}{4a^3c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^(I*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(2/(I - a*x) - (3*I)*Log[I - a*x] - I*Log[I + a*x]))/(4*
a^3*c*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.152, size = 86, normalized size = 0.6

$$\frac{3\,i\ln(-ax+i)\,xa+i\ln(ax+i)\,xa+3\,\ln(-ax+i)+\ln(ax+i)+2}{4\,c^2a^3\,(-ax+i)}\sqrt{c\left(a^2x^2+1\right)}\frac{1}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(3*I*ln(-a*x+I)*x*a+I*ln(a*x+I)
*x*a+3*ln(-a*x+I)+ln(a*x+I)+2)/c^2/a^3/(-a*x+I)
```

Maxima [A] time = 1.00657, size = 74, normalized size = 0.52

$$-\frac{\sqrt{c}}{2\,a^4c^2x-2i\,a^3c^2}-\frac{3i\,\log(ax-i)}{4\,a^3c^{\frac{3}{2}}}-\frac{i\,\log(i\,ax-1)}{4\,a^3c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -sqrt(c)/(2*a^4*c^2*x - 2*I*a^3*c^2) - 3/4*I*log(a*x - I)/(a^3*c^(3/2)) - 1
/4*I*log(I*a*x - 1)/(a^3*c^(3/2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] 1/2*((I*a^5*c^2*x^3 + a^4*c^2*x^2 + I*a^3*c^2*x + a^2*c^2)*sqrt(1/(a^6*c^3)
)*log((I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) +
I*a^2*x^3 + I*x)/(a^3*x^3 + I*a^2*x^2 + a*x + I)) + (-I*a^5*c^2*x^3 - a^4*c
^2*x^2 - I*a^3*c^2*x - a^2*c^2)*sqrt(1/(a^6*c^3))*log((-I*sqrt(a^2*c*x^2 +
c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) + I*a^2*x^3 + I*x)/(a^3*x^3
+ I*a^2*x^2 + a*x + I)) + (-3*I*a^5*c^2*x^3 - 3*a^4*c^2*x^2 - 3*I*a^3*c^2*x
 - 3*a^2*c^2)*sqrt(1/(a^6*c^3))*log((I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1
)*a^3*c*x*sqrt(1/(a^6*c^3)) - I*a^2*x^3 - I*x)/(a^3*x^3 - I*a^2*x^2 + a*x -
 I)) + (3*I*a^5*c^2*x^3 + 3*a^4*c^2*x^2 + 3*I*a^3*c^2*x + 3*a^2*c^2)*sqrt(1
/(a^6*c^3))*log((-I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a
^6*c^3)) - I*a^2*x^3 - I*x)/(a^3*x^3 - I*a^2*x^2 + a*x - I)) + (-4*I*a^5*c^
2*x^3 - 4*a^4*c^2*x^2 - 4*I*a^3*c^2*x - 4*a^2*c^2)*sqrt(1/(a^6*c^3))*log((s
qrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) + a^2*x^3 +
x)/(a^2*x^2 + 1)) + (4*I*a^5*c^2*x^3 + 4*a^4*c^2*x^2 + 4*I*a^3*c^2*x + 4*a^
2*c^2)*sqrt(1/(a^6*c^3))*log(-(sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*
x*sqrt(1/(a^6*c^3)) - a^2*x^3 - x)/(a^2*x^2 + 1)) + 4*I*sqrt(a^2*c*x^2 + c)
*sqrt(a^2*x^2 + 1)*x + 2*(4*a^5*c^2*x^3 - 4*I*a^4*c^2*x^2 + 4*a^3*c^2*x - 4
*I*a^2*c^2)*integral(1/2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*(-2*I*a*x +
1)/(a^6*c^2*x^4 + 2*a^4*c^2*x^2 + a^2*c^2), x))/(4*a^5*c^2*x^3 - 4*I*a^4*c^
2*x^2 + 4*a^3*c^2*x - 4*I*a^2*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{a^2x^2+1}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}(iax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*a*x)*(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*sqrt(a**2*x**2 + 1)/((c*(a**2*x**2 + 1))**(3/2)*(I*a*x + 1)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2x^2+1}x^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}(i\,ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)*x^2/((a^2*c*x^2 + c)^(3/2)*(I*a*x + 1)), x)
```

3.384 $\int \frac{e^{-3i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{11/2}}\,dx$

Optimal. Leaf size=65

$$\frac{(-3ax+i)\sqrt{a^2x^2+1}}{24a^3c^5(1-iax)^3(1+iax)^6\sqrt{a^2cx^2+c}}$$

```
[Out] ((I - 3*a*x)*Sqrt[1 + a^2*x^2])/(24*a^3*c^5*(1 - I*a*x)^3*(1 + I*a*x)^6*Sqr
t[c + a^2*c*x^2])
```

Rubi [A] time = 0.208288, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5085, 5082, 81}

$$\frac{(-3ax+i)\sqrt{a^2x^2+1}}{24a^3c^5(1-iax)^3(1+iax)^6\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^((3*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(11/2)),x]
```

```
[Out] ((I - 3*a*x)*Sqrt[1 + a^2*x^2])/(24*a^3*c^5*(1 - I*a*x)^3*(1 + I*a*x)^6*Sqr
t[c + a^2*c*x^2])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\int \frac{e^{-3i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{11/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{-3i\tan^{-1}(ax)}x^2}{\left(1+a^2x^2\right)^{11/2}}\,dx}{c^5\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)^4(1+iax)^7}\,dx}{c^5\sqrt{c+a^2cx^2}}$$
$$= \frac{(i-3ax)\sqrt{1+a^2x^2}}{24a^3c^5(1-iax)^3(1+iax)^6\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.100985, size = 65, normalized size = 1.

$$-\frac{i(3ax-i)\sqrt{a^2x^2+1}}{24a^3c^5(ax-i)^6(ax+i)^3\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^((3*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(11/2)),x]
```

```
[Out] ((-I/24)*(-I + 3*a*x)*Sqrt[1 + a^2*x^2])/(a^3*c^5*(-I + a*x)^6*(I + a*x)^3*
Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.074, size = 58, normalized size = 0.9

$$-\frac{(-ax+i)\,(ax+i)\,(i-3\,ax)}{24\,a^3\,(1+iax)^3}\left(a^2x^2+1\right)^{\frac{3}{2}}\left(a^2cx^2+c\right)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x)
```

```
[Out] -1/24*(a*x+I)*(-a*x+I)*(I-3*a*x)*(a^2*x^2+1)^(3/2)/a^3/(1+I*a*x)^3/(a^2*c*x
^2+c)^(11/2)
```

Maxima [A] time = 1.08692, size = 126, normalized size = 1.94

$$\frac{3\,ax-i}{24i\,a^{12}c^{\frac{11}{2}}x^9+72\,a^{11}c^{\frac{11}{2}}x^8+192\,a^9c^{\frac{11}{2}}x^6-144i\,a^8c^{\frac{11}{2}}x^5+144\,a^7c^{\frac{11}{2}}x^4-192i\,a^6c^{\frac{11}{2}}x^3-72i\,a^4c^{\frac{11}{2}}x-24\,a^3c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x, algorit
hm="maxima")
```

```
[Out] (3*a*x - I)/(24*I*a^12*c^(11/2)*x^9 + 72*a^11*c^(11/2)*x^8 + 192*a^9*c^(11/
2)*x^6 - 144*I*a^8*c^(11/2)*x^5 + 144*a^7*c^(11/2)*x^4 - 192*I*a^6*c^(11/2)
*x^3 - 72*I*a^4*c^(11/2)*x - 24*a^3*c^(11/2))
```

Fricas [B] time = 2.41703, size = 437, normalized size = 6.72

$$\frac{\left(-i\,a^6x^9-3\,a^5x^8-8\,a^3x^6+6i\,a^2x^5-6\,ax^4+8i\,x^3\right)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}}{24\,a^{11}c^6x^{11}-72i\,a^{10}c^6x^{10}+24\,a^9c^6x^9-264i\,a^8c^6x^8-144\,a^7c^6x^7-336i\,a^6c^6x^6-336\,a^5c^6x^5-144i\,a^4c^6x^4-264\,a^3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x, algorit
hm="fricas")
```

```
[Out] (-I*a^6*x^9 - 3*a^5*x^8 - 8*a^3*x^6 + 6*I*a^2*x^5 - 6*a*x^4 + 8*I*x^3)*sqrt
(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/(24*a^11*c^6*x^11 - 72*I*a^10*c^6*x^10 +
24*a^9*c^6*x^9 - 264*I*a^8*c^6*x^8 - 144*a^7*c^6*x^7 - 336*I*a^6*c^6*x^6 -
336*a^5*c^6*x^5 - 144*I*a^4*c^6*x^4 - 264*a^3*c^6*x^3 + 24*I*a^2*c^6*x^2 -
72*a*c^6*x + 24*I*c^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}x^2}{\left(a^2cx^2+c\right)^{\frac{11}{2}}(i\,ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x, algorit
hm="giac")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)*x^2/((a^2*c*x^2 + c)^(11/2)*(I*a*x + 1)^3), x
)
```

3.385 $\int \frac{e^{-5i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{27/2}}\,dx$

Optimal. Leaf size=65

$$\frac{(-5ax+i)\sqrt{a^2x^2+1}}{120a^3c^{13}(1-iax)^{10}(1+iax)^{15}\sqrt{a^2cx^2+c}}$$

```
[Out] ((I - 5*a*x)*Sqrt[1 + a^2*x^2])/(120*a^3*c^13*(1 - I*a*x)^10*(1 + I*a*x)^15
*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.208997, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5085, 5082, 81}

$$\frac{(-5ax+i)\sqrt{a^2x^2+1}}{120a^3c^{13}(1-iax)^{10}(1+iax)^{15}\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^((5*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(27/2)),x]
```

```
[Out] ((I - 5*a*x)*Sqrt[1 + a^2*x^2])/(120*a^3*c^13*(1 - I*a*x)^10*(1 + I*a*x)^15
*Sqrt[c + a^2*c*x^2])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\int \frac{e^{-5i\tan^{-1}(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{-5i\tan^{-1}(ax)}x^2}{(1+a^2x^2)^{27/2}}\,dx}{c^{13}\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)^{11}(1+iax)^{16}}\,dx}{c^{13}\sqrt{c+a^2cx^2}}$$
$$= \frac{(i-5ax)\sqrt{1+a^2x^2}}{120a^3c^{13}(1-iax)^{10}(1+iax)^{15}\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.493297, size = 63, normalized size = 0.97

$$\frac{(1+5iax)\sqrt{a^2x^2+1}}{120a^3c^{13}(ax-i)^{15}(ax+i)^{10}\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^((5*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(27/2)),x]
```

```
[Out] ((1 + (5*I)*a*x)*Sqrt[1 + a^2*x^2])/(120*a^3*c^13*(-I + a*x)^15*(I + a*x)^1
0*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.089, size = 58, normalized size = 0.9

$$-\frac{(-ax+i)(ax+i)(i-5\,ax)}{120\,a^3(1+iax)^5}\left(a^2x^2+1\right)^{\frac{5}{2}}\left(a^2cx^2+c\right)^{-\frac{27}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x)
```

```
[Out] -1/120*(a*x+I)*(-a*x+I)*(I-5*a*x)*(a^2*x^2+1)^(5/2)/a^3/(1+I*a*x)^5/(a^2*c*
x^2+c)^(27/2)
```

Maxima [B] time = 1.46, size = 370, normalized size = 5.69

$$\overline{120\,a^{28}c^{14}x^{25} - 600i\,a^{27}c^{14}x^{24} - 4800i\,a^{25}c^{14}x^{22} - 6000\,a^{24}c^{14}x^{21} - 15120i\,a^{23}c^{14}x^{20} - 33600\,a^{22}c^{14}x^{19} - 19200i\,a^{21}c^{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x, algorit
hm="maxima")
```

```
[Out] (5*I*a*sqrt(c)*x + sqrt(c))/(120*a^28*c^14*x^25 - 600*I*a^27*c^14*x^24 - 48
00*I*a^25*c^14*x^22 - 6000*a^24*c^14*x^21 - 15120*I*a^23*c^14*x^20 - 33600*
a^22*c^14*x^19 - 19200*I*a^21*c^14*x^18 - 91800*a^20*c^14*x^17 + 12600*I*a^
19*c^14*x^16 - 149760*a^18*c^14*x^15 + 86400*I*a^17*c^14*x^14 - 151200*a^16
*c^14*x^13 + 151200*I*a^15*c^14*x^12 - 86400*a^14*c^14*x^11 + 149760*I*a^13
*c^14*x^10 - 12600*a^12*c^14*x^9 + 91800*I*a^11*c^14*x^8 + 19200*a^10*c^14*
```

```
x^7 + 33600*I*a^9*c^14*x^6 + 15120*a^8*c^14*x^5 + 6000*I*a^7*c^14*x^4 + 480
0*a^6*c^14*x^3 + 600*a^4*c^14*x - 120*I*a^3*c^14)
```

Fricas [B] time = 4.28712, size = 1370, normalized size = 21.08

$$\frac{\left(-i\,a^{22}x^{25}-5\,a^{21}x^{24}-\cdots\right.}{120\,a^{27}c^{14}x^{27}-600i\,a^{26}c^{14}x^{26}+120\,a^{25}c^{14}x^{25}-5400i\,a^{24}c^{14}x^{24}-6000\,a^{23}c^{14}x^{23}-19920i\,a^{22}c^{14}x^{22}-39600\,a^{21}c^{14}x^{21}-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x, algorit
hm="fricas")
```

```
[Out] (-I*a^22*x^25 - 5*a^21*x^24 - 40*a^19*x^22 + 50*I*a^18*x^21 - 126*a^17*x^20
 + 280*I*a^16*x^19 - 160*a^15*x^18 + 765*I*a^14*x^17 + 105*a^13*x^16 + 1248
*I*a^12*x^15 + 720*a^11*x^14 + 1260*I*a^10*x^13 + 1260*a^9*x^12 + 720*I*a^8
*x^11 + 1248*a^7*x^10 + 105*I*a^6*x^9 + 765*a^5*x^8 - 160*I*a^4*x^7 + 280*a
^3*x^6 - 126*I*a^2*x^5 + 50*a*x^4 - 40*I*x^3)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*
x^2 + 1)/(120*a^27*c^14*x^27 - 600*I*a^26*c^14*x^26 + 120*a^25*c^14*x^25 -
5400*I*a^24*c^14*x^24 - 6000*a^23*c^14*x^23 - 19920*I*a^22*c^14*x^22 - 3960
0*a^21*c^14*x^21 - 34320*I*a^20*c^14*x^20 - 125400*a^19*c^14*x^19 - 6600*I*
a^18*c^14*x^18 - 241560*a^17*c^14*x^17 + 99000*I*a^16*c^14*x^16 - 300960*a^
15*c^14*x^15 + 237600*I*a^14*c^14*x^14 - 237600*a^13*c^14*x^13 + 300960*I*a
^12*c^14*x^12 - 99000*a^11*c^14*x^11 + 241560*I*a^10*c^14*x^10 + 6600*a^9*c
^14*x^9 + 125400*I*a^8*c^14*x^8 + 34320*a^7*c^14*x^7 + 39600*I*a^6*c^14*x^6
 + 19920*a^5*c^14*x^5 + 6000*I*a^4*c^14*x^4 + 5400*a^3*c^14*x^3 - 120*I*a^2
*c^14*x^2 + 600*a*c^14*x - 120*I*c^14)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*a*x)**5*(a**2*x**2+1)**(5/2)/(a**2*c*x**2+c)**(27/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2+1\right)^{\frac{5}{2}}x^2}{\left(a^2cx^2+c\right)^{\frac{27}{2}}(i\,ax+1)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x, algorit
hm="giac")
```

```
[Out] integrate((a^2*x^2 + 1)^(5/2)*x^2/((a^2*c*x^2 + c)^(27/2)*(I*a*x + 1)^5), x
)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
             fi;
             #both result and optimal complex
             if leaf_count_result<=2*leaf_count_optimal then
                return "A";
             else
                return "B";
             end if
          else #result contains complex but optimal is not
             if debug then
                print("result contains complex but optimal is not");
             fi;
             return "C";
          end if
       else # result do not contain complex
            # this assumes optimal do not as well
            if debug then
                print("result do not contain complex, this assumes optimal do
    not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
               if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
               fi;
               return "A";
            else
               if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
               fi;
               return "B";
            end if
       end if
  else #ExpnType(result) > ExpnType(optimal)
       if debug then
          print("ExpnType(result) > ExpnType(optimal)");
       fi;
       return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```