

# Computer algebra independent integration tests

5-Inverse-trig-functions/5.2-Inverse-cosine/5.2.5-Inverse-cosine-functions

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3.85	$\int \frac{1}{(a+b \cos^{-1}(-1+dx^2))^2} dx$	348
3.86	$\int \frac{1}{(a+b \cos^{-1}(-1+dx^2))^3} dx$	351
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3.108	$\int e^{\cos^{-1}(ax)} x^3 dx$	421
3.109	$\int e^{\cos^{-1}(ax)} x^2 dx$	424
3.110	$\int e^{\cos^{-1}(ax)} x dx$	427
3.111	$\int e^{\cos^{-1}(ax)} dx$	430
3.112	$\int \frac{e^{\cos^{-1}(ax)}}{x} dx$	433
3.113	$\int \frac{e^{\cos^{-1}(ax)}}{x^2} dx$	436
3.114	$\int \cos^{-1}\left(\frac{c}{a+bx}\right) dx$	439
3.115	$\int \frac{1}{\sqrt{1-x^2}\sqrt{\cos^{-1}(x)}} dx$	443
3.116	$\int \frac{1}{\sqrt{1-x^2}\cos^{-1}(x)} dx$	446
3.117	$\int \frac{\cos^{-1}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}} dx$	448
3.118	$\int \frac{1}{\sqrt{1+bx^2}\cos^{-1}\left(\sqrt{1+bx^2}\right)} dx$	451

#### 4 Listing of Grading functions

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 118 ]. This is test number [ 147 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

### 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. ( 118 )	% 0. ( 0 )
Mathematica	% 95.76 ( 113 )	% 4.24 ( 5 )
Maple	% 66.1 ( 78 )	% 33.9 ( 40 )
Maxima	% 22.03 ( 26 )	% 77.97 ( 92 )
Fricas	% 40.68 ( 48 )	% 59.32 ( 70 )
Sympy	% 26.27 ( 31 )	% 73.73 ( 87 )
Giac	% 42.37 ( 50 )	% 57.63 ( 68 )

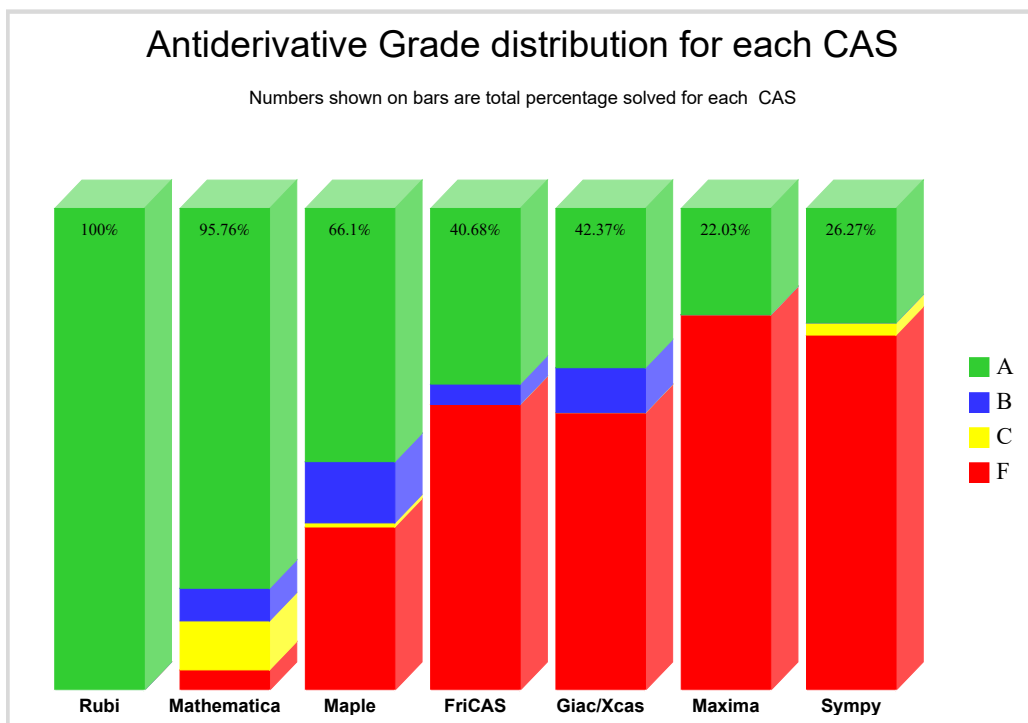
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

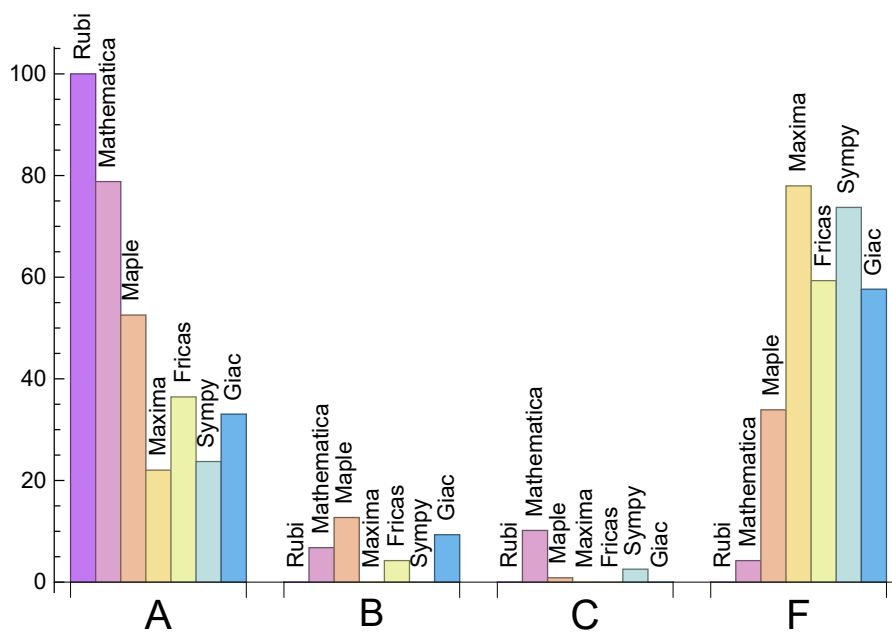
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	78.81	6.78	10.17	4.24
Maple	52.54	12.71	0.85	33.9
Maxima	22.03	0.	0.	77.97
Fricas	36.44	4.24	0.	59.32
Sympy	23.73	0.	2.54	73.73
Giac	33.05	9.32	0.	57.63

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.22	186.74	0.96	86.5	1.
Mathematica	0.93	262.63	1.12	85.	0.95
Maple	0.27	376.65	1.25	74.	1.16
Maxima	1.26	48.81	1.08	49.5	1.12
Fricas	2.31	191.46	2.74	104.5	2.18
Sympy	8.3	67.48	1.21	49.	1.28
Giac	1.24	106.2	1.6	71.	1.46

## 1.4 list of integrals that has no closed form antiderivative

{19, 23, 101, 105, 106}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {4, 5, 9, 13, 17, 18, 21, 28, 37, 38, 39, 40, 41, 42, 43, 44, 69, 77, 78, 79, 84, 85, 86, 112, 113, 115}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.



## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in->

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

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# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 19, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 45, 46, 47, 49, 51, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 118 }

B grade: { 9, 13, 17, 18, 21, 55, 69, 114 }

C grade: { 37, 38, 39, 40, 41, 42, 43, 44, 48, 50, 52, 115 }

F grade: { 20, 102, 103, 104, 107 }

#### 2.1.3 Maple

A grade: { 4, 10, 11, 17, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 76, 83, 101, 103, 104, 105, 106, 107, 114, 115, 116 }

B grade: { 1, 2, 3, 6, 7, 8, 9, 12, 13, 14, 15, 16, 18, 52, 102 }

C grade: { 5 }

F grade: { 20, 21, 22, 51, 70, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 108, 109, 110, 111, 112, 113, 117, 118 }

## 2.1.4 Maxima

A grade: { 23, 27, 46, 47, 49, 53, 54, 55, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 71, 72, 76, 83, 101, 105, 106 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 48, 50, 51, 52, 56, 63, 69, 70, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

## 2.1.5 FriCAS

A grade: { 19, 23, 24, 25, 26, 27, 32, 33, 46, 47, 49, 53, 54, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 80, 81, 82, 83, 101, 105, 106, 108, 109, 110, 111, 117, 118 }

B grade: { 29, 30, 31, 55, 114 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 28, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 48, 50, 51, 52, 56, 63, 69, 70, 77, 78, 79, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 107, 112, 113, 115, 116 }

## 2.1.6 Sympy

A grade: { 23, 24, 25, 26, 27, 32, 33, 46, 47, 48, 49, 50, 52, 54, 55, 57, 59, 60, 61, 62, 65, 66, 68, 71, 108, 109, 110, 111 }

B grade: { }

C grade: { 53, 58, 64 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 56, 63, 67, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116, 117, 118 }

## 2.1.7 Giac

A grade: { 19, 23, 25, 26, 27, 29, 32, 33, 34, 35, 36, 43, 44, 46, 47, 49, 53, 54, 55, 57, 58, 59, 60, 61, 62, 64, 68, 71, 72, 76, 83, 101, 105, 106, 108, 109, 110, 111, 116 }

B grade: { 24, 30, 31, 37, 38, 39, 40, 65, 66, 67, 115 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 28, 41, 42, 45, 48, 50, 51, 52, 56, 63, 69, 70, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 107, 112, 113, 114, 117, 118 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	670	670	442	1285	0	0	0	0
normalized size	1	1.	0.66	1.92	0.	0.	0.	0.
time (sec)	N/A	0.705	1.204	0.848	0.	0.	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	450	450	320	912	0	0	0	0
normalized size	1	1.	0.71	2.03	0.	0.	0.	0.
time (sec)	N/A	0.53	1.314	0.533	0.	0.	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	219	491	0	0	0	0
normalized size	1	1.	0.92	2.06	0.	0.	0.	0.
time (sec)	N/A	0.251	1.474	0.481	0.	0.	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	725	725	1095	1209	0	0	0	0
normalized size	1	1.	1.51	1.67	0.	0.	0.	0.
time (sec)	N/A	1.859	3.38	0.437	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	851	851	1130	1573	0	0	0	0
normalized size	1	1.	1.33	1.85	0.	0.	0.	0.
time (sec)	N/A	2.694	9.482	0.476	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	959	959	910	1734	0	0	0	0
normalized size	1	1.	0.95	1.81	0.	0.	0.	0.
time (sec)	N/A	0.962	4.572	0.811	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	680	680	591	1252	0	0	0	0
normalized size	1	1.	0.87	1.84	0.	0.	0.	0.
time (sec)	N/A	0.735	2.143	0.579	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	337	698	0	0	0	0
normalized size	1	1.	0.91	1.89	0.	0.	0.	0.
time (sec)	N/A	0.335	1.566	0.49	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1064	1064	3034	2132	0	0	0	0
normalized size	1	1.	2.85	2.	0.	0.	0.	0.
time (sec)	N/A	2.28	12.504	0.398	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1281	1281	1144	2236	0	0	0	0
normalized size	1	1.	0.89	1.75	0.	0.	0.	0.
time (sec)	N/A	1.177	7.462	0.959	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	940	940	794	1633	0	0	0	0
normalized size	1	1.	0.84	1.74	0.	0.	0.	0.
time (sec)	N/A	0.959	4.516	0.702	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	517	517	526	931	0	0	0	0
normalized size	1	1.	1.02	1.8	0.	0.	0.	0.
time (sec)	N/A	0.403	2.901	0.602	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1637	1637	6216	4692	0	0	0	0
normalized size	1	1.	3.8	2.87	0.	0.	0.	0.
time (sec)	N/A	2.73	19.605	0.531	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	450	450	342	845	0	0	0	0
normalized size	1	1.	0.76	1.88	0.	0.	0.	0.
time (sec)	N/A	0.585	1.182	0.605	0.	0.	0.	0.



Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	266	549	0	0	0	0
normalized size	1	1.	0.99	2.03	0.	0.	0.	0.
time (sec)	N/A	0.436	0.757	0.356	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	172	235	0	0	0	0
normalized size	1	1.	1.35	1.85	0.	0.	0.	0.
time (sec)	N/A	0.223	0.413	0.299	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	370	370	930	487	0	0	0	0
normalized size	1	1.	2.51	1.32	0.	0.	0.	0.
time (sec)	N/A	0.606	2.056	0.196	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	496	496	1108	1622	0	0	0	0
normalized size	1	1.	2.23	3.27	0.	0.	0.	0.
time (sec)	N/A	0.722	4.987	0.305	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	0.158	5.372	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	496	496	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.791	26.654	4.708	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	374	374	1248	0	0	0	0	0
normalized size	1	1.	3.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.611	5.566	3.134	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	246	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.338	0.022	0.015	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.2	0.199	2.447	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	104	235	0	219	255	327
normalized size	1	1.	0.76	1.72	0.	1.6	1.86	2.39
time (sec)	N/A	0.194	0.1	0.02	0.	2.479	1.757	1.316

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	83	161	0	173	170	211
normalized size	1	1.	0.88	1.71	0.	1.84	1.81	2.24
time (sec)	N/A	0.115	0.083	0.003	0.	2.421	0.82	1.306

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	69	78	0	135	104	119
normalized size	1	1.	0.86	0.98	0.	1.69	1.3	1.49
time (sec)	N/A	0.073	0.049	0.004	0.	2.551	0.361	1.322

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	47	33	43	92	46	43
normalized size	1	1.	1.31	0.92	1.19	2.56	1.28	1.19
time (sec)	N/A	0.017	0.037	0.002	1.41	2.507	0.174	1.281

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	228	199	0	0	0	0
normalized size	1	1.	1.29	1.12	0.	0.	0.	0.
time (sec)	N/A	0.276	0.192	0.104	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	79	77	0	857	0	107
normalized size	1	1.	1.25	1.22	0.	13.6	0.	1.7
time (sec)	N/A	0.076	0.055	0.003	0.	3.011	0.	1.316

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	126	118	0	1121	0	327
normalized size	1	1.	1.22	1.15	0.	10.88	0.	3.17
time (sec)	N/A	0.111	0.175	0.004	0.	2.947	0.	1.336

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	168	227	0	1328	0	752
normalized size	1	1.	1.17	1.58	0.	9.22	0.	5.22
time (sec)	N/A	0.179	0.184	0.005	0.	3.544	0.	1.356

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	74	71	0	170	109	105
normalized size	1	1.	0.9	0.87	0.	2.07	1.33	1.28
time (sec)	N/A	0.082	0.036	0.054	0.	2.559	0.693	1.43

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	49	48	0	130	63	70
normalized size	1	1.	1.04	1.02	0.	2.77	1.34	1.49
time (sec)	N/A	0.054	0.023	0.049	0.	2.336	0.283	1.342

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	0	0	16
normalized size	1	1.	1.	1.08	0.	0.	0.	1.33
time (sec)	N/A	0.022	0.026	0.046	0.	0.	0.	1.343

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	37	0	0	0	51
normalized size	1	1.	1.	0.92	0.	0.	0.	1.27
time (sec)	N/A	0.078	0.051	0.049	0.	0.	0.	1.33

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	53	0	0	0	77
normalized size	1	1.	1.	0.82	0.	0.	0.	1.18
time (sec)	N/A	0.081	0.05	0.051	0.	0.	0.	1.298

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	90	140	0	0	0	282
normalized size	1	1.	0.81	1.26	0.	0.	0.	2.54
time (sec)	N/A	0.147	0.049	0.092	0.	0.	0.	1.57

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	76	105	0	0	0	221
normalized size	1	1.	0.85	1.18	0.	0.	0.	2.48
time (sec)	N/A	0.092	0.035	0.082	0.	0.	0.	1.624

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	90	66	0	0	0	158
normalized size	1	1.	1.64	1.2	0.	0.	0.	2.87
time (sec)	N/A	0.081	0.039	0.073	0.	0.	0.	1.378

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	33	33	78	28	0	0	0	97
normalized size	1	1.	2.36	0.85	0.	0.	0.	2.94
time (sec)	N/A	0.03	0.032	0.055	0.	0.	0.	1.575

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	97	84	0	0	0	0
normalized size	1	1.	1.52	1.31	0.	0.	0.	0.
time (sec)	N/A	0.086	0.054	0.075	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	139	120	0	0	0	0
normalized size	1	1.	1.54	1.33	0.	0.	0.	0.
time (sec)	N/A	0.092	0.291	0.079	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	128	89	0	0	0	231
normalized size	1	1.	1.21	0.84	0.	0.	0.	2.18
time (sec)	N/A	0.13	0.105	0.071	0.	0.	0.	2.344

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	133	90	0	0	0	236
normalized size	1	1.	1.23	0.83	0.	0.	0.	2.19
time (sec)	N/A	0.116	0.102	0.064	0.	0.	0.	2.944

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	59	85	0	0	0	0
normalized size	1	1.	0.87	1.25	0.	0.	0.	0.
time (sec)	N/A	0.081	0.041	0.089	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	33	41	81	48	36
normalized size	1	1.	0.88	0.97	1.21	2.38	1.41	1.06
time (sec)	N/A	0.03	0.016	0.083	1.49	2.381	21.528	1.323

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	48	65	107	93	48	62
normalized size	1	1.	0.94	1.27	2.1	1.82	0.94	1.22
time (sec)	N/A	0.039	0.028	0.053	1.476	2.53	1.032	1.343

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	63	79	0	0	48	0
normalized size	1	1.	1.15	1.44	0.	0.	0.87	0.
time (sec)	N/A	0.028	0.155	0.007	0.	0.	1.427	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	42	68	32	42
normalized size	1	1.	1.	0.91	1.2	1.94	0.91	1.2
time (sec)	N/A	0.024	0.016	0.002	1.448	2.378	0.23	1.328

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	34	65	0	0	44	0
normalized size	1	1.	0.79	1.51	0.	0.	1.02	0.
time (sec)	N/A	0.032	0.005	0.007	0.	0.	1.111	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	56	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.031	0.066	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	40	57	0	0	44	0
normalized size	1	1.	1.38	1.97	0.	0.	1.52	0.
time (sec)	N/A	0.014	0.039	0.006	0.	0.	1.211	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	61	56	97	207	99	95
normalized size	1	1.	1.05	0.97	1.67	3.57	1.71	1.64
time (sec)	N/A	0.036	0.051	0.01	1.462	2.539	5.296	1.29

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	39	38	73	49	58
normalized size	1	1.	0.97	1.15	1.12	2.15	1.44	1.71
time (sec)	N/A	0.017	0.021	0.004	1.442	2.563	2.684	1.248

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	84	30	61	140	29	58
normalized size	1	1.	3.11	1.11	2.26	5.19	1.07	2.15
time (sec)	N/A	0.017	0.101	0.006	1.454	2.521	2.271	1.262

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	77	0	0	0	0
normalized size	1	1.	1.	1.28	0.	0.	0.	0.
time (sec)	N/A	0.055	0.021	0.055	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	32	42	69	26	42
normalized size	1	1.	1.	1.07	1.4	2.3	0.87	1.4
time (sec)	N/A	0.022	0.019	0.001	1.438	2.394	2.34	1.228

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	47	104	97	102	84
normalized size	1	1.	0.98	0.92	2.04	1.9	2.	1.65
time (sec)	N/A	0.033	0.024	0.006	1.465	2.47	5.305	1.28

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	47	55	66	104	102	88
normalized size	1	1.	0.84	0.98	1.18	1.86	1.82	1.57
time (sec)	N/A	0.038	0.031	0.006	1.445	2.462	6.366	1.314

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	46	53	70	115	73	70
normalized size	1	1.	0.59	0.68	0.9	1.47	0.94	0.9
time (sec)	N/A	0.028	0.038	0.004	1.436	2.632	10.162	1.128

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	41	41	54	99	58	54
normalized size	1	1.	0.68	0.68	0.9	1.65	0.97	0.9
time (sec)	N/A	0.019	0.029	0.003	1.465	2.585	3.901	1.277

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	38	26	34	78	29	34
normalized size	1	1.	1.03	0.7	0.92	2.11	0.78	0.92
time (sec)	N/A	0.011	0.015	0.002	1.524	2.612	0.337	1.193

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	59	0	0	0	0
normalized size	1	1.	0.96	1.05	0.	0.	0.	0.
time (sec)	N/A	0.056	0.026	0.002	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	22	28	59	44	54
normalized size	1	1.	0.89	0.81	1.04	2.19	1.63	2.
time (sec)	N/A	0.012	0.016	0.001	1.499	2.561	4.222	1.179

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	43	35	46	84	44	100
normalized size	1	1.	0.86	0.7	0.92	1.68	0.88	2.
time (sec)	N/A	0.018	0.021	0.004	1.473	2.628	24.914	1.281

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	37	47	62	97	60	143
normalized size	1	1.	0.54	0.69	0.91	1.43	0.88	2.1
time (sec)	N/A	0.022	0.038	0.003	1.476	2.631	150.038	1.312

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	42	59	78	111	0	186
normalized size	1	1.	0.49	0.69	0.91	1.29	0.	2.16
time (sec)	N/A	0.028	0.044	0.003	1.493	2.326	0.	1.285

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	59	20	26
normalized size	1	1.	1.	0.8	1.04	2.36	0.8	1.04
time (sec)	N/A	0.019	0.008	0.003	1.482	2.533	0.365	1.274

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	68	68	141	89	0	0	0	0
normalized size	1	1.	2.07	1.31	0.	0.	0.	0.
time (sec)	N/A	0.061	0.137	0.04	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	56	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.031	0.058	0.	0.	0.	0.



Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	43	40	53	105	61	53
normalized size	1	1.	0.91	0.85	1.13	2.23	1.3	1.13
time (sec)	N/A	0.051	0.028	0.003	1.612	2.439	1.143	1.261

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	43	0	55	132	0	55
normalized size	1	1.	0.9	0.	1.15	2.75	0.	1.15
time (sec)	N/A	0.054	0.041	0.03	1.606	2.646	0.	1.312

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	249	0	0	474	0	0
normalized size	1	1.	1.96	0.	0.	3.73	0.	0.
time (sec)	N/A	0.029	0.233	0.121	0.	2.605	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	162	0	0	328	0	0
normalized size	1	1.	1.47	0.	0.	2.98	0.	0.
time (sec)	N/A	0.057	0.126	0.116	0.	2.362	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	98	0	0	205	0	0
normalized size	1	1.	1.56	0.	0.	3.25	0.	0.
time (sec)	N/A	0.012	0.066	0.117	0.	2.5	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	41	45	61	103	0	78
normalized size	1	1.	0.95	1.05	1.42	2.4	0.	1.81
time (sec)	N/A	0.037	0.027	0.005	1.566	2.303	0.	1.349

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	99	99	85	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.104	0.066	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	133	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.332	0.066	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	147	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.246	0.067	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	249	0	0	474	0	0
normalized size	1	1.	1.96	0.	0.	3.73	0.	0.
time (sec)	N/A	0.028	0.239	0.117	0.	2.657	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	162	0	0	328	0	0
normalized size	1	1.	1.47	0.	0.	2.98	0.	0.
time (sec)	N/A	0.055	0.135	0.119	0.	2.573	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	98	0	0	205	0	0
normalized size	1	1.	1.56	0.	0.	3.25	0.	0.
time (sec)	N/A	0.012	0.066	0.118	0.	2.505	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	41	45	61	103	0	72
normalized size	1	1.	0.95	1.05	1.42	2.4	0.	1.67
time (sec)	N/A	0.036	0.029	0.004	1.527	2.682	0.	1.3

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	85	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.1	0.066	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	131	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.335	0.065	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	149	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.213	0.068	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	256	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	2.617	0.076	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	200	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.713	0.066	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	157	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.085	0.063	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	114	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.162	0.069	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	177	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.389	0.067	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	234	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.721	0.067	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	308	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.547	0.068	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	256	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	2.146	0.066	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	200	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.603	0.066	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	157	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.113	0.066	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	115	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.158	0.07	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	161	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.321	0.064	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	233	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.585	0.065	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	309	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.553	0.066	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.097	0.647	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	279	279	0	707	0	0	0	0
normalized size	1	1.	0.	2.53	0.	0.	0.	0.
time (sec)	N/A	0.207	0.296	0.974	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	207	0	401	0	0	0	0
normalized size	1	1.	0.	1.94	0.	0.	0.	0.
time (sec)	N/A	0.174	0.695	0.007	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	0	171	0	0	0	0
normalized size	1	1.	0.	1.21	0.	0.	0.	0.
time (sec)	N/A	0.104	0.372	0.005	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.101	0.344	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	1.693	0.342	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F(-1)	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	0	107	0	0	0	0
normalized size	1	1.	0.	1.27	0.	0.	0.	0.
time (sec)	N/A	0.066	0.947	0.004	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	50	0	0	130	105	111
normalized size	1	1.	0.62	0.	0.	1.6	1.3	1.37
time (sec)	N/A	0.064	0.139	0.01	0.	2.619	4.836	1.325

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	50	0	0	107	85	93
normalized size	1	1.	0.61	0.	0.	1.3	1.04	1.13
time (sec)	N/A	0.061	0.107	0.009	0.	2.566	1.716	1.376

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	30	0	0	89	58	59
normalized size	1	1.	0.73	0.	0.	2.17	1.41	1.44
time (sec)	N/A	0.034	0.037	0.007	0.	2.637	0.539	1.238

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	32	0	0	68	37	42
normalized size	1	1.	0.82	0.	0.	1.74	0.95	1.08
time (sec)	N/A	0.014	0.033	0.004	0.	2.6	0.201	1.277

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	79	0	0	0	0	0
normalized size	1	1.	1.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.056	0.007	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	55	0	0	0	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.062	0.007	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	141	45	0	305	0	0
normalized size	1	1.	2.94	0.94	0.	6.35	0.	0.
time (sec)	N/A	0.033	0.163	0.008	0.	2.529	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	26	26	56	21	0	0	0	78
normalized size	1	1.	2.15	0.81	0.	0.	0.	3.
time (sec)	N/A	0.071	0.085	0.106	0.	0.	0.	1.398

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	0	0	0	7
normalized size	1	1.	1.	1.2	0.	0.	0.	1.4
time (sec)	N/A	0.061	0.04	0.067	0.	0.	0.	1.378

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	0	107	0	0
normalized size	1	1.	1.	0.	0.	2.74	0.	0.
time (sec)	N/A	0.065	0.046	0.193	0.	2.995	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	25	0	0	68	0	0
normalized size	1	1.	0.81	0.	0.	2.19	0.	0.
time (sec)	N/A	0.061	0.024	0.184	0.	2.571	0.	0.

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of

the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [50] had the largest ratio of [ 1. ]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	16	12	1.	31	0.387
2	A	13	8	1.	31	0.258
3	A	8	6	1.	29	0.207
4	A	22	19	1.	31	0.613
5	A	35	22	1.	31	0.71
6	A	24	17	1.	31	0.548
7	A	20	12	1.	31	0.387
8	A	12	9	1.	29	0.31
9	A	29	23	1.	31	0.742
10	A	30	18	1.	31	0.581
11	A	26	15	1.	31	0.484
12	A	14	10	1.	29	0.345
13	A	37	28	1.	31	0.903
14	A	13	7	1.	31	0.226
15	A	9	7	1.	31	0.226
16	A	6	5	1.	29	0.172
17	A	10	7	1.	31	0.226
18	A	13	10	1.	31	0.323
19	A	0	0	0.	0	0.
20	A	13	9	1.	35	0.257
21	A	11	8	1.	33	0.242
22	A	9	7	1.	25	0.28
23	A	0	0	0.	0	0.
24	A	6	6	1.	10	0.6
25	A	5	5	1.	10	0.5
26	A	5	5	1.	8	0.625
27	A	3	3	1.	6	0.5
28	A	9	6	1.	10	0.6
29	A	4	4	1.	10	0.4
30	A	5	5	1.	10	0.5
31	A	6	6	1.	10	0.6
32	A	5	4	1.	8	0.5
33	A	4	4	1.	8	0.5
34	A	3	3	1.	8	0.375
35	A	4	4	1.	8	0.5
36	A	5	5	1.	8	0.625
37	A	7	6	1.	10	0.6
38	A	6	6	1.	10	0.6
39	A	5	5	1.	10	0.5
40	A	4	4	1.	10	0.4
41	A	5	5	1.	10	0.5

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	6	6	1.	10	0.6
43	A	7	7	1.	14	0.5
44	A	7	7	1.	15	0.467
45	A	7	7	1.	19	0.368
46	A	3	3	1.	14	0.214
47	A	5	5	1.	10	0.5
48	A	4	4	1.	10	0.4
49	A	3	3	1.	8	0.375
50	A	6	6	1.	6	1.
51	A	5	5	1.	10	0.5
52	A	3	3	1.	10	0.3
53	A	6	6	1.	10	0.6
54	A	3	3	1.	8	0.375
55	A	5	5	1.	6	0.833
56	A	5	5	1.	10	0.5
57	A	3	3	1.	10	0.3
58	A	5	5	1.	10	0.5
59	A	5	4	1.	10	0.4
60	A	8	6	1.	10	0.6
61	A	7	6	1.	8	0.75
62	A	6	6	1.	6	1.
63	A	5	5	1.	10	0.5
64	A	3	3	1.	10	0.3
65	A	4	4	1.	10	0.4
66	A	5	4	1.	10	0.4
67	A	6	4	1.	10	0.4
68	A	3	3	1.	12	0.25
69	A	5	5	1.	10	0.5
70	A	5	5	1.	10	0.5
71	A	4	4	1.	12	0.333
72	A	4	4	1.	14	0.286
73	A	3	2	1.	14	0.143
74	A	5	4	1.	14	0.286
75	A	2	2	1.	14	0.143
76	A	4	3	1.	12	0.25
77	A	1	1	1.	14	0.071
78	A	1	1	1.	14	0.071
79	A	2	2	1.	14	0.143
80	A	3	2	1.	14	0.143
81	A	5	4	1.	14	0.286
82	A	2	2	1.	14	0.143
83	A	4	3	1.	12	0.25
84	A	1	1	1.	14	0.071
85	A	1	1	1.	14	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.	14	0.143
87	A	2	2	1.	16	0.125
88	A	2	2	1.	16	0.125
89	A	1	1	1.	16	0.062
90	A	1	1	1.	16	0.062
91	A	1	1	1.	16	0.062
92	A	2	2	1.	16	0.125
93	A	2	2	1.	16	0.125
94	A	2	2	1.	16	0.125
95	A	2	2	1.	16	0.125
96	A	1	1	1.	16	0.062
97	A	1	1	1.	16	0.062
98	A	1	1	1.	16	0.062
99	A	2	2	1.	16	0.125
100	A	2	2	1.	16	0.125
101	A	0	0	0.	0	0.
102	A	8	8	1.	40	0.2
103	A	7	7	1.	40	0.175
104	A	6	7	1.	38	0.184
105	A	0	0	0.	0	0.
106	A	0	0	0.	0	0.
107	A	6	6	1.	10	0.6
108	A	6	4	1.	10	0.4
109	A	6	4	1.	10	0.4
110	A	5	4	1.	8	0.5
111	A	2	2	1.	6	0.333
112	A	6	5	1.	10	0.5
113	A	6	4	1.	10	0.4
114	A	6	6	1.	10	0.6
115	A	3	3	1.	19	0.158
116	A	2	2	1.	17	0.118
117	A	2	2	1.	26	0.077
118	A	2	2	1.	26	0.077

# Chapter 3

## Listing of integrals

### 3.1 $\int (f + gx)^3 \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) dx$

**Optimal.** Leaf size=670

$$\frac{f^2 g (1 - c^2 x^2) \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx))}{c^2} + \frac{1}{2} f^3 x \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) - \frac{f^3 \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx))^2}{4bc \sqrt{1 - c^2 x^2}}$$

```
[Out] -((b*f^2*g*x*Sqrt[d - c^2*d*x^2])/(c*Sqrt[1 - c^2*x^2])) - (2*b*g^3*x*Sqrt[d - c^2*d*x^2])/(15*c^3*Sqrt[1 - c^2*x^2]) + (b*c*f^3*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) - (3*b*f*g^2*x^2*Sqrt[d - c^2*d*x^2])/(16*c*Sqrt[1 - c^2*x^2]) + (b*c*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(3*Sqrt[1 - c^2*x^2]) - (b*g^3*x^3*Sqrt[d - c^2*d*x^2])/(45*c*Sqrt[1 - c^2*x^2]) + (3*b*c*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (b*c*g^3*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[1 - c^2*x^2]) + (f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (3*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(8*c^2) + (3*f*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 - (f^2*g*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/c^2 - (g^3*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*c^4) + (g^3*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^4) - (f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2]) - (3*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*c^3*Sqrt[1 - c^2*x^2])
```

---

**Rubi [A]** time = 0.705186, antiderivative size = 670, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$ , Rules used = {4778, 4764, 4648, 4642, 30, 4678, 4698, 4708, 266, 43, 4690, 12}

$$\frac{f^2 g (1 - c^2 x^2) \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx))}{c^2} + \frac{1}{2} f^3 x \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) - \frac{f^3 \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx))^2}{4bc \sqrt{1 - c^2 x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

```
[Out] -((b*f^2*g*x*Sqrt[d - c^2*d*x^2])/(c*Sqrt[1 - c^2*x^2])) - (2*b*g^3*x*Sqrt[d - c^2*d*x^2])/(15*c^3*Sqrt[1 - c^2*x^2]) + (b*c*f^3*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) - (3*b*f*g^2*x^2*Sqrt[d - c^2*d*x^2])/(16*c*Sqrt[1 - c^2*x^2]) + (b*c*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(3*Sqrt[1 - c^2*x^2]) - (b*g^3*x^3*Sqrt[d - c^2*d*x^2])/(45*c*Sqrt[1 - c^2*x^2]) + (3*b*c*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (b*c*g^3*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[1 - c^2*x^2]) + (f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (3*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(8*c^2) + (3*f*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 - (f^2*g*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/c^2 - (g^3*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*c^4) + (g^3*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^4) - (f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2]) - (3*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*c^3*Sqrt[1 - c^2*x^2])
```

$$\begin{aligned} & x)) / 2 - (3fg^2x\sqrt{d - c^2dx^2}(a + b\text{ArcCos}[cx])) / (8c^2) + (3f \\ & *g^2x^3\sqrt{d - c^2dx^2}(a + b\text{ArcCos}[cx])) / 4 - (f^2g(1 - c^2x^2) \\ & \sqrt{d - c^2dx^2}(a + b\text{ArcCos}[cx])) / c^2 - (g^3(1 - c^2x^2)\sqrt{d - \\ & c^2dx^2}(a + b\text{ArcCos}[cx])) / (3c^4) + (g^3(1 - c^2x^2)^2\sqrt{d - c^2 \\ & dx^2}(a + b\text{ArcCos}[cx])) / (5c^4) - (f^3\sqrt{d - c^2dx^2}(a + b\text{ArcC} \\ & \text{os}[cx])^2) / (4bc\sqrt{1 - c^2x^2}) - (3fg^2\sqrt{d - c^2dx^2}(a + b \\ & * \text{ArcCos}[cx])^2) / (16b^3c^3\sqrt{1 - c^2x^2}) \end{aligned}$$
Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_.))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] := Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
0] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]
```

Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_.))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^p*(a +
b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] := Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] := -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2e*(p +
1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcC
os[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x
^2]), Int[((f*x)^m*(a + b*ArcCos[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] + Dist[
```

$(b*c*n*\sqrt{d + e*x^2})/(f*(m + 2)*\sqrt{1 - c^2*x^2}), \text{Int}[(f*x)^{(m + 1)}*(a + b*\text{ArcCos}[c*x])^{(n - 1)}, x], x) /;$  FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])

#### Rule 4708

$\text{Int}[(a + \text{ArcCos}[c*x])^{(n)}*(f*x)^{(m)}/\sqrt{d + e*x^2}, x\_Symbol] := \text{Simp}[(f*(f*x)^{(m - 1)}*\sqrt{d + e*x^2}*(a + b*\text{ArcCos}[c*x])^{(n)})/(e*m), x] + (\text{Dist}[(f^2*(m - 1))/(c^2*m), \text{Int}[(f*x)^{(m - 2)}*(a + b*\text{ArcCos}[c*x])^{(n)}/\sqrt{d + e*x^2}, x], x] - \text{Dist}[(b*f*n*\sqrt{1 - c^2*x^2})/(c*m*\sqrt{d + e*x^2}), \text{Int}[(f*x)^{(m - 1)}*(a + b*\text{ArcCos}[c*x])^{(n - 1)}, x], x]) /;$  FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]

#### Rule 266

$\text{Int}[x^{(m)}*(a + b*x^{(n)})^{(p)}, x\_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)}*(a + b*x)^p, x], x, x^n], x] /;$  FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 43

$\text{Int}[(a + b*x)^{(m)}*(c + d*x)^{(n)}, x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 4690

$\text{Int}[(a + \text{ArcCos}[c*x])^{(m)}*(d + e*x^2)^{(p)}, x\_Symbol] := \text{With}[\{u = \text{IntHide}[x^m*(1 - c^2*x^2)^p, x]\}, \text{Dist}[d^p*(a + b*\text{ArcCos}[c*x]), u, x] + \text{Dist}[b*c*d^p, \text{Int}[\text{SimplifyIntegrand}[u/\sqrt{1 - c^2*x^2}, x], x], x]] /;$  FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2\*p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d, 0]

#### Rule 12

$\text{Int}[a*(u), x\_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rubi steps

$$\begin{aligned}
\int (f + gx)^3 \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) dx &= \frac{\sqrt{d - c^2 dx^2} \int (f + gx)^3 \sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{\sqrt{d - c^2 dx^2} \int (f^3 \sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) + 3f^2 gx \sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{(f^3 \sqrt{d - c^2 dx^2}) \int \sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} + \frac{(3f^2 g \sqrt{d - c^2 dx^2}) \int \sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{1}{2} f^3 x \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) + \frac{3}{4} f g^2 x^3 \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) \\
&= -\frac{b f^2 g x \sqrt{d - c^2 dx^2}}{c \sqrt{1 - c^2 x^2}} + \frac{b c f^3 x^2 \sqrt{d - c^2 dx^2}}{4 \sqrt{1 - c^2 x^2}} + \frac{b c f^2 g x^3 \sqrt{d - c^2 dx^2}}{3 \sqrt{1 - c^2 x^2}} + \frac{3 b c f g^2 x^4 \sqrt{d - c^2 dx^2}}{16 \sqrt{1 - c^2 x^2}} \\
&= -\frac{b f^2 g x \sqrt{d - c^2 dx^2}}{c \sqrt{1 - c^2 x^2}} - \frac{2 b g^3 x \sqrt{d - c^2 dx^2}}{15 c^3 \sqrt{1 - c^2 x^2}} + \frac{b c f^3 x^2 \sqrt{d - c^2 dx^2}}{4 \sqrt{1 - c^2 x^2}} - \frac{3 b f g^2 x^2 \sqrt{d - c^2 dx^2}}{16 c \sqrt{1 - c^2 x^2}}
\end{aligned}$$

**Mathematica [A]** time = 1.20368, size = 442, normalized size = 0.66

$$240a\sqrt{1 - c^2 x^2} \sqrt{d - c^2 dx^2} (6c^4 x (20f^2 gx + 10f^3 + 15fg^2 x^2 + 4g^3 x^3) - c^2 g (120f^2 + 45fgx + 8g^2 x^2) - 16g^3) - 3600ac \sqrt{1 - c^2 x^2} \sqrt{d - c^2 dx^2} (6c^4 x (20f^2 gx + 10f^3 + 15fg^2 x^2 + 4g^3 x^3) - c^2 g (120f^2 + 45fgx + 8g^2 x^2) - 16g^3) - 3600ac \sqrt{1 - c^2 x^2} \sqrt{d - c^2 dx^2} (6c^4 x (20f^2 gx + 10f^3 + 15fg^2 x^2 + 4g^3 x^3) - c^2 g (120f^2 + 45fgx + 8g^2 x^2) - 16g^3) - 3600ac \sqrt{1 - c^2 x^2} \sqrt{d - c^2 dx^2} (6c^4 x (20f^2 gx + 10f^3 + 15fg^2 x^2 + 4g^3 x^3) - c^2 g (120f^2 + 45fgx + 8g^2 x^2) - 16g^3)$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^3\*Sqrt[d - c^2\*d\*x^2]\*(a + b\*ArcCos[c\*x]),x]

[Out] (240\*a\*Sqrt[1 - c^2\*x^2]\*Sqrt[d - c^2\*d\*x^2]\*(-16\*g^3 - c^2\*g\*(120\*f^2 + 45\*f\*g\*x + 8\*g^2\*x^2) + 6\*c^4\*x\*(10\*f^3 + 20\*f^2\*g\*x + 15\*f\*g^2\*x^2 + 4\*g^3\*x^3)) - 3600\*a\*c\*Sqrt[d]\*f\*(4\*c^2\*f^2 + 3\*g^2)\*Sqrt[1 - c^2\*x^2]\*ArcTan[(c\*x\*Sqrt[d - c^2\*d\*x^2])/(Sqrt[d]\*(-1 + c^2\*x^2))] - 2400\*b\*c^2\*f^2\*g\*Sqrt[d - c^2\*d\*x^2]\*(9\*c\*x + 12\*(1 - c^2\*x^2)^(3/2)\*ArcCos[c\*x] - Cos[3\*ArcCos[c\*x]]) + 3600\*b\*c^3\*f^3\*Sqrt[d - c^2\*d\*x^2]\*(Cos[2\*ArcCos[c\*x]] + 2\*ArcCos[c\*x]\*(-ArcCos[c\*x] + Sin[2\*ArcCos[c\*x]])) + 675\*b\*c\*f\*g^2\*Sqrt[d - c^2\*d\*x^2]\*(-8\*ArcCos[c\*x]^2 + Cos[4\*ArcCos[c\*x]] + 4\*ArcCos[c\*x]\*Sin[4\*ArcCos[c\*x]]) - 8\*b\*g^3\*Sqrt[d - c^2\*d\*x^2]\*(16\*c\*x\*(30 + 5\*c^2\*x^2 - 9\*c^4\*x^4) + 15\*ArcCos[c\*x]\*(30\*Sqrt[1 - c^2\*x^2] - 5\*Sin[3\*ArcCos[c\*x]] - 3\*Sin[5\*ArcCos[c\*x]])))/(28800\*c^4\*Sqrt[1 - c^2\*x^2])

**Maple [B]** time = 0.848, size = 1285, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^3\*(a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2),x)

[Out] 1/8\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*f^3/c/(c^2\*x^2-1)\*(-c^2\*x^2+1)^(1/2)-1/5\*a\*g^3\*x^2\*(-c^2\*d\*x^2+d)^(3/2)/c^2/d+3/8\*a\*f\*g^2/c^2\*x\*(-c^2\*d\*x^2+d)^(1/2)-a\*f^2\*g/c^2/d\*(-c^2\*d\*x^2+d)^(3/2)+2/15\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*g^3/c^4/(c^2\*x^2-1)\*arccos(c\*x)-4/15\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*g^3/(c^2\*x^2-1)\*arccos(c\*x)\*x^4-1/2\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*f^3/(c^2\*x^2-1)\*arccos(c\*x)\*x-2/15\*a\*g^3/

$$\begin{aligned} & d/c^4*(-c^2*d*x^2+d)^{(3/2)}+3/16*b*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)} \\ & /c^3/(c^2*x^2-1)*\arccos(c*x)^2*f*g^2+3/4*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g^2*c^2 \\ & /c^2*x^2-1)*\arccos(c*x)*x^5+3/8*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g^2/c^2/(c^2*x^2-1) \\ & *x+b*(-d*(c^2*x^2-1))^{(1/2)}*g*c^2/(c^2*x^2-1)*\arccos(c*x)*x^4*f^2-1/3*b*(-d*(c^2*x^2-1))^{(1/2)} \\ & *g*c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^3*f^2+b*(-d*(c^2*x^2-1))^{(1/2)}*g/c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x*f^2-3/16*b \\ & *(-d*(c^2*x^2-1))^{(1/2)}*f*g^2*c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^4+3/16*b*(-d*(c^2*x^2-1))^{(1/2)} \\ & *f*g^2/c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^2-3/4*a*f*g^2*x*(-c^2*d*x^2+d)^{(3/2)}/c^2/d+3/8*a*f*g^2/c^2*d/(c^2*d)^{(1/2)} \\ & *\arctan((c^2*d)^{(1/2)}*x/(-c^2*d*x^2+d)^{(1/2)})-1/25*b*(-d*(c^2*x^2-1))^{(1/2)}*g^3*c/(c^2*x^2-1) \\ & *(-c^2*x^2+1)^{(1/2)}*x^5+1/45*b*(-d*(c^2*x^2-1))^{(1/2)}*g^3/c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^3+2/15*b \\ & *(-d*(c^2*x^2-1))^{(1/2)}*g^3/c^3/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x-3/128*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g^2/c^3/(c^2*x^2-1) \\ & *(-c^2*x^2+1)^{(1/2)}-1/4*b*(-d*(c^2*x^2-1))^{(1/2)}*f^3*c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^2-9/8*b \\ & *(-d*(c^2*x^2-1))^{(1/2)}*f*g^2/(c^2*x^2-1)*\arccos(c*x)*x^3-2*b*(-d*(c^2*x^2-1))^{(1/2)}*g/(c^2*x^2-1) \\ & *\arccos(c*x)*x^2*f^2+1/2*a*f^3*x*(-c^2*d*x^2+d)^{(1/2)}+1/4*b*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/c/(c^2*x^2-1) \\ & *\arccos(c*x)^2*f^3+1/5*b*(-d*(c^2*x^2-1))^{(1/2)}*g^3*c^2/(c^2*x^2-1)*\arccos(c*x)*x^6-1/15*b \\ & *(-d*(c^2*x^2-1))^{(1/2)}*g^3/c^2/(c^2*x^2-1)*\arccos(c*x)*x^2+b*(-d*(c^2*x^2-1))^{(1/2)}*g/c^2/(c^2*x^2-1) \\ & *\arccos(c*x)*f^2+1/2*b*(-d*(c^2*x^2-1))^{(1/2)}*f^3*c^2/(c^2*x^2-1)*\arccos(c*x)*x^3+1/2*a*f^3*d/(c^2*d)^{(1/2)} \\ & *\arctan((c^2*d)^{(1/2)}*x/(-c^2*d*x^2+d)^{(1/2)}) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral((ag^3\*x^3 + 3afg^2\*x^2 + 3af^2\*gx + af^3 + (bg^3\*x^3 + 3bfg^2\*x^2 + 3bf^2\*gx + bf^3) arccos(cx))sqrt(-c^2\*dx^2 + d), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral((a\*g^3\*x^3 + 3\*a\*f\*g^2\*x^2 + 3\*a\*f^2\*g\*x + a\*f^3 + (b\*g^3\*x^3 + 3\*b\*f\*g^2\*x^2 + 3\*b\*f^2\*g\*x + b\*f^3)\*arccos(c\*x))\*sqrt(-c^2\*d\*x^2 + d), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*acos(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c^2 dx^2 + d} (gx + f)^3 (b \arccos(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(g*x + f)^3*(b*arccos(c*x) + a), x)
```



### 3.2 $\int (f + gx)^2 \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) dx$

**Optimal.** Leaf size=450

$$\frac{1}{2} f^2 x \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) - \frac{f^2 \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx))^2}{4bc\sqrt{1 - c^2 x^2}} - \frac{2fg(1 - c^2 x^2) \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx))}{3c^2}$$

```
[Out] (-2*b*f*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[1 - c^2*x^2]) + (b*c*f^2*x^2*Sqr
t[d - c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) - (b*g^2*x^2*Sqrt[d - c^2*d*x^2])/(
16*c*Sqrt[1 - c^2*x^2]) + (2*b*c*f*g*x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt[1 - c
^2*x^2]) + (b*c*g^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (f^2*
x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (g^2*x*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCos[c*x]))/(8*c^2) + (g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*
x]))/4 - (2*f*g*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*c
^2) - (f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x
^2]) - (g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*c^3*Sqrt[1 - c
^2*x^2])
```

**Rubi [A]** time = 0.529749, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$ , Rules used = {4778, 4764, 4648, 4642, 30, 4678, 4698, 4708}

$$\frac{1}{2} f^2 x \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) - \frac{f^2 \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx))^2}{4bc\sqrt{1 - c^2 x^2}} - \frac{2fg(1 - c^2 x^2) \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx))}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-2*b*f*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[1 - c^2*x^2]) + (b*c*f^2*x^2*Sqr
t[d - c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) - (b*g^2*x^2*Sqrt[d - c^2*d*x^2])/(
16*c*Sqrt[1 - c^2*x^2]) + (2*b*c*f*g*x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt[1 - c
^2*x^2]) + (b*c*g^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (f^2*
x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (g^2*x*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCos[c*x]))/(8*c^2) + (g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*
x]))/4 - (2*f*g*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*c
^2) - (f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x
^2]) - (g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*c^3*Sqrt[1 - c
^2*x^2])
```

#### Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^ (n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
0] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]
```

#### Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^ (n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p + 1)), x] - Dist[(b*n*d*IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x^2]), Int[((f*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (f + gx)^2 \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) dx &= \frac{\sqrt{d - c^2 dx^2} \int (f + gx)^2 \sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{\sqrt{d - c^2 dx^2} \int (f^2 \sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) + 2fgx \sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) + g^2 x^3 \sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx))) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{(f^2 \sqrt{d - c^2 dx^2}) \int \sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} + \frac{(2fg \sqrt{d - c^2 dx^2}) \int \sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} + \frac{(g^2 \sqrt{d - c^2 dx^2}) \int x^3 \sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{1}{2} f^2 x \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) + \frac{1}{4} g^2 x^3 \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) \\
&= -\frac{2bfgx \sqrt{d - c^2 dx^2}}{3c \sqrt{1 - c^2 x^2}} + \frac{bcf^2 x^2 \sqrt{d - c^2 dx^2}}{4 \sqrt{1 - c^2 x^2}} + \frac{2bcfgx^3 \sqrt{d - c^2 dx^2}}{9 \sqrt{1 - c^2 x^2}} + \frac{bcg^2 x^4 \sqrt{d - c^2 dx^2}}{12 \sqrt{1 - c^2 x^2}} \\
&= -\frac{2bfgx \sqrt{d - c^2 dx^2}}{3c \sqrt{1 - c^2 x^2}} + \frac{bcf^2 x^2 \sqrt{d - c^2 dx^2}}{4 \sqrt{1 - c^2 x^2}} - \frac{bg^2 x^2 \sqrt{d - c^2 dx^2}}{16c \sqrt{1 - c^2 x^2}} + \frac{2bcfg^2 x^3 \sqrt{d - c^2 dx^2}}{9 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

**Mathematica [A]** time = 1.3145, size = 320, normalized size = 0.71

$$48ac \sqrt{1 - c^2 x^2} \sqrt{d - c^2 dx^2} (12c^2 f^2 x + 16fg(c^2 x^2 - 1) + 3g^2 x(2c^2 x^2 - 1)) - 144a \sqrt{d} \sqrt{1 - c^2 x^2} (4c^2 f^2 + g^2) \tan^{-1} \left( \frac{cx}{\sqrt{d - c^2 dx^2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2\*Sqrt[d - c^2\*d\*x^2]\*(a + b\*ArcCos[c\*x]),x]

[Out] (48\*a\*c\*Sqrt[1 - c^2\*x^2]\*Sqrt[d - c^2\*d\*x^2]\*(12\*c^2\*f^2\*x + 16\*f\*g\*(-1 + c^2\*x^2) + 3\*g^2\*x\*(-1 + 2\*c^2\*x^2)) - 144\*a\*Sqrt[d]\*(4\*c^2\*f^2 + g^2)\*Sqrt[1 - c^2\*x^2]\*ArcTan[(c\*x\*Sqrt[d - c^2\*d\*x^2])/(Sqrt[d]\*(-1 + c^2\*x^2))] - 64\*b\*c\*f\*g\*Sqrt[d - c^2\*d\*x^2]\*(9\*c\*x + 12\*(1 - c^2\*x^2)^(3/2)\*ArcCos[c\*x] - Cos[3\*ArcCos[c\*x]]) + 144\*b\*c^2\*f^2\*Sqrt[d - c^2\*d\*x^2]\*(Cos[2\*ArcCos[c\*x]] + 2\*ArcCos[c\*x]\*(-ArcCos[c\*x] + Sin[2\*ArcCos[c\*x]])) + 9\*b\*g^2\*Sqrt[d - c^2\*d\*x^2]\*(-8\*ArcCos[c\*x]^2 + Cos[4\*ArcCos[c\*x]] + 4\*ArcCos[c\*x]\*Sin[4\*ArcCos[c\*x]]))/(1152\*c^3\*Sqrt[1 - c^2\*x^2])

**Maple [B]** time = 0.533, size = 912, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2\*(a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2),x)

[Out] -1/4\*a\*g^2\*x\*(-c^2\*d\*x^2+d)^(3/2)/c^2/d+1/8\*a\*g^2/c^2\*x\*(-c^2\*d\*x^2+d)^(1/2)+1/8\*a\*g^2/c^2\*d/(c^2\*d)^(1/2)\*arctan((c^2\*d)^(1/2)\*x/(-c^2\*d\*x^2+d)^(1/2))-2/3\*a\*f\*g/c^2/d\*(-c^2\*d\*x^2+d)^(3/2)+1/2\*a\*f^2\*x\*(-c^2\*d\*x^2+d)^(1/2)+1/2\*a\*f^2\*d/(c^2\*d)^(1/2)\*arctan((c^2\*d)^(1/2)\*x/(-c^2\*d\*x^2+d)^(1/2))-2/9\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*f\*g\*c/(c^2\*x^2-1)\*(-c^2\*x^2+1)^(1/2)\*x^3+2/3\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*f\*g/c/(c^2\*x^2-1)\*(-c^2\*x^2+1)^(1/2)\*x+1/4\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*(-c^2\*x^2+1)^(1/2)/c/(c^2\*x^2-1)\*arccos(c\*x)^2\*f^2+1/16\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*(-c^2\*x^2+1)^(1/2)/c^3/(c^2\*x^2-1)\*arccos(c\*x)^2\*g^2+1/4\*b\*

$$(-d*(c^2*x^2-1))^{(1/2)}*g^2*c^2/(c^2*x^2-1)*\arccos(c*x)*x^5-3/8*b*(-d*(c^2*x^2-1))^{(1/2)}*g^2/(c^2*x^2-1)*\arccos(c*x)*x^3+1/8*b*(-d*(c^2*x^2-1))^{(1/2)}*g^2/c^2/(c^2*x^2-1)*\arccos(c*x)*x-1/128*b*(-d*(c^2*x^2-1))^{(1/2)}*g^2/c^3/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}+2/3*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g/c^2/(c^2*x^2-1)*\arccos(c*x)+1/2*b*(-d*(c^2*x^2-1))^{(1/2)}*f^2*c^2/(c^2*x^2-1)*\arccos(c*x)*x^3-1/2*b*(-d*(c^2*x^2-1))^{(1/2)}*f^2/(c^2*x^2-1)*\arccos(c*x)*x+1/8*b*(-d*(c^2*x^2-1))^{(1/2)}*f^2/c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}-1/16*b*(-d*(c^2*x^2-1))^{(1/2)}*g^2*c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^4+1/16*b*(-d*(c^2*x^2-1))^{(1/2)}*g^2/c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^2+2/3*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g*c^2/(c^2*x^2-1)*\arccos(c*x)*x^4-4/3*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g/(c^2*x^2-1)*\arccos(c*x)*x^2-1/4*b*(-d*(c^2*x^2-1))^{(1/2)}*f^2*c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^2$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2\*(a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-c^2dx^2+d}(ag^2x^2+2afgx+af^2+(bg^2x^2+2bfgx+bf^2)\arccos(cx)),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2\*(a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-c^2\*d\*x^2 + d)\*(a\*g^2\*x^2 + 2\*a\*f\*g\*x + a\*f^2 + (b\*g^2\*x^2 + 2\*b\*f\*g\*x + b\*f^2)\*arccos(c\*x)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2\*(a+b\*acos(c\*x))\*(-c\*\*2\*d\*x\*\*2+d)\*\*(1/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c^2dx^2+d}(gx+f)^2(b\arccos(cx)+a)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="gi  
ac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(g*x + f)^2*(b*arccos(c*x) + a), x)
```

### 3.3 $\int (f + gx)\sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) dx$

**Optimal.** Leaf size=238

$$\frac{1}{2}fx\sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) - \frac{f\sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx))^2}{4bc\sqrt{1 - c^2 x^2}} - \frac{g(1 - c^2 x^2)\sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx))}{3c^2} + \frac{bcfx^2}{4\sqrt{1 - c^2 x^2}}$$

```
[Out] -(b*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[1 - c^2*x^2]) + (b*c*f*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) + (b*c*g*x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt[1 - c^2*x^2]) + (f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (g*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*c^2) - (f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2])
```

**Rubi [A]** time = 0.251246, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {4778, 4764, 4648, 4642, 30, 4678}

$$\frac{1}{2}fx\sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) - \frac{f\sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx))^2}{4bc\sqrt{1 - c^2 x^2}} - \frac{g(1 - c^2 x^2)\sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx))}{3c^2} + \frac{bcfx^2}{4\sqrt{1 - c^2 x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

```
[Out] -(b*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[1 - c^2*x^2]) + (b*c*f*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) + (b*c*g*x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt[1 - c^2*x^2]) + (f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (g*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*c^2) - (f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2])
```

#### Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]
```

#### Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

#### Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4642

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] :> -Simp[(a + b\*ArcCos[c\*x])^(n + 1)/(b\*c\*Sqrt[d]\*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 4678

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCos[c\*x])^n)/(2\*e\*(p + 1)), x] - Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcCos[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int (f + gx)\sqrt{d - c^2dx^2} (a + b \cos^{-1}(cx)) dx &= \frac{\sqrt{d - c^2dx^2} \int (f + gx)\sqrt{1 - c^2x^2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2x^2}} \\
 &= \frac{\sqrt{d - c^2dx^2} \int (f\sqrt{1 - c^2x^2} (a + b \cos^{-1}(cx)) + gx\sqrt{1 - c^2x^2} (a + b \cos^{-1}(cx))) dx}{\sqrt{1 - c^2x^2}} \\
 &= \frac{(f\sqrt{d - c^2dx^2}) \int \sqrt{1 - c^2x^2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2x^2}} + \frac{(g\sqrt{d - c^2dx^2}) \int \sqrt{1 - c^2x^2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2x^2}} \\
 &= \frac{1}{2}fx\sqrt{d - c^2dx^2} (a + b \cos^{-1}(cx)) - \frac{g(1 - c^2x^2)\sqrt{d - c^2dx^2} (a + b \cos^{-1}(cx))}{3c^2} \\
 &= -\frac{bgx\sqrt{d - c^2dx^2}}{3c\sqrt{1 - c^2x^2}} + \frac{bcfx^2\sqrt{d - c^2dx^2}}{4\sqrt{1 - c^2x^2}} + \frac{bcgx^3\sqrt{d - c^2dx^2}}{9\sqrt{1 - c^2x^2}} + \frac{1}{2}fx\sqrt{d - c^2dx^2} (a + b \cos^{-1}(cx))
 \end{aligned}$$

**Mathematica [A]** time = 1.47429, size = 219, normalized size = 0.92

$$\frac{12a\sqrt{d - c^2dx^2} (3c^2fx + 2g(c^2x^2 - 1)) - 36ac\sqrt{d}f \tan^{-1}\left(\frac{cx\sqrt{d - c^2dx^2}}{\sqrt{d}(c^2x^2 - 1)}\right) + \frac{9bcf\sqrt{d - c^2dx^2}(-2\cos^{-1}(cx))^2 + \cos(2\cos^{-1}(cx)) + 2\cos^{-1}(cx)}{\sqrt{1 - c^2x^2}}}{72c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)\*Sqrt[d - c^2\*d\*x^2]\*(a + b\*ArcCos[c\*x]), x]

[Out] (12\*a\*Sqrt[d - c^2\*d\*x^2]\*(3\*c^2\*f\*x + 2\*g\*(-1 + c^2\*x^2)) - 36\*a\*c\*Sqrt[d]\*f\*ArcTan[(c\*x\*Sqrt[d - c^2\*d\*x^2])/(Sqrt[d]\*(-1 + c^2\*x^2))] + (2\*b\*g\*Sqrt[d - c^2\*d\*x^2]\*(-9\*c\*x - 12\*(1 - c^2\*x^2)^(3/2)\*ArcCos[c\*x] + Cos[3\*ArcCos[c\*x]]))/Sqrt[1 - c^2\*x^2] + (9\*b\*c\*f\*Sqrt[d - c^2\*d\*x^2]\*(-2\*ArcCos[c\*x]^2 + Cos[2\*ArcCos[c\*x]] + 2\*ArcCos[c\*x]\*Sin[2\*ArcCos[c\*x]]))/Sqrt[1 - c^2\*x^2])/(72\*c^2)

**Maple [B]** time = 0.481, size = 491, normalized size = 2.1

$$-\frac{ag}{3c^2d}(-c^2dx^2+d)^{\frac{3}{2}}+\frac{afx}{2}\sqrt{-c^2dx^2+d}+\frac{afd}{2}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}+\frac{bg\arccos(cx)}{3c^2(c^2x^2-1)}\sqrt{-d(c^2x^2-1)}+$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)\*(a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2),x)

[Out] 
$$-1/3*a*g/c^2/d*(-c^2*d*x^2+d)^{(3/2)}+1/2*a*f*x*(-c^2*d*x^2+d)^{(1/2)}+1/2*a*f*d/(c^2*d)^{(1/2)}*\arctan((c^2*d)^{(1/2)}*x/(-c^2*d*x^2+d)^{(1/2)})+1/3*b*(-d*(c^2*x^2-1))^{(1/2)}*g/c^2/(c^2*x^2-1)*\arccos(c*x)+1/3*b*(-d*(c^2*x^2-1))^{(1/2)}*g*c^2/(c^2*x^2-1)*\arccos(c*x)*x^4-2/3*b*(-d*(c^2*x^2-1))^{(1/2)}*g/(c^2*x^2-1)*\arccos(c*x)*x^2+1/2*b*(-d*(c^2*x^2-1))^{(1/2)}*f*c^2/(c^2*x^2-1)*\arccos(c*x)*x^3-1/2*b*(-d*(c^2*x^2-1))^{(1/2)}*f/(c^2*x^2-1)*\arccos(c*x)*x+1/8*b*(-d*(c^2*x^2-1))^{(1/2)}*f/c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}+1/4*b*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/c/(c^2*x^2-1)*\arccos(c*x)^2*f-1/9*b*(-d*(c^2*x^2-1))^{(1/2)}*g*c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^3+1/3*b*(-d*(c^2*x^2-1))^{(1/2)}*g/c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x-1/4*b*(-d*(c^2*x^2-1))^{(1/2)}*f*c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^2$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-c^2dx^2+d}\left(agx+af+(bgx+bf)\arccos(cx)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-c^2\*d\*x^2+d)\*(a\*g\*x+a\*f+(b\*g\*x+b\*f)\*arccos(c\*x)),x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(a+b\*acos(c\*x))\*(-c\*\*2\*d\*x\*\*2+d)\*\*(1/2),x)



[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c^2 dx^2 + d}(gx + f)(b \arccos(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-c^2\*d\*x^2 + d)\*(g\*x + f)\*(b\*arccos(c\*x) + a), x)

$$3.4 \quad \int \frac{\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))}{f+gx} dx$$

**Optimal.** Leaf size=725

$$\frac{b\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}\text{PolyLog}\left(2, -\frac{g^{e^{i\cos^{-1}(cx)}}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}\text{PolyLog}\left(2, -\frac{g^{e^{i\cos^{-1}(cx)}}}{\sqrt{c^2f^2-g^2}+cf}\right)}{g^2\sqrt{1-c^2x^2}} + \frac{\sqrt{d-c^2dx^2}}{2b}$$

```
[Out] (a*Sqrt[d - c^2*d*x^2])/g + (b*c*x*Sqrt[d - c^2*d*x^2])/(g*Sqrt[1 - c^2*x^2]) + (b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])/g - (c*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*g*Sqrt[1 - c^2*x^2]) + ((1 - (c^2*f^2)/g^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(f + g*x)*Sqrt[1 - c^2*x^2]) - (Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(f + g*x)) - (a*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])])/(g^2*Sqrt[1 - c^2*x^2]) - (I*b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[1 - c^2*x^2]) + (I*b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[1 - c^2*x^2]) - (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqrt[1 - c^2*x^2]) + (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqrt[1 - c^2*x^2])
```

**Rubi [A]** time = 1.85891, antiderivative size = 725, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 19, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.613$ , Rules used = {4778, 4766, 683, 4758, 6742, 725, 204, 1654, 12, 4800, 4798, 4678, 8, 4774, 3321, 2264, 2190, 2279, 2391}

$$\frac{b\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}\text{PolyLog}\left(2, -\frac{g^{e^{i\cos^{-1}(cx)}}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}\text{PolyLog}\left(2, -\frac{g^{e^{i\cos^{-1}(cx)}}}{\sqrt{c^2f^2-g^2}+cf}\right)}{g^2\sqrt{1-c^2x^2}} + \frac{\sqrt{d-c^2dx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(f + g*x), x]
```

```
[Out] (a*Sqrt[d - c^2*d*x^2])/g + (b*c*x*Sqrt[d - c^2*d*x^2])/(g*Sqrt[1 - c^2*x^2]) + (b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])/g - (c*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*g*Sqrt[1 - c^2*x^2]) + ((1 - (c^2*f^2)/g^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(f + g*x)*Sqrt[1 - c^2*x^2]) - (Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(f + g*x)) - (a*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])])/(g^2*Sqrt[1 - c^2*x^2]) - (I*b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[1 - c^2*x^2]) + (I*b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[1 - c^2*x^2]) - (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqrt[1 - c^2*x^2]) + (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqrt[1 - c^2*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] := Dist[(d*IntPart[p]*(d + e*x^2)^FracPart
[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
0] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]
```

#### Rule 4766

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*Sqrt[
(d_) + (e_.)*(x_)^2], x_Symbol] := -Simp[((f + g*x)^m*(d + e*x^2)*(a + b*Ar
cCos[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] + Dist[1/(b*c*Sqrt[d]*(n + 1)
), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCos[
c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e,
0] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

#### Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

#### Rule 4758

```
Int((((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_) + (h_.)*(x
_)^2)^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := With[{u = IntHide[(f + g*x
+ h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcCos[c*x])^n, u, x] + Dist[b*c*
n, Int[SimplifyIntegrand[(u*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ[p,
0] && EqQ[e*g - 2*d*h, 0]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

#### Rule 1654

```
Int[(Pq)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, c, d,
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
```

1/2, 0]))

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 4800

Int[(ArcCos[(c\_)\*(x\_)]\*(b\_) + (a\_))^(n\_)\*(RFx\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x^2)^p, RFx\*(a + b\*ArcCos[c\*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[c^2\*d + e, 0] && IntegerQ[p - 1/2]

### Rule 4798

Int[ArcCos[(c\_)\*(x\_)]^(n\_)\*(RFx\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{u = ExpandIntegrand[(d + e\*x^2)^p\*ArcCos[c\*x]^n, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[c^2\*d + e, 0] && IntegerQ[p - 1/2]

### Rule 4678

Int[((a\_) + ArcCos[(c\_)\*(x\_)]\*(b\_))^(n\_)\*(x\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCos[c\*x])^n)/(2\*e\*(p + 1)), x] - Dist[(b\*n\*d\*IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcCos[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 4774

Int[(((a\_) + ArcCos[(c\_)\*(x\_)]\*(b\_))^(n\_)\*((f\_) + (g\_)\*(x\_))^(m\_))/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := -Dist[(c^(m + 1)\*Sqrt[d])^(-1), Subst[Int[(a + b\*x)^n\*(c\*f + g\*Cos[x])^m, x], x, ArcCos[c\*x]], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[m] && GtQ[d, 0] && (GtQ[m, 0] || IGtQ[n, 0])

### Rule 3321

Int[((c\_) + (d\_)\*(x\_))^(m\_)/((a\_) + (b\_)\*sin[(e\_) + Pi\*(k\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(I\*Pi\*(k - 1/2))\*E^(I\*(e + f\*x)))/(b + 2\*a\*E^(I\*Pi\*(k - 1/2))\*E^(I\*(e + f\*x)) - b\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 2264

Int[((F\_)^(u\_)\*((f\_) + (g\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*(F\_)^(u\_) + (c\_)\*(F\_)^(v\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[((f + g\*x)^m\*F^u)/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[((f + g\*x)^m\*F^u)/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*(F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2279

```
Int[Log[(a_) + (b_)*(F_)^((e_)*((c_) + (d_)*(x_)))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

#### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))}{f+gx} dx &= \frac{\sqrt{d-c^2dx^2} \int \frac{\sqrt{1-c^2x^2}(a+b\cos^{-1}(cx))}{f+gx} dx}{\sqrt{1-c^2x^2}} \\
&= -\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(f+gx)} + \frac{\sqrt{d-c^2dx^2} \int \frac{(-g-2c^2fx-c^2gx^2)(a+b\cos^{-1}(cx))}{(f+gx)^2} dx}{2bc\sqrt{1-c^2x^2}} \\
&= -\frac{cx\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(f+gx)\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc\sqrt{1-c^2x^2}} \\
&= -\frac{cx\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(f+gx)\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc\sqrt{1-c^2x^2}} \\
&= -\frac{cx\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(f+gx)\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc\sqrt{1-c^2x^2}} \\
&= \frac{a\sqrt{d-c^2dx^2}}{g} - \frac{cx\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(f+gx)\sqrt{1-c^2x^2}} \\
&= \frac{a\sqrt{d-c^2dx^2}}{g} - \frac{cx\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(f+gx)\sqrt{1-c^2x^2}} \\
&= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(f+gx)\sqrt{1-c^2x^2}} \\
&= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}} \\
&= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}} \\
&= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}} \\
&= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}} \\
&= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}} \\
&= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 3.37971, size = 1095, normalized size = 1.51

$$-2a\sqrt{d-c^2dx^2}g + 2ac\sqrt{d}f \tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(c^2x^2-1)}\right) - 2a\sqrt{d}\sqrt{g^2-c^2f^2} \log(f+gx) + 2a\sqrt{d}\sqrt{g^2-c^2f^2} \log(d(fxc^2+g)) + \dots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(f + g*x), x]
```

```
[Out] -(2*a*g*Sqrt[d - c^2*d*x^2] + 2*a*c*Sqrt[d]*f*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))]) - 2*a*Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Log[f + g*x] + 2*a*Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Log[d*(g + c^2*f*x) + Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]] + b*Sqrt[d - c^2*d*x^2]*((-2*c*g*x)/Sqrt[1 - c^2*x^2] - 2*g*ArcCos[c*x] + (c*f*ArcCos[c*x]^2)/Sqrt[1 - c^2*x^2] + (2*(-(c*f) + g)*(c*f + g)*(2*ArcCos[c*x]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2)])/Sqrt[-(c^2*f^2) + g^2]) - 2*ArcCos[-((c*f)/g)]*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]) + (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]) + (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2])]*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^((I/2)*ArcCos[c*x])*Sqrt[g]*Sqrt[c*(f + g*x)])] + (ArcCos[-((c*f)/g)] + (2*I)*(ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2)])/Sqrt[-(c^2*f^2) + g^2]) - ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2])]*Log[(E^((I/2)*ArcCos[c*x])*Sqrt[-(c^2*f^2) + g^2])/Sqrt[2]*Sqrt[g]*Sqrt[c*(f + g*x)]] - (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2])]*Log[((c*f + g)*((-I)*c*f + I*g + Sqrt[-(c^2*f^2) + g^2])*(-I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - (ArcCos[-((c*f)/g)] + (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2])]*Log[((c*f + g)*(I*c*f - I*g + Sqrt[-(c^2*f^2) + g^2])*(I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))])]/Sqrt[-(c^2*f^2) + g^2]*Sqrt[1 - c^2*x^2]))/(2*g^2)
```

---

**Maple [A]** time = 0.437, size = 1209, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f), x)
```

```
[Out] a/g*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)+a/g^2*c^2*d*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+a/g^3*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2))*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))*c^2*f^2-a/g*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2))*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))+1/2*b*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^2*f*c/g^2+b*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/g*arccos(c*x)*x^2*c^2-b*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/g*(-c^2*x^2+1)^(1/2)*x*c-b*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/g*arccos(c*x)+I*b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^2*arccos(c*x)*ln((-c*x+I*(-c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2))-I*b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^2*arccos(c*x)*ln(((c*x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))+b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^2*dilog(-1/(-c*f+(c^2*f^2-g^2)^(1/2))*(c*x+I*(-c^2*x^2+1)^(1/2))*g-1/(-c*f+(c^2*f^2-g^2)^(1/2)))/g^2
```

$$2)^{(1/2)} * c * f + 1 / (-c * f + (c^2 * f^2 - g^2)^{(1/2)}) * (c^2 * f^2 - g^2)^{(1/2)} - b * (-d * (c^2 * x^2 - 1))^{(1/2)} * (c^2 * f^2 - g^2)^{(1/2)} * (-c^2 * x^2 + 1)^{(1/2)} / (c^2 * x^2 - 1) / g^2 * \text{dilog}((c * x + I * (-c^2 * x^2 + 1)^{(1/2)}) * g / (c * f + (c^2 * f^2 - g^2)^{(1/2)}) + 1 / (c * f + (c^2 * f^2 - g^2)^{(1/2)}) * c * f + 1 / (c * f + (c^2 * f^2 - g^2)^{(1/2)}) * (c^2 * f^2 - g^2)^{(1/2)})$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2)/(g\*x+f),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2 dx^2 + d}(b \arccos(cx) + a)}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2)/(g\*x+f),x, algorithm="fricas")

[Out] integral(sqrt(-c^2\*d\*x^2 + d)\*(b\*arccos(c\*x) + a)/(g\*x + f), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-d}(cx - 1)(cx + 1)(a + b \arccos(cx))}{f + gx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acos(c\*x))\*(-c\*\*2\*d\*x\*\*2+d)\*\*(1/2)/(g\*x+f),x)

[Out] Integral(sqrt(-d\*(c\*x - 1)\*(c\*x + 1))\*(a + b\*acos(c\*x))/(f + g\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2 dx^2 + d}(b \arccos(cx) + a)}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2)/(g\*x+f),x, algorithm="giac")

[Out] integrate(sqrt(-c^2\*d\*x^2 + d)\*(b\*arccos(c\*x) + a)/(g\*x + f), x)



$$3.5 \quad \int \frac{\sqrt{d-c^2x^2}(a+b\cos^{-1}(cx))}{(f+gx)^2} dx$$

**Optimal.** Leaf size=851

$$\frac{bf^2\sqrt{d-c^2x^2}\cos^{-1}(cx)c^3}{2g^2(c^2f^2-g^2)\sqrt{1-c^2x^2}} - \frac{af^2\sqrt{d-c^2x^2}\sin^{-1}(cx)c^3}{g^2(c^2f^2-g^2)\sqrt{1-c^2x^2}} + \frac{af\sqrt{d-c^2x^2}\tan^{-1}\left(\frac{fxc^2+g}{\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}\right)c^2}{g^2\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}} + \frac{ibf\sqrt{d-c^2x^2}}{g}$$

```
[Out] -((a*Sqrt[d - c^2*d*x^2])/(g*(f + g*x))) - (b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])/(g*(f + g*x)) + (b*c^3*f^2*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2)/(2*g^2*(c^2*f^2 - g^2)*Sqrt[1 - c^2*x^2]) - ((g + c^2*f*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(c^2*f^2 - g^2)*(f + g*x)^2*Sqrt[1 - c^2*x^2]) - (Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(f + g*x)^2) - (a*c^3*f^2*Sqrt[d - c^2*d*x^2]*ArcSin[c*x])/(g^2*(c^2*f^2 - g^2)*Sqrt[1 - c^2*x^2]) + (a*c^2*f*Sqrt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2]) + (I*b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2]) - (I*b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2]) - (b*c*Sqrt[d - c^2*d*x^2]*Log[f + g*x])/(g^2*Sqrt[1 - c^2*x^2]) + (b*c^2*f*Sqrt[d - c^2*d*x^2]*PolyLog[2, -(E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2]) - (b*c^2*f*Sqrt[d - c^2*d*x^2]*PolyLog[2, -(E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])
```

**Rubi [A]** time = 2.69367, antiderivative size = 851, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 22, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.71$ , Rules used = {4778, 4766, 37, 4756, 12, 1651, 844, 216, 725, 204, 4800, 4798, 4642, 4774, 3324, 3321, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{bf^2\sqrt{d-c^2x^2}\cos^{-1}(cx)c^3}{2g^2(c^2f^2-g^2)\sqrt{1-c^2x^2}} - \frac{af^2\sqrt{d-c^2x^2}\sin^{-1}(cx)c^3}{g^2(c^2f^2-g^2)\sqrt{1-c^2x^2}} + \frac{af\sqrt{d-c^2x^2}\tan^{-1}\left(\frac{fxc^2+g}{\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}\right)c^2}{g^2\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}} + \frac{ibf\sqrt{d-c^2x^2}}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(f + g*x)^2,x]
```

```
[Out] -((a*Sqrt[d - c^2*d*x^2])/(g*(f + g*x))) - (b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])/(g*(f + g*x)) + (b*c^3*f^2*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2)/(2*g^2*(c^2*f^2 - g^2)*Sqrt[1 - c^2*x^2]) - ((g + c^2*f*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(c^2*f^2 - g^2)*(f + g*x)^2*Sqrt[1 - c^2*x^2]) - (Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(f + g*x)^2) - (a*c^3*f^2*Sqrt[d - c^2*d*x^2]*ArcSin[c*x])/(g^2*(c^2*f^2 - g^2)*Sqrt[1 - c^2*x^2]) + (a*c^2*f*Sqrt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2]) + (I*b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2]) - (I*b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2]) - (b*c*Sqrt[d - c^2*d*x^2]*Log[f + g*x])/(g^2*Sqrt[1 - c^2*x^2]) + (b*c^2*f*Sqrt[d - c^2*d*x^2]*PolyLog[2, -(E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2]) - (b*c^2*f*Sqrt[d - c^2*d*x^2]*PolyLog[2, -(E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])
```

$\wedge 2)))] / (g^2 \sqrt{c^2 f^2 - g^2} \sqrt{1 - c^2 x^2})$

#### Rule 4778

$\text{Int}[(a_.) + \text{ArcCos}[c_.*x_)]*(b_.)^{(n_.)}*((f_.) + (g_.*x_))^{(m_.)}*((d_.) + (e_.*x_)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{Dist}[(d^{\text{IntPart}[p]}*(d + e*x^2)^{\text{FracPart}[p]}) / (1 - c^2*x^2)^{\text{FracPart}[p]}, \text{Int}[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*\text{ArcCos}[c*x])^n, x], x] /;$  FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]

#### Rule 4766

$\text{Int}[(a_.) + \text{ArcCos}[c_.*x_)]*(b_.)^{(n_.)}*((f_.) + (g_.*x_))^{(m_.)}*\sqrt{(d_.) + (e_.*x_)^2}, x\_Symbol] \rightarrow -\text{Simp}[(f + g*x)^m*(d + e*x^2)*(a + b*\text{ArcCos}[c*x])^{(n + 1)} / (b*c*\sqrt{d}*(n + 1)), x] + \text{Dist}[1 / (b*c*\sqrt{d}*(n + 1)), \text{Int}[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^{(m - 1)}*(a + b*\text{ArcCos}[c*x])^{(n + 1)}, x], x] /;$  FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2\*d + e, 0] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]

#### Rule 37

$\text{Int}[(a_.) + (b_.*x_))^{(m_.)}*((c_.) + (d_.*x_))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n + 1)} / ((b*c - a*d)*(m + 1)), x] /;$  FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

#### Rule 4756

$\text{Int}[(a_.) + \text{ArcCos}[c_.*x_)]*(b_.)^{(n_.)}*((d_.) + (e_.*x_))^{(m_.)}*((f_.) + (g_.*x_))^{(p_.)}, x\_Symbol] \rightarrow \text{With}[\{u = \text{IntHide}[(f + g*x)^p*(d + e*x)^m, x]\}, \text{Dist}[(a + b*\text{ArcCos}[c*x])^n, u, x] + \text{Dist}[b*c^n, \text{Int}[\text{SimplifyIntegrand}[(u*(a + b*\text{ArcCos}[c*x])^{(n - 1)}) / \sqrt{1 - c^2*x^2}], x], x] /;$  FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[m, 0] && LtQ[m + p + 1, 0]

#### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /;

 FreeQ[b, x]

#### Rule 1651

$\text{Int}[(Pq_)*((d_.) + (e_.*x_))^{(m_.)}*((a_.) + (c_.*x_)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{With}[\{Q = \text{PolynomialQuotient}[Pq, d + e*x, x], R = \text{PolynomialRemainder}[Pq, d + e*x, x]\}, \text{Simp}[(e*R*(d + e*x)^{(m + 1)}*(a + c*x^2)^{(p + 1)}) / ((m + 1)*(c*d^2 + a*e^2)), x] + \text{Dist}[1 / ((m + 1)*(c*d^2 + a*e^2)), \text{Int}[(d + e*x)^{(m + 1)}*(a + c*x^2)^p*\text{ExpandToSum}[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*R*(m + 2*p + 3)*x, x], x] /;$  FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[c\*d^2 + a\*e^2, 0] && LtQ[m, -1]

#### Rule 844

$\text{Int}[(d_.) + (e_.*x_))^{(m_.)}*((f_.) + (g_.*x_))*((a_.) + (c_.*x_)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{Dist}[g/e, \text{Int}[(d + e*x)^{(m + 1)}*(a + c*x^2)^p, x], x] + \text{Dist}[(e*f - d*g)/e, \text{Int}[(d + e*x)^m*(a + c*x^2)^p, x], x] /;$  FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

#### Rule 216

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x\_Symbol] \text{ :> } \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*x]/\text{Sqrt}[a]/\text{Rt}[-b, 2], x] \text{ /; } \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

#### Rule 725

$\text{Int}[1/(((d_) + (e_)*(x_))*\text{Sqrt}[(a_) + (c_)*(x_)^2]), x\_Symbol] \text{ :> } -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] \text{ /; } \text{FreeQ}[\{a, c, d, e\}, x]$

#### Rule 204

$\text{Int}[(a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] \text{ :> } -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] \text{ /; } \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

#### Rule 4800

$\text{Int}[(\text{ArcCos}[(c_)*(x_)]*(b_) + (a_))^{(n_)}*(\text{RFX}_)*((d_) + (e_)*(x_)^2)^{(p_)}, x\_Symbol] \text{ :> } \text{Int}[\text{ExpandIntegrand}[(d + e*x^2)^p, \text{RFX}*(a + b*\text{ArcCos}[c*x])^n, x], x] \text{ /; } \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{RationalFunctionQ}[\text{RFX}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IntegerQ}[p - 1/2]$

#### Rule 4798

$\text{Int}[\text{ArcCos}[(c_)*(x_)]^{(n_)}*(\text{RFX}_)*((d_) + (e_)*(x_)^2)^{(p_)}, x\_Symbol] \text{ :> } \text{With}[\{u = \text{ExpandIntegrand}[(d + e*x^2)^p*\text{ArcCos}[c*x]^n, \text{RFX}, x]\}, \text{Int}[u, x] \text{ /; } \text{SumQ}[u]] \text{ /; } \text{FreeQ}[\{c, d, e\}, x] \ \&\& \ \text{RationalFunctionQ}[\text{RFX}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IntegerQ}[p - 1/2]$

#### Rule 4642

$\text{Int}[(a_) + \text{ArcCos}[(c_)*(x_)]*(b_))^{(n_)} / \text{Sqrt}[(d_) + (e_)*(x_)^2], x\_Symbol] \text{ :> } -\text{Simp}[(a + b*\text{ArcCos}[c*x])^{(n + 1)} / (b*c*\text{Sqrt}[d]*(n + 1)), x] \text{ /; } \text{FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[d, 0] \ \&\& \ \text{NeQ}[n, -1]$

#### Rule 4774

$\text{Int}[(a_) + \text{ArcCos}[(c_)*(x_)]*(b_))^{(n_)}*((f_) + (g_)*(x_))^{(m_)}, x\_Symbol] \text{ :> } -\text{Dist}[(c^{(m + 1)}*\text{Sqrt}[d])^{-1}, \text{Subst}[\text{Int}[(a + b*x)^n*(c*f + g*\text{Cos}[x])^m, x], x, \text{ArcCos}[c*x]], x] \text{ /; } \text{FreeQ}[\{a, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{GtQ}[d, 0] \ \&\& \ (\text{GtQ}[m, 0] \ || \ \text{IGtQ}[n, 0])$

#### Rule 3324

$\text{Int}[(c_) + (d_)*(x_))^{(m_)} / ((a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)]^2, x\_Symbol] \text{ :> } \text{Simp}[(b*(c + d*x)^m*\text{Cos}[e + f*x]) / (f*(a^2 - b^2)*(a + b*\text{Sin}[e + f*x])), x] + (\text{Dist}[a/(a^2 - b^2), \text{Int}[(c + d*x)^m / (a + b*\text{Sin}[e + f*x]), x], x] - \text{Dist}[(b*d*m) / (f*(a^2 - b^2)), \text{Int}[(c + d*x)^{(m - 1)}*\text{Cos}[e + f*x]) / (a + b*\text{Sin}[e + f*x]), x], x)) \text{ /; } \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{IGtQ}[m, 0]$

#### Rule 3321

$\text{Int}[(c_) + (d_)*(x_))^{(m_)} / ((a_) + (b_)*\text{sin}[(e_) + \text{Pi}*(k_) + (f_)*(x_)]), x\_Symbol] \text{ :> } \text{Dist}[2, \text{Int}[(c + d*x)^m*\text{E}^{(I*\text{Pi}*(k - 1/2))}*\text{E}^{(I*(e + f*x))} / (b + 2*a*\text{E}^{(I*\text{Pi}*(k - 1/2))}*\text{E}^{(I*(e + f*x))} - b*\text{E}^{(2*I*k*\text{Pi})}*\text{E}^{(2*I*(e + f*x))}), x], x] \text{ /; } \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{IntegerQ}[2*k] \ \&\& \ \text{NeQ}[\text{E}^{(I*\text{Pi}*(k - 1/2))}, 1]$

$a^2 - b^2, 0]$  && IGtQ[m, 0]

#### Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] :=> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

#### Rule 2190

Int((((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.), x\_Symbol] :=> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_.)\*(F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)], x\_Symbol] :=> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :=> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 2668

Int[cos[(e\_.) + (f\_.)\*(x\_)]^(p\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(m\_.), x\_Symbol] :=> Dist[1/(b^p\*f), Subst[Int[(a + x)^m\*(b^2 - x^2)^((p - 1)/2), x], x, b\*Sine[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] :=> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))}{(f+gx)^2} dx &= \frac{\sqrt{d-c^2dx^2} \int \frac{\sqrt{1-c^2x^2}(a+b\cos^{-1}(cx))}{(f+gx)^2} dx}{\sqrt{1-c^2x^2}} \\
&= -\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(f+gx)^2} + \frac{\sqrt{d-c^2dx^2} \int \frac{(-2g-2c^2fx)(a+b\cos^{-1}(cx))}{(f+gx)^3} dx}{2bc\sqrt{1-c^2x^2}} \\
&= -\frac{(g+c^2fx)^2\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(c^2f^2-g^2)(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(f+gx)^2} \\
&= -\frac{(g+c^2fx)^2\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(c^2f^2-g^2)(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(f+gx)^2} \\
&= -\frac{(g+c^2fx)^2\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(c^2f^2-g^2)(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(f+gx)^2} \\
&= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{(g+c^2fx)^2\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(c^2f^2-g^2)(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(f+gx)^2} \\
&= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{(g+c^2fx)^2\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(c^2f^2-g^2)(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(f+gx)^2} \\
&= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\cos^{-1}(cx)^2}{2g^2(c^2f^2-g^2)\sqrt{1-c^2x^2}} - \frac{(g+c^2fx)^2\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(c^2f^2-g^2)(f+gx)^2\sqrt{1-c^2x^2}} \\
&= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g(f+gx)} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\cos^{-1}(cx)^2}{2g^2(c^2f^2-g^2)\sqrt{1-c^2x^2}} - \frac{(g+c^2fx)^2\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(c^2f^2-g^2)(f+gx)^2\sqrt{1-c^2x^2}} \\
&= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g(f+gx)} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\cos^{-1}(cx)^2}{2g^2(c^2f^2-g^2)\sqrt{1-c^2x^2}} - \frac{(g+c^2fx)^2\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(c^2f^2-g^2)(f+gx)^2\sqrt{1-c^2x^2}} \\
&= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g(f+gx)} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\cos^{-1}(cx)^2}{2g^2(c^2f^2-g^2)\sqrt{1-c^2x^2}} - \frac{(g+c^2fx)^2\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(c^2f^2-g^2)(f+gx)^2\sqrt{1-c^2x^2}} \\
&= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g(f+gx)} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\cos^{-1}(cx)^2}{2g^2(c^2f^2-g^2)\sqrt{1-c^2x^2}} - \frac{(g+c^2fx)^2\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(c^2f^2-g^2)(f+gx)^2\sqrt{1-c^2x^2}} \\
&= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g(f+gx)} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\cos^{-1}(cx)^2}{2g^2(c^2f^2-g^2)\sqrt{1-c^2x^2}} - \frac{(g+c^2fx)^2\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))^2}{2bc(c^2f^2-g^2)(f+gx)^2\sqrt{1-c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 9.48203, size = 1130, normalized size = 1.33

$$\frac{a\sqrt{d}f \log(f + gx)c^2}{g^2\sqrt{g^2 - c^2f^2}} - \frac{a\sqrt{d}f \log\left(dfxc^2 + dg + \sqrt{d}\sqrt{g^2 - c^2f^2}\sqrt{-d(c^2x^2 - 1)}\right)c^2}{g^2\sqrt{g^2 - c^2f^2}} + \frac{a\sqrt{d} \tan^{-1}\left(\frac{cx\sqrt{-d(c^2x^2 - 1)}}{\sqrt{d}(c^2x^2 - 1)}\right)c}{g^2} - \frac{b\sqrt{d}\left(\dots\right)}{g^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[d - c^2\*d\*x^2]\*(a + b\*ArcCos[c\*x]))/(f + g\*x)^2,x]

[Out]  $-\left(\frac{a \sqrt{-d(-1 + c^2 x^2)}}{g(f + gx)}\right) + \frac{a c \sqrt{d} \operatorname{ArcTan}\left[\frac{c x \sqrt{-d(-1 + c^2 x^2)}}{\sqrt{d}(-1 + c^2 x^2)}\right]}{g^2} + \frac{a c^2 \sqrt{d} \operatorname{Log}[f + gx]}{g^2 \sqrt{-c^2 f^2 + g^2}} - \frac{a c^2 \sqrt{d} f \operatorname{Log}[d g + c^2 d f x + \sqrt{d} \sqrt{-c^2 f^2 + g^2} \sqrt{-d(-1 + c^2 x^2)}]}{g^2 \sqrt{-c^2 f^2 + g^2}} - \frac{b c \sqrt{d} \operatorname{ArcCos}[c x]}{c f + c g x} - \frac{\operatorname{ArcCos}[c x]^2}{\sqrt{1 - c^2 x^2}} + \frac{2 \operatorname{Log}\left[1 + \frac{g x}{f}\right]}{\sqrt{1 - c^2 x^2}} + \frac{2 c f (2 \operatorname{ArcCos}[c x] \operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}[\operatorname{ArcCos}[c x] / 2]}{\sqrt{-c^2 f^2 + g^2}}\right] - 2 \operatorname{ArcCos}\left[-\frac{(c f)}{g}\right] \operatorname{ArcTanh}\left[\frac{(-c f) + g}{\sqrt{-c^2 f^2 + g^2}}\right] \operatorname{Tan}[\operatorname{ArcCos}[c x] / 2])}{\sqrt{-c^2 f^2 + g^2}} + \frac{\operatorname{ArcCos}\left[-\frac{(c f)}{g}\right] - (2 I) \operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}[\operatorname{ArcCos}[c x] / 2]}{\sqrt{-c^2 f^2 + g^2}}\right] + (2 I) \operatorname{ArcTanh}\left[\frac{(-c f) + g}{\sqrt{-c^2 f^2 + g^2}}\right] \operatorname{Tan}[\operatorname{ArcCos}[c x] / 2]}{\sqrt{-c^2 f^2 + g^2}} \operatorname{Log}\left[\frac{\sqrt{-c^2 f^2 + g^2}}{\sqrt{2}} e^{\left(\frac{I}{2} \operatorname{ArcCos}[c x]\right) \sqrt{g} \sqrt{c f + c g x}}\right] + \frac{\operatorname{ArcCos}\left[-\frac{(c f)}{g}\right] + (2 I) \operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}[\operatorname{ArcCos}[c x] / 2]}{\sqrt{-c^2 f^2 + g^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-c f) + g}{\sqrt{-c^2 f^2 + g^2}}\right] \operatorname{Tan}[\operatorname{ArcCos}[c x] / 2]}{\sqrt{-c^2 f^2 + g^2}} \operatorname{Log}\left[\frac{e^{\left(\frac{I}{2} \operatorname{ArcCos}[c x]\right) \sqrt{-c^2 f^2 + g^2}}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] - \frac{\operatorname{ArcCos}\left[-\frac{(c f)}{g}\right] - (2 I) \operatorname{ArcTanh}\left[\frac{(-c f) + g}{\sqrt{-c^2 f^2 + g^2}}\right] \operatorname{Tan}[\operatorname{ArcCos}[c x] / 2]}{\sqrt{-c^2 f^2 + g^2}} \operatorname{Log}\left[\frac{(c f + g) \left((-I) c f + I g + \sqrt{-c^2 f^2 + g^2}\right) \operatorname{Tan}[\operatorname{ArcCos}[c x] / 2]}{(-I + \operatorname{Tan}[\operatorname{ArcCos}[c x] / 2]) \sqrt{-c^2 f^2 + g^2}}\right] - \frac{\operatorname{ArcCos}\left[-\frac{(c f)}{g}\right] + (2 I) \operatorname{ArcTanh}\left[\frac{(-c f) + g}{\sqrt{-c^2 f^2 + g^2}}\right] \operatorname{Tan}[\operatorname{ArcCos}[c x] / 2]}{\sqrt{-c^2 f^2 + g^2}} \operatorname{Log}\left[\frac{(c f + g) \left(I c f - I g + \sqrt{-c^2 f^2 + g^2}\right) \left(I + \operatorname{Tan}[\operatorname{ArcCos}[c x] / 2]\right)}{(c f + g + \sqrt{-c^2 f^2 + g^2}) \operatorname{Tan}[\operatorname{ArcCos}[c x] / 2]}\right] + I \operatorname{PolyLog}\left[2, \frac{(c f - I \sqrt{-c^2 f^2 + g^2}) (c f + g - \sqrt{-c^2 f^2 + g^2}) \operatorname{Tan}[\operatorname{ArcCos}[c x] / 2]}{(c f + g + \sqrt{-c^2 f^2 + g^2}) (c f + g - \sqrt{-c^2 f^2 + g^2}) \operatorname{Tan}[\operatorname{ArcCos}[c x] / 2]}\right] - \operatorname{PolyLog}\left[2, \frac{(c f + I \sqrt{-c^2 f^2 + g^2}) (c f + g - \sqrt{-c^2 f^2 + g^2}) \operatorname{Tan}[\operatorname{ArcCos}[c x] / 2]}{(c f + g + \sqrt{-c^2 f^2 + g^2}) (c f + g - \sqrt{-c^2 f^2 + g^2}) \operatorname{Tan}[\operatorname{ArcCos}[c x] / 2]}\right] \Big) / \left(\sqrt{-c^2 f^2 + g^2} \sqrt{1 - c^2 x^2}\right) \Big) / (2 g^2)$

**Maple [C]** time = 0.476, size = 1573, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2)/(g\*x+f)^2,x)

[Out]  $\frac{a/d}{(c^2 f^2 - g^2)^{3/2}} \frac{1}{x + f/g} - \frac{d c^2 (x + f/g)^2 + 2 c^2 d f/g (x + f/g) - d (c^2 f^2 - g^2)}{g^2 (c^2 f^2 - g^2)^{3/2}} - \frac{a}{g c^2 f} \frac{1}{(c^2 f^2 - g^2)^{3/2}} - \frac{d c^2 (x + f/g)^2 + 2 c^2 d f/g (x + f/g) - d (c^2 f^2 - g^2)}{g^2 (c^2 f^2 - g^2)^{3/2}} - \frac{a}{g^2 c^4 f^2} \frac{1}{(c^2 f^2 - g^2)^{3/2}} \frac{d}{(c^2 d)^{1/2}} \operatorname{arctan}\left(\frac{(c^2 d)^{1/2} x}{-d c^2 (x + f/g)^2 + 2 c^2 d f/g (x + f/g) - d (c^2 f^2 - g^2)}\right) - \frac{a}{g^3 c^4 f^3} \frac{1}{(c^2 f^2 - g^2)^{3/2}} \frac{d}{(-d c^2 f^2 - g^2)^{1/2}} \ln\left(\frac{-2 d (c^2 f^2 - g^2) / g^2 + 2 c^2 d f/g (x + f/g) + 2 (-d c^2 f^2 - g^2) / g^2}{-d c^2 (x + f/g)^2 + 2 c^2 d f/g (x + f/g) - d (c^2 f^2 - g^2) / g^2}\right) \Big) / (x + f/g) + \dots$

$$\begin{aligned} & a/g*c^2*f/(c^2*f^2-g^2)*d/(-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*\ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)})/(x+f/g))+a*c^2/(c^2*f^2-g^2)*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*x+a*c^2/(c^2*f^2-g^2)*d/(c^2*d)^{(1/2)}*\arctan((c^2*d)^{(1/2)}*x/(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)})+b*(-1/2*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)})/(c^2*x^2-1)*\arccos(c*x)^2*c/g^2-(-d*(c^2*x^2-1))^{(1/2)}*(c^2*x^2-I*(-c^2*x^2+1)^{(1/2)}*x*c-1)*\arccos(c*x)*(c^2*f*x+g+I*(-c^2*x^2+1)^{(1/2)}*c*f)/(c^2*x^2-1)/g^2/(g*x+f)-c*(I*\arccos(c*x)*\ln((-c*x+I*(-c^2*x^2+1))^{(1/2)})*g-c*f+(c^2*f^2-g^2)^{(1/2)})/(-c*f+(c^2*f^2-g^2)^{(1/2)}))*c^2*f^2-g^2)^{(1/2)}*c*f-I*\arccos(c*x)*\ln(((c*x+I*(-c^2*x^2+1))^{(1/2)})*g+c*f+(c^2*f^2-g^2)^{(1/2)})/(c*f+(c^2*f^2-g^2)^{(1/2)}))*c^2*f^2-g^2)^{(1/2)}*c*f-2*Im(\arccos(c*x))*c^2*f^2+2*\ln(\exp(I*Re(\arccos(c*x)))))*c^2*f^2-\ln((c*x+I*(-c^2*x^2+1))^{(1/2)})^2*g+2*c*f*(c*x+I*(-c^2*x^2+1))^{(1/2)})+g)*c^2*f^2+dilog(-1/(-c*f+(c^2*f^2-g^2)^{(1/2)}))*c*x+I*(-c^2*x^2+1))^{(1/2)})*g-1/(-c*f+(c^2*f^2-g^2)^{(1/2)}))*c*f+1/(-c*f+(c^2*f^2-g^2)^{(1/2)}))*c^2*f^2-g^2)^{(1/2)})*c^2*f^2-g^2)^{(1/2)}*c*f-dilog((c*x+I*(-c^2*x^2+1))^{(1/2)})*g/(c*f+(c^2*f^2-g^2)^{(1/2)})+1/(c*f+(c^2*f^2-g^2)^{(1/2)}))*c*f+1/(c*f+(c^2*f^2-g^2)^{(1/2)}))*c^2*f^2-g^2)^{(1/2)})*c^2*f^2-g^2)^{(1/2)}*c*f+2*Im(\arccos(c*x))*g^2-2*\ln(\exp(I*Re(\arccos(c*x)))))*g^2+\ln((c*x+I*(-c^2*x^2+1))^{(1/2)})^2*g+2*c*f*(c*x+I*(-c^2*x^2+1))^{(1/2)})+g)*g^2*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)})/(c^2*f^2-g^2)/(c^2*x^2-1)/g^2 \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2)/(g\*x+f)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)}{g^2x^2+2fgx+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2)/(g\*x+f)^2,x, algorithm="fricas")

[Out] integral(sqrt(-c^2\*d\*x^2 + d)\*(b\*arccos(c\*x) + a)/(g^2\*x^2 + 2\*f\*g\*x + f^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-d}(cx-1)(cx+1)(a+b\arccos(cx))}{(f+gx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acos(c\*x))\*(-c\*\*2\*d\*x\*\*2+d)\*\*(1/2)/(g\*x+f)\*\*2,x)

[Out] Integral(sqrt(-d\*(c\*x - 1)\*(c\*x + 1))\*(a + b\*acos(c\*x))/(f + g\*x)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2 dx^2 + d}(b \arccos(cx) + a)}{(gx + f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(c\*x))\*(-c^2\*d\*x^2+d)^(1/2)/(g\*x+f)^2,x, algorithm="giac")

[Out] integrate(sqrt(-c^2\*d\*x^2 + d)\*(b\*arccos(c\*x) + a)/(g\*x + f)^2, x)



### 3.6 $\int (f + gx)^3 (d - c^2 dx^2)^{3/2} (a + b \cos^{-1}(cx)) dx$

**Optimal.** Leaf size=959

$$\frac{bc^3 dg^3 \sqrt{d - c^2 dx^2} x^7}{49\sqrt{1 - c^2 x^2}} - \frac{bc^3 df g^2 \sqrt{d - c^2 dx^2} x^6}{12\sqrt{1 - c^2 x^2}} + \frac{8bcdg^3 \sqrt{d - c^2 dx^2} x^5}{175\sqrt{1 - c^2 x^2}} - \frac{3bc^3 df^2 g \sqrt{d - c^2 dx^2} x^5}{25\sqrt{1 - c^2 x^2}} - \frac{bc^3 df^3 \sqrt{d - c^2 dx^2}}{16\sqrt{1 - c^2 x^2}}$$

[Out]  $(-3*b*d*f^2*g*x*\text{Sqrt}[d - c^2*d*x^2])/(5*c*\text{Sqrt}[1 - c^2*x^2]) - (2*b*d*g^3*x*\text{Sqrt}[d - c^2*d*x^2])/(35*c^3*\text{Sqrt}[1 - c^2*x^2]) + (5*b*c*d*f^3*x^2*\text{Sqrt}[d - c^2*d*x^2])/(16*\text{Sqrt}[1 - c^2*x^2]) - (3*b*d*f*g^2*x^2*\text{Sqrt}[d - c^2*d*x^2])/(32*c*\text{Sqrt}[1 - c^2*x^2]) + (2*b*c*d*f^2*g*x^3*\text{Sqrt}[d - c^2*d*x^2])/(5*\text{Sqrt}[1 - c^2*x^2]) - (b*d*g^3*x^3*\text{Sqrt}[d - c^2*d*x^2])/(105*c*\text{Sqrt}[1 - c^2*x^2]) - (b*c^3*d*f^3*x^4*\text{Sqrt}[d - c^2*d*x^2])/(16*\text{Sqrt}[1 - c^2*x^2]) + (7*b*c*d*f*g^2*x^4*\text{Sqrt}[d - c^2*d*x^2])/(32*\text{Sqrt}[1 - c^2*x^2]) - (3*b*c^3*d*f^2*g*x^5*\text{Sqrt}[d - c^2*d*x^2])/(25*\text{Sqrt}[1 - c^2*x^2]) + (8*b*c*d*g^3*x^5*\text{Sqrt}[d - c^2*d*x^2])/(175*\text{Sqrt}[1 - c^2*x^2]) - (b*c^3*d*f*g^2*x^6*\text{Sqrt}[d - c^2*d*x^2])/(12*\text{Sqrt}[1 - c^2*x^2]) - (b*c^3*d*g^3*x^7*\text{Sqrt}[d - c^2*d*x^2])/(49*\text{Sqrt}[1 - c^2*x^2]) + (3*d*f^3*x*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/8 - (3*d*f*g^2*x*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/(16*c^2) + (3*d*f*g^2*x^3*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/8 + (d*f^3*x*(1 - c^2*x^2)*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/4 + (d*f*g^2*x^3*(1 - c^2*x^2)*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/2 - (3*d*f^2*g*(1 - c^2*x^2)^2*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/(5*c^2) - (d*g^3*(1 - c^2*x^2)^2*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/(5*c^4) + (d*g^3*(1 - c^2*x^2)^3*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/(7*c^4) - (3*d*f^3*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x])^2)/(16*b*c*\text{Sqrt}[1 - c^2*x^2]) - (3*d*f*g^2*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x])^2)/(32*b*c^3*\text{Sqrt}[1 - c^2*x^2])$

**Rubi [A]** time = 0.96175, antiderivative size = 959, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 17, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.548$ , Rules used = {4778, 4764, 4650, 4648, 4642, 30, 14, 4678, 194, 4700, 4698, 4708, 266, 43, 4690, 12, 373}

$$\frac{bc^3 dg^3 \sqrt{d - c^2 dx^2} x^7}{49\sqrt{1 - c^2 x^2}} - \frac{bc^3 df g^2 \sqrt{d - c^2 dx^2} x^6}{12\sqrt{1 - c^2 x^2}} + \frac{8bcdg^3 \sqrt{d - c^2 dx^2} x^5}{175\sqrt{1 - c^2 x^2}} - \frac{3bc^3 df^2 g \sqrt{d - c^2 dx^2} x^5}{25\sqrt{1 - c^2 x^2}} - \frac{bc^3 df^3 \sqrt{d - c^2 dx^2}}{16\sqrt{1 - c^2 x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)^3*(d - c^2*d*x^2)^{(3/2)}*(a + b*\text{ArcCos}[c*x]), x]$

[Out]  $(-3*b*d*f^2*g*x*\text{Sqrt}[d - c^2*d*x^2])/(5*c*\text{Sqrt}[1 - c^2*x^2]) - (2*b*d*g^3*x*\text{Sqrt}[d - c^2*d*x^2])/(35*c^3*\text{Sqrt}[1 - c^2*x^2]) + (5*b*c*d*f^3*x^2*\text{Sqrt}[d - c^2*d*x^2])/(16*\text{Sqrt}[1 - c^2*x^2]) - (3*b*d*f*g^2*x^2*\text{Sqrt}[d - c^2*d*x^2])/(32*c*\text{Sqrt}[1 - c^2*x^2]) + (2*b*c*d*f^2*g*x^3*\text{Sqrt}[d - c^2*d*x^2])/(5*\text{Sqrt}[1 - c^2*x^2]) - (b*d*g^3*x^3*\text{Sqrt}[d - c^2*d*x^2])/(105*c*\text{Sqrt}[1 - c^2*x^2]) - (b*c^3*d*f^3*x^4*\text{Sqrt}[d - c^2*d*x^2])/(16*\text{Sqrt}[1 - c^2*x^2]) + (7*b*c*d*f*g^2*x^4*\text{Sqrt}[d - c^2*d*x^2])/(32*\text{Sqrt}[1 - c^2*x^2]) - (3*b*c^3*d*f^2*g*x^5*\text{Sqrt}[d - c^2*d*x^2])/(25*\text{Sqrt}[1 - c^2*x^2]) + (8*b*c*d*g^3*x^5*\text{Sqrt}[d - c^2*d*x^2])/(175*\text{Sqrt}[1 - c^2*x^2]) - (b*c^3*d*f*g^2*x^6*\text{Sqrt}[d - c^2*d*x^2])/(12*\text{Sqrt}[1 - c^2*x^2]) - (b*c^3*d*g^3*x^7*\text{Sqrt}[d - c^2*d*x^2])/(49*\text{Sqrt}[1 - c^2*x^2]) + (3*d*f^3*x*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/8 - (3*d*f*g^2*x*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/(16*c^2) + (3*d*f*g^2*x^3*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/8 + (d*f^3*x*(1 - c^2*x^2)*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/4 + (d*f*g^2*x^3*(1 - c^2*x^2)*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/2 - (3*d*f^2*g*(1 - c^2*x^2)^2*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/(5*c^2) - (d*g^3*(1 - c^2*x^2)^2*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/(5*c^4) + (d*g^3*(1 - c^2*x^2)^3*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/(7*c^4) - (3*d*f^3*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x])^2)/(16*b*c*\text{Sqrt}[1 - c^2*x^2]) - (3*d*f*g^2*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x])^2)/(32*b*c^3*\text{Sqrt}[1 - c^2*x^2])$

$$\begin{aligned} &^2*d*x^2)*(a + b*ArcCos[c*x])/(5*c^2) - (d*g^3*(1 - c^2*x^2)^2*sqrt[d - c^2 \\ &2*d*x^2)*(a + b*ArcCos[c*x])/(5*c^4) + (d*g^3*(1 - c^2*x^2)^3*sqrt[d - c^2 \\ &*d*x^2)*(a + b*ArcCos[c*x])/(7*c^4) - (3*d*f^3*sqrt[d - c^2*d*x^2)*(a + b* \\ &ArcCos[c*x])^2)/(16*b*c*sqrt[1 - c^2*x^2]) - (3*d*f*g^2*sqrt[d - c^2*d*x^2] \\ &*(a + b*ArcCos[c*x])^2)/(32*b*c^3*sqrt[1 - c^2*x^2]) \end{aligned}$$
Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] := Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
0] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]
```

Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^p*(a +
b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4650

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] := Simp[(x*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n)/(2*p + 1), x] + (D
ist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])]/((2*p + 1)*(1 - c^2*x
^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x)) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
GtQ[p, 0]
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] := Simp[(x*sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*sqrt[d + e*x^2])/(2*sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)/sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] := -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

#### Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

#### Rule 4700

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

#### Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_.))*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^m*(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

#### Rule 4708

```
Int((((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[(f*x)^(m - 2)*(a + b*ArcCos[c*x])^n/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 4690

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.)
```

```
, x_Symbol] := With[{u = IntHide[x^m*(1 - c^2*x^2)^p, x]}, Dist[d^p*(a + b*
ArcCos[c*x]), u, x] + Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^
2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Intege
rQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -
2^(-1)] && GtQ[d, 0]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] := Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

### Rubi steps

$$\begin{aligned}
\int (f + gx)^3 (d - c^2 dx^2)^{3/2} (a + b \cos^{-1}(cx)) dx &= \frac{(d\sqrt{d - c^2 dx^2}) \int (f + gx)^3 (1 - c^2 x^2)^{3/2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{(d\sqrt{d - c^2 dx^2}) \int (f^3 (1 - c^2 x^2)^{3/2} (a + b \cos^{-1}(cx)) + 3f^2 gx (1 - c^2 x^2)^{3/2} (a + b \cos^{-1}(cx))) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{(df^3 \sqrt{d - c^2 dx^2}) \int (1 - c^2 x^2)^{3/2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} + \frac{(3df^2 g \sqrt{d - c^2 dx^2}) \int (1 - c^2 x^2)^{3/2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{1}{4} df^3 x (1 - c^2 x^2) \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) + \frac{1}{2} df g^2 x^3 (1 - c^2 x^2) \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) \\
&= \frac{3}{8} df^3 x \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) + \frac{3}{8} df g^2 x^3 \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) \\
&= -\frac{3bd f^2 gx \sqrt{d - c^2 dx^2}}{5c \sqrt{1 - c^2 x^2}} + \frac{5bcd f^3 x^2 \sqrt{d - c^2 dx^2}}{16 \sqrt{1 - c^2 x^2}} + \frac{2bcd f^2 gx^3 \sqrt{d - c^2 dx^2}}{5 \sqrt{1 - c^2 x^2}} \\
&= -\frac{3bd f^2 gx \sqrt{d - c^2 dx^2}}{5c \sqrt{1 - c^2 x^2}} - \frac{2bd g^3 x \sqrt{d - c^2 dx^2}}{35c^3 \sqrt{1 - c^2 x^2}} + \frac{5bcd f^3 x^2 \sqrt{d - c^2 dx^2}}{16 \sqrt{1 - c^2 x^2}} - \frac{3bd f^2 g^2 x^3 \sqrt{d - c^2 dx^2}}{5c^3 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

**Mathematica [A]** time = 4.57178, size = 910, normalized size = 0.95

$$-88200bcd f (2c^2 f^2 + g^2) \sqrt{d - c^2 dx^2} \cos^{-1}(cx)^2 + 140bd \sqrt{d - c^2 dx^2} \left( 6720f^2 g x^2 \sqrt{1 - c^2 x^2} c^4 + 1680f^3 \sin(2 \cos^{-1}(cx)) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]), x]
```

```
[Out] (-88200*b*c*d*f*(2*c^2*f^2 + g^2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 17640
0*a*c*d^(3/2)*f*(2*c^2*f^2 + g^2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^
2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - d*Sqrt[d - c^2*d*x^2]*(352800*b*c^3*f
^2*g*x + 44100*b*c*g^3*x + 564480*a*c^2*f^2*g*Sqrt[1 - c^2*x^2] + 53760*a*g
^3*Sqrt[1 - c^2*x^2] - 588000*a*c^4*f^3*x*Sqrt[1 - c^2*x^2] + 176400*a*c^2*
```

$$\begin{aligned}
& f^2 g^2 x \sqrt{1 - c^2 x^2} - 1128960 a^4 c^4 f^2 g^2 x^2 \sqrt{1 - c^2 x^2} + 26880 a^4 c^2 g^3 x^2 \sqrt{1 - c^2 x^2} + 235200 a^4 c^6 f^3 x^3 \sqrt{1 - c^2 x^2} \\
& - 823200 a^4 c^4 f g^2 x^3 \sqrt{1 - c^2 x^2} + 564480 a^4 c^6 f^2 g x^4 \sqrt{1 - c^2 x^2} - 215040 a^4 c^4 g^3 x^4 \sqrt{1 - c^2 x^2} + 470400 a^4 c^6 f g^2 x^5 \sqrt{1 - c^2 x^2} \\
& + 134400 a^4 c^6 g^3 x^6 \sqrt{1 - c^2 x^2} - 7350 b^2 c f (16 c^2 f^2 + 3 g^2) \cos[2 \operatorname{ArcCos}[c x]] - 4900 b^2 g (12 c^2 f^2 + g^2) \cos[3 \operatorname{ArcCos}[c x]] \\
& + 7350 b^2 c^3 f^3 \cos[4 \operatorname{ArcCos}[c x]] - 11025 b^2 c f g^2 \cos[4 \operatorname{ArcCos}[c x]] + 7056 b^2 c^2 f^2 g \cos[5 \operatorname{ArcCos}[c x]] - 588 b^2 g^3 \cos[5 \operatorname{ArcCos}[c x]] \\
& + 2450 b^2 c f g^2 \cos[6 \operatorname{ArcCos}[c x]] + 300 b^2 g^3 \cos[7 \operatorname{ArcCos}[c x]]) + 140 b^2 d \sqrt{d - c^2 d x^2} \operatorname{ArcCos}[c x] * (-4200 c^2 f^2 g \sqrt{1 - c^2 x^2} \\
& + 416 g^3 \sqrt{1 - c^2 x^2} + 6720 c^4 f^2 g x^2 \sqrt{1 - c^2 x^2} - 1256 c^2 g^3 x^2 \sqrt{1 - c^2 x^2} + 864 g^3 (1 - c^2 x^2)^{(3/2)} \cos[2 \operatorname{ArcCos}[c x]] \\
& + 120 g^3 (1 - c^2 x^2)^{(3/2)} \cos[4 \operatorname{ArcCos}[c x]] + 1680 c^3 f^3 \sin[2 \operatorname{ArcCos}[c x]] + 315 c f g^2 \sin[2 \operatorname{ArcCos}[c x]] - 420 c^2 f^2 g \sin[3 \operatorname{ArcCos}[c x]] \\
& + 140 g^3 \sin[3 \operatorname{ArcCos}[c x]] - 210 c^3 f^3 \sin[4 \operatorname{ArcCos}[c x]] + 315 c f g^2 \sin[4 \operatorname{ArcCos}[c x]] - 252 c^2 f^2 g \sin[5 \operatorname{ArcCos}[c x]] + 84 g^3 \sin[5 \operatorname{ArcCos}[c x]] \\
& - 105 c f g^2 \sin[6 \operatorname{ArcCos}[c x]]) / (940800 c^4 \sqrt{1 - c^2 x^2})
\end{aligned}$$

**Maple [B]** time = 0.811, size = 1734, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (g x + f)^3 (-c^2 d x^2 + d)^{(3/2)} (a + b \arccos(c x)) dx$

[Out]  $\begin{aligned}
& 3/5 b (-d (c^2 x^2 - 1))^{(1/2)} g d / c^2 / (c^2 x^2 - 1) \arccos(c x) f^2 - 1/4 b (-d (c^2 x^2 - 1))^{(1/2)} f^3 d c^4 / (c^2 x^2 - 1) \arccos(c x) x^5 + 7/8 b (-d (c^2 x^2 - 1))^{(1/2)} f^3 d c^2 / (c^2 x^2 - 1) \arccos(c x) x^3 - 17/16 b (-d (c^2 x^2 - 1))^{(1/2)} f g^2 d / (c^2 x^2 - 1) \arccos(c x) x^3 - 9/5 b (-d (c^2 x^2 - 1))^{(1/2)} g d / (c^2 x^2 - 1) \arccos(c x) x^2 f^2 + 3/16 b (-d (c^2 x^2 - 1))^{(1/2)} (-c^2 x^2 + 1)^{(1/2)} / c / (c^2 x^2 - 1) \arccos(c x)^2 f^3 d - 1/7 b (-d (c^2 x^2 - 1))^{(1/2)} g^3 d c^4 / (c^2 x^2 - 1) \arccos(c x) x^8 + 13/35 b (-d (c^2 x^2 - 1))^{(1/2)} g^3 d c^2 / (c^2 x^2 - 1) \arccos(c x) x^6 - 1/35 b (-d (c^2 x^2 - 1))^{(1/2)} g^3 d / c^2 / (c^2 x^2 - 1) \arccos(c x) x^2 + 3/8 a f^3 d x (-c^2 d x^2 + d)^{(1/2)} + 3/8 a f^3 d^2 / (c^2 d)^{(1/2)} \arctan((c^2 d)^{(1/2)} x / (-c^2 d x^2 + d)^{(1/2)}) - 2/35 a g^3 d / c^4 (-c^2 d x^2 + d)^{(5/2)} - 9/35 b (-d (c^2 x^2 - 1))^{(1/2)} g^3 d / (c^2 x^2 - 1) \arccos(c x) x^4 - 5/8 b (-d (c^2 x^2 - 1))^{(1/2)} f^3 d / (c^2 x^2 - 1) \arccos(c x) x^2 + 3/35 b (-d (c^2 x^2 - 1))^{(1/2)} g^3 d / c^4 / (c^2 x^2 - 1) \arccos(c x) - 1/7 a g^3 x^2 (-c^2 d x^2 + d)^{(5/2)} / c^2 / d + 1/8 a f g^2 / c^2 x (-c^2 d x^2 + d)^{(3/2)} - 3/5 a f^2 g / c^2 / d (-c^2 d x^2 + d)^{(5/2)} + 1/4 a f^3 x (-c^2 d x^2 + d)^{(3/2)} - 1/2 b (-d (c^2 x^2 - 1))^{(1/2)} f g^2 d c^4 / (c^2 x^2 - 1) \arccos(c x) x^7 + 11/8 b (-d (c^2 x^2 - 1))^{(1/2)} f g^2 d c^2 / (c^2 x^2 - 1) \arccos(c x) x^5 + 3/16 b (-d (c^2 x^2 - 1))^{(1/2)} f g^2 d / c^2 / (c^2 x^2 - 1) \arccos(c x) x^3 - 3/5 b (-d (c^2 x^2 - 1))^{(1/2)} g d c^4 / (c^2 x^2 - 1) \arccos(c x) x^6 f^2 + 9/5 b (-d (c^2 x^2 - 1))^{(1/2)} g d c^2 / (c^2 x^2 - 1) \arccos(c x) x^4 f^2 + 3/32 b (-d (c^2 x^2 - 1))^{(1/2)} (-c^2 x^2 + 1)^{(1/2)} / c^3 / (c^2 x^2 - 1) \arccos(c x)^2 f d g^2 - 7/32 b (-d (c^2 x^2 - 1))^{(1/2)} f g^2 d c / (c^2 x^2 - 1) (-c^2 x^2 + 1)^{(1/2)} x^4 + 3/32 b (-d (c^2 x^2 - 1))^{(1/2)} f g^2 d / c / (c^2 x^2 - 1) (-c^2 x^2 + 1)^{(1/2)} x^2 + 3/25 b (-d (c^2 x^2 - 1))^{(1/2)} g d c^3 / (c^2 x^2 - 1) (-c^2 x^2 + 1)^{(1/2)} x^5 f^2 - 2/5 b (-d (c^2 x^2 - 1))^{(1/2)} g d c / (c^2 x^2 - 1) (-c^2 x^2 + 1)^{(1/2)} x^3 f^2 + 3/5 b (-d (c^2 x^2 - 1))^{(1/2)} g d / c / (c^2 x^2 - 1) (-c^2 x^2 + 1)^{(1/2)} x f^2 + 1/12 b (-d (c^2 x^2 - 1))^{(1/2)} f g^2 d c^3 / (c^2 x^2 - 1) (-c^2 x^2 + 1)^{(1/2)} x^6 - 1/2 a f g^2 x (-c^2 d x^2 + d)^{(5/2)} / c^2 / d + 3/16 a f g^2 / c^2 d x (-c^2 d x^2 + d)^{(1/2)} + 1/49 b (-d (c^2 x^2 - 1))^{(1/2)} g^3 d c^3 / (c^2 x^2 - 1) (-c^2 x^2 + 1)^{(1/2)} x^7 - 8/175 b (-d (c^2 x^2 - 1))^{(1/2)} g^3 d c / (c^2 x^2 - 1) (-c^2 x^2 + 1)^{(1/2)} x^5 + 1/105 b (-d (c^2 x^2 - 1))^{(1/2)} *
\end{aligned}$

$$g^3 d/c/(c^2 x^2 - 1) * (-c^2 x^2 + 1)^{(1/2)} * x^3 + 2/35 * b * (-d * (c^2 x^2 - 1))^{(1/2)} * g^3 d/c^3 / (c^2 x^2 - 1) * (-c^2 x^2 + 1)^{(1/2)} * x + 7/768 * b * (-d * (c^2 x^2 - 1))^{(1/2)} * f * g^2 * d/c^3 / (c^2 x^2 - 1) * (-c^2 x^2 + 1)^{(1/2)} + 1/16 * b * (-d * (c^2 x^2 - 1))^{(1/2)} * f^3 * d * c^3 / (c^2 x^2 - 1) * (-c^2 x^2 + 1)^{(1/2)} * x^4 - 5/16 * b * (-d * (c^2 x^2 - 1))^{(1/2)} * f^3 * d * c / (c^2 x^2 - 1) * (-c^2 x^2 + 1)^{(1/2)} * x^2 + 3/16 * a * f * g^2 / c^2 * d^2 / (c^2 * d)^{(1/2)} * \arctan((c^2 * d)^{(1/2)} * x / (-c^2 * d * x^2 + d)^{(1/2)}) + 17/128 * b * (-d * (c^2 x^2 - 1))^{(1/2)} * f^3 * d / c / (c^2 x^2 - 1) * (-c^2 x^2 + 1)^{(1/2)}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(-c^2\*d\*x^2+d)^(3/2)\*(a+b\*arccos(c\*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(ac^2 dg^3 x^5 + 3ac^2 dfg^2 x^4 - 3adf^2 gx - adf^3 + \left(3ac^2 df^2 g - adg^3\right)x^3 + \left(ac^2 df^3 - 3adfg^2\right)x^2 + \left(bc^2 dg^3 x^5 + 3\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(-c^2\*d\*x^2+d)^(3/2)\*(a+b\*arccos(c\*x)),x, algorithm="fricas")

[Out] integral(-(a\*c^2\*d\*g^3\*x^5 + 3\*a\*c^2\*d\*f\*g^2\*x^4 - 3\*a\*d\*f^2\*g\*x - a\*d\*f^3 + (3\*a\*c^2\*d\*f^2\*g - a\*d\*g^3)\*x^3 + (a\*c^2\*d\*f^3 - 3\*a\*d\*f\*g^2)\*x^2 + (b\*c^2\*d\*g^3\*x^5 + 3\*b\*c^2\*d\*f\*g^2\*x^4 - 3\*b\*d\*f^2\*g\*x - b\*d\*f^3 + (3\*b\*c^2\*d\*f^2\*g - b\*d\*g^3)\*x^3 + (b\*c^2\*d\*f^3 - 3\*b\*d\*f\*g^2)\*x^2)\*arccos(c\*x))\*sqrt(-c^2\*d\*x^2 + d), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*3\*(-c\*\*2\*d\*x\*\*2+d)\*\*(3/2)\*(a+b\*acos(c\*x)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (-c^2 dx^2 + d)^{\frac{3}{2}} (gx + f)^3 (b \arccos(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="gi  
ac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(g*x + f)^3*(b*arccos(c*x) + a), x)
```

### 3.7 $\int (f + gx)^2 (d - c^2 dx^2)^{3/2} (a + b \cos^{-1}(cx)) dx$

**Optimal.** Leaf size=680

$$\frac{3}{8}df^2x\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx)) + \frac{1}{4}df^2x(1-c^2x^2)\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx)) - \frac{3df^2\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))}{16bc\sqrt{1-c^2x^2}}$$

```
[Out] (-2*b*d*f*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[1 - c^2*x^2]) + (5*b*c*d*f^2*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) - (b*d*g^2*x^2*Sqrt[d - c^2*d*x^2])/(32*c*Sqrt[1 - c^2*x^2]) + (4*b*c*d*f*g*x^3*Sqrt[d - c^2*d*x^2])/(15*Sqrt[1 - c^2*x^2]) - (b*c^3*d*f^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (7*b*c*d*g^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) - (2*b*c^3*d*f*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[1 - c^2*x^2]) - (b*c^3*d*g^2*x^6*Sqrt[d - c^2*d*x^2])/(36*Sqrt[1 - c^2*x^2]) + (3*d*f^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 - (d*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(16*c^2) + (d*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 + (d*f^2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 + (d*g^2*x^3*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/6 - (2*d*f*g*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^2) - (3*d*f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*c*Sqrt[1 - c^2*x^2]) - (d*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(32*b*c^3*Sqrt[1 - c^2*x^2])
```

**Rubi [A]** time = 0.735067, antiderivative size = 680, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$ , Rules used = {4778, 4764, 4650, 4648, 4642, 30, 14, 4678, 194, 4700, 4698, 4708}

$$\frac{3}{8}df^2x\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx)) + \frac{1}{4}df^2x(1-c^2x^2)\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx)) - \frac{3df^2\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))}{16bc\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-2*b*d*f*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[1 - c^2*x^2]) + (5*b*c*d*f^2*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) - (b*d*g^2*x^2*Sqrt[d - c^2*d*x^2])/(32*c*Sqrt[1 - c^2*x^2]) + (4*b*c*d*f*g*x^3*Sqrt[d - c^2*d*x^2])/(15*Sqrt[1 - c^2*x^2]) - (b*c^3*d*f^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (7*b*c*d*g^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) - (2*b*c^3*d*f*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[1 - c^2*x^2]) - (b*c^3*d*g^2*x^6*Sqrt[d - c^2*d*x^2])/(36*Sqrt[1 - c^2*x^2]) + (3*d*f^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 - (d*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(16*c^2) + (d*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 + (d*f^2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 + (d*g^2*x^3*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/6 - (2*d*f*g*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^2) - (3*d*f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*c*Sqrt[1 - c^2*x^2]) - (d*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(32*b*c^3*Sqrt[1 - c^2*x^2])
```

**Rule 4778**

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^ (n_.)*((f_.) + (g_.)*(x_.))^ (m_.)*((d_.) + (e_.)*(x_.)^2)^ (p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]
```



Rule 4764

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((f\_) + (g\_.)\*(x\_))^(m\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x^2)^p\*(a + b\*ArcCos[c\*x])^n, (f + g\*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2\*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))

Rule 4650

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(x\*(d + e\*x^2)^p\*(a + b\*ArcCos[c\*x])^n)/(2\*p + 1), x] + (Dist[(2\*d\*p)/(2\*p + 1), Int[(d + e\*x^2)^(p - 1)\*(a + b\*ArcCos[c\*x])^n, x], x] + Dist[(b\*c\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p]]/((2\*p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[x\*(1 - c^2\*x^2)^(p - 1/2)\*(a + b\*ArcCos[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]

Rule 4648

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[(x\*Sqrt[d + e\*x^2]\*(a + b\*ArcCos[c\*x])^n)/2, x] + (Dist[Sqrt[d + e\*x^2]/(2\*Sqrt[1 - c^2\*x^2]), Int[(a + b\*ArcCos[c\*x])^n/Sqrt[1 - c^2\*x^2], x], x] + Dist[(b\*c\*n\*Sqrt[d + e\*x^2])/(2\*Sqrt[1 - c^2\*x^2]), Int[x\*(a + b\*ArcCos[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0]

Rule 4642

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := -Simp[(a + b\*ArcCos[c\*x])^(n + 1)/(b\*c\*Sqrt[d]\*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 14

Int[(u\_)\*((c\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_) + (b\_.)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 4678

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*(x\_)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCos[c\*x])^n)/(2\*e\*(p + 1)), x] - Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p]]/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcCos[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

Rule 194

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4700

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcC
os[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x)^
m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart
[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 - c^2*x^2)^FracPart[p]), I
nt[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
GtQ[p, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

### Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcC
os[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x
^2]), Int[((f*x)^m*(a + b*ArcCos[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] + Dist[
(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && Eq
Q[c^2*d + e, 0] && GtQ[n, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

### Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] := Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

### Rubi steps

$$\begin{aligned}
\int (f + gx)^2 (d - c^2 dx^2)^{3/2} (a + b \cos^{-1}(cx)) dx &= \frac{(d\sqrt{d - c^2 dx^2}) \int (f + gx)^2 (1 - c^2 x^2)^{3/2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{(d\sqrt{d - c^2 dx^2}) \int (f^2 (1 - c^2 x^2)^{3/2} (a + b \cos^{-1}(cx)) + 2fgx (1 - c^2 x^2)^{3/2}) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{(df^2 \sqrt{d - c^2 dx^2}) \int (1 - c^2 x^2)^{3/2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} + \frac{(2dfg \sqrt{d - c^2 dx^2}) \int (1 - c^2 x^2)^{3/2} dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{1}{4} df^2 x (1 - c^2 x^2) \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) + \frac{1}{6} dg^2 x^3 (1 - c^2 x^2) \sqrt{d - c^2 dx^2} \\
&= \frac{3}{8} df^2 x \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) + \frac{1}{8} dg^2 x^3 \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) \\
&= -\frac{2bdfgx \sqrt{d - c^2 dx^2}}{5c\sqrt{1 - c^2 x^2}} + \frac{5bcd f^2 x^2 \sqrt{d - c^2 dx^2}}{16\sqrt{1 - c^2 x^2}} + \frac{4bcd fgx^3 \sqrt{d - c^2 dx^2}}{15\sqrt{1 - c^2 x^2}} \\
&= -\frac{2bdfgx \sqrt{d - c^2 dx^2}}{5c\sqrt{1 - c^2 x^2}} + \frac{5bcd f^2 x^2 \sqrt{d - c^2 dx^2}}{16\sqrt{1 - c^2 x^2}} - \frac{bdg^2 x^2 \sqrt{d - c^2 dx^2}}{32c\sqrt{1 - c^2 x^2}} + \frac{4bcd fgx^3 \sqrt{d - c^2 dx^2}}{15\sqrt{1 - c^2 x^2}}
\end{aligned}$$

**Mathematica [A]** time = 2.14273, size = 591, normalized size = 0.87

$$-d\sqrt{d - c^2 dx^2} \left( 14400ac^5 f^2 x^3 \sqrt{1 - c^2 x^2} - 36000ac^3 f^2 x \sqrt{1 - c^2 x^2} + 23040ac^5 fgx^4 \sqrt{1 - c^2 x^2} - 46080ac^3 fgx^2 \sqrt{1 - c^2 x^2} + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2\*(d - c^2\*d\*x^2)^(3/2)\*(a + b\*ArcCos[c\*x]),x]

[Out] 
$$\begin{aligned} & (-1800*b*d*(6*c^2*f^2 + g^2)*\text{Sqrt}[d - c^2*d*x^2]*\text{ArcCos}[c*x]^2 - 3600*a*d^{(3/2)} \\ & (6*c^2*f^2 + g^2)*\text{Sqrt}[1 - c^2*x^2]*\text{ArcTan}[(c*x*\text{Sqrt}[d - c^2*d*x^2])/(\text{Sqrt}[d] \\ & *(-1 + c^2*x^2))] - d*\text{Sqrt}[d - c^2*d*x^2]*(14400*b*c^2*f*g*x + 23040 \\ & *a*c*f*g*\text{Sqrt}[1 - c^2*x^2] - 36000*a*c^3*f^2*x*\text{Sqrt}[1 - c^2*x^2] + 3600*a*c \\ & *g^2*x*\text{Sqrt}[1 - c^2*x^2] - 46080*a*c^3*f*g*x^2*\text{Sqrt}[1 - c^2*x^2] + 14400*a* \\ & c^5*f^2*x^3*\text{Sqrt}[1 - c^2*x^2] - 16800*a*c^3*g^2*x^3*\text{Sqrt}[1 - c^2*x^2] + 230 \\ & 40*a*c^5*f*g*x^4*\text{Sqrt}[1 - c^2*x^2] + 9600*a*c^5*g^2*x^5*\text{Sqrt}[1 - c^2*x^2] - \\ & 450*b*(16*c^2*f^2 + g^2)*\text{Cos}[2*\text{ArcCos}[c*x]] - 2400*b*c*f*g*\text{Cos}[3*\text{ArcCos}[c* \\ & x]] + 450*b*c^2*f^2*\text{Cos}[4*\text{ArcCos}[c*x]] - 225*b*g^2*\text{Cos}[4*\text{ArcCos}[c*x]] + 288 \\ & *b*c*f*g*\text{Cos}[5*\text{ArcCos}[c*x]] + 50*b*g^2*\text{Cos}[6*\text{ArcCos}[c*x]]) + 60*b*d*\text{Sqrt}[d \\ & - c^2*d*x^2]*\text{ArcCos}[c*x]*(-400*c*f*g*\text{Sqrt}[1 - c^2*x^2] + 640*c^3*f*g*x^2*\text{Sqrt}[1 - c^2*x^2] \\ & + 15*(16*c^2*f^2 + g^2)*\text{Sin}[2*\text{ArcCos}[c*x]] - 40*c*f*g*\text{Sin}[3 \\ & *\text{ArcCos}[c*x]] - 30*c^2*f^2*\text{Sin}[4*\text{ArcCos}[c*x]] + 15*g^2*\text{Sin}[4*\text{ArcCos}[c*x]] - \\ & 24*c*f*g*\text{Sin}[5*\text{ArcCos}[c*x]] - 5*g^2*\text{Sin}[6*\text{ArcCos}[c*x]])/(57600*c^3*\text{Sqrt}[1 \\ & - c^2*x^2]) \end{aligned}$$

**Maple [B]** time = 0.579, size = 1252, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2\*(-c^2\*d\*x^2+d)^(3/2)\*(a+b\*arccos(c\*x)),x)

[Out] 
$$\begin{aligned} & 1/24*a*g^2/c^2*x*(-c^2*d*x^2+d)^{(3/2)} - 5/8*b*(-d*(c^2*x^2-1))^{(1/2)}*d/(c^2*x \\ & ^2-1)*\text{arccos}(c*x)*x*f^2+17/128*b*(-d*(c^2*x^2-1))^{(1/2)}*d/c/(c^2*x^2-1)*(-c \\ & ^2*x^2+1)^{(1/2)}*f^2-17/48*b*(-d*(c^2*x^2-1))^{(1/2)}*g^2*d/(c^2*x^2-1)*\text{arccos} \\ & (c*x)*x^3+7/2304*b*(-d*(c^2*x^2-1))^{(1/2)}*g^2*d/c^3/(c^2*x^2-1)*(-c^2*x^2+1 \\ & )^{(1/2)}+3/8*a*f^2*d*x*(-c^2*d*x^2+d)^{(1/2)}-2/5*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g \\ & *d*c^4/(c^2*x^2-1)*\text{arccos}(c*x)*x^6+6/5*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g*d*c^2/( \\ & c^2*x^2-1)*\text{arccos}(c*x)*x^4+2/25*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g*d*c^3/(c^2*x^2 \\ & -1)*(-c^2*x^2+1)^{(1/2)}*x^5-4/15*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g*d*c/(c^2*x^2-1 \\ & )*(-c^2*x^2+1)^{(1/2)}*x^3+2/5*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g*d/c/(c^2*x^2-1)* \\ & (-c^2*x^2+1)^{(1/2)}*x+1/4*a*f^2*x*(-c^2*d*x^2+d)^{(3/2)}+1/36*b*(-d*(c^2*x^2-1) \\ & )^{(1/2)}*g^2*d*c^3/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^6-7/96*b*(-d*(c^2*x^2-1) \\ & )^{(1/2)}*g^2*d*c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^4+1/32*b*(-d*(c^2*x^2-1))^{(1/2)} \\ & *g^2*d/c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^2-6/5*b*(-d*(c^2*x^2-1))^{(1/2)} \\ & *f*g*d/(c^2*x^2-1)*\text{arccos}(c*x)*x^2+1/16*b*(-d*(c^2*x^2-1))^{(1/2)}*d*c^3/(c \\ & ^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^4*f^2-5/16*b*(-d*(c^2*x^2-1))^{(1/2)}*d*c/(c^2 \\ & *x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^2*f^2+3/16*b*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+ \\ & 1)^{(1/2)}/c/(c^2*x^2-1)*\text{arccos}(c*x)^2*d*f^2+1/32*b*(-d*(c^2*x^2-1))^{(1/2)}*(- \\ & c^2*x^2+1)^{(1/2)}/c^3/(c^2*x^2-1)*\text{arccos}(c*x)^2*d*g^2-1/6*b*(-d*(c^2*x^2-1)) \\ & ^{(1/2)}*g^2*d*c^4/(c^2*x^2-1)*\text{arccos}(c*x)*x^7+11/24*b*(-d*(c^2*x^2-1))^{(1/2)} \\ & *g^2*d*c^2/(c^2*x^2-1)*\text{arccos}(c*x)*x^5+1/16*b*(-d*(c^2*x^2-1))^{(1/2)}*g^2*d/ \\ & c^2/(c^2*x^2-1)*\text{arccos}(c*x)*x^2+5*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g*d/c^2/(c^2*x \\ & ^2-1)*\text{arccos}(c*x)-1/4*b*(-d*(c^2*x^2-1))^{(1/2)}*d*c^4/(c^2*x^2-1)*\text{arccos}(c*x) \\ & )*x^5*f^2+7/8*b*(-d*(c^2*x^2-1))^{(1/2)}*d*c^2/(c^2*x^2-1)*\text{arccos}(c*x)*x^3*f^ \\ & 2+3/8*a*f^2*d^2/(c^2*d)^{(1/2)}*\text{arctan}((c^2*d)^{(1/2)}*x/(-c^2*d*x^2+d)^{(1/2)})+ \\ & 1/16*a*g^2/c^2*d*x*(-c^2*d*x^2+d)^{(1/2)}+1/16*a*g^2/c^2*d^2/(c^2*d)^{(1/2)}*\text{ar} \\ & \text{ctan}((c^2*d)^{(1/2)}*x/(-c^2*d*x^2+d)^{(1/2)})-2/5*a*f*g/c^2/d*(-c^2*d*x^2+d)^{(5/2)}-1/6*a*g^2*x*(-c^2*d*x^2+d)^{(5/2)}/c^2/d \end{aligned}$$

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2\*(-c^2\*d\*x^2+d)^(3/2)\*(a+b\*arccos(c\*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral(-(ac^2dg^2x^4 + 2ac^2dfgx^3 - 2adfgx - adf^2 + (ac^2df^2 - adg^2)x^2 + (bc^2dg^2x^4 + 2bc^2dfgx^3 - 2bdfgx - bdf^2 +

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2\*(-c^2\*d\*x^2+d)^(3/2)\*(a+b\*arccos(c\*x)),x, algorithm="fricas")

[Out] integral(-(a\*c^2\*d\*g^2\*x^4 + 2\*a\*c^2\*d\*f\*g\*x^3 - 2\*a\*d\*f\*g\*x - a\*d\*f^2 + (a\*c^2\*d\*f^2 - a\*d\*g^2)\*x^2 + (b\*c^2\*d\*g^2\*x^4 + 2\*b\*c^2\*d\*f\*g\*x^3 - 2\*b\*d\*f\*g\*x - b\*d\*f^2 + (b\*c^2\*d\*f^2 - b\*d\*g^2)\*x^2)\*arccos(c\*x))\*sqrt(-c^2\*d\*x^2 + d), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2\*(-c\*\*2\*d\*x\*\*2+d)\*\*(3/2)\*(a+b\*acos(c\*x)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (-c^2dx^2 + d)^{\frac{3}{2}}(gx + f)^2(b \arccos(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2\*(-c^2\*d\*x^2+d)^(3/2)\*(a+b\*arccos(c\*x)),x, algorithm="giac")

[Out] integrate((-c^2\*d\*x^2 + d)^(3/2)\*(g\*x + f)^2\*(b\*arccos(c\*x) + a), x)

### 3.8 $\int (f + gx) (d - c^2 dx^2)^{3/2} (a + b \cos^{-1}(cx)) dx$

**Optimal.** Leaf size=370

$$\frac{3}{8}dfx\sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) + \frac{1}{4}dfx(1 - c^2 x^2) \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) - \frac{3df\sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx))}{16bc\sqrt{1 - c^2 x^2}}$$

```
[Out] -(b*d*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[1 - c^2*x^2]) + (5*b*c*d*f*x^2*Sqr
t[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (2*b*c*d*g*x^3*Sqrt[d - c^2*d*x^
2])/(15*Sqrt[1 - c^2*x^2]) - (b*c^3*d*f*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1
 - c^2*x^2]) - (b*c^3*d*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[1 - c^2*x^2]) +
(3*d*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 + (d*f*x*(1 - c^2*x^2)
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 - (d*g*(1 - c^2*x^2)^2*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^2) - (3*d*f*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCos[c*x])^2)/(16*b*c*Sqrt[1 - c^2*x^2])
```

**Rubi [A]** time = 0.334807, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.31$ , Rules used = {4778, 4764, 4650, 4648, 4642, 30, 14, 4678, 194}

$$\frac{3}{8}dfx\sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) + \frac{1}{4}dfx(1 - c^2 x^2) \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) - \frac{3df\sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx))}{16bc\sqrt{1 - c^2 x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] -(b*d*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[1 - c^2*x^2]) + (5*b*c*d*f*x^2*Sqr
t[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (2*b*c*d*g*x^3*Sqrt[d - c^2*d*x^
2])/(15*Sqrt[1 - c^2*x^2]) - (b*c^3*d*f*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1
 - c^2*x^2]) - (b*c^3*d*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[1 - c^2*x^2]) +
(3*d*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 + (d*f*x*(1 - c^2*x^2)
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 - (d*g*(1 - c^2*x^2)^2*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^2) - (3*d*f*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCos[c*x])^2)/(16*b*c*Sqrt[1 - c^2*x^2])
```

#### Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.)^ (n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
0] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]
```

#### Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.)^ (n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

#### Rule 4650

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.)^ (n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n)/(2*p + 1), x] + (D
```

```
Int[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 - c^2*x^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.)^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.)^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.)^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*c*(p + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (f + gx) (d - c^2 dx^2)^{3/2} (a + b \cos^{-1}(cx)) dx &= \frac{(d\sqrt{d - c^2 dx^2}) \int (f + gx) (1 - c^2 x^2)^{3/2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{(d\sqrt{d - c^2 dx^2}) \int (f(1 - c^2 x^2)^{3/2} (a + b \cos^{-1}(cx)) + gx(1 - c^2 x^2)^{3/2}) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{(df\sqrt{d - c^2 dx^2}) \int (1 - c^2 x^2)^{3/2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} + \frac{(dg\sqrt{d - c^2 dx^2}) \int (1 - c^2 x^2)^{3/2} dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{1}{4} dfx (1 - c^2 x^2) \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) - \frac{dg (1 - c^2 x^2)^2 \sqrt{d - c^2 dx^2}}{5} \\
&= \frac{3}{8} dfx \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) + \frac{1}{4} dfx (1 - c^2 x^2) \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) \\
&= -\frac{bdgx\sqrt{d - c^2 dx^2}}{5c\sqrt{1 - c^2 x^2}} + \frac{5bcdfx^2\sqrt{d - c^2 dx^2}}{16\sqrt{1 - c^2 x^2}} + \frac{2bcdgx^3\sqrt{d - c^2 dx^2}}{15\sqrt{1 - c^2 x^2}} - \frac{bdg}{5}
\end{aligned}$$

**Mathematica [A]** time = 1.56584, size = 337, normalized size = 0.91

$$-d\sqrt{d - c^2 dx^2} \left( 3 \left( 80a\sqrt{1 - c^2 x^2} \left( 5c^2 fx (2c^2 x^2 - 5) + 8g (c^2 x^2 - 1)^2 \right) + 25bcf \cos(4 \cos^{-1}(cx)) + 400bcgx + 8bg \cos^{-1}(cx) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)\*(d - c^2\*d\*x^2)^(3/2)\*(a + b\*ArcCos[c\*x]),x]

[Out] (-1800\*b\*c\*d\*f\*Sqrt[d - c^2\*d\*x^2]\*ArcCos[c\*x]^2 - 3600\*a\*c\*d^(3/2)\*f\*Sqrt[1 - c^2\*x^2]\*ArcTan[(c\*x\*Sqrt[d - c^2\*d\*x^2])/(Sqrt[d]\*(-1 + c^2\*x^2))] - d\*Sqrt[d - c^2\*d\*x^2]\*(-1200\*b\*c\*f\*Cos[2\*ArcCos[c\*x]] - 200\*b\*g\*Cos[3\*ArcCos[c\*x]] + 3\*(400\*b\*c\*g\*x + 80\*a\*Sqrt[1 - c^2\*x^2]\*(8\*g\*(-1 + c^2\*x^2)^2 + 5\*c^2\*f\*x\*(-5 + 2\*c^2\*x^2)) + 25\*b\*c\*f\*Cos[4\*ArcCos[c\*x]] + 8\*b\*g\*Cos[5\*ArcCos[c\*x]]) + 20\*b\*d\*Sqrt[d - c^2\*d\*x^2]\*ArcCos[c\*x]\*(-100\*g\*Sqrt[1 - c^2\*x^2] + 160\*c^2\*g\*x^2\*Sqrt[1 - c^2\*x^2] + 120\*c\*f\*Sin[2\*ArcCos[c\*x]] - 10\*g\*Sin[3\*ArcCos[c\*x]] - 15\*c\*f\*Sin[4\*ArcCos[c\*x]] - 6\*g\*Sin[5\*ArcCos[c\*x]]))/(960\*c^2\*Sqrt[1 - c^2\*x^2])

**Maple [B]** time = 0.49, size = 698, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)\*(-c^2\*d\*x^2+d)^(3/2)\*(a+b\*arccos(c\*x)),x)

[Out] -1/5\*a\*g/c^2/d\*(-c^2\*d\*x^2+d)^(5/2)+1/4\*a\*f\*x\*(-c^2\*d\*x^2+d)^(3/2)+3/8\*a\*f\*d\*x\*(-c^2\*d\*x^2+d)^(1/2)+3/8\*a\*f\*d^2/(c^2\*d)^(1/2)\*arctan((c^2\*d)^(1/2)\*x/(-c^2\*d\*x^2+d)^(1/2))+3/16\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*(-c^2\*x^2+1)^(1/2)/c/(c^2\*x^2-1)\*arccos(c\*x)^2\*f\*d+1/25\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*g\*d\*c^3/(c^2\*x^2-1)\*(-c^2\*x^2+1)^(1/2)\*x^5-2/15\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*g\*d\*c/(c^2\*x^2-1)\*(-c^2\*x^2+1)^(1/2)\*x^3+1/5\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*g\*d/c/(c^2\*x^2-1)\*(-c^2\*x^2+1)^(1/2)\*x+1/16\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*f\*d\*c^3/(c^2\*x^2-1)\*(-c^2\*x^2+1)^(1/2)

$$\begin{aligned} &)^{(1/2)} * x^4 - 5/16 * b * (-d * (c^2 * x^2 - 1))^{(1/2)} * f * d * c / (c^2 * x^2 - 1) * (-c^2 * x^2 + 1)^{(1/2)} * x^2 + 1/5 * b * (-d * (c^2 * x^2 - 1))^{(1/2)} * g * d / c^2 / (c^2 * x^2 - 1) * \arccos(c * x) - 1/5 * b * \\ &(-d * (c^2 * x^2 - 1))^{(1/2)} * g * d * c^4 / (c^2 * x^2 - 1) * \arccos(c * x) * x^6 + 3/5 * b * (-d * (c^2 * x^2 - 1))^{(1/2)} * g * d * c^2 / (c^2 * x^2 - 1) * \arccos(c * x) * x^4 - 3/5 * b * (-d * (c^2 * x^2 - 1))^{(1/2)} * g * d / (c^2 * x^2 - 1) * \arccos(c * x) * x^2 - 1/4 * b * (-d * (c^2 * x^2 - 1))^{(1/2)} * f * d * c^4 / (c^2 * x^2 - 1) * \arccos(c * x) * x^5 + 7/8 * b * (-d * (c^2 * x^2 - 1))^{(1/2)} * f * d * c^2 / (c^2 * x^2 - 1) * \arccos(c * x) * x^3 - 5/8 * b * (-d * (c^2 * x^2 - 1))^{(1/2)} * f * d / (c^2 * x^2 - 1) * \arccos(c * x) * x + 1/7 * 128 * b * (-d * (c^2 * x^2 - 1))^{(1/2)} * f * d / c / (c^2 * x^2 - 1) * (-c^2 * x^2 + 1)^{(1/2)} \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(-c^2\*d\*x^2+d)^(3/2)\*(a+b\*arccos(c\*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(ac^2d gx^3 + ac^2d f x^2 - adgx - adf + (bc^2d gx^3 + bc^2d f x^2 - bdgx - bdf) \arccos(cx)\right) \sqrt{-c^2 dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(-c^2\*d\*x^2+d)^(3/2)\*(a+b\*arccos(c\*x)),x, algorithm="fricas")

[Out] integral(-(a\*c^2\*d\*g\*x^3 + a\*c^2\*d\*f\*x^2 - a\*d\*g\*x - a\*d\*f + (b\*c^2\*d\*g\*x^3 + b\*c^2\*d\*f\*x^2 - b\*d\*g\*x - b\*d\*f)\*arccos(c\*x))\*sqrt(-c^2\*d\*x^2 + d), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(-c\*\*2\*d\*x\*\*2+d)\*\*(3/2)\*(a+b\*acos(c\*x)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (-c^2 dx^2 + d)^{\frac{3}{2}} (gx + f) (b \arccos(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(g*x + f)*(b*arccos(c*x) + a), x)
```

$$3.9 \quad \int \frac{(d-c^2 dx^2)^{3/2} (a+b \cos^{-1}(cx))}{f+gx} dx$$

**Optimal.** Leaf size=1064

$$-\frac{bdx^3\sqrt{d-c^2dx^2}c^3}{9g\sqrt{1-c^2x^2}} + \frac{bdfx^2\sqrt{d-c^2dx^2}c^3}{4g^2\sqrt{1-c^2x^2}} + \frac{dfx\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))c^2}{2g^2} + \frac{d(cf-g)(cf+g)x\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))}{2bg^3\sqrt{1-c^2x^2}}$$

```
[Out] -((a*d*(c*f - g)*(c*f + g)*Sqrt[d - c^2*d*x^2])/g^3) + (b*c*d*x*Sqrt[d - c^2*d*x^2])/(3*g*Sqrt[1 - c^2*x^2]) - (b*c*d*(c*f - g)*(c*f + g)*x*Sqrt[d - c^2*d*x^2])/(g^3*Sqrt[1 - c^2*x^2]) + (b*c^3*d*f*x^2*Sqrt[d - c^2*d*x^2])/(4*g^2*Sqrt[1 - c^2*x^2]) - (b*c^3*d*x^3*Sqrt[d - c^2*d*x^2])/(9*g*Sqrt[1 - c^2*x^2]) - (b*d*(c*f - g)*(c*f + g)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])/g^3 + (c^2*d*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(2*g^2) + (d*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*g) - (c*d*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*g^2*Sqrt[1 - c^2*x^2]) + (c*d*(c*f - g)*(c*f + g)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*g^3*Sqrt[1 - c^2*x^2]) + (d*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*g^4*(f + g*x)*Sqrt[1 - c^2*x^2]) + (d*(c*f - g)*(c*f + g)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*g^2*(f + g*x)) + (a*d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])])/(g^4*Sqrt[1 - c^2*x^2]) + (I*b*d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[1 - c^2*x^2]) - (I*b*d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[1 - c^2*x^2]) + (b*d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -(E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[1 - c^2*x^2]) - (b*d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -(E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[1 - c^2*x^2])
```

**Rubi [A]** time = 2.28, antiderivative size = 1064, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 23, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.742$ , Rules used = {4778, 4768, 4648, 4642, 30, 4678, 4766, 683, 4758, 6742, 725, 204, 1654, 12, 4800, 4798, 8, 4774, 3321, 2264, 2190, 2279, 2391}

$$-\frac{bdx^3\sqrt{d-c^2dx^2}c^3}{9g\sqrt{1-c^2x^2}} + \frac{bdfx^2\sqrt{d-c^2dx^2}c^3}{4g^2\sqrt{1-c^2x^2}} + \frac{dfx\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))c^2}{2g^2} + \frac{d(cf-g)(cf+g)x\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))}{2bg^3\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(f + g*x), x]
```

```
[Out] -((a*d*(c*f - g)*(c*f + g)*Sqrt[d - c^2*d*x^2])/g^3) + (b*c*d*x*Sqrt[d - c^2*d*x^2])/(3*g*Sqrt[1 - c^2*x^2]) - (b*c*d*(c*f - g)*(c*f + g)*x*Sqrt[d - c^2*d*x^2])/(g^3*Sqrt[1 - c^2*x^2]) + (b*c^3*d*f*x^2*Sqrt[d - c^2*d*x^2])/(4*g^2*Sqrt[1 - c^2*x^2]) - (b*c^3*d*x^3*Sqrt[d - c^2*d*x^2])/(9*g*Sqrt[1 - c^2*x^2]) - (b*d*(c*f - g)*(c*f + g)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])/g^3 + (c^2*d*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(2*g^2) + (d*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*g) - (c*d*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*g^2*Sqrt[1 - c^2*x^2]) + (c*d*(c*f - g)*(c*f + g)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*g^3*Sqrt[1 - c^2*x^2]) + (d*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*g^4*(f + g*x)*Sqrt[1 - c^2*x^2]) + (d*(c*f - g)*(c*f + g)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*g^2*(f + g*x)) + (a*d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])])/(g^4*Sqrt[1 - c^2*x^2]) + (I*b*d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[1 - c^2*x^2]) - (I*b*d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[1 - c^2*x^2]) + (b*d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -(E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[1 - c^2*x^2]) - (b*d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -(E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[1 - c^2*x^2])
```

$$x^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCos}[c x])^2 / (2 b c g^2 (f + g x)) + (a d (c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2} \operatorname{ArcTan}[(g + c^2 f x) / (\sqrt{c^2 f^2 - g^2} \sqrt{1 - c^2 x^2})]) / (g^4 \sqrt{1 - c^2 x^2}) + (I b d (c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2} \operatorname{ArcCos}[c x] \operatorname{Log}[1 + (E^{(I \operatorname{ArcCos}[c x])} g) / (c f - \sqrt{c^2 f^2 - g^2})]) / (g^4 \sqrt{1 - c^2 x^2}) - (I b d (c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2} \operatorname{ArcCos}[c x] \operatorname{Log}[1 + (E^{(I \operatorname{ArcCos}[c x])} g) / (c f + \sqrt{c^2 f^2 - g^2})]) / (g^4 \sqrt{1 - c^2 x^2}) + (b d (c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}[2, -(E^{(I \operatorname{ArcCos}[c x])} g) / (c f - \sqrt{c^2 f^2 - g^2})]) / (g^4 \sqrt{1 - c^2 x^2}) - (b d (c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}[2, -(E^{(I \operatorname{ArcCos}[c x])} g) / (c f + \sqrt{c^2 f^2 - g^2})]) / (g^4 \sqrt{1 - c^2 x^2})$$
**Rule 4778**

$$\operatorname{Int}[(a + \operatorname{ArcCos}[c x])^n (f + g x)^m (d + e x^2)^p, x] \rightarrow \operatorname{Dist}[(d + e x^2)^p \operatorname{IntPart}[p] / (1 - c^2 x^2)^{\operatorname{FracPart}[p]}, \operatorname{Int}[(f + g x)^m (1 - c^2 x^2)^p (a + b \operatorname{ArcCos}[c x])^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, n\}, x] \&\& \operatorname{EqQ}[c^2 d + e, 0] \&\& \operatorname{IntegerQ}[m] \&\& \operatorname{IntegerQ}[p - 1/2] \&\& !\operatorname{GtQ}[d, 0]$$
**Rule 4768**

$$\operatorname{Int}[(a + \operatorname{ArcCos}[c x])^n (f + g x)^m (d + e x^2)^p, x] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[\sqrt{d + e x^2} (a + b \operatorname{ArcCos}[c x])^n (f + g x)^m (d + e x^2)^{p - 1/2}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g\}, x] \&\& \operatorname{EqQ}[c^2 d + e, 0] \&\& \operatorname{IntegerQ}[m] \&\& \operatorname{IGtQ}[p + 1/2, 0] \&\& \operatorname{GtQ}[d, 0] \&\& \operatorname{IGtQ}[n, 0]$$
**Rule 4648**

$$\operatorname{Int}[(a + \operatorname{ArcCos}[c x])^n \sqrt{d + e x^2}, x] \rightarrow \operatorname{Simp}[(x \sqrt{d + e x^2} (a + b \operatorname{ArcCos}[c x])^n) / 2, x] + (\operatorname{Dist}[\sqrt{d + e x^2} / (2 \sqrt{1 - c^2 x^2}), \operatorname{Int}[(a + b \operatorname{ArcCos}[c x])^n / \sqrt{1 - c^2 x^2}, x], x] + \operatorname{Dist}[(b c n \sqrt{d + e x^2}) / (2 \sqrt{1 - c^2 x^2}), \operatorname{Int}[x (a + b \operatorname{ArcCos}[c x])^{n - 1}, x], x]) /; \operatorname{FreeQ}\{a, b, c, d, e\}, x] \&\& \operatorname{EqQ}[c^2 d + e, 0] \&\& \operatorname{GtQ}[n, 0]$$
**Rule 4642**

$$\operatorname{Int}[(a + \operatorname{ArcCos}[c x])^n / \sqrt{d + e x^2}, x] \rightarrow -\operatorname{Simp}[(a + b \operatorname{ArcCos}[c x])^{n + 1} / (b c \sqrt{d + e x^2} (n + 1)), x] /; \operatorname{FreeQ}\{a, b, c, d, e, n\}, x] \&\& \operatorname{EqQ}[c^2 d + e, 0] \&\& \operatorname{GtQ}[d, 0] \&\& \operatorname{NeQ}[n, -1]$$
**Rule 30**

$$\operatorname{Int}[x^m, x] \rightarrow \operatorname{Simp}[x^{m + 1} / (m + 1), x] /; \operatorname{FreeQ}[m, x] \&\& \operatorname{NeQ}[m, -1]$$
**Rule 4678**

$$\operatorname{Int}[(a + \operatorname{ArcCos}[c x])^n (f + g x)^m (d + e x^2)^p, x] \rightarrow \operatorname{Simp}[(d + e x^2)^{p + 1} (a + b \operatorname{ArcCos}[c x])^n / (2 e (p + 1)), x] - \operatorname{Dist}[(b n d \operatorname{IntPart}[p] (d + e x^2)^{\operatorname{FracPart}[p]}] / (2 c (p + 1) (1 - c^2 x^2)^{\operatorname{FracPart}[p]}), \operatorname{Int}[(1 - c^2 x^2)^{p + 1/2} (a + b \operatorname{ArcCos}[c x])^{n - 1}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, p\}, x] \&\& \operatorname{EqQ}[c^2 d + e, 0] \&\& \operatorname{GtQ}[n, 0] \&\& \operatorname{NeQ}[p, -1]$$
**Rule 4766**

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^ (n_.)*((f_) + (g_.)*(x_))^(m_)*Sqrt[
(d_) + (e_.)*(x_)^2], x_Symbol] := -Simp[((f + g*x)^m*(d + e*x^2)*(a + b*Ar
cCos[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] + Dist[1/(b*c*Sqrt[d]*(n + 1)
), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCos[
c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e,
0] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

### Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

### Rule 4758

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.) + (g_.)*(x_) + (h_.)*(x
_)^2)^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := With[{u = IntHide[(f + g*x
+ h*x^2)^p/(d + e*x^2), x]}, Dist[(a + b*ArcCos[c*x])^n, u, x] + Dist[b*c*
n, Int[SimplifyIntegrand[(u*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ[p,
0] && EqQ[e*g - 2*d*h, 0]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, c, d,
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 4800

```
Int[(ArcCos[(c_.)*(x_)]*(b_.) + (a_.))^(n_.)*(RFX_)*((d_) + (e_.)*(x_)^2)^(p
_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^p, RFX*(a + b*ArcCos[c*x])
^m, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFX, x] && IGt
Q[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

#### Rule 4798

```
Int[ArcCos[(c_.)*(x_)]^(n_.)*(RFX_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :
> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcCos[c*x]^n, RFX, x]}, Int[u, x
] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFX, x] && IGtQ[n
, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

#### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

#### Rule 4774

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_)^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] := -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst
[Int[(a + b*x)^n*(c*f + g*cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &&
(GtQ[m, 0] || IGtQ[n, 0])
```

#### Rule 3321

```
Int[(((c_.) + (d_.)*(x_)^(m_.))/((a_.) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] := Dist[2, Int[(((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

#### Rule 2264

```
Int[(((F_)^(u_)*((f_.) + (g_.)*(x_)^(m_.)))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_)^(m_.))/
((a_.) + (b_.)*(F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

## Rubi steps

$$\begin{aligned}
\int \frac{(d - c^2 dx^2)^{3/2} (a + b \cos^{-1}(cx))}{f + gx} dx &= \frac{(d\sqrt{d - c^2 dx^2}) \int \frac{(1 - c^2 x^2)^{3/2} (a + b \cos^{-1}(cx))}{f + gx} dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{(d\sqrt{d - c^2 dx^2}) \int \left( \frac{c^2 f \sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx))}{g^2} - \frac{c^2 x \sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx))}{g} + \frac{(-c^2 f^2 + g^2) \sqrt{1 - c^2 x^2}}{g^2} \right) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{\left( d \left( 1 - \frac{c^2 f^2}{g^2} \right) \sqrt{d - c^2 dx^2} \right) \int \frac{\sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx))}{f + gx} dx}{\sqrt{1 - c^2 x^2}} + \frac{(c^2 d f \sqrt{d - c^2 dx^2}) \int \sqrt{1 - c^2 x^2} dx}{g^2 \sqrt{1 - c^2 x^2}} \\
&= \frac{c^2 d f x \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx))}{2g^2} + \frac{d (1 - c^2 x^2) \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx))}{3g} \\
&= \frac{bcdx\sqrt{d - c^2 dx^2}}{3g\sqrt{1 - c^2 x^2}} + \frac{bc^3 d f x^2 \sqrt{d - c^2 dx^2}}{4g^2 \sqrt{1 - c^2 x^2}} - \frac{bc^3 dx^3 \sqrt{d - c^2 dx^2}}{9g\sqrt{1 - c^2 x^2}} + \frac{c^2 d f x \sqrt{d - c^2 dx^2}}{2g^2} \\
&= \frac{bcdx\sqrt{d - c^2 dx^2}}{3g\sqrt{1 - c^2 x^2}} + \frac{bc^3 d f x^2 \sqrt{d - c^2 dx^2}}{4g^2 \sqrt{1 - c^2 x^2}} - \frac{bc^3 dx^3 \sqrt{d - c^2 dx^2}}{9g\sqrt{1 - c^2 x^2}} + \frac{c^2 d f x \sqrt{d - c^2 dx^2}}{2g^2} \\
&= \frac{bcdx\sqrt{d - c^2 dx^2}}{3g\sqrt{1 - c^2 x^2}} + \frac{bc^3 d f x^2 \sqrt{d - c^2 dx^2}}{4g^2 \sqrt{1 - c^2 x^2}} - \frac{bc^3 dx^3 \sqrt{d - c^2 dx^2}}{9g\sqrt{1 - c^2 x^2}} + \frac{c^2 d f x \sqrt{d - c^2 dx^2}}{2g^2} \\
&= -\frac{ad(cf - g)(cf + g)\sqrt{d - c^2 dx^2}}{g^3} + \frac{bcdx\sqrt{d - c^2 dx^2}}{3g\sqrt{1 - c^2 x^2}} + \frac{bc^3 d f x^2 \sqrt{d - c^2 dx^2}}{4g^2 \sqrt{1 - c^2 x^2}} - \frac{bc^3 dx^3 \sqrt{d - c^2 dx^2}}{9g\sqrt{1 - c^2 x^2}} \\
&= -\frac{ad(cf - g)(cf + g)\sqrt{d - c^2 dx^2}}{g^3} + \frac{bcdx\sqrt{d - c^2 dx^2}}{3g\sqrt{1 - c^2 x^2}} + \frac{bc^3 d f x^2 \sqrt{d - c^2 dx^2}}{4g^2 \sqrt{1 - c^2 x^2}} - \frac{bc^3 dx^3 \sqrt{d - c^2 dx^2}}{9g\sqrt{1 - c^2 x^2}} \\
&= -\frac{ad(cf - g)(cf + g)\sqrt{d - c^2 dx^2}}{g^3} + \frac{bcdx\sqrt{d - c^2 dx^2}}{3g\sqrt{1 - c^2 x^2}} + \frac{bc^3 d f x^2 \sqrt{d - c^2 dx^2}}{4g^2 \sqrt{1 - c^2 x^2}} - \frac{bc^3 dx^3 \sqrt{d - c^2 dx^2}}{9g\sqrt{1 - c^2 x^2}} \\
&= -\frac{ad(cf - g)(cf + g)\sqrt{d - c^2 dx^2}}{g^3} + \frac{bcdx\sqrt{d - c^2 dx^2}}{3g\sqrt{1 - c^2 x^2}} + \frac{bcd \left( 1 - \frac{c^2 f^2}{g^2} \right) x \sqrt{d - c^2 dx^2}}{g\sqrt{1 - c^2 x^2}} \\
&= -\frac{ad(cf - g)(cf + g)\sqrt{d - c^2 dx^2}}{g^3} + \frac{bcdx\sqrt{d - c^2 dx^2}}{3g\sqrt{1 - c^2 x^2}} + \frac{bcd \left( 1 - \frac{c^2 f^2}{g^2} \right) x \sqrt{d - c^2 dx^2}}{g\sqrt{1 - c^2 x^2}} \\
&= -\frac{ad(cf - g)(cf + g)\sqrt{d - c^2 dx^2}}{g^3} + \frac{bcdx\sqrt{d - c^2 dx^2}}{3g\sqrt{1 - c^2 x^2}} + \frac{bcd \left( 1 - \frac{c^2 f^2}{g^2} \right) x \sqrt{d - c^2 dx^2}}{g\sqrt{1 - c^2 x^2}} \\
&= -\frac{ad(cf - g)(cf + g)\sqrt{d - c^2 dx^2}}{g^3} + \frac{bcdx\sqrt{d - c^2 dx^2}}{3g\sqrt{1 - c^2 x^2}} + \frac{bcd \left( 1 - \frac{c^2 f^2}{g^2} \right) x \sqrt{d - c^2 dx^2}}{g\sqrt{1 - c^2 x^2}} \\
&= -\frac{ad(cf - g)(cf + g)\sqrt{d - c^2 dx^2}}{g^3} + \frac{bcdx\sqrt{d - c^2 dx^2}}{3g\sqrt{1 - c^2 x^2}} + \frac{bcd \left( 1 - \frac{c^2 f^2}{g^2} \right) x \sqrt{d - c^2 dx^2}}{g\sqrt{1 - c^2 x^2}}
\end{aligned}$$

**Mathematica [B]** time = 12.5042, size = 3034, normalized size = 2.85

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((d - c^2\*d\*x^2)^(3/2)\*(a + b\*ArcCos[c\*x]))/(f + g\*x),x]

[Out]  $\sqrt{-(d(-1 + c^2x^2))} \left( \frac{(ad(-3c^2f^2 + 4g^2))/(3g^3) + (ac^2d*fx)/(2g^2) - (ac^2d*x^2)/(3g)}{(d - c^2d*x^2)^{3/2}} + \frac{(ac*d^{3/2}*f*(2c^2f^2 - 3g^2)*\text{ArcTan}[c*x*\sqrt{-(d(-1 + c^2x^2))}]/(\sqrt{d}*(-1 + c^2x^2))}{(2g^4)} + \frac{ad^{3/2}*(-(c^2f^2) + g^2)^{3/2}*\text{Log}[f + g*x]/g^4 - (ad^{3/2}*(-(c^2f^2) + g^2)^{3/2}*\text{Log}[d*g + c^2d*fx + \sqrt{d}*\sqrt{-(c^2f^2) + g^2}]*\sqrt{-(d(-1 + c^2x^2))}]/g^4 - (b*d*\sqrt{d*(1 - c^2x^2)}*((-2c*gx)/\sqrt{1 - c^2x^2}) - 2g*\text{ArcCos}[c*x] + (c*f*\text{ArcCos}[c*x]^2)/\sqrt{1 - c^2x^2} + (2*(-(c*f) + g)*(c*f + g)*(2*\text{ArcCos}[c*x]*\text{ArcTanh}[\frac{(c*f + g)*\text{Cot}[\text{ArcCos}[c*x]/2}])/\sqrt{-(c^2f^2) + g^2}] - 2*\text{ArcCos}[-((c*f)/g)]*\text{ArcTanh}[\frac{(-(c*f) + g)*\text{Tan}[\text{ArcCos}[c*x]/2}])/\sqrt{-(c^2f^2) + g^2}] + (\text{ArcCos}[-((c*f)/g)] - (2*I)*\text{ArcTanh}[\frac{(c*f + g)*\text{Cot}[\text{ArcCos}[c*x]/2}])/\sqrt{-(c^2f^2) + g^2}] + (2*I)*\text{ArcTanh}[\frac{(-(c*f) + g)*\text{Tan}[\text{ArcCos}[c*x]/2}])/\sqrt{-(c^2f^2) + g^2}])*\text{Log}[\sqrt{-(c^2f^2) + g^2}]/(\sqrt{2}*E^{(I/2)*\text{ArcCos}[c*x]}*\sqrt{g}*\sqrt{c*f + c*gx}) + (\text{ArcCos}[-((c*f)/g)] + (2*I)*(\text{ArcTanh}[\frac{(c*f + g)*\text{Cot}[\text{ArcCos}[c*x]/2}])/\sqrt{-(c^2f^2) + g^2}] - \text{ArcTanh}[\frac{(-(c*f) + g)*\text{Tan}[\text{ArcCos}[c*x]/2}])/\sqrt{-(c^2f^2) + g^2}])*\text{Log}[(E^{(I/2)*\text{ArcCos}[c*x]}*\sqrt{-(c^2f^2) + g^2})/(\sqrt{2}*\sqrt{g}*\sqrt{c*f + c*gx})] - (\text{ArcCos}[-((c*f)/g)] - (2*I)*\text{ArcTanh}[\frac{(-(c*f) + g)*\text{Tan}[\text{ArcCos}[c*x]/2}])/\sqrt{-(c^2f^2) + g^2}])*\text{Log}[\frac{(c*f + g)*((-I)*c*f + I*g + \sqrt{-(c^2f^2) + g^2})*(-I + \text{Tan}[\text{ArcCos}[c*x]/2])}{(g*(c*f + g + \sqrt{-(c^2f^2) + g^2})*\text{Tan}[\text{ArcCos}[c*x]/2])}] - (\text{ArcCos}[-((c*f)/g)] + (2*I)*\text{ArcTanh}[\frac{(-(c*f) + g)*\text{Tan}[\text{ArcCos}[c*x]/2}])/\sqrt{-(c^2f^2) + g^2}])*\text{Log}[\frac{(c*f + g)*(I*c*f - I*g + \sqrt{-(c^2f^2) + g^2})*(I + \text{Tan}[\text{ArcCos}[c*x]/2])}{(g*(c*f + g + \sqrt{-(c^2f^2) + g^2})*\text{Tan}[\text{ArcCos}[c*x]/2])}] + I*(\text{PolyLog}[2, \frac{(c*f - I*\sqrt{-(c^2f^2) + g^2})*(c*f + g - \sqrt{-(c^2f^2) + g^2})*\text{Tan}[\text{ArcCos}[c*x]/2])}{(g*(c*f + g + \sqrt{-(c^2f^2) + g^2})*\text{Tan}[\text{ArcCos}[c*x]/2])}] - \text{PolyLog}[2, \frac{(c*f + I*\sqrt{-(c^2f^2) + g^2})*(c*f + g - \sqrt{-(c^2f^2) + g^2})*\text{Tan}[\text{ArcCos}[c*x]/2])}{(g*(c*f + g + \sqrt{-(c^2f^2) + g^2})*\text{Tan}[\text{ArcCos}[c*x]/2])}]) \right) / (\sqrt{-(c^2f^2) + g^2} * \sqrt{1 - c^2x^2}) + (b*d*\sqrt{d*(1 - c^2x^2)}) * ((9*(2*\text{ArcCos}[c*x]*\text{ArcTanh}[\frac{(c*f + g)*\text{Cot}[\text{ArcCos}[c*x]/2}])/\sqrt{-(c^2f^2) + g^2}] - 2*\text{ArcCos}[-((c*f)/g)]*\text{ArcTanh}[\frac{(-(c*f) + g)*\text{Tan}[\text{ArcCos}[c*x]/2}])/\sqrt{-(c^2f^2) + g^2}] + (\text{ArcCos}[-((c*f)/g)] - (2*I)*\text{ArcTanh}[\frac{(c*f + g)*\text{Cot}[\text{ArcCos}[c*x]/2}])/\sqrt{-(c^2f^2) + g^2}] + (2*I)*\text{ArcTanh}[\frac{(-(c*f) + g)*\text{Tan}[\text{ArcCos}[c*x]/2}])/\sqrt{-(c^2f^2) + g^2}])*\text{Log}[\sqrt{-(c^2f^2) + g^2}]/(\sqrt{2}*E^{(I/2)*\text{ArcCos}[c*x]}*\sqrt{g}*\sqrt{c*f + c*gx}) + (\text{ArcCos}[-((c*f)/g)] + (2*I)*(\text{ArcTanh}[\frac{(c*f + g)*\text{Cot}[\text{ArcCos}[c*x]/2}])/\sqrt{-(c^2f^2) + g^2}] - \text{ArcTanh}[\frac{(-(c*f) + g)*\text{Tan}[\text{ArcCos}[c*x]/2}])/\sqrt{-(c^2f^2) + g^2}])*\text{Log}[(E^{(I/2)*\text{ArcCos}[c*x]}*\sqrt{-(c^2f^2) + g^2})/(\sqrt{2}*\sqrt{g}*\sqrt{c*f + c*gx})] - (\text{ArcCos}[-((c*f)/g)] - (2*I)*\text{ArcTanh}[\frac{(-(c*f) + g)*\text{Tan}[\text{ArcCos}[c*x]/2}])/\sqrt{-(c^2f^2) + g^2}])*\text{Log}[\frac{(c*f + g)*((-I)*c*f + I*g + \sqrt{-(c^2f^2) + g^2})*(-I + \text{Tan}[\text{ArcCos}[c*x]/2])}{(g*(c*f + g + \sqrt{-(c^2f^2) + g^2})*\text{Tan}[\text{ArcCos}[c*x]/2])}] - (\text{ArcCos}[-((c*f)/g)] + (2*I)*\text{ArcTanh}[\frac{(-(c*f) + g)*\text{Tan}[\text{ArcCos}[c*x]/2}])/\sqrt{-(c^2f^2) + g^2}])*\text{Log}[\frac{(c*f + g)*(I*c*f - I*g + \sqrt{-(c^2f^2) + g^2})*(I + \text{Tan}[\text{ArcCos}[c*x]/2])}{(g*(c*f + g + \sqrt{-(c^2f^2) + g^2})*\text{Tan}[\text{ArcCos}[c*x]/2])}] + I*(\text{PolyLog}[2, \frac{(c*f - I*\sqrt{-(c^2f^2) + g^2})*(c*f + g - \sqrt{-(c^2f^2) + g^2})*\text{Tan}[\text{ArcCos}[c*x]/2])}{(g*(c*f + g + \sqrt{-(c^2f^2) + g^2})*\text{Tan}[\text{ArcCos}[c*x]/2])}] - \text{PolyLog}[2, \frac{(c*f + I*\sqrt{-(c^2f^2) + g^2})*(c*f + g - \sqrt{-(c^2f^2) + g^2})*\text{Tan}[\text{ArcCos}[c*x]/2])}{(g*(c*f + g + \sqrt{-(c^2f^2) + g^2})*\text{Tan}[\text{ArcCos}[c*x]/2])}]) \right) / \sqrt{-(c^2f^2) + g^2} + (18*c*g*(-4*c^2*f^2 + g^2)*x + 18*g*(-4*c^2*f^2 + g^2)*\sqrt{1 - c^2*x^2}*\text{ArcCos}[c*x] + 18*c*f*(2*c^2*f^2 - g^2)*\text{ArcCos}[c*x]^2 + 9*c*f*g^2*\text{Cos}[2*\text{ArcCos}[c*x]] - 2*g^3*\text{Cos}[3*\text{ArcCos}[c*x]] - (9*(8*c^4*f^4 - 8*c^2*f^2*g^2$

$$\begin{aligned}
& + g^4 * (2 * \text{ArcCos}[c*x] * \text{ArcTanh}[\frac{(c*f + g) * \text{Cot}[\text{ArcCos}[c*x]/2]}{\sqrt{-(c^2*f^2) + g^2}}] - 2 * \text{ArcCos}[-\frac{(c*f)}{g}] * \text{ArcTanh}[\frac{(-c*f) + g * \text{Tan}[\text{ArcCos}[c*x]/2]}{\sqrt{-(c^2*f^2) + g^2}}] + (\text{ArcCos}[-\frac{(c*f)}{g}]) - (2*I) * \text{ArcTanh}[\frac{(c*f + g) * \text{Cot}[\text{ArcCos}[c*x]/2]}{\sqrt{-(c^2*f^2) + g^2}}] + (2*I) * \text{ArcTanh}[\frac{(-c*f) + g * \text{Tan}[\text{ArcCos}[c*x]/2]}{\sqrt{-(c^2*f^2) + g^2}}]) * \text{Log}[\frac{\sqrt{-(c^2*f^2) + g^2}}{\sqrt{2} * E^{(I/2) * \text{ArcCos}[c*x]} * \sqrt{g} * \sqrt{c*f + c*g*x}}] + (\text{ArcCos}[-\frac{(c*f)}{g}]) \\
& + (2*I) * (\text{ArcTanh}[\frac{(c*f + g) * \text{Cot}[\text{ArcCos}[c*x]/2]}{\sqrt{-(c^2*f^2) + g^2}}] - \text{ArcTanh}[\frac{(-c*f) + g * \text{Tan}[\text{ArcCos}[c*x]/2]}{\sqrt{-(c^2*f^2) + g^2}}]) * \text{Log}[(E^{(I/2) * \text{ArcCos}[c*x]} * \sqrt{-(c^2*f^2) + g^2}) / (\sqrt{2} * \sqrt{g} * \sqrt{c*f + c*g*x})] - (\text{ArcCos}[-\frac{(c*f)}{g}]) - (2*I) * \text{ArcTanh}[\frac{(-c*f) + g * \text{Tan}[\text{ArcCos}[c*x]/2]}{\sqrt{-(c^2*f^2) + g^2}}] * \text{Log}[\frac{(c*f + g) * ((-I) * c*f + I * g + \sqrt{-(c^2*f^2) + g^2}) * (-I + \text{Tan}[\text{ArcCos}[c*x]/2])}{g * (c*f + g + \sqrt{-(c^2*f^2) + g^2}) * \text{Tan}[\text{ArcCos}[c*x]/2]}] - (\text{ArcCos}[-\frac{(c*f)}{g}]) + (2*I) * \text{ArcTanh}[\frac{(-c*f) + g * \text{Tan}[\text{ArcCos}[c*x]/2]}{\sqrt{-(c^2*f^2) + g^2}}] * \text{Log}[\frac{(c*f + g) * (I * c*f - I * g + \sqrt{-(c^2*f^2) + g^2}) * (I + \text{Tan}[\text{ArcCos}[c*x]/2])}{g * (c*f + g + \sqrt{-(c^2*f^2) + g^2}) * \text{Tan}[\text{ArcCos}[c*x]/2]}] + I * (\text{PolyLog}[2, ((c*f - I * \sqrt{-(c^2*f^2) + g^2}) * (c*f + g - \sqrt{-(c^2*f^2) + g^2}) * \text{Tan}[\text{ArcCos}[c*x]/2])] / (g * (c*f + g + \sqrt{-(c^2*f^2) + g^2}) * \text{Tan}[\text{ArcCos}[c*x]/2])) - \text{PolyLog}[2, ((c*f + I * \sqrt{-(c^2*f^2) + g^2}) * (c*f + g - \sqrt{-(c^2*f^2) + g^2}) * \text{Tan}[\text{ArcCos}[c*x]/2])] / (g * (c*f + g + \sqrt{-(c^2*f^2) + g^2}) * \text{Tan}[\text{ArcCos}[c*x]/2])))] / \sqrt{-(c^2*f^2) + g^2} + 18 * c*f*g^2 * \text{ArcCos}[c*x] * \text{Sin}[2 * \text{ArcCos}[c*x]] - 6 * g^3 * \text{ArcCos}[c*x] * \text{Sin}[3 * \text{ArcCos}[c*x]] / g^4) / (72 * \sqrt{1 - c^2*x^2})
\end{aligned}$$

**Maple [B]** time = 0.398, size = 2132, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (-c^2*d*x^2+d)^{(3/2)}*(a+b*\arccos(c*x))/(g*x+f), x$

[Out]  $a/g*d*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)}+1/8*b*(-d*(c^2*x^2-1))^{(1/2)}*f*d*c/(c^2*x^2-1)/g^2*(-c^2*x^2+1)^{(1/2)}+1/9*b*(-d*(c^2*x^2-1))^{(1/2)}*d/(c^2*x^2-1)/g*(-c^2*x^2+1)^{(1/2)}*x^3*c^3-4/3*b*(-d*(c^2*x^2-1))^{(1/2)}*d/(c^2*x^2-1)/g*(-c^2*x^2+1)^{(1/2)}*x*c+b*(-d*(c^2*x^2-1))^{(1/2)}*d/(c^2*x^2-1)/g^3*\arccos(c*x)*c^2*f^2-b*(c^2*f^2-g^2)^{(3/2)}*d*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x^2-1)/g^4*\text{dilog}(-1/(-c*f+(c^2*f^2-g^2)^{(1/2)})*(c*x+I*(-c^2*x^2+1)^{(1/2)})*g-1/(-c*f+(c^2*f^2-g^2)^{(1/2)})*c*f+1/(-c*f+(c^2*f^2-g^2)^{(1/2)})*(c^2*f^2-g^2)^{(1/2)})+b*(c^2*f^2-g^2)^{(3/2)}*d*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x^2-1)/g^4*\text{dilog}((c*x+I*(-c^2*x^2+1)^{(1/2)})*g/(c*f+(c^2*f^2-g^2)^{(1/2)})+1/(c*f+(c^2*f^2-g^2)^{(1/2)})*c*f+1/(c*f+(c^2*f^2-g^2)^{(1/2)})*(c^2*f^2-g^2)^{(1/2)})-1/3*b*(-d*(c^2*x^2-1))^{(1/2)}*d/(c^2*x^2-1)/g*\arccos(c*x)*x^4*c^4+5/3*b*(-d*(c^2*x^2-1))^{(1/2)}*d/(c^2*x^2-1)/g*\arccos(c*x)*x^2*c^2+1/3*a/g*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(3/2)}+2*a/g^3*d^2/(-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*\ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)})/(x+f/g)*c^2*f^2+1/2*a/g^2*c^2*d*f*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*x+3/2*a/g^2*c^2*d^2*f/(c^2*d)^{(1/2)}*\arctan((c^2*d)^{(1/2)}*x/(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)})-a/g^4*d^2*c^4*f^3/(c^2*d)^{(1/2)}*\arctan((c^2*d)^{(1/2)}*x/(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)})-a/g^5*d^2/(-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*\ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)})/(x+f/g)*c^4*f^4-a/g^3*d*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*c^2*f^2-4/3*b*(-d*(c^2*x^2-1))^{(1/2)}*d/(c^2*x^2-1)/g*\arccos(c*x)+I*b*(c^2*f^2-g^2)^{(3/2)}*d*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x^2-1)/$



$$g^4 \arccos(cx) \ln\left(\frac{(cx + I(-c^2x^2 + 1)^{1/2}) * g + c * f + (c^2 * f^2 - g^2)^{1/2}}{c * f + (c^2 * f^2 - g^2)^{1/2}}\right) - I * b * (c^2 * f^2 - g^2)^{3/2} * d * (-d * (c^2 * x^2 - 1))^{1/2} * (-c^2 * x^2 + 1)^{1/2} / (c^2 * x^2 - 1) / g^4 \arccos(cx) \ln\left(\frac{(-cx + I(-c^2 * x^2 + 1)^{1/2}) * g - c * f + (c^2 * f^2 - g^2)^{1/2}}{-c * f + (c^2 * f^2 - g^2)^{1/2}}\right) - 1/4 * b * (-d * (c^2 * x^2 - 1))^{1/2} * f * d * c^3 / (c^2 * x^2 - 1) / g^2 * (-c^2 * x^2 + 1)^{1/2} * x^2 + b * (-d * (c^2 * x^2 - 1))^{1/2} * d / (c^2 * x^2 - 1) / g^3 * (-c^2 * x^2 + 1)^{1/2} * x * c^3 * f^2 - 1/2 * b * (-d * (c^2 * x^2 - 1))^{1/2} * f * d * c^2 / (c^2 * x^2 - 1) / g^2 \arccos(cx) * x - b * (-d * (c^2 * x^2 - 1))^{1/2} * d / (c^2 * x^2 - 1) / g^3 \arccos(cx) * x^2 * c^4 * f^2 - 1/2 * b * (-d * (c^2 * x^2 - 1))^{1/2} * (-c^2 * x^2 + 1)^{1/2} / (c^2 * x^2 - 1) * \arccos(cx)^2 * f^3 * d * c^3 / g^4 + 3/4 * b * (-d * (c^2 * x^2 - 1))^{1/2} * (-c^2 * x^2 + 1)^{1/2} / (c^2 * x^2 - 1) * \arccos(cx)^2 * f * d * c / g^2 + 1/2 * b * (-d * (c^2 * x^2 - 1))^{1/2} * f * d * c^4 / (c^2 * x^2 - 1) / g^2 \arccos(cx) * x^3 - a / g * d^2 / (-d * (c^2 * f^2 - g^2) / g^2)^{1/2} * \ln\left(\frac{-2 * d * (c^2 * f^2 - g^2) / g^2 + 2 * c^2 * d * f / g * (x + f / g) + 2 * (-d * (c^2 * f^2 - g^2) / g^2)^{1/2} * (-d * c^2 * (x + f / g)^2 + 2 * c^2 * d * f / g * (x + f / g) - d * (c^2 * f^2 - g^2) / g^2)^{1/2}}{x + f / g}\right)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2\*d\*x^2+d)^(3/2)\*(a+b\*arccos(c\*x))/(g\*x+f),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ac^2dx^2 - ad + (bc^2dx^2 - bd) \arccos(cx))\sqrt{-c^2dx^2 + d}}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2\*d\*x^2+d)^(3/2)\*(a+b\*arccos(c\*x))/(g\*x+f),x, algorithm="fricas")

[Out] integral(-(a\*c^2\*d\*x^2 - a\*d + (b\*c^2\*d\*x^2 - b\*d)\*arccos(c\*x))\*sqrt(-c^2\*d\*x^2 + d)/(g\*x + f), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-d(cx - 1)(cx + 1))^{3/2} (a + b \arccos(cx))}{f + gx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c\*\*2\*d\*x\*\*2+d)\*\*(3/2)\*(a+b\*acos(c\*x))/(g\*x+f),x)

[Out] Integral((-d\*(c\*x - 1)\*(c\*x + 1))\*\*(3/2)\*(a + b\*acos(c\*x))/(f + g\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c^2 dx^2 + d)^{\frac{3}{2}} (b \arccos(cx) + a)}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2\*d\*x^2+d)^(3/2)\*(a+b\*arccos(c\*x))/(g\*x+f),x, algorithm="giac")

[Out] integrate((-c^2\*d\*x^2 + d)^(3/2)\*(b\*arccos(c\*x) + a)/(g\*x + f), x)

### 3.10 $\int (f + gx)^3 (d - c^2 dx^2)^{5/2} (a + b \cos^{-1}(cx)) dx$

**Optimal.** Leaf size=1281

result too large to display

```
[Out] (-3*b*d^2*f^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[1 - c^2*x^2]) - (2*b*d^2*g^3*x*Sqrt[d - c^2*d*x^2])/(63*c^3*Sqrt[1 - c^2*x^2]) + (25*b*c*d^2*f^3*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) - (15*b*d^2*f*g^2*x^2*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[1 - c^2*x^2]) + (3*b*c*d^2*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[1 - c^2*x^2]) - (b*d^2*g^3*x^3*Sqrt[d - c^2*d*x^2])/(189*c*Sqrt[1 - c^2*x^2]) - (5*b*c^3*d^2*f^3*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) + (59*b*c*d^2*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(256*Sqrt[1 - c^2*x^2]) - (9*b*c^3*d^2*f^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[1 - c^2*x^2]) + (b*c*d^2*g^3*x^5*Sqrt[d - c^2*d*x^2])/(21*Sqrt[1 - c^2*x^2]) - (17*b*c^3*d^2*f*g^2*x^6*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) + (3*b*c^5*d^2*f^2*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[1 - c^2*x^2]) - (19*b*c^3*d^2*g^3*x^7*Sqrt[d - c^2*d*x^2])/(441*Sqrt[1 - c^2*x^2]) + (3*b*c^5*d^2*f*g^2*x^8*Sqrt[d - c^2*d*x^2])/(64*Sqrt[1 - c^2*x^2]) + (b*c^5*d^2*g^3*x^9*Sqrt[d - c^2*d*x^2])/(81*Sqrt[1 - c^2*x^2]) - (b*d^2*f^3*(1 - c^2*x^2)^(5/2)*Sqrt[d - c^2*d*x^2])/(36*c) + (5*d^2*f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 - (15*d^2*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(128*c^2) + (15*d^2*f*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/64 + (5*d^2*f^3*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/24 + (5*d^2*f*g^2*x^3*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 + (d^2*f^3*x*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/6 + (3*d^2*f*g^2*x^3*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 - (3*d^2*f^2*g*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^2) - (d^2*g^3*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^4) + (d^2*g^3*(1 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(9*c^4) - (5*d^2*f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(32*b*c*Sqrt[1 - c^2*x^2]) - (15*d^2*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(256*b*c^3*Sqrt[1 - c^2*x^2])
```

**Rubi [A]** time = 1.17746, antiderivative size = 1281, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 18, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$ , Rules used = {4778, 4764, 4650, 4648, 4642, 30, 14, 261, 4678, 194, 4700, 4698, 4708, 266, 43, 4690, 12, 373}

$$\frac{bc^5d^2g^3\sqrt{d-c^2dx^2}x^9}{81\sqrt{1-c^2x^2}} + \frac{3bc^5d^2fg^2\sqrt{d-c^2dx^2}x^8}{64\sqrt{1-c^2x^2}} - \frac{19bc^3d^2g^3\sqrt{d-c^2dx^2}x^7}{441\sqrt{1-c^2x^2}} + \frac{3bc^5d^2f^2g\sqrt{d-c^2dx^2}x^7}{49\sqrt{1-c^2x^2}} - \frac{17bc^3d^2fg^2}{96\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-3*b*d^2*f^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[1 - c^2*x^2]) - (2*b*d^2*g^3*x*Sqrt[d - c^2*d*x^2])/(63*c^3*Sqrt[1 - c^2*x^2]) + (25*b*c*d^2*f^3*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) - (15*b*d^2*f*g^2*x^2*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[1 - c^2*x^2]) + (3*b*c*d^2*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[1 - c^2*x^2]) - (b*d^2*g^3*x^3*Sqrt[d - c^2*d*x^2])/(189*c*Sqrt[1 - c^2*x^2]) - (5*b*c^3*d^2*f^3*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) + (59*b*c*d^2*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(256*Sqrt[1 - c^2*x^2]) - (9*b*c^3*d^2*f^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[1 - c^2*x^2]) + (b*c*d^2*g^3*x^5*Sqrt[d - c^2*d*x^2])/(21*Sqrt[1 - c^2*x^2]) - (17*b*c^3*d^2*f*g^2*x^6*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) + (3*b*c^5*d^2*f^2*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[1 - c^2*x^2]) - (19*b*c^3*d^2*g^3*x^7*Sqrt[d - c^2*d*x^2])/(441*Sqrt[1 - c^2*x^2]) + (3*b*c^5*d^2*f*g^2*x^8*Sqrt[d - c^2*d*x^2])/(64*Sqrt[1 - c^2*x^2]) + (b*c^5*d^2*g^3*x^9*Sqrt[d - c^2*d*x^2])/(81*Sqrt[1 - c^2*x^2]) - (b*d^2*f^3*(1 - c^2*x^2)^(5/2)*Sqrt[d - c^2*d*x^2])/(36*c) + (5*d^2*f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 - (15*d^2*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(128*c^2) + (15*d^2*f*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/64 + (5*d^2*f^3*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/24 + (5*d^2*f*g^2*x^3*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 + (d^2*f^3*x*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/6 + (3*d^2*f*g^2*x^3*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 - (3*d^2*f^2*g*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^2) - (d^2*g^3*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^4) + (d^2*g^3*(1 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(9*c^4) - (5*d^2*f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(32*b*c*Sqrt[1 - c^2*x^2]) - (15*d^2*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(256*b*c^3*Sqrt[1 - c^2*x^2])
```

```

g*x^7*Sqrt[d - c^2*d*x^2]/(49*Sqrt[1 - c^2*x^2]) - (19*b*c^3*d^2*g^3*x^7*S
qrt[d - c^2*d*x^2]/(441*Sqrt[1 - c^2*x^2]) + (3*b*c^5*d^2*f*g^2*x^8*Sqrt[d
- c^2*d*x^2]/(64*Sqrt[1 - c^2*x^2]) + (b*c^5*d^2*g^3*x^9*Sqrt[d - c^2*d*x
^2]/(81*Sqrt[1 - c^2*x^2]) - (b*d^2*f^3*(1 - c^2*x^2)^(5/2)*Sqrt[d - c^2*d
*x^2]/(36*c) + (5*d^2*f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 -
(15*d^2*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(128*c^2) + (15*d^
2*f*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/64 + (5*d^2*f^3*x*(1 -
c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/24 + (5*d^2*f*g^2*x^3*(1
- c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 + (d^2*f^3*x*(1 - c
^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/6 + (3*d^2*f*g^2*x^3*(1
- c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 - (3*d^2*f^2*g*(1 -
c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^2) - (d^2*g^3*(1
- c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^4) + (d^2*g^3*(1
- c^2*x^2)^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(9*c^4) - (5*d^2*f^3
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(32*b*c*Sqrt[1 - c^2*x^2]) - (1
5*d^2*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(256*b*c^3*Sqrt[1 -
c^2*x^2])

```

#### Rule 4778

```

Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] := Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
0] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]

```

#### Rule 4764

```

Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^p*(a +
b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))

```

#### Rule 4650

```

Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] := Simp[(x*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n)/(2*p + 1), x] + (D
ist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 - c^2*x
^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
GtQ[p, 0]

```

#### Rule 4648

```

Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] := Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/((2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0]

```

#### Rule 4642

```

Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] := -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

```

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_) + (b\_)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 261

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 4678

Int[((a\_) + ArcCos[(c\_)\*(x\_)]\*(b\_))^(n\_)\*(x\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCos[c\*x])^n)/(2\*e\*(p + 1)), x] - Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcCos[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

Rule 194

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4700

Int[((a\_) + ArcCos[(c\_)\*(x\_)]\*(b\_))^(n\_)\*((f\_)\*(x\_))^(m\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(d + e\*x^2)^p\*(a + b\*ArcCos[c\*x])^n)/(f\*(m + 2\*p + 1)), x] + (Dist[(2\*d\*p)/(m + 2\*p + 1), Int[(f\*x)^m\*(d + e\*x^2)^(p - 1)\*(a + b\*ArcCos[c\*x])^n, x], x] + Dist[(b\*c\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(f\*(m + 2\*p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(f\*x)^(m + 1)\*(1 - c^2\*x^2)^(p - 1/2)\*(a + b\*ArcCos[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])

Rule 4698

Int[((a\_) + ArcCos[(c\_)\*(x\_)]\*(b\_))^(n\_)\*((f\_)\*(x\_))^(m\_)\*Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := Simp[((f\*x)^(m + 1)\*Sqrt[d + e\*x^2]\*(a + b\*ArcCos[c\*x])^n)/(f\*(m + 2)), x] + (Dist[Sqrt[d + e\*x^2]/((m + 2)\*Sqrt[1 - c^2\*x^2]), Int[(f\*x)^m\*(a + b\*ArcCos[c\*x])^n/Sqrt[1 - c^2\*x^2], x], x] + Dist[(b\*c\*n\*Sqrt[d + e\*x^2])/(f\*(m + 2)\*Sqrt[1 - c^2\*x^2]), Int[(f\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])

Rule 4708

Int((((a\_) + ArcCos[(c\_)\*(x\_)]\*(b\_))^(n\_)\*((f\_)\*(x\_))^(m\_))/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := Simp[(f\*(f\*x)^(m - 1)\*Sqrt[d + e\*x^2]\*(a + b\*ArcCos[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m - 1))/(c^2\*m), Int[(f\*x)^(m - 2)\*(a + b\*ArcCos[c\*x])^n/Sqrt[d + e\*x^2], x], x] - Dist[(b\*f\*n\*Sqrt[1 - c^2\*x^2])/(c\*m\*Sqrt[d + e\*x^2]), Int[(f\*x)^(m - 1)\*(a + b\*ArcCos[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0]

&& GtQ[m, 1] && IntegerQ[m]

### Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[  
Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b,  
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 43

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int  
[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n},  
x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le  
Q[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 4690

Int[((a\_) + ArcCos[(c\_)\*(x\_)])\*(b\_)\*(x\_)^(m\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_)  
, x\_Symbol] := With[{u = IntHide[x^m\*(1 - c^2\*x^2)^p, x]}, Dist[d^p\*(a + b\*  
ArcCos[c\*x]), u, x] + Dist[b\*c\*d^p, Int[SimplifyIntegrand[u/Sqrt[1 - c^2\*x^  
2], x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && Intege  
rQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2\*p + 3)/2, 0]) && NeQ[p, -  
2^(-1)] && GtQ[d, 0]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match  
Q[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 373

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol  
] := Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b,  
c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

### Rubi steps

$$\begin{aligned}
\int (f + gx)^3 (d - c^2 dx^2)^{5/2} (a + b \cos^{-1}(cx)) dx &= \frac{(d^2 \sqrt{d - c^2 dx^2}) \int (f + gx)^3 (1 - c^2 x^2)^{5/2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{(d^2 \sqrt{d - c^2 dx^2}) \int (f^3 (1 - c^2 x^2)^{5/2} (a + b \cos^{-1}(cx)) + 3f^2 gx (1 - c^2 x^2)^{5/2} (a + b \cos^{-1}(cx)) + 3fg^2 x^2 (1 - c^2 x^2)^{5/2} (a + b \cos^{-1}(cx)) + 3g^3 x^3 (1 - c^2 x^2)^{5/2} (a + b \cos^{-1}(cx))) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{(d^2 f^3 \sqrt{d - c^2 dx^2}) \int (1 - c^2 x^2)^{5/2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} + \frac{(3d^2 f^2 g \sqrt{d - c^2 dx^2}) \int (1 - c^2 x^2)^{5/2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{1}{6} d^2 f^3 x (1 - c^2 x^2)^2 \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) + \frac{3}{8} d^2 f g^2 x^3 (1 - c^2 x^2)^2 \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) \\
&= -\frac{bd^2 f^3 (1 - c^2 x^2)^{5/2} \sqrt{d - c^2 dx^2}}{36c} + \frac{5}{24} d^2 f^3 x (1 - c^2 x^2) \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) \\
&= -\frac{3bd^2 f^2 gx \sqrt{d - c^2 dx^2}}{7c \sqrt{1 - c^2 x^2}} + \frac{3bcd^2 f^2 gx^3 \sqrt{d - c^2 dx^2}}{7 \sqrt{1 - c^2 x^2}} - \frac{9bc^3 d^2 f^2 gx^5 \sqrt{d - c^2 dx^2}}{35 \sqrt{1 - c^2 x^2}} \\
&= -\frac{3bd^2 f^2 gx \sqrt{d - c^2 dx^2}}{7c \sqrt{1 - c^2 x^2}} - \frac{2bd^2 g^3 x \sqrt{d - c^2 dx^2}}{63c^3 \sqrt{1 - c^2 x^2}} + \frac{25bcd^2 f^3 x^2 \sqrt{d - c^2 dx^2}}{96 \sqrt{1 - c^2 x^2}} \\
&= -\frac{3bd^2 f^2 gx \sqrt{d - c^2 dx^2}}{7c \sqrt{1 - c^2 x^2}} - \frac{2bd^2 g^3 x \sqrt{d - c^2 dx^2}}{63c^3 \sqrt{1 - c^2 x^2}} + \frac{25bcd^2 f^3 x^2 \sqrt{d - c^2 dx^2}}{96 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

**Mathematica [A]** time = 7.46235, size = 1144, normalized size = 0.89

$$d^2 \left( -3175200bcf(8c^2f^2 + 3g^2) \sqrt{d - c^2 dx^2} \cos^{-1}(cx)^2 + 504b \sqrt{d - c^2 dx^2} \left( 503424f^2gx^2 \sqrt{1 - c^2x^2}c^4 + 75600f^3 \sin(2 \right. \right.$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^3\*(d - c^2\*d\*x^2)^(5/2)\*(a + b\*ArcCos[c\*x]),x]

[Out] (d^2\*(-3175200\*b\*c\*f\*(8\*c^2\*f^2 + 3\*g^2)\*Sqrt[d - c^2\*d\*x^2]\*ArcCos[c\*x]^2 - 6350400\*a\*c\*Sqrt[d]\*f\*(8\*c^2\*f^2 + 3\*g^2)\*Sqrt[1 - c^2\*x^2]\*ArcTan[(c\*x\*Sqrt[d - c^2\*d\*x^2])/(Sqrt[d]\*(-1 + c^2\*x^2))] + Sqrt[d - c^2\*d\*x^2]\*(-38102400\*b\*c^3\*f^2\*g\*x - 3810240\*b\*c\*g^3\*x - 69672960\*a\*c^2\*f^2\*g\*Sqrt[1 - c^2\*x^2] - 5160960\*a\*g^3\*Sqrt[1 - c^2\*x^2] + 111767040\*a\*c^4\*f^3\*x\*Sqrt[1 - c^2\*x^2] - 19051200\*a\*c^2\*f\*g^2\*x\*Sqrt[1 - c^2\*x^2] + 209018880\*a\*c^4\*f^2\*g\*x^2\*Sqrt[1 - c^2\*x^2] - 2580480\*a\*c^2\*g^3\*x^2\*Sqrt[1 - c^2\*x^2] - 88058880\*a\*c^6\*f^3\*x^3\*Sqrt[1 - c^2\*x^2] + 149869440\*a\*c^4\*f\*g^2\*x^3\*Sqrt[1 - c^2\*x^2] - 209018880\*a\*c^6\*f^2\*g\*x^4\*Sqrt[1 - c^2\*x^2] + 38707200\*a\*c^4\*g^3\*x^4\*Sqrt[1 - c^2\*x^2] + 27095040\*a\*c^8\*f^3\*x^5\*Sqrt[1 - c^2\*x^2] - 172730880\*a\*c^6\*f\*g^2\*x^5\*Sqrt[1 - c^2\*x^2] + 69672960\*a\*c^8\*f^2\*g\*x^6\*Sqrt[1 - c^2\*x^2] - 49029120\*a\*c^6\*g^3\*x^6\*Sqrt[1 - c^2\*x^2] + 60963840\*a\*c^8\*f\*g^2\*x^7\*Sqrt[1 - c^2\*x^2] + 18063360\*a\*c^8\*g^3\*x^8\*Sqrt[1 - c^2\*x^2] + 3810240\*b\*c\*f\*(5\*c^2\*f^2 + g^2)\*Cos[2\*ArcCos[c\*x]] + 282240\*b\*g\*(27\*c^2\*f^2 + 2\*g^2)\*Cos[3\*ArcCos[c\*x]] - 1905120\*b\*c^3\*f^3\*Cos[4\*ArcCos[c\*x]] + 952560\*b\*c\*f\*g^2\*Cos[4\*ArcCos[c\*x]] - 1524096\*b\*c^2\*f^2\*g\*Cos[5\*ArcCos[c\*x]] + 141120\*b\*c^3\*f^3\*Cos[6\*ArcCos[c\*x]] - 423360\*b\*c\*f\*g^2\*Cos[6\*ArcCos[c\*x]] + 155520\*b\*c^2\*f^2\*g\*Cos[7\*ArcCos[c\*x]] - 38880\*b\*g^3\*Cos[7\*ArcCos[c\*x]] + 59535\*b\*c\*f\*g^2\*Cos[8\*ArcCos[c\*x]] + 7840\*b\*g^3\*Cos[9\*ArcCos[c\*x]]) + 504\*b\*Sqrt[d - c^2\*d\*x^2]\*ArcCos[c\*x]\*(-261504\*c^2\*f^2\*g\*Sqrt[1 - c^2\*x^2] + 62616\*g^3\*Sqrt[1 - c^2\*x^2] + 503424\*c^4\*f^2\*g\*x^2\*Sqrt[1 - c^2\*x^2] - 120576\*c^2\*g^3\*x^2\*Sqrt[1 -

$$c^2x^2 - 41472g(3c^2f^2 - 2g^2)(1 - c^2x^2)^{3/2}\cos[2\arccos[cx]] - 5760g(3c^2f^2 - 2g^2)(1 - c^2x^2)^{3/2}\cos[4\arccos[cx]] + 75600c^3f^3\sin[2\arccos[cx]] + 15120cf^2g^2\sin[2\arccos[cx]] - 40320c^2f^2g\sin[3\arccos[cx]] + 6720g^3\sin[3\arccos[cx]] - 15120c^3f^3\sin[4\arccos[cx]] + 7560cf^2g^2\sin[4\arccos[cx]] - 24192c^2f^2g\sin[5\arccos[cx]] + 6048g^3\sin[5\arccos[cx]] + 1680c^3f^3\sin[6\arccos[cx]] - 5040cf^2g^2\sin[6\arccos[cx]] + 900g^3\sin[7\arccos[cx]] + 945cf^2g^2\sin[8\arccos[cx]] + 140g^3\sin[9\arccos[cx]])) / (162570240c^4\sqrt{1 - c^2x^2})$$

**Maple [A]** time = 0.959, size = 2236, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((gx+f)^3(-c^2dx^2+d)^{5/2}(a+b\arccos(cx)), x)$

[Out]  $\frac{5}{16}af^3d^2x(-c^2dx^2+d)^{1/2} + \frac{5}{16}af^3d^3/(c^2d)^{1/2}\arctan((c^2d)^{1/2}x/(-c^2dx^2+d)^{1/2}) - \frac{23}{16}b(-d(c^2x^2-1))^{1/2}fg^2d^2c^4/(c^2x^2-1)\arccos(cx)x^7 + \frac{3}{7}b(-d(c^2x^2-1))^{1/2}gd^2c^6/(c^2x^2-1)\arccos(cx)x^8 + \frac{12}{7}b(-d(c^2x^2-1))^{1/2}gd^2c^4/(c^2x^2-1)\arccos(cx)x^6 + \frac{18}{7}b(-d(c^2x^2-1))^{1/2}gd^2c^2/(c^2x^2-1)\arccos(cx)x^4 + \frac{2}{63}ag^3d/c^4(-c^2dx^2+d)^{7/2} + \frac{3}{7}b(-d(c^2x^2-1))^{1/2}gd^2/c^2/(c^2x^2-1)\arccos(cx)f^2 + \frac{1}{6}b(-d(c^2x^2-1))^{1/2}f^3d^2c^6/(c^2x^2-1)\arccos(cx)x^7 - \frac{17}{24}b(-d(c^2x^2-1))^{1/2}f^3d^2c^4/(c^2x^2-1)\arccos(cx)x^5 + \frac{59}{48}b(-d(c^2x^2-1))^{1/2}f^3d^2c^2/(c^2x^2-1)\arccos(cx)x^3 - \frac{1}{36}b(-d(c^2x^2-1))^{1/2}f^3d^2c^5/(c^2x^2-1)(-c^2x^2+1)^{1/2}x^6 + \frac{13}{96}b(-d(c^2x^2-1))^{1/2}f^3d^2c^3/(c^2x^2-1)(-c^2x^2+1)^{1/2}x^4 - \frac{11}{32}b(-d(c^2x^2-1))^{1/2}f^3d^2c/(c^2x^2-1)(-c^2x^2+1)^{1/2}x^2 - \frac{1}{81}b(-d(c^2x^2-1))^{1/2}g^3d^2c^5/(c^2x^2-1)(-c^2x^2+1)^{1/2}x^9 + \frac{19}{441}b(-d(c^2x^2-1))^{1/2}g^3d^2c^3/(c^2x^2-1)(-c^2x^2+1)^{1/2}x^7 - \frac{1}{21}b(-d(c^2x^2-1))^{1/2}g^3d^2c/(c^2x^2-1)(-c^2x^2+1)^{1/2}x^5 + \frac{1}{189}b(-d(c^2x^2-1))^{1/2}g^3d^2/c/(c^2x^2-1)(-c^2x^2+1)^{1/2}x^3 + \frac{2}{63}b(-d(c^2x^2-1))^{1/2}g^3d^2/c^3/(c^2x^2-1)(-c^2x^2+1)^{1/2}x + \frac{359}{24576}b(-d(c^2x^2-1))^{1/2}fg^2d^2/c^3/(c^2x^2-1)(-c^2x^2+1)^{1/2} - \frac{133}{128}b(-d(c^2x^2-1))^{1/2}fg^2d^2/(c^2x^2-1)\arccos(cx)x^3 - \frac{12}{7}b(-d(c^2x^2-1))^{1/2}gd^2/(c^2x^2-1)\arccos(cx)x^2 + \frac{5}{32}b(-d(c^2x^2-1))^{1/2}(-c^2x^2+1)^{1/2}/(c^2x^2-1)\arccos(cx)^2 + \frac{1}{9}b(-d(c^2x^2-1))^{1/2}g^3d^2c^6/(c^2x^2-1)\arccos(cx)x^{10} - \frac{26}{63}b(-d(c^2x^2-1))^{1/2}g^3d^2c^4/(c^2x^2-1)\arccos(cx)x^8 + \frac{34}{63}b(-d(c^2x^2-1))^{1/2}g^3d^2c^2/(c^2x^2-1)\arccos(cx)x^6 - \frac{1}{63}b(-d(c^2x^2-1))^{1/2}g^3d^2/c^2/(c^2x^2-1)\arccos(cx)x^2 + \frac{3}{7}b(-d(c^2x^2-1))^{1/2}gd^2/c/(c^2x^2-1)(-c^2x^2+1)^{1/2}x^2 + \frac{3}{64}b(-d(c^2x^2-1))^{1/2}fg^2d^2c^5/(c^2x^2-1)(-c^2x^2+1)^{1/2}x^8 + \frac{17}{96}b(-d(c^2x^2-1))^{1/2}fg^2d^2c^3/(c^2x^2-1)(-c^2x^2+1)^{1/2}x^6 - \frac{3}{49}b(-d(c^2x^2-1))^{1/2}gd^2c^5/(c^2x^2-1)(-c^2x^2+1)^{1/2}x^7 + \frac{3}{7}b(-d(c^2x^2-1))^{1/2}gd^2c/(c^2x^2-1)(-c^2x^2+1)^{1/2}x^3 + \frac{3}{7}b(-d(c^2x^2-1))^{1/2}fg^2d^2c/(c^2x^2-1)(-c^2x^2+1)^{1/2}x^4 + \frac{15}{256}b(-d(c^2x^2-1))^{1/2}fg^2d^2/c/(c^2x^2-1)(-c^2x^2+1)^{1/2}x^2 + \frac{15}{256}b(-d(c^2x^2-1))^{1/2}(-c^2x^2+1)^{1/2}/c^3/(c^2x^2-1)\arccos(cx)^2 + \frac{2}{3}b(-d(c^2x^2-1))^{1/2}fg^2d^2c^6/(c^2x^2-1)\arccos(cx)x^9 + \frac{7}{64}b(-d(c^2x^2-1))^{1/2}fg^2d^2c^2/(c^2x^2-1)\arccos(cx)x^5 + \frac{15}{128}b(-d(c^2x^2-1))^{1/2}fg^2d^2/c^2/(c^2x^2-1)\arccos(cx)x^9 + \frac{35}{b}(-d(c^2x^2-1))^{1/2}gd^2c^3/(c^2x^2-1)(-c^2x^2+1)^{1/2}x^5 + \frac{5}{24}af^3d^2x(-c^2dx^2+d)^{3/2} - \frac{1}{9}ag^3x^2(-c^2dx^2+d)^{7/2}/c^2/d +$



$$\begin{aligned} & 1/16*a*f*g^2/c^2*x*(-c^2*d*x^2+d)^{(5/2)}-3/7*a*f^2*g*(-c^2*d*x^2+d)^{(7/2)}/c^2/d+1/6*a*f^3*x*(-c^2*d*x^2+d)^{(5/2)}-3/8*a*f*g^2*x*(-c^2*d*x^2+d)^{(7/2)}/c^2/d+5/64*a*f*g^2/c^2*d*x*(-c^2*d*x^2+d)^{(3/2)}+15/128*a*f*g^2/c^2*d^2*x*(-c^2*d*x^2+d)^{(1/2)}+15/128*a*f*g^2/c^2*d^3/(c^2*d)^{(1/2)}*\arctan((c^2*d)^{(1/2)}*x/(-c^2*d*x^2+d)^{(1/2)})+299/2304*b*(-d*(c^2*x^2-1))^{(1/2)}*f^3*d^2/c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}+2/63*b*(-d*(c^2*x^2-1))^{(1/2)}*g^3*d^2/c^4/(c^2*x^2-1)*\arccos(c*x)-16/63*b*(-d*(c^2*x^2-1))^{(1/2)}*g^3*d^2/(c^2*x^2-1)*\arccos(c*x)*x^4-11/16*b*(-d*(c^2*x^2-1))^{(1/2)}*f^3*d^2/(c^2*x^2-1)*\arccos(c*x)*x \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(-c^2\*d\*x^2+d)^{(5/2)}\*(a+b\*arccos(c\*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ac^4d^2g^3x^7 + 3ac^4d^2fg^2x^6 + 3ad^2f^2gx + ad^2f^3 + (3ac^4d^2f^2g - 2ac^2d^2g^3)x^5 + (ac^4d^2f^3 - 6ac^2d^2fg^2)x^4\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(-c^2\*d\*x^2+d)^{(5/2)}\*(a+b\*arccos(c\*x)),x, algorithm="fricas")

[Out] integral((a\*c^4\*d^2\*g^3\*x^7 + 3\*a\*c^4\*d^2\*f\*g^2\*x^6 + 3\*a\*d^2\*f^2\*g\*x + a\*d^2\*f^3 + (3\*a\*c^4\*d^2\*f^2\*g - 2\*a\*c^2\*d^2\*g^3)\*x^5 + (a\*c^4\*d^2\*f^3 - 6\*a\*c^2\*d^2\*f\*g^2)\*x^4 - (6\*a\*c^2\*d^2\*f^2\*g - a\*d^2\*g^3)\*x^3 - (2\*a\*c^2\*d^2\*f^3 - 3\*a\*d^2\*f\*g^2)\*x^2 + (b\*c^4\*d^2\*g^3\*x^7 + 3\*b\*c^4\*d^2\*f\*g^2\*x^6 + 3\*b\*d^2\*f^2\*g\*x + b\*d^2\*f^3 + (3\*b\*c^4\*d^2\*f^2\*g - 2\*b\*c^2\*d^2\*g^3)\*x^5 + (b\*c^4\*d^2\*f^3 - 6\*b\*c^2\*d^2\*f\*g^2)\*x^4 - (6\*b\*c^2\*d^2\*f^2\*g - b\*d^2\*g^3)\*x^3 - (2\*b\*c^2\*d^2\*f^3 - 3\*b\*d^2\*f\*g^2)\*x^2)\*arccos(c\*x))\*sqrt(-c^2\*d\*x^2 + d), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*3\*(-c\*\*2\*d\*x\*\*2+d)\*\*(5/2)\*(a+b\*acos(c\*x)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (-c^2dx^2 + d)^{\frac{5}{2}}(gx + f)^3(b \arccos(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="gi  
ac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(g*x + f)^3*(b*arccos(c*x) + a), x)
```

### 3.11 $\int (f + gx)^2 (d - c^2 dx^2)^{5/2} (a + b \cos^{-1}(cx)) dx$

**Optimal.** Leaf size=940

$$\frac{bc^5 d^2 g^2 \sqrt{d - c^2 dx^2} x^8}{64 \sqrt{1 - c^2 x^2}} + \frac{2bc^5 d^2 fg \sqrt{d - c^2 dx^2} x^7}{49 \sqrt{1 - c^2 x^2}} - \frac{17bc^3 d^2 g^2 \sqrt{d - c^2 dx^2} x^6}{288 \sqrt{1 - c^2 x^2}} - \frac{6bc^3 d^2 fg \sqrt{d - c^2 dx^2} x^5}{35 \sqrt{1 - c^2 x^2}} - \frac{5bc^3 d^2 f^2 \sqrt{d - c^2 dx^2} x^4}{96 \sqrt{1 - c^2 x^2}}$$

[Out]  $(-2*b*d^2*f*g*x*\text{Sqrt}[d - c^2*d*x^2])/(7*c*\text{Sqrt}[1 - c^2*x^2]) + (25*b*c*d^2*f^2*x^2*\text{Sqrt}[d - c^2*d*x^2])/(96*\text{Sqrt}[1 - c^2*x^2]) - (5*b*d^2*g^2*x^2*\text{Sqrt}[d - c^2*d*x^2])/(256*c*\text{Sqrt}[1 - c^2*x^2]) + (2*b*c*d^2*f*g*x^3*\text{Sqrt}[d - c^2*d*x^2])/(7*\text{Sqrt}[1 - c^2*x^2]) - (5*b*c^3*d^2*f^2*x^4*\text{Sqrt}[d - c^2*d*x^2])/(96*\text{Sqrt}[1 - c^2*x^2]) + (59*b*c*d^2*g^2*x^4*\text{Sqrt}[d - c^2*d*x^2])/(768*\text{Sqrt}[1 - c^2*x^2]) - (6*b*c^3*d^2*f*g*x^5*\text{Sqrt}[d - c^2*d*x^2])/(35*\text{Sqrt}[1 - c^2*x^2]) - (17*b*c^3*d^2*g^2*x^6*\text{Sqrt}[d - c^2*d*x^2])/(288*\text{Sqrt}[1 - c^2*x^2]) + (2*b*c^5*d^2*f*g*x^7*\text{Sqrt}[d - c^2*d*x^2])/(49*\text{Sqrt}[1 - c^2*x^2]) + (b*c^5*d^2*g^2*x^8*\text{Sqrt}[d - c^2*d*x^2])/(64*\text{Sqrt}[1 - c^2*x^2]) - (b*d^2*f^2*(1 - c^2*x^2)^(5/2)*\text{Sqrt}[d - c^2*d*x^2])/(36*c) + (5*d^2*f^2*x*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/16 - (5*d^2*g^2*x*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/(128*c^2) + (5*d^2*g^2*x^3*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/64 + (5*d^2*f^2*x*(1 - c^2*x^2)*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/24 + (5*d^2*g^2*x^3*(1 - c^2*x^2)*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/48 + (d^2*f^2*x*(1 - c^2*x^2)^2*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/6 + (d^2*g^2*x^3*(1 - c^2*x^2)^2*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/8 - (2*d^2*f*g*(1 - c^2*x^2)^3*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/(7*c^2) - (5*d^2*f^2*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x])^2)/(32*b*c*\text{Sqrt}[1 - c^2*x^2]) - (5*d^2*g^2*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x])^2)/(256*b*c^3*\text{Sqrt}[1 - c^2*x^2])$

**Rubi [A]** time = 0.95887, antiderivative size = 940, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$ , Rules used = {4778, 4764, 4650, 4648, 4642, 30, 14, 261, 4678, 194, 4700, 4698, 4708, 266, 43}

$$\frac{bc^5 d^2 g^2 \sqrt{d - c^2 dx^2} x^8}{64 \sqrt{1 - c^2 x^2}} + \frac{2bc^5 d^2 fg \sqrt{d - c^2 dx^2} x^7}{49 \sqrt{1 - c^2 x^2}} - \frac{17bc^3 d^2 g^2 \sqrt{d - c^2 dx^2} x^6}{288 \sqrt{1 - c^2 x^2}} - \frac{6bc^3 d^2 fg \sqrt{d - c^2 dx^2} x^5}{35 \sqrt{1 - c^2 x^2}} - \frac{5bc^3 d^2 f^2 \sqrt{d - c^2 dx^2} x^4}{96 \sqrt{1 - c^2 x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)^2*(d - c^2*d*x^2)^(5/2)*(a + b*\text{ArcCos}[c*x]), x]$

[Out]  $(-2*b*d^2*f*g*x*\text{Sqrt}[d - c^2*d*x^2])/(7*c*\text{Sqrt}[1 - c^2*x^2]) + (25*b*c*d^2*f^2*x^2*\text{Sqrt}[d - c^2*d*x^2])/(96*\text{Sqrt}[1 - c^2*x^2]) - (5*b*d^2*g^2*x^2*\text{Sqrt}[d - c^2*d*x^2])/(256*c*\text{Sqrt}[1 - c^2*x^2]) + (2*b*c*d^2*f*g*x^3*\text{Sqrt}[d - c^2*d*x^2])/(7*\text{Sqrt}[1 - c^2*x^2]) - (5*b*c^3*d^2*f^2*x^4*\text{Sqrt}[d - c^2*d*x^2])/(96*\text{Sqrt}[1 - c^2*x^2]) + (59*b*c*d^2*g^2*x^4*\text{Sqrt}[d - c^2*d*x^2])/(768*\text{Sqrt}[1 - c^2*x^2]) - (6*b*c^3*d^2*f*g*x^5*\text{Sqrt}[d - c^2*d*x^2])/(35*\text{Sqrt}[1 - c^2*x^2]) - (17*b*c^3*d^2*g^2*x^6*\text{Sqrt}[d - c^2*d*x^2])/(288*\text{Sqrt}[1 - c^2*x^2]) + (2*b*c^5*d^2*f*g*x^7*\text{Sqrt}[d - c^2*d*x^2])/(49*\text{Sqrt}[1 - c^2*x^2]) + (b*c^5*d^2*g^2*x^8*\text{Sqrt}[d - c^2*d*x^2])/(64*\text{Sqrt}[1 - c^2*x^2]) - (b*d^2*f^2*(1 - c^2*x^2)^(5/2)*\text{Sqrt}[d - c^2*d*x^2])/(36*c) + (5*d^2*f^2*x*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/16 - (5*d^2*g^2*x*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/(128*c^2) + (5*d^2*g^2*x^3*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/64 + (5*d^2*f^2*x*(1 - c^2*x^2)*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/24 + (5*d^2*g^2*x^3*(1 - c^2*x^2)*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/48 + (d^2*f^2*x*(1 - c^2*x^2)^2*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcCos}[c*x]))/6$

$$+ (d^2 g^2 x^3 (1 - c^2 x^2)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCos}[c x])) / 8 - (2 d^2 f g (1 - c^2 x^2)^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCos}[c x])) / (7 c^2) - (5 d^2 f^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCos}[c x])^2) / (32 b c \sqrt{1 - c^2 x^2}) - (5 d^2 g^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCos}[c x])^2) / (256 b c^3 \sqrt{1 - c^2 x^2})$$
Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]
```

Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4650

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n)/(2*p + 1), x] + (Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])]/((2*p + 1)*(1 - c^2*x^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x)) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 261

$\text{Int}[(x_)^{(m_)} * ((a_) + (b_.) * (x_)^{(n_)})^{(p_)}, x\_Symbol] := \text{Simp}[(a + b*x^n)^{(p + 1)} / (b*n*(p + 1)), x] /;$  FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 4678

$\text{Int}[(a_.) + \text{ArcCos}[(c_.) * (x_)] * (b_.)^{(n_)} * (x_.) * ((d_.) + (e_.) * (x_)^2)^{(p_)}], x\_Symbol] := \text{Simp}[(d + e*x^2)^{(p + 1)} * (a + b*\text{ArcCos}[c*x])^n / (2*e*(p + 1)), x] - \text{Dist}[(b*n*d^{\text{IntPart}[p]} * (d + e*x^2)^{\text{FracPart}[p]}] / (2*c*(p + 1)*(1 - c^2*x^2)^{\text{FracPart}[p]}), \text{Int}[(1 - c^2*x^2)^{(p + 1/2)} * (a + b*\text{ArcCos}[c*x])^{(n - 1)}, x], x] /;$  FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

Rule 194

$\text{Int}[(a_.) + (b_.) * (x_)^{(n_)})^{(p_)}, x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x^n)^p, x], x] /;$  FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4700

$\text{Int}[(a_.) + \text{ArcCos}[(c_.) * (x_)] * (b_.)^{(n_)} * ((f_.) * (x_))^{(m_)} * ((d_.) + (e_.) * (x_)^2)^{(p_)}], x\_Symbol] := \text{Simp}[(f*x)^{(m + 1)} * (d + e*x^2)^p * (a + b*\text{ArcCos}[c*x])^n / (f*(m + 2*p + 1)), x] + (\text{Dist}[(2*d*p) / (m + 2*p + 1), \text{Int}[(f*x)^m * (d + e*x^2)^{(p - 1)} * (a + b*\text{ArcCos}[c*x])^n, x], x] + \text{Dist}[(b*c*n*d^{\text{IntPart}[p]} * (d + e*x^2)^{\text{FracPart}[p]}] / (f*(m + 2*p + 1)*(1 - c^2*x^2)^{\text{FracPart}[p]}), \text{Int}[(f*x)^{(m + 1)} * (1 - c^2*x^2)^{(p - 1/2)} * (a + b*\text{ArcCos}[c*x])^{(n - 1)}, x], x]) /;$  FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])

Rule 4698

$\text{Int}[(a_.) + \text{ArcCos}[(c_.) * (x_)] * (b_.)^{(n_)} * ((f_.) * (x_))^{(m_)} * \text{Sqrt}[(d_.) + (e_.) * (x_)^2], x\_Symbol] := \text{Simp}[(f*x)^{(m + 1)} * \text{Sqrt}[d + e*x^2] * (a + b*\text{ArcCos}[c*x])^n / (f*(m + 2)), x] + (\text{Dist}[\text{Sqrt}[d + e*x^2] / ((m + 2)*\text{Sqrt}[1 - c^2*x^2]), \text{Int}[(f*x)^m * (a + b*\text{ArcCos}[c*x])^n / \text{Sqrt}[1 - c^2*x^2], x], x] + \text{Dist}[(b*c*n*\text{Sqrt}[d + e*x^2]) / (f*(m + 2)*\text{Sqrt}[1 - c^2*x^2]), \text{Int}[(f*x)^{(m + 1)} * (a + b*\text{ArcCos}[c*x])^{(n - 1)}, x], x]) /;$  FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])

Rule 4708

$\text{Int}[(a_.) + \text{ArcCos}[(c_.) * (x_)] * (b_.)^{(n_)} * ((f_.) * (x_))^{(m_)}] / \text{Sqrt}[(d_.) + (e_.) * (x_)^2], x\_Symbol] := \text{Simp}[(f*(f*x)^{(m - 1)} * \text{Sqrt}[d + e*x^2] * (a + b*\text{ArcCos}[c*x])^n) / (e*m), x] + (\text{Dist}[(f^2*(m - 1)) / (c^2*m), \text{Int}[(f*x)^{(m - 2)} * (a + b*\text{ArcCos}[c*x])^n / \text{Sqrt}[d + e*x^2], x], x] - \text{Dist}[(b*f*n*\text{Sqrt}[1 - c^2*x^2]) / (c*m*\text{Sqrt}[d + e*x^2]), \text{Int}[(f*x)^{(m - 1)} * (a + b*\text{ArcCos}[c*x])^{(n - 1)}, x], x]) /;$  FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]

Rule 266

$\text{Int}[(x_)^{(m_)} * ((a_) + (b_.) * (x_)^{(n_)})^{(p_)}, x\_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)} * (a + b*x)^p, x], x, x^n], x] /;$  FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 43

$\text{Int}[(a_.) + (b_.) * (x_))^{(m_)} * ((c_.) + (d_.) * (x_))^{(n_)}], x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n},

$x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& ( !\text{IntegerQ}[n] \parallel (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) \parallel \text{LtQ}[9*m + 5*(n + 1), 0] \parallel \text{GtQ}[m + n + 2, 0])$

### Rubi steps

$$\begin{aligned}
 \int (f + gx)^2 (d - c^2 dx^2)^{5/2} (a + b \cos^{-1}(cx)) dx &= \frac{(d^2 \sqrt{d - c^2 dx^2}) \int (f + gx)^2 (1 - c^2 x^2)^{5/2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} \\
 &= \frac{(d^2 \sqrt{d - c^2 dx^2}) \int (f^2 (1 - c^2 x^2)^{5/2} (a + b \cos^{-1}(cx)) + 2fgx (1 - c^2 x^2)) dx}{\sqrt{1 - c^2 x^2}} \\
 &= \frac{(d^2 f^2 \sqrt{d - c^2 dx^2}) \int (1 - c^2 x^2)^{5/2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} + \frac{(2d^2 fg \sqrt{d - c^2 dx^2}) \int (1 - c^2 x^2)^{5/2} dx}{\sqrt{1 - c^2 x^2}} \\
 &= \frac{1}{6} d^2 f^2 x (1 - c^2 x^2)^2 \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) + \frac{1}{8} d^2 g^2 x^3 (1 - c^2 x^2)^2 \sqrt{d - c^2 dx^2} \\
 &= -\frac{bd^2 f^2 (1 - c^2 x^2)^{5/2} \sqrt{d - c^2 dx^2}}{36c} + \frac{5}{24} d^2 f^2 x (1 - c^2 x^2) \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) \\
 &= -\frac{2bd^2 fgx \sqrt{d - c^2 dx^2}}{7c \sqrt{1 - c^2 x^2}} + \frac{2bcd^2 fgx^3 \sqrt{d - c^2 dx^2}}{7 \sqrt{1 - c^2 x^2}} - \frac{6bc^3 d^2 fgx^5 \sqrt{d - c^2 dx^2}}{35 \sqrt{1 - c^2 x^2}} \\
 &= -\frac{2bd^2 fgx \sqrt{d - c^2 dx^2}}{7c \sqrt{1 - c^2 x^2}} + \frac{25bcd^2 f^2 x^2 \sqrt{d - c^2 dx^2}}{96 \sqrt{1 - c^2 x^2}} + \frac{2bcd^2 fgx^3 \sqrt{d - c^2 dx^2}}{7 \sqrt{1 - c^2 x^2}} \\
 &= -\frac{2bd^2 fgx \sqrt{d - c^2 dx^2}}{7c \sqrt{1 - c^2 x^2}} + \frac{25bcd^2 f^2 x^2 \sqrt{d - c^2 dx^2}}{96 \sqrt{1 - c^2 x^2}} - \frac{5bd^2 g^2 x^2 \sqrt{d - c^2 dx^2}}{256c \sqrt{1 - c^2 x^2}}
 \end{aligned}$$

**Mathematica [A]** time = 4.516, size = 794, normalized size = 0.84

$$d^2 \left( \sqrt{d - c^2 dx^2} \left( 3010560ac^7 f^2 x^5 \sqrt{1 - c^2 x^2} - 9784320ac^5 f^2 x^3 \sqrt{1 - c^2 x^2} + 12418560ac^3 f^2 x \sqrt{1 - c^2 x^2} + 5160960ac^7 fgx^6 \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2\*(d - c^2\*d\*x^2)^(5/2)\*(a + b\*ArcCos[c\*x]), x]

[Out] (d^2\*(-352800\*b\*(8\*c^2\*f^2 + g^2)\*Sqrt[d - c^2\*d\*x^2]\*ArcCos[c\*x]^2 - 705600\*a\*Sqrt[d]\*(8\*c^2\*f^2 + g^2)\*Sqrt[1 - c^2\*x^2]\*ArcTan[(c\*x\*Sqrt[d - c^2\*d\*x^2])/(Sqrt[d]\*(-1 + c^2\*x^2))] + Sqrt[d - c^2\*d\*x^2]\*(-2822400\*b\*c^2\*f\*g\*x - 5160960\*a\*c\*f\*g\*Sqrt[1 - c^2\*x^2] + 12418560\*a\*c^3\*f^2\*x\*Sqrt[1 - c^2\*x^2] - 705600\*a\*c\*g^2\*x\*Sqrt[1 - c^2\*x^2] + 15482880\*a\*c^3\*f\*g\*x^2\*Sqrt[1 - c^2\*x^2] - 9784320\*a\*c^5\*f^2\*x^3\*Sqrt[1 - c^2\*x^2] + 5550720\*a\*c^3\*g^2\*x^3\*Sqrt[1 - c^2\*x^2] - 15482880\*a\*c^5\*f\*g\*x^4\*Sqrt[1 - c^2\*x^2] + 3010560\*a\*c^7\*f^2\*x^5\*Sqrt[1 - c^2\*x^2] - 6397440\*a\*c^5\*g^2\*x^5\*Sqrt[1 - c^2\*x^2] + 5160960\*a\*c^7\*f\*g\*x^6\*Sqrt[1 - c^2\*x^2] + 2257920\*a\*c^7\*g^2\*x^7\*Sqrt[1 - c^2\*x^2] + 141120\*b\*(15\*c^2\*f^2 + g^2)\*Cos[2\*ArcCos[c\*x]] + 564480\*b\*c\*f\*g\*Cos[3\*ArcCos[c\*x]] - 211680\*b\*c^2\*f^2\*Cos[4\*ArcCos[c\*x]] + 35280\*b\*g^2\*Cos[4\*ArcCos[c\*x]] - 112896\*b\*c\*f\*g\*Cos[5\*ArcCos[c\*x]] + 15680\*b\*c^2\*f^2\*Cos[6\*ArcCos[c\*x]] - 15680\*b\*g^2\*Cos[6\*ArcCos[c\*x]] + 11520\*b\*c\*f\*g\*Cos[7\*ArcCos[c\*x]] + 2205\*b\*g^2\*Cos[8\*ArcCos[c\*x]]) + 168\*b\*Sqrt[d - c^2\*d\*x^2]\*ArcCos[c\*x]\*(-58112\*c\*f\*g\*Sqrt[1 - c^2\*x^2] + 111872\*c^3\*f\*g\*x^2\*Sqrt[1 - c^2\*x^2] - 27648\*c\*f\*g\*(1 - c^2\*x^2)^(3/2)\*Cos[2\*ArcCos[c\*x]] - 3840\*c\*f\*g\*(1 - c^2\*x^2)^(

$$\frac{3/2 * \text{Cos}[4 * \text{ArcCos}[c * x]] + 25200 * c^2 * f^2 * \text{Sin}[2 * \text{ArcCos}[c * x]] + 1680 * g^2 * \text{Sin}[2 * \text{ArcCos}[c * x]] - 8960 * c * f * g * \text{Sin}[3 * \text{ArcCos}[c * x]] - 5040 * c^2 * f^2 * \text{Sin}[4 * \text{ArcCos}[c * x]] + 840 * g^2 * \text{Sin}[4 * \text{ArcCos}[c * x]] - 5376 * c * f * g * \text{Sin}[5 * \text{ArcCos}[c * x]] + 560 * c^2 * f^2 * \text{Sin}[6 * \text{ArcCos}[c * x]] - 560 * g^2 * \text{Sin}[6 * \text{ArcCos}[c * x]] + 105 * g^2 * \text{Sin}[8 * \text{ArcCos}[c * x]]}{(18063360 * c^3 * \text{Sqrt}[1 - c^2 * x^2])}$$

**Maple [A]** time = 0.702, size = 1633, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((g*x+f)^2*(-c^2*d*x^2+d)^{(5/2)}*(a+b*\arccos(c*x)),x)$

[Out] 
$$\begin{aligned} & -2/7*a*f*g*(-c^2*d*x^2+d)^{(7/2)}/c^2/d-1/8*a*g^2*x*(-c^2*d*x^2+d)^{(7/2)}/c^2/d+5/192*a*g^2/c^2*d*x*(-c^2*d*x^2+d)^{(3/2)}+5/128*a*g^2/c^2*d^2*x*(-c^2*d*x^2+d)^{(1/2)}+5/128*a*g^2/c^2*d^3/(c^2*d)^{(1/2)}*\arctan((c^2*d)^{(1/2)}*x/(-c^2*d*x^2+d)^{(1/2)})+13/96*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2*c^3/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^4*f^2-59/768*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2*c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^4*g^2-11/32*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2*c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^2*f^2+5/256*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^2*g^2-1/64*b*(-d*(c^2*x^2-1))^{(1/2)}*g^2*d^2*c^5/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^8+17/288*b*(-d*(c^2*x^2-1))^{(1/2)}*g^2*d^2*c^3/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^6-1/36*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2*c^5/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^6*f^2+1/48*a*g^2/c^2*x*(-c^2*d*x^2+d)^{(5/2)}-133/384*b*(-d*(c^2*x^2-1))^{(1/2)}*g^2*d^2/(c^2*x^2-1)*\arccos(c*x)*x^3-11/16*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(c^2*x^2-1)*\arccos(c*x)*x*f^2+1/6*a*f^2*x*(-c^2*d*x^2+d)^{(5/2)}-2/49*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g*d^2*c^5/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^7+6/35*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g*d^2*c^3/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^5-2/7*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g*d^2*c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^3+2/7*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g*d^2/c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x+2/7*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g*d^2*c^6/(c^2*x^2-1)*\arccos(c*x)*x^8-8/7*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g*d^2*c^4/(c^2*x^2-1)*\arccos(c*x)*x^6+12/7*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g*d^2*c^2/(c^2*x^2-1)*\arccos(c*x)*x^4+5/128*b*(-d*(c^2*x^2-1))^{(1/2)}*g^2*d^2/c^2/(c^2*x^2-1)*\arccos(c*x)*x+2/7*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g*d^2/c^2/(c^2*x^2-1)*\arccos(c*x)+1/6*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2*c^6/(c^2*x^2-1)*\arccos(c*x)*x^7*f^2-17/24*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2*c^4/(c^2*x^2-1)*\arccos(c*x)*x^5*f^2+59/48*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2*c^2/(c^2*x^2-1)*\arccos(c*x)*x^3*f^2-8/7*b*(-d*(c^2*x^2-1))^{(1/2)}*f*g*d^2/(c^2*x^2-1)*\arccos(c*x)*x^2+5/32*b*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/c/(c^2*x^2-1)*\arccos(c*x)^2*d^2*f^2+5/256*b*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/c^3/(c^2*x^2-1)*\arccos(c*x)^2*d^2*g^2+1/8*b*(-d*(c^2*x^2-1))^{(1/2)}*g^2*d^2*c^6/(c^2*x^2-1)*\arccos(c*x)*x^9-23/48*b*(-d*(c^2*x^2-1))^{(1/2)}*g^2*d^2*c^4/(c^2*x^2-1)*\arccos(c*x)*x^7+127/192*b*(-d*(c^2*x^2-1))^{(1/2)}*g^2*d^2*c^2/(c^2*x^2-1)*\arccos(c*x)*x^5+5/16*a*f^2*d^2*x*(-c^2*d*x^2+d)^{(1/2)}+5/16*a*f^2*d^3/(c^2*d)^{(1/2)}*\arctan((c^2*d)^{(1/2)}*x/(-c^2*d*x^2+d)^{(1/2)})+5/24*a*f^2*d*x*(-c^2*d*x^2+d)^{(3/2)}+299/2304*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*f^2+359/73728*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/c^3/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*g^2 \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

```
integral((ac^4*d^2*g^2*x^6 + 2*ac^4*d^2*f*g*x^5 - 4*ac^2*d^2*f*g*x^3 + 2*ad^2*f*g*x + ad^2*f^2 + (ac^4*d^2*f^2 - 2*ac^2*d^2*g^2)*x^4 - (2*ac^2*d^2*f^2 - ad^2*g^2)
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*g^2*x^6 + 2*a*c^4*d^2*f*g*x^5 - 4*a*c^2*d^2*f*g*x^3 + 2*a*d^2*f*g*x + a*d^2*f^2 + (a*c^4*d^2*f^2 - 2*a*c^2*d^2*g^2)*x^4 - (2*a*c^2*d^2*f^2 - a*d^2*g^2)*x^2 + (b*c^4*d^2*g^2*x^6 + 2*b*c^4*d^2*f*g*x^5 - 4*b*c^2*d^2*f*g*x^3 + 2*b*d^2*f*g*x + b*d^2*f^2 + (b*c^4*d^2*f^2 - 2*b*c^2*d^2*g^2)*x^4 - (2*b*c^2*d^2*f^2 - b*d^2*g^2)*x^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (-c^2 dx^2 + d)^{\frac{5}{2}} (gx + f)^2 (b \arccos(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(g*x + f)^2*(b*arccos(c*x) + a), x)
```



### 3.12 $\int (f + gx) (d - c^2 dx^2)^{5/2} (a + b \cos^{-1}(cx)) dx$

**Optimal.** Leaf size=517

$$\frac{1}{6}d^2fx(1-c^2x^2)^2\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx)) + \frac{5}{16}d^2fx\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx)) + \frac{5}{24}d^2fx(1-c^2x^2)\sqrt{d-c^2d}$$

```
[Out] -(b*d^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[1 - c^2*x^2]) + (25*b*c*d^2*f*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) + (b*c*d^2*g*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[1 - c^2*x^2]) - (5*b*c^3*d^2*f*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) - (3*b*c^3*d^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[1 - c^2*x^2]) + (b*c^5*d^2*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[1 - c^2*x^2]) - (b*d^2*f*(1 - c^2*x^2)^(5/2)*Sqrt[d - c^2*d*x^2])/(36*c) + (5*d^2*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 + (5*d^2*f*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/24 + (d^2*f*x*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/6 - (d^2*g*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^2) - (5*d^2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(32*b*c*Sqrt[1 - c^2*x^2])
```

---

**Rubi [A]** time = 0.402964, antiderivative size = 517, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$ , Rules used = {4778, 4764, 4650, 4648, 4642, 30, 14, 261, 4678, 194}

$$\frac{1}{6}d^2fx(1-c^2x^2)^2\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx)) + \frac{5}{16}d^2fx\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx)) + \frac{5}{24}d^2fx(1-c^2x^2)\sqrt{d-c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]), x]
```

```
[Out] -(b*d^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[1 - c^2*x^2]) + (25*b*c*d^2*f*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) + (b*c*d^2*g*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[1 - c^2*x^2]) - (5*b*c^3*d^2*f*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) - (3*b*c^3*d^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[1 - c^2*x^2]) + (b*c^5*d^2*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[1 - c^2*x^2]) - (b*d^2*f*(1 - c^2*x^2)^(5/2)*Sqrt[d - c^2*d*x^2])/(36*c) + (5*d^2*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 + (5*d^2*f*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/24 + (d^2*f*x*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/6 - (d^2*g*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^2) - (5*d^2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(32*b*c*Sqrt[1 - c^2*x^2])
```

#### Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_), x_Symbol] := Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]
```

#### Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
```

& EqQ[c^2\*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))

#### Rule 4650

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^ (n\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(x\*(d + e\*x^2)^p\*(a + b\*ArcCos[c\*x])^n)/(2\*p + 1), x] + (Dist[(2\*d\*p)/(2\*p + 1), Int[(d + e\*x^2)^(p - 1)\*(a + b\*ArcCos[c\*x])^n, x], x] + Dist[(b\*c\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/((2\*p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[x\*(1 - c^2\*x^2)^(p - 1/2)\*(a + b\*ArcCos[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]

#### Rule 4648

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^ (n\_.)\*Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[(x\*Sqrt[d + e\*x^2]\*(a + b\*ArcCos[c\*x])^n)/2, x] + (Dist[Sqrt[d + e\*x^2]/(2\*Sqrt[1 - c^2\*x^2]), Int[(a + b\*ArcCos[c\*x])^n/Sqrt[1 - c^2\*x^2], x], x] + Dist[(b\*c\*n\*Sqrt[d + e\*x^2])/((2\*Sqrt[1 - c^2\*x^2]), Int[x\*(a + b\*ArcCos[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0]

#### Rule 4642

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^ (n\_.)/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := -Simp[(a + b\*ArcCos[c\*x])^(n + 1)/(b\*c\*Sqrt[d]\*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 14

Int[(u\_)\*((c\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_) + (b\_.)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

#### Rule 261

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rule 4678

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^ (n\_.)\*(x\_)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCos[c\*x])^n)/(2\*e\*(p + 1)), x] - Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/((2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcCos[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

#### Rule 194

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int (f + gx) (d - c^2 dx^2)^{5/2} (a + b \cos^{-1}(cx)) dx &= \frac{(d^2 \sqrt{d - c^2 dx^2}) \int (f + gx) (1 - c^2 x^2)^{5/2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{(d^2 \sqrt{d - c^2 dx^2}) \int (f (1 - c^2 x^2)^{5/2} (a + b \cos^{-1}(cx)) + gx (1 - c^2 x^2)^{5/2}) dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{(d^2 f \sqrt{d - c^2 dx^2}) \int (1 - c^2 x^2)^{5/2} (a + b \cos^{-1}(cx)) dx}{\sqrt{1 - c^2 x^2}} + \frac{(d^2 g \sqrt{d - c^2 dx^2}) \int (1 - c^2 x^2)^{5/2} dx}{\sqrt{1 - c^2 x^2}} \\
&= \frac{1}{6} d^2 f x (1 - c^2 x^2)^2 \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) - \frac{d^2 g (1 - c^2 x^2)^3 \sqrt{d - c^2 dx^2}}{36c} \\
&= -\frac{bd^2 f (1 - c^2 x^2)^{5/2} \sqrt{d - c^2 dx^2}}{36c} + \frac{5}{24} d^2 f x (1 - c^2 x^2) \sqrt{d - c^2 dx^2} (a + b \cos^{-1}(cx)) \\
&= -\frac{bd^2 g x \sqrt{d - c^2 dx^2}}{7c \sqrt{1 - c^2 x^2}} + \frac{bcd^2 g x^3 \sqrt{d - c^2 dx^2}}{7 \sqrt{1 - c^2 x^2}} - \frac{3bc^3 d^2 g x^5 \sqrt{d - c^2 dx^2}}{35 \sqrt{1 - c^2 x^2}} + \\
&= -\frac{bd^2 g x \sqrt{d - c^2 dx^2}}{7c \sqrt{1 - c^2 x^2}} + \frac{25bcd^2 f x^2 \sqrt{d - c^2 dx^2}}{96 \sqrt{1 - c^2 x^2}} + \frac{bcd^2 g x^3 \sqrt{d - c^2 dx^2}}{7 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

**Mathematica [A]** time = 2.90072, size = 526, normalized size = 1.02

$$d^2 \left( \sqrt{d - c^2 dx^2} \left( 94080ac^6 fx^5 \sqrt{1 - c^2 x^2} - 305760ac^4 fx^3 \sqrt{1 - c^2 x^2} + 388080ac^2 fx \sqrt{1 - c^2 x^2} + 80640ac^6 gx^6 \sqrt{1 - c^2 x^2} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)\*(d - c^2\*d\*x^2)^(5/2)\*(a + b\*ArcCos[c\*x]),x]

[Out] (d^2\*(-88200\*b\*c\*f\*Sqrt[d - c^2\*d\*x^2]\*ArcCos[c\*x]^2 - 176400\*a\*c\*Sqrt[d]\*f\*Sqrt[1 - c^2\*x^2]\*ArcTan[(c\*x\*Sqrt[d - c^2\*d\*x^2])/(Sqrt[d]\*(-1 + c^2\*x^2))] + Sqrt[d - c^2\*d\*x^2]\*(-44100\*b\*c\*g\*x - 80640\*a\*g\*Sqrt[1 - c^2\*x^2] + 388080\*a\*c^2\*f\*x\*Sqrt[1 - c^2\*x^2] + 241920\*a\*c^2\*g\*x^2\*Sqrt[1 - c^2\*x^2] - 305760\*a\*c^4\*f\*x^3\*Sqrt[1 - c^2\*x^2] - 241920\*a\*c^4\*g\*x^4\*Sqrt[1 - c^2\*x^2] + 94080\*a\*c^6\*f\*x^5\*Sqrt[1 - c^2\*x^2] + 80640\*a\*c^6\*g\*x^6\*Sqrt[1 - c^2\*x^2] + 66150\*b\*c\*f\*Cos[2\*ArcCos[c\*x]] + 8820\*b\*g\*Cos[3\*ArcCos[c\*x]] - 6615\*b\*c\*f\*Cos[4\*ArcCos[c\*x]] - 1764\*b\*g\*Cos[5\*ArcCos[c\*x]] + 490\*b\*c\*f\*Cos[6\*ArcCos[c\*x]] + 180\*b\*g\*Cos[7\*ArcCos[c\*x]]) + 84\*b\*Sqrt[d - c^2\*d\*x^2]\*ArcCos[c\*x]\*(-1816\*g\*Sqrt[1 - c^2\*x^2] + 3496\*c^2\*g\*x^2\*Sqrt[1 - c^2\*x^2] - 864\*g\*(1 - c^2\*x^2)^(3/2)\*Cos[2\*ArcCos[c\*x]] - 120\*g\*(1 - c^2\*x^2)^(3/2)\*Cos[4\*ArcCos[c\*x]] + 1575\*c\*f\*Ssin[2\*ArcCos[c\*x]] - 280\*g\*Ssin[3\*ArcCos[c\*x]] - 315\*c\*f\*Ssin[4\*ArcCos[c\*x]] - 168\*g\*Ssin[5\*ArcCos[c\*x]] + 35\*c\*f\*Ssin[6\*ArcCos[c\*x]])))/(564480\*c^2\*Sqrt[1 - c^2\*x^2])

**Maple [B]** time = 0.602, size = 931, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((g*x+f)*(-c^2*d*x^2+d)^{(5/2)}*(a+b*\arccos(c*x)),x)$

[Out] 
$$\begin{aligned} & -1/7*a*g*(-c^2*d*x^2+d)^{(7/2)}/c^2/d+1/6*a*f*x*(-c^2*d*x^2+d)^{(5/2)}+5/24*a*f \\ & *d*x*(-c^2*d*x^2+d)^{(3/2)}+5/16*a*f*d^2*x*(-c^2*d*x^2+d)^{(1/2)}+5/16*a*f*d^3/ \\ & (c^2*d)^{(1/2)}*\arctan((c^2*d)^{(1/2)}*x/(-c^2*d*x^2+d)^{(1/2)})+5/32*b*(-d*(c^2* \\ & x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/c/(c^2*x^2-1)*\arccos(c*x)^2*f*d^2-1/36*b*( \\ & -d*(c^2*x^2-1))^{(1/2)}*f*d^2*c^5/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^6+13/96*b* \\ & (-d*(c^2*x^2-1))^{(1/2)}*f*d^2*c^3/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^4-11/32*b \\ & *(-d*(c^2*x^2-1))^{(1/2)}*f*d^2*c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^2-1/49*b*( \\ & -d*(c^2*x^2-1))^{(1/2)}*g*d^2*c^5/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^7+3/35*b*( \\ & -d*(c^2*x^2-1))^{(1/2)}*g*d^2*c^3/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^5-1/7*b*( \\ & -d*(c^2*x^2-1))^{(1/2)}*g*d^2*c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x^3+1/7*b*(-d*( \\ & c^2*x^2-1))^{(1/2)}*g*d^2/c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*x+1/7*b*(-d*(c^2*x \\ & ^2-1))^{(1/2)}*g*d^2/c^2/(c^2*x^2-1)*\arccos(c*x)+1/7*b*(-d*(c^2*x^2-1))^{(1/2)} \\ & *g*d^2*c^6/(c^2*x^2-1)*\arccos(c*x)*x^8-4/7*b*(-d*(c^2*x^2-1))^{(1/2)}*g*d^2*c \\ & ^4/(c^2*x^2-1)*\arccos(c*x)*x^6+6/7*b*(-d*(c^2*x^2-1))^{(1/2)}*g*d^2*c^2/(c^2* \\ & x^2-1)*\arccos(c*x)*x^4-4/7*b*(-d*(c^2*x^2-1))^{(1/2)}*g*d^2/(c^2*x^2-1)*\arcco \\ & s(c*x)*x^2+1/6*b*(-d*(c^2*x^2-1))^{(1/2)}*f*d^2*c^6/(c^2*x^2-1)*\arccos(c*x)*x \\ & ^7-17/24*b*(-d*(c^2*x^2-1))^{(1/2)}*f*d^2*c^4/(c^2*x^2-1)*\arccos(c*x)*x^5+59/ \\ & 48*b*(-d*(c^2*x^2-1))^{(1/2)}*f*d^2*c^2/(c^2*x^2-1)*\arccos(c*x)*x^3-11/16*b*( \\ & -d*(c^2*x^2-1))^{(1/2)}*f*d^2/(c^2*x^2-1)*\arccos(c*x)*x+299/2304*b*(-d*(c^2*x \\ & ^2-1))^{(1/2)}*f*d^2/c/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)} \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((g*x+f)*(-c^2*d*x^2+d)^{(5/2)}*(a+b*\arccos(c*x)),x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$\text{integral}\left(\left(ac^4d^2gx^5 + ac^4d^2fx^4 - 2ac^2d^2gx^3 - 2ac^2d^2fx^2 + ad^2gx + ad^2f + (bc^4d^2gx^5 + bc^4d^2fx^4 - 2bc^2d^2gx^3 - 2bc^2d^2fx^2 + ad^2gx + ad^2f)*\arccos(c*x)\right)\sqrt{-c^2*d*x^2 + d}, x\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((g*x+f)*(-c^2*d*x^2+d)^{(5/2)}*(a+b*\arccos(c*x)),x, \text{algorithm}="fricas")$

[Out] 
$$\text{integral}\left(\left(a*c^4*d^2*g*x^5 + a*c^4*d^2*f*x^4 - 2*a*c^2*d^2*g*x^3 - 2*a*c^2*d^2*f*x^2 + a*d^2*g*x + a*d^2*f + (b*c^4*d^2*g*x^5 + b*c^4*d^2*f*x^4 - 2*b*c^2*d^2*g*x^3 - 2*b*c^2*d^2*f*x^2 + b*d^2*g*x + b*d^2*f)*\arccos(c*x)\right)\sqrt{-c^2*d*x^2 + d}, x\right)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(-c\*\*2\*d\*x\*\*2+d)\*\*(5/2)\*(a+b\*acos(c\*x)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (-c^2 dx^2 + d)^{\frac{5}{2}} (gx + f)(b \arccos(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(-c^2\*d\*x^2+d)^(5/2)\*(a+b\*arccos(c\*x)),x, algorithm="giac")

[Out] integrate((-c^2\*d\*x^2 + d)^(5/2)\*(g\*x + f)\*(b\*arccos(c\*x) + a), x)

$$3.13 \quad \int \frac{(d-c^2 dx^2)^{5/2} (a+b \cos^{-1}(cx))}{f+gx} dx$$

**Optimal.** Leaf size=1637

result too large to display

```
[Out] (a*d^2*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2])/g^5 - (2*b*c*d^2*x*Sqrt[d - c^2*d*x^2])/(15*g*Sqrt[1 - c^2*x^2]) - (b*c*d^2*(c^2*f^2 - 2*g^2)*x*Sqrt[d - c^2*d*x^2])/(3*g^3*Sqrt[1 - c^2*x^2]) + (b*c*d^2*(c^2*f^2 - g^2)^2*x*Sqrt[d - c^2*d*x^2])/(g^5*Sqrt[1 - c^2*x^2]) + (b*c^3*d^2*f*x^2*Sqrt[d - c^2*d*x^2])/(16*g^2*Sqrt[1 - c^2*x^2]) - (b*c^3*d^2*f*(c^2*f^2 - 2*g^2)*x^2*Sqrt[d - c^2*d*x^2])/(4*g^4*Sqrt[1 - c^2*x^2]) - (b*c^3*d^2*x^3*Sqrt[d - c^2*d*x^2])/(45*g*Sqrt[1 - c^2*x^2]) + (b*c^3*d^2*(c^2*f^2 - 2*g^2)*x^3*Sqrt[d - c^2*d*x^2])/(9*g^3*Sqrt[1 - c^2*x^2]) - (b*c^5*d^2*f*x^4*Sqrt[d - c^2*d*x^2])/(16*g^2*Sqrt[1 - c^2*x^2]) + (b*c^5*d^2*x^5*Sqrt[d - c^2*d*x^2])/(25*g*Sqrt[1 - c^2*x^2]) + (b*d^2*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])/g^5 + (c^2*d^2*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(8*g^2) - (c^2*d^2*f*(c^2*f^2 - 2*g^2)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(2*g^4) - (c^4*d^2*f*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(4*g^2) - (d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*g) - (d^2*(c^2*f^2 - 2*g^2)*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*g^3) + (d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*g) + (c*d^2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*g^2*Sqrt[1 - c^2*x^2]) + (c*d^2*f*(c^2*f^2 - 2*g^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*g^4*Sqrt[1 - c^2*x^2]) - (c*d^2*(c^2*f^2 - g^2)^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*g^5*Sqrt[1 - c^2*x^2]) - (d^2*(c^2*f^2 - g^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*g^6*(f + g*x)*Sqrt[1 - c^2*x^2]) - (d^2*(c^2*f^2 - g^2)^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*g^4*(f + g*x)) - (a*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])])/(g^6*Sqrt[1 - c^2*x^2]) - (I*b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^6*Sqrt[1 - c^2*x^2]) + (I*b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^6*Sqrt[1 - c^2*x^2]) - (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -(E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^6*Sqrt[1 - c^2*x^2]) + (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -(E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^6*Sqrt[1 - c^2*x^2])
```

**Rubi [A]** time = 2.73, antiderivative size = 1637, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 28, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.903$ , Rules used = {4778, 4768, 4648, 4642, 30, 4678, 4698, 4708, 266, 43, 4690, 12, 4766, 683, 4758, 6742, 725, 204, 1654, 4800, 4798, 8, 4774, 3321, 2264, 2190, 2279, 2391}

$$\frac{bd^2x^5\sqrt{d-c^2dx^2}c^5}{25g\sqrt{1-c^2x^2}} - \frac{bd^2fx^4\sqrt{d-c^2dx^2}c^5}{16g^2\sqrt{1-c^2x^2}} - \frac{d^2fx^3\sqrt{d-c^2dx^2}(a+b\cos^{-1}(cx))c^4}{4g^2} + \frac{bd^2(c^2f^2-2g^2)x^3\sqrt{d-c^2dx^2}c^3}{9g^3\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(f + g*x), x]
```

```
[Out] (a*d^2*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2])/g^5 - (2*b*c*d^2*x*Sqrt[d - c^2*d*x^2])/(15*g*Sqrt[1 - c^2*x^2]) - (b*c*d^2*(c^2*f^2 - 2*g^2)*x*Sqrt[d -
```

$$\begin{aligned} & c^2*d*x^2)/(3*g^3*sqrt[1 - c^2*x^2]) + (b*c*d^2*(c^2*f^2 - g^2)^2*x*sqrt[ \\ & d - c^2*d*x^2])/(g^5*sqrt[1 - c^2*x^2]) + (b*c^3*d^2*f*x^2*sqrt[d - c^2*d*x \\ & ^2])/(16*g^2*sqrt[1 - c^2*x^2]) - (b*c^3*d^2*f*(c^2*f^2 - 2*g^2)*x^2*sqrt[d \\ & - c^2*d*x^2])/(4*g^4*sqrt[1 - c^2*x^2]) - (b*c^3*d^2*x^3*sqrt[d - c^2*d*x^ \\ & 2])/(45*g*sqrt[1 - c^2*x^2]) + (b*c^3*d^2*(c^2*f^2 - 2*g^2)*x^3*sqrt[d - c^ \\ & 2*d*x^2])/(9*g^3*sqrt[1 - c^2*x^2]) - (b*c^5*d^2*f*x^4*sqrt[d - c^2*d*x^2]) \\ & /((16*g^2*sqrt[1 - c^2*x^2]) + (b*c^5*d^2*x^5*sqrt[d - c^2*d*x^2]))/(25*g*sqrt \\ & [1 - c^2*x^2]) + (b*d^2*(c^2*f^2 - g^2)^2*sqrt[d - c^2*d*x^2]*ArcCos[c*x]) \\ & /g^5 + (c^2*d^2*f*x*sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(8*g^2) - (c^2 \\ & *d^2*f*(c^2*f^2 - 2*g^2)*x*sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(2*g^4) \\ & - (c^4*d^2*f*x^3*sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(4*g^2) - (d^2*( \\ & 1 - c^2*x^2)*sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*g) - (d^2*(c^2*f^2 \\ & - 2*g^2)*(1 - c^2*x^2)*sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*g^3) + \\ & (d^2*(1 - c^2*x^2)^2*sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*g) + (c*d^ \\ & 2*f*sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*g^2*sqrt[1 - c^2*x^2]) \\ & + (c*d^2*f*(c^2*f^2 - 2*g^2)*sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4 \\ & *b*g^4*sqrt[1 - c^2*x^2]) - (c*d^2*(c^2*f^2 - g^2)^2*x*sqrt[d - c^2*d*x^2]* \\ & (a + b*ArcCos[c*x])^2)/(2*b*g^5*sqrt[1 - c^2*x^2]) - (d^2*(c^2*f^2 - g^2)^3 \\ & *sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*g^6*(f + g*x)*sqrt[1 - c \\ & ^2*x^2]) - (d^2*(c^2*f^2 - g^2)^2*sqrt[1 - c^2*x^2]*sqrt[d - c^2*d*x^2]*(a \\ & + b*ArcCos[c*x])^2)/(2*b*c*g^4*(f + g*x)) - (a*d^2*(c^2*f^2 - g^2)^(5/2)*sqrt \\ & [d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(sqrt[c^2*f^2 - g^2]*sqrt[1 - c^2*x^ \\ & 2])])/(g^6*sqrt[1 - c^2*x^2]) - (I*b*d^2*(c^2*f^2 - g^2)^(5/2)*sqrt[d - c^2 \\ & *d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - sqrt[c^2*f^2 - g^2 \\ & ])])/(g^6*sqrt[1 - c^2*x^2]) + (I*b*d^2*(c^2*f^2 - g^2)^(5/2)*sqrt[d - c^2* \\ & d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + sqrt[c^2*f^2 - g^2 \\ & ])])/(g^6*sqrt[1 - c^2*x^2]) - (b*d^2*(c^2*f^2 - g^2)^(5/2)*sqrt[d - c^2*d*x \\ & ^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - sqrt[c^2*f^2 - g^2]))])/(g^6* \\ & sqrt[1 - c^2*x^2]) + (b*d^2*(c^2*f^2 - g^2)^(5/2)*sqrt[d - c^2*d*x^2]*PolyL \\ & og[2, -((E^(I*ArcCos[c*x])*g)/(c*f + sqrt[c^2*f^2 - g^2]))])/(g^6*sqrt[1 - \\ & c^2*x^2]) \end{aligned}$$
**Rule 4778**

$$\text{Int}[(a + \text{ArcCos}[c(x)](b))^n * ((f) + (g)(x))^m * ((d) + (e)(x)^2)^p, x\_Symbol] \rightarrow \text{Dist}[(d \text{IntPart}[p] * (d + e*x^2)^{\text{FracPart}[p]} / (1 - c^2*x^2)^{\text{FracPart}[p]}, \text{Int}[(f + g*x)^m * (1 - c^2*x^2)^p * (a + b*\text{ArcCos}[c*x])^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[p - 1/2] \&\& !\text{GtQ}[d, 0]$$
**Rule 4768**

$$\text{Int}[(a + \text{ArcCos}[c(x)](b))^n * ((f) + (g)(x))^m * ((d) + (e)(x)^2)^p, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{sqrt}[d + e*x^2] * (a + b*\text{ArcCos}[c*x])^n, (f + g*x)^m * (d + e*x^2)^{p - 1/2}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IntegerQ}[m] \&\& \text{IGtQ}[p + 1/2, 0] \&\& \text{GtQ}[d, 0] \&\& \text{IGtQ}[n, 0]$$
**Rule 4648**

$$\text{Int}[(a + \text{ArcCos}[c(x)](b))^n * \text{sqrt}[(d) + (e)(x)^2], x\_Symbol] \rightarrow \text{Simp}[(x*\text{sqrt}[d + e*x^2] * (a + b*\text{ArcCos}[c*x])^n) / 2, x] + (\text{Dist}[\text{sqrt}[d + e*x^2] / (2*\text{sqrt}[1 - c^2*x^2]), \text{Int}[(a + b*\text{ArcCos}[c*x])^n / \text{sqrt}[1 - c^2*x^2], x], x] + \text{Dist}[(b*c*n*\text{sqrt}[d + e*x^2]) / (2*\text{sqrt}[1 - c^2*x^2]), \text{Int}[x*(a + b*\text{ArcCos}[c*x])^{n - 1}, x], x]) /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0]$$
**Rule 4642**

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)/Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol]
:> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x]
&& EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

### Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

### Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

### Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_)^(m_)*Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol]
:> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^m*(a + b*ArcCos[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

### Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_)^(m_))/Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol]
:> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[(f*x)^(m - 2)*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]
```

### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 4690

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))*(x_)^(m_)*((d_.) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = IntHide[x^m*(1 - c^2*x^2)^p, x]}, Dist[d^p*(a + b*ArcCos[c*x]), u, x] + Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -
```



$2^{(-1)}$  && GtQ[d, 0]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 4766

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)]\*(b\_.))^ (n\_.)\*((f\_.) + (g\_.)\*(x\_))^ (m\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] := -Simp[((f + g\*x)^m\*(d + e\*x^2)\*(a + b\*ArcCos[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] + Dist[1/(b\*c\*Sqrt[d]\*(n + 1)), Int[(d\*g\*m + 2\*e\*f\*x + e\*g\*(m + 2)\*x^2)\*(f + g\*x)^(m - 1)\*(a + b\*ArcCos[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2\*d + e, 0] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]

### Rule 683

Int[((d\_.) + (e\_.)\*(x\_))^ (m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^ (p\_.), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[2\*c\*d - b\*e, 0] && IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])

### Rule 4758

Int((((a\_.) + ArcCos[(c\_.)\*(x\_)]\*(b\_.))^ (n\_.)\*((f\_.) + (g\_.)\*(x\_) + (h\_.)\*(x\_)^2)^ (p\_.))/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := With[{u = IntHide[(f + g\*x + h\*x^2)^p/(d + e\*x)^2, x]}, Dist[(a + b\*ArcCos[c\*x])^n, u, x] + Dist[b\*c^n, Int[SimplifyIntegrand[(u\*(a + b\*ArcCos[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ[p, 0] && EqQ[e\*g - 2\*d\*h, 0]

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^ (-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 1654

Int[(Pq\_)\*((d\_) + (e\_.)\*(x\_))^ (m\_.)\*((a\_) + (c\_.)\*(x\_)^2)^ (p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - 2\*c\*d\*e\*(m + q + p)\*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[c\*d^2 + a\*e^2, 0] && !(EqQ[d, 0] && T

```
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

#### Rule 4800

```
Int[(ArcCos[(c_.)*(x_)]*(b_.) + (a_.))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

#### Rule 4798

```
Int[ArcCos[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcCos[c*x]^n, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

#### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

#### Rule 4774

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] && (GtQ[m, 0] || IGtQ[n, 0])
```

#### Rule 3321

```
Int[(((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

#### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps



**Mathematica [B]** time = 19.6051, size = 6216, normalized size = 3.8

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((d - c^2\*d\*x^2)^(5/2)\*(a + b\*ArcCos[c\*x]))/(f + g\*x), x]

[Out] Result too large to show

**Maple [B]** time = 0.531, size = 4692, normalized size = 2.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-c^2\*d\*x^2+d)^(5/2)\*(a+b\*arccos(c\*x))/(g\*x+f), x)

[Out] 
$$\begin{aligned} & -5/4*b*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x^2-1)*\arccos(c*x)^2* \\ & f^3*d^2*c^3/g^4-8/3*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(c^2*x^2-1)/g^3*\arccos(c*x) \\ & *x^2*c^4*f^2-1/2*b*(-d*(c^2*x^2-1))^{(1/2)}*f^3*d^2*c^6/(c^2*x^2-1)/g^4*\arcc \\ & \cos(c*x)*x^3+1/2*b*(-d*(c^2*x^2-1))^{(1/2)}*f^3*d^2*c^4/(c^2*x^2-1)/g^4*\arcc \\ & \cos(c*x)*x+b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(c^2*x^2-1)/g^5*\arccos(c*x)*x^2*c^6*f^ \\ & 4+1/2*b*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x^2-1)*\arccos(c*x)^2 \\ & *f^5*d^2*c^5/g^6+1/5*a/g*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g \\ & ^2)/g^2)^{(5/2)}+1/3*a/g*d*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g \\ & ^2)/g^2)^{(3/2)}+a/g*d^2*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2 \\ & )/g^2)^{(1/2)}-a/g*d^3/(-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*\ln((-2*d*(c^2*f^2-g^2)/g^ \\ & 2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*(-d*c^2*(x+f/g)^2+2*c^ \\ & 2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)})/(x+f/g))+15/16*b*(-d*(c^2*x^2-1 \\ & ))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x^2-1)*\arccos(c*x)^2*f*d^2*c/g^2-1/4*b*(-d \\ & *(c^2*x^2-1))^{(1/2)}*f*d^2*c^6/(c^2*x^2-1)/g^2*\arccos(c*x)*x^5+11/8*b*(-d*(c \\ & ^2*x^2-1))^{(1/2)}*f*d^2*c^4/(c^2*x^2-1)/g^2*\arccos(c*x)*x^3-9/8*b*(-d*(c^2*x \\ & ^2-1))^{(1/2)}*f*d^2*c^2/(c^2*x^2-1)/g^2*\arccos(c*x)*x+1/3*b*(-d*(c^2*x^2-1)) \\ & ^{(1/2)}*d^2/(c^2*x^2-1)/g^3*\arccos(c*x)*x^4*c^6*f^2+a/g^6*d^3*c^6*f^5/(c^2*d \\ & )^{(1/2)}*\arctan((c^2*d)^{(1/2)}*x/(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2 \\ & *f^2-g^2)/g^2)^{(1/2)})+a/g^7*d^3/(-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*\ln((-2*d*(c^2* \\ & f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*(-d*c^2*(x+ \\ & f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)})/(x+f/g))*c^6*f^6-3*a \\ & /g^5*d^3/(-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*\ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/ \\ & g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f \\ & /g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)})/(x+f/g))*c^4*f^4+3*a/g^3*d^3/(-d*(c^2*f^2-g \\ & ^2)/g^2)^{(1/2)}*\ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^ \\ & 2-g^2)/g^2)^{(1/2)}*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2 \\ & )^{(1/2)})/(x+f/g))*c^2*f^2+15/8*a/g^2*c^2*d^3*f/(c^2*d)^{(1/2)}*\arctan((c^2*d) \\ & ^{(1/2)}*x/(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)})- \\ & 1/2*a/g^4*d^2*c^4*f^3*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2) \\ & /g^2)^{(1/2)}*x-5/2*a/g^4*d^3*c^4*f^3/(c^2*d)^{(1/2)}*\arctan((c^2*d)^{(1/2)}*x/(- \\ & d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)})+1/4*a/g^2*c \\ & ^2*d*f*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(3/2)}*x+7 \\ & /8*a/g^2*c^2*d^2*f*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^ \\ & 2)^{(1/2)}*x-1/25*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(c^2*x^2-1)/g*(-c^2*x^2+1)^{(1/ \\ & 2)}*x^5*c^5+33/128*b*(-d*(c^2*x^2-1))^{(1/2)}*f*d^2*c/(c^2*x^2-1)/g^2*(-c^2*x^ \\ & 2+1)^{(1/2)}+11/45*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(c^2*x^2-1)/g*(-c^2*x^2+1)^{(1 \\ & /2)}*x^3*c^3-23/15*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(c^2*x^2-1)/g*(-c^2*x^2+1)^{( \\ & 1/2)}*x*c-1/8*b*(-d*(c^2*x^2-1))^{(1/2)}*f^3*d^2*c^3/(c^2*x^2-1)/g^4*(-c^2*x^2 \end{aligned}$$

$$\begin{aligned}
& +1)^{(1/2)} - 2*I*b*d^2*(-d*(c^2*x^2-1))^{(1/2)}*(c^2*f^2-g^2)^{(1/2)}*(-c^2*x^2+1) \\
& ^{(1/2)}/(c^2*x^2-1)/g^4*\arccos(c*x)*\ln((-c*x+I*(-c^2*x^2+1))^{(1/2)})*g-c*f+(c \\
& ^2*f^2-g^2)^{(1/2)}/(-c*f+(c^2*f^2-g^2)^{(1/2)}))*c^2*f^2+2*I*b*d^2*(-d*(c^2*x \\
& ^2-1))^{(1/2)}*(c^2*f^2-g^2)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x^2-1)/g^4*\arccos( \\
& c*x)*\ln(((c*x+I*(-c^2*x^2+1))^{(1/2)})*g+c*f+(c^2*f^2-g^2)^{(1/2)})/(c*f+(c^2*f^ \\
& 2-g^2)^{(1/2)}))*c^2*f^2+34/15*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(c^2*x^2-1)/g*\arccos(c*x) \\
& *x^2*c^2+7/3*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(c^2*x^2-1)/g^3*\arccos(c*x) \\
& *c^2*f^2-b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(c^2*x^2-1)/g^5*\arccos(c*x)*c^4*f^4 \\
& +b*d^2*(-d*(c^2*x^2-1))^{(1/2)}*(c^2*f^2-g^2)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x \\
& ^2-1)/g^2*\operatorname{dilog}(-1/(-c*f+(c^2*f^2-g^2)^{(1/2)}))*(c*x+I*(-c^2*x^2+1))^{(1/2)}*g- \\
& 1/(-c*f+(c^2*f^2-g^2)^{(1/2)})*c*f+1/(-c*f+(c^2*f^2-g^2)^{(1/2)})*(c^2*f^2-g^2) \\
& ^{(1/2)}-b*d^2*(-d*(c^2*x^2-1))^{(1/2)}*(c^2*f^2-g^2)^{(1/2)}*(-c^2*x^2+1)^{(1/2)} \\
& /((c^2*x^2-1)/g^2*\operatorname{dilog}((c*x+I*(-c^2*x^2+1))^{(1/2)}*g/(c*f+(c^2*f^2-g^2)^{(1/2)})) \\
& +1/(c*f+(c^2*f^2-g^2)^{(1/2)})*c*f+1/(c*f+(c^2*f^2-g^2)^{(1/2)})*(c^2*f^2-g^2) \\
& ^{(1/2)}))+1/5*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(c^2*x^2-1)/g*\arccos(c*x)*x^6*c^6 \\
& -14/15*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(c^2*x^2-1)/g*\arccos(c*x)*x^4*c^4-1/3*a \\
& /g^3*d*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(3/2)}*c^2 \\
& *f^2+a/g^5*d^2*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*c^4*f^4-2*a/g^3*d^2*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*c^2*f^2+I*b*d^2*(-d*(c^2*x^2-1))^{(1/2)}*(c^2*f^2-g^2)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x^2-1)/g^6*\arccos(c*x)*\ln((-c*x+I*(-c^2*x^2+1))^{(1/2)})*g-c*f+(c^2*f^2-g^2)^{(1/2)}/(-c*f+(c^2*f^2-g^2)^{(1/2)}))*c^4*f^4-I*b*d^2*(-d*(c^2*x^2-1))^{(1/2)}*(c^2*f^2-g^2)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x^2-1)/g^6*\arccos(c*x)*\ln(((c*x+I*(-c^2*x^2+1))^{(1/2)})*g+c*f+(c^2*f^2-g^2)^{(1/2)})/(c*f+(c^2*f^2-g^2)^{(1/2)}))*c^4*f^4+2*b*d^2*(-d*(c^2*x^2-1))^{(1/2)}*(c^2*f^2-g^2)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x^2-1)/g^4*\operatorname{dilog}((c*x+I*(-c^2*x^2+1))^{(1/2)}*g/(c*f+(c^2*f^2-g^2)^{(1/2)}))+1/(c*f+(c^2*f^2-g^2)^{(1/2)})*c*f+1/(c*f+(c^2*f^2-g^2)^{(1/2)})*(c^2*f^2-g^2)^{(1/2)}*c^2*f^2+b*d^2*(-d*(c^2*x^2-1))^{(1/2)}*(c^2*f^2-g^2)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x^2-1)/g^6*\operatorname{dilog}(-1/(-c*f+(c^2*f^2-g^2)^{(1/2)}))*(c*x+I*(-c^2*x^2+1))^{(1/2)}*g-1/(-c*f+(c^2*f^2-g^2)^{(1/2)})*c*f+1/(-c*f+(c^2*f^2-g^2)^{(1/2)})*(c^2*f^2-g^2)^{(1/2)}))*c^4*f^4-b*d^2*(-d*(c^2*x^2-1))^{(1/2)}*(c^2*f^2-g^2)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x^2-1)/g^6*\operatorname{dilog}((c*x+I*(-c^2*x^2+1))^{(1/2)}*g/(c*f+(c^2*f^2-g^2)^{(1/2)}))+1/(c*f+(c^2*f^2-g^2)^{(1/2)})*c*f+1/(c*f+(c^2*f^2-g^2)^{(1/2)})*(c^2*f^2-g^2)^{(1/2)}*c^4*f^4+I*b*d^2*(-d*(c^2*x^2-1))^{(1/2)}*(c^2*f^2-g^2)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x^2-1)/g^2*\arccos(c*x)*\ln((-c*x+I*(-c^2*x^2+1))^{(1/2)})*g-c*f+(c^2*f^2-g^2)^{(1/2)}/(-c*f+(c^2*f^2-g^2)^{(1/2)}))-2*b*d^2*(-d*(c^2*x^2-1))^{(1/2)}*(c^2*f^2-g^2)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x^2-1)/g^4*\operatorname{dilog}(-1/(-c*f+(c^2*f^2-g^2)^{(1/2)}))*(c*x+I*(-c^2*x^2+1))^{(1/2)}*g-1/(-c*f+(c^2*f^2-g^2)^{(1/2)})*c*f+1/(-c*f+(c^2*f^2-g^2)^{(1/2)})*(c^2*f^2-g^2)^{(1/2)}*c^2*f^2-I*b*d^2*(-d*(c^2*x^2-1))^{(1/2)}*(c^2*f^2-g^2)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x^2-1)/g^2*\arccos(c*x)*\ln(((c*x+I*(-c^2*x^2+1))^{(1/2)})*g+c*f+(c^2*f^2-g^2)^{(1/2)})/(c*f+(c^2*f^2-g^2)^{(1/2)}))-23/15*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(c^2*x^2-1)/g*\arccos(c*x)-9/16*b*(-d*(c^2*x^2-1))^{(1/2)}*f*d^2*c^3/(c^2*x^2-1)/g^2*(-c^2*x^2+1)^{(1/2)}*x^2-1/9*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(c^2*x^2-1)/g^3*(-c^2*x^2+1)^{(1/2)}*x^3*c^5*f^2+7/3*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(c^2*x^2-1)/g^3*(-c^2*x^2+1)^{(1/2)}*x*c^3*f^2+1/4*b*(-d*(c^2*x^2-1))^{(1/2)}*f^3*d^2*c^5/(c^2*x^2-1)/g^4*(-c^2*x^2+1)^{(1/2)}*x^2-b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(c^2*x^2-1)/g^5*(-c^2*x^2+1)^{(1/2)}*x*c^5*f^4+1/16*b*(-d*(c^2*x^2-1))^{(1/2)}*f*d^2*c^5/(c^2*x^2-1)/g^2*(-c^2*x^2+1)^{(1/2)}*x^4
\end{aligned}$$


---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2\*d\*x^2+d)^(5/2)\*(a+b\*arccos(c\*x))/(g\*x+f),x, algorithm="maxi

ma")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ac^4d^2x^4 - 2ac^2d^2x^2 + ad^2 + (bc^4d^2x^4 - 2bc^2d^2x^2 + bd^2)\arccos(cx))\sqrt{-c^2dx^2 + d}}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2\*d\*x^2+d)^(5/2)\*(a+b\*arccos(c\*x))/(g\*x+f),x, algorithm="fricas")

[Out] integral((a\*c^4\*d^2\*x^4 - 2\*a\*c^2\*d^2\*x^2 + a\*d^2 + (b\*c^4\*d^2\*x^4 - 2\*b\*c^2\*d^2\*x^2 + b\*d^2)\*arccos(c\*x))\*sqrt(-c^2\*d\*x^2 + d)/(g\*x + f), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c\*\*2\*d\*x\*\*2+d)\*\*(5/2)\*(a+b\*acos(c\*x))/(g\*x+f),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c^2dx^2 + d)^{\frac{5}{2}}(b \arccos(cx) + a)}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2\*d\*x^2+d)^(5/2)\*(a+b\*arccos(c\*x))/(g\*x+f),x, algorithm="giac")

[Out] integrate((-c^2\*d\*x^2 + d)^(5/2)\*(b\*arccos(c\*x) + a)/(g\*x + f), x)

$$3.14 \quad \int \frac{(f+gx)^3(a+b \cos^{-1}(cx))}{\sqrt{d-c^2dx^2}} dx$$

**Optimal.** Leaf size=450

$$\frac{3f^2g(1-c^2x^2)(a+b \cos^{-1}(cx))}{c^2\sqrt{d-c^2dx^2}} - \frac{f^3\sqrt{1-c^2x^2}(a+b \cos^{-1}(cx))^2}{2bc\sqrt{d-c^2dx^2}} - \frac{3fg^2\sqrt{1-c^2x^2}(a+b \cos^{-1}(cx))^2}{4bc^3\sqrt{d-c^2dx^2}} - \frac{3fg^2x(1-c^2x^2)}{2c^2\sqrt{d-c^2dx^2}}$$

[Out]  $(-3*b*f^2*g*x*\text{Sqrt}[1 - c^2*x^2])/(c*\text{Sqrt}[d - c^2*d*x^2]) - (2*b*g^3*x*\text{Sqrt}[1 - c^2*x^2])/(3*c^3*\text{Sqrt}[d - c^2*d*x^2]) - (3*b*f*g^2*x^2*\text{Sqrt}[1 - c^2*x^2])/(4*c*\text{Sqrt}[d - c^2*d*x^2]) - (b*g^3*x^3*\text{Sqrt}[1 - c^2*x^2])/(9*c*\text{Sqrt}[d - c^2*d*x^2]) - (3*f^2*g*(1 - c^2*x^2)*(a + b*\text{ArcCos}[c*x]))/(c^2*\text{Sqrt}[d - c^2*d*x^2]) - (2*g^3*(1 - c^2*x^2)*(a + b*\text{ArcCos}[c*x]))/(3*c^4*\text{Sqrt}[d - c^2*d*x^2]) - (3*f*g^2*x*(1 - c^2*x^2)*(a + b*\text{ArcCos}[c*x]))/(2*c^2*\text{Sqrt}[d - c^2*d*x^2]) - (g^3*x^2*(1 - c^2*x^2)*(a + b*\text{ArcCos}[c*x]))/(3*c^2*\text{Sqrt}[d - c^2*d*x^2]) - (f^3*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcCos}[c*x])^2)/(2*b*c*\text{Sqrt}[d - c^2*d*x^2]) - (3*f*g^2*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcCos}[c*x])^2)/(4*b*c^3*\text{Sqrt}[d - c^2*d*x^2])$

**Rubi [A]** time = 0.585273, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {4778, 4764, 4642, 4678, 8, 4708, 30}

$$\frac{3f^2g(1-c^2x^2)(a+b \cos^{-1}(cx))}{c^2\sqrt{d-c^2dx^2}} - \frac{f^3\sqrt{1-c^2x^2}(a+b \cos^{-1}(cx))^2}{2bc\sqrt{d-c^2dx^2}} - \frac{3fg^2\sqrt{1-c^2x^2}(a+b \cos^{-1}(cx))^2}{4bc^3\sqrt{d-c^2dx^2}} - \frac{3fg^2x(1-c^2x^2)}{2c^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)^3*(a + b*\text{ArcCos}[c*x])/\text{Sqrt}[d - c^2*d*x^2], x]$

[Out]  $(-3*b*f^2*g*x*\text{Sqrt}[1 - c^2*x^2])/(c*\text{Sqrt}[d - c^2*d*x^2]) - (2*b*g^3*x*\text{Sqrt}[1 - c^2*x^2])/(3*c^3*\text{Sqrt}[d - c^2*d*x^2]) - (3*b*f*g^2*x^2*\text{Sqrt}[1 - c^2*x^2])/(4*c*\text{Sqrt}[d - c^2*d*x^2]) - (b*g^3*x^3*\text{Sqrt}[1 - c^2*x^2])/(9*c*\text{Sqrt}[d - c^2*d*x^2]) - (3*f^2*g*(1 - c^2*x^2)*(a + b*\text{ArcCos}[c*x]))/(c^2*\text{Sqrt}[d - c^2*d*x^2]) - (2*g^3*(1 - c^2*x^2)*(a + b*\text{ArcCos}[c*x]))/(3*c^4*\text{Sqrt}[d - c^2*d*x^2]) - (3*f*g^2*x*(1 - c^2*x^2)*(a + b*\text{ArcCos}[c*x]))/(2*c^2*\text{Sqrt}[d - c^2*d*x^2]) - (g^3*x^2*(1 - c^2*x^2)*(a + b*\text{ArcCos}[c*x]))/(3*c^2*\text{Sqrt}[d - c^2*d*x^2]) - (f^3*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcCos}[c*x])^2)/(2*b*c*\text{Sqrt}[d - c^2*d*x^2]) - (3*f*g^2*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcCos}[c*x])^2)/(4*b*c^3*\text{Sqrt}[d - c^2*d*x^2])$

#### Rule 4778

$\text{Int}[(a_. + \text{ArcCos}[(c_.)*(x_.)]*(b_.))^n*((f_.) + (g_.)*(x_.))^m*((d_.) + (e_.)*(x_.)^2)^p], x\_Symbol] \rightarrow \text{Dist}[(d^{\text{IntPart}[p]}*(d + e*x^2)^{\text{FracPart}[p]})/(1 - c^2*x^2)^{\text{FracPart}[p]}, \text{Int}[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*\text{ArcCos}[c*x])^n, x], x] /;$  FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]

#### Rule 4764

$\text{Int}[(a_. + \text{ArcCos}[(c_.)*(x_.)]*(b_.))^n*((f_.) + (g_.)*(x_.))^m*((d_.) + (e_.)*(x_.)^2)^p], x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x^2)^p*(a + b*\text{ArcCos}[c*x])^n, (f + g*x)^m, x], x] /;$  FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2\*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ



[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))

#### Rule 4642

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)/Sqrt[(d\_.) + (e\_.)\*(x\_.)^2], x\_Symbol] :> -Simp[(a + b\*ArcCos[c\*x])^(n + 1)/(b\*c\*Sqrt[d]\*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

#### Rule 4678

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)\*((d\_.) + (e\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCos[c\*x])^n)/(2\*e\*(p + 1)), x] - Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcCos[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

#### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

#### Rule 4708

Int((((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*((f\_.)\*(x\_.))^(m\_.))/Sqrt[(d\_.) + (e\_.)\*(x\_.)^2], x\_Symbol] :> Simp[(f\*(f\*x)^(m - 1)\*Sqrt[d + e\*x^2]\*(a + b\*ArcCos[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m - 1))/(c^2\*m), Int[((f\*x)^(m - 2)\*(a + b\*ArcCos[c\*x])^n)/Sqrt[d + e\*x^2], x], x] - Dist[(b\*f\*n\*Sqrt[1 - c^2\*x^2])/(c\*m\*Sqrt[d + e\*x^2]), Int[(f\*x)^(m - 1)\*(a + b\*ArcCos[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned}
 \int \frac{(f + gx)^3 (a + b \cos^{-1}(cx))}{\sqrt{d - c^2 dx^2}} dx &= \frac{\sqrt{1 - c^2 x^2} \int \frac{(f + gx)^3 (a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} dx}{\sqrt{d - c^2 dx^2}} \\
 &= \frac{\sqrt{1 - c^2 x^2} \int \left( \frac{f^3 (a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} + \frac{3f^2 gx (a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} + \frac{3fg^2 x^2 (a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} + \frac{g^3 x^3 (a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} \right) dx}{\sqrt{d - c^2 dx^2}} \\
 &= \frac{(f^3 \sqrt{1 - c^2 x^2}) \int \frac{a + b \cos^{-1}(cx)}{\sqrt{1 - c^2 x^2}} dx}{\sqrt{d - c^2 dx^2}} + \frac{(3f^2 g \sqrt{1 - c^2 x^2}) \int \frac{x (a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} dx}{\sqrt{d - c^2 dx^2}} + \frac{(3fg^2 \sqrt{1 - c^2 x^2}) \int \frac{x^2 (a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} dx}{\sqrt{d - c^2 dx^2}} + \frac{(g^3 \sqrt{1 - c^2 x^2}) \int \frac{x^3 (a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} dx}{\sqrt{d - c^2 dx^2}} \\
 &= \frac{3f^2 g (1 - c^2 x^2) (a + b \cos^{-1}(cx))}{c^2 \sqrt{d - c^2 dx^2}} - \frac{3fg^2 x (1 - c^2 x^2) (a + b \cos^{-1}(cx))}{2c^2 \sqrt{d - c^2 dx^2}} - \frac{g^3 x^2 (1 - c^2 x^2) (a + b \cos^{-1}(cx))}{c^2 \sqrt{d - c^2 dx^2}} \\
 &= \frac{3bf^2 gx \sqrt{1 - c^2 x^2}}{c \sqrt{d - c^2 dx^2}} - \frac{3bf^2 g^2 x^2 \sqrt{1 - c^2 x^2}}{4c \sqrt{d - c^2 dx^2}} - \frac{bg^3 x^3 \sqrt{1 - c^2 x^2}}{9c \sqrt{d - c^2 dx^2}} - \frac{3f^2 g (1 - c^2 x^2) (a + b \cos^{-1}(cx))}{c^2 \sqrt{d - c^2 dx^2}} \\
 &= \frac{3bf^2 gx \sqrt{1 - c^2 x^2}}{c \sqrt{d - c^2 dx^2}} - \frac{2bg^3 x \sqrt{1 - c^2 x^2}}{3c^3 \sqrt{d - c^2 dx^2}} - \frac{3bf^2 g^2 x^2 \sqrt{1 - c^2 x^2}}{4c \sqrt{d - c^2 dx^2}} - \frac{bg^3 x^3 \sqrt{1 - c^2 x^2}}{9c \sqrt{d - c^2 dx^2}} - \frac{3f^2 g (1 - c^2 x^2) (a + b \cos^{-1}(cx))}{c^2 \sqrt{d - c^2 dx^2}}
 \end{aligned}$$

**Mathematica [A]** time = 1.18172, size = 342, normalized size = 0.76

$$\sqrt{d}g(c^2x^2 - 1) \left( 12a\sqrt{1 - c^2x^2} (c^2(18f^2 + 9fgx + 2g^2x^2) + 4g^2) + 8bcx (c^2(27f^2 + g^2x^2) + 6g^2) + 27bcfg \cos(2 \cos^{-1}(c^2x^2)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((f + g\*x)^3\*(a + b\*ArcCos[c\*x]))/Sqrt[d - c^2\*d\*x^2], x]

[Out] (18\*b\*c\*Sqrt[d]\*f\*(2\*c^2\*f^2 + 3\*g^2)\*(-1 + c^2\*x^2)\*ArcCos[c\*x]^2 - 36\*a\*c\*f\*(2\*c^2\*f^2 + 3\*g^2)\*Sqrt[1 - c^2\*x^2]\*Sqrt[d - c^2\*d\*x^2]\*ArcTan[(c\*x\*Sqrt[d - c^2\*d\*x^2])/(Sqrt[d]\*(-1 + c^2\*x^2))] + Sqrt[d]\*g\*(-1 + c^2\*x^2)\*(8\*b\*c\*x\*(6\*g^2 + c^2\*(27\*f^2 + g^2\*x^2)) + 12\*a\*Sqrt[1 - c^2\*x^2]\*(4\*g^2 + c^2\*(18\*f^2 + 9\*f\*g\*x + 2\*g^2\*x^2)) + 27\*b\*c\*f\*g\*Cos[2\*ArcCos[c\*x]]) + 6\*b\*Sqrt[d]\*g\*(-1 + c^2\*x^2)\*ArcCos[c\*x]\*(4\*Sqrt[1 - c^2\*x^2]\*(2\*g^2 + c^2\*(9\*f^2 + g^2\*x^2)) + 9\*c\*f\*g\*Sin[2\*ArcCos[c\*x]]))/(72\*c^4\*Sqrt[d]\*Sqrt[1 - c^2\*x^2]\*Sqrt[d - c^2\*d\*x^2])

**Maple [B]** time = 0.605, size = 845, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^3\*(a+b\*arccos(c\*x))/(-c^2\*d\*x^2+d)^(1/2), x)

[Out] -1/3\*a\*g^3\*x^2/c^2/d\*(-c^2\*d\*x^2+d)^(1/2)-2/3\*a\*g^3/d/c^4\*(-c^2\*d\*x^2+d)^(1/2)-3/2\*a\*f\*g^2\*x/c^2/d\*(-c^2\*d\*x^2+d)^(1/2)+3/2\*a\*f\*g^2/c^2/(c^2\*d)^(1/2)\*arctan((c^2\*d)^(1/2)\*x/(-c^2\*d\*x^2+d)^(1/2))-3\*a\*f^2\*g/c^2/d\*(-c^2\*d\*x^2+d)^(1/2)+a\*f^3/(c^2\*d)^(1/2)\*arctan((c^2\*d)^(1/2)\*x/(-c^2\*d\*x^2+d)^(1/2))+1/2\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*(-c^2\*x^2+1)^(1/2)/c/d/(c^2\*x^2-1)\*arccos(c\*x)^2\*f^3-1/3\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*g^3/d/(c^2\*x^2-1)\*arccos(c\*x)\*x^4-1/3\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*g^3/c^2/d/(c^2\*x^2-1)\*arccos(c\*x)\*x^2+3\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*g/c^2/d/(c^2\*x^2-1)\*arccos(c\*x)\*f^2+1/9\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*g^3/c/d/(c^2\*x^2-1)\*(-c^2\*x^2+1)^(1/2)\*x^3+2/3\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*g^3/c^3/d/(c^2\*x^2-1)\*(-c^2\*x^2+1)^(1/2)\*x-3/2\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*f\*g^2/d/(c^2\*x^2-1)\*arccos(c\*x)\*x^3+3/2\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*f\*g^2/c^2/d/(c^2\*x^2-1)\*arccos(c\*x)\*x-3/8\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*f\*g^2/c^3/d/(c^2\*x^2-1)\*(-c^2\*x^2+1)^(1/2)-3\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*g/d/(c^2\*x^2-1)\*arccos(c\*x)\*x^2\*f^2+3/4\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*(-c^2\*x^2+1)^(1/2)/c^3/d/(c^2\*x^2-1)\*arccos(c\*x)^2\*f\*g^2+3/4\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*f\*g^2/c/d/(c^2\*x^2-1)\*(-c^2\*x^2+1)^(1/2)\*x^2+3\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*g/c/d/(c^2\*x^2-1)\*(-c^2\*x^2+1)^(1/2)\*x\*f^2+2/3\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*g^3/c^4/d/(c^2\*x^2-1)\*arccos(c\*x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(a+b\*arccos(c\*x))/(-c^2\*d\*x^2+d)^(1/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ag^3x^3 + 3afg^2x^2 + 3af^2gx + af^3 + (bg^3x^3 + 3bfg^2x^2 + 3bf^2gx + bf^3)\arccos(cx))\sqrt{-c^2dx^2 + d}}{c^2dx^2 - d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(a+b\*arccos(c\*x))/(-c^2\*d\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(-(a\*g^3\*x^3 + 3\*a\*f\*g^2\*x^2 + 3\*a\*f^2\*g\*x + a\*f^3 + (b\*g^3\*x^3 + 3\*b\*f\*g^2\*x^2 + 3\*b\*f^2\*g\*x + b\*f^3)\*arccos(c\*x))\*sqrt(-c^2\*d\*x^2 + d)/(c^2\*d\*x^2 - d), x)

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*3\*(a+b\*acos(c\*x))/(-c\*\*2\*d\*x\*\*2+d)\*\*(1/2),x)

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^3 (b \arccos(cx) + a)}{\sqrt{-c^2 dx^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(a+b\*arccos(c\*x))/(-c^2\*d\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((g\*x + f)^3\*(b\*arccos(c\*x) + a)/sqrt(-c^2\*d\*x^2 + d), x)

$$3.15 \quad \int \frac{(f+gx)^2(a+b \cos^{-1}(cx))}{\sqrt{d-c^2dx^2}} dx$$

**Optimal.** Leaf size=270

$$\frac{f^2\sqrt{1-c^2x^2}(a+b \cos^{-1}(cx))^2}{2bc\sqrt{d-c^2dx^2}} - \frac{2fg(1-c^2x^2)(a+b \cos^{-1}(cx))}{c^2\sqrt{d-c^2dx^2}} - \frac{g^2\sqrt{1-c^2x^2}(a+b \cos^{-1}(cx))^2}{4bc^3\sqrt{d-c^2dx^2}} - \frac{g^2x(1-c^2x^2)(a+b \cos^{-1}(cx))}{2c^2\sqrt{d-c^2dx^2}}$$

[Out]  $(-2*b*f*g*x*\text{Sqrt}[1 - c^2*x^2])/(c*\text{Sqrt}[d - c^2*d*x^2]) - (b*g^2*x^2*\text{Sqrt}[1 - c^2*x^2])/(4*c*\text{Sqrt}[d - c^2*d*x^2]) - (2*f*g*(1 - c^2*x^2)*(a + b*\text{ArcCos}[c*x]))/(c^2*\text{Sqrt}[d - c^2*d*x^2]) - (g^2*x*(1 - c^2*x^2)*(a + b*\text{ArcCos}[c*x]))/(2*c^2*\text{Sqrt}[d - c^2*d*x^2]) - (f^2*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcCos}[c*x])^2)/(2*b*c*\text{Sqrt}[d - c^2*d*x^2]) - (g^2*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcCos}[c*x])^2)/(4*b*c^3*\text{Sqrt}[d - c^2*d*x^2])$

**Rubi [A]** time = 0.43615, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {4778, 4764, 4642, 4678, 8, 4708, 30}

$$\frac{f^2\sqrt{1-c^2x^2}(a+b \cos^{-1}(cx))^2}{2bc\sqrt{d-c^2dx^2}} - \frac{2fg(1-c^2x^2)(a+b \cos^{-1}(cx))}{c^2\sqrt{d-c^2dx^2}} - \frac{g^2\sqrt{1-c^2x^2}(a+b \cos^{-1}(cx))^2}{4bc^3\sqrt{d-c^2dx^2}} - \frac{g^2x(1-c^2x^2)(a+b \cos^{-1}(cx))}{2c^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)^2*(a + b*\text{ArcCos}[c*x])/ \text{Sqrt}[d - c^2*d*x^2], x]$

[Out]  $(-2*b*f*g*x*\text{Sqrt}[1 - c^2*x^2])/(c*\text{Sqrt}[d - c^2*d*x^2]) - (b*g^2*x^2*\text{Sqrt}[1 - c^2*x^2])/(4*c*\text{Sqrt}[d - c^2*d*x^2]) - (2*f*g*(1 - c^2*x^2)*(a + b*\text{ArcCos}[c*x]))/(c^2*\text{Sqrt}[d - c^2*d*x^2]) - (g^2*x*(1 - c^2*x^2)*(a + b*\text{ArcCos}[c*x]))/(2*c^2*\text{Sqrt}[d - c^2*d*x^2]) - (f^2*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcCos}[c*x])^2)/(2*b*c*\text{Sqrt}[d - c^2*d*x^2]) - (g^2*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcCos}[c*x])^2)/(4*b*c^3*\text{Sqrt}[d - c^2*d*x^2])$

#### Rule 4778

$\text{Int}[(a_.) + \text{ArcCos}[(c_.)*(x_)]*(b_.)]^{(n_.)}*((f_.) + (g_.)*(x_))^{(m_.)}*((d_.) + (e_.)*(x_)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{Dist}[(d^{\text{IntPart}[p]}*(d + e*x^2)^{\text{FracPart}[p]})/(1 - c^2*x^2)^{\text{FracPart}[p]}, \text{Int}[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*\text{ArcCos}[c*x])^n, x], x] /;$  FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]

#### Rule 4764

$\text{Int}[(a_.) + \text{ArcCos}[(c_.)*(x_)]*(b_.)]^{(n_.)}*((f_.) + (g_.)*(x_))^{(m_.)}*((d_.) + (e_.)*(x_)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x^2)^p*(a + b*\text{ArcCos}[c*x])^n, (f + g*x)^m, x], x] /;$  FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2\*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))

#### Rule 4642

$\text{Int}[(a_.) + \text{ArcCos}[(c_.)*(x_)]*(b_.)]^{(n_.)}/\text{Sqrt}[(d_.) + (e_.)*(x_)^2], x\_Symbol] \rightarrow -\text{Simp}[(a + b*\text{ArcCos}[c*x])^{(n + 1)}/(b*c*\text{Sqrt}[d]*(n + 1)), x] /;$  FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)*((d_.) + (e_.)*(x_.)^2)^(p_.), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4708

```
Int((((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.))/Sqrt[(d_.) + (e_.)*(x_.)^2], x_Symbol] := Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(f + gx)^2 (a + b \cos^{-1}(cx))}{\sqrt{d - c^2 dx^2}} dx &= \frac{\sqrt{1 - c^2 x^2} \int \frac{(f + gx)^2 (a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} dx}{\sqrt{d - c^2 dx^2}} \\ &= \frac{\sqrt{1 - c^2 x^2} \int \left( \frac{f^2 (a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} + \frac{2fgx(a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} + \frac{g^2 x^2 (a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} \right) dx}{\sqrt{d - c^2 dx^2}} \\ &= \frac{(f^2 \sqrt{1 - c^2 x^2}) \int \frac{a + b \cos^{-1}(cx)}{\sqrt{1 - c^2 x^2}} dx}{\sqrt{d - c^2 dx^2}} + \frac{(2fg \sqrt{1 - c^2 x^2}) \int \frac{x(a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} dx}{\sqrt{d - c^2 dx^2}} + \frac{(g^2 \sqrt{1 - c^2 x^2}) \int \frac{x^2(a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} dx}{\sqrt{d - c^2 dx^2}} \\ &= \frac{2fg(1 - c^2 x^2)(a + b \cos^{-1}(cx))}{c^2 \sqrt{d - c^2 dx^2}} - \frac{g^2 x(1 - c^2 x^2)(a + b \cos^{-1}(cx))}{2c^2 \sqrt{d - c^2 dx^2}} - \frac{f^2 \sqrt{1 - c^2 x^2}}{2b} \\ &= \frac{2bfgx\sqrt{1 - c^2 x^2}}{c\sqrt{d - c^2 dx^2}} - \frac{bg^2 x^2 \sqrt{1 - c^2 x^2}}{4c\sqrt{d - c^2 dx^2}} - \frac{2fg(1 - c^2 x^2)(a + b \cos^{-1}(cx))}{c^2 \sqrt{d - c^2 dx^2}} - \frac{g^2 x(1 - c^2 x^2)}{2b} \end{aligned}$$

**Mathematica [A]** time = 0.757153, size = 266, normalized size = 0.99

$$\frac{\sqrt{d}g(c^2x^2 - 1)\left(4c\left(a\sqrt{1 - c^2x^2}(4f + gx) + 4bcfx\right) + bg\cos\left(2\cos^{-1}(cx)\right)\right) - 4a\sqrt{1 - c^2x^2}\sqrt{d - c^2dx^2}\left(2c^2f^2 + g^2\right)\tan^{-1}\left(\frac{\sqrt{d - c^2dx^2}}{c\sqrt{d - c^2dx^2}}\right) - 8c^3\sqrt{d}\sqrt{1 - c^2x^2}}{8c^3\sqrt{d}\sqrt{1 - c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2], x]
```

```
[Out] (2*b*Sqrt[d]*(2*c^2*f^2 + g^2)*(-1 + c^2*x^2)*ArcCos[c*x]^2 - 4*a*(2*c^2*f^2 + g^2)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d]*g*(-1 + c^2*x^2)*(4*c*(4*b*c*f*x + a*(4*f + g*x)*Sqrt[1 - c^2*x^2]) + b*g*Cos[2*ArcCos[c*x]]) + 2*b*Sqrt[d]*g*(-1 + c^2*x^2)*ArcCos[c*x]*(8*c*f*Sqrt[1 - c^2*x^2] + g*Sin[2*ArcCos[c*x]])/(8*c^3*Sqrt[d]*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2])
```

**Maple [B]** time = 0.356, size = 549, normalized size = 2.

$$-\frac{ag^2x}{2c^2d}\sqrt{-c^2dx^2+d} + \frac{ag^2}{2c^2}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}} - 2\frac{afg\sqrt{-c^2dx^2+d}}{c^2d} + af^2\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2), x)
```

```
[Out] -1/2*a*g^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a*g^2/c^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-2*a*f*g/c^2/d*(-c^2*d*x^2+d)^(1/2)+a*f^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/2*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d/(c^2*x^2-1)*arccos(c*x)^2*f^2+1/4*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*arccos(c*x)^2*g^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)*g^2/d/(c^2*x^2-1)*arccos(c*x)*x^3+1/2*b*(-d*(c^2*x^2-1))^(1/2)*g^2/c^2/d/(c^2*x^2-1)*arccos(c*x)*x-1/8*b*(-d*(c^2*x^2-1))^(1/2)*g^2/c^3/d/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)+2*b*(-d*(c^2*x^2-1))^(1/2)*f*g/c^2/d/(c^2*x^2-1)*arccos(c*x)-2*b*(-d*(c^2*x^2-1))^(1/2)*f*g/d/(c^2*x^2-1)*arccos(c*x)*x^2+1/4*b*(-d*(c^2*x^2-1))^(1/2)*g^2/c/d/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^2+2*b*(-d*(c^2*x^2-1))^(1/2)*f*g/c/d/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2), x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}(ag^2x^2 + 2afgx + af^2 + (bg^2x^2 + 2bfgx + bf^2)\arccos(cx))}{c^2dx^2 - d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2), x, algorithm="fricas")
```

[Out]  $\text{integral}(-\sqrt{-c^2dx^2 + d})(ag^2x^2 + 2afgx + af^2 + (bg^2x^2 + 2bfgx + bf^2)\arccos(cx))/(c^2dx^2 - d), x)$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((gx+f)^2(a+b\arccos(cx))/(-c^2dx^2+d)^{(1/2)}, x)$

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^2 (b \arccos(cx) + a)}{\sqrt{-c^2dx^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((gx+f)^2(a+b\arccos(cx))/(-c^2dx^2+d)^{(1/2)}, x, \text{algorithm}="giac")$

[Out]  $\text{integrate}((gx + f)^2(b\arccos(cx) + a)/\sqrt{-c^2dx^2 + d}, x)$

$$3.16 \quad \int \frac{(f+gx)(a+b \cos^{-1}(cx))}{\sqrt{d-c^2dx^2}} dx$$

**Optimal.** Leaf size=127

$$-\frac{f\sqrt{1-c^2x^2}(a+b \cos^{-1}(cx))^2}{2bc\sqrt{d-c^2dx^2}} - \frac{g(1-c^2x^2)(a+b \cos^{-1}(cx))}{c^2\sqrt{d-c^2dx^2}} - \frac{bgx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}}$$

[Out]  $-\left(\frac{b*g*x*\text{Sqrt}[1 - c^2*x^2]}{c*\text{Sqrt}[d - c^2*d*x^2]}\right) - \left(\frac{g*(1 - c^2*x^2)*(a + b*\text{ArcCos}[c*x])}{c^2*\text{Sqrt}[d - c^2*d*x^2]}\right) - \left(\frac{f*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcCos}[c*x])^2}{2*b*c*\text{Sqrt}[d - c^2*d*x^2]}\right)$

**Rubi [A]** time = 0.223334, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {4778, 4764, 4642, 4678, 8}

$$-\frac{f\sqrt{1-c^2x^2}(a+b \cos^{-1}(cx))^2}{2bc\sqrt{d-c^2dx^2}} - \frac{g(1-c^2x^2)(a+b \cos^{-1}(cx))}{c^2\sqrt{d-c^2dx^2}} - \frac{bgx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(f + g*x)*(a + b*\text{ArcCos}[c*x])}{\text{Sqrt}[d - c^2*d*x^2]}, x]$

[Out]  $-\left(\frac{b*g*x*\text{Sqrt}[1 - c^2*x^2]}{c*\text{Sqrt}[d - c^2*d*x^2]}\right) - \left(\frac{g*(1 - c^2*x^2)*(a + b*\text{ArcCos}[c*x])}{c^2*\text{Sqrt}[d - c^2*d*x^2]}\right) - \left(\frac{f*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcCos}[c*x])^2}{2*b*c*\text{Sqrt}[d - c^2*d*x^2]}\right)$

#### Rule 4778

$\text{Int}[\frac{(a_.) + \text{ArcCos}[(c_.)*(x_)]*(b_.)^{(n_.)}*((f_.) + (g_.)*(x_))^{(m_.)}*((d_.) + (e_.)*(x_)^2)^{(p_.)}}{\text{Sqrt}[d - c^2*d*x^2]}, x\_Symbol] \rightarrow \text{Dist}[(d^{\text{IntPart}[p]}*(d + e*x^2)^{\text{FracPart}[p]}), \text{Int}[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*\text{ArcCos}[c*x])^n, x], x] /;$  FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]

#### Rule 4764

$\text{Int}[\frac{(a_.) + \text{ArcCos}[(c_.)*(x_)]*(b_.)^{(n_.)}*((f_.) + (g_.)*(x_))^{(m_.)}*((d_.) + (e_.)*(x_)^2)^{(p_.)}}{\text{Sqrt}[d - c^2*d*x^2]}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x^2)^p*(a + b*\text{ArcCos}[c*x])^n, (f + g*x)^m, x], x] /;$  FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2\*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))

#### Rule 4642

$\text{Int}[\frac{(a_.) + \text{ArcCos}[(c_.)*(x_)]*(b_.)^{(n_.)}}{\text{Sqrt}[(d_.) + (e_.)*(x_)^2]}, x\_Symbol] \rightarrow -\text{Simp}[\frac{(a + b*\text{ArcCos}[c*x])^{(n+1)}}{(b*c*\text{Sqrt}[d]*(n+1))}, x] /;$  FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

#### Rule 4678

$\text{Int}[\frac{(a_.) + \text{ArcCos}[(c_.)*(x_)]*(b_.)^{(n_.)}*(x_)*((d_.) + (e_.)*(x_)^2)^{(p_.)}}{\text{Sqrt}[d - c^2*d*x^2]}, x\_Symbol] \rightarrow \text{Simp}[\frac{(d + e*x^2)^{(p+1)}*(a + b*\text{ArcCos}[c*x])^n}{2*e*(p+1)}, x] - \text{Dist}[\frac{(b*n*d^{\text{IntPart}[p]}*(d + e*x^2)^{\text{FracPart}[p]})}{2*c*(p+1)*(1 - c^2*x^2)^{\text{FracPart}[p]}}, \text{Int}[(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcCos}[c*x])^{(n-1)}, x], x] /;$  FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n



, 0] && NeQ[p, -1]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned} \int \frac{(f + gx)(a + b \cos^{-1}(cx))}{\sqrt{d - c^2 dx^2}} dx &= \frac{\sqrt{1 - c^2 x^2} \int \frac{(f + gx)(a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} dx}{\sqrt{d - c^2 dx^2}} \\ &= \frac{\sqrt{1 - c^2 x^2} \int \left( \frac{f(a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} + \frac{gx(a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} \right) dx}{\sqrt{d - c^2 dx^2}} \\ &= \frac{(f\sqrt{1 - c^2 x^2}) \int \frac{a + b \cos^{-1}(cx)}{\sqrt{1 - c^2 x^2}} dx}{\sqrt{d - c^2 dx^2}} + \frac{(g\sqrt{1 - c^2 x^2}) \int \frac{x(a + b \cos^{-1}(cx))}{\sqrt{1 - c^2 x^2}} dx}{\sqrt{d - c^2 dx^2}} \\ &= \frac{g(1 - c^2 x^2)(a + b \cos^{-1}(cx))}{c^2 \sqrt{d - c^2 dx^2}} - \frac{f\sqrt{1 - c^2 x^2}(a + b \cos^{-1}(cx))^2}{2bc\sqrt{d - c^2 dx^2}} - \frac{(bg\sqrt{1 - c^2 x^2})}{c\sqrt{d - c^2 dx^2}} \\ &= \frac{bgx\sqrt{1 - c^2 x^2}}{c\sqrt{d - c^2 dx^2}} - \frac{g(1 - c^2 x^2)(a + b \cos^{-1}(cx))}{c^2 \sqrt{d - c^2 dx^2}} - \frac{f\sqrt{1 - c^2 x^2}(a + b \cos^{-1}(cx))^2}{2bc\sqrt{d - c^2 dx^2}} \end{aligned}$$

**Mathematica [A]** time = 0.413451, size = 172, normalized size = 1.35

$$\frac{-2\sqrt{d}g(-ac^2x^2 + a + bcx\sqrt{1 - c^2x^2}) - 2acf\sqrt{d - c^2dx^2} \tan^{-1}\left(\frac{cx\sqrt{d - c^2dx^2}}{\sqrt{d}(c^2x^2 - 1)}\right) - bc\sqrt{d}f\sqrt{1 - c^2x^2} \cos^{-1}(cx)^2 + 2b\sqrt{d}g(c^2x^2 - 1)}{2c^2\sqrt{d}\sqrt{d - c^2dx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((f + g\*x)\*(a + b\*ArcCos[c\*x]))/Sqrt[d - c^2\*d\*x^2], x]

[Out] (-2\*Sqrt[d]\*g\*(a - a\*c^2\*x^2 + b\*c\*x\*Sqrt[1 - c^2\*x^2]) + 2\*b\*Sqrt[d]\*g\*(-1 + c^2\*x^2)\*ArcCos[c\*x] - b\*c\*Sqrt[d]\*f\*Sqrt[1 - c^2\*x^2]\*ArcCos[c\*x]^2 - 2\*a\*c\*f\*Sqrt[d - c^2\*d\*x^2]\*ArcTan[(c\*x\*Sqrt[d - c^2\*d\*x^2])/(Sqrt[d]\*(-1 + c^2\*x^2))])/(2\*c^2\*Sqrt[d]\*Sqrt[d - c^2\*d\*x^2])

**Maple [B]** time = 0.299, size = 235, normalized size = 1.9

$$-\frac{ag}{c^2d}\sqrt{-c^2dx^2 + d} + af \arctan\left(x\sqrt{cd}\frac{1}{\sqrt{-c^2dx^2 + d}}\right) + \frac{1}{\sqrt{c^2d}} + \frac{b(\arccos(cx))^2 f}{2dc(c^2x^2 - 1)}\sqrt{-d(c^2x^2 - 1)}\sqrt{-c^2x^2 + 1} - \frac{bg \arccos(cx)}{d(c^2x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)\*(a+b\*arccos(c\*x))/(-c^2\*d\*x^2+d)^(1/2), x)

[Out] -a\*g/c^2/d\*(-c^2\*d\*x^2+d)^(1/2)+a\*f/(c^2\*d)^(1/2)\*arctan((c^2\*d)^(1/2)\*x/(-c^2\*d\*x^2+d)^(1/2))+1/2\*b\*(-d\*(c^2\*x^2-1))^(1/2)\*(-c^2\*x^2+1)^(1/2)/c/d/(c^2\*x^2-1)\*arccos(c\*x)^2\*f-b\*(-d\*(c^2\*x^2-1))^(1/2)\*g/d/(c^2\*x^2-1)\*arccos(c\*x)\*x^2+b\*(-d\*(c^2\*x^2-1))^(1/2)\*g/c/d/(c^2\*x^2-1)\*(-c^2\*x^2+1)^(1/2)\*x+b\*(-

$$d*(c^2*x^2-1)^{(1/2)}*g/c^2/d/(c^2*x^2-1)*arccos(c*x)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(a+b\*arccos(c\*x))/(-c^2\*d\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}(agx+af+(bgx+bf)\arccos(cx))}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(a+b\*arccos(c\*x))/(-c^2\*d\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c^2\*d\*x^2 + d)\*(a\*g\*x + a\*f + (b\*g\*x + b\*f)\*arccos(c\*x))/(c^2\*d\*x^2 - d), x)

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(a+b\*acos(c\*x))/(-c\*\*2\*d\*x\*\*2+d)\*\*(1/2),x)

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)(b\arccos(cx)+a)}{\sqrt{-c^2dx^2+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(a+b\*arccos(c\*x))/(-c^2\*d\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((g\*x + f)\*(b\*arccos(c\*x) + a)/sqrt(-c^2\*d\*x^2 + d), x)

$$3.17 \quad \int \frac{a+b \cos^{-1}(cx)}{(f+gx)\sqrt{d-c^2dx^2}} dx$$

**Optimal.** Leaf size=370

$$\frac{b\sqrt{1-c^2x^2}\text{PolyLog}\left(2, -\frac{g^{e^{i\cos^{-1}(cx)}}}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}} - \frac{b\sqrt{1-c^2x^2}\text{PolyLog}\left(2, -\frac{g^{e^{i\cos^{-1}(cx)}}}{\sqrt{c^2f^2-g^2}+cf}\right)}{\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}} + \frac{i\sqrt{1-c^2x^2}(a+b\cos^{-1}(cx))\log\left(1\right)}{\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}}$$

```
[Out] (I*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) - (I*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[1 - c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2])
```

**Rubi [A]** time = 0.606007, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {4778, 4774, 3321, 2264, 2190, 2279, 2391}

$$\frac{b\sqrt{1-c^2x^2}\text{PolyLog}\left(2, -\frac{g^{e^{i\cos^{-1}(cx)}}}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}} - \frac{b\sqrt{1-c^2x^2}\text{PolyLog}\left(2, -\frac{g^{e^{i\cos^{-1}(cx)}}}{\sqrt{c^2f^2-g^2}+cf}\right)}{\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}} + \frac{i\sqrt{1-c^2x^2}(a+b\cos^{-1}(cx))\log\left(1\right)}{\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])/((f + g*x)*Sqrt[d - c^2*d*x^2]), x]
```

```
[Out] (I*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) - (I*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[1 - c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2])
```

#### Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]
```

#### Rule 4774

```
Int[(((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] && (GtQ[m, 0] || IGtQ[n, 0])
```

#### Rule 3321

```
Int[((c_.) + (d_.)*(x_)^(m_.))/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

#### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_)^(m_.)))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_)^(m_.)))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \cos^{-1}(cx)}{(f + gx)\sqrt{d - c^2 dx^2}} dx &= \frac{\sqrt{1 - c^2 x^2} \int \frac{a + b \cos^{-1}(cx)}{(f + gx)\sqrt{1 - c^2 x^2}} dx}{\sqrt{d - c^2 dx^2}} \\
&= -\frac{\sqrt{1 - c^2 x^2} \operatorname{Subst}\left(\int \frac{a + bx}{cf + g \cos(x)} dx, x, \cos^{-1}(cx)\right)}{\sqrt{d - c^2 dx^2}} \\
&= -\frac{\left(2\sqrt{1 - c^2 x^2}\right) \operatorname{Subst}\left(\int \frac{e^{ix}(a + bx)}{2ce^{ix}f + g + e^{2ix}g} dx, x, \cos^{-1}(cx)\right)}{\sqrt{d - c^2 dx^2}} \\
&= -\frac{\left(2g\sqrt{1 - c^2 x^2}\right) \operatorname{Subst}\left(\int \frac{e^{ix}(a + bx)}{2cf + 2e^{ix}g - 2\sqrt{c^2 f^2 - g^2}} dx, x, \cos^{-1}(cx)\right)}{\sqrt{c^2 f^2 - g^2}\sqrt{d - c^2 dx^2}} + \frac{\left(2g\sqrt{1 - c^2 x^2}\right) \operatorname{Subst}\left(\int \frac{e^{ix}(a + bx)}{2cf + 2e^{ix}g - 2\sqrt{c^2 f^2 - g^2}} dx, x, \cos^{-1}(cx)\right)}{\sqrt{c^2 f^2 - g^2}\sqrt{d - c^2 dx^2}} \\
&= \frac{i\sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) \log\left(1 + \frac{e^{i \cos^{-1}(cx)} g}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{\sqrt{c^2 f^2 - g^2}\sqrt{d - c^2 dx^2}} - \frac{i\sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) \log\left(1 - \frac{e^{i \cos^{-1}(cx)} g}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{\sqrt{c^2 f^2 - g^2}\sqrt{d - c^2 dx^2}} \\
&= \frac{i\sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) \log\left(1 + \frac{e^{i \cos^{-1}(cx)} g}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{\sqrt{c^2 f^2 - g^2}\sqrt{d - c^2 dx^2}} - \frac{i\sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) \log\left(1 - \frac{e^{i \cos^{-1}(cx)} g}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{\sqrt{c^2 f^2 - g^2}\sqrt{d - c^2 dx^2}} \\
&= \frac{i\sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) \log\left(1 + \frac{e^{i \cos^{-1}(cx)} g}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{\sqrt{c^2 f^2 - g^2}\sqrt{d - c^2 dx^2}} - \frac{i\sqrt{1 - c^2 x^2} (a + b \cos^{-1}(cx)) \log\left(1 - \frac{e^{i \cos^{-1}(cx)} g}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{\sqrt{c^2 f^2 - g^2}\sqrt{d - c^2 dx^2}}
\end{aligned}$$

**Mathematica [B]** time = 2.056, size = 930, normalized size = 2.51

$$\frac{a \log(f + gx)}{\sqrt{d}} - \frac{a \log(d(fxc^2 + g) + \sqrt{d}\sqrt{g^2 - c^2 f^2}\sqrt{d - c^2 dx^2})}{\sqrt{d}} - \frac{b\sqrt{1 - c^2 x^2} \left( 2 \cos^{-1}(cx) \tanh^{-1}\left(\frac{(cf + g) \cot\left(\frac{1}{2} \cos^{-1}(cx)\right)}{\sqrt{g^2 - c^2 f^2}}\right) - 2 \cos^{-1}\left(-\frac{cf}{g}\right) \tanh^{-1}\left(\frac{(g - cf) \tan\left(\frac{1}{2} \cos^{-1}(cx)\right)}{\sqrt{g^2 - c^2 f^2}}\right) \right)}{\sqrt{d}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCos[c\*x])/((f + g\*x)\*Sqrt[d - c^2\*d\*x^2]),x]

[Out] ((a\*Log[f + g\*x])/Sqrt[d] - (a\*Log[d\*(g + c^2\*f\*x) + Sqrt[d]\*Sqrt[-(c^2\*f^2 + g^2)\*Sqrt[d - c^2\*d\*x^2]])/Sqrt[d] - (b\*Sqrt[1 - c^2\*x^2]\*(2\*ArcCos[c\*x]\*ArcTanh[((c\*f + g)\*Cot[ArcCos[c\*x]/2)]/Sqrt[-(c^2\*f^2 + g^2)] - 2\*ArcCos[-((c\*f)/g)]\*ArcTanh[((-c\*f) + g)\*Tan[ArcCos[c\*x]/2)]/Sqrt[-(c^2\*f^2 + g^2)] + (ArcCos[-((c\*f)/g)] - (2\*I)\*ArcTanh[((c\*f + g)\*Cot[ArcCos[c\*x]/2)]/Sqrt[-(c^2\*f^2 + g^2)] + (2\*I)\*ArcTanh[((-c\*f) + g)\*Tan[ArcCos[c\*x]/2)]/Sqrt[-(c^2\*f^2 + g^2)])\*Log[Sqrt[-(c^2\*f^2 + g^2)]/(Sqrt[2]\*E^((I/2)\*ArcCos[c\*x])\*Sqrt[g]\*Sqrt[c\*(f + g\*x)])] + (ArcCos[-((c\*f)/g)] + (2\*I)\*(ArcTanh[((c\*f + g)\*Cot[ArcCos[c\*x]/2)]/Sqrt[-(c^2\*f^2 + g^2)] - ArcTanh[((-c\*f) + g)\*Tan[ArcCos[c\*x]/2)]/Sqrt[-(c^2\*f^2 + g^2)])\*Log[(E^((I/2)\*ArcCos[c\*x])\*Sqrt[-(c^2\*f^2 + g^2)]/(Sqrt[2]\*Sqrt[g]\*Sqrt[c\*(f + g\*x)])] - (ArcCos[-((c\*f)/g)] - (2\*I)\*ArcTanh[((-c\*f) + g)\*Tan[ArcCos[c\*x]/2)]/Sqrt[-(c^2\*f^2 + g^2)] + (2\*I)\*ArcTanh[((-c\*f) + g)\*Tan[ArcCos[c\*x]/2)]/Sqrt[-(c^2\*f^2 + g^2)])\*Log[((c\*f + g)\*((-I)\*c\*f + I\*g + Sqrt[-(c^2\*f^2 + g^2)])\*(-I + Tan[ArcCos[c\*x]/2]))/(g\*(c\*f + g + Sqrt[-(c^2\*f^2 + g^2)]\*Tan[ArcCos[c\*x]/2]))] - (ArcCos[-((c\*f)/g)] + (2\*I)\*ArcTanh[((-c\*f) + g)\*Tan[ArcCos[c\*x]/2)]/Sqrt[-(c^2\*f^2 + g^2)]\*Log[((c\*f + g)\*(I\*c\*f - I\*g + Sqrt[-(c^2\*f^2 + g^2)])\*(I + Tan[ArcCos[c\*x]/2]))/(g\*(c\*f + g + Sqrt[-(c^2\*f^2 + g^2)]\*Tan[ArcCos[c\*x]/2]))] + I\*(PolyLog[2, ((c\*f - I\*Sqrt[-(c^2\*f^2 + g^2)])\*(c\*f + g - Sqrt[-(c^2\*f^2 + g^2)]\*Tan[ArcCos[c\*x]/2]))/(g\*(c\*f + g + Sqrt[-(c^2\*f^2 + g^2)]\*Tan[ArcCos[c\*x]/2]))])

2]\*Tan[ArcCos[c\*x]/2]))] - PolyLog[2, ((c\*f + I\*Sqrt[-(c^2\*f^2) + g^2])\*(c\*f + g - Sqrt[-(c^2\*f^2) + g^2]\*Tan[ArcCos[c\*x]/2]))/(g\*(c\*f + g + Sqrt[-(c^2\*f^2) + g^2]\*Tan[ArcCos[c\*x]/2])))]/Sqrt[d - c^2\*d\*x^2])/Sqrt[-(c^2\*f^2) + g^2]

**Maple [A]** time = 0.196, size = 487, normalized size = 1.3

$$-\frac{a}{g} \ln \left( \left( -2 \frac{d(c^2 f^2 - g^2)}{g^2} + 2 \frac{c^2 f d}{g} \left( x + \frac{f}{g} \right) + 2 \sqrt{-\frac{d(c^2 f^2 - g^2)}{g^2}} \sqrt{-d c^2 \left( x + \frac{f}{g} \right)^2 + 2 \frac{c^2 f d}{g} \left( x + \frac{f}{g} \right) - \frac{d(c^2 f^2 - g^2)}{g^2}} \right) \right) (x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos(c\*x))/(g\*x+f)/(-c^2\*d\*x^2+d)^(1/2), x)

[Out] -a/g/(-d\*(c^2\*f^2-g^2)/g^2)^(1/2)\*ln((-2\*d\*(c^2\*f^2-g^2)/g^2+2\*c^2\*d\*f/g\*(x+f/g)+2\*(-d\*(c^2\*f^2-g^2)/g^2)^(1/2)\*(-d\*c^2\*(x+f/g)^2+2\*c^2\*d\*f/g\*(x+f/g)-d\*(c^2\*f^2-g^2)/g^2)^(1/2))/(x+f/g)-b\*(-d\*(c^2\*x^2-1))^(1/2)/(c^2\*f^2-g^2)^(1/2)\*(-c^2\*x^2+1)^(1/2)\*(I\*arccos(c\*x)\*ln((-c\*x+I\*(-c^2\*x^2+1)^(1/2))\*g-c\*f+(c^2\*f^2-g^2)^(1/2))/(-c\*f+(c^2\*f^2-g^2)^(1/2)))-I\*arccos(c\*x)\*ln(((c\*x+I\*(-c^2\*x^2+1)^(1/2))\*g+c\*f+(c^2\*f^2-g^2)^(1/2))/(c\*f+(c^2\*f^2-g^2)^(1/2)))+dilog((-c\*x+I\*(-c^2\*x^2+1)^(1/2))\*g-c\*f+(c^2\*f^2-g^2)^(1/2))/(-c\*f+(c^2\*f^2-g^2)^(1/2)))-dilog(((c\*x+I\*(-c^2\*x^2+1)^(1/2))\*g+c\*f+(c^2\*f^2-g^2)^(1/2))/(c\*f+(c^2\*f^2-g^2)^(1/2)))/d/(c^2\*x^2-1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arccos(cx) + a}{\sqrt{-c^2 dx^2 + d}(gx + f)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(c\*x))/(g\*x+f)/(-c^2\*d\*x^2+d)^(1/2), x, algorithm="maxima")

[Out] integrate((b\*arccos(c\*x) + a)/(sqrt(-c^2\*d\*x^2 + d)\*(g\*x + f)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{\sqrt{-c^2 dx^2 + d}(b \arccos(cx) + a)}{c^2 d g x^3 + c^2 d f x^2 - d g x - d f}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(c\*x))/(g\*x+f)/(-c^2\*d\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] integral(-sqrt(-c^2\*d\*x^2 + d)\*(b\*arccos(c\*x) + a)/(c^2\*d\*g\*x^3 + c^2\*d\*f\*x^2 - d\*g\*x - d\*f), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \arccos(cx)}{\sqrt{-d(cx-1)(cx+1)}(f+gx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acos(c\*x))/(g\*x+f)/(-c\*\*2\*d\*x\*\*2+d)\*\*(1/2),x)

[Out] Integral((a + b\*acos(c\*x))/(sqrt(-d\*(c\*x - 1)\*(c\*x + 1))\*(f + g\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arccos(cx) + a}{\sqrt{-c^2 dx^2 + d}(gx + f)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(c\*x))/(g\*x+f)/(-c^2\*d\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccos(c\*x) + a)/(sqrt(-c^2\*d\*x^2 + d)\*(g\*x + f)), x)

$$3.18 \quad \int \frac{a+b \cos^{-1}(cx)}{(f+gx)^2 \sqrt{d-c^2 dx^2}} dx$$

**Optimal.** Leaf size=496

$$\frac{bc^2 f \sqrt{1-c^2 x^2} \text{PolyLog}\left(2, -\frac{g e^{i \cos^{-1}(cx)}}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{\sqrt{d-c^2 dx^2} (c^2 f^2 - g^2)^{3/2}} - \frac{bc^2 f \sqrt{1-c^2 x^2} \text{PolyLog}\left(2, -\frac{g e^{i \cos^{-1}(cx)}}{\sqrt{c^2 f^2 - g^2} + cf}\right)}{\sqrt{d-c^2 dx^2} (c^2 f^2 - g^2)^{3/2}} + \frac{g(1-c^2 x^2)(a+b \cos^{-1}(cx))}{\sqrt{d-c^2 dx^2} (c^2 f^2 - g^2)(f+gx)}$$

```
[Out] (g*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/((c^2*f^2 - g^2)*(f + g*x)*Sqrt[d - c^2*d*x^2]) + (I*c^2*f*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]) - (I*c^2*f*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[1 - c^2*x^2]*Log[f + g*x])/((c^2*f^2 - g^2)*Sqrt[d - c^2*d*x^2]) + (b*c^2*f*Sqrt[1 - c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]) - (b*c^2*f*Sqrt[1 - c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2])
```

**Rubi [A]** time = 0.721545, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$ , Rules used = {4778, 4774, 3324, 3321, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{bc^2 f \sqrt{1-c^2 x^2} \text{PolyLog}\left(2, -\frac{g e^{i \cos^{-1}(cx)}}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{\sqrt{d-c^2 dx^2} (c^2 f^2 - g^2)^{3/2}} - \frac{bc^2 f \sqrt{1-c^2 x^2} \text{PolyLog}\left(2, -\frac{g e^{i \cos^{-1}(cx)}}{\sqrt{c^2 f^2 - g^2} + cf}\right)}{\sqrt{d-c^2 dx^2} (c^2 f^2 - g^2)^{3/2}} + \frac{g(1-c^2 x^2)(a+b \cos^{-1}(cx))}{\sqrt{d-c^2 dx^2} (c^2 f^2 - g^2)(f+gx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])/((f + g*x)^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (g*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/((c^2*f^2 - g^2)*(f + g*x)*Sqrt[d - c^2*d*x^2]) + (I*c^2*f*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]) - (I*c^2*f*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[1 - c^2*x^2]*Log[f + g*x])/((c^2*f^2 - g^2)*Sqrt[d - c^2*d*x^2]) + (b*c^2*f*Sqrt[1 - c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]) - (b*c^2*f*Sqrt[1 - c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2])
```

**Rule 4778**

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^ (n_.)*((f_.) + (g_.)*(x_.))^ (m_.)*((d_.) + (e_.)*(x_.)^2)^ (p_.), x_Symbol] := Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]
```

**Rule 4774**

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^ (n_.)*((f_.) + (g_.)*(x_.))^ (m_.))/Sqrt[(d_.) + (e_.)*(x_.)^2], x_Symbol] := -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b
```



, c, d, e, f, g, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[m] && GtQ[d, 0] && (GtQ[m, 0] || IGtQ[n, 0])

#### Rule 3324

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]^2, x\_Symbol] := Simp[(b\*(c + d\*x)^m\*cos[e + f\*x]/(f\*(a^2 - b^2)\*(a + b\*sin[e + f\*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d\*x)^m/(a + b\*sin[e + f\*x]), x], x] - Dist[(b\*d\*m)/(f\*(a^2 - b^2)), Int[((c + d\*x)^(m - 1)\*cos[e + f\*x])/(a + b\*sin[e + f\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

#### Rule 3321

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Dist[2, Int[(c + d\*x)^m\*E^(I\*Pi\*(k - 1/2))\*E^(I\*(e + f\*x))]/(b + 2\*a\*E^(I\*Pi\*(k - 1/2))\*E^(I\*(e + f\*x)) - b\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

#### Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_.))^(m\_.)/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

#### Rule 2190

Int((((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_.)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_.)))^(n\_.)), x\_Symbol] := Simp[(c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_.)))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_.)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 2668

Int[cos[(e\_.) + (f\_.)\*(x\_.)]^(p\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]^(m\_.)), x\_Symbol] := Dist[1/(b^p\*f), Subst[Int[(a + x)^m\*(b^2 - x^2)^((p - 1)/2), x], x, b\*sin[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_.))^(-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \cos^{-1}(cx)}{(f + gx)^2 \sqrt{d - c^2 dx^2}} dx &= \frac{\sqrt{1 - c^2 x^2} \int \frac{a + b \cos^{-1}(cx)}{(f + gx)^2 \sqrt{1 - c^2 x^2}} dx}{\sqrt{d - c^2 dx^2}} \\
 &= \frac{\left(c\sqrt{1 - c^2 x^2}\right) \text{Subst}\left(\int \frac{a + bx}{(cf + g \cos(x))^2} dx, x, \cos^{-1}(cx)\right)}{\sqrt{d - c^2 dx^2}} \\
 &= \frac{g(1 - c^2 x^2)(a + b \cos^{-1}(cx))}{(c^2 f^2 - g^2)(f + gx)\sqrt{d - c^2 dx^2}} - \frac{\left(c^2 f \sqrt{1 - c^2 x^2}\right) \text{Subst}\left(\int \frac{a + bx}{cf + g \cos(x)} dx, x, \cos^{-1}(cx)\right)}{(c^2 f^2 - g^2)\sqrt{d - c^2 dx^2}} \\
 &= \frac{g(1 - c^2 x^2)(a + b \cos^{-1}(cx))}{(c^2 f^2 - g^2)(f + gx)\sqrt{d - c^2 dx^2}} + \frac{\left(bc\sqrt{1 - c^2 x^2}\right) \text{Subst}\left(\int \frac{1}{cf + x} dx, x, cgx\right)}{(c^2 f^2 - g^2)\sqrt{d - c^2 dx^2}} - \frac{\left(2c^2 f \sqrt{1 - c^2 x^2}\right) \text{Subst}\left(\int \frac{1}{cf + x} dx, x, cgx\right)}{(c^2 f^2 - g^2)\sqrt{d - c^2 dx^2}} \\
 &= \frac{g(1 - c^2 x^2)(a + b \cos^{-1}(cx))}{(c^2 f^2 - g^2)(f + gx)\sqrt{d - c^2 dx^2}} + \frac{bc\sqrt{1 - c^2 x^2} \log(f + gx)}{(c^2 f^2 - g^2)\sqrt{d - c^2 dx^2}} - \frac{\left(2c^2 f g \sqrt{1 - c^2 x^2}\right) \text{Subst}\left(\int \frac{1}{cf + x} dx, x, cgx\right)}{(c^2 f^2 - g^2)\sqrt{d - c^2 dx^2}} \\
 &= \frac{g(1 - c^2 x^2)(a + b \cos^{-1}(cx))}{(c^2 f^2 - g^2)(f + gx)\sqrt{d - c^2 dx^2}} + \frac{ic^2 f \sqrt{1 - c^2 x^2}(a + b \cos^{-1}(cx)) \log\left(1 + \frac{e^{i \cos^{-1}(cx)} g}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{(c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 dx^2}} \\
 &= \frac{g(1 - c^2 x^2)(a + b \cos^{-1}(cx))}{(c^2 f^2 - g^2)(f + gx)\sqrt{d - c^2 dx^2}} + \frac{ic^2 f \sqrt{1 - c^2 x^2}(a + b \cos^{-1}(cx)) \log\left(1 + \frac{e^{i \cos^{-1}(cx)} g}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{(c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 dx^2}} \\
 &= \frac{g(1 - c^2 x^2)(a + b \cos^{-1}(cx))}{(c^2 f^2 - g^2)(f + gx)\sqrt{d - c^2 dx^2}} + \frac{ic^2 f \sqrt{1 - c^2 x^2}(a + b \cos^{-1}(cx)) \log\left(1 + \frac{e^{i \cos^{-1}(cx)} g}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{(c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 dx^2}}
 \end{aligned}$$

**Mathematica [B]** time = 4.98671, size = 1108, normalized size = 2.23

$$\frac{af \log(f + gx)c^2}{\sqrt{d}(g^2 - c^2 f^2)^{3/2}} - \frac{af \log\left(d(fxc^2 + g) + \sqrt{d}\sqrt{g^2 - c^2 f^2}\sqrt{d - c^2 dx^2}\right)c^2}{\sqrt{d}(cf - g)(cf + g)\sqrt{g^2 - c^2 f^2}} - \frac{b\sqrt{1 - c^2 x^2} \left( -\frac{g\sqrt{1 - c^2 x^2} \cos^{-1}(cx)}{(cf - g)(cf + g)(cf + cgx)} - \frac{\log\left(\frac{gx}{f} + 1\right)}{c^2 f^2 - g^2} \right)}{\sqrt{d}(g^2 - c^2 f^2)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])/((f + g*x)^2*Sqrt[d - c^2*d*x^2]), x]
```

```
[Out] -((a*g*Sqrt[d - c^2*d*x^2])/(d*(-(c^2*f^2) + g^2)*(f + g*x))) - (a*c^2*f*Log[g[f + g*x]/(Sqrt[d]*(-(c^2*f^2) + g^2)^(3/2)) - (a*c^2*f*Log[d*(g + c^2*f*x) + Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]])/(Sqrt[d]*(c*f - g)*(c*f + g)*Sqrt[-(c^2*f^2) + g^2]) - (b*c*Sqrt[1 - c^2*x^2]*(-(g*Sqrt[1 - c^2*x^2]*ArcCos[c*x])/(c*f - g)*(c*f + g)*(c*f + c*g*x))) - Log[1 + (g*x)/f]/(c^2*f^2 - g^2) - (c*f*(2*ArcCos[c*x]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - 2*ArcCos[-((c*f)/g)]*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[Sqrt[-(c^2*f^2) + g^2]]
```

$$\begin{aligned}
& 2*f^2 + g^2) / (\text{Sqrt}[2]*E^{(I/2)*\text{ArcCos}[c*x]}*\text{Sqrt}[g]*\text{Sqrt}[c*(f + g*x)]) + \\
& (\text{ArcCos}[-((c*f)/g)] + (2*I)*(\text{ArcTanh}[\frac{(c*f + g)*\text{Cot}[\text{ArcCos}[c*x]/2]}{\text{Sqrt}[-(c^2*f^2 + g^2)}] - \text{ArcTanh}[\frac{(-(c*f) + g)*\text{Tan}[\text{ArcCos}[c*x]/2]}{\text{Sqrt}[-(c^2*f^2 + g^2)}]}) * \text{Log}[(E^{(I/2)*\text{ArcCos}[c*x]}*\text{Sqrt}[-(c^2*f^2 + g^2)}) / (\text{Sqrt}[2]*\text{Sqrt}[g]*\text{Sqrt}[c*(f + g*x)])]) - (\text{ArcCos}[-((c*f)/g)] - (2*I)*\text{ArcTanh}[\frac{(-(c*f) + g)*\text{Tan}[\text{ArcCos}[c*x]/2]}{\text{Sqrt}[-(c^2*f^2 + g^2)}]}) * \text{Log}[\frac{(c*f + g)*(-I)*c*f + I*g + \text{Sqrt}[-(c^2*f^2 + g^2)}]{(g*(c*f + g + \text{Sqrt}[-(c^2*f^2 + g^2)*\text{Tan}[\text{ArcCos}[c*x]/2])}] - (\text{ArcCos}[-((c*f)/g)] + (2*I)*\text{ArcTanh}[\frac{(-(c*f) + g)*\text{Tan}[\text{ArcCos}[c*x]/2]}{\text{Sqrt}[-(c^2*f^2 + g^2)}]}) * \text{Log}[\frac{(c*f + g)*(I*c*f - I*g + \text{Sqrt}[-(c^2*f^2 + g^2)}]{(g*(c*f + g + \text{Sqrt}[-(c^2*f^2 + g^2)*\text{Tan}[\text{ArcCos}[c*x]/2])}] + I*(\text{PolyLog}[2, \frac{(c*f - I*\text{Sqrt}[-(c^2*f^2 + g^2)}]{(g*(c*f + g + \text{Sqrt}[-(c^2*f^2 + g^2)*\text{Tan}[\text{ArcCos}[c*x]/2])})] - \text{PolyLog}[2, \frac{(c*f + I*\text{Sqrt}[-(c^2*f^2 + g^2)}]{(g*(c*f + g + \text{Sqrt}[-(c^2*f^2 + g^2)*\text{Tan}[\text{ArcCos}[c*x]/2])})] / (- (c^2*f^2 + g^2)^{(3/2)})) / \text{Sqrt}[d - c^2*d*x^2]
\end{aligned}$$

**Maple [B]** time = 0.305, size = 1622, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccos(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2), x)`

[Out] 
$$\begin{aligned}
& a/d/(c^2*f^2-g^2)/(x+f/g)*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)}-a/g*c^2*f/(c^2*f^2-g^2)/(-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*\ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^{(1/2)}*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^{(1/2)})/(x+f/g))+b*(-d*(c^2*x^2-1))^{(1/2)}*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*(-c^2*x^2+1)*x*c^2*f+b*(-d*(c^2*x^2-1))^{(1/2)}*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*x^3*c^4*f+I*b*c^2*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/d/(c^2*x^2-1)/(c^2*f^2-g^2)^{(3/2)}*\ln(((c*x+I*(-c^2*x^2+1))^{(1/2)}*g+c*f+(c^2*f^2-g^2)^{(1/2)})/(c*f+(c^2*f^2-g^2)^{(1/2)}))*arccos(c*x)*f+b*(-d*(c^2*x^2-1))^{(1/2)}*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*x^2*c^2*g-I*b*(-d*(c^2*x^2-1))^{(1/2)}*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*x*c^2*f-b*(-d*(c^2*x^2-1))^{(1/2)}*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*g-I*b*(-d*(c^2*x^2-1))^{(1/2)}*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*(-c^2*x^2+1)^{(1/2)}*c*f-I*b*c^2*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/d/(c^2*x^2-1)/(c^2*f^2-g^2)^{(3/2)}*\ln((-c*x+I*(-c^2*x^2+1))^{(1/2)}*g-c*f+(c^2*f^2-g^2)^{(1/2)})/(-c*f+(c^2*f^2-g^2)^{(1/2)}))*arccos(c*x)*f-b*c^3*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/d/(c^2*x^2-1)/(c^2*f^2-g^2)^2*\ln((c*x+I*(-c^2*x^2+1))^{(1/2)})*g+2*c*f*(c*x+I*(-c^2*x^2+1))^{(1/2)}+g)*f^2+2*b*c^3*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/d/(c^2*x^2-1)/(c^2*f^2-g^2)^2*\ln(c*x+I*(-c^2*x^2+1))^{(1/2)}*f^2-b*c^2*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/d/(c^2*x^2-1)/(c^2*f^2-g^2)^{(3/2)}*dilog((-c*x+I*(-c^2*x^2+1))^{(1/2)}*g-c*f+(c^2*f^2-g^2)^{(1/2)})/(-c*f+(c^2*f^2-g^2)^{(1/2)}))*f+b*c^2*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/d/(c^2*x^2-1)/(c^2*f^2-g^2)^2*\ln((c*x+I*(-c^2*x^2+1))^{(1/2)})*g+2*c*f*(c*x+I*(-c^2*x^2+1))^{(1/2)}+g)*g^2-2*b*c*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/d/(c^2*x^2-1)/(c^2*f^2-g^2)^2*\ln(c*x+I*(-c^2*x^2+1))^{(1/2)})*g^2
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arccos(cx) + a}{\sqrt{-c^2 dx^2 + d}(gx + f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(c\*x))/(g\*x+f)^2/(-c^2\*d\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] integrate((b\*arccos(c\*x) + a)/(sqrt(-c^2\*d\*x^2 + d)\*(g\*x + f)^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2 dx^2 + d}(b \arccos(cx) + a)}{c^2 dg^2 x^4 + 2c^2 df gx^3 - 2df gx - df^2 + (c^2 df^2 - dg^2)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(c\*x))/(g\*x+f)^2/(-c^2\*d\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c^2\*d\*x^2 + d)\*(b\*arccos(c\*x) + a)/(c^2\*d\*g^2\*x^4 + 2\*c^2\*d\*f\*g\*x^3 - 2\*d\*f\*g\*x - d\*f^2 + (c^2\*d\*f^2 - d\*g^2)\*x^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \arccos(cx)}{\sqrt{-d}(cx - 1)(cx + 1)(f + gx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acos(c\*x))/(g\*x+f)\*\*2/(-c\*\*2\*d\*x\*\*2+d)\*\*(1/2),x)

[Out] Integral((a + b\*acos(c\*x))/(sqrt(-d\*(c\*x - 1)\*(c\*x + 1))\*(f + g\*x)\*\*2), x)

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(c\*x))/(g\*x+f)^2/(-c^2\*d\*x^2+d)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.19 \quad \int \frac{(a+b \cos^{-1}(cx))^n \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}} dx$$

**Optimal.** Leaf size=37

$$\text{Unintegrable} \left( \frac{(a+b \cos^{-1}(cx))^n \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}, x \right)$$

[Out] Unintegrable[((a + b\*ArcCos[c\*x])^n\*Log[h\*(f + g\*x)^m])/Sqrt[1 - c^2\*x^2], x]

**Rubi [A]** time = 0.192177, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a+b \cos^{-1}(cx))^n \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}} dx$$

Verification is Not applicable to the result.

[In] Int[((a + b\*ArcCos[c\*x])^n\*Log[h\*(f + g\*x)^m])/Sqrt[1 - c^2\*x^2], x]

[Out] Defer[Int] [((a + b\*ArcCos[c\*x])^n\*Log[h\*(f + g\*x)^m])/Sqrt[1 - c^2\*x^2], x]

Rubi steps

$$\int \frac{(a+b \cos^{-1}(cx))^n \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}} dx = \int \frac{(a+b \cos^{-1}(cx))^n \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}} dx$$

**Mathematica [A]** time = 0.15798, size = 0, normalized size = 0.

$$\int \frac{(a+b \cos^{-1}(cx))^n \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[((a + b\*ArcCos[c\*x])^n\*Log[h\*(f + g\*x)^m])/Sqrt[1 - c^2\*x^2], x]

[Out] Integrate[((a + b\*ArcCos[c\*x])^n\*Log[h\*(f + g\*x)^m])/Sqrt[1 - c^2\*x^2], x]

**Maple [A]** time = 5.372, size = 0, normalized size = 0.

$$\int (a+b \arccos(cx))^n \ln(h(gx+f)^m) \frac{1}{\sqrt{-c^2x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos(c\*x))^n\*ln(h\*(g\*x+f)^m)/(-c^2\*x^2+1)^(1/2), x)

[Out] `int((a+b*arccos(c*x))^n*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccos(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)^n\log\left((gx+f)^mh\right)}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccos(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")`

[Out] `integral(-sqrt(-c^2*x^2+1)*(b*arccos(c*x)+a)^n*log((g*x+f)^m*h)/(c^2*x^2-1),x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acos(c*x))^n*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arccos(cx)+a)^n\log\left((gx+f)^mh\right)}{\sqrt{-c^2x^2+1}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccos(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="giac")`

[Out] `integrate((b*arccos(c*x)+a)^n*log((g*x+f)^m*h)/sqrt(-c^2*x^2+1),x)`

$$3.20 \quad \int \frac{(a+b \cos^{-1}(cx))^2 \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}} dx$$

**Optimal.** Leaf size=496

$$\frac{im(a+b \cos^{-1}(cx))^2 \operatorname{PolyLog}\left(2, -\frac{ge^{i \cos^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{im(a+b \cos^{-1}(cx))^2 \operatorname{PolyLog}\left(2, -\frac{ge^{i \cos^{-1}(cx)}}{\sqrt{c^2f^2-g^2+cf}}\right)}{c} + \frac{2bm(a+b \cos^{-1}(cx))}{c}$$

[Out]  $((-I/12)*m*(a + b*\operatorname{ArcCos}[c*x])^4)/(b^2*c) + (m*(a + b*\operatorname{ArcCos}[c*x])^3*\operatorname{Log}[1 + (E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f - \operatorname{Sqrt}[c^2*f^2 - g^2])])/(3*b*c) + (m*(a + b*\operatorname{ArcCos}[c*x])^3*\operatorname{Log}[1 + (E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f + \operatorname{Sqrt}[c^2*f^2 - g^2])])/(3*b*c) - ((a + b*\operatorname{ArcCos}[c*x])^3*\operatorname{Log}[h*(f + g*x)^m])/(3*b*c) - (I*m*(a + b*\operatorname{ArcCos}[c*x])^2*\operatorname{PolyLog}[2, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f - \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c - (I*m*(a + b*\operatorname{ArcCos}[c*x])^2*\operatorname{PolyLog}[2, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f + \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c + (2*b*m*(a + b*\operatorname{ArcCos}[c*x])* \operatorname{PolyLog}[3, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f - \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c + (2*b*m*(a + b*\operatorname{ArcCos}[c*x])* \operatorname{PolyLog}[3, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f + \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c + ((2*I)*b^2*m*\operatorname{PolyLog}[4, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f - \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c + ((2*I)*b^2*m*\operatorname{PolyLog}[4, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f + \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c$

**Rubi [A]** time = 0.791464, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$ , Rules used = {4642, 4780, 4742, 4520, 2190, 2531, 6609, 2282, 6589}

$$\frac{im(a+b \cos^{-1}(cx))^2 \operatorname{PolyLog}\left(2, -\frac{ge^{i \cos^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{im(a+b \cos^{-1}(cx))^2 \operatorname{PolyLog}\left(2, -\frac{ge^{i \cos^{-1}(cx)}}{\sqrt{c^2f^2-g^2+cf}}\right)}{c} + \frac{2bm(a+b \cos^{-1}(cx))}{c}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCos}[c*x])^2*\operatorname{Log}[h*(f + g*x)^m]/\operatorname{Sqrt}[1 - c^2*x^2], x]$

[Out]  $((-I/12)*m*(a + b*\operatorname{ArcCos}[c*x])^4)/(b^2*c) + (m*(a + b*\operatorname{ArcCos}[c*x])^3*\operatorname{Log}[1 + (E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f - \operatorname{Sqrt}[c^2*f^2 - g^2])])/(3*b*c) + (m*(a + b*\operatorname{ArcCos}[c*x])^3*\operatorname{Log}[1 + (E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f + \operatorname{Sqrt}[c^2*f^2 - g^2])])/(3*b*c) - ((a + b*\operatorname{ArcCos}[c*x])^3*\operatorname{Log}[h*(f + g*x)^m])/(3*b*c) - (I*m*(a + b*\operatorname{ArcCos}[c*x])^2*\operatorname{PolyLog}[2, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f - \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c - (I*m*(a + b*\operatorname{ArcCos}[c*x])^2*\operatorname{PolyLog}[2, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f + \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c + (2*b*m*(a + b*\operatorname{ArcCos}[c*x])* \operatorname{PolyLog}[3, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f - \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c + (2*b*m*(a + b*\operatorname{ArcCos}[c*x])* \operatorname{PolyLog}[3, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f + \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c + ((2*I)*b^2*m*\operatorname{PolyLog}[4, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f - \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c + ((2*I)*b^2*m*\operatorname{PolyLog}[4, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f + \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c$

**Rule 4642**

$\operatorname{Int}[(a + \operatorname{ArcCos}[(c_*)(x)]*(b_*)^{(n_*)}/\operatorname{Sqrt}[(d_*) + (e_*)(x)^2], x_{\text{Symbol}}] :> -\operatorname{Simp}[(a + b*\operatorname{ArcCos}[c*x])^{(n+1)}/(b*c*\operatorname{Sqrt}[d]*(n+1)), x] /;$  FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

**Rule 4780**

$\operatorname{Int}[(\operatorname{Log}[(h_*)(f_*) + (g_*)(x)]^{(m_*)})*(a + \operatorname{ArcCos}[(c_*)(x)]*(b_*)^{(n_*)})/\operatorname{Sqrt}[(d_*) + (e_*)(x)^2], x_{\text{Symbol}}] :> -\operatorname{Simp}[(\operatorname{Log}[h*(f + g*x)^m]$

```
*(a + b*ArcCos[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] + Dist[(g*m)/(b*c*S
qrt[d]*(n + 1)), Int[(a + b*ArcCos[c*x])^(n + 1)/(f + g*x), x], x] /; FreeQ
[{a, b, c, d, e, f, g, h, m}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && IGtQ[
n, 0]
```

#### Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol]
:> -Subst[Int[((a + b*x)^n*Sin[x])/(c*d + e*Cos[x]), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

#### Rule 4520

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_.)), x_Symbol] :> Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x))], x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))], x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

#### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

#### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a +
b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

#### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rubi steps



$$\begin{aligned}
\int \frac{(a + b \cos^{-1}(cx))^2 \log(h(f + gx)^m)}{\sqrt{1 - c^2x^2}} dx &= -\frac{(a + b \cos^{-1}(cx))^3 \log(h(f + gx)^m)}{3bc} + \frac{(gm) \int \frac{(a+b \cos^{-1}(cx))^3}{f+gx} dx}{3bc} \\
&= -\frac{(a + b \cos^{-1}(cx))^3 \log(h(f + gx)^m)}{3bc} - \frac{(gm) \text{Subst}\left(\int \frac{(a+bx)^3 \sin(x)}{cf+g \cos(x)} dx, x, \right)}{3bc} \\
&= -\frac{im(a + b \cos^{-1}(cx))^4}{12b^2c} - \frac{(a + b \cos^{-1}(cx))^3 \log(h(f + gx)^m)}{3bc} + \frac{(igm) \text{Subst}\left(\int \frac{(a+bx)^3 \sin(x)}{cf+g \cos(x)} dx, x, \right)}{3bc} \\
&= -\frac{im(a + b \cos^{-1}(cx))^4}{12b^2c} + \frac{m(a + b \cos^{-1}(cx))^3 \log\left(1 + \frac{e^{i \cos^{-1}(cx)}g}{cf - \sqrt{c^2f^2 - g^2}}\right)}{3bc} + \frac{(igm) \text{Subst}\left(\int \frac{(a+bx)^3 \sin(x)}{cf+g \cos(x)} dx, x, \right)}{3bc} \\
&= -\frac{im(a + b \cos^{-1}(cx))^4}{12b^2c} + \frac{m(a + b \cos^{-1}(cx))^3 \log\left(1 + \frac{e^{i \cos^{-1}(cx)}g}{cf - \sqrt{c^2f^2 - g^2}}\right)}{3bc} + \frac{(igm) \text{Subst}\left(\int \frac{(a+bx)^3 \sin(x)}{cf+g \cos(x)} dx, x, \right)}{3bc} \\
&= -\frac{im(a + b \cos^{-1}(cx))^4}{12b^2c} + \frac{m(a + b \cos^{-1}(cx))^3 \log\left(1 + \frac{e^{i \cos^{-1}(cx)}g}{cf - \sqrt{c^2f^2 - g^2}}\right)}{3bc} + \frac{(igm) \text{Subst}\left(\int \frac{(a+bx)^3 \sin(x)}{cf+g \cos(x)} dx, x, \right)}{3bc} \\
&= -\frac{im(a + b \cos^{-1}(cx))^4}{12b^2c} + \frac{m(a + b \cos^{-1}(cx))^3 \log\left(1 + \frac{e^{i \cos^{-1}(cx)}g}{cf - \sqrt{c^2f^2 - g^2}}\right)}{3bc} + \frac{(igm) \text{Subst}\left(\int \frac{(a+bx)^3 \sin(x)}{cf+g \cos(x)} dx, x, \right)}{3bc} \\
&= -\frac{im(a + b \cos^{-1}(cx))^4}{12b^2c} + \frac{m(a + b \cos^{-1}(cx))^3 \log\left(1 + \frac{e^{i \cos^{-1}(cx)}g}{cf - \sqrt{c^2f^2 - g^2}}\right)}{3bc} + \frac{(igm) \text{Subst}\left(\int \frac{(a+bx)^3 \sin(x)}{cf+g \cos(x)} dx, x, \right)}{3bc} \\
&= -\frac{im(a + b \cos^{-1}(cx))^4}{12b^2c} + \frac{m(a + b \cos^{-1}(cx))^3 \log\left(1 + \frac{e^{i \cos^{-1}(cx)}g}{cf - \sqrt{c^2f^2 - g^2}}\right)}{3bc} + \frac{(igm) \text{Subst}\left(\int \frac{(a+bx)^3 \sin(x)}{cf+g \cos(x)} dx, x, \right)}{3bc}
\end{aligned}$$

**Mathematica [F]** time = 26.654, size = 0, normalized size = 0.

$$\int \frac{(a + b \cos^{-1}(cx))^2 \log(h(f + gx)^m)}{\sqrt{1 - c^2x^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[((a + b\*ArcCos[c\*x])^2\*Log[h\*(f + g\*x)^m])/Sqrt[1 - c^2\*x^2], x]

[Out] Integrate[((a + b\*ArcCos[c\*x])^2\*Log[h\*(f + g\*x)^m])/Sqrt[1 - c^2\*x^2], x]

**Maple [F]** time = 4.708, size = 0, normalized size = 0.

$$\int (a + b \arccos(cx))^2 \ln(h(gx + f)^m) \frac{1}{\sqrt{-c^2x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos(c\*x))^2\*ln(h\*(g\*x+f)^m)/(-c^2\*x^2+1)^(1/2), x)

[Out] int((a+b\*arccos(c\*x))^2\*ln(h\*(g\*x+f)^m)/(-c^2\*x^2+1)^(1/2), x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{\sqrt{-c^2x^2+1} (b^2 \arccos(cx)^2 + 2ab \arccos(cx) + a^2) \log((gx+f)^m h)}{c^2x^2-1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*log((g*x + f)^m*h)/(c^2*x^2 - 1), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \arccos(cx))^2 \log(h(f + gx)^m)}{\sqrt{-(cx-1)(cx+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))^2*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acos(c*x))^2*log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arccos(cx) + a)^2 \log((gx+f)^m h)}{\sqrt{-c^2x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^2*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

$$3.21 \quad \int \frac{(a+b \cos^{-1}(cx)) \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}} dx$$

**Optimal.** Leaf size=374

$$\frac{im(a+b \cos^{-1}(cx)) \operatorname{PolyLog}\left(2, -\frac{g e^{i \cos^{-1}(cx)}}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{c} - \frac{im(a+b \cos^{-1}(cx)) \operatorname{PolyLog}\left(2, -\frac{g e^{i \cos^{-1}(cx)}}{\sqrt{c^2 f^2 - g^2} + cf}\right)}{c} + \frac{bm \operatorname{PolyLog}\left(3, -\frac{g e^{i \cos^{-1}(cx)}}{\sqrt{c^2 f^2 - g^2} + cf}\right)}{c}$$

[Out]  $((-I/6)*m*(a + b*\operatorname{ArcCos}[c*x])^3)/(b^2*c) + (m*(a + b*\operatorname{ArcCos}[c*x])^2*\operatorname{Log}[1 + (E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f - \operatorname{Sqrt}[c^2*f^2 - g^2])])/(2*b*c) + (m*(a + b*\operatorname{ArcCos}[c*x])^2*\operatorname{Log}[1 + (E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f + \operatorname{Sqrt}[c^2*f^2 - g^2])])/(2*b*c) - ((a + b*\operatorname{ArcCos}[c*x])^2*\operatorname{Log}[h*(f + g*x)^m])/(2*b*c) - (I*m*(a + b*\operatorname{ArcCos}[c*x])* \operatorname{PolyLog}[2, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f - \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c - (I*m*(a + b*\operatorname{ArcCos}[c*x])* \operatorname{PolyLog}[2, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f + \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c + (b*m*\operatorname{PolyLog}[3, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f - \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c + (b*m*\operatorname{PolyLog}[3, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f + \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c$

**Rubi [A]** time = 0.611272, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {4642, 4780, 4742, 4520, 2190, 2531, 2282, 6589}

$$\frac{im(a+b \cos^{-1}(cx)) \operatorname{PolyLog}\left(2, -\frac{g e^{i \cos^{-1}(cx)}}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{c} - \frac{im(a+b \cos^{-1}(cx)) \operatorname{PolyLog}\left(2, -\frac{g e^{i \cos^{-1}(cx)}}{\sqrt{c^2 f^2 - g^2} + cf}\right)}{c} + \frac{bm \operatorname{PolyLog}\left(3, -\frac{g e^{i \cos^{-1}(cx)}}{\sqrt{c^2 f^2 - g^2} + cf}\right)}{c}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCos}[c*x])* \operatorname{Log}[h*(f + g*x)^m]/\operatorname{Sqrt}[1 - c^2*x^2], x]$

[Out]  $((-I/6)*m*(a + b*\operatorname{ArcCos}[c*x])^3)/(b^2*c) + (m*(a + b*\operatorname{ArcCos}[c*x])^2*\operatorname{Log}[1 + (E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f - \operatorname{Sqrt}[c^2*f^2 - g^2])])/(2*b*c) + (m*(a + b*\operatorname{ArcCos}[c*x])^2*\operatorname{Log}[1 + (E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f + \operatorname{Sqrt}[c^2*f^2 - g^2])])/(2*b*c) - ((a + b*\operatorname{ArcCos}[c*x])^2*\operatorname{Log}[h*(f + g*x)^m])/(2*b*c) - (I*m*(a + b*\operatorname{ArcCos}[c*x])* \operatorname{PolyLog}[2, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f - \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c - (I*m*(a + b*\operatorname{ArcCos}[c*x])* \operatorname{PolyLog}[2, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f + \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c + (b*m*\operatorname{PolyLog}[3, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f - \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c + (b*m*\operatorname{PolyLog}[3, -((E^{(I*\operatorname{ArcCos}[c*x])*g})/(c*f + \operatorname{Sqrt}[c^2*f^2 - g^2]))])/c$

#### Rule 4642

$\operatorname{Int}[(a + b*\operatorname{ArcCos}[c*x])^n/\operatorname{Sqrt}[d + e*(x)^2], x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[(a + b*\operatorname{ArcCos}[c*x])^{n+1}/(b*c*\operatorname{Sqrt}[d]*(n+1)), x] /; \operatorname{FreeQ}\{a, b, c, d, e, n\}, x] \&\& \operatorname{EqQ}[c^2*d + e, 0] \&\& \operatorname{GtQ}[d, 0] \&\& \operatorname{NeQ}[n, -1]$

#### Rule 4780

$\operatorname{Int}[(\operatorname{Log}[h*(f + g*x)^m])*(a + b*\operatorname{ArcCos}[c*x])^n/\operatorname{Sqrt}[d + e*(x)^2], x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[(\operatorname{Log}[h*(f + g*x)^m]*(a + b*\operatorname{ArcCos}[c*x])^{n+1})/(b*c*\operatorname{Sqrt}[d]*(n+1)), x] + \operatorname{Dist}[(g*m)/(b*c*\operatorname{Sqrt}[d]*(n+1)), \operatorname{Int}[(a + b*\operatorname{ArcCos}[c*x])^{n+1}/(f + g*x), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, h, m\}, x] \&\& \operatorname{EqQ}[c^2*d + e, 0] \&\& \operatorname{GtQ}[d, 0] \&\& \operatorname{IGtQ}[n, 0]$

#### Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^n_]/((d_.) + (e_.)*(x_)), x_Symbol]
:> -Subst[Int[((a + b*x)^n*Sin[x])/(c*d + e*Cos[x]), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

#### Rule 4520

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x))], x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))], x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

#### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

#### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \cos^{-1}(cx)) \log(h(f + gx)^m)}{\sqrt{1 - c^2x^2}} dx &= -\frac{(a + b \cos^{-1}(cx))^2 \log(h(f + gx)^m)}{2bc} + \frac{(gm) \int \frac{(a+b \cos^{-1}(cx))^2}{f+gx} dx}{2bc} \\
&= -\frac{(a + b \cos^{-1}(cx))^2 \log(h(f + gx)^m)}{2bc} - \frac{(gm) \operatorname{Subst}\left(\int \frac{(a+bx)^2 \sin(x)}{cf+g \cos(x)} dx, x, c\right)}{2bc} \\
&= -\frac{im(a + b \cos^{-1}(cx))^3}{6b^2c} - \frac{(a + b \cos^{-1}(cx))^2 \log(h(f + gx)^m)}{2bc} + \frac{(igm) \operatorname{Subst}\left(\int \frac{(a+bx)^2 \sin(x)}{cf+g \cos(x)} dx, x, c\right)}{2bc} \\
&= -\frac{im(a + b \cos^{-1}(cx))^3}{6b^2c} + \frac{m(a + b \cos^{-1}(cx))^2 \log\left(1 + \frac{e^{i \cos^{-1}(cx)}g}{cf - \sqrt{c^2f^2 - g^2}}\right)}{2bc} + \frac{(igm) \operatorname{Subst}\left(\int \frac{(a+bx)^2 \sin(x)}{cf+g \cos(x)} dx, x, c\right)}{2bc} \\
&= -\frac{im(a + b \cos^{-1}(cx))^3}{6b^2c} + \frac{m(a + b \cos^{-1}(cx))^2 \log\left(1 + \frac{e^{i \cos^{-1}(cx)}g}{cf - \sqrt{c^2f^2 - g^2}}\right)}{2bc} + \frac{(igm) \operatorname{Subst}\left(\int \frac{(a+bx)^2 \sin(x)}{cf+g \cos(x)} dx, x, c\right)}{2bc} \\
&= -\frac{im(a + b \cos^{-1}(cx))^3}{6b^2c} + \frac{m(a + b \cos^{-1}(cx))^2 \log\left(1 + \frac{e^{i \cos^{-1}(cx)}g}{cf - \sqrt{c^2f^2 - g^2}}\right)}{2bc} + \frac{(igm) \operatorname{Subst}\left(\int \frac{(a+bx)^2 \sin(x)}{cf+g \cos(x)} dx, x, c\right)}{2bc} \\
&= -\frac{im(a + b \cos^{-1}(cx))^3}{6b^2c} + \frac{m(a + b \cos^{-1}(cx))^2 \log\left(1 + \frac{e^{i \cos^{-1}(cx)}g}{cf - \sqrt{c^2f^2 - g^2}}\right)}{2bc} + \frac{(igm) \operatorname{Subst}\left(\int \frac{(a+bx)^2 \sin(x)}{cf+g \cos(x)} dx, x, c\right)}{2bc} \\
&= -\frac{im(a + b \cos^{-1}(cx))^3}{6b^2c} + \frac{m(a + b \cos^{-1}(cx))^2 \log\left(1 + \frac{e^{i \cos^{-1}(cx)}g}{cf - \sqrt{c^2f^2 - g^2}}\right)}{2bc} + \frac{(igm) \operatorname{Subst}\left(\int \frac{(a+bx)^2 \sin(x)}{cf+g \cos(x)} dx, x, c\right)}{2bc}
\end{aligned}$$

**Mathematica [B]** time = 5.56597, size = 1248, normalized size = 3.34

$$-ibm \cos^{-1}(cx)^3 - 3iam \cos^{-1}(cx)^2 + 3bm \log\left(\frac{e^{i \cos^{-1}(cx)}g}{cf - \sqrt{c^2f^2 - g^2}} + 1\right) \cos^{-1}(cx)^2 + 3bm \log\left(\frac{e^{i \cos^{-1}(cx)}(cf - \sqrt{c^2f^2 - g^2})}{g} + 1\right) \cos^{-1}(cx)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b\*ArcCos[c\*x])\*Log[h\*(f + g\*x)^m])/Sqrt[1 - c^2\*x^2], x]

[Out] ((-3\*I)\*a\*m\*ArcCos[c\*x]^2 - I\*b\*m\*ArcCos[c\*x]^3 + (24\*I)\*a\*m\*ArcSin[Sqrt[1 + (c\*f)/g]/Sqrt[2]]\*ArcTan[((c\*f - g)\*Tan[ArcCos[c\*x]/2])/Sqrt[c^2\*f^2 - g^2]] + 3\*b\*m\*ArcCos[c\*x]^2\*Log[1 + (E^(I\*ArcCos[c\*x])\*g)/(c\*f - Sqrt[c^2\*f^2 - g^2])] + 6\*a\*m\*ArcCos[c\*x]\*Log[1 + (E^(I\*ArcCos[c\*x])\*(c\*f - Sqrt[c^2\*f^2 - g^2]))/g] + 3\*b\*m\*ArcCos[c\*x]^2\*Log[1 + (E^(I\*ArcCos[c\*x])\*(c\*f - Sqrt[c^2\*f^2 - g^2]))/g] + 12\*a\*m\*ArcSin[Sqrt[1 + (c\*f)/g]/Sqrt[2]]\*Log[1 + (E^(I\*ArcCos[c\*x])\*(c\*f - Sqrt[c^2\*f^2 - g^2]))/g] + 12\*b\*m\*ArcCos[c\*x]\*ArcSin[Sqrt[1 + (c\*f)/g]/Sqrt[2]]\*Log[1 + (E^(I\*ArcCos[c\*x])\*(c\*f - Sqrt[c^2\*f^2 - g^2]))/g] + 3\*b\*m\*ArcCos[c\*x]^2\*Log[1 + (E^(I\*ArcCos[c\*x])\*g)/(c\*f + Sqrt[c^2\*f^2 - g^2])] + 6\*a\*m\*ArcCos[c\*x]\*Log[1 + (E^(I\*ArcCos[c\*x])\*(c\*f + Sqrt[c^2\*f^2 - g^2]))/g] + 3\*b\*m\*ArcCos[c\*x]^2\*Log[1 + (E^(I\*ArcCos[c\*x])\*(c\*f + Sqrt[c^2\*f^2 - g^2]))/g] - 12\*a\*m\*ArcSin[Sqrt[1 + (c\*f)/g]/Sqrt[2]]\*Log[1 + (E^(I\*ArcCos[c\*x])\*(c\*f + Sqrt[c^2\*f^2 - g^2]))/g] - 12\*b\*m\*ArcCos[c\*x]\*ArcSin[Sqrt[1 + (c\*f)/g]/Sqrt[2]]\*Log[1 + ((c\*f - Sqrt[c^2\*f^2 - g^2])\*(c\*x + I\*Sqrt[1 - c^2\*x^2]))/g] - 6\*a\*m\*ArcCos[c\*x]\*Log[f + g\*x] - 6\*a\*m\*ArcSin[c\*x]\*Log[h\*(f + g\*x)^m] - 3\*b\*ArcCos[c\*x]^2\*Log[h\*(f + g\*x)^m] + 6\*a\*ArcSin[c\*x]\*Log[h\*(f + g\*x)^m] - 3\*b\*m\*ArcCos[c\*x]^2\*Log[1 + ((c\*f - Sqrt[c^2\*f^2 - g^2])\*(c\*x + I\*Sqrt[1 - c^2\*x^2]))/g] - 12\*b\*m\*ArcCos[c\*x]\*ArcSin[Sqrt[1 + (c\*f)/g]/Sqrt[2]]\*Log[1 + ((c\*f - Sqrt[c^2\*f^2 - g^2])\*(c\*x + I\*Sqrt[1 - c^2\*x^2]))/g] - 3\*b\*m\*ArcCos[c\*x]^2\*Log[1 + ((c\*f + Sqrt[c^2\*f^2 - g^2])\*(c\*x + I\*Sqrt[1 - c^2\*x^2]))/g] + 12\*b\*m\*ArcCos[c\*x]\*ArcSin[Sqrt[1 + (c\*f)/g]/Sqrt[2]]\*Log[1 + ((c\*f + Sqrt[c^2\*f^2 - g^2])\*(c\*x + I\*Sqrt[1 - c^2\*x^2]))/g]

$$1 + ((c*f + \text{Sqrt}[c^2*f^2 - g^2])*(c*x + I*\text{Sqrt}[1 - c^2*x^2]))/g] - (6*I)*b*m*\text{ArcCos}[c*x]*\text{PolyLog}[2, (E^{(I*\text{ArcCos}[c*x])*g})/(-(c*f) + \text{Sqrt}[c^2*f^2 - g^2])] - (6*I)*a*m*\text{PolyLog}[2, (E^{(I*\text{ArcCos}[c*x])*(-(c*f) + \text{Sqrt}[c^2*f^2 - g^2])})/g] - (6*I)*b*m*\text{ArcCos}[c*x]*\text{PolyLog}[2, -((E^{(I*\text{ArcCos}[c*x])*g})/(c*f + \text{Sqrt}[c^2*f^2 - g^2]))] - (6*I)*a*m*\text{PolyLog}[2, -((E^{(I*\text{ArcCos}[c*x])*(c*f + \text{Sqrt}[c^2*f^2 - g^2])})/g)] + 6*b*m*\text{PolyLog}[3, (E^{(I*\text{ArcCos}[c*x])*g})/(-(c*f) + \text{Sqrt}[c^2*f^2 - g^2])] + 6*b*m*\text{PolyLog}[3, -((E^{(I*\text{ArcCos}[c*x])*g})/(c*f + \text{Sqrt}[c^2*f^2 - g^2])))]/(6*c)$$

**Maple [F]** time = 3.134, size = 0, normalized size = 0.

$$\int (a + b \arccos(cx)) \ln(h(gx + f)^m) \frac{1}{\sqrt{-c^2x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos(c\*x))\*ln(h\*(g\*x+f)^m)/(-c^2\*x^2+1)^(1/2),x)

[Out] int((a+b\*arccos(c\*x))\*ln(h\*(g\*x+f)^m)/(-c^2\*x^2+1)^(1/2),x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(c\*x))\*log(h\*(g\*x+f)^m)/(-c^2\*x^2+1)^(1/2),x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{\sqrt{-c^2x^2 + 1}(b \arccos(cx) + a) \log((gx + f)^m h)}{c^2x^2 - 1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(c\*x))\*log(h\*(g\*x+f)^m)/(-c^2\*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c^2\*x^2 + 1)\*(b\*arccos(c\*x) + a)\*log((g\*x + f)^m\*h)/(c^2\*x^2 - 1), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \arccos(cx)) \log(h(f + gx)^m)}{\sqrt{-(cx - 1)(cx + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acos(c\*x))\*ln(h\*(g\*x+f)\*\*m)/(-c\*\*2\*x\*\*2+1)\*\*(1/2),x)

[Out] Integral((a + b\*acos(c\*x))\*log(h\*(f + g\*x)\*\*m)/sqrt(-(c\*x - 1)\*(c\*x + 1)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arccos(cx) + a) \log\left(\left(gx + f\right)^m h\right)}{\sqrt{-c^2x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(c\*x))\*log(h\*(g\*x+f)^m)/(-c^2\*x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccos(c\*x) + a)\*log((g\*x + f)^m\*h)/sqrt(-c^2\*x^2 + 1), x)

$$3.22 \quad \int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}} dx$$

**Optimal.** Leaf size=237

$$\frac{\operatorname{imPolyLog}\left(2, \frac{ig^i \sin^{-1}(cx)}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{c} + \frac{\operatorname{imPolyLog}\left(2, \frac{ig^i \sin^{-1}(cx)}{\sqrt{c^2 f^2 - g^2} + cf}\right)}{c} - \frac{m \sin^{-1}(cx) \log\left(1 - \frac{ig^i \sin^{-1}(cx)}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{c} - \frac{m \sin^{-1}(cx) \log\left(1 - \frac{ig^i \sin^{-1}(cx)}{\sqrt{c^2 f^2 - g^2} + cf}\right)}{c}$$

[Out] ((I/2)\*m\*ArcSin[c\*x]^2)/c - (m\*ArcSin[c\*x]\*Log[1 - (I\*E^(I\*ArcSin[c\*x]))\*g]/(c\*f - Sqrt[c^2\*f^2 - g^2]))/c - (m\*ArcSin[c\*x]\*Log[1 - (I\*E^(I\*ArcSin[c\*x]))\*g]/(c\*f + Sqrt[c^2\*f^2 - g^2]))/c + (ArcSin[c\*x]\*Log[h\*(f + g\*x)^m])/c + (I\*m\*PolyLog[2, (I\*E^(I\*ArcSin[c\*x]))\*g]/(c\*f - Sqrt[c^2\*f^2 - g^2]))/c + (I\*m\*PolyLog[2, (I\*E^(I\*ArcSin[c\*x]))\*g]/(c\*f + Sqrt[c^2\*f^2 - g^2]))/c

**Rubi [A]** time = 0.338099, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$ , Rules used = {216, 2404, 4741, 4519, 2190, 2279, 2391}

$$\frac{\operatorname{imPolyLog}\left(2, \frac{ig^i \sin^{-1}(cx)}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{c} + \frac{\operatorname{imPolyLog}\left(2, \frac{ig^i \sin^{-1}(cx)}{\sqrt{c^2 f^2 - g^2} + cf}\right)}{c} - \frac{m \sin^{-1}(cx) \log\left(1 - \frac{ig^i \sin^{-1}(cx)}{cf - \sqrt{c^2 f^2 - g^2}}\right)}{c} - \frac{m \sin^{-1}(cx) \log\left(1 - \frac{ig^i \sin^{-1}(cx)}{\sqrt{c^2 f^2 - g^2} + cf}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[Log[h\*(f + g\*x)^m]/Sqrt[1 - c^2\*x^2], x]

[Out] ((I/2)\*m\*ArcSin[c\*x]^2)/c - (m\*ArcSin[c\*x]\*Log[1 - (I\*E^(I\*ArcSin[c\*x]))\*g]/(c\*f - Sqrt[c^2\*f^2 - g^2]))/c - (m\*ArcSin[c\*x]\*Log[1 - (I\*E^(I\*ArcSin[c\*x]))\*g]/(c\*f + Sqrt[c^2\*f^2 - g^2]))/c + (ArcSin[c\*x]\*Log[h\*(f + g\*x)^m])/c + (I\*m\*PolyLog[2, (I\*E^(I\*ArcSin[c\*x]))\*g]/(c\*f - Sqrt[c^2\*f^2 - g^2]))/c + (I\*m\*PolyLog[2, (I\*E^(I\*ArcSin[c\*x]))\*g]/(c\*f + Sqrt[c^2\*f^2 - g^2]))/c

### Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

### Rule 2404

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/Sqrt[(f\_) + (g\_.)\*(x\_)^2], x\_Symbol] := With[{u = IntHide[1/Sqrt[f + g\*x^2], x]}, Simp[u\*(a + b\*Log[c\*(d + e\*x)^n]), x] - Dist[b\*e\*n, Int[SimplifyIntegrand[u/(d + e\*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]

### Rule 4741

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Subst[Int[((a + b\*x)^n \* Cos[x])/(c\*d + e \* Sin[x]), x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

### Rule 4519

Int[(Cos[(c\_.) + (d\_.)\*(x\_)])\*(e\_.) + (f\_.)\*(x\_))^(m\_.)/((a\_) + (b\_.)\*Sin[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Simp[(I\*(e + f\*x)^(m + 1))/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m \* E^(I\*(c + d\*x)))/(a - Rt[a^2 - b^2, 2] - I\*b \* E^(I\*(c + d\*x))), x] + Int[((e + f\*x)^m \* E^(I\*(c + d\*x)))/(a + Rt[a^2 - b^2, 2])



- I\*b\*E^(I\*(c + d\*x)), x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &  
& PosQ[a^2 - b^2]

### Rule 2190

Int[(((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/  
((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] := Simp  
[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]/(b\*f\*g\*n\*Log[F]), x] - Di  
st[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x  
)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)], x\_Symbol]  
:= Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x))  
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2  
, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned} \int \frac{\log(h(f + gx)^m)}{\sqrt{1 - c^2x^2}} dx &= \frac{\sin^{-1}(cx) \log(h(f + gx)^m)}{c} - (gm) \int \frac{\sin^{-1}(cx)}{cf + cgx} dx \\ &= \frac{\sin^{-1}(cx) \log(h(f + gx)^m)}{c} - (gm) \text{Subst} \left( \int \frac{x \cos(x)}{c^2f + cg \sin(x)} dx, x, \sin^{-1}(cx) \right) \\ &= \frac{im \sin^{-1}(cx)^2}{2c} + \frac{\sin^{-1}(cx) \log(h(f + gx)^m)}{c} - (gm) \text{Subst} \left( \int \frac{e^{ix} x}{c^2f - ice^{ix}g - c\sqrt{c^2f^2 - g^2}} dx \right) \\ &= \frac{im \sin^{-1}(cx)^2}{2c} - \frac{m \sin^{-1}(cx) \log \left( 1 - \frac{ie^{i \sin^{-1}(cx)}g}{cf - \sqrt{c^2f^2 - g^2}} \right)}{c} - \frac{m \sin^{-1}(cx) \log \left( 1 - \frac{ie^{i \sin^{-1}(cx)}g}{cf + \sqrt{c^2f^2 - g^2}} \right)}{c} + \sin^{-1}(cx) \log(h(f + gx)^m) \\ &= \frac{im \sin^{-1}(cx)^2}{2c} - \frac{m \sin^{-1}(cx) \log \left( 1 - \frac{ie^{i \sin^{-1}(cx)}g}{cf - \sqrt{c^2f^2 - g^2}} \right)}{c} - \frac{m \sin^{-1}(cx) \log \left( 1 - \frac{ie^{i \sin^{-1}(cx)}g}{cf + \sqrt{c^2f^2 - g^2}} \right)}{c} + \sin^{-1}(cx) \log(h(f + gx)^m) \\ &= \frac{im \sin^{-1}(cx)^2}{2c} - \frac{m \sin^{-1}(cx) \log \left( 1 - \frac{ie^{i \sin^{-1}(cx)}g}{cf - \sqrt{c^2f^2 - g^2}} \right)}{c} - \frac{m \sin^{-1}(cx) \log \left( 1 - \frac{ie^{i \sin^{-1}(cx)}g}{cf + \sqrt{c^2f^2 - g^2}} \right)}{c} + \sin^{-1}(cx) \log(h(f + gx)^m) \end{aligned}$$

**Mathematica [A]** time = 0.0218738, size = 246, normalized size = 1.04

$$\frac{im \text{PolyLog} \left( 2, \frac{ig e^{i \sin^{-1}(cx)}}{cf - \sqrt{c^2f^2 - g^2}} \right)}{c} + \frac{im \text{PolyLog} \left( 2, \frac{ig e^{i \sin^{-1}(cx)}}{\sqrt{c^2f^2 - g^2} + cf} \right)}{c} - \frac{m \sin^{-1}(cx) \log \left( 1 - \frac{icg e^{i \sin^{-1}(cx)}}{c^2f - c\sqrt{c^2f^2 - g^2}} \right)}{c} - \frac{m \sin^{-1}(cx) \log \left( 1 - \frac{icg e^{i \sin^{-1}(cx)}}{c^2f + c\sqrt{c^2f^2 - g^2}} \right)}{c} + \sin^{-1}(cx) \log(h(f + gx)^m)$$

Antiderivative was successfully verified.

[In] Integrate[Log[h\*(f + g\*x)^m]/Sqrt[1 - c^2\*x^2], x]

[Out] ((I/2)\*m\*ArcSin[c\*x]^2)/c - (m\*ArcSin[c\*x]\*Log[1 - (I\*c\*E^(I\*ArcSin[c\*x]))\*g  
) / (c^2\*f - c\*Sqrt[c^2\*f^2 - g^2]))/c - (m\*ArcSin[c\*x]\*Log[1 - (I\*c\*E^(I\*Ar

$c\sin[cx]g/(c^2f + c\sqrt{c^2f^2 - g^2})/c + (\arcsin[cx]\log[h(f + gx)^m])/c + (I^m \text{PolyLog}[2, (I^E(I\arcsin[cx])g)/(cf - \sqrt{c^2f^2 - g^2})])/c + (I^m \text{PolyLog}[2, (I^E(I\arcsin[cx])g)/(cf + \sqrt{c^2f^2 - g^2})])/c$

**Maple [F]** time = 0.015, size = 0, normalized size = 0.

$$\int \ln(h(gx + f)^m) \frac{1}{\sqrt{-c^2x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(h\*(g\*x+f)^m)/(-c^2\*x^2+1)^(1/2),x)

[Out] int(ln(h\*(g\*x+f)^m)/(-c^2\*x^2+1)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log((gx + f)^m h)}{\sqrt{-c^2x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(h\*(g\*x+f)^m)/(-c^2\*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(log((g\*x + f)^m\*h)/sqrt(-c^2\*x^2 + 1), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2 + 1} \log((gx + f)^m h)}{c^2x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(h\*(g\*x+f)^m)/(-c^2\*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c^2\*x^2 + 1)\*log((g\*x + f)^m\*h)/(c^2\*x^2 - 1), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(h(f + gx)^m)}{\sqrt{-(cx - 1)(cx + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(h\*(g\*x+f)\*\*m)/(-c\*\*2\*x\*\*2+1)\*\*(1/2),x)

[Out] Integral(log(h\*(f + g\*x)\*\*m)/sqrt(-(c\*x - 1)\*(c\*x + 1)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((gx + f)^m h\right)}{\sqrt{-c^2x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(h\*(g\*x+f)^m)/(-c^2\*x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(log((g\*x + f)^m\*h)/sqrt(-c^2\*x^2 + 1), x)

$$3.23 \quad \int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}(a+b\cos^{-1}(cx))} dx$$

**Optimal.** Leaf size=37

$$\text{Unintegrable} \left( \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}(a+b\cos^{-1}(cx))}, x \right)$$

[Out] Unintegrable[Log[h\*(f + g\*x)^m]/(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcCos[c\*x])), x]

**Rubi [A]** time = 0.200123, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}(a+b\cos^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Int[Log[h\*(f + g\*x)^m]/(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcCos[c\*x])), x]

[Out] Defer[Int][Log[h\*(f + g\*x)^m]/(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcCos[c\*x])), x]

Rubi steps

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}(a+b\cos^{-1}(cx))} dx = \int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}(a+b\cos^{-1}(cx))} dx$$

**Mathematica [A]** time = 0.199132, size = 0, normalized size = 0.

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}(a+b\cos^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Log[h\*(f + g\*x)^m]/(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcCos[c\*x])), x]

[Out] Integrate[Log[h\*(f + g\*x)^m]/(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcCos[c\*x])), x]

**Maple [A]** time = 2.447, size = 0, normalized size = 0.

$$\int \frac{\ln(h(gx+f)^m)}{a+b\arccos(cx)} \frac{1}{\sqrt{-c^2x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(h\*(g\*x+f)^m)/(a+b\*arccos(c\*x))/(-c^2\*x^2+1)^(1/2), x)

[Out] `int(ln(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((gx+f)^m h\right)}{\sqrt{-c^2x^2+1}(b \arccos(cx)+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")`

[Out] `integrate(log((g*x + f)^m*h)/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1} \log\left((gx+f)^m h\right)}{ac^2x^2 + (bc^2x^2 - b) \arccos(cx) - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")`

[Out] `integral(-sqrt(-c^2*x^2 + 1)*log((g*x + f)^m*h)/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccos(c*x) - a), x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{(cx-1)(cx+1)}(a+b \arccos(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(h*(g*x+f)**m)/(a+b*acos(c*x))/(-c**2*x**2+1)**(1/2),x)`

[Out] `Integral(log(h*(f + g*x)**m)/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))), x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left((gx+f)^m h\right)}{\sqrt{-c^2x^2+1}(b \arccos(cx)+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm  
m="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)), x)
```

### 3.24 $\int x^3 \cos^{-1}(a + bx) dx$

**Optimal.** Leaf size=137

$$\frac{(4a(19a^2 + 16) - (26a^2 + 9)(a + bx))\sqrt{1 - (a + bx)^2}}{96b^4} + \frac{(8a^4 + 24a^2 + 3)\sin^{-1}(a + bx)}{32b^4} + \frac{7ax^2\sqrt{1 - (a + bx)^2}}{48b^2} - \frac{x^3\sqrt{1 - (a + bx)^2}}{16b}$$

[Out] (7\*a\*x^2\*Sqrt[1 - (a + b\*x)^2])/(48\*b^2) - (x^3\*Sqrt[1 - (a + b\*x)^2])/(16\*b) + ((4\*a\*(16 + 19\*a^2) - (9 + 26\*a^2)\*(a + b\*x))\*Sqrt[1 - (a + b\*x)^2])/(96\*b^4) + (x^4\*ArcCos[a + b\*x])/4 + ((3 + 24\*a^2 + 8\*a^4)\*ArcSin[a + b\*x])/(32\*b^4)

**Rubi [A]** time = 0.194006, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {4806, 4744, 743, 833, 780, 216}

$$\frac{(4a(19a^2 + 16) - (26a^2 + 9)(a + bx))\sqrt{1 - (a + bx)^2}}{96b^4} + \frac{(8a^4 + 24a^2 + 3)\sin^{-1}(a + bx)}{32b^4} + \frac{7ax^2\sqrt{1 - (a + bx)^2}}{48b^2} - \frac{x^3\sqrt{1 - (a + bx)^2}}{16b}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcCos[a + b\*x],x]

[Out] (7\*a\*x^2\*Sqrt[1 - (a + b\*x)^2])/(48\*b^2) - (x^3\*Sqrt[1 - (a + b\*x)^2])/(16\*b) + ((4\*a\*(16 + 19\*a^2) - (9 + 26\*a^2)\*(a + b\*x))\*Sqrt[1 - (a + b\*x)^2])/(96\*b^4) + (x^4\*ArcCos[a + b\*x])/4 + ((3 + 24\*a^2 + 8\*a^4)\*ArcSin[a + b\*x])/(32\*b^4)

#### Rule 4806

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]

#### Rule 4744

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^n)/(e\*(m + 1)), x] + Dist[(b\*c\*n)/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 743

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(e\*(d + e\*x)^(m - 1)\*(a + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 1)), x] + Dist[1/(c\*(m + 2\*p + 1)), Int[(d + e\*x)^(m - 2)\*Simp[c\*d^2\*(m + 2\*p + 1) - a\*e^2\*(m - 1) + 2\*c\*d\*e\*(m + p)\*x, x]\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2\*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

#### Rule 833

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(g\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 2)), x] + Dist[1/(c\*(m + 2\*p + 2)), Int[(d + e\*x)^(m - 1)\*(a + c\*x^2)^p\*Simp[c\*d\*f\*(m + 2\*p + 2) - a\*e\*g\*m + c\*(e\*f\*(m + 2\*p + 2) + d\*g\*m)\*x, x], x], x]

```

/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])

```

### Rule 780

```

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]

```

### Rule 216

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

```

### Rubi steps

$$\begin{aligned}
\int x^3 \cos^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right)^3 \cos^{-1}(x) dx, x, a + bx\right)}{b} \\
&= \frac{1}{4} x^4 \cos^{-1}(a + bx) + \frac{1}{4} \text{Subst}\left(\int \frac{\left(-\frac{a}{b} + \frac{x}{b}\right)^4}{\sqrt{1-x^2}} dx, x, a + bx\right) \\
&= -\frac{x^3 \sqrt{1-(a+bx)^2}}{16b} + \frac{1}{4} x^4 \cos^{-1}(a + bx) - \frac{1}{16} \text{Subst}\left(\int \frac{\left(-\frac{3+4a^2}{b^2} + \frac{7ax}{b^2}\right) \left(-\frac{a}{b} + \frac{x}{b}\right)^2}{\sqrt{1-x^2}} dx, x, a + bx\right) \\
&= \frac{7ax^2 \sqrt{1-(a+bx)^2}}{48b^2} - \frac{x^3 \sqrt{1-(a+bx)^2}}{16b} + \frac{1}{4} x^4 \cos^{-1}(a + bx) + \frac{1}{48} \text{Subst}\left(\int \frac{\left(-\frac{a(23+12a^2)}{b^3} + \frac{(9+12a^2)x}{b^3}\right)}{\sqrt{1-x^2}} dx, x, a + bx\right) \\
&= \frac{7ax^2 \sqrt{1-(a+bx)^2}}{48b^2} - \frac{x^3 \sqrt{1-(a+bx)^2}}{16b} + \frac{(4a(16+19a^2) - (9+26a^2)(a+bx) \sqrt{1-(a+bx)^2})}{96b^4} \\
&= \frac{7ax^2 \sqrt{1-(a+bx)^2}}{48b^2} - \frac{x^3 \sqrt{1-(a+bx)^2}}{16b} + \frac{(4a(16+19a^2) - (9+26a^2)(a+bx) \sqrt{1-(a+bx)^2})}{96b^4}
\end{aligned}$$

**Mathematica [A]** time = 0.100011, size = 104, normalized size = 0.76

$$\frac{\sqrt{-a^2 - 2abx - b^2x^2 + 1}(-26a^2bx + 50a^3 + 14ab^2x^2 + 55a - 6b^3x^3 - 9bx) + 3(8a^4 + 24a^2 + 3) \sin^{-1}(a + bx) + 24b^4x^4}{96b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCos[a + b*x], x]
```

```
[Out] (Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*(55*a + 50*a^3 - 9*b*x - 26*a^2*b*x + 14
*a*b^2*x^2 - 6*b^3*x^3) + 24*b^4*x^4*ArcCos[a + b*x] + 3*(3 + 24*a^2 + 8*a^
4)*ArcSin[a + b*x])/(96*b^4)
```

**Maple [A]** time = 0.02, size = 235, normalized size = 1.7

$$\frac{1}{b^4} \left( \frac{\arccos(bx + a)(bx + a)^4}{4} - \arccos(bx + a)(bx + a)^3 a + \frac{3 \arccos(bx + a)(bx + a)^2 a^2}{2} - \arccos(bx + a)(bx + a) a^3 \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*arccos(b*x+a),x)`

[Out]  $\frac{1}{b^4} \left( \frac{1}{4} \arccos(bx+a) (bx+a)^4 - \arccos(bx+a) (bx+a)^3 + \frac{3}{2} \arccos(bx+a) (bx+a)^2 a^2 - \arccos(bx+a) (bx+a) a^3 + \frac{1}{4} \arccos(bx+a) a^4 - \frac{1}{16} (bx+a)^3 (1-(bx+a)^2)^{1/2} - \frac{3}{32} (bx+a) (1-(bx+a)^2)^{1/2} + \frac{3}{32} \arcsin(bx+a) - a \left( -\frac{1}{3} (bx+a)^2 (1-(bx+a)^2)^{1/2} - \frac{2}{3} (1-(bx+a)^2)^{1/2} \right) + \frac{3}{2} a^2 \left( -\frac{1}{2} (bx+a) (1-(bx+a)^2)^{1/2} + \frac{1}{2} \arcsin(bx+a) \right) + a^3 (1-(bx+a)^2)^{1/2} + \frac{1}{4} \arcsin(bx+a) a^4 \right)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccos(b*x+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.47944, size = 219, normalized size = 1.6

$$\frac{3(8b^4x^4 - 8a^4 - 24a^2 - 3)\arccos(bx+a) - (6b^3x^3 - 14ab^2x^2 - 50a^3 + (26a^2 + 9)bx - 55a)\sqrt{-b^2x^2 - 2abx - a^2}}{96b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccos(b*x+a),x, algorithm="fricas")`

[Out]  $\frac{1}{96} \left( 3(8b^4x^4 - 8a^4 - 24a^2 - 3)\arccos(bx+a) - (6b^3x^3 - 14ab^2x^2 - 50a^3 + (26a^2 + 9)bx - 55a)\sqrt{-b^2x^2 - 2abx - a^2} + 1 \right) / b^4$

**Sympy [A]** time = 1.75661, size = 255, normalized size = 1.86

$$\left\{ \begin{array}{l} -\frac{a^4 \operatorname{acos}(a+bx)}{4b^4} + \frac{25a^3 \sqrt{-a^2-2abx-b^2x^2+1}}{48b^4} - \frac{13a^2x \sqrt{-a^2-2abx-b^2x^2+1}}{48b^3} - \frac{3a^2 \operatorname{acos}(a+bx)}{4b^4} + \frac{7ax^2 \sqrt{-a^2-2abx-b^2x^2+1}}{48b^2} + \frac{55a \sqrt{-a^2-2abx-b^2x^2+1}}{96b^4} \\ \frac{x^4 \operatorname{acos}(a)}{4} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*acos(b*x+a),x)`

[Out] `Piecewise((-a**4*acos(a + b*x)/(4*b**4) + 25*a**3*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(48*b**4) - 13*a**2*x*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(48*b**3) - 3*a**2*acos(a + b*x)/(4*b**4) + 7*a*x**2*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(48*b**2) + 55*a*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(96*b**4) + x**4*acos(a + b*x)/4 - x**3*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(16*b) - 3*x*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(32*b**3) - 3*acos(a + b*x)/(32*b**4), Ne(b, 0)), (x**4*acos(a)/4, True))`

---

**Giac [B]** time = 1.31574, size = 327, normalized size = 2.39

$$\frac{(bx+a)^4 \arccos(bx+a)}{4b^4} - \frac{(bx+a)^3 a \arccos(bx+a)}{b^4} + \frac{3(bx+a)^2 a^2 \arccos(bx+a)}{2b^4} - \frac{(bx+a)a^3 \arccos(bx+a)}{b^4} - \sqrt{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccos(b\*x+a),x, algorithm="giac")

[Out] 1/4\*(b\*x + a)^4\*arccos(b\*x + a)/b^4 - (b\*x + a)^3\*a\*arccos(b\*x + a)/b^4 + 3/2\*(b\*x + a)^2\*a^2\*arccos(b\*x + a)/b^4 - (b\*x + a)\*a^3\*arccos(b\*x + a)/b^4 - 1/16\*sqrt(-(b\*x + a)^2 + 1)\*(b\*x + a)^3/b^4 + 1/3\*sqrt(-(b\*x + a)^2 + 1)\*(b\*x + a)^2\*a/b^4 - 3/4\*sqrt(-(b\*x + a)^2 + 1)\*(b\*x + a)\*a^2/b^4 + sqrt(-(b\*x + a)^2 + 1)\*a^3/b^4 - 3/4\*a^2\*arccos(b\*x + a)/b^4 - 3/32\*sqrt(-(b\*x + a)^2 + 1)\*(b\*x + a)/b^4 + 2/3\*sqrt(-(b\*x + a)^2 + 1)\*a/b^4 - 3/32\*arccos(b\*x + a)/b^4

### 3.25 $\int x^2 \cos^{-1}(a + bx) dx$

**Optimal.** Leaf size=94

$$\frac{(11a^2 - 5abx + 4)\sqrt{1 - (a + bx)^2}}{18b^3} - \frac{a(2a^2 + 3)\sin^{-1}(a + bx)}{6b^3} - \frac{x^2\sqrt{1 - (a + bx)^2}}{9b} + \frac{1}{3}x^3 \cos^{-1}(a + bx)$$

[Out]  $-(x^2\sqrt{1 - (a + b*x)^2})/(9*b) - ((4 + 11*a^2 - 5*a*b*x)*\sqrt{1 - (a + b*x)^2})/(18*b^3) + (x^3*\text{ArcCos}[a + b*x])/3 - (a*(3 + 2*a^2)*\text{ArcSin}[a + b*x])/ (6*b^3)$

**Rubi [A]** time = 0.114887, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4806, 4744, 743, 780, 216}

$$\frac{(11a^2 - 5abx + 4)\sqrt{1 - (a + bx)^2}}{18b^3} - \frac{a(2a^2 + 3)\sin^{-1}(a + bx)}{6b^3} - \frac{x^2\sqrt{1 - (a + bx)^2}}{9b} + \frac{1}{3}x^3 \cos^{-1}(a + bx)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*\text{ArcCos}[a + b*x], x]$

[Out]  $-(x^2*\sqrt{1 - (a + b*x)^2})/(9*b) - ((4 + 11*a^2 - 5*a*b*x)*\sqrt{1 - (a + b*x)^2})/(18*b^3) + (x^3*\text{ArcCos}[a + b*x])/3 - (a*(3 + 2*a^2)*\text{ArcSin}[a + b*x])/ (6*b^3)$

#### Rule 4806

$\text{Int}[(a + \text{ArcCos}[c + (d)*(x)]*(b))^n * ((e) + (f)*(x))^m, x\_Symbol] := \text{Dist}[1/d, \text{Subst}[\text{Int}[(d*e - c*f)/d + (f*x)/d]^m * (a + b*\text{ArcCos}[x])^n, x], x, c + d*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x]$

#### Rule 4744

$\text{Int}[(a + \text{ArcCos}[c*(x)]*(b))^n * ((d) + (e)*(x))^m, x\_Symbol] := \text{Simp}[(d + e*x)^(m + 1) * (a + b*\text{ArcCos}[c*x])^n / (e*(m + 1)), x] + \text{Dist}[(b*c*n) / (e*(m + 1)), \text{Int}[(d + e*x)^(m + 1) * (a + b*\text{ArcCos}[c*x])^(n - 1)] / \text{Sqrt}[1 - c^2*x^2], x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$

#### Rule 743

$\text{Int}[(d + (e)*(x))^m * (a + (c)*(x)^2)^p, x\_Symbol] := \text{Simp}[(e*(d + e*x)^(m - 1) * (a + c*x^2)^(p + 1)) / (c*(m + 2*p + 1)), x] + \text{Dist}[1 / (c*(m + 2*p + 1)), \text{Int}[(d + e*x)^(m - 2) * \text{Simp}[c*d^2*(m + 2*p + 1) - a*e^2*(m - 1) + 2*c*d*e*(m + p)*x, x] * (a + c*x^2)^p, x], x] /; \text{FreeQ}\{a, c, d, e, m, p\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{If}[\text{RationalQ}[m], \text{GtQ}[m, 1], \text{SumSimplerQ}[m, -2]] \&\& \text{NeQ}[m + 2*p + 1, 0] \&\& \text{IntQuadraticQ}[a, 0, c, d, e, m, p, x]$

#### Rule 780

$\text{Int}[(d + (e)*(x)) * ((f) + (g)*(x)) * (a + (c)*(x)^2)^p, x\_Symbol] := \text{Simp}[(e*f + d*g) * (2*p + 3) + 2*e*g*(p + 1)*x * (a + c*x^2)^(p + 1) / (2*c*(p + 1) * (2*p + 3)), x] - \text{Dist}[(a*e*g - c*d*f * (2*p + 3)) / (c*(2*p + 3)), \text{Int}[(a + c*x^2)^p, x], x] /; \text{FreeQ}\{a, c, d, e, f, g, p\}, x] \&\& !\text{LeQ}[p, -1]$

**Rule 216**

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

**Rubi steps**

$$\begin{aligned} \int x^2 \cos^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right)^2 \cos^{-1}(x) dx, x, a + bx\right)}{b} \\ &= \frac{1}{3} x^3 \cos^{-1}(a + bx) + \frac{1}{3} \text{Subst}\left(\int \frac{\left(-\frac{a}{b} + \frac{x}{b}\right)^3}{\sqrt{1-x^2}} dx, x, a + bx\right) \\ &= -\frac{x^2 \sqrt{1-(a+bx)^2}}{9b} + \frac{1}{3} x^3 \cos^{-1}(a + bx) - \frac{1}{9} \text{Subst}\left(\int \frac{\left(-\frac{2+3a^2}{b^2} + \frac{5ax}{b^2}\right) \left(-\frac{a}{b} + \frac{x}{b}\right)}{\sqrt{1-x^2}} dx, x, a + bx\right) \\ &= -\frac{x^2 \sqrt{1-(a+bx)^2}}{9b} - \frac{(4 + 11a^2 - 5abx) \sqrt{1-(a+bx)^2}}{18b^3} + \frac{1}{3} x^3 \cos^{-1}(a + bx) - \frac{(a(3 + 2a^2)) \sin^{-1}\left(\frac{a+bx}{b}\right)}{6b^3} \\ &= -\frac{x^2 \sqrt{1-(a+bx)^2}}{9b} - \frac{(4 + 11a^2 - 5abx) \sqrt{1-(a+bx)^2}}{18b^3} + \frac{1}{3} x^3 \cos^{-1}(a + bx) - \frac{a(3 + 2a^2) \sin^{-1}\left(\frac{a+bx}{b}\right)}{6b^3} \end{aligned}$$

**Mathematica [A]** time = 0.0828341, size = 83, normalized size = 0.88

$$\frac{\sqrt{-a^2 - 2abx - b^2x^2 + 1} (11a^2 - 5abx + 2b^2x^2 + 4) + 3a(2a^2 + 3) \sin^{-1}(a + bx) - 6b^3x^3 \cos^{-1}(a + bx)}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[a + b*x], x]
```

```
[Out] -(Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*(4 + 11*a^2 - 5*a*b*x + 2*b^2*x^2) - 6*b^3*x^3*ArcCos[a + b*x] + 3*a*(3 + 2*a^2)*ArcSin[a + b*x])/(18*b^3)
```

**Maple [A]** time = 0.003, size = 161, normalized size = 1.7

$$\frac{1}{b^3} \left( \frac{\arccos(bx + a)(bx + a)^3}{3} - \arccos(bx + a)(bx + a)^2 a + \arccos(bx + a)(bx + a) a^2 - \frac{\arccos(bx + a) a^3}{3} - \frac{(bx + a)^2}{9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(b*x+a), x)
```

```
[Out] 1/b^3*(1/3*arccos(b*x+a)*(b*x+a)^3-arccos(b*x+a)*(b*x+a)^2*a+arccos(b*x+a)*(b*x+a)*a^2-1/3*arccos(b*x+a)*a^3-1/9*(b*x+a)^2*(1-(b*x+a)^2)^(1/2)-2/9*(1-(b*x+a)^2)^(1/2)-a*(-1/2*(b*x+a)*(1-(b*x+a)^2)^(1/2)+1/2*arcsin(b*x+a))-a^2*(1-(b*x+a)^2)^(1/2)-1/3*arcsin(b*x+a)*a^3)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccos(b\*x+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.42065, size = 173, normalized size = 1.84

$$\frac{3(2b^3x^3 + 2a^3 + 3a)\arccos(bx + a) - (2b^2x^2 - 5abx + 11a^2 + 4)\sqrt{-b^2x^2 - 2abx - a^2 + 1}}{18b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccos(b\*x+a),x, algorithm="fricas")

[Out] 1/18\*(3\*(2\*b^3\*x^3 + 2\*a^3 + 3\*a)\*arccos(b\*x + a) - (2\*b^2\*x^2 - 5\*a\*b\*x + 11\*a^2 + 4)\*sqrt(-b^2\*x^2 - 2\*a\*b\*x - a^2 + 1))/b^3

**Sympy [A]** time = 0.819965, size = 170, normalized size = 1.81

$$\left\{ \begin{array}{l} \frac{a^3 \arccos(a+bx)}{3b^3} - \frac{11a^2\sqrt{-a^2-2abx-b^2x^2+1}}{18b^3} + \frac{5ax\sqrt{-a^2-2abx-b^2x^2+1}}{18b^2} + \frac{a \arccos(a+bx)}{2b^3} + \frac{x^3 \arccos(a+bx)}{3} - \frac{x^2\sqrt{-a^2-2abx-b^2x^2+1}}{9b} - \frac{2\sqrt{-a^2-2abx-b^2x^2+1}}{9b^3} \\ \frac{x^3 \arccos(a)}{3} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acos(b\*x+a),x)

[Out] Piecewise((a\*\*3\*acos(a + b\*x)/(3\*b\*\*3) - 11\*a\*\*2\*sqrt(-a\*\*2 - 2\*a\*b\*x - b\*\*2\*x\*\*2 + 1)/(18\*b\*\*3) + 5\*a\*x\*sqrt(-a\*\*2 - 2\*a\*b\*x - b\*\*2\*x\*\*2 + 1)/(18\*b\*\*2) + a\*acos(a + b\*x)/(2\*b\*\*3) + x\*\*3\*acos(a + b\*x)/3 - x\*\*2\*sqrt(-a\*\*2 - 2\*a\*b\*x - b\*\*2\*x\*\*2 + 1)/(9\*b) - 2\*sqrt(-a\*\*2 - 2\*a\*b\*x - b\*\*2\*x\*\*2 + 1)/(9\*b\*\*3), Ne(b, 0)), (x\*\*3\*acos(a)/3, True))

**Giac [A]** time = 1.30621, size = 211, normalized size = 2.24

$$\frac{(bx + a)^3 \arccos(bx + a)}{3b^3} - \frac{(bx + a)^2 a \arccos(bx + a)}{b^3} + \frac{(bx + a)a^2 \arccos(bx + a)}{b^3} - \frac{\sqrt{-(bx + a)^2 + 1}(bx + a)^2}{9b^3} + \sqrt{-(bx + a)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccos(b\*x+a),x, algorithm="giac")

[Out] 1/3\*(b\*x + a)^3\*arccos(b\*x + a)/b^3 - (b\*x + a)^2\*a\*arccos(b\*x + a)/b^3 + (b\*x + a)\*a^2\*arccos(b\*x + a)/b^3 - 1/9\*sqrt(-(b\*x + a)^2 + 1)\*(b\*x + a)^2/b^3 + 1/2\*sqrt(-(b\*x + a)^2 + 1)\*(b\*x + a)\*a/b^3 - sqrt(-(b\*x + a)^2 + 1)\*a^2/b^3 + 1/2\*a\*arccos(b\*x + a)/b^3 - 2/9\*sqrt(-(b\*x + a)^2 + 1)/b^3

### 3.26 $\int x \cos^{-1}(a + bx) dx$

**Optimal.** Leaf size=80

$$\frac{(2a^2 + 1) \sin^{-1}(a + bx)}{4b^2} + \frac{3a\sqrt{1 - (a + bx)^2}}{4b^2} + \frac{1}{2}x^2 \cos^{-1}(a + bx) - \frac{x\sqrt{1 - (a + bx)^2}}{4b}$$

[Out] (3\*a\*Sqrt[1 - (a + b\*x)^2])/(4\*b^2) - (x\*Sqrt[1 - (a + b\*x)^2])/(4\*b) + (x^2\*ArcCos[a + b\*x])/2 + ((1 + 2\*a^2)\*ArcSin[a + b\*x])/(4\*b^2)

**Rubi [A]** time = 0.0731415, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {4806, 4744, 743, 641, 216}

$$\frac{(2a^2 + 1) \sin^{-1}(a + bx)}{4b^2} + \frac{3a\sqrt{1 - (a + bx)^2}}{4b^2} + \frac{1}{2}x^2 \cos^{-1}(a + bx) - \frac{x\sqrt{1 - (a + bx)^2}}{4b}$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCos[a + b\*x], x]

[Out] (3\*a\*Sqrt[1 - (a + b\*x)^2])/(4\*b^2) - (x\*Sqrt[1 - (a + b\*x)^2])/(4\*b) + (x^2\*ArcCos[a + b\*x])/2 + ((1 + 2\*a^2)\*ArcSin[a + b\*x])/(4\*b^2)

#### Rule 4806

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^n\_]\*((e\_.) + (f\_.)\*(x\_.))^m\_., x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]

#### Rule 4744

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^n\_]\*((d\_.) + (e\_.)\*(x\_.))^m\_., x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^n)/(e\*(m + 1)), x] + Dist[(b\*c^n)/(e\*(m + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^(n - 1)]/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 743

Int[((d\_.) + (e\_.)\*(x\_.))^m\_]\*((a\_.) + (c\_.)\*(x\_.)^2)^p\_., x\_Symbol] := Simp[(e\*(d + e\*x)^(m - 1)\*(a + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 1)), x] + Dist[1/(c\*(m + 2\*p + 1)), Int[(d + e\*x)^(m - 2)\*Simp[c\*d^2\*(m + 2\*p + 1) - a\*e^2\*(m - 1) + 2\*c\*d\*e\*(m + p)\*x, x]\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2\*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

#### Rule 641

Int[((d\_.) + (e\_.)\*(x\_.))\*((a\_.) + (c\_.)\*(x\_.)^2)^p\_., x\_Symbol] := Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

#### Rule 216

Int[1/Sqrt[(a\_.) + (b\_.)\*(x\_.)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned}
\int x \cos^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right) \cos^{-1}(x) dx, x, a + bx\right)}{b} \\
&= \frac{1}{2} x^2 \cos^{-1}(a + bx) + \frac{1}{2} \text{Subst}\left(\int \frac{\left(-\frac{a}{b} + \frac{x}{b}\right)^2}{\sqrt{1-x^2}} dx, x, a + bx\right) \\
&= -\frac{x\sqrt{1-(a+bx)^2}}{4b} + \frac{1}{2} x^2 \cos^{-1}(a + bx) - \frac{1}{4} \text{Subst}\left(\int \frac{-\frac{1+2a^2}{b^2} + \frac{3ax}{b^2}}{\sqrt{1-x^2}} dx, x, a + bx\right) \\
&= \frac{3a\sqrt{1-(a+bx)^2}}{4b^2} - \frac{x\sqrt{1-(a+bx)^2}}{4b} + \frac{1}{2} x^2 \cos^{-1}(a + bx) + \frac{(1+2a^2) \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, a + bx\right)}{4b^2} \\
&= \frac{3a\sqrt{1-(a+bx)^2}}{4b^2} - \frac{x\sqrt{1-(a+bx)^2}}{4b} + \frac{1}{2} x^2 \cos^{-1}(a + bx) + \frac{(1+2a^2) \sin^{-1}(a + bx)}{4b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0494883, size = 69, normalized size = 0.86

$$\frac{(3a - bx)\sqrt{-a^2 - 2abx - b^2x^2 + 1} + (2a^2 + 1)\sin^{-1}(a + bx) + 2b^2x^2 \cos^{-1}(a + bx)}{4b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCos[a + b\*x], x]

[Out] ((3\*a - b\*x)\*Sqrt[1 - a^2 - 2\*a\*b\*x - b^2\*x^2] + 2\*b^2\*x^2\*ArcCos[a + b\*x] + (1 + 2\*a^2)\*ArcSin[a + b\*x])/(4\*b^2)

**Maple [A]** time = 0.004, size = 78, normalized size = 1.

$$\frac{1}{b^2} \left( \frac{\arccos(bx + a)(bx + a)^2}{2} - \arccos(bx + a)a(bx + a) - \frac{bx + a}{4} \sqrt{1 - (bx + a)^2} + \frac{\arcsin(bx + a)}{4} + a\sqrt{1 - (bx + a)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccos(b\*x+a), x)

[Out] 1/b^2\*(1/2\*arccos(b\*x+a)\*(b\*x+a)^2-arccos(b\*x+a)\*a\*(b\*x+a)-1/4\*(b\*x+a)\*(1-(b\*x+a)^2)^(1/2)+1/4\*arcsin(b\*x+a)+a\*(1-(b\*x+a)^2)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccos(b\*x+a), x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 2.55125, size = 135, normalized size = 1.69

$$\frac{(2b^2x^2 - 2a^2 - 1)\arccos(bx + a) - \sqrt{-b^2x^2 - 2abx - a^2 + 1}(bx - 3a)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccos(b\*x+a),x, algorithm="fricas")

[Out] 1/4\*((2\*b^2\*x^2 - 2\*a^2 - 1)\*arccos(b\*x + a) - sqrt(-b^2\*x^2 - 2\*a\*b\*x - a^2 + 1)\*(b\*x - 3\*a))/b^2

---

**Sympy [A]** time = 0.361235, size = 104, normalized size = 1.3

$$\begin{cases} -\frac{a^2 \arccos(a+bx)}{2b^2} + \frac{3a\sqrt{-a^2-2abx-b^2x^2+1}}{4b^2} + \frac{x^2 \arccos(a+bx)}{2} - \frac{x\sqrt{-a^2-2abx-b^2x^2+1}}{4b} - \frac{\arccos(a+bx)}{4b^2} & \text{for } b \neq 0 \\ \frac{x^2 \arccos(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acos(b\*x+a),x)

[Out] Piecewise((-a\*\*2\*acos(a + b\*x)/(2\*b\*\*2) + 3\*a\*sqrt(-a\*\*2 - 2\*a\*b\*x - b\*\*2\*x\*\*2 + 1)/(4\*b\*\*2) + x\*\*2\*acos(a + b\*x)/2 - x\*sqrt(-a\*\*2 - 2\*a\*b\*x - b\*\*2\*x\*\*2 + 1)/(4\*b) - acos(a + b\*x)/(4\*b\*\*2), Ne(b, 0)), (x\*\*2\*acos(a)/2, True))

---

**Giac [A]** time = 1.32168, size = 119, normalized size = 1.49

$$\frac{(bx + a)^2 \arccos(bx + a)}{2b^2} - \frac{(bx + a)a \arccos(bx + a)}{b^2} - \frac{\sqrt{-(bx + a)^2 + 1}(bx + a)}{4b^2} + \frac{\sqrt{-(bx + a)^2 + 1}a}{b^2} - \frac{\arccos(bx + a)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccos(b\*x+a),x, algorithm="giac")

[Out] 1/2\*(b\*x + a)^2\*arccos(b\*x + a)/b^2 - (b\*x + a)\*a\*arccos(b\*x + a)/b^2 - 1/4\*sqrt(-(b\*x + a)^2 + 1)\*(b\*x + a)/b^2 + sqrt(-(b\*x + a)^2 + 1)\*a/b^2 - 1/4\*arccos(b\*x + a)/b^2



### 3.27 $\int \cos^{-1}(a + bx) dx$

**Optimal.** Leaf size=36

$$\frac{(a + bx) \cos^{-1}(a + bx)}{b} - \frac{\sqrt{1 - (a + bx)^2}}{b}$$

[Out]  $-(\text{Sqrt}[1 - (a + b*x)^2]/b) + ((a + b*x)*\text{ArcCos}[a + b*x])/b$

**Rubi [A]** time = 0.0168144, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4804, 4620, 261}

$$\frac{(a + bx) \cos^{-1}(a + bx)}{b} - \frac{\sqrt{1 - (a + bx)^2}}{b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCos}[a + b*x], x]$

[Out]  $-(\text{Sqrt}[1 - (a + b*x)^2]/b) + ((a + b*x)*\text{ArcCos}[a + b*x])/b$

#### Rule 4804

$\text{Int}[(a + \text{ArcCos}[c + (d*x)]*(b))^n, x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Subst}[\text{Int}[(a + b*\text{ArcCos}[x])^n, x], x, c + d*x], x] /;$   $\text{FreeQ}\{a, b, c, d, n\}, x]$

#### Rule 4620

$\text{Int}[(a + \text{ArcCos}[c*(x)]*(b))^n, x\_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcCos}[c*x])^n, x] + \text{Dist}[b*c*n, \text{Int}[(x*(a + b*\text{ArcCos}[c*x])^{n-1})/\text{Sqrt}[1 - c^2*x^2], x], x] /;$   $\text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{GtQ}[n, 0]$

#### Rule 261

$\text{Int}[(x)^m*(a + (b*x)^n)^p, x\_Symbol] \rightarrow \text{Simp}[(a + b*x^n)^{p+1}/(b*n*(p+1)), x] /;$   $\text{FreeQ}\{a, b, m, n, p\}, x \ \&\& \ \text{EqQ}[m, n-1] \ \&\& \ \text{NeQ}[p, -1]$

#### Rubi steps

$$\begin{aligned} \int \cos^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int \cos^{-1}(x) dx, x, a + bx\right)}{b} \\ &= \frac{(a + bx) \cos^{-1}(a + bx)}{b} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}} dx, x, a + bx\right)}{b} \\ &= -\frac{\sqrt{1 - (a + bx)^2}}{b} + \frac{(a + bx) \cos^{-1}(a + bx)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.036752, size = 47, normalized size = 1.31

$$x \cos^{-1}(a + bx) - \frac{\sqrt{-a^2 - 2abx - b^2x^2 + 1} + a \sin^{-1}(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a + b\*x], x]

[Out]  $x \cdot \text{ArcCos}[a + b \cdot x] - (\text{Sqrt}[1 - a^2 - 2 \cdot a \cdot b \cdot x - b^2 \cdot x^2] + a \cdot \text{ArcSin}[a + b \cdot x]) / b$

**Maple [A]** time = 0.002, size = 33, normalized size = 0.9

$$\frac{1}{b} \left( (bx + a) \arccos(bx + a) - \sqrt{1 - (bx + a)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(b\*x+a), x)

[Out]  $1/b * ((b*x+a) * \arccos(b*x+a) - (1 - (b*x+a)^2)^{(1/2)})$

**Maxima [A]** time = 1.41026, size = 43, normalized size = 1.19

$$\frac{(bx + a) \arccos(bx + a) - \sqrt{-(bx + a)^2 + 1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a), x, algorithm="maxima")

[Out]  $((b*x + a) * \arccos(b*x + a) - \text{sqrt}(-(b*x + a)^2 + 1)) / b$

**Fricas [A]** time = 2.50697, size = 92, normalized size = 2.56

$$\frac{(bx + a) \arccos(bx + a) - \sqrt{-b^2x^2 - 2abx - a^2 + 1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a), x, algorithm="fricas")

[Out]  $((b*x + a) * \arccos(b*x + a) - \text{sqrt}(-b^2*x^2 - 2*a*b*x - a^2 + 1)) / b$

**Sympy [A]** time = 0.174177, size = 46, normalized size = 1.28

$$\begin{cases} \frac{a \arccos(a+bx)}{b} + x \arccos(a + bx) - \frac{\sqrt{-a^2 - 2abx - b^2x^2 + 1}}{b} & \text{for } b \neq 0 \\ x \arccos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(b\*x+a), x)

```
[Out] Piecewise((a*acos(a + b*x)/b + x*acos(a + b*x) - sqrt(-a**2 - 2*a*b*x - b**
2*x**2 + 1)/b, Ne(b, 0)), (x*acos(a), True))
```

**Giac [A]** time = 1.2806, size = 43, normalized size = 1.19

$$\frac{(bx + a) \arccos(bx + a) - \sqrt{-(bx + a)^2 + 1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a),x, algorithm="giac")
```

```
[Out] ((b*x + a)*arccos(b*x + a) - sqrt(-(b*x + a)^2 + 1))/b
```

### 3.28 $\int \frac{\cos^{-1}(a+bx)}{x} dx$

**Optimal.** Leaf size=177

$$-i\text{PolyLog}\left(2, \frac{e^{i\cos^{-1}(a+bx)}}{a-i\sqrt{1-a^2}}\right) - i\text{PolyLog}\left(2, \frac{e^{i\cos^{-1}(a+bx)}}{a+i\sqrt{1-a^2}}\right) + \cos^{-1}(a+bx)\log\left(1 - \frac{e^{i\cos^{-1}(a+bx)}}{a-i\sqrt{1-a^2}}\right) + \cos^{-1}(a+bx)\log\left(1 - \frac{e^{i\cos^{-1}(a+bx)}}{a+i\sqrt{1-a^2}}\right)$$

[Out]  $(-I/2)*\text{ArcCos}[a + b*x]^2 + \text{ArcCos}[a + b*x]*\text{Log}[1 - E^{(I*\text{ArcCos}[a + b*x])}]/(a - I*\text{Sqrt}[1 - a^2]) + \text{ArcCos}[a + b*x]*\text{Log}[1 - E^{(I*\text{ArcCos}[a + b*x])}]/(a + I*\text{Sqrt}[1 - a^2]) - I*\text{PolyLog}[2, E^{(I*\text{ArcCos}[a + b*x])}]/(a - I*\text{Sqrt}[1 - a^2]) - I*\text{PolyLog}[2, E^{(I*\text{ArcCos}[a + b*x])}]/(a + I*\text{Sqrt}[1 - a^2])$

**Rubi [A]** time = 0.276321, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {4806, 4742, 4522, 2190, 2279, 2391}

$$-i\text{PolyLog}\left(2, \frac{e^{i\cos^{-1}(a+bx)}}{a-i\sqrt{1-a^2}}\right) - i\text{PolyLog}\left(2, \frac{e^{i\cos^{-1}(a+bx)}}{a+i\sqrt{1-a^2}}\right) + \cos^{-1}(a+bx)\log\left(1 - \frac{e^{i\cos^{-1}(a+bx)}}{a-i\sqrt{1-a^2}}\right) + \cos^{-1}(a+bx)\log\left(1 - \frac{e^{i\cos^{-1}(a+bx)}}{a+i\sqrt{1-a^2}}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a + b\*x]/x, x]

[Out]  $(-I/2)*\text{ArcCos}[a + b*x]^2 + \text{ArcCos}[a + b*x]*\text{Log}[1 - E^{(I*\text{ArcCos}[a + b*x])}]/(a - I*\text{Sqrt}[1 - a^2]) + \text{ArcCos}[a + b*x]*\text{Log}[1 - E^{(I*\text{ArcCos}[a + b*x])}]/(a + I*\text{Sqrt}[1 - a^2]) - I*\text{PolyLog}[2, E^{(I*\text{ArcCos}[a + b*x])}]/(a - I*\text{Sqrt}[1 - a^2]) - I*\text{PolyLog}[2, E^{(I*\text{ArcCos}[a + b*x])}]/(a + I*\text{Sqrt}[1 - a^2])$

#### Rule 4806

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]

#### Rule 4742

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> -Subst[Int[((a + b\*x)^n\*Sin[x])/(c\*d + e\*Cos[x]), x], x, ArcCos[c\*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

#### Rule 4522

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sin[(c\_.) + (d\_.)\*(x\_)])/(Cos[(c\_.) + (d\_.)\*(x\_)]\*(b\_.) + (a\_.)), x\_Symbol] :> Simp[(I\*(e + f\*x)^(m + 1))/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(I\*(c + d\*x)))/(I\*a - Rt[-a^2 + b^2, 2] + I\*b\*E^(I\*(c + d\*x))), x] + Int[((e + f\*x)^m\*E^(I\*(c + d\*x)))/(I\*a + Rt[-a^2 + b^2, 2] + I\*b\*E^(I\*(c + d\*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]

#### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n], x]

)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol]  
 :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x))  
 )^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2  
 , -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned} \int \frac{\cos^{-1}(a+bx)}{x} dx &= \frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{-\frac{a}{b} + \frac{x}{b}} dx, x, a+bx\right)}{b} \\ &= -\frac{\text{Subst}\left(\int \frac{x \sin(x)}{-\frac{a}{b} + \frac{\cos(x)}{b}} dx, x, \cos^{-1}(a+bx)\right)}{b} \\ &= -\frac{1}{2} i \cos^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int \frac{e^{ix}}{-\frac{ia}{b} - \frac{\sqrt{1-a^2}}{b} + \frac{ie^{ix}}{b}} dx, x, \cos^{-1}(a+bx)\right)}{b} - \frac{\text{Subst}\left(\int \frac{e^{ix}}{-\frac{ia}{b} + \frac{\sqrt{1-a^2}}{b} + \frac{ie^{ix}}{b}} dx, x, \cos^{-1}(a+bx)\right)}{b} \\ &= -\frac{1}{2} i \cos^{-1}(a+bx)^2 + \cos^{-1}(a+bx) \log\left(1 - \frac{e^{i \cos^{-1}(a+bx)}}{a - i\sqrt{1-a^2}}\right) + \cos^{-1}(a+bx) \log\left(1 - \frac{e^{i \cos^{-1}(a+bx)}}{a + i\sqrt{1-a^2}}\right) \\ &= -\frac{1}{2} i \cos^{-1}(a+bx)^2 + \cos^{-1}(a+bx) \log\left(1 - \frac{e^{i \cos^{-1}(a+bx)}}{a - i\sqrt{1-a^2}}\right) + \cos^{-1}(a+bx) \log\left(1 - \frac{e^{i \cos^{-1}(a+bx)}}{a + i\sqrt{1-a^2}}\right) \\ &= -\frac{1}{2} i \cos^{-1}(a+bx)^2 + \cos^{-1}(a+bx) \log\left(1 - \frac{e^{i \cos^{-1}(a+bx)}}{a - i\sqrt{1-a^2}}\right) + \cos^{-1}(a+bx) \log\left(1 - \frac{e^{i \cos^{-1}(a+bx)}}{a + i\sqrt{1-a^2}}\right) \end{aligned}$$

**Mathematica [A]** time = 0.191965, size = 228, normalized size = 1.29

$-i \left( \text{PolyLog}\left(2, \left(a - \sqrt{a^2 - 1}\right) e^{i \cos^{-1}(a+bx)}\right) + \text{PolyLog}\left(2, \left(\sqrt{a^2 - 1} + a\right) e^{i \cos^{-1}(a+bx)}\right) \right) + \log\left(1 + \left(\sqrt{a^2 - 1} - a\right) e^{i \cos^{-1}(a+bx)}\right) + \log\left(1 + \left(\sqrt{a^2 - 1} + a\right) e^{i \cos^{-1}(a+bx)}\right)$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCos[a + b\*x]/x, x]

[Out]  $(-I/2) \text{ArcCos}[a + b*x]^2 - (4*I) \text{ArcSin}[\text{Sqrt}[1 - a]/\text{Sqrt}[2]] \text{ArcTan}[\left(\frac{(1 + a) \text{Tan}[\text{ArcCos}[a + b*x]/2]}{\text{Sqrt}[-1 + a^2]}\right) / \text{Sqrt}[-1 + a^2]] + (\text{ArcCos}[a + b*x] - 2 \text{ArcSin}[\text{Sqrt}[1 - a]/\text{Sqrt}[2]]) \text{Log}[1 + (-a + \text{Sqrt}[-1 + a^2]) \text{E}^{(I \text{ArcCos}[a + b*x])}] + (\text{ArcCos}[a + b*x] + 2 \text{ArcSin}[\text{Sqrt}[1 - a]/\text{Sqrt}[2]]) \text{Log}[1 - (a + \text{Sqrt}[-1 + a^2]) \text{E}^{(I \text{ArcCos}[a + b*x])}] - I \text{PolyLog}[2, (a - \text{Sqrt}[-1 + a^2]) \text{E}^{(I \text{ArcCos}[a + b*x])}] + \text{PolyLog}[2, (a + \text{Sqrt}[-1 + a^2]) \text{E}^{(I \text{ArcCos}[a + b*x])}]$

---

**Maple [A]** time = 0.104, size = 199, normalized size = 1.1

$$-\frac{i}{2} (\arccos(bx + a))^2 + \arccos(bx + a) \ln\left(\left(\sqrt{a^2 - 1} + bx + i\sqrt{1 - (bx + a)^2}\right)\left(-a + \sqrt{a^2 - 1}\right)^{-1}\right) + \arccos(bx + a) \ln\left(\left(\sqrt{a^2 - 1} - bx + i\sqrt{1 - (bx + a)^2}\right)\left(-a - \sqrt{a^2 - 1}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(b\*x+a)/x,x)

[Out]  $-1/2*I*\arccos(b*x+a)^2 + \arccos(b*x+a)*\ln\left(\left(a^2-1\right)^{1/2} + b*x + I*\left(1 - (b*x+a)^2\right)^{1/2}\right) / \left(-a + \left(a^2-1\right)^{1/2}\right) + \arccos(b*x+a)*\ln\left(\left(a^2-1\right)^{1/2} - b*x - I*\left(1 - (b*x+a)^2\right)^{1/2}\right) / \left(a + \left(a^2-1\right)^{1/2}\right) - I*\operatorname{dilog}\left(\left(a^2-1\right)^{1/2} + b*x + I*\left(1 - (b*x+a)^2\right)^{1/2}\right) / \left(-a + \left(a^2-1\right)^{1/2}\right) - I*\operatorname{dilog}\left(\left(a^2-1\right)^{1/2} - b*x - I*\left(1 - (b*x+a)^2\right)^{1/2}\right) / \left(a + \left(a^2-1\right)^{1/2}\right)$

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)/x,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\arccos(bx + a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)/x,x, algorithm="fricas")

[Out] integral(arccos(b\*x + a)/x, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(a + bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(b\*x+a)/x,x)

[Out] Integral(acos(a + b\*x)/x, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(bx + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(b*x + a)/x, x)
```

### 3.29 $\int \frac{\cos^{-1}(a+bx)}{x^2} dx$

**Optimal.** Leaf size=63

$$\frac{b \tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{\sqrt{1-a^2}} - \frac{\cos^{-1}(a+bx)}{x}$$

[Out] -(ArcCos[a + b\*x]/x) + (b\*ArcTanh[(1 - a\*(a + b\*x))/(Sqrt[1 - a^2]\*Sqrt[1 - (a + b\*x)^2]])/Sqrt[1 - a^2]

**Rubi [A]** time = 0.075788, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4806, 4744, 725, 206}

$$\frac{b \tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{\sqrt{1-a^2}} - \frac{\cos^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a + b\*x]/x^2, x]

[Out] -(ArcCos[a + b\*x]/x) + (b\*ArcTanh[(1 - a\*(a + b\*x))/(Sqrt[1 - a^2]\*Sqrt[1 - (a + b\*x)^2]])/Sqrt[1 - a^2]

#### Rule 4806

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^n\_.]\*((e\_.) + (f\_.)\*(x\_.))^m\_. , x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]

#### Rule 4744

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^n\_.]\*((d\_.) + (e\_.)\*(x\_.))^m\_. , x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^n)/(e\*(m + 1)), x] + Dist[(b\*c\*n)/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 725

Int[1/(((d\_.) + (e\_.)\*(x\_.))\*Sqrt[(a\_.) + (c\_.)\*(x\_.)^2]), x\_Symbol] :> -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

#### Rule 206

Int[((a\_.) + (b\_.)\*(x\_.)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps



$$\begin{aligned}
\int \frac{\cos^{-1}(a+bx)}{x^2} dx &= \frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{\left(-\frac{a}{b} + \frac{x}{b}\right)^2} dx, x, a+bx\right)}{b} \\
&= -\frac{\cos^{-1}(a+bx)}{x} - \text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b} + \frac{x}{b}\right)\sqrt{1-x^2}} dx, x, a+bx\right) \\
&= -\frac{\cos^{-1}(a+bx)}{x} + \text{Subst}\left(\int \frac{1}{\frac{1}{b^2} - \frac{a^2}{b^2} - x^2} dx, x, \frac{\frac{1}{b} - \frac{a(a+bx)}{b}}{\sqrt{1-(a+bx)^2}}\right) \\
&= -\frac{\cos^{-1}(a+bx)}{x} + \frac{b \tanh^{-1}\left(\frac{b\left(\frac{1}{b} - \frac{a(a+bx)}{b}\right)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{\sqrt{1-a^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0550105, size = 79, normalized size = 1.25

$$\frac{b\left(\log\left(\sqrt{1-a^2}\sqrt{-a^2-2abx-b^2x^2+1}-a^2-abx+1\right)-\log(x)\right)}{\sqrt{1-a^2}} - \frac{\cos^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a + b\*x]/x^2,x]

[Out] -(ArcCos[a + b\*x]/x) + (b\*(-Log[x] + Log[1 - a^2 - a\*b\*x + Sqrt[1 - a^2]\*Sqrt[1 - a^2 - 2\*a\*b\*x - b^2\*x^2]]))/Sqrt[1 - a^2]

**Maple [A]** time = 0.003, size = 77, normalized size = 1.2

$$-\frac{\arccos(bx+a)}{x} + b \ln\left(\frac{1}{bx} \left(-2a^2 + 2 - 2xab + 2\sqrt{-a^2+1}\sqrt{-b^2x^2-2xab-a^2+1}\right)\right) \frac{1}{\sqrt{-a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(b\*x+a)/x^2,x)

[Out] -arccos(b\*x+a)/x+b/(-a^2+1)^(1/2)\*ln((-2\*a^2+2-2\*x\*a\*b+2\*(-a^2+1)^(1/2)\*(-b^2\*x^2-2\*a\*b\*x-a^2+1)^(1/2))/b/x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)/x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 3.01077, size = 857, normalized size = 13.6

$$\left[ \frac{\sqrt{-a^2 + 1}bx \log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx-2\sqrt{-b^2x^2-2abx-a^2+1}(abx+a^2-1)\sqrt{-a^2+1}-4a^2+2}{x^2}\right) + 2(a^2-1)x \arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{b^2x^2+2abx+a^2-1}\right)}{2(a^2-1)x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)/x^2,x, algorithm="fricas")

[Out] [-1/2\*(sqrt(-a^2 + 1)\*b\*x\*log(((2\*a^2 - 1)\*b^2\*x^2 + 2\*a^4 + 4\*(a^3 - a)\*b\*x - 2\*sqrt(-b^2\*x^2 - 2\*a\*b\*x - a^2 + 1)\*(a\*b\*x + a^2 - 1)\*sqrt(-a^2 + 1) - 4\*a^2 + 2)/x^2) + 2\*(a^2 - 1)\*x\*arctan(sqrt(-b^2\*x^2 - 2\*a\*b\*x - a^2 + 1)\*(b\*x + a)/(b^2\*x^2 + 2\*a\*b\*x + a^2 - 1)) + 2\*(a^2 - (a^2 - 1)\*x - 1)\*arccos(b\*x + a)/((a^2 - 1)\*x), -(sqrt(a^2 - 1)\*b\*x\*arctan(sqrt(-b^2\*x^2 - 2\*a\*b\*x - a^2 + 1)\*(a\*b\*x + a^2 - 1)\*sqrt(a^2 - 1)/((a^2 - 1)\*b^2\*x^2 + a^4 + 2\*(a^3 - a)\*b\*x - 2\*a^2 + 1)) + (a^2 - 1)\*x\*arctan(sqrt(-b^2\*x^2 - 2\*a\*b\*x - a^2 + 1)\*(b\*x + a)/(b^2\*x^2 + 2\*a\*b\*x + a^2 - 1)) + (a^2 - (a^2 - 1)\*x - 1)\*arccos(b\*x + a)/((a^2 - 1)\*x)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(a + bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(b\*x+a)/x\*\*2,x)

[Out] Integral(acos(a + b\*x)/x\*\*2, x)

**Giac [A]** time = 1.31559, size = 107, normalized size = 1.7

$$-\frac{2b^2 \arctan\left(\frac{(\sqrt{-b^2x^2-2abx-a^2+1}|b|+b)a-1}{b^2x+ab}\right)}{\sqrt{a^2-1}|b|} - \frac{\arccos(bx+a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)/x^2,x, algorithm="giac")

[Out] -2\*b^2\*arctan(((sqrt(-b^2\*x^2 - 2\*a\*b\*x - a^2 + 1)\*abs(b) + b)\*a/(b^2\*x + a\*b) - 1)/sqrt(a^2 - 1))/(sqrt(a^2 - 1)\*abs(b)) - arccos(b\*x + a)/x

### 3.30 $\int \frac{\cos^{-1}(a+bx)}{x^3} dx$

**Optimal.** Leaf size=103

$$\frac{ab^2 \tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{2(1-a^2)^{3/2}} + \frac{b\sqrt{1-(a+bx)^2}}{2(1-a^2)x} - \frac{\cos^{-1}(a+bx)}{2x^2}$$

[Out] (b\*Sqrt[1 - (a + b\*x)^2])/(2\*(1 - a^2)\*x) - ArcCos[a + b\*x]/(2\*x^2) + (a\*b^2\*ArcTanh[(1 - a\*(a + b\*x))/(Sqrt[1 - a^2]\*Sqrt[1 - (a + b\*x)^2]])/(2\*(1 - a^2)^(3/2))

**Rubi [A]** time = 0.111094, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4806, 4744, 731, 725, 206}

$$\frac{ab^2 \tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{2(1-a^2)^{3/2}} + \frac{b\sqrt{1-(a+bx)^2}}{2(1-a^2)x} - \frac{\cos^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a + b\*x]/x^3,x]

[Out] (b\*Sqrt[1 - (a + b\*x)^2])/(2\*(1 - a^2)\*x) - ArcCos[a + b\*x]/(2\*x^2) + (a\*b^2\*ArcTanh[(1 - a\*(a + b\*x))/(Sqrt[1 - a^2]\*Sqrt[1 - (a + b\*x)^2]])/(2\*(1 - a^2)^(3/2))

#### Rule 4806

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]

#### Rule 4744

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^n)/(e\*(m + 1)), x] + Dist[(b\*c\*n)/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 731

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(e\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[(c\*d)/(c\*d^2 + a\*e^2), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m + 2\*p + 3, 0]

#### Rule 725

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\cos^{-1}(a+bx)}{x^3} dx &= \frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{\left(-\frac{a}{b} + \frac{x}{b}\right)^3} dx, x, a+bx\right)}{b} \\ &= -\frac{\cos^{-1}(a+bx)}{2x^2} - \frac{1}{2} \text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b} + \frac{x}{b}\right)^2 \sqrt{1-x^2}} dx, x, a+bx\right) \\ &= \frac{b\sqrt{1-(a+bx)^2}}{2(1-a^2)x} - \frac{\cos^{-1}(a+bx)}{2x^2} - \frac{(ab) \text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b} + \frac{x}{b}\right)\sqrt{1-x^2}} dx, x, a+bx\right)}{2(1-a^2)} \\ &= \frac{b\sqrt{1-(a+bx)^2}}{2(1-a^2)x} - \frac{\cos^{-1}(a+bx)}{2x^2} + \frac{(ab) \text{Subst}\left(\int \frac{1}{\frac{1}{b^2} - \frac{a^2}{b^2} - x^2} dx, x, \frac{\frac{1}{b} - \frac{a+bx}{b}}{\sqrt{1-(a+bx)^2}}\right)}{2(1-a^2)} \\ &= \frac{b\sqrt{1-(a+bx)^2}}{2(1-a^2)x} - \frac{\cos^{-1}(a+bx)}{2x^2} + \frac{ab^2 \tanh^{-1}\left(\frac{1-a+bx}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{2(1-a^2)^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.174607, size = 126, normalized size = 1.22

$$\frac{\cos^{-1}(a+bx) - \frac{bx\left(\sqrt{1-a^2}\sqrt{-a^2-2abx-b^2x^2+1}+abx \log\left(\frac{\sqrt{1-a^2}\sqrt{-a^2-2abx-b^2x^2+1}-a^2-abx+1}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)-abx \log(x)\right)}{(1-a^2)^{3/2}}}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a + b\*x]/x^3, x]

[Out] -(ArcCos[a + b\*x] - (b\*x\*(Sqrt[1 - a^2]\*Sqrt[1 - a^2 - 2\*a\*b\*x - b^2\*x^2] - a\*b\*x\*Log[x] + a\*b\*x\*Log[1 - a^2 - a\*b\*x + Sqrt[1 - a^2]\*Sqrt[1 - a^2 - 2\*a\*b\*x - b^2\*x^2]])))/(1 - a^2)^(3/2))/(2\*x^2)

**Maple [A]** time = 0.004, size = 118, normalized size = 1.2

$$-\frac{\arccos(bx+a)}{2x^2} + \frac{b}{(-2a^2+2)x} \sqrt{-b^2x^2-2xab-a^2+1} + \frac{b^2a}{2} \ln\left(\frac{1}{bx} \left(-2a^2+2-2xab+2\sqrt{-a^2+1}\sqrt{-b^2x^2-2xab}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(b\*x+a)/x^3, x)

[Out] -1/2\*arccos(b\*x+a)/x^2+1/2\*b/(-a^2+1)/x\*(-b^2\*x^2-2\*a\*b\*x-a^2+1)^(1/2)+1/2\*b^2\*a/(-a^2+1)^(3/2)\*ln((-2\*a^2+2-2\*x\*a\*b+2\*(-a^2+1)^(1/2)\*(-b^2\*x^2-2\*a\*b\*x-a^2+1)^(1/2))/b/x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)/x^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 2.94729, size = 1121, normalized size = 10.88

$$\left[ \frac{\sqrt{-a^2 + 1} ab^2 x^2 \log\left(\frac{(2a^2 - 1)b^2 x^2 + 2a^4 + 4(a^3 - a)bx + 2\sqrt{-b^2 x^2 - 2abx - a^2 + 1}(abx + a^2 - 1)\sqrt{-a^2 + 1 - 4a^2 + 2}}{x^2}\right) + 2(a^4 - 2a^2 + 1)x^2 \arctan\left(\frac{\sqrt{-b^2 x^2 - 2abx - a^2 + 1}(abx + a^2 - 1)}{x}\right)}{4(a^4 - 2a^2 + 1)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)/x^3,x, algorithm="fricas")

[Out] [-1/4\*(sqrt(-a^2 + 1)\*a\*b^2\*x^2\*log(((2\*a^2 - 1)\*b^2\*x^2 + 2\*a^4 + 4\*(a^3 - a)\*b\*x + 2\*sqrt(-b^2\*x^2 - 2\*a\*b\*x - a^2 + 1)\*(a\*b\*x + a^2 - 1)\*sqrt(-a^2 + 1) - 4\*a^2 + 2)/x^2) + 2\*(a^4 - 2\*a^2 + 1)\*x^2\*arctan(sqrt(-b^2\*x^2 - 2\*a\*b\*x - a^2 + 1)\*(b\*x + a)/(b^2\*x^2 + 2\*a\*b\*x + a^2 - 1)) + 2\*sqrt(-b^2\*x^2 - 2\*a\*b\*x - a^2 + 1)\*(a^2 - 1)\*b\*x + 2\*(a^4 - (a^4 - 2\*a^2 + 1)\*x^2 - 2\*a^2 + 1)\*arccos(b\*x + a))/((a^4 - 2\*a^2 + 1)\*x^2), 1/2\*(sqrt(a^2 - 1)\*a\*b^2\*x^2\*arctan(sqrt(-b^2\*x^2 - 2\*a\*b\*x - a^2 + 1)\*(a\*b\*x + a^2 - 1)\*sqrt(a^2 - 1)/((a^2 - 1)\*b^2\*x^2 + a^4 + 2\*(a^3 - a)\*b\*x - 2\*a^2 + 1)) - (a^4 - 2\*a^2 + 1)\*x^2\*arctan(sqrt(-b^2\*x^2 - 2\*a\*b\*x - a^2 + 1)\*(b\*x + a)/(b^2\*x^2 + 2\*a\*b\*x + a^2 - 1)) - sqrt(-b^2\*x^2 - 2\*a\*b\*x - a^2 + 1)\*(a^2 - 1)\*b\*x - (a^4 - (a^4 - 2\*a^2 + 1)\*x^2 - 2\*a^2 + 1)\*arccos(b\*x + a))/((a^4 - 2\*a^2 + 1)\*x^2)]

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(a + bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(b\*x+a)/x\*\*3,x)

[Out] Integral(acos(a + b\*x)/x\*\*3, x)

---

**Giac [B]** time = 1.33583, size = 327, normalized size = 3.17

$$\left( \frac{ab^2 \arctan\left(\frac{\left(\frac{\sqrt{-b^2 x^2 - 2abx - a^2 + 1}|b| + b\right)a}{b^2 x + ab} - 1\right)}{\sqrt{a^2 - 1}} \right) - \frac{ab^2 - \frac{\left(\sqrt{-b^2 x^2 - 2abx - a^2 + 1}|b| + b\right)b^2}{b^2 x + ab}}{\left(a^3 |b| - a|b|\right) \left(\frac{\left(\frac{\sqrt{-b^2 x^2 - 2abx - a^2 + 1}|b| + b\right)^2 a}{(b^2 x + ab)^2} + a - \frac{2\left(\sqrt{-b^2 x^2 - 2abx - a^2 + 1}|b| + b\right)}{b^2 x + ab}\right)} \right) b - \arccos\left(\frac{a + bx}{|b|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)/x^3,x, algorithm="giac")

[Out]  $(a*b^2*\arctan(((\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1})*\text{abs}(b) + b)*a/(b^2*x + a*b) - 1)/\sqrt{a^2 - 1})/((a^2*\text{abs}(b) - \text{abs}(b))*\sqrt{a^2 - 1}) - (a*b^2 - (\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1})*\text{abs}(b) + b)*b^2/(b^2*x + a*b))/((a^3*\text{abs}(b) - a*\text{abs}(b))*((\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1})*\text{abs}(b) + b)^2*a/(b^2*x + a*b)^2 + a - 2*(\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1})*\text{abs}(b) + b)/(b^2*x + a*b))) * b - 1/2*\arccos(b*x + a)/x^2$

### 3.31 $\int \frac{\cos^{-1}(a+bx)}{x^4} dx$

**Optimal.** Leaf size=144

$$\frac{ab^2\sqrt{1-(a+bx)^2}}{2(1-a^2)^2x} + \frac{(2a^2+1)b^3 \tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{6(1-a^2)^{5/2}} + \frac{b\sqrt{1-(a+bx)^2}}{6(1-a^2)x^2} - \frac{\cos^{-1}(a+bx)}{3x^3}$$

[Out] (b\*Sqrt[1 - (a + b\*x)^2])/(6\*(1 - a^2)\*x^2) + (a\*b^2\*Sqrt[1 - (a + b\*x)^2])/(2\*(1 - a^2)^2\*x) - ArcCos[a + b\*x]/(3\*x^3) + ((1 + 2\*a^2)\*b^3\*ArcTanh[(1 - a\*(a + b\*x))/(Sqrt[1 - a^2]\*Sqrt[1 - (a + b\*x)^2]])/(6\*(1 - a^2)^(5/2))

**Rubi [A]** time = 0.179291, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {4806, 4744, 745, 807, 725, 206}

$$\frac{ab^2\sqrt{1-(a+bx)^2}}{2(1-a^2)^2x} + \frac{(2a^2+1)b^3 \tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{6(1-a^2)^{5/2}} + \frac{b\sqrt{1-(a+bx)^2}}{6(1-a^2)x^2} - \frac{\cos^{-1}(a+bx)}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a + b\*x]/x^4, x]

[Out] (b\*Sqrt[1 - (a + b\*x)^2])/(6\*(1 - a^2)\*x^2) + (a\*b^2\*Sqrt[1 - (a + b\*x)^2])/(2\*(1 - a^2)^2\*x) - ArcCos[a + b\*x]/(3\*x^3) + ((1 + 2\*a^2)\*b^3\*ArcTanh[(1 - a\*(a + b\*x))/(Sqrt[1 - a^2]\*Sqrt[1 - (a + b\*x)^2]])/(6\*(1 - a^2)^(5/2))

#### Rule 4806

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^n\_.\*((e\_.) + (f\_.)\*(x\_.))^m\_., x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]

#### Rule 4744

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^n\_.\*((d\_.) + (e\_.)\*(x\_.))^m\_., x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^n)/(e\*(m + 1)), x] + Dist[(b\*c\*n)/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 745

Int[((d\_.) + (e\_.)\*(x\_.))^m\_.\*((a\_.) + (c\_.)\*(x\_.)^2)^p\_., x\_Symbol] :> Simp[(e\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[c/((m + 1)\*(c\*d^2 + a\*e^2)), Int[(d + e\*x)^(m + 1)\*Simp[d\*(m + 1) - e\*(m + 2\*p + 3)\*x, x]\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2\*p + 3], 0])

#### Rule 807

Int[((d\_.) + (e\_.)\*(x\_.))^m\_.\*((f\_.) + (g\_.)\*(x\_.))\*((a\_.) + (c\_.)\*(x\_.)^2)^p\_., x\_Symbol] :> -Simp[(e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1)]

$\int \frac{1}{(2(p+1)(cd^2 + ae^2))x} + \text{Dist}[(c*d*f + a*e*g)/(c*d^2 + a*e^2), \text{Int}[(d + e*x)^{(m+1)}(a + c*x^2)^p, x], x] /; \text{FreeQ}\{a, c, d, e, f, g, m, p\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{EqQ}[\text{Simplify}[m + 2*p + 3], 0]$

### Rule 725

$\text{Int}[1/((d_.) + (e_.)*(x_))*\text{Sqrt}[(a_.) + (c_.)*(x_)^2], x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] /; \text{FreeQ}\{a, c, d, e\}, x]$

### Rule 206

$\text{Int}[(a_.) + (b_.)*(x_)^2]^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rubi steps

$$\begin{aligned} \int \frac{\cos^{-1}(a+bx)}{x^4} dx &= \frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{\left(-\frac{a}{b} + \frac{x}{b}\right)^4} dx, x, a+bx\right)}{b} \\ &= -\frac{\cos^{-1}(a+bx)}{3x^3} - \frac{1}{3} \text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b} + \frac{x}{b}\right)^3 \sqrt{1-x^2}} dx, x, a+bx\right) \\ &= \frac{b\sqrt{1-(a+bx)^2}}{6(1-a^2)x^2} - \frac{\cos^{-1}(a+bx)}{3x^3} - \frac{b^2 \text{Subst}\left(\int \frac{\frac{2a}{b} + \frac{x}{b}}{\left(-\frac{a}{b} + \frac{x}{b}\right)^2 \sqrt{1-x^2}} dx, x, a+bx\right)}{6(1-a^2)} \\ &= \frac{b\sqrt{1-(a+bx)^2}}{6(1-a^2)x^2} + \frac{ab^2\sqrt{1-(a+bx)^2}}{2(1-a^2)^2 x} - \frac{\cos^{-1}(a+bx)}{3x^3} - \frac{((1+2a^2)b^2) \text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b} + \frac{x}{b}\right)\sqrt{1-x^2}} dx, x, a+bx\right)}{6(1-a^2)^2} \\ &= \frac{b\sqrt{1-(a+bx)^2}}{6(1-a^2)x^2} + \frac{ab^2\sqrt{1-(a+bx)^2}}{2(1-a^2)^2 x} - \frac{\cos^{-1}(a+bx)}{3x^3} + \frac{((1+2a^2)b^2) \text{Subst}\left(\int \frac{1}{\frac{1}{b^2} - \frac{a^2}{b^2} - x^2} dx, x, a+bx\right)}{6(1-a^2)^2} \\ &= \frac{b\sqrt{1-(a+bx)^2}}{6(1-a^2)x^2} + \frac{ab^2\sqrt{1-(a+bx)^2}}{2(1-a^2)^2 x} - \frac{\cos^{-1}(a+bx)}{3x^3} + \frac{(1+2a^2)b^3 \tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{6(1-a^2)^{5/2}} \end{aligned}$$

**Mathematica [A]** time = 0.184182, size = 168, normalized size = 1.17

$$\frac{\sqrt{1-a^2}bx(-a^2+3abx+1)\sqrt{-a^2-2abx-b^2x^2+1} - (2a^2+1)b^3x^3 \log(x) + (2a^2+1)b^3x^3 \log\left(\sqrt{1-a^2}\sqrt{-a^2-2abx-b^2x^2+1}\right)}{6(1-a^2)^{5/2}x^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a + b\*x]/x^4, x]

[Out] (Sqrt[1 - a^2]\*b\*x\*(1 - a^2 + 3\*a\*b\*x)\*Sqrt[1 - a^2 - 2\*a\*b\*x - b^2\*x^2] - 2\*(1 - a^2)^(5/2)\*ArcCos[a + b\*x] - (1 + 2\*a^2)\*b^3\*x^3\*Log[x] + (1 + 2\*a^2)\*b^3\*x^3\*Log[1 - a^2 - a\*b\*x + Sqrt[1 - a^2]\*Sqrt[1 - a^2 - 2\*a\*b\*x - b^2\*x^2]])/(6\*(1 - a^2)^(5/2)\*x^3)



---

**Maple [A]** time = 0.005, size = 227, normalized size = 1.6

$$-\frac{\arccos(bx+a)}{3x^3} + \frac{b}{(-6a^2+6)x^2} \sqrt{-b^2x^2-2xab-a^2+1} + \frac{b^2a}{2(-a^2+1)^2x} \sqrt{-b^2x^2-2xab-a^2+1} + \frac{b^3a^2}{2} \ln\left(\frac{1}{bx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(b\*x+a)/x^4,x)

[Out] 
$$-1/3*\arccos(b*x+a)/x^3+1/6*b/(-a^2+1)/x^2*(-b^2*x^2-2*a*b*x-a^2+1)^{(1/2)+1/2*b^2*a/(-a^2+1)^2/x*(-b^2*x^2-2*a*b*x-a^2+1)^{(1/2)+1/2*b^3*a^2/(-a^2+1)^{(5/2)*\ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^{(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^{(1/2))}/b/x)+1/6*b^3/(-a^2+1)^{(3/2)*\ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^{(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^{(1/2))}/b/x)}$$

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)/x^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 3.54357, size = 1328, normalized size = 9.22

$$\left[ \frac{(2a^2+1)\sqrt{-a^2+1}b^3x^3 \log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx-2\sqrt{-b^2x^2-2abx-a^2+1}(abx+a^2-1)\sqrt{-a^2+1-4a^2+2}}{x^2}\right) + 4(a^6-3a^4+3a^2)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)/x^4,x, algorithm="fricas")

[Out] 
$$[-1/12*((2*a^2+1)*\sqrt{-a^2+1}*b^3*x^3*\log(((2*a^2-1)*b^2*x^2+2*a^4+4*(a^3-a)*b*x-2*\sqrt{-b^2*x^2-2*a*b*x-a^2+1}*(a*b*x+a^2-1))*\sqrt{-a^2+1}-4*a^2+2)/x^2)+4*(a^6-3*a^4+3*a^2-1)*x^3*\arctan(\sqrt{-b^2*x^2-2*a*b*x-a^2+1}*(b*x+a)/(b^2*x^2+2*a*b*x+a^2-1))+4*(a^6-3*a^4-(a^6-3*a^4+3*a^2-1)*x^3+3*a^2-1)*\arccos(b*x+a)-2*(3*(a^3-a)*b^2*x^2-(a^4-2*a^2+1)*b*x)*\sqrt{-b^2*x^2-2*a*b*x-a^2+1})/((a^6-3*a^4+3*a^2-1)*x^3), -1/6*((2*a^2+1)*\sqrt{-a^2+1}*b^3*x^3*\arctan(\sqrt{-b^2*x^2-2*a*b*x-a^2+1}*(a*b*x+a^2-1))*\sqrt{-a^2+1}/((a^2-1)*b^2*x^2+a^4+2*(a^3-a)*b*x-2*a^2+1))+2*(a^6-3*a^4+3*a^2-1)*x^3*\arctan(\sqrt{-b^2*x^2-2*a*b*x-a^2+1}*(b*x+a)/(b^2*x^2+2*a*b*x+a^2-1))+2*(a^6-3*a^4-(a^6-3*a^4+3*a^2-1)*x^3+3*a^2-1)*\arccos(b*x+a)-(3*(a^3-a)*b^2*x^2-(a^4-2*a^2+1)*b*x)*\sqrt{-b^2*x^2-2*a*b*x-a^2+1})/((a^6-3*a^4+3*a^2-1)*x^3)]$$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(a + bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)/x\*\*4,x)

[Out] Integral(arccos(a + b\*x)/x\*\*4, x)

---

**Giac [B]** time = 1.35558, size = 752, normalized size = 5.22

$$-\frac{1}{3} b \left( \frac{(2 a^2 b^3 + b^3) \arctan\left(\frac{(\sqrt{-b^2 x^2 - 2 a b x - a^2 + 1} |b| + b) a}{b^2 x + a b} - 1\right)}{(a^4 |b| - 2 a^2 |b| + |b|) \sqrt{a^2 - 1}} - \frac{4 (\sqrt{-b^2 x^2 - 2 a b x - a^2 + 1} |b| + b)^2 a^4 b^3}{(b^2 x + a b)^2} + 4 a^4 b^3 - \frac{11 (\sqrt{-b^2 x^2 - 2 a b x - a^2 + 1} |b| + b) a^3 b^3}{b^2 x + a b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)/x^4,x, algorithm="giac")

[Out]  $-1/3*b*((2*a^2*b^3 + b^3)*\arctan(((\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1})*\text{abs}(b) + b)*a/(b^2*x + a*b) - 1)/\sqrt{a^2 - 1})/((a^4*\text{abs}(b) - 2*a^2*\text{abs}(b) + \text{abs}(b))*\sqrt{a^2 - 1}) - (4*(\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1})*\text{abs}(b) + b)^2*a^4*b^3/(b^2*x + a*b)^2 + 4*a^4*b^3 - 11*(\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1})*\text{abs}(b) + b)*a^3*b^3/(b^2*x + a*b) - 5*(\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1})*\text{abs}(b) + b)^3*a^3*b^3/(b^2*x + a*b)^3 + 7*(\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1})*\text{abs}(b) + b)^2*a^2*b^3/(b^2*x + a*b)^2 - a^2*b^3 + 2*(\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1})*\text{abs}(b) + b)*a*b^3/(b^2*x + a*b) + 2*(\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1})*\text{abs}(b) + b)^3*a*b^3/(b^2*x + a*b)^3 - 2*(\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1})*\text{abs}(b) + b)^2*b^3/(b^2*x + a*b)^2)/((a^6*\text{abs}(b) - 2*a^4*\text{abs}(b) + a^2*\text{abs}(b))*((\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1})*\text{abs}(b) + b)^2*a/(b^2*x + a*b)^2 + a - 2*(\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1})*\text{abs}(b) + b)/(b^2*x + a*b))^2) - 1/3*\arccos(b*x + a)/x^3$

### 3.32 $\int \cos^{-1}(a + bx)^3 dx$

**Optimal.** Leaf size=82

$$\frac{6\sqrt{1-(a+bx)^2}}{b} + \frac{(a+bx)\cos^{-1}(a+bx)^3}{b} - \frac{3\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)^2}{b} - \frac{6(a+bx)\cos^{-1}(a+bx)}{b}$$

[Out] (6\*Sqrt[1 - (a + b\*x)^2])/b - (6\*(a + b\*x)\*ArcCos[a + b\*x])/b - (3\*Sqrt[1 - (a + b\*x)^2]\*ArcCos[a + b\*x]^2)/b + ((a + b\*x)\*ArcCos[a + b\*x]^3)/b

**Rubi [A]** time = 0.0820063, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4804, 4620, 4678, 261}

$$\frac{6\sqrt{1-(a+bx)^2}}{b} + \frac{(a+bx)\cos^{-1}(a+bx)^3}{b} - \frac{3\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)^2}{b} - \frac{6(a+bx)\cos^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a + b\*x]^3,x]

[Out] (6\*Sqrt[1 - (a + b\*x)^2])/b - (6\*(a + b\*x)\*ArcCos[a + b\*x])/b - (3\*Sqrt[1 - (a + b\*x)^2]\*ArcCos[a + b\*x]^2)/b + ((a + b\*x)\*ArcCos[a + b\*x]^3)/b

#### Rule 4804

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, n}, x]

#### Rule 4620

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcCos[c\*x])^n, x] + Dist[b\*c\*n, Int[(x\*(a + b\*ArcCos[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4678

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCos[c\*x])^n)/(2\*e\*(p + 1)), x] - Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcCos[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

#### Rule 261

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \cos^{-1}(a + bx)^3 dx &= \frac{\text{Subst}\left(\int \cos^{-1}(x)^3 dx, x, a + bx\right)}{b} \\
&= \frac{(a + bx) \cos^{-1}(a + bx)^3}{b} + \frac{3 \text{Subst}\left(\int \frac{x \cos^{-1}(x)^2}{\sqrt{1-x^2}} dx, x, a + bx\right)}{b} \\
&= -\frac{3\sqrt{1 - (a + bx)^2} \cos^{-1}(a + bx)^2}{b} + \frac{(a + bx) \cos^{-1}(a + bx)^3}{b} - \frac{6 \text{Subst}\left(\int \cos^{-1}(x) dx, x, a + bx\right)}{b} \\
&= -\frac{6(a + bx) \cos^{-1}(a + bx)}{b} - \frac{3\sqrt{1 - (a + bx)^2} \cos^{-1}(a + bx)^2}{b} + \frac{(a + bx) \cos^{-1}(a + bx)^3}{b} - \frac{6 \text{Subst}\left(\int \cos^{-1}(x) dx, x, a + bx\right)}{b} \\
&= \frac{6\sqrt{1 - (a + bx)^2}}{b} - \frac{6(a + bx) \cos^{-1}(a + bx)}{b} - \frac{3\sqrt{1 - (a + bx)^2} \cos^{-1}(a + bx)^2}{b} + \frac{(a + bx) \cos^{-1}(a + bx)^3}{b}
\end{aligned}$$

**Mathematica [A]** time = 0.0356647, size = 74, normalized size = 0.9

$$\frac{6\sqrt{1 - (a + bx)^2} + (a + bx) \cos^{-1}(a + bx)^3 - 3\sqrt{1 - (a + bx)^2} \cos^{-1}(a + bx)^2 - 6(a + bx) \cos^{-1}(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a + b\*x]^3, x]

[Out] (6\*Sqrt[1 - (a + b\*x)^2] - 6\*(a + b\*x)\*ArcCos[a + b\*x] - 3\*Sqrt[1 - (a + b\*x)^2]\*ArcCos[a + b\*x]^2 + (a + b\*x)\*ArcCos[a + b\*x]^3)/b

**Maple [A]** time = 0.054, size = 71, normalized size = 0.9

$$\frac{1}{b} \left( (\arccos(bx + a))^3 (bx + a) - 3 (\arccos(bx + a))^2 \sqrt{1 - (bx + a)^2} + 6 \sqrt{1 - (bx + a)^2} - 6 (bx + a) \arccos(bx + a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(b\*x+a)^3, x)

[Out] 1/b\*(arccos(b\*x+a)^3\*(b\*x+a)-3\*arccos(b\*x+a)^2\*(1-(b\*x+a)^2)^(1/2)+6\*(1-(b\*x+a)^2)^(1/2)-6\*(b\*x+a)\*arccos(b\*x+a))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)^3, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.55929, size = 170, normalized size = 2.07

$$\frac{(bx + a) \arccos(bx + a)^3 - 6(bx + a) \arccos(bx + a) - 3\sqrt{-b^2x^2 - 2abx - a^2 + 1}(\arccos(bx + a)^2 - 2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)^3,x, algorithm="fricas")

[Out]  $((b*x + a)*\arccos(b*x + a)^3 - 6*(b*x + a)*\arccos(b*x + a) - 3*\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1}*(\arccos(b*x + a)^2 - 2))/b$

**Sympy [A]** time = 0.692653, size = 109, normalized size = 1.33

$$\left\{ \begin{array}{l} \frac{a \arccos^3(a+bx)}{b} - \frac{6a \arccos(a+bx)}{b} + x \arccos^3(a+bx) - 6x \arccos(a+bx) - \frac{3\sqrt{-a^2-2abx-b^2x^2+1} \arccos^2(a+bx)}{b} + \frac{6\sqrt{-a^2-2abx-b^2x^2+1}}{b} \\ x \arccos^3(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(b\*x+a)\*\*3,x)

[Out] Piecewise((a\*acos(a + b\*x)\*\*3/b - 6\*a\*acos(a + b\*x)/b + x\*acos(a + b\*x)\*\*3 - 6\*x\*acos(a + b\*x) - 3\*sqrt(-a\*\*2 - 2\*a\*b\*x - b\*\*2\*x\*\*2 + 1)\*acos(a + b\*x)\*\*2/b + 6\*sqrt(-a\*\*2 - 2\*a\*b\*x - b\*\*2\*x\*\*2 + 1)/b, Ne(b, 0)), (x\*acos(a)\*\*3, True))

**Giac [A]** time = 1.42999, size = 105, normalized size = 1.28

$$\frac{(bx + a) \arccos(bx + a)^3}{b} - \frac{3 \sqrt{-(bx + a)^2 + 1} \arccos(bx + a)^2}{b} - \frac{6(bx + a) \arccos(bx + a)}{b} + \frac{6 \sqrt{-(bx + a)^2 + 1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)^3,x, algorithm="giac")

[Out]  $(b*x + a)*\arccos(b*x + a)^3/b - 3*\sqrt{-(b*x + a)^2 + 1}*\arccos(b*x + a)^2/b - 6*(b*x + a)*\arccos(b*x + a)/b + 6*\sqrt{-(b*x + a)^2 + 1}/b$

### 3.33 $\int \cos^{-1}(a + bx)^2 dx$

**Optimal.** Leaf size=47

$$\frac{(a + bx) \cos^{-1}(a + bx)^2}{b} - \frac{2\sqrt{1 - (a + bx)^2} \cos^{-1}(a + bx)}{b} - 2x$$

[Out]  $-2*x - (2*\text{Sqrt}[1 - (a + b*x)^2]*\text{ArcCos}[a + b*x])/b + ((a + b*x)*\text{ArcCos}[a + b*x]^2)/b$

**Rubi [A]** time = 0.0544189, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4804, 4620, 4678, 8}

$$\frac{(a + bx) \cos^{-1}(a + bx)^2}{b} - \frac{2\sqrt{1 - (a + bx)^2} \cos^{-1}(a + bx)}{b} - 2x$$

Antiderivative was successfully verified.

[In] `Int[ArcCos[a + b*x]^2,x]`

[Out]  $-2*x - (2*\text{Sqrt}[1 - (a + b*x)^2]*\text{ArcCos}[a + b*x])/b + ((a + b*x)*\text{ArcCos}[a + b*x]^2)/b$

#### Rule 4804

`Int[((a_.) + ArcCos[(c_.) + (d_.)*(x_.)]*(b_.))^n], x_Symbol] := Dist[1/d, Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n}, x]`

#### Rule 4620

`Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^n], x_Symbol] := Simp[x*(a + b*ArcCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]`

#### Rule 4678

`Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^n*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

#### Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

#### Rubi steps

$$\begin{aligned}
\int \cos^{-1}(a+bx)^2 dx &= \frac{\text{Subst}\left(\int \cos^{-1}(x)^2 dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx) \cos^{-1}(a+bx)^2}{b} + \frac{2 \text{Subst}\left(\int \frac{x \cos^{-1}(x)}{\sqrt{1-x^2}} dx, x, a+bx\right)}{b} \\
&= -\frac{2\sqrt{1-(a+bx)^2} \cos^{-1}(a+bx)}{b} + \frac{(a+bx) \cos^{-1}(a+bx)^2}{b} - \frac{2 \text{Subst}\left(\int 1 dx, x, a+bx\right)}{b} \\
&= -2x - \frac{2\sqrt{1-(a+bx)^2} \cos^{-1}(a+bx)}{b} + \frac{(a+bx) \cos^{-1}(a+bx)^2}{b}
\end{aligned}$$

**Mathematica [A]** time = 0.0228701, size = 49, normalized size = 1.04

$$\frac{-2(a+bx) + (a+bx) \cos^{-1}(a+bx)^2 - 2\sqrt{1-(a+bx)^2} \cos^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a + b\*x]^2,x]

[Out] (-2\*(a + b\*x) - 2\*Sqrt[1 - (a + b\*x)^2]\*ArcCos[a + b\*x] + (a + b\*x)\*ArcCos[a + b\*x]^2)/b

**Maple [A]** time = 0.049, size = 48, normalized size = 1.

$$\frac{1}{b} \left( (\arccos(bx+a))^2 (bx+a) - 2bx - 2a - 2 \arccos(bx+a) \sqrt{1-(bx+a)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(b\*x+a)^2,x)

[Out] 1/b\*(arccos(b\*x+a)^2\*(b\*x+a)-2\*b\*x-2\*a-2\*arccos(b\*x+a)\*(1-(b\*x+a)^2)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.33554, size = 130, normalized size = 2.77

$$\frac{(bx+a) \arccos(bx+a)^2 - 2bx - 2\sqrt{-b^2x^2 - 2abx - a^2 + 1} \arccos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)^2,x, algorithm="fricas")

[Out] ((b\*x + a)\*arccos(b\*x + a)^2 - 2\*b\*x - 2\*sqrt(-b^2\*x^2 - 2\*a\*b\*x - a^2 + 1)\*arccos(b\*x + a))/b

**Sympy [A]** time = 0.282811, size = 63, normalized size = 1.34

$$\begin{cases} \frac{a \operatorname{acos}^2(a+bx)}{b} + x \operatorname{acos}^2(a+bx) - 2x - \frac{2\sqrt{-a^2-2abx-b^2x^2+1} \operatorname{acos}(a+bx)}{b} & \text{for } b \neq 0 \\ x \operatorname{acos}^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(b\*x+a)\*\*2,x)

[Out] Piecewise((a\*acos(a + b\*x)\*\*2/b + x\*acos(a + b\*x)\*\*2 - 2\*x - 2\*sqrt(-a\*\*2 - 2\*a\*b\*x - b\*\*2\*x\*\*2 + 1)\*acos(a + b\*x)/b, Ne(b, 0)), (x\*acos(a)\*\*2, True))

**Giac [A]** time = 1.34207, size = 70, normalized size = 1.49

$$\frac{(bx + a) \operatorname{arccos}(bx + a)^2}{b} - \frac{2\sqrt{-(bx + a)^2 + 1} \operatorname{arccos}(bx + a)}{b} - \frac{2(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)^2,x, algorithm="giac")

[Out] (b\*x + a)\*arccos(b\*x + a)^2/b - 2\*sqrt(-(b\*x + a)^2 + 1)\*arccos(b\*x + a)/b - 2\*(b\*x + a)/b



$$3.34 \quad \int \frac{1}{\cos^{-1}(a+bx)} dx$$

**Optimal.** Leaf size=12

$$-\frac{\text{Si}(\cos^{-1}(a+bx))}{b}$$

[Out] -(SinIntegral[ArcCos[a + b\*x]]/b)

**Rubi [A]** time = 0.0215678, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4804, 4624, 3299}

$$-\frac{\text{Si}(\cos^{-1}(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a + b\*x]^(-1), x]

[Out] -(SinIntegral[ArcCos[a + b\*x]]/b)

#### Rule 4804

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, n}, x]

#### Rule 4624

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] :> Dist[1/(b\*c), Subst[Int[x^n\*Sin[a/b - x/b], x], x, a + b\*ArcCos[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

#### Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{\cos^{-1}(a+bx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)} dx, x, a+bx\right)}{b} \\ &= -\frac{\text{Subst}\left(\int \frac{\sin(x)}{x} dx, x, \cos^{-1}(a+bx)\right)}{b} \\ &= -\frac{\text{Si}(\cos^{-1}(a+bx))}{b} \end{aligned}$$

**Mathematica [A]** time = 0.0259629, size = 12, normalized size = 1.

$$-\frac{\text{Si}(\cos^{-1}(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a + b\*x]^(-1),x]

[Out] -(SinIntegral[ArcCos[a + b\*x]]/b)

**Maple [A]** time = 0.046, size = 13, normalized size = 1.1

$$-\frac{\text{Si}(\arccos(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arccos(b\*x+a),x)

[Out] -Si(arccos(b\*x+a))/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\arccos(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(b\*x+a),x, algorithm="maxima")

[Out] integrate(1/arccos(b\*x + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\arccos(bx + a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(b\*x+a),x, algorithm="fricas")

[Out] integral(1/arccos(b\*x + a), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\text{acos}(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/acos(b\*x+a),x)

[Out] Integral(1/acos(a + b\*x), x)

---

**Giac [A]** time = 1.34309, size = 16, normalized size = 1.33

$$-\frac{\text{Si}(\arccos(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(b\*x+a),x, algorithm="giac")

[Out] -sin\_integral(arccos(b\*x + a))/b

$$3.35 \quad \int \frac{1}{\cos^{-1}(a+bx)^2} dx$$

**Optimal.** Leaf size=40

$$\frac{\sqrt{1-(a+bx)^2}}{b \cos^{-1}(a+bx)} - \frac{\text{CosIntegral}(\cos^{-1}(a+bx))}{b}$$

[Out] Sqrt[1 - (a + b\*x)^2]/(b\*ArcCos[a + b\*x]) - CosIntegral[ArcCos[a + b\*x]]/b

**Rubi [A]** time = 0.0784772, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4804, 4622, 4724, 3302}

$$\frac{\sqrt{1-(a+bx)^2}}{b \cos^{-1}(a+bx)} - \frac{\text{CosIntegral}(\cos^{-1}(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a + b\*x]^(-2), x]

[Out] Sqrt[1 - (a + b\*x)^2]/(b\*ArcCos[a + b\*x]) - CosIntegral[ArcCos[a + b\*x]]/b

#### Rule 4804

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^ (n\_.), x\_Symbol] := Dist[1/d, Subst[Int[(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, n}, x]

#### Rule 4622

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^ (n\_.), x\_Symbol] := -Simp[(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcCos[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[c/(b\*(n + 1)), Int[(x\*(a + b\*ArcCos[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

#### Rule 4724

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^ (n\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := -Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Cos[x]^m\*Sin[x]^(2\*p + 1), x], x, ArcCos[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

#### Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] := Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{-1}(a+bx)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)^2} dx, x, a+bx\right)}{b} \\
&= \frac{\sqrt{1-(a+bx)^2}}{b \cos^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2} \cos^{-1}(x)} dx, x, a+bx\right)}{b} \\
&= \frac{\sqrt{1-(a+bx)^2}}{b \cos^{-1}(a+bx)} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{x} dx, x, \cos^{-1}(a+bx)\right)}{b} \\
&= \frac{\sqrt{1-(a+bx)^2}}{b \cos^{-1}(a+bx)} - \frac{\text{Ci}\left(\cos^{-1}(a+bx)\right)}{b}
\end{aligned}$$

**Mathematica [A]** time = 0.0508103, size = 40, normalized size = 1.

$$\frac{\sqrt{1-(a+bx)^2}}{b \cos^{-1}(a+bx)} - \frac{\text{CosIntegral}\left(\cos^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a + b\*x]^(-2), x]

[Out] Sqrt[1 - (a + b\*x)^2]/(b\*ArcCos[a + b\*x]) - CosIntegral[ArcCos[a + b\*x]]/b

**Maple [A]** time = 0.049, size = 37, normalized size = 0.9

$$\frac{1}{b} \left( \frac{1}{\arccos(bx+a)} \sqrt{1-(bx+a)^2} - \text{Ci}(\arccos(bx+a)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arccos(b\*x+a)^2, x)

[Out] 1/b\*(1/arccos(b\*x+a)\*(1-(b\*x+a)^2)^(1/2)-Ci(arccos(b\*x+a)))

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(b\*x+a)^2, x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\arccos(bx+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(b\*x+a)^2,x, algorithm="fricas")

[Out] integral(arccos(b\*x + a)^(-2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\arccos^2(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/acos(b\*x+a)\*\*2,x)

[Out] Integral(acos(a + b\*x)\*\*(-2), x)

**Giac [A]** time = 1.32966, size = 51, normalized size = 1.27

$$-\frac{\text{Ci}(\arccos(bx + a))}{b} + \frac{\sqrt{-(bx + a)^2 + 1}}{b \arccos(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(b\*x+a)^2,x, algorithm="giac")

[Out] -cos\_integral(arccos(b\*x + a))/b + sqrt(-(b\*x + a)^2 + 1)/(b\*arccos(b\*x + a))

$$3.36 \quad \int \frac{1}{\cos^{-1}(a+bx)^3} dx$$

**Optimal.** Leaf size=65

$$\frac{\text{Si}(\cos^{-1}(a+bx))}{2b} + \frac{a+bx}{2b \cos^{-1}(a+bx)} + \frac{\sqrt{1-(a+bx)^2}}{2b \cos^{-1}(a+bx)^2}$$

[Out] Sqrt[1 - (a + b\*x)^2]/(2\*b\*ArcCos[a + b\*x]^2) + (a + b\*x)/(2\*b\*ArcCos[a + b\*x]) + SinIntegral[ArcCos[a + b\*x]]/(2\*b)

**Rubi [A]** time = 0.08065, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {4804, 4622, 4720, 4624, 3299}

$$\frac{\text{Si}(\cos^{-1}(a+bx))}{2b} + \frac{a+bx}{2b \cos^{-1}(a+bx)} + \frac{\sqrt{1-(a+bx)^2}}{2b \cos^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a + b\*x]^(-3), x]

[Out] Sqrt[1 - (a + b\*x)^2]/(2\*b\*ArcCos[a + b\*x]^2) + (a + b\*x)/(2\*b\*ArcCos[a + b\*x]) + SinIntegral[ArcCos[a + b\*x]]/(2\*b)

#### Rule 4804

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^n], x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, n}, x]

#### Rule 4622

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^n], x\_Symbol] :> -Simp[(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcCos[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[c/(b\*(n + 1)), Int[(x\*(a + b\*ArcCos[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

#### Rule 4720

Int[(((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^n)\*((f\_.)\*(x\_.))^m]/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] :> -Simp[((f\*x)^m\*(a + b\*ArcCos[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] + Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)^(m - 1)\*(a + b\*ArcCos[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

#### Rule 4624

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^n], x\_Symbol] :> Dist[1/(b\*c), Subst[Int[x^n\*Sin[a/b - x/b], x], x, a + b\*ArcCos[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

#### Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] :> Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{-1}(a+bx)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)^3} dx, x, a+bx\right)}{b} \\
&= \frac{\sqrt{1-(a+bx)^2}}{2b \cos^{-1}(a+bx)^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2} \cos^{-1}(x)^2} dx, x, a+bx\right)}{2b} \\
&= \frac{\sqrt{1-(a+bx)^2}}{2b \cos^{-1}(a+bx)^2} + \frac{a+bx}{2b \cos^{-1}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)} dx, x, a+bx\right)}{2b} \\
&= \frac{\sqrt{1-(a+bx)^2}}{2b \cos^{-1}(a+bx)^2} + \frac{a+bx}{2b \cos^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\sin(x)}{x} dx, x, \cos^{-1}(a+bx)\right)}{2b} \\
&= \frac{\sqrt{1-(a+bx)^2}}{2b \cos^{-1}(a+bx)^2} + \frac{a+bx}{2b \cos^{-1}(a+bx)} + \frac{\text{Si}\left(\cos^{-1}(a+bx)\right)}{2b}
\end{aligned}$$

**Mathematica [A]** time = 0.0501086, size = 65, normalized size = 1.

$$\frac{\text{Si}\left(\cos^{-1}(a+bx)\right)}{2b} + \frac{a+bx}{2b \cos^{-1}(a+bx)} + \frac{\sqrt{1-(a+bx)^2}}{2b \cos^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a + b\*x]^(-3), x]

[Out] Sqrt[1 - (a + b\*x)^2]/(2\*b\*ArcCos[a + b\*x]^2) + (a + b\*x)/(2\*b\*ArcCos[a + b\*x]) + SinIntegral[ArcCos[a + b\*x]]/(2\*b)

**Maple [A]** time = 0.051, size = 53, normalized size = 0.8

$$\frac{1}{b} \left( \frac{1}{2 (\arccos(bx+a))^2} \sqrt{1-(bx+a)^2} + \frac{bx+a}{2 \arccos(bx+a)} + \frac{\text{Si}(\arccos(bx+a))}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arccos(b\*x+a)^3, x)

[Out] 1/b\*(1/2/arccos(b\*x+a)^2\*(1-(b\*x+a)^2)^(1/2)+1/2/arccos(b\*x+a)\*(b\*x+a)+1/2\*Si(arccos(b\*x+a)))

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(b\*x+a)^3, x, algorithm="maxima")

[Out] Timed out



---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\arccos(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(b\*x+a)^3,x, algorithm="fricas")

[Out] integral(arccos(b\*x + a)^(-3), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\arccos^3(a+bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/acos(b\*x+a)\*\*3,x)

[Out] Integral(acos(a + b\*x)\*\*(-3), x)

---

**Giac [A]** time = 1.29833, size = 77, normalized size = 1.18

$$\frac{\text{Si}(\arccos(bx+a))}{2b} + \frac{bx+a}{2b \arccos(bx+a)} + \frac{\sqrt{-(bx+a)^2+1}}{2b \arccos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(b\*x+a)^3,x, algorithm="giac")

[Out] 1/2\*sin\_integral(arccos(b\*x + a))/b + 1/2\*(b\*x + a)/(b\*arccos(b\*x + a)) + 1/2\*sqrt(-(b\*x + a)^2 + 1)/(b\*arccos(b\*x + a)^2)

### 3.37 $\int \cos^{-1}(a + bx)^{5/2} dx$

**Optimal.** Leaf size=111

$$\frac{15\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a + bx)}\right)}{4b} + \frac{(a + bx)\cos^{-1}(a + bx)^{5/2}}{b} - \frac{5\sqrt{1 - (a + bx)^2}\cos^{-1}(a + bx)^{3/2}}{2b} - \frac{15(a + bx)\sqrt{\cos^{-1}(a + bx)}}{4b}$$

[Out] (-15\*(a + b\*x)\*Sqrt[ArcCos[a + b\*x]]/(4\*b) - (5\*Sqrt[1 - (a + b\*x)^2]\*ArcCos[a + b\*x]^(3/2))/(2\*b) + ((a + b\*x)\*ArcCos[a + b\*x]^(5/2))/b + (15\*Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Sqrt[ArcCos[a + b\*x]]])/(4\*b)

**Rubi [A]** time = 0.14736, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {4804, 4620, 4678, 4724, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a + bx)}\right)}{4b} + \frac{(a + bx)\cos^{-1}(a + bx)^{5/2}}{b} - \frac{5\sqrt{1 - (a + bx)^2}\cos^{-1}(a + bx)^{3/2}}{2b} - \frac{15(a + bx)\sqrt{\cos^{-1}(a + bx)}}{4b}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a + b\*x]^(5/2), x]

[Out] (-15\*(a + b\*x)\*Sqrt[ArcCos[a + b\*x]]/(4\*b) - (5\*Sqrt[1 - (a + b\*x)^2]\*ArcCos[a + b\*x]^(3/2))/(2\*b) + ((a + b\*x)\*ArcCos[a + b\*x]^(5/2))/b + (15\*Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Sqrt[ArcCos[a + b\*x]]])/(4\*b)

#### Rule 4804

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_.), x\_Symbol] := Dist[1/d, Subst[Int[(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, n}, x]

#### Rule 4620

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCos[c\*x])^n, x] + Dist[b\*c\*n, Int[(x\*(a + b\*ArcCos[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4678

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)\*((d\_.) + (e\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCos[c\*x])^n)/(2\*e\*(p + 1)), x] - Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcCos[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

#### Rule 4724

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)^(m\_.)\*((d\_.) + (e\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] := -Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Cos[x]^m\*Sin[x]^(2\*p + 1), x], x, ArcCos[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

Rule 3304

`Int[sin[Pi/2 + (e_.) + (f_.)*(x_.)]/Sqrt[(c_.) + (d_.)*(x_.)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]`

Rule 3352

`Int[Cos[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]`

Rubi steps

$$\begin{aligned} \int \cos^{-1}(a + bx)^{5/2} dx &= \frac{\text{Subst}\left(\int \cos^{-1}(x)^{5/2} dx, x, a + bx\right)}{b} \\ &= \frac{(a + bx) \cos^{-1}(a + bx)^{5/2}}{b} + \frac{5 \text{Subst}\left(\int \frac{x \cos^{-1}(x)^{3/2}}{\sqrt{1-x^2}} dx, x, a + bx\right)}{2b} \\ &= -\frac{5\sqrt{1 - (a + bx)^2} \cos^{-1}(a + bx)^{3/2}}{2b} + \frac{(a + bx) \cos^{-1}(a + bx)^{5/2}}{b} - \frac{15 \text{Subst}\left(\int \sqrt{\cos^{-1}(x)} dx, x, a + bx\right)}{4b} \\ &= -\frac{15(a + bx)\sqrt{\cos^{-1}(a + bx)}}{4b} - \frac{5\sqrt{1 - (a + bx)^2} \cos^{-1}(a + bx)^{3/2}}{2b} + \frac{(a + bx) \cos^{-1}(a + bx)^{5/2}}{b} \\ &= -\frac{15(a + bx)\sqrt{\cos^{-1}(a + bx)}}{4b} - \frac{5\sqrt{1 - (a + bx)^2} \cos^{-1}(a + bx)^{3/2}}{2b} + \frac{(a + bx) \cos^{-1}(a + bx)^{5/2}}{b} \\ &= -\frac{15(a + bx)\sqrt{\cos^{-1}(a + bx)}}{4b} - \frac{5\sqrt{1 - (a + bx)^2} \cos^{-1}(a + bx)^{3/2}}{2b} + \frac{(a + bx) \cos^{-1}(a + bx)^{5/2}}{b} \\ &= -\frac{15(a + bx)\sqrt{\cos^{-1}(a + bx)}}{4b} - \frac{5\sqrt{1 - (a + bx)^2} \cos^{-1}(a + bx)^{3/2}}{2b} + \frac{(a + bx) \cos^{-1}(a + bx)^{5/2}}{b} \end{aligned}$$

**Mathematica [C]** time = 0.0488174, size = 90, normalized size = 0.81

$$\frac{\frac{\sqrt{\cos^{-1}(a+bx)}\Gamma\left(\frac{7}{2}, -i\cos^{-1}(a+bx)\right)}{2\sqrt{-i\cos^{-1}(a+bx)}} + \frac{\sqrt{\cos^{-1}(a+bx)}\Gamma\left(\frac{7}{2}, i\cos^{-1}(a+bx)\right)}{2\sqrt{i\cos^{-1}(a+bx)}}}{b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCos[a + b\*x]^(5/2), x]

[Out] -(((Sqrt[ArcCos[a + b\*x]]\*Gamma[7/2, (-I)\*ArcCos[a + b\*x]])/(2\*Sqrt[(-I)\*ArcCos[a + b\*x]]) + (Sqrt[ArcCos[a + b\*x]]\*Gamma[7/2, I\*ArcCos[a + b\*x]])/(2\*Sqrt[I\*ArcCos[a + b\*x]]))/b)

**Maple [A]** time = 0.092, size = 140, normalized size = 1.3

$$-\frac{\sqrt{2}}{8b\sqrt{\pi}} \left( -4 (\arccos(bx + a))^{5/2} \sqrt{2}\sqrt{\pi}xb - 4 (\arccos(bx + a))^{5/2} \sqrt{2}\sqrt{\pi}a + 10 (\arccos(bx + a))^{3/2} \sqrt{2}\sqrt{\pi}\sqrt{-b^2x^2 - 2ax - a^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(b\*x+a)^(5/2), x)

```
[Out] -1/8/b*2^(1/2)*(-4*arccos(b*x+a)^(5/2)*2^(1/2)*Pi^(1/2)*x*b-4*arccos(b*x+a)
^(5/2)*2^(1/2)*Pi^(1/2)*a+10*arccos(b*x+a)^(3/2)*2^(1/2)*Pi^(1/2)*(-b^2*x^2
-2*a*b*x-a^2+1)^(1/2)+15*2^(1/2)*arccos(b*x+a)^(1/2)*Pi^(1/2)*x*b+15*2^(1/2
)*arccos(b*x+a)^(1/2)*Pi^(1/2)*a-15*Pi*FresnelC(2^(1/2)/Pi^(1/2)*arccos(b*x
+a)^(1/2)))/Pi^(1/2)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

**Giac [B]** time = 1.57023, size = 282, normalized size = 2.54

$$\frac{5i \arccos(bx+a)^{\frac{3}{2}} e^{i \arccos(bx+a)}}{4b} + \frac{\arccos(bx+a)^{\frac{5}{2}} e^{i \arccos(bx+a)}}{2b} - \frac{5i \arccos(bx+a)^{\frac{3}{2}} e^{-i \arccos(bx+a)}}{4b} + \frac{\arccos(bx+a)^{\frac{5}{2}} e^{-i \arccos(bx+a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5/4*i*arccos(b*x + a)^(3/2)*e^(i*arccos(b*x + a))/b + 1/2*arccos(b*x + a)^(
5/2)*e^(i*arccos(b*x + a))/b - 5/4*i*arccos(b*x + a)^(3/2)*e^(-i*arccos(b*x
+ a))/b + 1/2*arccos(b*x + a)^(5/2)*e^(-i*arccos(b*x + a))/b - 15/16*sqrt(
2)*sqrt(pi)*i*erf(sqrt(2)*sqrt(arccos(b*x + a)))/(i - 1)/(b*(i - 1)) - 15/8
```

```
*sqrt(arccos(b*x + a))*e^(i*arccos(b*x + a))/b - 15/8*sqrt(arccos(b*x + a))
*e^(-i*arccos(b*x + a))/b + 15/16*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*i*sqrt(arcc
os(b*x + a))/(i - 1))/(b*(i - 1))
```

### 3.38 $\int \cos^{-1}(a + bx)^{3/2} dx$

**Optimal.** Leaf size=89

$$\frac{3\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{2b} + \frac{(a+bx)\cos^{-1}(a+bx)^{3/2}}{b} - \frac{3\sqrt{1-(a+bx)^2}\sqrt{\cos^{-1}(a+bx)}}{2b}$$

[Out] (-3\*Sqrt[1 - (a + b\*x)^2]\*Sqrt[ArcCos[a + b\*x]])/(2\*b) + ((a + b\*x)\*ArcCos[a + b\*x]^(3/2))/b + (3\*Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Sqrt[ArcCos[a + b\*x]]])/(2\*b)

**Rubi [A]** time = 0.09211, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {4804, 4620, 4678, 4624, 3305, 3351}

$$\frac{3\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{2b} + \frac{(a+bx)\cos^{-1}(a+bx)^{3/2}}{b} - \frac{3\sqrt{1-(a+bx)^2}\sqrt{\cos^{-1}(a+bx)}}{2b}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a + b\*x]^(3/2), x]

[Out] (-3\*Sqrt[1 - (a + b\*x)^2]\*Sqrt[ArcCos[a + b\*x]])/(2\*b) + ((a + b\*x)\*ArcCos[a + b\*x]^(3/2))/b + (3\*Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Sqrt[ArcCos[a + b\*x]]])/(2\*b)

#### Rule 4804

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] := Dist[1/d, Subst[Int[(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, n}, x]

#### Rule 4620

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCos[c\*x])^n, x] + Dist[b\*c\*n, Int[(x\*(a + b\*ArcCos[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4678

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCos[c\*x])^n)/(2\*e\*(p + 1)), x] - Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcCos[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

#### Rule 4624

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] := Dist[1/(b\*c), Subst[Int[x^n\*Sin[a/b - x/b], x], x, a + b\*ArcCos[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

#### Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

### Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

### Rubi steps

$$\begin{aligned} \int \cos^{-1}(a + bx)^{3/2} dx &= \frac{\text{Subst}\left(\int \cos^{-1}(x)^{3/2} dx, x, a + bx\right)}{b} \\ &= \frac{(a + bx) \cos^{-1}(a + bx)^{3/2}}{b} + \frac{3 \text{Subst}\left(\int \frac{x \sqrt{\cos^{-1}(x)}}{\sqrt{1-x^2}} dx, x, a + bx\right)}{2b} \\ &= -\frac{3\sqrt{1 - (a + bx)^2} \sqrt{\cos^{-1}(a + bx)}}{2b} + \frac{(a + bx) \cos^{-1}(a + bx)^{3/2}}{b} - \frac{3 \text{Subst}\left(\int \frac{1}{\sqrt{\cos^{-1}(x)}} dx, x, a + bx\right)}{4b} \\ &= -\frac{3\sqrt{1 - (a + bx)^2} \sqrt{\cos^{-1}(a + bx)}}{2b} + \frac{(a + bx) \cos^{-1}(a + bx)^{3/2}}{b} + \frac{3 \text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}} dx, x, \cos^{-1}(a + bx)\right)}{4b} \\ &= -\frac{3\sqrt{1 - (a + bx)^2} \sqrt{\cos^{-1}(a + bx)}}{2b} + \frac{(a + bx) \cos^{-1}(a + bx)^{3/2}}{b} + \frac{3 \text{Subst}\left(\int \sin(x^2) dx, x, \sqrt{\cos^{-1}(a + bx)}\right)}{2b} \\ &= -\frac{3\sqrt{1 - (a + bx)^2} \sqrt{\cos^{-1}(a + bx)}}{2b} + \frac{(a + bx) \cos^{-1}(a + bx)^{3/2}}{b} + \frac{3\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\cos^{-1}(a + bx)}\right)}{2b} \end{aligned}$$

**Mathematica [C]** time = 0.0353963, size = 76, normalized size = 0.85

$$\frac{\sqrt{-i \cos^{-1}(a + bx)} \Gamma\left(\frac{5}{2}, -i \cos^{-1}(a + bx)\right) + \sqrt{i \cos^{-1}(a + bx)} \Gamma\left(\frac{5}{2}, i \cos^{-1}(a + bx)\right)}{2b \sqrt{\cos^{-1}(a + bx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a + b*x]^(3/2), x]
```

```
[Out] -(Sqrt[(-I)*ArcCos[a + b*x]]*Gamma[5/2, (-I)*ArcCos[a + b*x]] + Sqrt[I*ArcC
os[a + b*x]]*Gamma[5/2, I*ArcCos[a + b*x]])/(2*b*Sqrt[ArcCos[a + b*x]])
```

**Maple [A]** time = 0.082, size = 105, normalized size = 1.2

$$\frac{\sqrt{2}}{4b\sqrt{\pi}} \left( 2 (\arccos(bx + a))^{3/2} \sqrt{2}\sqrt{\pi}xb + 2 (\arccos(bx + a))^{3/2} \sqrt{2}\sqrt{\pi}a - 3 \sqrt{2}\sqrt{\pi} \sqrt{\arccos(bx + a)} \sqrt{-b^2x^2 - 2xab - a^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)^(3/2), x)
```

```
[Out] 1/4/b*2^(1/2)/Pi^(1/2)*(2*arccos(b*x+a)^(3/2)*2^(1/2)*Pi^(1/2)*x*b+2*arccos
(b*x+a)^(3/2)*2^(1/2)*Pi^(1/2)*a-3*2^(1/2)*Pi^(1/2)*arccos(b*x+a)^(1/2)*(-b
^2*x^2-2*a*b*x-a^2+1)^(1/2)+3*Pi*FresnelS(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1
```

/2)))

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \arccos^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(b\*x+a)\*\*(3/2),x)

[Out] Integral(acos(a + b\*x)\*\*(3/2), x)

---

**Giac [B]** time = 1.62439, size = 221, normalized size = 2.48

$$\frac{3i\sqrt{\arccos(bx+a)}e^{i\arccos(bx+a)}}{4b} + \frac{\arccos(bx+a)^{\frac{3}{2}}e^{i\arccos(bx+a)}}{2b} - \frac{3i\sqrt{\arccos(bx+a)}e^{-i\arccos(bx+a)}}{4b} + \frac{\arccos(bx+a)^{\frac{3}{2}}e^{-i\arccos(bx+a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)^(3/2),x, algorithm="giac")

[Out]  $\frac{3}{4}i\sqrt{\arccos(bx+a)}e^{i\arccos(bx+a)}/b + \frac{1}{2}\arccos(bx+a)^{\frac{3}{2}}e^{i\arccos(bx+a)}/b - \frac{3}{4}i\sqrt{\arccos(bx+a)}e^{-i\arccos(bx+a)}/b + \frac{1}{2}\arccos(bx+a)^{\frac{3}{2}}e^{-i\arccos(bx+a)}/b - \frac{3}{8}\sqrt{2}\sqrt{\pi}i\operatorname{erf}(-\sqrt{2}i\sqrt{\arccos(bx+a)})/(i-1)/(b(i-1)) + \frac{3}{8}\sqrt{2}\sqrt{\pi}\operatorname{erf}(\sqrt{2}\sqrt{\arccos(bx+a)})/(i-1)/(b(i-1))$



### 3.39 $\int \sqrt{\cos^{-1}(a + bx)} dx$

**Optimal.** Leaf size=55

$$\frac{(a + bx)\sqrt{\cos^{-1}(a + bx)}}{b} - \frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a + bx)}\right)}{b}$$

[Out] ((a + b\*x)\*Sqrt[ArcCos[a + b\*x]])/b - (Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Sqrt[ArcCos[a + b\*x]]])/b

**Rubi [A]** time = 0.0806047, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4804, 4620, 4724, 3304, 3352}

$$\frac{(a + bx)\sqrt{\cos^{-1}(a + bx)}}{b} - \frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a + bx)}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[ArcCos[a + b\*x]], x]

[Out] ((a + b\*x)\*Sqrt[ArcCos[a + b\*x]])/b - (Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Sqrt[ArcCos[a + b\*x]]])/b

#### Rule 4804

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, n}, x]

#### Rule 4620

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcCos[c\*x])^n, x] + Dist[b\*c\*n, Int[(x\*(a + b\*ArcCos[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4724

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Cos[x]^m\*Sin[x]^(2\*p + 1), x], x, ArcCos[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_)^2)], x\_Symbol] :> Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/ (f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos^{-1}(a+bx)} dx &= \frac{\text{Subst}\left(\int \sqrt{\cos^{-1}(x)} dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)\sqrt{\cos^{-1}(a+bx)}}{b} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{\cos^{-1}(x)}} dx, x, a+bx\right)}{2b} \\
&= \frac{(a+bx)\sqrt{\cos^{-1}(a+bx)}}{b} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}} dx, x, \cos^{-1}(a+bx)\right)}{2b} \\
&= \frac{(a+bx)\sqrt{\cos^{-1}(a+bx)}}{b} - \frac{\text{Subst}\left(\int \cos(x^2) dx, x, \sqrt{\cos^{-1}(a+bx)}\right)}{b} \\
&= \frac{(a+bx)\sqrt{\cos^{-1}(a+bx)}}{b} - \frac{\sqrt{\frac{\pi}{2}} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\cos^{-1}(a+bx)}\right)}{b}
\end{aligned}$$

**Mathematica [C]** time = 0.0390328, size = 90, normalized size = 1.64

$$-\frac{\sqrt{\cos^{-1}(a+bx)}\Gamma\left(\frac{3}{2}, -i\cos^{-1}(a+bx)\right)}{2\sqrt{-i\cos^{-1}(a+bx)}} - \frac{\sqrt{\cos^{-1}(a+bx)}\Gamma\left(\frac{3}{2}, i\cos^{-1}(a+bx)\right)}{2\sqrt{i\cos^{-1}(a+bx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[ArcCos[a + b\*x]], x]

[Out] -((- (Sqrt[ArcCos[a + b\*x]]\*Gamma[3/2, (-I)\*ArcCos[a + b\*x]])/(2\*Sqrt[(-I)\*ArcCos[a + b\*x]]) - (Sqrt[ArcCos[a + b\*x]]\*Gamma[3/2, I\*ArcCos[a + b\*x]])/(2\*Sqrt[I\*ArcCos[a + b\*x]]))/b)

**Maple [A]** time = 0.073, size = 66, normalized size = 1.2

$$\frac{1}{2b} \left( -\sqrt{2}\sqrt{\arccos(bx+a)}\sqrt{\pi}\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\sqrt{\arccos(bx+a)}\right) + 2\arccos(bx+a)xb + 2\arccos(bx+a)a \right) \frac{1}{\sqrt{\arccos(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(b\*x+a)^(1/2), x)

[Out] 1/2/b/arccos(b\*x+a)^(1/2)\*(-2^(1/2)\*arccos(b\*x+a)^(1/2)\*Pi^(1/2)\*FresnelC(2^(1/2)/Pi^(1/2)\*arccos(b\*x+a)^(1/2))+2\*arccos(b\*x+a)\*x\*b+2\*arccos(b\*x+a)\*a)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)^(1/2), x, algorithm="maxima")

[Out] Exception raised: RuntimeError

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\arccos(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(b\*x+a)\*\*(1/2),x)

[Out] Integral(sqrt(acos(a + b\*x)), x)

---

**Giac [B]** time = 1.37766, size = 158, normalized size = 2.87

$$\frac{\sqrt{2}\sqrt{\pi}i \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\arccos(bx+a)}}{i-1}\right)}{4b(i-1)} + \frac{\sqrt{\arccos(bx+a)}e^{i\arccos(bx+a)}}{2b} + \frac{\sqrt{\arccos(bx+a)}e^{-i\arccos(bx+a)}}{2b} - \frac{\sqrt{2}\sqrt{\pi} \operatorname{erf}\left(-\frac{\sqrt{2}i}{4b(i-1)}\right)}{4b(i-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(b\*x+a)^(1/2),x, algorithm="giac")

[Out] 1/4\*sqrt(2)\*sqrt(pi)\*i\*erf(sqrt(2)\*sqrt(arccos(b\*x + a))/(i - 1))/(b\*(i - 1)) + 1/2\*sqrt(arccos(b\*x + a))\*e^(i\*arccos(b\*x + a))/b + 1/2\*sqrt(arccos(b\*x + a))\*e^(-i\*arccos(b\*x + a))/b - 1/4\*sqrt(2)\*sqrt(pi)\*erf(-sqrt(2)\*i\*sqrt(arccos(b\*x + a))/(i - 1))/(b\*(i - 1))

$$3.40 \quad \int \frac{1}{\sqrt{\cos^{-1}(a+bx)}} dx$$

**Optimal.** Leaf size=33

$$-\frac{\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{b}$$

[Out] -((Sqrt[2\*Pi]\*FresnelS[Sqrt[2/Pi]\*Sqrt[ArcCos[a + b\*x]]])/b)

**Rubi [A]** time = 0.0297642, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4804, 4624, 3305, 3351}

$$-\frac{\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[ArcCos[a + b\*x]],x]

[Out] -((Sqrt[2\*Pi]\*FresnelS[Sqrt[2/Pi]\*Sqrt[ArcCos[a + b\*x]]])/b)

#### Rule 4804

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_.), x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, n}, x]

#### Rule 4624

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.), x\_Symbol] :> Dist[1/(b\*c), Subst[Int[x^n\*Sin[a/b - x/b], x], x, a + b\*ArcCos[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/Sqrt[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_.))^2], x\_Symbol] :> Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\cos^{-1}(a+bx)}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cos^{-1}(x)}} dx, x, a+bx\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}} dx, x, \cos^{-1}(a+bx)\right)}{b} \\
&= -\frac{2 \text{Subst}\left(\int \sin(x^2) dx, x, \sqrt{\cos^{-1}(a+bx)}\right)}{b} \\
&= -\frac{\sqrt{2\pi} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\cos^{-1}(a+bx)}\right)}{b}
\end{aligned}$$

**Mathematica [C]** time = 0.0316839, size = 78, normalized size = 2.36

$$\frac{-\sqrt{-i \cos^{-1}(a+bx)} \text{Gamma}\left(\frac{1}{2}, -i \cos^{-1}(a+bx)\right) - \sqrt{i \cos^{-1}(a+bx)} \text{Gamma}\left(\frac{1}{2}, i \cos^{-1}(a+bx)\right)}{2b \sqrt{\cos^{-1}(a+bx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/Sqrt[ArcCos[a + b\*x]], x]

[Out] -(-(Sqrt[(-I)\*ArcCos[a + b\*x]]\*Gamma[1/2, (-I)\*ArcCos[a + b\*x]]) - Sqrt[I\*ArcCos[a + b\*x]]\*Gamma[1/2, I\*ArcCos[a + b\*x]])/(2\*b\*Sqrt[ArcCos[a + b\*x]])

**Maple [A]** time = 0.055, size = 28, normalized size = 0.9

$$-\frac{\sqrt{2}\sqrt{\pi}}{b} \text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}} \sqrt{\arccos(bx+a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arccos(b\*x+a)^(1/2), x)

[Out] -FresnelS(2^(1/2)/Pi^(1/2)\*arccos(b\*x+a)^(1/2))\*2^(1/2)\*Pi^(1/2)/b

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(b\*x+a)^(1/2), x, algorithm="maxima")

[Out] Exception raised: RuntimeError

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/arccos(b*x+a)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\arccos(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/acos(b*x+a)**(1/2),x)`

[Out] `Integral(1/sqrt(acos(a + b*x)), x)`

**Giac [B]** time = 1.57481, size = 97, normalized size = 2.94

$$\frac{\sqrt{2}\sqrt{\pi}i \operatorname{erf}\left(-\frac{\sqrt{2}i\sqrt{\arccos(bx+a)}}{i-1}\right)}{2b(i-1)} - \frac{\sqrt{2}\sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\arccos(bx+a)}}{i-1}\right)}{2b(i-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/arccos(b*x+a)^(1/2),x, algorithm="giac")`

[Out] `1/2*sqrt(2)*sqrt(pi)*i*erf(-sqrt(2)*i*sqrt(arccos(b*x + a))/(i - 1))/(b*(i - 1)) - 1/2*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(arccos(b*x + a))/(i - 1))/(b*(i - 1))`

$$3.41 \quad \int \frac{1}{\cos^{-1}(a+bx)^{3/2}} dx$$

**Optimal.** Leaf size=64

$$\frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}} - \frac{2\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{b}$$

[Out] (2\*Sqrt[1 - (a + b\*x)^2])/(b\*Sqrt[ArcCos[a + b\*x]]) - (2\*Sqrt[2\*Pi]\*FresnelC[Sqrt[2/Pi]\*Sqrt[ArcCos[a + b\*x]]])/b

**Rubi [A]** time = 0.0860013, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4804, 4622, 4724, 3304, 3352}

$$\frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}} - \frac{2\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a + b\*x]^(-3/2), x]

[Out] (2\*Sqrt[1 - (a + b\*x)^2])/(b\*Sqrt[ArcCos[a + b\*x]]) - (2\*Sqrt[2\*Pi]\*FresnelC[Sqrt[2/Pi]\*Sqrt[ArcCos[a + b\*x]]])/b

#### Rule 4804

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^ (n\_.), x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, n}, x]

#### Rule 4622

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^ (n\_.), x\_Symbol] :> -Simp[(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcCos[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[c/(b\*(n + 1)), Int[(x\*(a + b\*ArcCos[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

#### Rule 4724

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^ (n\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Cos[x]^m\*Sin[x]^(2\*p + 1), x], x, ArcCos[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_.)]/Sqrt[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{\cos^{-1}(a+bx)^{3/2}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)^{3/2}} dx, x, a+bx\right)}{b} \\ &= \frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{\cos^{-1}(x)}} dx, x, a+bx\right)}{b} \\ &= \frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}} - \frac{2\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}} dx, x, \cos^{-1}(a+bx)\right)}{b} \\ &= \frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}} - \frac{4\text{Subst}\left(\int \cos(x^2) dx, x, \sqrt{\cos^{-1}(a+bx)}\right)}{b} \\ &= \frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}} - \frac{2\sqrt{2}\pi C\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{b} \end{aligned}$$

**Mathematica [C]** time = 0.0535818, size = 97, normalized size = 1.52

$$\frac{-i\sqrt{-i\cos^{-1}(a+bx)}\Gamma\left(\frac{1}{2}, -i\cos^{-1}(a+bx)\right) + i\sqrt{i\cos^{-1}(a+bx)}\Gamma\left(\frac{1}{2}, i\cos^{-1}(a+bx)\right) - 2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a + b*x]^(-3/2), x]
```

```
[Out] -((-2*Sqrt[1 - (a + b*x)^2] - I*Sqrt[(-I)*ArcCos[a + b*x]]*Gamma[1/2, (-I)*ArcCos[a + b*x]] + I*Sqrt[I*ArcCos[a + b*x]]*Gamma[1/2, I*ArcCos[a + b*x]])/(b*Sqrt[ArcCos[a + b*x]])
```

**Maple [A]** time = 0.075, size = 84, normalized size = 1.3

$$-\frac{\sqrt{2}}{b\sqrt{\pi}\arccos(bx+a)}\left(2\arccos(bx+a)\pi\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{\arccos(bx+a)}}{\sqrt{\pi}}\right) - \sqrt{2}\sqrt{\pi}\sqrt{\arccos(bx+a)}\sqrt{-b^2x^2 - 2xab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(b*x+a)^(3/2), x)
```

```
[Out] -1/b*2^(1/2)/Pi^(1/2)/arccos(b*x+a)*(2*arccos(b*x+a)*Pi*FresnelC(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2))-2^(1/2)*Pi^(1/2)*arccos(b*x+a)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(b\*x+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(b\*x+a)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acos}^{\frac{3}{2}}(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/acos(b\*x+a)\*\*(3/2),x)

[Out] Integral(acos(a + b\*x)\*\*(-3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\arccos(bx + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(b\*x+a)^(3/2),x, algorithm="giac")

[Out] integrate(arccos(b\*x + a)^(-3/2), x)

### 3.42 $\int \frac{1}{\cos^{-1}(a+bx)^{5/2}} dx$

**Optimal.** Leaf size=90

$$\frac{4\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{3b} + \frac{4(a+bx)}{3b\sqrt{\cos^{-1}(a+bx)}} + \frac{2\sqrt{1-(a+bx)^2}}{3b\cos^{-1}(a+bx)^{3/2}}$$

[Out] (2\*Sqrt[1 - (a + b\*x)^2])/(3\*b\*ArcCos[a + b\*x]^(3/2)) + (4\*(a + b\*x))/(3\*b\*Sqrt[ArcCos[a + b\*x]]) + (4\*Sqrt[2\*Pi]\*FresnelS[Sqrt[2/Pi]\*Sqrt[ArcCos[a + b\*x]]])/(3\*b)

**Rubi [A]** time = 0.0920133, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {4804, 4622, 4720, 4624, 3305, 3351}

$$\frac{4\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{3b} + \frac{4(a+bx)}{3b\sqrt{\cos^{-1}(a+bx)}} + \frac{2\sqrt{1-(a+bx)^2}}{3b\cos^{-1}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a + b\*x]^(-5/2), x]

[Out] (2\*Sqrt[1 - (a + b\*x)^2])/(3\*b\*ArcCos[a + b\*x]^(3/2)) + (4\*(a + b\*x))/(3\*b\*Sqrt[ArcCos[a + b\*x]]) + (4\*Sqrt[2\*Pi]\*FresnelS[Sqrt[2/Pi]\*Sqrt[ArcCos[a + b\*x]]])/(3\*b)

#### Rule 4804

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^ (n\_.), x\_Symbol] := Dist[1/d, Subst[Int[(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, n}, x]

#### Rule 4622

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^ (n\_.), x\_Symbol] := -Simp[(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcCos[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[c/(b\*(n + 1)), Int[(x\*(a + b\*ArcCos[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

#### Rule 4720

Int[(((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^ (n\_.)\*((f\_.)\*(x\_.))^ (m\_.))/Sqrt[(d\_.) + (e\_.)\*(x\_.)^2], x\_Symbol] := -Simp[((f\*x)^m\*(a + b\*ArcCos[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] + Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)^(m - 1)\*(a + b\*ArcCos[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

#### Rule 4624

Int[((a\_.) + ArcCos[(c\_.)\*(x\_.)]\*(b\_.))^ (n\_.), x\_Symbol] := Dist[1/(b\*c), Subst[Int[x^n\*Sin[a/b - x/b], x], x, a + b\*ArcCos[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

#### Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

### Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{\cos^{-1}(a+bx)^{5/2}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)^{5/2}} dx, x, a+bx\right)}{b} \\ &= \frac{2\sqrt{1-(a+bx)^2}}{3b \cos^{-1}(a+bx)^{3/2}} + \frac{2 \text{Subst}\left(\int \frac{x}{\sqrt{1-x^2} \cos^{-1}(x)^{3/2}} dx, x, a+bx\right)}{3b} \\ &= \frac{2\sqrt{1-(a+bx)^2}}{3b \cos^{-1}(a+bx)^{3/2}} + \frac{4(a+bx)}{3b\sqrt{\cos^{-1}(a+bx)}} - \frac{4 \text{Subst}\left(\int \frac{1}{\sqrt{\cos^{-1}(x)}} dx, x, a+bx\right)}{3b} \\ &= \frac{2\sqrt{1-(a+bx)^2}}{3b \cos^{-1}(a+bx)^{3/2}} + \frac{4(a+bx)}{3b\sqrt{\cos^{-1}(a+bx)}} + \frac{4 \text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}} dx, x, \cos^{-1}(a+bx)\right)}{3b} \\ &= \frac{2\sqrt{1-(a+bx)^2}}{3b \cos^{-1}(a+bx)^{3/2}} + \frac{4(a+bx)}{3b\sqrt{\cos^{-1}(a+bx)}} + \frac{8 \text{Subst}\left(\int \sin(x^2) dx, x, \sqrt{\cos^{-1}(a+bx)}\right)}{3b} \\ &= \frac{2\sqrt{1-(a+bx)^2}}{3b \cos^{-1}(a+bx)^{3/2}} + \frac{4(a+bx)}{3b\sqrt{\cos^{-1}(a+bx)}} + \frac{4\sqrt{2\pi} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\cos^{-1}(a+bx)}\right)}{3b} \end{aligned}$$

**Mathematica [C]** time = 0.291454, size = 139, normalized size = 1.54

$$\frac{2 \left( i \left( -i \cos^{-1}(a+bx) \right)^{3/2} \text{Gamma}\left(\frac{1}{2}, -i \cos^{-1}(a+bx)\right) - i \left( i \cos^{-1}(a+bx) \right)^{3/2} \text{Gamma}\left(\frac{1}{2}, i \cos^{-1}(a+bx)\right) - \sqrt{1-(a+bx)^2} \right)}{3b \cos^{-1}(a+bx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a + b*x]^(-5/2), x]
```

```
[Out] (-2*(-Sqrt[1 - (a + b*x)^2] - ArcCos[a + b*x])/E^(I*ArcCos[a + b*x]) - E^(I*ArcCos[a + b*x])*ArcCos[a + b*x] + I*((-I)*ArcCos[a + b*x])^(3/2)*Gamma[1/2, (-I)*ArcCos[a + b*x]] - I*(I*ArcCos[a + b*x])^(3/2)*Gamma[1/2, I*ArcCos[a + b*x]])/(3*b*ArcCos[a + b*x]^(3/2))
```

**Maple [A]** time = 0.079, size = 120, normalized size = 1.3

$$\frac{\sqrt{2}}{3b\sqrt{\pi}(\arccos(bx+a))^2} \left( 4\pi(\arccos(bx+a))^2 \text{FresnelS}\left(\frac{\sqrt{2}\sqrt{\arccos(bx+a)}}{\sqrt{\pi}}\right) + 2(\arccos(bx+a))^{3/2} \sqrt{2}\sqrt{\pi}xb \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(b*x+a)^(5/2), x)
```

[Out]  $\frac{1}{3} \frac{1}{b} 2^{(1/2)} / \pi^{(1/2)} * (4 * \pi * \arccos(b*x+a)^2 * \text{FresnelS}(2^{(1/2)} / \pi^{(1/2)} * \arccos(b*x+a)^{(1/2)}) + 2 * \arccos(b*x+a)^{(3/2)} * 2^{(1/2)} * \pi^{(1/2)} * x * b + 2 * \arccos(b*x+a)^{(3/2)} * 2^{(1/2)} * \pi^{(1/2)} * a + 2^{(1/2)} * \pi^{(1/2)} * \arccos(b*x+a)^{(1/2)} * (-b^2 * x^2 - 2 * a * b * x - a^2 + 1)^{(1/2)}) / \arccos(b*x+a)^2$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(b\*x+a)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(b\*x+a)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\arccos^2(a + bx)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/acos(b\*x+a)\*\*(5/2),x)

[Out] Integral(acos(a + b\*x)\*\*(-5/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\arccos(bx + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(b\*x+a)^(5/2),x, algorithm="giac")

[Out] integrate(arccos(b\*x + a)^(-5/2), x)

$$3.43 \quad \int \frac{1}{\sqrt{a+b \cos^{-1}(c+dx)}} dx$$

**Optimal.** Leaf size=106

$$\frac{\sqrt{2\pi} \sin\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{bd}} - \frac{\sqrt{2\pi} \cos\left(\frac{a}{b}\right) \text{S}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{bd}}$$

[Out]  $-\left(\frac{\text{Sqrt}[2*\text{Pi}]*\text{Cos}[a/b]*\text{FresnelS}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcCos}[c + d*x]])/\text{Sqrt}[b]]}{(\text{Sqrt}[b]*d)} + \frac{\text{Sqrt}[2*\text{Pi}]*\text{FresnelC}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcCos}[c + d*x]])/\text{Sqrt}[b]]*\text{Sin}[a/b]}{(\text{Sqrt}[b]*d)}\right)$

**Rubi [A]** time = 0.130424, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4804, 4624, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{2\pi} \sin\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{bd}} - \frac{\sqrt{2\pi} \cos\left(\frac{a}{b}\right) \text{S}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{bd}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b\*ArcCos[c + d\*x]], x]

[Out]  $-\left(\frac{\text{Sqrt}[2*\text{Pi}]*\text{Cos}[a/b]*\text{FresnelS}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcCos}[c + d*x]])/\text{Sqrt}[b]]}{(\text{Sqrt}[b]*d)} + \frac{\text{Sqrt}[2*\text{Pi}]*\text{FresnelC}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcCos}[c + d*x]])/\text{Sqrt}[b]]*\text{Sin}[a/b]}{(\text{Sqrt}[b]*d)}\right)$

#### Rule 4804

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, n}, x]

#### Rule 4624

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] :> Dist[1/(b\*c), Subst[Int[x^n\*Sin[a/b - x/b], x], x, a + b\*ArcCos[c\*x], x] /; FreeQ[{a, b, c, n}, x]

#### Rule 3306

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d\*e - c\*f, 0]

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)]]/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x<sup>2</sup>)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)]]/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a + b \cos^{-1}(c + dx)}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a + b \cos^{-1}(x)}} dx, x, c + dx\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{a-x}{b}\right)}{\sqrt{x}} dx, x, a + b \cos^{-1}(c + dx)\right)}{bd} \\ &= -\frac{\cos\left(\frac{a}{b}\right) \text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}} dx, x, a + b \cos^{-1}(c + dx)\right)}{bd} + \frac{\sin\left(\frac{a}{b}\right) \text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}} dx, x, a + b \cos^{-1}(c + dx)\right)}{bd} \\ &= -\frac{\left(2 \cos\left(\frac{a}{b}\right)\right) \text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right) dx, x, \sqrt{a + b \cos^{-1}(c + dx)}\right)}{bd} + \frac{\left(2 \sin\left(\frac{a}{b}\right)\right) \text{Subst}\left(\int \cos\left(\frac{x^2}{b}\right) dx, x, \sqrt{a + b \cos^{-1}(c + dx)}\right)}{bd} \\ &= -\frac{\sqrt{2\pi} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a + b \cos^{-1}(c + dx)}}{\sqrt{b}}\right)}{\sqrt{bd}} + \frac{\sqrt{2\pi} C\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a + b \cos^{-1}(c + dx)}}{\sqrt{b}}\right) \sin\left(\frac{a}{b}\right)}{\sqrt{bd}} \end{aligned}$$

**Mathematica [C]** time = 0.104934, size = 128, normalized size = 1.21

$$\frac{e^{-\frac{ia}{b}} \left( \sqrt{-\frac{i(a+b \cos^{-1}(c+dx))}{b}} \text{Gamma}\left(\frac{1}{2}, -\frac{i(a+b \cos^{-1}(c+dx))}{b}\right) + e^{\frac{2ia}{b}} \sqrt{\frac{i(a+b \cos^{-1}(c+dx))}{b}} \text{Gamma}\left(\frac{1}{2}, \frac{i(a+b \cos^{-1}(c+dx))}{b}\right) \right)}{2d\sqrt{a + b \cos^{-1}(c + dx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/Sqrt[a + b\*ArcCos[c + d\*x]], x]

[Out] (Sqrt[((-I)\*(a + b\*ArcCos[c + d\*x]))/b]\*Gamma[1/2, ((-I)\*(a + b\*ArcCos[c + d\*x]))/b] + E^(((2\*I)\*a)/b)\*Sqrt[(I\*(a + b\*ArcCos[c + d\*x]))/b]\*Gamma[1/2, (I\*(a + b\*ArcCos[c + d\*x]))/b])/(2\*d\*E^((I\*a)/b)\*Sqrt[a + b\*ArcCos[c + d\*x]])

**Maple [A]** time = 0.071, size = 89, normalized size = 0.8

$$-\frac{\sqrt{2}\sqrt{\pi}}{d}\sqrt{b^{-1}}\left(\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi b}}\sqrt{a + b \arccos(dx + c)}\frac{1}{\sqrt{b^{-1}}}\right) - \sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi b}}\sqrt{a + b \arccos(dx + c)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*arccos(d*x+c))^(1/2),x)`

[Out] 
$$-2^{1/2} \pi^{1/2} (1/b)^{1/2} (\cos(a/b) \operatorname{FresnelS}(2^{1/2}/\pi^{1/2}) / (1/b)^{1/2} - \sin(a/b) \operatorname{FresnelC}(2^{1/2}/\pi^{1/2}) / (1/b)^{1/2}) (a+b \arccos(dx+c))^{1/2} / b - \sin(a/b) \operatorname{FresnelC}(2^{1/2}/\pi^{1/2}) / (1/b)^{1/2} (a+b \arccos(dx+c))^{1/2} / b) / d$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \arccos(dx+c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*arccos(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt(b*arccos(d*x + c) + a), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*arccos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \arccos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*acos(d*x+c))**(1/2),x)`

[Out] `Integral(1/sqrt(a + b*acos(c + d*x)), x)`

**Giac [A]** time = 2.34393, size = 231, normalized size = 2.18

$$\frac{\sqrt{\pi} i \operatorname{erf}\left(-\frac{\sqrt{2} \sqrt{b \arccos(dx+c)+ai}}{2 \sqrt{|b|}} - \frac{\sqrt{2} \sqrt{b \arccos(dx+c)+a \sqrt{|b|}}}{2b}\right) e^{\left(\frac{ai}{b}\right)} + \sqrt{\pi} i \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{b \arccos(dx+c)+ai}}{2 \sqrt{|b|}} - \frac{\sqrt{2} \sqrt{b \arccos(dx+c)+a \sqrt{|b|}}}{2b}\right) e^{\left(-\frac{ai}{b}\right)}}{\left(\frac{\sqrt{2} bi}{\sqrt{|b|}} + \sqrt{2} \sqrt{|b|}\right) d} + \frac{\sqrt{\pi} i \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{b \arccos(dx+c)+ai}}{2 \sqrt{|b|}} - \frac{\sqrt{2} \sqrt{b \arccos(dx+c)+a \sqrt{|b|}}}{2b}\right) e^{\left(-\frac{ai}{b}\right)}}{\left(\frac{\sqrt{2} bi}{\sqrt{|b|}} - \sqrt{2} \sqrt{|b|}\right) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*arccos(d*x+c))^(1/2),x, algorithm="giac")`

```
[Out] sqrt(pi)*i*erf(-1/2*sqrt(2)*sqrt(b*arccos(d*x + c) + a)*i/sqrt(abs(b)) - 1/
2*sqrt(2)*sqrt(b*arccos(d*x + c) + a)*sqrt(abs(b))/b)*e^(a*i/b)/((sqrt(2)*b
*i/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))*d) + sqrt(pi)*i*erf(1/2*sqrt(2)*sqr
t(b*arccos(d*x + c) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(d*x + c
) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((sqrt(2)*b*i/sqrt(abs(b)) - sqrt(2)*sqrt
(abs(b)))*d)
```



$$3.44 \quad \int \frac{1}{\sqrt{a-b \cos^{-1}(c+dx)}} dx$$

**Optimal.** Leaf size=108

$$\frac{\sqrt{2\pi} \sin\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a-b \cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{bd}} - \frac{\sqrt{2\pi} \cos\left(\frac{a}{b}\right) \text{S}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a-b \cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{bd}}$$

[Out] -((Sqrt[2\*Pi]\*Cos[a/b]\*FresnelS[(Sqrt[2/Pi]\*Sqrt[a - b\*ArcCos[c + d\*x]])/Sqrt[b]])/(Sqrt[b]\*d)) + (Sqrt[2\*Pi]\*FresnelC[(Sqrt[2/Pi]\*Sqrt[a - b\*ArcCos[c + d\*x]])/Sqrt[b]]\*Sin[a/b])/(Sqrt[b]\*d)

**Rubi [A]** time = 0.115801, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$ , Rules used = {4804, 4624, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{2\pi} \sin\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a-b \cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{bd}} - \frac{\sqrt{2\pi} \cos\left(\frac{a}{b}\right) \text{S}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a-b \cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{bd}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a - b\*ArcCos[c + d\*x]], x]

[Out] -((Sqrt[2\*Pi]\*Cos[a/b]\*FresnelS[(Sqrt[2/Pi]\*Sqrt[a - b\*ArcCos[c + d\*x]])/Sqrt[b]])/(Sqrt[b]\*d)) + (Sqrt[2\*Pi]\*FresnelC[(Sqrt[2/Pi]\*Sqrt[a - b\*ArcCos[c + d\*x]])/Sqrt[b]]\*Sin[a/b])/(Sqrt[b]\*d)

#### Rule 4804

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcCos[x])^n, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, n}, x]

#### Rule 4624

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] :> Dist[1/(b\*c), Subst[Int[x^n\*Sin[a/b - x/b], x], x, a + b\*ArcCos[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

#### Rule 3306

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d\*e - c\*f, 0]

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]]/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

### Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

### Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]]/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a-b \cos^{-1}(c+dx)}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-b \cos^{-1}(x)}} dx, x, c+dx\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{a-x}{b}\right)}{\sqrt{x}} dx, x, a-b \cos^{-1}(c+dx)\right)}{bd} \\ &= -\frac{\cos\left(\frac{a}{b}\right) \text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}} dx, x, a-b \cos^{-1}(c+dx)\right)}{bd} + \frac{\sin\left(\frac{a}{b}\right) \text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}} dx, x, a-b \cos^{-1}(c+dx)\right)}{bd} \\ &= -\frac{\left(2 \cos\left(\frac{a}{b}\right)\right) \text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right) dx, x, \sqrt{a-b \cos^{-1}(c+dx)}\right)}{bd} + \frac{\left(2 \sin\left(\frac{a}{b}\right)\right) \text{Subst}\left(\int \cos\left(\frac{x^2}{b}\right) dx, x, \sqrt{a-b \cos^{-1}(c+dx)}\right)}{bd} \\ &= -\frac{\sqrt{2\pi} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a-b \cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{bd}} + \frac{\sqrt{2\pi} C\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a-b \cos^{-1}(c+dx)}}{\sqrt{b}}\right) \sin\left(\frac{a}{b}\right)}{\sqrt{bd}} \end{aligned}$$

**Mathematica [C]** time = 0.101873, size = 133, normalized size = 1.23

$$\frac{e^{-\frac{ia}{b}} \left( \sqrt{-\frac{i(a-b \cos^{-1}(c+dx))}{b}} \Gamma\left(\frac{1}{2}, -\frac{i(a-b \cos^{-1}(c+dx))}{b}\right) + e^{\frac{2ia}{b}} \sqrt{\frac{i(a-b \cos^{-1}(c+dx))}{b}} \Gamma\left(\frac{1}{2}, \frac{i(a-b \cos^{-1}(c+dx))}{b}\right) \right)}{2d\sqrt{a-b \cos^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a - b*ArcCos[c + d*x]], x]
```

```
[Out] (Sqrt[((-I)*(a - b*ArcCos[c + d*x]))/b]*Gamma[1/2, ((-I)*(a - b*ArcCos[c + d*x]))/b] + E^(((2*I)*a)/b)*Sqrt[(I*(a - b*ArcCos[c + d*x]))/b]*Gamma[1/2, (I*(a - b*ArcCos[c + d*x]))/b])/(2*d*E^((I*a)/b)*Sqrt[a - b*ArcCos[c + d*x]])
```

**Maple [A]** time = 0.064, size = 90, normalized size = 0.8

$$\frac{\sqrt{2}\sqrt{\pi}}{d} \sqrt{b^{-1}} \left( \sin\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi b}} \sqrt{a-b \arccos(dx+c)} \frac{1}{\sqrt{b^{-1}}}\right) - \cos\left(\frac{a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi b}} \sqrt{a-b \arccos(dx+c)} \frac{1}{\sqrt{b^{-1}}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a-b\*arccos(d\*x+c))^(1/2),x)

[Out]  $2^{(1/2)} * \pi^{(1/2)} * (1/b)^{(1/2)} * (\sin(a/b) * \text{FresnelC}(2^{(1/2)}/\pi^{(1/2)}) / (1/b)^{(1/2)}) * (a-b * \arccos(d*x+c))^{(1/2)}/b - \cos(a/b) * \text{FresnelS}(2^{(1/2)}/\pi^{(1/2)}) / (1/b)^{(1/2)} * (a-b * \arccos(d*x+c))^{(1/2)}/b) / d$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b \arccos(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-b\*arccos(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(-b\*arccos(d\*x + c) + a), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-b\*arccos(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a - b \arccos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-b\*arccos(d\*x+c))^(1/2),x)

[Out] Integral(1/sqrt(a - b\*arccos(c + d\*x)), x)

**Giac [A]** time = 2.94386, size = 236, normalized size = 2.19

$$\frac{\sqrt{\pi}i \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{-b \arccos(dx+c)+ai}}{2\sqrt{|b|}} - \frac{\sqrt{2}\sqrt{-b \arccos(dx+c)+a\sqrt{|b|}}}{2b}\right) e^{\left(\frac{ai}{b}\right)} + \sqrt{\pi}i \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{-b \arccos(dx+c)+ai}}{2\sqrt{|b|}} - \frac{\sqrt{2}\sqrt{-b \arccos(dx+c)+a\sqrt{|b|}}}{2b}\right)}{\left(\frac{\sqrt{2}bi}{\sqrt{|b|}} + \sqrt{2}\sqrt{|b|}\right)d} + \frac{\sqrt{\pi}i \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{-b \arccos(dx+c)+ai}}{2\sqrt{|b|}} - \frac{\sqrt{2}\sqrt{-b \arccos(dx+c)+a\sqrt{|b|}}}{2b}\right)}{\left(\frac{\sqrt{2}bi}{\sqrt{|b|}} - \sqrt{2}\sqrt{|b|}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-b\*arccos(d\*x+c))^(1/2),x, algorithm="giac")

```
[Out] sqrt(pi)*i*erf(-1/2*sqrt(2)*sqrt(-b*arccos(d*x + c) + a)*i/sqrt(abs(b)) - 1
/2*sqrt(2)*sqrt(-b*arccos(d*x + c) + a)*sqrt(abs(b))/b)*e^(a*i/b)/((sqrt(2)
*b*i/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))*d) + sqrt(pi)*i*erf(1/2*sqrt(2)*s
qrt(-b*arccos(d*x + c) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(-b*arccos(d*x
+ c) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((sqrt(2)*b*i/sqrt(abs(b)) - sqrt(2)*
sqrt(abs(b)))*d)
```

$$3.45 \quad \int \frac{\cos^{-1}(a+bx)}{\frac{ad}{b}+dx} dx$$

**Optimal.** Leaf size=68

$$-\frac{i \operatorname{PolyLog}\left(2, -e^{2i \cos^{-1}(a+bx)}\right)}{2d} - \frac{i \cos^{-1}(a+bx)^2}{2d} + \frac{\cos^{-1}(a+bx) \log\left(1 + e^{2i \cos^{-1}(a+bx)}\right)}{d}$$

[Out]  $((-I/2)*\operatorname{ArcCos}[a + b*x]^2)/d + (\operatorname{ArcCos}[a + b*x]*\operatorname{Log}[1 + E^{((2*I)*\operatorname{ArcCos}[a + b*x])}])/d - ((I/2)*\operatorname{PolyLog}[2, -E^{((2*I)*\operatorname{ArcCos}[a + b*x])}])/d$

**Rubi [A]** time = 0.0813552, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {4806, 12, 4626, 3719, 2190, 2279, 2391}

$$-\frac{i \operatorname{PolyLog}\left(2, -e^{2i \cos^{-1}(a+bx)}\right)}{2d} - \frac{i \cos^{-1}(a+bx)^2}{2d} + \frac{\cos^{-1}(a+bx) \log\left(1 + e^{2i \cos^{-1}(a+bx)}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{ArcCos}[a + b*x]/((a*d)/b + d*x), x]$

[Out]  $((-I/2)*\operatorname{ArcCos}[a + b*x]^2)/d + (\operatorname{ArcCos}[a + b*x]*\operatorname{Log}[1 + E^{((2*I)*\operatorname{ArcCos}[a + b*x])}])/d - ((I/2)*\operatorname{PolyLog}[2, -E^{((2*I)*\operatorname{ArcCos}[a + b*x])}])/d$

#### Rule 4806

$\operatorname{Int}[(a + \operatorname{ArcCos}[c + (d)*(x)]*(b))^n * ((e) + (f)*(x))^m, x\_Symbol] \rightarrow \operatorname{Dist}[1/d, \operatorname{Subst}[\operatorname{Int}[(d*e - c*f)/d + (f*x)/d]^m * (a + b*\operatorname{ArcCos}[x])^n, x], x, c + d*x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m, n\}, x]$

#### Rule 12

$\operatorname{Int}[(a)*(u), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b)*(v)] /; \operatorname{FreeQ}[b, x]$

#### Rule 4626

$\operatorname{Int}[(a + \operatorname{ArcCos}[c*(x)]*(b))^n / (x), x\_Symbol] \rightarrow -\operatorname{Subst}[\operatorname{Int}[(a + b*x)^n / \operatorname{Cot}[x], x], x, \operatorname{ArcCos}[c*x]] /; \operatorname{FreeQ}\{a, b, c\}, x] \&\& \operatorname{IGtQ}[n, 0]$

#### Rule 3719

$\operatorname{Int}[(c + (d)*(x))^m * \tan[(e) + (f)*(x)], x\_Symbol] \rightarrow \operatorname{Simp}[(I*(c + d*x)^{m+1}) / (d*(m+1)), x] - \operatorname{Dist}[2*I, \operatorname{Int}[(c + d*x)^m * E^{(2*I*(e + f*x))} / (1 + E^{(2*I*(e + f*x))}), x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2190

$\operatorname{Int}[(F)^{(g*(e) + (f)*(x))})^n * (c + (d)*(x))^m / ((a) + (b)*(F)^{(g*(e) + (f)*(x))})^n, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^m * \operatorname{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a] / (b*f*g*n * \operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m) / (b*f*g*n * \operatorname{Log}[F]), \operatorname{Int}[(c + d*x)^{m-1} * \operatorname{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \operatorname{IGtQ}[m, 0]$

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^{-1}(a+bx)}{\frac{ad}{b} + dx} dx &= \frac{\text{Subst}\left(\int \frac{b \cos^{-1}(x)}{dx} dx, x, a+bx\right)}{b} \\ &= \frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{x} dx, x, a+bx\right)}{d} \\ &= -\frac{\text{Subst}\left(\int x \tan(x) dx, x, \cos^{-1}(a+bx)\right)}{d} \\ &= -\frac{i \cos^{-1}(a+bx)^2}{2d} + \frac{(2i) \text{Subst}\left(\int \frac{e^{2ix}}{1+e^{2ix}} dx, x, \cos^{-1}(a+bx)\right)}{d} \\ &= -\frac{i \cos^{-1}(a+bx)^2}{2d} + \frac{\cos^{-1}(a+bx) \log\left(1 + e^{2i \cos^{-1}(a+bx)}\right)}{d} - \frac{\text{Subst}\left(\int \log\left(1 + e^{2ix}\right) dx, x, \cos^{-1}(a+bx)\right)}{d} \\ &= -\frac{i \cos^{-1}(a+bx)^2}{2d} + \frac{\cos^{-1}(a+bx) \log\left(1 + e^{2i \cos^{-1}(a+bx)}\right)}{d} + \frac{i \text{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^{2i \cos^{-1}(a+bx)}\right)}{2d} \\ &= -\frac{i \cos^{-1}(a+bx)^2}{2d} + \frac{\cos^{-1}(a+bx) \log\left(1 + e^{2i \cos^{-1}(a+bx)}\right)}{d} - \frac{i \text{Li}_2\left(-e^{2i \cos^{-1}(a+bx)}\right)}{2d} \end{aligned}$$

**Mathematica [A]** time = 0.0414374, size = 59, normalized size = 0.87

$$\frac{i \left( \text{PolyLog}\left(2, -e^{2i \cos^{-1}(a+bx)}\right) + \cos^{-1}(a+bx) \left( \cos^{-1}(a+bx) + 2i \log\left(1 + e^{2i \cos^{-1}(a+bx)}\right) \right) \right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]/((a*d)/b + d*x), x]
```

```
[Out] ((-I/2)*(ArcCos[a + b*x]*(ArcCos[a + b*x] + (2*I)*Log[1 + E^((2*I)*ArcCos[a + b*x])]) + PolyLog[2, -E^((2*I)*ArcCos[a + b*x])]))/d
```

**Maple [A]** time = 0.089, size = 85, normalized size = 1.3

$$\frac{-\frac{i}{2} (\arccos(bx+a))^2}{d} + \frac{\arccos(bx+a)}{d} \ln\left(1 + \left(bx+a + i\sqrt{1-(bx+a)^2}\right)^2\right) - \frac{i}{2d} \text{polylog}\left(2, -\left(bx+a + i\sqrt{1-(bx+a)^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)/(a*d/b+d*x), x)
```

[Out]  $-1/2*I*\arccos(b*x+a)^2/d+\arccos(b*x+a)*\ln(1+(b*x+a+I*(1-(b*x+a)^2)^{(1/2)})^2)/d-1/2*I*\text{polylog}(2,-(b*x+a+I*(1-(b*x+a)^2)^{(1/2)})^2)/d$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \arccos(bx + a)}{bdx + ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")`

[Out] `integral(b*arccos(b*x + a)/(b*d*x + a*d), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{b \int \frac{\arccos(a+bx)}{a+bx} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acos(b*x+a)/(a*d/b+d*x),x)`

[Out] `b*Integral(acos(a + b*x)/(a + b*x), x)/d`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(bx + a)}{dx + \frac{ad}{b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(b*x+a)/(a*d/b+d*x),x, algorithm="giac")`

[Out] `integrate(arccos(b*x + a)/(d*x + a*d/b), x)`

### 3.46 $\int \sqrt{1-x^2} \cos^{-1}(x) dx$

**Optimal.** Leaf size=34

$$\frac{x^2}{4} + \frac{1}{2}\sqrt{1-x^2}x \cos^{-1}(x) - \frac{1}{4} \cos^{-1}(x)^2$$

[Out]  $x^2/4 + (x*\text{Sqrt}[1 - x^2]*\text{ArcCos}[x])/2 - \text{ArcCos}[x]^2/4$

**Rubi [A]** time = 0.0297441, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {4648, 4642, 30}

$$\frac{x^2}{4} + \frac{1}{2}\sqrt{1-x^2}x \cos^{-1}(x) - \frac{1}{4} \cos^{-1}(x)^2$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[1 - x^2]*\text{ArcCos}[x], x]$

[Out]  $x^2/4 + (x*\text{Sqrt}[1 - x^2]*\text{ArcCos}[x])/2 - \text{ArcCos}[x]^2/4$

#### Rule 4648

$\text{Int}[(a + \text{ArcCos}[c*x])*(b*x)^n*\text{Sqrt}[d + (e*x)^2], x\_Symbol] \rightarrow \text{Simp}[(x*\text{Sqrt}[d + e*x^2]*(a + b*\text{ArcCos}[c*x])^n)/2, x] + (\text{Dist}[\text{Sqrt}[d + e*x^2]/(2*\text{Sqrt}[1 - c^2*x^2]), \text{Int}[(a + b*\text{ArcCos}[c*x])^n/\text{Sqrt}[1 - c^2*x^2], x], x] + \text{Dist}[(b*c*n*\text{Sqrt}[d + e*x^2])/(2*\text{Sqrt}[1 - c^2*x^2]), \text{Int}[x*(a + b*\text{ArcCos}[c*x])^{n-1}, x], x]) /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0]$

#### Rule 4642

$\text{Int}[(a + \text{ArcCos}[c*x])*(b*x)^n/\text{Sqrt}[d + (e*x)^2], x\_Symbol] \rightarrow -\text{Simp}[(a + b*\text{ArcCos}[c*x])^{n+1}/(b*c*\text{Sqrt}[d + e*x^2]), x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[d, 0] \&\& \text{NeQ}[n, -1]$

#### Rule 30

$\text{Int}[x^m, x\_Symbol] \rightarrow \text{Simp}[x^{m+1}/(m+1), x] /; \text{FreeQ}[m, x] \&\& \text{NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned} \int \sqrt{1-x^2} \cos^{-1}(x) dx &= \frac{1}{2}x\sqrt{1-x^2} \cos^{-1}(x) + \frac{\int x dx}{2} + \frac{1}{2} \int \frac{\cos^{-1}(x)}{\sqrt{1-x^2}} dx \\ &= \frac{x^2}{4} + \frac{1}{2}x\sqrt{1-x^2} \cos^{-1}(x) - \frac{1}{4} \cos^{-1}(x)^2 \end{aligned}$$

**Mathematica [A]** time = 0.0157414, size = 30, normalized size = 0.88

$$\frac{1}{4} \left( x^2 + 2\sqrt{1-x^2}x \cos^{-1}(x) - \cos^{-1}(x)^2 \right)$$



Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x^2]\*ArcCos[x], x]

[Out] (x^2 + 2\*x\*Sqrt[1 - x^2]\*ArcCos[x] - ArcCos[x]^2)/4

**Maple [A]** time = 0.083, size = 33, normalized size = 1.

$$-\frac{\arccos(x)}{2} \left( -x\sqrt{-x^2+1} + \arccos(x) \right) + \frac{(\arccos(x))^2}{4} + \frac{x^2}{4} - \frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(x)\*(-x^2+1)^(1/2), x)

[Out] -1/2\*arccos(x)\*(-x\*(-x^2+1)^(1/2)+arccos(x))+1/4\*arccos(x)^2+1/4\*x^2-1/4

**Maxima [A]** time = 1.49029, size = 41, normalized size = 1.21

$$\frac{1}{4}x^2 + \frac{1}{2} \left( \sqrt{-x^2+1}x + \arcsin(x) \right) \arccos(x) + \frac{1}{4} \arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x)\*(-x^2+1)^(1/2), x, algorithm="maxima")

[Out] 1/4\*x^2 + 1/2\*(sqrt(-x^2 + 1)\*x + arcsin(x))\*arccos(x) + 1/4\*arcsin(x)^2

**Fricas [A]** time = 2.38116, size = 81, normalized size = 2.38

$$\frac{1}{2} \sqrt{-x^2+1}x \arccos(x) + \frac{1}{4}x^2 - \frac{1}{4} \arccos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x)\*(-x^2+1)^(1/2), x, algorithm="fricas")

[Out] 1/2\*sqrt(-x^2 + 1)\*x\*arccos(x) + 1/4\*x^2 - 1/4\*arccos(x)^2

**Sympy [A]** time = 21.5284, size = 48, normalized size = 1.41

$$\left( \left( \frac{x\sqrt{1-x^2}}{2} + \frac{\arcsin(x)}{2} \quad \text{for } x > -1 \wedge x < 1 \right) \arccos(x) + \begin{cases} \text{NaN} & \text{for } x < -1 \\ \frac{x^2}{4} + \frac{\arcsin^2(x)}{4} - \frac{\pi^2}{16} - \frac{1}{4} & \text{for } x < 1 \\ \text{NaN} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(x)\*(-x\*\*2+1)\*\*(1/2), x)

```
[Out] Piecewise((x*sqrt(1 - x**2)/2 + asin(x)/2, (x > -1) & (x < 1)))*acos(x) + Piecewise((nan, x < -1), (x**2/4 + asin(x)**2/4 - pi**2/16 - 1/4, x < 1), (nan, True))
```

**Giac [A]** time = 1.32261, size = 36, normalized size = 1.06

$$\frac{1}{2} \sqrt{-x^2 + 1} x \arccos(x) + \frac{1}{4} x^2 - \frac{1}{4} \arccos(x)^2 - \frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2 - 1/8
```

### 3.47 $\int x^3 \cos^{-1}(ax^2) dx$

**Optimal.** Leaf size=51

$$-\frac{x^2\sqrt{1-a^2x^4}}{8a} + \frac{\sin^{-1}(ax^2)}{8a^2} + \frac{1}{4}x^4 \cos^{-1}(ax^2)$$

[Out]  $-(x^2\sqrt{1-a^2x^4})/(8*a) + (x^4*\text{ArcCos}[a*x^2])/4 + \text{ArcSin}[a*x^2]/(8*a^2)$

**Rubi [A]** time = 0.0385854, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4843, 12, 275, 321, 216}

$$-\frac{x^2\sqrt{1-a^2x^4}}{8a} + \frac{\sin^{-1}(ax^2)}{8a^2} + \frac{1}{4}x^4 \cos^{-1}(ax^2)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*\text{ArcCos}[a*x^2], x]$

[Out]  $-(x^2*\sqrt{1-a^2*x^4})/(8*a) + (x^4*\text{ArcCos}[a*x^2])/4 + \text{ArcSin}[a*x^2]/(8*a^2)$

#### Rule 4843

$\text{Int}[(c + d*x)^m * \text{ArcCos}[u] * (b + d*x)^n, x\_Symbol] := \text{Simp}[(c + d*x)^{m+1} * (a + b*\text{ArcCos}[u]) / (d*(m+1)), x] + \text{Dist}[b/(d*(m+1)), \text{Int}[\text{SimplifyIntegrand}[(c + d*x)^{m+1} * D[u, x] / \sqrt{1-u^2}], x], x] /;$  FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d\*x)^{m+1}, u, x] && !FunctionOfExponentialQ[u, x]

#### Rule 12

$\text{Int}[a*(u), x\_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 275

$\text{Int}[(x)^m * (a + (b)*(x)^n)^p, x\_Symbol] := \text{With}[\{k = \text{GCD}[m+1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{(m+1)/k - 1} * (a + b*x^{n/k})^p, x], x, x^k], x] /;$  k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

#### Rule 321

$\text{Int}[(c + d*x)^m * (a + (b)*(x)^n)^p, x\_Symbol] := \text{Simp}[(c + d*x)^{m-n+1} * (a + b*x^n)^{p+1} / (b*(m+n*p+1)), x] - \text{Dist}[(a*c^n * (m-n+1)) / (b*(m+n*p+1)), \text{Int}[(c*x)^{m-n} * (a + b*x^n)^p, x], x] /;$  FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 216

$\text{Int}[1/\sqrt{(a + (b)*(x)^2)}, x\_Symbol] := \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*x] / \sqrt{a}] / \text{Rt}[-b, 2], x] /;$  FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned}
\int x^3 \cos^{-1}(ax^2) dx &= \frac{1}{4}x^4 \cos^{-1}(ax^2) + \frac{1}{4} \int \frac{2ax^5}{\sqrt{1-a^2x^4}} dx \\
&= \frac{1}{4}x^4 \cos^{-1}(ax^2) + \frac{1}{2}a \int \frac{x^5}{\sqrt{1-a^2x^4}} dx \\
&= \frac{1}{4}x^4 \cos^{-1}(ax^2) + \frac{1}{4}a \operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1-a^2x^2}} dx, x, x^2\right) \\
&= -\frac{x^2\sqrt{1-a^2x^4}}{8a} + \frac{1}{4}x^4 \cos^{-1}(ax^2) + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-a^2x^2}} dx, x, x^2\right)}{8a} \\
&= -\frac{x^2\sqrt{1-a^2x^4}}{8a} + \frac{1}{4}x^4 \cos^{-1}(ax^2) + \frac{\sin^{-1}(ax^2)}{8a^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0275862, size = 48, normalized size = 0.94

$$\frac{-ax^2\sqrt{1-a^2x^4} + 2a^2x^4 \cos^{-1}(ax^2) + \sin^{-1}(ax^2)}{8a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCos[a\*x^2], x]

[Out]  $(-(a*x^2*\sqrt{1 - a^2*x^4}) + 2*a^2*x^4*ArcCos[a*x^2] + ArcSin[a*x^2])/(8*a^2)$

**Maple [A]** time = 0.053, size = 65, normalized size = 1.3

$$\frac{x^4 \arccos(ax^2)}{4} - \frac{x^2 \sqrt{-a^2x^4 + 1}}{8a} + \frac{1}{8a} \arctan\left(x^2 \sqrt{a^2} \frac{1}{\sqrt{-a^2x^4 + 1}}\right) \frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arccos(a\*x^2), x)

[Out]  $1/4*x^4*arccos(a*x^2) - 1/8*x^2*(-a^2*x^4+1)^{(1/2)}/a + 1/8/a/(a^2)^{(1/2)}*arctan((a^2)^{(1/2)}*x^2/(-a^2*x^4+1)^{(1/2)})$

**Maxima [A]** time = 1.47624, size = 107, normalized size = 2.1

$$\frac{1}{4}x^4 \arccos(ax^2) - \frac{1}{8}a \left( \frac{\arctan\left(\frac{\sqrt{-a^2x^4+1}}{ax^2}\right)}{a^3} + \frac{\sqrt{-a^2x^4+1}}{\left(a^4 - \frac{(a^2x^4-1)a^2}{x^4}\right)x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccos(a\*x^2), x, algorithm="maxima")

[Out]  $\frac{1}{4}x^4 \arccos(ax^2) - \frac{1}{8}a \left( \arctan\left(\frac{\sqrt{-a^2x^4 + 1}}{ax^2}\right) \right) / a^3 + \sqrt{-a^2x^4 + 1} / ((a^4 - (a^2x^4 - 1)a^2/x^4)x^2)$

**Fricas [A]** time = 2.52952, size = 93, normalized size = 1.82

$$\frac{\sqrt{-a^2x^4 + 1}ax^2 - (2a^2x^4 - 1)\arccos(ax^2)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccos(a*x^2),x, algorithm="fricas")`

[Out]  $-1/8*(\sqrt{-a^2x^4 + 1})ax^2 - (2a^2x^4 - 1)\arccos(ax^2)/a^2$

**Sympy [A]** time = 1.03189, size = 48, normalized size = 0.94

$$\begin{cases} \frac{x^4 \arccos(ax^2)}{4} - \frac{x^2 \sqrt{-a^2x^4 + 1}}{8a} - \frac{\arccos(ax^2)}{8a^2} & \text{for } a \neq 0 \\ \frac{\pi x^4}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*acos(a*x**2),x)`

[Out] `Piecewise((x**4*acos(a*x**2)/4 - x**2*sqrt(-a**2*x**4 + 1)/(8*a) - acos(a*x**2)/(8*a**2), Ne(a, 0)), (pi*x**4/8, True))`

**Giac [A]** time = 1.34345, size = 62, normalized size = 1.22

$$\frac{2a^2x^4 \arccos(ax^2) - \sqrt{-a^2x^4 + 1}ax^2 - \arccos(ax^2)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccos(a*x^2),x, algorithm="giac")`

[Out]  $1/8*(2a^2x^4 \arccos(ax^2) - \sqrt{-a^2x^4 + 1})ax^2 - \arccos(ax^2)/a^2$

### 3.48 $\int x^2 \cos^{-1}(ax^2) dx$

**Optimal.** Leaf size=55

$$\frac{2\text{EllipticF}(\sin^{-1}(\sqrt{ax}), -1)}{9a^{3/2}} - \frac{2x\sqrt{1-a^2x^4}}{9a} + \frac{1}{3}x^3 \cos^{-1}(ax^2)$$

[Out]  $(-2*x*\text{Sqrt}[1 - a^2*x^4])/(9*a) + (x^3*\text{ArcCos}[a*x^2])/3 + (2*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a]*x], -1])/(9*a^{(3/2)})$

**Rubi [A]** time = 0.0284355, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4843, 12, 321, 221}

$$-\frac{2x\sqrt{1-a^2x^4}}{9a} + \frac{2F(\sin^{-1}(\sqrt{ax})|-1)}{9a^{3/2}} + \frac{1}{3}x^3 \cos^{-1}(ax^2)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*\text{ArcCos}[a*x^2], x]$

[Out]  $(-2*x*\text{Sqrt}[1 - a^2*x^4])/(9*a) + (x^3*\text{ArcCos}[a*x^2])/3 + (2*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a]*x], -1])/(9*a^{(3/2)})$

#### Rule 4843

$\text{Int}[(a + \text{ArcCos}[u]*(b))*(c + (d)*(x))^m, x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{m+1}*(a + b*\text{ArcCos}[u])/(d*(m+1)), x] + \text{Dist}[b/(d*(m+1)), \text{Int}[\text{SimplifyIntegrand}[(c + d*x)^{m+1}*D[u, x]/\text{Sqrt}[1 - u^2], x], x] /;$   $\text{FreeQ}\{a, b, c, d, m\}, x \&\& \text{NeQ}[m, -1] \&\& \text{InverseFunctionFreeQ}[u, x] \&\& \text{!FunctionOfQ}[(c + d*x)^{m+1}, u, x] \&\& \text{!FunctionOfExponentialQ}[u, x]$

#### Rule 12

$\text{Int}[(a)*(u), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$   $\text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b)*(v)] /;$   $\text{FreeQ}[b, x]$

#### Rule 321

$\text{Int}[(c*(x))^m*(a + (b)*(x)^n)^p, x\_Symbol] \rightarrow \text{Simp}[(c^{n-1}*(c*x)^{m-n+1}*(a + b*x^n)^{p+1})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^{n-1}*(m-n+1))/(b*(m+n*p+1)), \text{Int}[(c*x)^{m-n}*(a + b*x^n)^p, x] /;$   $\text{FreeQ}\{a, b, c, p\}, x \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, n-1] \&\& \text{NeQ}[m+n*p+1, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

#### Rule 221

$\text{Int}[1/\text{Sqrt}[(a) + (b)*(x)^4], x\_Symbol] \rightarrow \text{Simp}[\text{EllipticF}[\text{ArcSin}[(\text{Rt}[-b, 4]*x)/\text{Rt}[a, 4]], -1]/(\text{Rt}[a, 4]*\text{Rt}[-b, 4]), x] /;$   $\text{FreeQ}\{a, b\}, x \&\& \text{NegQ}[b/a] \&\& \text{GtQ}[a, 0]$

#### Rubi steps

$$\begin{aligned}
\int x^2 \cos^{-1}(ax^2) dx &= \frac{1}{3}x^3 \cos^{-1}(ax^2) + \frac{1}{3} \int \frac{2ax^4}{\sqrt{1-a^2x^4}} dx \\
&= \frac{1}{3}x^3 \cos^{-1}(ax^2) + \frac{1}{3}(2a) \int \frac{x^4}{\sqrt{1-a^2x^4}} dx \\
&= -\frac{2x\sqrt{1-a^2x^4}}{9a} + \frac{1}{3}x^3 \cos^{-1}(ax^2) + \frac{2 \int \frac{1}{\sqrt{1-a^2x^4}} dx}{9a} \\
&= -\frac{2x\sqrt{1-a^2x^4}}{9a} + \frac{1}{3}x^3 \cos^{-1}(ax^2) + \frac{2F(\sin^{-1}(\sqrt{ax})|-1)}{9a^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.155284, size = 63, normalized size = 1.15

$$\frac{1}{9} \left( \frac{2i \operatorname{EllipticF}(i \sinh^{-1}(\sqrt{-ax}), -1)}{(-a)^{3/2}} - \frac{2x\sqrt{1-a^2x^4}}{a} + 3x^3 \cos^{-1}(ax^2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCos[a\*x^2], x]

[Out] ((-2\*x\*Sqrt[1 - a^2\*x^4])/a + 3\*x^3\*ArcCos[a\*x^2] + ((2\*I)\*EllipticF[I\*ArcSinh[Sqrt[-a]\*x], -1])/(-a)^(3/2))/9

**Maple [A]** time = 0.007, size = 79, normalized size = 1.4

$$\frac{x^3 \arccos(ax^2)}{3} + \frac{2a}{3} \left( -\frac{x}{3a^2} \sqrt{-a^2x^4+1} + \frac{1}{3} \sqrt{-ax^2+1} \sqrt{ax^2+1} \operatorname{EllipticF}(x\sqrt{a}, i) a^{-\frac{5}{2}} \frac{1}{\sqrt{-a^2x^4+1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccos(a\*x^2), x)

[Out] 1/3\*x^3\*arccos(a\*x^2)+2/3\*a\*(-1/3/a^2\*x\*(-a^2\*x^4+1)^(1/2)+1/3/a^(5/2)\*(-a\*x^2+1)^(1/2)\*(a\*x^2+1)^(1/2)/(-a^2\*x^4+1)^(1/2)\*EllipticF(x\*a^(1/2), I))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccos(a\*x^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral(x^2 arccos(ax^2), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccos(a\*x^2),x, algorithm="fricas")

[Out] integral(x^2\*arccos(a\*x^2), x)

**Sympy [A]** time = 1.42681, size = 48, normalized size = 0.87

$$\frac{ax^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{5}{4} \middle| \frac{9}{4} \middle| a^2x^4e^{2i\pi}\right)}{6\Gamma\left(\frac{9}{4}\right)} + \frac{x^3 \operatorname{acos}(ax^2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acos(a\*x\*\*2),x)

[Out] a\*x\*\*5\*gamma(5/4)\*hyper((1/2, 5/4), (9/4,), a\*\*2\*x\*\*4\*exp\_polar(2\*I\*pi))/(6\*gamma(9/4)) + x\*\*3\*acos(a\*x\*\*2)/3

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \arccos(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccos(a\*x^2),x, algorithm="giac")

[Out] integrate(x^2\*arccos(a\*x^2), x)



### 3.49 $\int x \cos^{-1}(ax^2) dx$

**Optimal.** Leaf size=35

$$\frac{1}{2}x^2 \cos^{-1}(ax^2) - \frac{\sqrt{1-a^2x^4}}{2a}$$

[Out]  $-\text{Sqrt}[1 - a^2x^4]/(2a) + (x^2\text{ArcCos}[ax^2])/2$

**Rubi [A]** time = 0.0236618, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {6715, 4620, 261}

$$\frac{1}{2}x^2 \cos^{-1}(ax^2) - \frac{\sqrt{1-a^2x^4}}{2a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x\text{ArcCos}[ax^2], x]$

[Out]  $-\text{Sqrt}[1 - a^2x^4]/(2a) + (x^2\text{ArcCos}[ax^2])/2$

#### Rule 6715

$\text{Int}[(u_*)(x_)^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/(m + 1), \text{Subst}[\text{Int}[\text{SubstFor}[x^{(m + 1)}, u, x], x, x^{(m + 1)}], x] /; \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{FunctionOfQ}[x^{(m + 1)}, u, x]$

#### Rule 4620

$\text{Int}[(a_. + \text{ArcCos}[(c_.)(x_)])(b_.)^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcCos}[c*x])^n, x] + \text{Dist}[b*c^n, \text{Int}[(x*(a + b*\text{ArcCos}[c*x])^{(n - 1)})/\text{Sqrt}[1 - c^2*x^2], x], x] /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{GtQ}[n, 0]$

#### Rule 261

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)(x_)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x^n)^{(p + 1)}/(b*n*(p + 1)), x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \ \&\& \ \text{EqQ}[m, n - 1] \ \&\& \ \text{NeQ}[p, -1]$

#### Rubi steps

$$\begin{aligned} \int x \cos^{-1}(ax^2) dx &= \frac{1}{2} \text{Subst} \left( \int \cos^{-1}(ax) dx, x, x^2 \right) \\ &= \frac{1}{2} x^2 \cos^{-1}(ax^2) + \frac{1}{2} a \text{Subst} \left( \int \frac{x}{\sqrt{1-a^2x^2}} dx, x, x^2 \right) \\ &= -\frac{\sqrt{1-a^2x^4}}{2a} + \frac{1}{2} x^2 \cos^{-1}(ax^2) \end{aligned}$$

**Mathematica [A]** time = 0.0161599, size = 35, normalized size = 1.

$$\frac{1}{2}x^2 \cos^{-1}(ax^2) - \frac{\sqrt{1-a^2x^4}}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCos[a\*x^2],x]

[Out] -Sqrt[1 - a^2\*x^4]/(2\*a) + (x^2\*ArcCos[a\*x^2])/2

**Maple [A]** time = 0.002, size = 32, normalized size = 0.9

$$\frac{1}{2a} \left( x^2 a \arccos(ax^2) - \sqrt{-a^2x^4 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccos(a\*x^2),x)

[Out] 1/2/a\*(x^2\*a\*arccos(a\*x^2)-(-a^2\*x^4+1)^(1/2))

**Maxima [A]** time = 1.44838, size = 42, normalized size = 1.2

$$\frac{ax^2 \arccos(ax^2) - \sqrt{-a^2x^4 + 1}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccos(a\*x^2),x, algorithm="maxima")

[Out] 1/2\*(a\*x^2\*arccos(a\*x^2) - sqrt(-a^2\*x^4 + 1))/a

**Fricas [A]** time = 2.37848, size = 68, normalized size = 1.94

$$\frac{ax^2 \arccos(ax^2) - \sqrt{-a^2x^4 + 1}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccos(a\*x^2),x, algorithm="fricas")

[Out] 1/2\*(a\*x^2\*arccos(a\*x^2) - sqrt(-a^2\*x^4 + 1))/a

**Sympy [A]** time = 0.229752, size = 32, normalized size = 0.91

$$\begin{cases} \frac{x^2 \arccos(ax^2)}{2} - \frac{\sqrt{-a^2x^4+1}}{2a} & \text{for } a \neq 0 \\ \frac{\pi x^2}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acos(a\*x\*\*2),x)

```
[Out] Piecewise((x**2*acos(a*x**2)/2 - sqrt(-a**2*x**4 + 1)/(2*a), Ne(a, 0)), (pi
*x**2/4, True))
```

**Giac [A]** time = 1.32775, size = 42, normalized size = 1.2

$$\frac{ax^2 \arccos(ax^2) - \sqrt{-a^2x^4 + 1}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*x^2*arccos(a*x^2) - sqrt(-a^2*x^4 + 1))/a
```

### 3.50 $\int \cos^{-1}(ax^2) dx$

**Optimal.** Leaf size=43

$$-\frac{2\text{EllipticF}(\sin^{-1}(\sqrt{ax}), -1)}{\sqrt{a}} + x \cos^{-1}(ax^2) + \frac{2E(\sin^{-1}(\sqrt{ax})|-1)}{\sqrt{a}}$$

[Out] x\*ArcCos[a\*x^2] + (2\*EllipticE[ArcSin[Sqrt[a]\*x], -1])/Sqrt[a] - (2\*EllipticF[ArcSin[Sqrt[a]\*x], -1])/Sqrt[a]

**Rubi [A]** time = 0.0324736, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.$ , Rules used = {4841, 12, 307, 221, 1199, 424}

$$x \cos^{-1}(ax^2) - \frac{2F(\sin^{-1}(\sqrt{ax})|-1)}{\sqrt{a}} + \frac{2E(\sin^{-1}(\sqrt{ax})|-1)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a\*x^2], x]

[Out] x\*ArcCos[a\*x^2] + (2\*EllipticE[ArcSin[Sqrt[a]\*x], -1])/Sqrt[a] - (2\*EllipticF[ArcSin[Sqrt[a]\*x], -1])/Sqrt[a]

#### Rule 4841

Int[ArcCos[u\_], x\_Symbol] :> Simp[x\*ArcCos[u], x] + Int[SimplifyIntegrand[(x\*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] && !FunctionOfExponentialQ[u, x]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 307

Int[(x\_)^2/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] :> With[{q = Rt[-(b/a), 2]}, -Dist[q^(-1), Int[1/Sqrt[a + b\*x^4], x], x] + Dist[1/q, Int[(1 + q\*x^2)/Sqrt[a + b\*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]

#### Rule 221

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

#### Rule 1199

Int[((d\_) + (e\_.)\*(x\_)^2)/Sqrt[(a\_) + (c\_.)\*(x\_)^4], x\_Symbol] :> Dist[d/Sqrt[a], Int[Sqrt[1 + (e\*x^2)/d]/Sqrt[1 - (e\*x^2)/d], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c\*d^2 + a\*e^2, 0] && GtQ[a, 0]

#### Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c)
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

### Rubi steps

$$\begin{aligned}
\int \cos^{-1}(ax^2) dx &= x \cos^{-1}(ax^2) + \int \frac{2ax^2}{\sqrt{1-a^2x^4}} dx \\
&= x \cos^{-1}(ax^2) + (2a) \int \frac{x^2}{\sqrt{1-a^2x^4}} dx \\
&= x \cos^{-1}(ax^2) - 2 \int \frac{1}{\sqrt{1-a^2x^4}} dx + 2 \int \frac{1+ax^2}{\sqrt{1-a^2x^4}} dx \\
&= x \cos^{-1}(ax^2) - \frac{2F(\sin^{-1}(\sqrt{ax})| -1)}{\sqrt{a}} + 2 \int \frac{\sqrt{1+ax^2}}{\sqrt{1-a^2x^4}} dx \\
&= x \cos^{-1}(ax^2) + \frac{2E(\sin^{-1}(\sqrt{ax})| -1)}{\sqrt{a}} - \frac{2F(\sin^{-1}(\sqrt{ax})| -1)}{\sqrt{a}}
\end{aligned}$$

**Mathematica [C]** time = 0.0047055, size = 34, normalized size = 0.79

$$\frac{2}{3}ax^3\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, a^2x^4\right) + x \cos^{-1}(ax^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x^2], x]
```

```
[Out] x*ArcCos[a*x^2] + (2*a*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, a^2*x^4])/3
```

**Maple [A]** time = 0.007, size = 65, normalized size = 1.5

$$x \arccos(ax^2) - 2 \frac{\sqrt{-ax^2+1}\sqrt{ax^2+1}(\text{EllipticF}(x\sqrt{a}, i) - \text{EllipticE}(x\sqrt{a}, i))}{\sqrt{a}\sqrt{-a^2x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x^2), x)
```

```
[Out] x*arccos(a*x^2)-2/a^(1/2)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)/(-a^2*x^4+1)^(1/2)*
(EllipticF(x*a^(1/2), I)-EllipticE(x*a^(1/2), I))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2), x, algorithm="maxima")
```

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\arccos(ax^2), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a\*x^2), x, algorithm="fricas")

[Out] integral(arccos(a\*x^2), x)

**Sympy [A]** time = 1.11102, size = 44, normalized size = 1.02

$$\frac{ax^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{7}{4}, a^2x^4e^{2i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)} + x \arccos(ax^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(a\*x\*\*2), x)

[Out] a\*x\*\*3\*gamma(3/4)\*hyper((1/2, 3/4), (7/4,), a\*\*2\*x\*\*4\*exp\_polar(2\*I\*pi))/(2\*gamma(7/4)) + x\*acos(a\*x\*\*2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \arccos(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a\*x^2), x, algorithm="giac")

[Out] integrate(arccos(a\*x^2), x)

### 3.51 $\int \frac{\cos^{-1}(ax^2)}{x} dx$

**Optimal.** Leaf size=62

$$-\frac{1}{4}i\text{PolyLog}\left(2, -e^{2i\cos^{-1}(ax^2)}\right) - \frac{1}{4}i\cos^{-1}(ax^2)^2 + \frac{1}{2}\cos^{-1}(ax^2)\log\left(1 + e^{2i\cos^{-1}(ax^2)}\right)$$

[Out]  $(-I/4)*\text{ArcCos}[a*x^2]^2 + (\text{ArcCos}[a*x^2]*\text{Log}[1 + E^((2*I)*\text{ArcCos}[a*x^2])])/2 - (I/4)*\text{PolyLog}[2, -E^((2*I)*\text{ArcCos}[a*x^2])]$

**Rubi [A]** time = 0.0584569, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4831, 3719, 2190, 2279, 2391}

$$-\frac{1}{4}i\text{PolyLog}\left(2, -e^{2i\cos^{-1}(ax^2)}\right) - \frac{1}{4}i\cos^{-1}(ax^2)^2 + \frac{1}{2}\cos^{-1}(ax^2)\log\left(1 + e^{2i\cos^{-1}(ax^2)}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a\*x^2]/x,x]

[Out]  $(-I/4)*\text{ArcCos}[a*x^2]^2 + (\text{ArcCos}[a*x^2]*\text{Log}[1 + E^((2*I)*\text{ArcCos}[a*x^2])])/2 - (I/4)*\text{PolyLog}[2, -E^((2*I)*\text{ArcCos}[a*x^2])]$

#### Rule 4831

Int[ArcCos[(a\_.)\*(x\_)^(p\_)]^(n\_.)/(x\_), x\_Symbol] :> -Dist[p^(-1), Subst[Int[x^n\*Tan[x], x], x, ArcCos[a\*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]

#### Rule 3719

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))\*tan[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

#### Rule 2190

Int[((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_)^(m\_.))/((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(ax^2)}{x} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int x \tan(x) dx, x, \cos^{-1}(ax^2)\right)\right) \\
&= -\frac{1}{4}i \cos^{-1}(ax^2)^2 + i \text{Subst}\left(\int \frac{e^{2ix}}{1+e^{2ix}} dx, x, \cos^{-1}(ax^2)\right) \\
&= -\frac{1}{4}i \cos^{-1}(ax^2)^2 + \frac{1}{2} \cos^{-1}(ax^2) \log\left(1+e^{2i\cos^{-1}(ax^2)}\right) - \frac{1}{2} \text{Subst}\left(\int \log(1+e^{2ix}) dx, x, \cos^{-1}(ax^2)\right) \\
&= -\frac{1}{4}i \cos^{-1}(ax^2)^2 + \frac{1}{2} \cos^{-1}(ax^2) \log\left(1+e^{2i\cos^{-1}(ax^2)}\right) + \frac{1}{4}i \text{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^{2i\cos^{-1}(ax^2)}\right) \\
&= -\frac{1}{4}i \cos^{-1}(ax^2)^2 + \frac{1}{2} \cos^{-1}(ax^2) \log\left(1+e^{2i\cos^{-1}(ax^2)}\right) - \frac{1}{4}i \text{Li}_2\left(-e^{2i\cos^{-1}(ax^2)}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0309198, size = 56, normalized size = 0.9

$$-\frac{1}{4}i \left( \text{PolyLog}\left(2, -e^{2i\cos^{-1}(ax^2)}\right) + \cos^{-1}(ax^2) \left( \cos^{-1}(ax^2) + 2i \log\left(1 + e^{2i\cos^{-1}(ax^2)}\right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a\*x^2]/x,x]

[Out] (-I/4)\*(ArcCos[a\*x^2]\*(ArcCos[a\*x^2] + (2\*I)\*Log[1 + E^((2\*I)\*ArcCos[a\*x^2])]) + PolyLog[2, -E^((2\*I)\*ArcCos[a\*x^2])])

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int \frac{\arccos(ax^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(a\*x^2)/x,x)

[Out] int(arccos(a\*x^2)/x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a\*x^2)/x,x, algorithm="maxima")

[Out] integrate(arccos(a\*x^2)/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arccos(ax^2)}{x}, x\right)$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x^2)/x, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x**2)/x,x)
```

```
[Out] Integral(acos(a*x**2)/x, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x^2)/x, x)
```

$$3.52 \quad \int \frac{\cos^{-1}(ax^2)}{x^2} dx$$

**Optimal.** Leaf size=29

$$-2\sqrt{a}\text{EllipticF}(\sin^{-1}(\sqrt{ax}), -1) - \frac{\cos^{-1}(ax^2)}{x}$$

[Out] -(ArcCos[a\*x^2]/x) - 2\*Sqrt[a]\*EllipticF[ArcSin[Sqrt[a]\*x], -1]

**Rubi [A]** time = 0.014442, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {4843, 12, 221}

$$-\frac{\cos^{-1}(ax^2)}{x} - 2\sqrt{a}F(\sin^{-1}(\sqrt{ax})|-1)$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a\*x^2]/x^2,x]

[Out] -(ArcCos[a\*x^2]/x) - 2\*Sqrt[a]\*EllipticF[ArcSin[Sqrt[a]\*x], -1]

#### Rule 4843

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcCos[u]))/(d*(m + 1)), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 - u^2], x], x],
x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u,
x]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

#### Rubi steps

$$\begin{aligned} \int \frac{\cos^{-1}(ax^2)}{x^2} dx &= -\frac{\cos^{-1}(ax^2)}{x} - \int \frac{2a}{\sqrt{1-a^2x^4}} dx \\ &= -\frac{\cos^{-1}(ax^2)}{x} - (2a) \int \frac{1}{\sqrt{1-a^2x^4}} dx \\ &= -\frac{\cos^{-1}(ax^2)}{x} - 2\sqrt{a}F(\sin^{-1}(\sqrt{ax})|-1) \end{aligned}$$

**Mathematica [C]** time = 0.0385381, size = 40, normalized size = 1.38

$$\frac{\cos^{-1}(ax^2) + 2i\sqrt{-ax}\text{EllipticF}\left(i\sinh^{-1}(\sqrt{-ax}), -1\right)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a\*x^2]/x^2,x]

[Out] -((ArcCos[a\*x^2] + (2\*I)\*Sqrt[-a]\*x\*EllipticF[I\*ArcSinh[Sqrt[-a]\*x], -1])/x)

**Maple [B]** time = 0.006, size = 57, normalized size = 2.

$$-\frac{\arccos(ax^2)}{x} - 2\frac{\sqrt{a}\sqrt{-ax^2+1}\sqrt{ax^2+1}\text{EllipticF}(x\sqrt{a}, i)}{\sqrt{-a^2x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(a\*x^2)/x^2,x)

[Out] -arccos(a\*x^2)/x-2\*a^(1/2)\*(-a\*x^2+1)^(1/2)\*(a\*x^2+1)^(1/2)/(-a^2\*x^4+1)^(1/2)\*EllipticF(x\*a^(1/2),I)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a\*x^2)/x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arccos(ax^2)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a\*x^2)/x^2,x, algorithm="fricas")

[Out] integral(arccos(a\*x^2)/x^2, x)

**Sympy [A]** time = 1.21139, size = 44, normalized size = 1.52

$$\frac{ax\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{5}{4}, a^2x^4e^{2i\pi}\right)}{2\Gamma\left(\frac{5}{4}\right)} - \frac{\arccos(ax^2)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(a\*x\*\*2)/x\*\*2,x)

[Out] -a\*x\*gamma(1/4)\*hyper((1/4, 1/2), (5/4,), a\*\*2\*x\*\*4\*exp\_polar(2\*I\*pi))/(2\*gamma(5/4)) - acos(a\*x\*\*2)/x

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a\*x^2)/x^2,x, algorithm="giac")

[Out] integrate(arccos(a\*x^2)/x^2, x)

### 3.53 $\int x^2 \cos^{-1}\left(\frac{a}{x}\right) dx$

**Optimal.** Leaf size=58

$$-\frac{1}{6}ax^2\sqrt{1-\frac{a^2}{x^2}} - \frac{1}{6}a^3 \tanh^{-1}\left(\sqrt{1-\frac{a^2}{x^2}}\right) + \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right)$$

[Out]  $-(a*\text{Sqrt}[1 - a^2/x^2]*x^2)/6 + (x^3*\text{ArcSec}[x/a])/3 - (a^3*\text{ArcTanh}[\text{Sqrt}[1 - a^2/x^2]])/6$

**Rubi [A]** time = 0.036197, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {4833, 5220, 266, 51, 63, 208}

$$-\frac{1}{6}ax^2\sqrt{1-\frac{a^2}{x^2}} - \frac{1}{6}a^3 \tanh^{-1}\left(\sqrt{1-\frac{a^2}{x^2}}\right) + \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*\text{ArcCos}[a/x], x]$

[Out]  $-(a*\text{Sqrt}[1 - a^2/x^2]*x^2)/6 + (x^3*\text{ArcSec}[x/a])/3 - (a^3*\text{ArcTanh}[\text{Sqrt}[1 - a^2/x^2]])/6$

#### Rule 4833

$\text{Int}[\text{ArcCos}[(c\_)/((a\_)+(b\_)*(x\_)^{(n\_)})]^{(m\_)}*(u\_), x\_Symbol] \rightarrow \text{Int}[u*\text{ArcSec}[a/c + (b*x^n)/c]^{m}, x] /; \text{FreeQ}\{a, b, c, n, m\}, x]$

#### Rule 5220

$\text{Int}[(a\_ + \text{ArcSec}[(c\_)*(x\_)]*(b\_))*((d\_)*(x\_))^{(m\_)}, x\_Symbol] \rightarrow \text{Simp}[\text{p}[\text{((d*x)^{(m+1)}*(a + b*\text{ArcSec}[c*x]))}/(\text{d*(m+1)}), x] - \text{Dist}[(b*d)/(c*(m+1)), \text{Int}[(d*x)^{(m-1)}/\text{Sqrt}[1 - 1/(c^2*x^2)], x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{NeQ}[m, -1]$

#### Rule 266

$\text{Int}[(x\_)^{(m\_)}*((a\_)+(b\_)*(x\_)^{(n\_)})^{(p\_)}, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

#### Rule 51

$\text{Int}[(a\_ + (b\_)*(x\_))^{(m\_)}*((c\_)+(d\_)*(x\_))^{(n\_)}, x\_Symbol] \rightarrow \text{Simp}[\text{((a + b*x)^{(m+1)}*(c + d*x)^{(n+1))}/((b*c - a*d)*(m+1)), x] - \text{Dist}[(d*(m+n+2))/((b*c - a*d)*(m+1)), \text{Int}[(a + b*x)^{(m+1)}*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[m, -1] \&\& !(\text{LtQ}[n, -1] \&\& (\text{EqQ}[a, 0] \mid (\text{NeQ}[c, 0] \&\& \text{LtQ}[m-n, 0] \&\& \text{IntegerQ}[n]))) \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 63

$\text{Int}[(a\_ + (b\_)*(x\_))^{(m\_)}*((c\_)+(d\_)*(x\_))^{(n\_)}, x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1) - 1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}$

`[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

### Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

### Rubi steps

$$\begin{aligned}
 \int x^2 \cos^{-1}\left(\frac{a}{x}\right) dx &= \int x^2 \sec^{-1}\left(\frac{x}{a}\right) dx \\
 &= \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{3}a \int \frac{x}{\sqrt{1-\frac{a^2}{x^2}}} dx \\
 &= \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right) + \frac{1}{6}a \operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}} dx, x, \frac{1}{x^2}\right) \\
 &= -\frac{1}{6}a\sqrt{1-\frac{a^2}{x^2}}x^2 + \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right) + \frac{1}{12}a^3 \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}} dx, x, \frac{1}{x^2}\right) \\
 &= -\frac{1}{6}a\sqrt{1-\frac{a^2}{x^2}}x^2 + \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{6}a \operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}} dx, x, \sqrt{1-\frac{a^2}{x^2}}\right) \\
 &= -\frac{1}{6}a\sqrt{1-\frac{a^2}{x^2}}x^2 + \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{6}a^3 \tanh^{-1}\left(\sqrt{1-\frac{a^2}{x^2}}\right)
 \end{aligned}$$

**Mathematica [A]** time = 0.0505763, size = 61, normalized size = 1.05

$$\frac{1}{3}x^3 \cos^{-1}\left(\frac{a}{x}\right) - \frac{1}{6}a \left( x^2 \sqrt{1-\frac{a^2}{x^2}} + a^2 \log\left(x \left(\sqrt{1-\frac{a^2}{x^2}} + 1\right)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCos[a/x], x]

[Out] (x^3\*ArcCos[a/x])/3 - (a\*(Sqrt[1 - a^2/x^2]\*x^2 + a^2\*Log[(1 + Sqrt[1 - a^2/x^2])\*x]))/6

**Maple [A]** time = 0.01, size = 56, normalized size = 1.

$$-a^3 \left( -\frac{x^3}{3a^3} \arccos\left(\frac{a}{x}\right) + \frac{x^2}{6a^2} \sqrt{1-\frac{a^2}{x^2}} + \frac{1}{6} \operatorname{Artanh}\left(\frac{1}{\sqrt{1-\frac{a^2}{x^2}}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccos(a/x), x)

[Out] -a^3\*(-1/3/a^3\*x^3\*arccos(a/x)+1/6/a^2\*x^2\*(1-a^2/x^2)^(1/2)+1/6\*arctanh(1/(1-a^2/x^2)^(1/2)))

**Maxima [A]** time = 1.46236, size = 97, normalized size = 1.67

$$\frac{1}{3} x^3 \arccos\left(\frac{a}{x}\right) - \frac{1}{12} \left( a^2 \log\left(\sqrt{-\frac{a^2}{x^2} + 1} + 1\right) - a^2 \log\left(\sqrt{-\frac{a^2}{x^2} + 1} - 1\right) + 2x^2 \sqrt{-\frac{a^2}{x^2} + 1} \right) a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccos(a/x),x, algorithm="maxima")

[Out] 1/3\*x^3\*arccos(a/x) - 1/12\*(a^2\*log(sqrt(-a^2/x^2 + 1) + 1) - a^2\*log(sqrt(-a^2/x^2 + 1) - 1) + 2\*x^2\*sqrt(-a^2/x^2 + 1))\*a

**Fricas [A]** time = 2.53943, size = 207, normalized size = 3.57

$$\frac{1}{6} a^3 \log\left(x \sqrt{-\frac{a^2 - x^2}{x^2}} - x\right) - \frac{1}{6} a x^2 \sqrt{-\frac{a^2 - x^2}{x^2}} + \frac{1}{3} (x^3 - 1) \arccos\left(\frac{a}{x}\right) + \frac{2}{3} \arctan\left(\frac{x \sqrt{-\frac{a^2 - x^2}{x^2}} - x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccos(a/x),x, algorithm="fricas")

[Out] 1/6\*a^3\*log(x\*sqrt(-(a^2 - x^2)/x^2) - x) - 1/6\*a\*x^2\*sqrt(-(a^2 - x^2)/x^2) + 1/3\*(x^3 - 1)\*arccos(a/x) + 2/3\*arctan((x\*sqrt(-(a^2 - x^2)/x^2) - x)/a)

**Sympy [C]** time = 5.29622, size = 99, normalized size = 1.71

$$-\frac{a \left( \begin{array}{l} \frac{a^2 \operatorname{acosh}\left(\frac{x}{a}\right)}{2} + \frac{ax \sqrt{-1 + \frac{x^2}{a^2}}}{2} \quad \text{for } \frac{|x^2|}{|a^2|} > 1 \\ -\frac{ia^2 \operatorname{asin}\left(\frac{x}{a}\right)}{2} + \frac{iax}{2\sqrt{1 - \frac{x^2}{a^2}}} - \frac{ix^3}{2a\sqrt{1 - \frac{x^2}{a^2}}} \quad \text{otherwise} \end{array} \right)}{3} + \frac{x^3 \operatorname{acos}\left(\frac{a}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acos(a/x),x)

[Out] -a\*Piecewise((a\*\*2\*acosh(x/a)/2 + a\*x\*sqrt(-1 + x\*\*2/a\*\*2)/2, Abs(x\*\*2)/Abs(a\*\*2) > 1), (-I\*a\*\*2\*asin(x/a)/2 + I\*a\*x/(2\*sqrt(1 - x\*\*2/a\*\*2)) - I\*x\*\*3/(2\*a\*sqrt(1 - x\*\*2/a\*\*2)), True))/3 + x\*\*3\*acos(a/x)/3

**Giac [A]** time = 1.29039, size = 95, normalized size = 1.64

$$\frac{1}{3} x^3 \arccos\left(\frac{a}{x}\right) - \frac{1}{12} \left( a^2 \log(a^2) \operatorname{sgn}(x) - \frac{2a^2 \log\left(\left|-x + \sqrt{-a^2 + x^2}\right|\right)}{\operatorname{sgn}(x)} + \frac{2\sqrt{-a^2 + x^2}x}{\operatorname{sgn}(x)} \right) a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a/x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arccos(a/x) - 1/12*(a^2*log(a^2)*sgn(x) - 2*a^2*log(abs(-x + sqrt(-a^2 + x^2))))/sgn(x) + 2*sqrt(-a^2 + x^2)*x/sgn(x)*a
```



### 3.54 $\int x \cos^{-1}\left(\frac{a}{x}\right) dx$

**Optimal.** Leaf size=34

$$\frac{1}{2}x^2 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{2}ax\sqrt{1 - \frac{a^2}{x^2}}$$

[Out]  $-(a\sqrt{1 - a^2/x^2})x/2 + (x^2\text{ArcSec}[x/a])/2$

**Rubi [A]** time = 0.0165922, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4833, 5220, 191}

$$\frac{1}{2}x^2 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{2}ax\sqrt{1 - \frac{a^2}{x^2}}$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCos[a/x],x]

[Out]  $-(a\sqrt{1 - a^2/x^2})x/2 + (x^2\text{ArcSec}[x/a])/2$

#### Rule 4833

Int[ArcCos[(c\_.)/((a\_.) + (b\_.)\*(x\_.)^(n\_.))]^(m\_.)\*(u\_.), x\_Symbol] := Int[u\*ArcSec[a/c + (b\*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]

#### Rule 5220

Int[((a\_.) + ArcSec[(c\_.)\*(x\_.)]\*(b\_.))\*((d\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[(d\*x)^(m+1)\*(a + b\*ArcSec[c\*x])/(d\*(m+1)), x] - Dist[(b\*d)/(c\*(m+1)), Int[(d\*x)^(m-1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

#### Rule 191

Int[((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p+1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rubi steps

$$\begin{aligned} \int x \cos^{-1}\left(\frac{a}{x}\right) dx &= \int x \sec^{-1}\left(\frac{x}{a}\right) dx \\ &= \frac{1}{2}x^2 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{2}a \int \frac{1}{\sqrt{1 - \frac{a^2}{x^2}}} dx \\ &= -\frac{1}{2}a\sqrt{1 - \frac{a^2}{x^2}}x + \frac{1}{2}x^2 \sec^{-1}\left(\frac{x}{a}\right) \end{aligned}$$

**Mathematica [A]** time = 0.021294, size = 33, normalized size = 0.97

$$\frac{1}{2}\left(x^2 \cos^{-1}\left(\frac{a}{x}\right) - ax\sqrt{1 - \frac{a^2}{x^2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCos[a/x],x]

[Out]  $(-(a*\text{Sqrt}[1 - a^2/x^2]*x) + x^2*\text{ArcCos}[a/x])/2$

**Maple [A]** time = 0.004, size = 39, normalized size = 1.2

$$-a^2 \left( -\frac{x^2}{2a^2} \arccos\left(\frac{a}{x}\right) + \frac{x}{2a} \sqrt{1 - \frac{a^2}{x^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccos(a/x),x)

[Out]  $-a^2*(-1/2/a^2*x^2*\arccos(a/x)+1/2*(1-a^2/x^2)^(1/2)/a*x)$

**Maxima [A]** time = 1.44162, size = 38, normalized size = 1.12

$$\frac{1}{2} x^2 \arccos\left(\frac{a}{x}\right) - \frac{1}{2} ax \sqrt{-\frac{a^2}{x^2} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccos(a/x),x, algorithm="maxima")

[Out]  $1/2*x^2*\arccos(a/x) - 1/2*a*x*\text{sqrt}(-a^2/x^2 + 1)$

**Fricas [A]** time = 2.56326, size = 73, normalized size = 2.15

$$\frac{1}{2} x^2 \arccos\left(\frac{a}{x}\right) - \frac{1}{2} ax \sqrt{-\frac{a^2 - x^2}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccos(a/x),x, algorithm="fricas")

[Out]  $1/2*x^2*\arccos(a/x) - 1/2*a*x*\text{sqrt}(-(a^2 - x^2)/x^2)$

**Sympy [A]** time = 2.68407, size = 49, normalized size = 1.44

$$-\frac{a \left( \begin{cases} a \sqrt{-1 + \frac{x^2}{a^2}} & \text{for } \frac{|x^2|}{|a^2|} > 1 \\ ia \sqrt{1 - \frac{x^2}{a^2}} & \text{otherwise} \end{cases} \right)}{2} + \frac{x^2 \arccos\left(\frac{a}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acos(a/x),x)

[Out] -a\*Piecewise((a\*sqrt(-1 + x\*\*2/a\*\*2), Abs(x\*\*2)/Abs(a\*\*2) > 1), (I\*a\*sqrt(1 - x\*\*2/a\*\*2), True))/2 + x\*\*2\*acos(a/x)/2

**Giac [A]** time = 1.24789, size = 58, normalized size = 1.71

$$\frac{1}{2} x^2 \arccos\left(\frac{a}{x}\right) + \frac{1}{2} \left( \sqrt{-a^2} \operatorname{sgn}(x) - \frac{\sqrt{-a^2 + x^2}}{\operatorname{sgn}(x)} \right) a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccos(a/x),x, algorithm="giac")

[Out] 1/2\*x^2\*arccos(a/x) + 1/2\*(sqrt(-a^2)\*sgn(x) - sqrt(-a^2 + x^2)/sgn(x))\*a

### 3.55 $\int \cos^{-1}\left(\frac{a}{x}\right) dx$

**Optimal.** Leaf size=27

$$x \sec^{-1}\left(\frac{x}{a}\right) - a \tanh^{-1}\left(\sqrt{1 - \frac{a^2}{x^2}}\right)$$

[Out] x\*ArcSec[x/a] - a\*ArcTanh[Sqrt[1 - a^2/x^2]]

**Rubi [A]** time = 0.0169255, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$ , Rules used = {4833, 5214, 266, 63, 208}

$$x \sec^{-1}\left(\frac{x}{a}\right) - a \tanh^{-1}\left(\sqrt{1 - \frac{a^2}{x^2}}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a/x], x]

[Out] x\*ArcSec[x/a] - a\*ArcTanh[Sqrt[1 - a^2/x^2]]

#### Rule 4833

Int[ArcCos[(c\_.)/((a\_.) + (b\_.)\*(x\_)^(n\_.))]^(m\_.)\*(u\_.), x\_Symbol] :> Int[u\*ArcSec[a/c + (b\*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]

#### Rule 5214

Int[ArcSec[(c\_.)\*(x\_)], x\_Symbol] :> Simp[x\*ArcSec[c\*x], x] - Dist[1/c, Int[1/(x\*Sqrt[1 - 1/(c^2\*x^2)]), x], x] /; FreeQ[c, x]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rubi steps

$$\begin{aligned}
\int \cos^{-1}\left(\frac{a}{x}\right) dx &= \int \sec^{-1}\left(\frac{x}{a}\right) dx \\
&= x \sec^{-1}\left(\frac{x}{a}\right) - a \int \frac{1}{\sqrt{1 - \frac{a^2}{x^2}}} dx \\
&= x \sec^{-1}\left(\frac{x}{a}\right) + \frac{1}{2}a \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1 - a^2x}} dx, x, \frac{1}{x^2}\right) \\
&= x \sec^{-1}\left(\frac{x}{a}\right) - \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2} - \frac{x^2}{a^2}} dx, x, \sqrt{1 - \frac{a^2}{x^2}}\right)}{a} \\
&= x \sec^{-1}\left(\frac{x}{a}\right) - a \tanh^{-1}\left(\sqrt{1 - \frac{a^2}{x^2}}\right)
\end{aligned}$$

**Mathematica [B]** time = 0.1011, size = 84, normalized size = 3.11

$$x \cos^{-1}\left(\frac{a}{x}\right) - \frac{a\sqrt{x^2 - a^2} \left(\log\left(\frac{x}{\sqrt{x^2 - a^2}} + 1\right) - \log\left(1 - \frac{x}{\sqrt{x^2 - a^2}}\right)\right)}{2x\sqrt{1 - \frac{a^2}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a/x], x]

[Out] x\*ArcCos[a/x] - (a\*Sqrt[-a^2 + x^2]\*(-Log[1 - x/Sqrt[-a^2 + x^2]] + Log[1 + x/Sqrt[-a^2 + x^2]]))/(2\*Sqrt[1 - a^2/x^2]\*x)

**Maple [A]** time = 0.006, size = 30, normalized size = 1.1

$$-a \left( -\frac{x}{a} \arccos\left(\frac{a}{x}\right) + \operatorname{Artanh}\left(\frac{1}{\sqrt{1 - \frac{a^2}{x^2}}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(a/x), x)

[Out] -a\*(-1/a\*x\*arccos(a/x)+arctanh(1/(1-a^2/x^2)^(1/2)))

**Maxima [A]** time = 1.45402, size = 61, normalized size = 2.26

$$-\frac{1}{2}a \left( \log\left(\sqrt{\frac{a^2}{x^2} + 1} + 1\right) - \log\left(\sqrt{\frac{a^2}{x^2} + 1} - 1\right) \right) + x \arccos\left(\frac{a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a/x), x, algorithm="maxima")

[Out] -1/2\*a\*(log(sqrt(-a^2/x^2 + 1) + 1) - log(sqrt(-a^2/x^2 + 1) - 1)) + x\*arccos(a/x)

---

**Fricas [B]** time = 2.52093, size = 140, normalized size = 5.19

$$(x-1) \arccos\left(\frac{a}{x}\right) + a \log\left(x\sqrt{-\frac{a^2-x^2}{x^2}} - x\right) + 2 \arctan\left(\frac{x\sqrt{-\frac{a^2-x^2}{x^2}} - x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a/x),x, algorithm="fricas")

[Out] (x - 1)\*arccos(a/x) + a\*log(x\*sqrt(-(a^2 - x^2)/x^2) - x) + 2\*arctan((x\*sqrt(-(a^2 - x^2)/x^2) - x)/a)

---

**Sympy [A]** time = 2.27124, size = 29, normalized size = 1.07

$$-a \left( \begin{cases} \operatorname{acosh}\left(\frac{x}{a}\right) & \text{for } \frac{|x^2|}{|a^2|} > 1 \\ -i \operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases} \right) + x \operatorname{acos}\left(\frac{a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(a/x),x)

[Out] -a\*Piecewise((acosh(x/a), Abs(x\*\*2)/Abs(a\*\*2) > 1), (-I\*asin(x/a), True)) + x\*acos(a/x)

---

**Giac [A]** time = 1.26154, size = 58, normalized size = 2.15

$$-\frac{1}{2} \left( \log(a^2) \operatorname{sgn}(x) - \frac{2 \log\left(\left| -x + \sqrt{-a^2 + x^2} \right| \right)}{\operatorname{sgn}(x)} \right) a + x \arccos\left(\frac{a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a/x),x, algorithm="giac")

[Out] -1/2\*(log(a^2)\*sgn(x) - 2\*log(abs(-x + sqrt(-a^2 + x^2)))/sgn(x))\*a + x\*arccos(a/x)

$$3.56 \quad \int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x} dx$$

**Optimal.** Leaf size=60

$$\frac{1}{2}i\text{PolyLog}\left(2, -e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right) + \frac{1}{2}i\cos^{-1}\left(\frac{a}{x}\right)^2 - \cos^{-1}\left(\frac{a}{x}\right)\log\left(1 + e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right)$$

[Out] (I/2)\*ArcCos[a/x]^2 - ArcCos[a/x]\*Log[1 + E^((2\*I)\*ArcCos[a/x])] + (I/2)\*PolyLog[2, -E^((2\*I)\*ArcCos[a/x])]

**Rubi [A]** time = 0.0546249, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4831, 3719, 2190, 2279, 2391}

$$\frac{1}{2}i\text{PolyLog}\left(2, -e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right) + \frac{1}{2}i\cos^{-1}\left(\frac{a}{x}\right)^2 - \cos^{-1}\left(\frac{a}{x}\right)\log\left(1 + e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a/x]/x,x]

[Out] (I/2)\*ArcCos[a/x]^2 - ArcCos[a/x]\*Log[1 + E^((2\*I)\*ArcCos[a/x])] + (I/2)\*PolyLog[2, -E^((2\*I)\*ArcCos[a/x])]

#### Rule 4831

Int[ArcCos[(a\_.)\*(x\_)^(p\_)]^(n\_.)/(x\_), x\_Symbol] :> -Dist[p^(-1), Subst[Int[x^n\*Tan[x], x], x, ArcCos[a\*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]

#### Rule 3719

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))\*tan[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

#### Rule 2190

Int[((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_)^(m\_.))/((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x} dx &= \text{Subst}\left(\int x \tan(x) dx, x, \cos^{-1}\left(\frac{a}{x}\right)\right) \\
&= \frac{1}{2}i \cos^{-1}\left(\frac{a}{x}\right)^2 - 2i \text{Subst}\left(\int \frac{e^{2ix}}{1+e^{2ix}} dx, x, \cos^{-1}\left(\frac{a}{x}\right)\right) \\
&= \frac{1}{2}i \cos^{-1}\left(\frac{a}{x}\right)^2 - \cos^{-1}\left(\frac{a}{x}\right) \log\left(1 + e^{2i \cos^{-1}\left(\frac{a}{x}\right)}\right) + \text{Subst}\left(\int \log(1 + e^{2ix}) dx, x, \cos^{-1}\left(\frac{a}{x}\right)\right) \\
&= \frac{1}{2}i \cos^{-1}\left(\frac{a}{x}\right)^2 - \cos^{-1}\left(\frac{a}{x}\right) \log\left(1 + e^{2i \cos^{-1}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}i \text{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^{2i \cos^{-1}\left(\frac{a}{x}\right)}\right) \\
&= \frac{1}{2}i \cos^{-1}\left(\frac{a}{x}\right)^2 - \cos^{-1}\left(\frac{a}{x}\right) \log\left(1 + e^{2i \cos^{-1}\left(\frac{a}{x}\right)}\right) + \frac{1}{2}i \text{Li}_2\left(-e^{2i \cos^{-1}\left(\frac{a}{x}\right)}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0208701, size = 60, normalized size = 1.

$$\frac{1}{2}i \text{PolyLog}\left(2, -e^{2i \cos^{-1}\left(\frac{a}{x}\right)}\right) + \frac{1}{2}i \cos^{-1}\left(\frac{a}{x}\right)^2 - \cos^{-1}\left(\frac{a}{x}\right) \log\left(1 + e^{2i \cos^{-1}\left(\frac{a}{x}\right)}\right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a/x]/x,x]

[Out] (I/2)\*ArcCos[a/x]^2 - ArcCos[a/x]\*Log[1 + E^((2\*I)\*ArcCos[a/x])] + (I/2)\*PolyLog[2, -E^((2\*I)\*ArcCos[a/x])]

**Maple [A]** time = 0.055, size = 77, normalized size = 1.3

$$\frac{i}{2} \left( \arccos\left(\frac{a}{x}\right)^2 - \arccos\left(\frac{a}{x}\right) \ln\left(1 + \left(\frac{a}{x} + i\sqrt{1 - \frac{a^2}{x^2}}\right)^2\right) \right) + \frac{i}{2} \text{polylog}\left(2, -\left(\frac{a}{x} + i\sqrt{1 - \frac{a^2}{x^2}}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(a/x)/x,x)

[Out] 1/2\*I\*arccos(a/x)^2-arccos(a/x)\*ln(1+(a/x+I\*(1-a^2/x^2)^(1/2))^2)+1/2\*I\*polylg(2,-(a/x+I\*(1-a^2/x^2)^(1/2))^2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-i a^2 \int -\frac{\log(x)}{a^2 x - x^3} dx - a \int -\frac{\sqrt{a+x}\sqrt{-a+x} \log(x)}{a^2 x - x^3} dx + \arctan(\sqrt{a+x}\sqrt{-a+x}, a) \log(x) - \frac{1}{2}i \log(x^2) \log(x) + \frac{1}{2}i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a/x)/x,x, algorithm="maxima")

[Out] -I\*a^2\*integrate(-log(x)/(a^2\*x - x^3), x) - a\*integrate(-sqrt(a+x)\*sqrt(-a+x)\*log(x)/(a^2\*x - x^3), x) + arctan2(sqrt(a+x)\*sqrt(-a+x), a)\*log(x) - 1/2\*I\*log(x^2)\*log(x) + 1/2\*I\*log(x)^2 + 1/2\*I\*log(x)\*log((a+x)/a) + 1/2\*I\*log(x)\*log((a-x)/a) + 1/2\*I\*dilog(x/a) + 1/2\*I\*dilog(-x/a)



---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arccos\left(\frac{a}{x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a/x)/x,x, algorithm="fricas")

[Out] integral(arccos(a/x)/x, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(a/x)/x,x)

[Out] Integral(acos(a/x)/x, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a/x)/x,x, algorithm="giac")

[Out] integrate(arccos(a/x)/x, x)

$$3.57 \quad \int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x^2} dx$$

**Optimal.** Leaf size=30

$$\frac{\sqrt{1 - \frac{a^2}{x^2}}}{a} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x}$$

[Out] Sqrt[1 - a^2/x^2]/a - ArcSec[x/a]/x

**Rubi [A]** time = 0.0218937, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {4833, 5220, 261}

$$\frac{\sqrt{1 - \frac{a^2}{x^2}}}{a} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a/x]/x^2,x]

[Out] Sqrt[1 - a^2/x^2]/a - ArcSec[x/a]/x

#### Rule 4833

Int[ArcCos[(c\_)/((a\_) + (b\_)\*(x\_)^(n\_))]^(m\_)\*(u\_), x\_Symbol] :> Int[u\*ArcSec[a/c + (b\*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]

#### Rule 5220

Int[((a\_) + ArcSec[(c\_)\*(x\_)]\*(b\_))\*((d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcSec[c\*x]))/(d\*(m + 1)), x] - Dist[(b\*d)/(c\*(m + 1)), Int[(d\*x)^(m - 1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

#### Rule 261

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x^2} dx &= \int \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x^2} dx \\ &= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{x} + a \int \frac{1}{\sqrt{1 - \frac{a^2}{x^2}} x^3} dx \\ &= \frac{\sqrt{1 - \frac{a^2}{x^2}}}{a} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x} \end{aligned}$$

**Mathematica [A]** time = 0.0185694, size = 30, normalized size = 1.

$$\frac{\sqrt{1 - \frac{a^2}{x^2}}}{a} - \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a/x]/x^2,x]

[Out] Sqrt[1 - a^2/x^2]/a - ArcCos[a/x]/x

**Maple [A]** time = 0.001, size = 32, normalized size = 1.1

$$-\frac{1}{a} \left( \frac{a}{x} \arccos\left(\frac{a}{x}\right) - \sqrt{1 - \frac{a^2}{x^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(a/x)/x^2,x)

[Out] -1/a\*(1/x\*a\*arccos(a/x)-(1-a^2/x^2)^(1/2))

**Maxima [A]** time = 1.43795, size = 42, normalized size = 1.4

$$-\frac{\frac{a \arccos\left(\frac{a}{x}\right)}{x} - \sqrt{-\frac{a^2}{x^2} + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a/x)/x^2,x, algorithm="maxima")

[Out] -(a\*arccos(a/x)/x - sqrt(-a^2/x^2 + 1))/a

**Fricas [A]** time = 2.39408, size = 69, normalized size = 2.3

$$-\frac{a \arccos\left(\frac{a}{x}\right) - x \sqrt{-\frac{a^2 - x^2}{x^2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a/x)/x^2,x, algorithm="fricas")

[Out] -(a\*arccos(a/x) - x\*sqrt(-(a^2 - x^2)/x^2))/(a\*x)

**Sympy [A]** time = 2.34016, size = 26, normalized size = 0.87

$$\begin{cases} -\frac{\arccos\left(\frac{a}{x}\right)}{x} + \frac{\sqrt{-\frac{a^2}{x^2} + 1}}{a} & \text{for } a \neq 0 \\ -\frac{\pi}{2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(a/x)/x\*\*2,x)

[Out] Piecewise((-acos(a/x)/x + sqrt(-a\*\*2/x\*\*2 + 1)/a, Ne(a, 0)), (-pi/(2\*x), True))

**Giac [A]** time = 1.2283, size = 42, normalized size = 1.4

$$-\frac{\frac{a \arccos\left(\frac{a}{x}\right)}{x} - \sqrt{-\frac{a^2}{x^2} + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a/x)/x^2,x, algorithm="giac")

[Out] -(a\*arccos(a/x)/x - sqrt(-a^2/x^2 + 1))/a

$$3.58 \quad \int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x^3} dx$$

**Optimal.** Leaf size=51

$$\frac{\sqrt{1 - \frac{a^2}{x^2}}}{4ax} - \frac{\csc^{-1}\left(\frac{x}{a}\right)}{4a^2} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

[Out] Sqrt[1 - a^2/x^2]/(4\*a\*x) - ArcCsc[x/a]/(4\*a^2) - ArcSec[x/a]/(2\*x^2)

**Rubi [A]** time = 0.0325045, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4833, 5220, 335, 321, 216}

$$\frac{\sqrt{1 - \frac{a^2}{x^2}}}{4ax} - \frac{\csc^{-1}\left(\frac{x}{a}\right)}{4a^2} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a/x]/x^3, x]

[Out] Sqrt[1 - a^2/x^2]/(4\*a\*x) - ArcCsc[x/a]/(4\*a^2) - ArcSec[x/a]/(2\*x^2)

#### Rule 4833

Int[ArcCos[(c\_.)/((a\_.) + (b\_.)\*(x\_)^(n\_.))]^(m\_.)\*(u\_.), x\_Symbol] := Int[u\*ArcSec[a/c + (b\*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]

#### Rule 5220

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)^(b\_.)]\*(d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[(d\*x)^(m+1)\*(a + b\*ArcSec[c\*x])/d^(m+1), x] - Dist[(b\*d)/(c\*(m+1)), Int[(d\*x)^(m-1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

#### Rule 335

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := -Subst[Int[(a + b/x^n)^p/x^(m+2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]

#### Rule 321

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a + b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x^3} dx &= \int \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x^3} dx \\
&= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2} + \frac{1}{2}a \int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}x^4} dx \\
&= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2} - \frac{1}{2}a \operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1-a^2x^2}} dx, x, \frac{1}{x}\right) \\
&= \frac{\sqrt{1-\frac{a^2}{x^2}}}{4ax} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2} - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-a^2x^2}} dx, x, \frac{1}{x}\right)}{4a} \\
&= \frac{\sqrt{1-\frac{a^2}{x^2}}}{4ax} - \frac{\csc^{-1}\left(\frac{x}{a}\right)}{4a^2} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0244137, size = 50, normalized size = 0.98

$$\frac{ax\sqrt{1-\frac{a^2}{x^2}} - 2a^2\cos^{-1}\left(\frac{a}{x}\right) - x^2\sin^{-1}\left(\frac{a}{x}\right)}{4a^2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a/x]/x^3,x]

[Out] (a\*Sqrt[1 - a^2/x^2]\*x - 2\*a^2\*ArcCos[a/x] - x^2\*ArcSin[a/x])/(4\*a^2\*x^2)

**Maple [A]** time = 0.006, size = 47, normalized size = 0.9

$$-\frac{1}{a^2} \left( \frac{a^2}{2x^2} \arccos\left(\frac{a}{x}\right) - \frac{a}{4x} \sqrt{1-\frac{a^2}{x^2}} + \frac{1}{4} \arcsin\left(\frac{a}{x}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(a/x)/x^3,x)

[Out] -1/a^2\*(1/2\*arccos(a/x)\*a^2/x^2-1/4\*a/x\*(1-a^2/x^2)^(1/2)+1/4\*arcsin(a/x))

**Maxima [A]** time = 1.46544, size = 104, normalized size = 2.04

$$-\frac{1}{4}a \left( \frac{x\sqrt{-\frac{a^2}{x^2}+1}}{a^2x^2\left(\frac{a^2}{x^2}-1\right)-a^4} - \frac{\arctan\left(\frac{x\sqrt{-\frac{a^2}{x^2}+1}}{a}\right)}{a^3} \right) - \frac{\arccos\left(\frac{a}{x}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a/x)/x^3,x, algorithm="maxima")

[Out]  $-1/4*a*(x*\sqrt{-a^2/x^2 + 1})/(a^2*x^2*(a^2/x^2 - 1) - a^4) - \arctan(x*\sqrt{-a^2/x^2 + 1}/a)/a^3) - 1/2*\arccos(a/x)/x^2$

**Fricas [A]** time = 2.47003, size = 97, normalized size = 1.9

$$\frac{ax\sqrt{-\frac{a^2-x^2}{x^2}} - (2a^2 - x^2)\arccos\left(\frac{a}{x}\right)}{4a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(a/x)/x^3,x, algorithm="fricas")`

[Out]  $1/4*(a*x*\sqrt{-(a^2 - x^2)/x^2} - (2*a^2 - x^2)*\arccos(a/x))/(a^2*x^2)$

**Sympy [C]** time = 5.30487, size = 102, normalized size = 2.

$$a \left( \begin{array}{l} \left( \frac{i\sqrt{\frac{a^2}{x^2}-1}}{2a^2x} + \frac{i\operatorname{acosh}\left(\frac{a}{x}\right)}{2a^3} \right) \quad \text{for } \frac{|a^2|}{|x^2|} > 1 \\ \left( -\frac{1}{2x^3\sqrt{-\frac{a^2}{x^2}+1}} + \frac{1}{2a^2x\sqrt{-\frac{a^2}{x^2}+1}} - \frac{\operatorname{asin}\left(\frac{a}{x}\right)}{2a^3} \right) \quad \text{otherwise} \end{array} \right) - \frac{\operatorname{acos}\left(\frac{a}{x}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acos(a/x)/x**3,x)`

[Out]  $a*\operatorname{Piecewise}\left(\left(I*\sqrt{a^{**2}/x^{**2} - 1}\right)/\left(2*a^{**2}*x\right) + I*\operatorname{acosh}\left(a/x\right)/\left(2*a^{**3}\right), \operatorname{Abs}\left(a^{**2}\right)/\operatorname{Abs}\left(x^{**2}\right) > 1\right), \left(-1/\left(2*x^{**3}*\sqrt{-a^{**2}/x^{**2} + 1}\right) + 1/\left(2*a^{**2}*x*\sqrt{-a^{**2}/x^{**2} + 1}\right) - \operatorname{asin}\left(a/x\right)/\left(2*a^{**3}\right), \operatorname{True}\right)/2 - \operatorname{acos}\left(a/x\right)/\left(2*x^{**2}\right)$

**Giac [A]** time = 1.28026, size = 84, normalized size = 1.65

$$\frac{1}{4}a \left( \frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a^3\operatorname{sgn}(x)} + \frac{\sqrt{-a^2+x^2}}{a^2x^2\operatorname{sgn}(x)} \right) - \frac{\arccos\left(\frac{a}{x}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(a/x)/x^3,x, algorithm="giac")`

[Out]  $1/4*a*(\arctan(\sqrt{-a^2 + x^2}/a)/(a^3*\operatorname{sgn}(x)) + \sqrt{-a^2 + x^2}/(a^2*x^2*\operatorname{sgn}(x))) - 1/2*\arccos(a/x)/x^2$

$$3.59 \quad \int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x^4} dx$$

**Optimal.** Leaf size=56

$$-\frac{\left(1 - \frac{a^2}{x^2}\right)^{3/2}}{9a^3} + \frac{\sqrt{1 - \frac{a^2}{x^2}}}{3a^3} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

[Out] Sqrt[1 - a^2/x^2]/(3\*a^3) - (1 - a^2/x^2)^(3/2)/(9\*a^3) - ArcSec[x/a]/(3\*x^3)

**Rubi [A]** time = 0.0383783, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4833, 5220, 266, 43}

$$-\frac{\left(1 - \frac{a^2}{x^2}\right)^{3/2}}{9a^3} + \frac{\sqrt{1 - \frac{a^2}{x^2}}}{3a^3} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a/x]/x^4,x]

[Out] Sqrt[1 - a^2/x^2]/(3\*a^3) - (1 - a^2/x^2)^(3/2)/(9\*a^3) - ArcSec[x/a]/(3\*x^3)

#### Rule 4833

Int[ArcCos[(c\_)/((a\_) + (b\_)\*(x\_)^(n\_))]^(m\_)\*(u\_), x\_Symbol] :> Int[u\*ArcSec[a/c + (b\*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]

#### Rule 5220

Int[((a\_) + ArcSec[(c\_)\*(x\_)]\*(b\_))\*((d\_)\*(x\_)^(m\_)), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*ArcSec[c\*x]))/(d\*(m+1)), x] - Dist[(b\*d)/(c\*(m+1)), Int[(d\*x)^(m-1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

#### Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m+1)/n]]

#### Rule 43

Int[((a\_) + (b\_)\*(x\_)^(m\_))\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n+1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps



$$\begin{aligned}
\int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x^4} dx &= \int \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x^4} dx \\
&= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3} + \frac{1}{3}a \int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}x^5} dx \\
&= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3} - \frac{1}{6}a \operatorname{Subst}\left(\int \frac{x}{\sqrt{1-a^2x}} dx, x, \frac{1}{x^2}\right) \\
&= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3} - \frac{1}{6}a \operatorname{Subst}\left(\int \left(\frac{1}{a^2\sqrt{1-a^2x}} - \frac{\sqrt{1-a^2x}}{a^2}\right) dx, x, \frac{1}{x^2}\right) \\
&= \frac{\sqrt{1-\frac{a^2}{x^2}}}{3a^3} - \frac{\left(1-\frac{a^2}{x^2}\right)^{3/2}}{9a^3} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.0308231, size = 47, normalized size = 0.84

$$\frac{x\sqrt{1-\frac{a^2}{x^2}}(a^2+2x^2)-3a^3\cos^{-1}\left(\frac{a}{x}\right)}{9a^3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a/x]/x^4,x]

[Out] (Sqrt[1 - a^2/x^2]\*x\*(a^2 + 2\*x^2) - 3\*a^3\*ArcCos[a/x])/(9\*a^3\*x^3)

**Maple [A]** time = 0.006, size = 55, normalized size = 1.

$$-\frac{1}{a^3}\left(\frac{a^3}{3x^3}\arccos\left(\frac{a}{x}\right)-\frac{a^2}{9x^2}\sqrt{1-\frac{a^2}{x^2}}-\frac{2}{9}\sqrt{1-\frac{a^2}{x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(a/x)/x^4,x)

[Out] -1/a^3\*(1/3\*arccos(a/x)\*a^3/x^3-1/9\*a^2/x^2\*(1-a^2/x^2)^(1/2)-2/9\*(1-a^2/x^2)^(1/2))

**Maxima [A]** time = 1.44467, size = 66, normalized size = 1.18

$$-\frac{1}{9}a\left(\frac{\left(-\frac{a^2}{x^2}+1\right)^{\frac{3}{2}}}{a^4}-\frac{3\sqrt{-\frac{a^2}{x^2}+1}}{a^4}\right)-\frac{\arccos\left(\frac{a}{x}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a/x)/x^4,x, algorithm="maxima")

[Out]  $-1/9*a*((-a^2/x^2 + 1)^{(3/2)}/a^4 - 3*\sqrt{-a^2/x^2 + 1}/a^4) - 1/3*\arccos(a/x)/x^3$

**Fricas [A]** time = 2.46169, size = 104, normalized size = 1.86

$$\frac{3a^3 \arccos\left(\frac{a}{x}\right) - (a^2x + 2x^3)\sqrt{-\frac{a^2-x^2}{x^2}}}{9a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(a/x)/x^4,x, algorithm="fricas")`

[Out]  $-1/9*(3*a^3*\arccos(a/x) - (a^2*x + 2*x^3)*\sqrt{-(a^2 - x^2)/x^2})/(a^3*x^3)$

**Sympy [A]** time = 6.3656, size = 102, normalized size = 1.82

$$a \left( \begin{array}{l} \frac{\sqrt{-1+\frac{x^2}{a^2}}}{3ax^3} + \frac{2\sqrt{-1+\frac{x^2}{a^2}}}{3a^3x} \quad \text{for } \frac{|x^2|}{|a^2|} > 1 \\ \frac{i\sqrt{1-\frac{x^2}{a^2}}}{3ax^3} + \frac{2i\sqrt{1-\frac{x^2}{a^2}}}{3a^3x} \quad \text{otherwise} \end{array} \right) - \frac{\arccos\left(\frac{a}{x}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acos(a/x)/x**4,x)`

[Out] `a*Piecewise((sqrt(-1 + x**2/a**2)/(3*a*x**3) + 2*sqrt(-1 + x**2/a**2)/(3*a**3*x), Abs(x**2)/Abs(a**2) > 1), (I*sqrt(1 - x**2/a**2)/(3*a*x**3) + 2*I*sqrt(1 - x**2/a**2)/(3*a**3*x), True))/3 - acos(a/x)/(3*x**3)`

**Giac [A]** time = 1.31389, size = 88, normalized size = 1.57

$$-\frac{\arccos\left(\frac{a}{x}\right)}{3x^3} + \frac{4\left(a^2 + 3\left(x - \sqrt{-a^2 + x^2}\right)^2\right)a}{9\left(a^2 + \left(x - \sqrt{-a^2 + x^2}\right)^2\right)^3 \operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(a/x)/x^4,x, algorithm="giac")`

[Out]  $-1/3*\arccos(a/x)/x^3 + 4/9*(a^2 + 3*(x - \sqrt{-a^2 + x^2})^2)*a/((a^2 + (x - \sqrt{-a^2 + x^2})^2)^3*\operatorname{sgn}(x))$

### 3.60 $\int x^2 \cos^{-1}(\sqrt{x}) dx$

**Optimal.** Leaf size=78

$$-\frac{1}{18}\sqrt{1-xx^{5/2}} - \frac{5}{72}\sqrt{1-xx^{3/2}} + \frac{1}{3}x^3 \cos^{-1}(\sqrt{x}) - \frac{5}{48}\sqrt{1-x}\sqrt{x} - \frac{5}{96} \sin^{-1}(1-2x)$$

[Out]  $(-5*\text{Sqrt}[1-x]*\text{Sqrt}[x])/48 - (5*\text{Sqrt}[1-x]*x^{(3/2)})/72 - (\text{Sqrt}[1-x]*x^{(5/2)})/18 + (x^3*\text{ArcCos}[\text{Sqrt}[x]])/3 - (5*\text{ArcSin}[1-2*x])/96$

**Rubi [A]** time = 0.028212, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {4843, 12, 50, 53, 619, 216}

$$-\frac{1}{18}\sqrt{1-xx^{5/2}} - \frac{5}{72}\sqrt{1-xx^{3/2}} + \frac{1}{3}x^3 \cos^{-1}(\sqrt{x}) - \frac{5}{48}\sqrt{1-x}\sqrt{x} - \frac{5}{96} \sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*\text{ArcCos}[\text{Sqrt}[x]], x]$

[Out]  $(-5*\text{Sqrt}[1-x]*\text{Sqrt}[x])/48 - (5*\text{Sqrt}[1-x]*x^{(3/2)})/72 - (\text{Sqrt}[1-x]*x^{(5/2)})/18 + (x^3*\text{ArcCos}[\text{Sqrt}[x]])/3 - (5*\text{ArcSin}[1-2*x])/96$

#### Rule 4843

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Sim
p[((c + d*x)^(m + 1)*(a + b*ArcCos[u]))/(d*(m + 1)), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 - u^2], x], x],
x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u,
x]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] := Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

#### Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

+ 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

### Rubi steps

$$\begin{aligned}
 \int x^2 \cos^{-1}(\sqrt{x}) dx &= \frac{1}{3}x^3 \cos^{-1}(\sqrt{x}) + \frac{1}{3} \int \frac{x^{5/2}}{2\sqrt{1-x}} dx \\
 &= \frac{1}{3}x^3 \cos^{-1}(\sqrt{x}) + \frac{1}{6} \int \frac{x^{5/2}}{\sqrt{1-x}} dx \\
 &= -\frac{1}{18}\sqrt{1-xx^{5/2}} + \frac{1}{3}x^3 \cos^{-1}(\sqrt{x}) + \frac{5}{36} \int \frac{x^{3/2}}{\sqrt{1-x}} dx \\
 &= -\frac{5}{72}\sqrt{1-xx^{3/2}} - \frac{1}{18}\sqrt{1-xx^{5/2}} + \frac{1}{3}x^3 \cos^{-1}(\sqrt{x}) + \frac{5}{48} \int \frac{\sqrt{x}}{\sqrt{1-x}} dx \\
 &= -\frac{5}{48}\sqrt{1-x}\sqrt{x} - \frac{5}{72}\sqrt{1-xx^{3/2}} - \frac{1}{18}\sqrt{1-xx^{5/2}} + \frac{1}{3}x^3 \cos^{-1}(\sqrt{x}) + \frac{5}{96} \int \frac{1}{\sqrt{1-x}\sqrt{x}} dx \\
 &= -\frac{5}{48}\sqrt{1-x}\sqrt{x} - \frac{5}{72}\sqrt{1-xx^{3/2}} - \frac{1}{18}\sqrt{1-xx^{5/2}} + \frac{1}{3}x^3 \cos^{-1}(\sqrt{x}) + \frac{5}{96} \int \frac{1}{\sqrt{x-x^2}} dx \\
 &= -\frac{5}{48}\sqrt{1-x}\sqrt{x} - \frac{5}{72}\sqrt{1-xx^{3/2}} - \frac{1}{18}\sqrt{1-xx^{5/2}} + \frac{1}{3}x^3 \cos^{-1}(\sqrt{x}) - \frac{5}{96} \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, 1\right) \\
 &= -\frac{5}{48}\sqrt{1-x}\sqrt{x} - \frac{5}{72}\sqrt{1-xx^{3/2}} - \frac{1}{18}\sqrt{1-xx^{5/2}} + \frac{1}{3}x^3 \cos^{-1}(\sqrt{x}) - \frac{5}{96} \sin^{-1}(1-2x)
 \end{aligned}$$

**Mathematica [A]** time = 0.0375544, size = 46, normalized size = 0.59

$$\frac{1}{144} \left( -\sqrt{-(x-1)x} (8x^2 + 10x + 15) + 48x^3 \cos^{-1}(\sqrt{x}) + 15 \sin^{-1}(\sqrt{x}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCos[Sqrt[x]], x]

[Out] (-(Sqrt[-((-1 + x)\*x)]\*(15 + 10\*x + 8\*x^2)) + 48\*x^3\*ArcCos[Sqrt[x]] + 15\*ArcSin[Sqrt[x]])/144

**Maple [A]** time = 0.004, size = 53, normalized size = 0.7

$$\frac{x^3}{3} \arccos(\sqrt{x}) - \frac{1}{18}x^{\frac{5}{2}}\sqrt{1-x} - \frac{5}{72}x^{\frac{3}{2}}\sqrt{1-x} - \frac{5}{48}\sqrt{1-x}\sqrt{x} + \frac{5}{48} \arcsin(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccos(x^(1/2)), x)

[Out] 1/3\*x^3\*arccos(x^(1/2))-1/18\*x^(5/2)\*(1-x)^(1/2)-5/72\*x^(3/2)\*(1-x)^(1/2)-5/48\*(1-x)^(1/2)\*x^(1/2)+5/48\*arcsin(x^(1/2))

**Maxima [A]** time = 1.43558, size = 70, normalized size = 0.9

$$\frac{1}{3} x^3 \arccos(\sqrt{x}) - \frac{1}{18} x^2 \sqrt{-x+1} - \frac{5}{72} x^{\frac{3}{2}} \sqrt{-x+1} - \frac{5}{48} \sqrt{x} \sqrt{-x+1} + \frac{5}{48} \arcsin(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccos(x^(1/2)),x, algorithm="maxima")

[Out] 1/3\*x^3\*arccos(sqrt(x)) - 1/18\*x^(5/2)\*sqrt(-x + 1) - 5/72\*x^(3/2)\*sqrt(-x + 1) - 5/48\*sqrt(x)\*sqrt(-x + 1) + 5/48\*arcsin(sqrt(x))

**Fricas [A]** time = 2.63229, size = 115, normalized size = 1.47

$$-\frac{1}{144} (8x^2 + 10x + 15) \sqrt{x} \sqrt{-x+1} + \frac{1}{48} (16x^3 - 5) \arccos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccos(x^(1/2)),x, algorithm="fricas")

[Out] -1/144\*(8\*x^2 + 10\*x + 15)\*sqrt(x)\*sqrt(-x + 1) + 1/48\*(16\*x^3 - 5)\*arccos(sqrt(x))

**Sympy [A]** time = 10.1623, size = 73, normalized size = 0.94

$$\frac{x^3 \arccos(\sqrt{x})}{3} + \frac{\left\{ \frac{x^{\frac{3}{2}}(1-x)^{\frac{3}{2}}}{6} + \frac{3\sqrt{x}(1-2x)\sqrt{1-x}}{16} - \frac{\sqrt{x}\sqrt{1-x}}{2} + \frac{5 \arcsin(\sqrt{x})}{16} \right\}}{3} \text{ for } x \geq 0 \wedge x < 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acos(x\*\*(1/2)),x)

[Out] x\*\*3\*acos(sqrt(x))/3 + Piecewise(((x\*\*(3/2)\*(1 - x)\*\*(3/2)/6 + 3\*sqrt(x)\*(1 - 2\*x)\*sqrt(1 - x)/16 - sqrt(x)\*sqrt(1 - x)/2 + 5\*asin(sqrt(x))/16, (x >= 0) & (x < 1)))/3

**Giac [A]** time = 1.12811, size = 70, normalized size = 0.9

$$\frac{1}{3} x^3 \arccos(\sqrt{x}) - \frac{1}{18} x^2 \sqrt{-x+1} - \frac{5}{72} x^{\frac{3}{2}} \sqrt{-x+1} - \frac{5}{48} \sqrt{x} \sqrt{-x+1} - \frac{5}{48} \arccos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccos(x^(1/2)),x, algorithm="giac")

[Out] 1/3\*x^3\*arccos(sqrt(x)) - 1/18\*x^(5/2)\*sqrt(-x + 1) - 5/72\*x^(3/2)\*sqrt(-x + 1) - 5/48\*sqrt(x)\*sqrt(-x + 1) - 5/48\*arccos(sqrt(x))

### 3.61 $\int x \cos^{-1}(\sqrt{x}) dx$

**Optimal.** Leaf size=60

$$-\frac{1}{8}\sqrt{1-xx^{3/2}} + \frac{1}{2}x^2 \cos^{-1}(\sqrt{x}) - \frac{3}{16}\sqrt{1-x}\sqrt{x} - \frac{3}{32}\sin^{-1}(1-2x)$$

[Out]  $(-3\sqrt{1-x}\sqrt{x})/16 - (\sqrt{1-x}x^{3/2})/8 + (x^2\text{ArcCos}[\sqrt{x}])/2 - (3\text{ArcSin}[1-2x])/32$

**Rubi [A]** time = 0.0191523, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$ , Rules used = {4843, 12, 50, 53, 619, 216}

$$-\frac{1}{8}\sqrt{1-xx^{3/2}} + \frac{1}{2}x^2 \cos^{-1}(\sqrt{x}) - \frac{3}{16}\sqrt{1-x}\sqrt{x} - \frac{3}{32}\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x\text{ArcCos}[\sqrt{x}], x]$

[Out]  $(-3\sqrt{1-x}\sqrt{x})/16 - (\sqrt{1-x}x^{3/2})/8 + (x^2\text{ArcCos}[\sqrt{x}])/2 - (3\text{ArcSin}[1-2x])/32$

#### Rule 4843

$\text{Int}[(a + \text{ArcCos}[u])*(b + (c + d*x)^m), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{m+1}*(a + b*\text{ArcCos}[u])/(d*(m+1)), x] + \text{Dist}[b/(d*(m+1)), \text{Int}[\text{SimplifyIntegrand}[(c + d*x)^{m+1}*D[u, x]/\sqrt{1-u^2}], x], x] /;$   $\text{FreeQ}\{a, b, c, d, m\}, x \&\& \text{NeQ}[m, -1] \&\& \text{InverseFunctionFreeQ}[u, x] \&\& \text{!FunctionOfQ}[(c + d*x)^{m+1}, u, x] \&\& \text{!FunctionOfExponentialQ}[u, x]$

#### Rule 12

$\text{Int}[a*(u), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$   $\text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b)*(v)] /;$   $\text{FreeQ}[b, x]$

#### Rule 50

$\text{Int}[(a + (b*x)^m)*(c + d*x)^n], x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{m+1}*(c + d*x)^n/(b*(m+n+1)), x] + \text{Dist}[(n*(b*c - a*d))/(b*(m+n+1)), \text{Int}[(a + b*x)^m*(c + d*x)^{n-1}], x] /;$   $\text{FreeQ}\{a, b, c, d\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{GtQ}[n, 0] \&\& \text{NeQ}[m+n+1, 0] \&\& \text{!(IGtQ}[m, 0] \&\& (\text{!IntegerQ}[n] \mid\mid (\text{GtQ}[m, 0] \&\& \text{LtQ}[m-n, 0]))) \&\& \text{!ILtQ}[m+n+2, 0] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 53

$\text{Int}[1/(\sqrt{(a + (b*x)^m)*\sqrt{(c + d*x)^n}}), x\_Symbol] \rightarrow \text{Int}[1/\sqrt{a*c - b*(a-c)*x - b^2*x^2}], x] /;$   $\text{FreeQ}\{a, b, c, d\}, x \&\& \text{EqQ}[b+d, 0] \&\& \text{GtQ}[a+c, 0]$

#### Rule 619

$\text{Int}[(a + (b*x + (c*x)^2)^p), x\_Symbol] \rightarrow \text{Dist}[1/(2*c*(-4*c)/(b^2 - 4*a*c))^p], \text{Subst}[\text{Int}[\text{Simp}[1-x^2/(b^2 - 4*a*c)], x]^p, x], x, b$

+ 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

### Rubi steps

$$\begin{aligned}
 \int x \cos^{-1}(\sqrt{x}) dx &= \frac{1}{2}x^2 \cos^{-1}(\sqrt{x}) + \frac{1}{2} \int \frac{x^{3/2}}{2\sqrt{1-x}} dx \\
 &= \frac{1}{2}x^2 \cos^{-1}(\sqrt{x}) + \frac{1}{4} \int \frac{x^{3/2}}{\sqrt{1-x}} dx \\
 &= -\frac{1}{8}\sqrt{1-xx^{3/2}} + \frac{1}{2}x^2 \cos^{-1}(\sqrt{x}) + \frac{3}{16} \int \frac{\sqrt{x}}{\sqrt{1-x}} dx \\
 &= -\frac{3}{16}\sqrt{1-x}\sqrt{x} - \frac{1}{8}\sqrt{1-xx^{3/2}} + \frac{1}{2}x^2 \cos^{-1}(\sqrt{x}) + \frac{3}{32} \int \frac{1}{\sqrt{1-x}\sqrt{x}} dx \\
 &= -\frac{3}{16}\sqrt{1-x}\sqrt{x} - \frac{1}{8}\sqrt{1-xx^{3/2}} + \frac{1}{2}x^2 \cos^{-1}(\sqrt{x}) + \frac{3}{32} \int \frac{1}{\sqrt{x-x^2}} dx \\
 &= -\frac{3}{16}\sqrt{1-x}\sqrt{x} - \frac{1}{8}\sqrt{1-xx^{3/2}} + \frac{1}{2}x^2 \cos^{-1}(\sqrt{x}) - \frac{3}{32} \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, 1-2x\right) \\
 &= -\frac{3}{16}\sqrt{1-x}\sqrt{x} - \frac{1}{8}\sqrt{1-xx^{3/2}} + \frac{1}{2}x^2 \cos^{-1}(\sqrt{x}) - \frac{3}{32} \sin^{-1}(1-2x)
 \end{aligned}$$

**Mathematica [A]** time = 0.0293534, size = 41, normalized size = 0.68

$$\frac{1}{16} \left( 8x^2 \cos^{-1}(\sqrt{x}) - \sqrt{-(x-1)x}(2x+3) + 3 \sin^{-1}(\sqrt{x}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCos[Sqrt[x]], x]

[Out] (-(Sqrt[-((-1 + x)\*x)]\*(3 + 2\*x)) + 8\*x^2\*ArcCos[Sqrt[x]] + 3\*ArcSin[Sqrt[x]])/16

**Maple [A]** time = 0.003, size = 41, normalized size = 0.7

$$\frac{x^2}{2} \arccos(\sqrt{x}) - \frac{1}{8}x^{\frac{3}{2}}\sqrt{1-x} - \frac{3}{16}\sqrt{1-x}\sqrt{x} + \frac{3}{16} \arcsin(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccos(x^(1/2)), x)

[Out] 1/2\*x^2\*arccos(x^(1/2))-1/8\*x^(3/2)\*(1-x)^(1/2)-3/16\*(1-x)^(1/2)\*x^(1/2)+3/16\*arcsin(x^(1/2))

**Maxima [A]** time = 1.46528, size = 54, normalized size = 0.9

$$\frac{1}{2}x^2 \arccos(\sqrt{x}) - \frac{1}{8}x^{\frac{3}{2}}\sqrt{-x+1} - \frac{3}{16}\sqrt{x}\sqrt{-x+1} + \frac{3}{16} \arcsin(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccos(x^(1/2)),x, algorithm="maxima")

[Out] 1/2\*x^2\*arccos(sqrt(x)) - 1/8\*x^(3/2)\*sqrt(-x + 1) - 3/16\*sqrt(x)\*sqrt(-x + 1) + 3/16\*arcsin(sqrt(x))

**Fricas [A]** time = 2.5852, size = 99, normalized size = 1.65

$$-\frac{1}{16}(2x+3)\sqrt{x}\sqrt{-x+1} + \frac{1}{16}(8x^2-3)\arccos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccos(x^(1/2)),x, algorithm="fricas")

[Out] -1/16\*(2\*x + 3)\*sqrt(x)\*sqrt(-x + 1) + 1/16\*(8\*x^2 - 3)\*arccos(sqrt(x))

**Sympy [A]** time = 3.90069, size = 58, normalized size = 0.97

$$\frac{x^2 \arccos(\sqrt{x})}{2} + \frac{\begin{cases} \frac{\sqrt{x}(1-2x)\sqrt{1-x}}{8} - \frac{\sqrt{x}\sqrt{1-x}}{2} + \frac{3\arcsin(\sqrt{x})}{8} & \text{for } x \geq 0 \wedge x < 1 \end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acos(x\*\*(1/2)),x)

[Out] x\*\*2\*acos(sqrt(x))/2 + Piecewise((sqrt(x)\*(1 - 2\*x)\*sqrt(1 - x)/8 - sqrt(x)\*sqrt(1 - x)/2 + 3\*asin(sqrt(x))/8, (x >= 0) & (x < 1)))/2

**Giac [A]** time = 1.2774, size = 54, normalized size = 0.9

$$\frac{1}{2}x^2 \arccos(\sqrt{x}) - \frac{1}{8}x^{\frac{3}{2}}\sqrt{-x+1} - \frac{3}{16}\sqrt{x}\sqrt{-x+1} - \frac{3}{16}\arccos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccos(x^(1/2)),x, algorithm="giac")

[Out] 1/2\*x^2\*arccos(sqrt(x)) - 1/8\*x^(3/2)\*sqrt(-x + 1) - 3/16\*sqrt(x)\*sqrt(-x + 1) - 3/16\*arccos(sqrt(x))



### 3.62 $\int \cos^{-1}(\sqrt{x}) dx$

**Optimal.** Leaf size=37

$$-\frac{1}{2}\sqrt{1-x}\sqrt{x} - \frac{1}{4}\sin^{-1}(1-2x) + x\cos^{-1}(\sqrt{x})$$

[Out]  $-(\text{Sqrt}[1-x]*\text{Sqrt}[x])/2 + x*\text{ArcCos}[\text{Sqrt}[x]] - \text{ArcSin}[1-2*x]/4$

**Rubi [A]** time = 0.0107611, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.$ , Rules used = {4841, 12, 50, 53, 619, 216}

$$-\frac{1}{2}\sqrt{1-x}\sqrt{x} - \frac{1}{4}\sin^{-1}(1-2x) + x\cos^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCos}[\text{Sqrt}[x]], x]$

[Out]  $-(\text{Sqrt}[1-x]*\text{Sqrt}[x])/2 + x*\text{ArcCos}[\text{Sqrt}[x]] - \text{ArcSin}[1-2*x]/4$

#### Rule 4841

$\text{Int}[\text{ArcCos}[u], x\_Symbol] \rightarrow \text{Simp}[x*\text{ArcCos}[u], x] + \text{Int}[\text{SimplifyIntegrand}[(x*D[u, x])/Sqrt[1-u^2], x], x] /;$  InverseFunctionFreeQ[u, x] && !FunctionOfExponentialQ[u, x]

#### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 50

$\text{Int}[(a_.) + (b_.)*(x_)^(m_)*((c_.) + (d_.)*(x_))^(n_), x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^(m+1)*(c + d*x)^n/(b*(m+n+1)), x] + \text{Dist}[(n*(b*c - a*d))/(b*(m+n+1)), \text{Int}[(a + b*x)^m*(c + d*x)^(n-1), x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 53

$\text{Int}[1/(\text{Sqrt}[(a_.) + (b_.)*(x_)]*\text{Sqrt}[(c_.) + (d_.)*(x_)]), x\_Symbol] \rightarrow \text{Int}[1/\text{Sqrt}[a*c - b*(a-c)*x - b^2*x^2], x] /;$  FreeQ[{a, b, c, d}, x] && EqQ[b+d, 0] && GtQ[a+c, 0]

#### Rule 619

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^(p_), x\_Symbol] \rightarrow \text{Dist}[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), \text{Subst}[\text{Int}[\text{Simp}[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /;$  FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

### Rubi steps

$$\begin{aligned}
 \int \cos^{-1}(\sqrt{x}) dx &= x \cos^{-1}(\sqrt{x}) + \int \frac{\sqrt{x}}{2\sqrt{1-x}} dx \\
 &= x \cos^{-1}(\sqrt{x}) + \frac{1}{2} \int \frac{\sqrt{x}}{\sqrt{1-x}} dx \\
 &= -\frac{1}{2}\sqrt{1-x}\sqrt{x} + x \cos^{-1}(\sqrt{x}) + \frac{1}{4} \int \frac{1}{\sqrt{1-x}\sqrt{x}} dx \\
 &= -\frac{1}{2}\sqrt{1-x}\sqrt{x} + x \cos^{-1}(\sqrt{x}) + \frac{1}{4} \int \frac{1}{\sqrt{x-x^2}} dx \\
 &= -\frac{1}{2}\sqrt{1-x}\sqrt{x} + x \cos^{-1}(\sqrt{x}) - \frac{1}{4} \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, 1-2x\right) \\
 &= -\frac{1}{2}\sqrt{1-x}\sqrt{x} + x \cos^{-1}(\sqrt{x}) - \frac{1}{4} \sin^{-1}(1-2x)
 \end{aligned}$$

**Mathematica [A]** time = 0.0150251, size = 38, normalized size = 1.03

$$\frac{1}{2} \left( -\sqrt{-(x-1)x} - \sin^{-1}(\sqrt{1-x}) \right) + x \cos^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x]], x]
```

```
[Out] x*ArcCos[Sqrt[x]] + (-Sqrt[-((-1 + x)*x)] - ArcSin[Sqrt[1 - x]])/2
```

**Maple [A]** time = 0.002, size = 26, normalized size = 0.7

$$x \arccos(\sqrt{x}) - \frac{1}{2}\sqrt{1-x}\sqrt{x} + \frac{1}{2} \arcsin(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x^(1/2)), x)
```

```
[Out] x*arccos(x^(1/2))-1/2*(1-x)^(1/2)*x^(1/2)+1/2*arcsin(x^(1/2))
```

**Maxima [A]** time = 1.52363, size = 34, normalized size = 0.92

$$x \arccos(\sqrt{x}) - \frac{1}{2}\sqrt{x}\sqrt{-x+1} + \frac{1}{2} \arcsin(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2)), x, algorithm="maxima")
```

```
[Out] x*arccos(sqrt(x)) - 1/2*sqrt(x)*sqrt(-x + 1) + 1/2*arcsin(sqrt(x))
```

---

**Fricas [A]** time = 2.61165, size = 78, normalized size = 2.11

$$\frac{1}{2}(2x-1)\arccos(\sqrt{x}) - \frac{1}{2}\sqrt{x}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x^(1/2)),x, algorithm="fricas")

[Out] 1/2\*(2\*x - 1)\*arccos(sqrt(x)) - 1/2\*sqrt(x)\*sqrt(-x + 1)

---

**Sympy [A]** time = 0.337239, size = 29, normalized size = 0.78

$$-\frac{\sqrt{x}\sqrt{1-x}}{2} + x\arccos(\sqrt{x}) - \frac{\arccos(\sqrt{x})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(x\*\*(1/2)),x)

[Out] -sqrt(x)\*sqrt(1 - x)/2 + x\*acos(sqrt(x)) - acos(sqrt(x))/2

---

**Giac [A]** time = 1.19294, size = 34, normalized size = 0.92

$$x\arccos(\sqrt{x}) - \frac{1}{2}\sqrt{x}\sqrt{-x+1} - \frac{1}{2}\arccos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x^(1/2)),x, algorithm="giac")

[Out] x\*arccos(sqrt(x)) - 1/2\*sqrt(x)\*sqrt(-x + 1) - 1/2\*arccos(sqrt(x))

### 3.63 $\int \frac{\cos^{-1}(\sqrt{x})}{x} dx$

**Optimal.** Leaf size=56

$$-i\text{PolyLog}\left(2, -e^{2i\cos^{-1}(\sqrt{x})}\right) - i\cos^{-1}(\sqrt{x})^2 + 2\cos^{-1}(\sqrt{x})\log\left(1 + e^{2i\cos^{-1}(\sqrt{x})}\right)$$

[Out] (-I)\*ArcCos[Sqrt[x]]^2 + 2\*ArcCos[Sqrt[x]]\*Log[1 + E^((2\*I)\*ArcCos[Sqrt[x]])] - I\*PolyLog[2, -E^((2\*I)\*ArcCos[Sqrt[x]])]

**Rubi [A]** time = 0.0558409, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4831, 3719, 2190, 2279, 2391}

$$-i\text{PolyLog}\left(2, -e^{2i\cos^{-1}(\sqrt{x})}\right) - i\cos^{-1}(\sqrt{x})^2 + 2\cos^{-1}(\sqrt{x})\log\left(1 + e^{2i\cos^{-1}(\sqrt{x})}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCos[Sqrt[x]]/x, x]

[Out] (-I)\*ArcCos[Sqrt[x]]^2 + 2\*ArcCos[Sqrt[x]]\*Log[1 + E^((2\*I)\*ArcCos[Sqrt[x]])] - I\*PolyLog[2, -E^((2\*I)\*ArcCos[Sqrt[x]])]

#### Rule 4831

Int[ArcCos[(a\_.)\*(x\_)^(p\_)]^(n\_.)/(x\_), x\_Symbol] :> -Dist[p^(-1), Subst[Int[x^n\*Tan[x], x], x, ArcCos[a\*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]

#### Rule 3719

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_)))^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(\sqrt{x})}{x} dx &= -\left(2 \operatorname{Subst}\left(\int x \tan(x) dx, x, \cos^{-1}(\sqrt{x})\right)\right) \\
&= -i \cos^{-1}(\sqrt{x})^2 + 4i \operatorname{Subst}\left(\int \frac{e^{2ix}}{1+e^{2ix}} dx, x, \cos^{-1}(\sqrt{x})\right) \\
&= -i \cos^{-1}(\sqrt{x})^2 + 2 \cos^{-1}(\sqrt{x}) \log\left(1 + e^{2i \cos^{-1}(\sqrt{x})}\right) - 2 \operatorname{Subst}\left(\int \log(1 + e^{2ix}) dx, x, \cos^{-1}(\sqrt{x})\right) \\
&= -i \cos^{-1}(\sqrt{x})^2 + 2 \cos^{-1}(\sqrt{x}) \log\left(1 + e^{2i \cos^{-1}(\sqrt{x})}\right) + i \operatorname{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^{2i \cos^{-1}(\sqrt{x})}\right) \\
&= -i \cos^{-1}(\sqrt{x})^2 + 2 \cos^{-1}(\sqrt{x}) \log\left(1 + e^{2i \cos^{-1}(\sqrt{x})}\right) - i \operatorname{Li}_2\left(-e^{2i \cos^{-1}(\sqrt{x})}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0260489, size = 54, normalized size = 0.96

$$-i \left( \operatorname{PolyLog}\left(2, -e^{2i \cos^{-1}(\sqrt{x})}\right) + \cos^{-1}(\sqrt{x}) \left( \cos^{-1}(\sqrt{x}) + 2i \log\left(1 + e^{2i \cos^{-1}(\sqrt{x})}\right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[Sqrt[x]]/x,x]

[Out] (-I)\*(ArcCos[Sqrt[x]]\*(ArcCos[Sqrt[x]] + (2\*I)\*Log[1 + E^((2\*I)\*ArcCos[Sqrt[x]])]) + PolyLog[2, -E^((2\*I)\*ArcCos[Sqrt[x]])])

**Maple [A]** time = 0.002, size = 59, normalized size = 1.1

$$-i \left( \arccos(\sqrt{x})^2 + 2 \arccos(\sqrt{x}) \ln\left(1 + \left(\sqrt{x} + i\sqrt{1-x}\right)^2\right) - i \operatorname{polylog}\left(2, -\left(\sqrt{x} + i\sqrt{1-x}\right)^2\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(x^(1/2))/x,x)

[Out] -I\*arccos(x^(1/2))^2+2\*arccos(x^(1/2))\*ln(1+(x^(1/2)+I\*(1-x)^(1/2))^2)-I\*polylog(2,-(x^(1/2)+I\*(1-x)^(1/2))^2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(\sqrt{x})}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x^(1/2))/x,x, algorithm="maxima")

[Out] integrate(arccos(sqrt(x))/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\arccos(\sqrt{x})}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arccos(sqrt(x))/x, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(\sqrt{x})}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x**(1/2))/x,x)
```

```
[Out] Integral(acos(sqrt(x))/x, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(\sqrt{x})}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(sqrt(x))/x, x)
```

$$3.64 \quad \int \frac{\cos^{-1}(\sqrt{x})}{x^2} dx$$

**Optimal.** Leaf size=27

$$\frac{\sqrt{1-x}}{\sqrt{x}} - \frac{\cos^{-1}(\sqrt{x})}{x}$$

[Out] Sqrt[1 - x]/Sqrt[x] - ArcCos[Sqrt[x]]/x

**Rubi [A]** time = 0.0119337, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {4843, 12, 37}

$$\frac{\sqrt{1-x}}{\sqrt{x}} - \frac{\cos^{-1}(\sqrt{x})}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[Sqrt[x]]/x^2,x]

[Out] Sqrt[1 - x]/Sqrt[x] - ArcCos[Sqrt[x]]/x

#### Rule 4843

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Sim
p[(((c + d*x)^(m + 1)*(a + b*ArcCos[u]))/(d*(m + 1)), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 - u^2], x], x],
x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u,
x]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp
[(((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

#### Rubi steps

$$\begin{aligned} \int \frac{\cos^{-1}(\sqrt{x})}{x^2} dx &= -\frac{\cos^{-1}(\sqrt{x})}{x} - \int \frac{1}{2\sqrt{1-xx^{3/2}}} dx \\ &= -\frac{\cos^{-1}(\sqrt{x})}{x} - \frac{1}{2} \int \frac{1}{\sqrt{1-xx^{3/2}}} dx \\ &= \frac{\sqrt{1-x}}{\sqrt{x}} - \frac{\cos^{-1}(\sqrt{x})}{x} \end{aligned}$$

**Mathematica [A]** time = 0.0156122, size = 24, normalized size = 0.89

$$\frac{\sqrt{x-x^2} - \cos^{-1}(\sqrt{x})}{x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[Sqrt[x]]/x^2,x]

[Out] (Sqrt[x - x^2] - ArcCos[Sqrt[x]])/x

**Maple [A]** time = 0.001, size = 22, normalized size = 0.8

$$-\frac{1}{x} \arccos(\sqrt{x}) + \sqrt{1-x} \frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(x^(1/2))/x^2,x)

[Out] -arccos(x^(1/2))/x+(1-x)^(1/2)/x^(1/2)

**Maxima [A]** time = 1.49926, size = 28, normalized size = 1.04

$$\frac{\sqrt{-x+1}}{\sqrt{x}} - \frac{\arccos(\sqrt{x})}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x^(1/2))/x^2,x, algorithm="maxima")

[Out] sqrt(-x + 1)/sqrt(x) - arccos(sqrt(x))/x

**Fricas [A]** time = 2.56133, size = 59, normalized size = 2.19

$$\frac{\sqrt{x}\sqrt{-x+1} - \arccos(\sqrt{x})}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x^(1/2))/x^2,x, algorithm="fricas")

[Out] (sqrt(x)\*sqrt(-x + 1) - arccos(sqrt(x)))/x

**Sympy [C]** time = 4.22227, size = 44, normalized size = 1.63

$$\frac{\begin{cases} -\frac{2i\sqrt{x-1}}{\sqrt{x}} & \text{for } |x| > 1 \\ -\frac{2\sqrt{1-x}}{\sqrt{x}} & \text{otherwise} \end{cases} - \arccos(\sqrt{x})}{2x}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(x\*\*(1/2))/x\*\*2,x)

[Out] -Piecewise((-2\*I\*sqrt(x - 1)/sqrt(x), Abs(x) > 1), (-2\*sqrt(1 - x)/sqrt(x), True))/2 - acos(sqrt(x))/x

**Giac [A]** time = 1.17879, size = 54, normalized size = 2.

$$\frac{\sqrt{-x+1}-1}{2\sqrt{x}} - \frac{\arccos(\sqrt{x})}{x} - \frac{\sqrt{x}}{2(\sqrt{-x+1}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x^(1/2))/x^2,x, algorithm="giac")

[Out] 1/2\*(sqrt(-x + 1) - 1)/sqrt(x) - arccos(sqrt(x))/x - 1/2\*sqrt(x)/(sqrt(-x + 1) - 1)

$$3.65 \quad \int \frac{\cos^{-1}(\sqrt{x})}{x^3} dx$$

**Optimal.** Leaf size=50

$$\frac{\sqrt{1-x}}{6x^{3/2}} - \frac{\cos^{-1}(\sqrt{x})}{2x^2} + \frac{\sqrt{1-x}}{3\sqrt{x}}$$

[Out] Sqrt[1 - x]/(6\*x^(3/2)) + Sqrt[1 - x]/(3\*Sqrt[x]) - ArcCos[Sqrt[x]]/(2\*x^2)

**Rubi [A]** time = 0.0176729, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4843, 12, 45, 37}

$$\frac{\sqrt{1-x}}{6x^{3/2}} - \frac{\cos^{-1}(\sqrt{x})}{2x^2} + \frac{\sqrt{1-x}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[Sqrt[x]]/x^3, x]

[Out] Sqrt[1 - x]/(6\*x^(3/2)) + Sqrt[1 - x]/(3\*Sqrt[x]) - ArcCos[Sqrt[x]]/(2\*x^2)

#### Rule 4843

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(((c + d*x)^(m + 1)*(a + b*ArcCos[u]))/(d*(m + 1)), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 - u^2], x], x],
x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u,
x]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
+ d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] &&
(EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] || !SumSimplerQ[n, 1])
```

#### Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(\sqrt{x})}{x^3} dx &= -\frac{\cos^{-1}(\sqrt{x})}{2x^2} - \frac{1}{2} \int \frac{1}{2\sqrt{1-xx^{5/2}}} dx \\
&= -\frac{\cos^{-1}(\sqrt{x})}{2x^2} - \frac{1}{4} \int \frac{1}{\sqrt{1-xx^{5/2}}} dx \\
&= \frac{\sqrt{1-x}}{6x^{3/2}} - \frac{\cos^{-1}(\sqrt{x})}{2x^2} - \frac{1}{6} \int \frac{1}{\sqrt{1-xx^{3/2}}} dx \\
&= \frac{\sqrt{1-x}}{6x^{3/2}} + \frac{\sqrt{1-x}}{3\sqrt{x}} - \frac{\cos^{-1}(\sqrt{x})}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0214374, size = 43, normalized size = 0.86

$$\left(\frac{1}{6x^{3/2}} + \frac{1}{3\sqrt{x}}\right)\sqrt{1-x} - \frac{\cos^{-1}(\sqrt{x})}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[Sqrt[x]]/x^3,x]

[Out] (1/(6\*x^(3/2)) + 1/(3\*Sqrt[x]))\*Sqrt[1 - x] - ArcCos[Sqrt[x]]/(2\*x^2)

**Maple [A]** time = 0.004, size = 35, normalized size = 0.7

$$-\frac{1}{2x^2} \arccos(\sqrt{x}) + \frac{1}{6} \sqrt{1-xx^{-\frac{3}{2}}} + \frac{1}{3} \sqrt{1-x} \frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(x^(1/2))/x^3,x)

[Out] -1/2\*arccos(x^(1/2))/x^2+1/6\*(1-x)^(1/2)/x^(3/2)+1/3\*(1-x)^(1/2)/x^(1/2)

**Maxima [A]** time = 1.47328, size = 46, normalized size = 0.92

$$\frac{\sqrt{-x+1}}{3\sqrt{x}} + \frac{\sqrt{-x+1}}{6x^{\frac{3}{2}}} - \frac{\arccos(\sqrt{x})}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x^(1/2))/x^3,x, algorithm="maxima")

[Out] 1/3\*sqrt(-x + 1)/sqrt(x) + 1/6\*sqrt(-x + 1)/x^(3/2) - 1/2\*arccos(sqrt(x))/x^2

**Fricas [A]** time = 2.6279, size = 84, normalized size = 1.68

$$\frac{(2x+1)\sqrt{x}\sqrt{-x+1} - 3 \arccos(\sqrt{x})}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x^(1/2))/x^3,x, algorithm="fricas")

[Out] 1/6\*((2\*x + 1)\*sqrt(x)\*sqrt(-x + 1) - 3\*arccos(sqrt(x)))/x^2

**Sympy [A]** time = 24.9142, size = 44, normalized size = 0.88

$$\frac{\begin{cases} -\frac{\sqrt{1-x}}{\sqrt{x}} - \frac{(1-x)^{\frac{3}{2}}}{3x^{\frac{3}{2}}} & \text{for } x \geq 0 \wedge x < 1 \\ \arccos(\sqrt{x}) \end{cases}}{2} - \frac{\arccos(\sqrt{x})}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(x\*\*(1/2))/x\*\*3,x)

[Out] -Piecewise((-sqrt(1 - x)/sqrt(x) - (1 - x)\*\*(3/2)/(3\*x\*\*(3/2)), (x >= 0) & (x < 1))/2 - acos(sqrt(x))/(2\*x\*\*2)

**Giac [B]** time = 1.28093, size = 100, normalized size = 2.

$$\frac{(\sqrt{-x+1}-1)^3}{48x^{\frac{3}{2}}} + \frac{3(\sqrt{-x+1}-1)}{16\sqrt{x}} - \frac{x^{\frac{3}{2}}\left(\frac{9(\sqrt{-x+1}-1)^2}{x} + 1\right)}{48(\sqrt{-x+1}-1)^3} - \frac{\arccos(\sqrt{x})}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x^(1/2))/x^3,x, algorithm="giac")

[Out] 1/48\*(sqrt(-x + 1) - 1)^3/x^(3/2) + 3/16\*(sqrt(-x + 1) - 1)/sqrt(x) - 1/48\*x^(3/2)\*(9\*(sqrt(-x + 1) - 1)^2/x + 1)/(sqrt(-x + 1) - 1)^3 - 1/2\*arccos(sqrt(x))/x^2

$$3.66 \quad \int \frac{\cos^{-1}(\sqrt{x})}{x^4} dx$$

**Optimal.** Leaf size=68

$$\frac{4\sqrt{1-x}}{45x^{3/2}} + \frac{\sqrt{1-x}}{15x^{5/2}} - \frac{\cos^{-1}(\sqrt{x})}{3x^3} + \frac{8\sqrt{1-x}}{45\sqrt{x}}$$

[Out] Sqrt[1 - x]/(15\*x^(5/2)) + (4\*Sqrt[1 - x])/(45\*x^(3/2)) + (8\*Sqrt[1 - x])/(45\*Sqrt[x]) - ArcCos[Sqrt[x]]/(3\*x^3)

**Rubi [A]** time = 0.0218177, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4843, 12, 45, 37}

$$\frac{4\sqrt{1-x}}{45x^{3/2}} + \frac{\sqrt{1-x}}{15x^{5/2}} - \frac{\cos^{-1}(\sqrt{x})}{3x^3} + \frac{8\sqrt{1-x}}{45\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[Sqrt[x]]/x^4,x]

[Out] Sqrt[1 - x]/(15\*x^(5/2)) + (4\*Sqrt[1 - x])/(45\*x^(3/2)) + (8\*Sqrt[1 - x])/(45\*Sqrt[x]) - ArcCos[Sqrt[x]]/(3\*x^3)

#### Rule 4843

Int[((a\_.) + ArcCos[u\_]\*(b\_.))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[ ((c + d\*x)^(m + 1)\*(a + b\*ArcCos[u]))/(d\*(m + 1)), x] + Dist[b/(d\*(m + 1)), Int[SimplifyIntegrand[((c + d\*x)^(m + 1)\*D[u, x])/Sqrt[1 - u^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d\*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u, x]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 45

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] - Dist[(d\*Simplify[m + n + 2])/((b\*c - a\*d)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && !LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

#### Rule 37

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(\sqrt{x})}{x^4} dx &= -\frac{\cos^{-1}(\sqrt{x})}{3x^3} - \frac{1}{3} \int \frac{1}{2\sqrt{1-xx^{7/2}}} dx \\
&= -\frac{\cos^{-1}(\sqrt{x})}{3x^3} - \frac{1}{6} \int \frac{1}{\sqrt{1-xx^{7/2}}} dx \\
&= \frac{\sqrt{1-x}}{15x^{5/2}} - \frac{\cos^{-1}(\sqrt{x})}{3x^3} - \frac{2}{15} \int \frac{1}{\sqrt{1-xx^{5/2}}} dx \\
&= \frac{\sqrt{1-x}}{15x^{5/2}} + \frac{4\sqrt{1-x}}{45x^{3/2}} - \frac{\cos^{-1}(\sqrt{x})}{3x^3} - \frac{4}{45} \int \frac{1}{\sqrt{1-xx^{3/2}}} dx \\
&= \frac{\sqrt{1-x}}{15x^{5/2}} + \frac{4\sqrt{1-x}}{45x^{3/2}} + \frac{8\sqrt{1-x}}{45\sqrt{x}} - \frac{\cos^{-1}(\sqrt{x})}{3x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.0376777, size = 37, normalized size = 0.54

$$\frac{\sqrt{-(x-1)x}(8x^2+4x+3)-15\cos^{-1}(\sqrt{x})}{45x^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[Sqrt[x]]/x^4,x]

[Out] (Sqrt[-((-1+x)\*x)]\*(3+4\*x+8\*x^2)-15\*ArcCos[Sqrt[x]])/(45\*x^3)

**Maple [A]** time = 0.003, size = 47, normalized size = 0.7

$$-\frac{1}{3x^3} \arccos(\sqrt{x}) + \frac{1}{15} \sqrt{1-xx}^{-\frac{5}{2}} + \frac{4}{45} \sqrt{1-xx}^{-\frac{3}{2}} + \frac{8}{45} \sqrt{1-x} \frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(x^(1/2))/x^4,x)

[Out] -1/3\*arccos(x^(1/2))/x^3+1/15\*(1-x)^(1/2)/x^(5/2)+4/45\*(1-x)^(1/2)/x^(3/2)+8/45\*(1-x)^(1/2)/x^(1/2)

**Maxima [A]** time = 1.4762, size = 62, normalized size = 0.91

$$\frac{8\sqrt{-x+1}}{45\sqrt{x}} + \frac{4\sqrt{-x+1}}{45x^{\frac{3}{2}}} + \frac{\sqrt{-x+1}}{15x^{\frac{5}{2}}} - \frac{\arccos(\sqrt{x})}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x^(1/2))/x^4,x, algorithm="maxima")

[Out] 8/45\*sqrt(-x+1)/sqrt(x)+4/45\*sqrt(-x+1)/x^(3/2)+1/15\*sqrt(-x+1)/x^(5/2)-1/3\*arccos(sqrt(x))/x^3

---

**Fricas [A]** time = 2.63136, size = 97, normalized size = 1.43

$$\frac{(8x^2 + 4x + 3)\sqrt{x}\sqrt{-x+1} - 15 \arccos(\sqrt{x})}{45x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x^(1/2))/x^4,x, algorithm="fricas")

[Out] 1/45\*((8\*x^2 + 4\*x + 3)\*sqrt(x)\*sqrt(-x + 1) - 15\*arccos(sqrt(x)))/x^3

---

**Sympy [A]** time = 150.038, size = 60, normalized size = 0.88

$$\frac{\begin{cases} -\frac{\sqrt{1-x}}{\sqrt{x}} - \frac{2(1-x)^{\frac{3}{2}}}{3x^2} - \frac{(1-x)^{\frac{5}{2}}}{5x^2} & \text{for } x \geq 0 \wedge x < 1 \\ \arccos(\sqrt{x}) \end{cases}}{3} - \frac{\arccos(\sqrt{x})}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(x\*\*(1/2))/x\*\*4,x)

[Out] -Piecewise((-sqrt(1 - x)/sqrt(x) - 2\*(1 - x)\*\*(3/2)/(3\*x\*\*(3/2)) - (1 - x)\*  
\*(5/2)/(5\*x\*\*(5/2)), (x >= 0) & (x < 1))/3 - acos(sqrt(x))/(3\*x\*\*3)

---

**Giac [B]** time = 1.31196, size = 143, normalized size = 2.1

$$\frac{(\sqrt{-x+1}-1)^5}{480x^{\frac{5}{2}}} + \frac{5(\sqrt{-x+1}-1)^3}{288x^{\frac{3}{2}}} + \frac{5(\sqrt{-x+1}-1)}{48\sqrt{x}} - \frac{\left(\frac{150(\sqrt{-x+1}-1)^4}{x^2} + \frac{25(\sqrt{-x+1}-1)^2}{x} + 3\right)x^{\frac{5}{2}}}{1440(\sqrt{-x+1}-1)^5} - \frac{\arccos(\sqrt{x})}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x^(1/2))/x^4,x, algorithm="giac")

[Out] 1/480\*(sqrt(-x + 1) - 1)^5/x^(5/2) + 5/288\*(sqrt(-x + 1) - 1)^3/x^(3/2) + 5/  
48\*(sqrt(-x + 1) - 1)/sqrt(x) - 1/1440\*(150\*(sqrt(-x + 1) - 1)^4/x^2 + 25\*  
(sqrt(-x + 1) - 1)^2/x + 3)\*x^(5/2)/(sqrt(-x + 1) - 1)^5 - 1/3\*arccos(sqrt(  
x))/x^3

$$3.67 \quad \int \frac{\cos^{-1}(\sqrt{x})}{x^5} dx$$

**Optimal.** Leaf size=86

$$\frac{2\sqrt{1-x}}{35x^{3/2}} + \frac{3\sqrt{1-x}}{70x^{5/2}} + \frac{\sqrt{1-x}}{28x^{7/2}} - \frac{\cos^{-1}(\sqrt{x})}{4x^4} + \frac{4\sqrt{1-x}}{35\sqrt{x}}$$

[Out] Sqrt[1 - x]/(28\*x^(7/2)) + (3\*Sqrt[1 - x])/(70\*x^(5/2)) + (2\*Sqrt[1 - x])/(35\*x^(3/2)) + (4\*Sqrt[1 - x])/(35\*Sqrt[x]) - ArcCos[Sqrt[x]]/(4\*x^4)

**Rubi [A]** time = 0.0277524, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4843, 12, 45, 37}

$$\frac{2\sqrt{1-x}}{35x^{3/2}} + \frac{3\sqrt{1-x}}{70x^{5/2}} + \frac{\sqrt{1-x}}{28x^{7/2}} - \frac{\cos^{-1}(\sqrt{x})}{4x^4} + \frac{4\sqrt{1-x}}{35\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[Sqrt[x]]/x^5,x]

[Out] Sqrt[1 - x]/(28\*x^(7/2)) + (3\*Sqrt[1 - x])/(70\*x^(5/2)) + (2\*Sqrt[1 - x])/(35\*x^(3/2)) + (4\*Sqrt[1 - x])/(35\*Sqrt[x]) - ArcCos[Sqrt[x]]/(4\*x^4)

#### Rule 4843

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[
((c + d*x)^(m + 1)*(a + b*ArcCos[u])/(d*(m + 1)), x] + Dist[b/(d*(m + 1)),
Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 - u^2], x], x],
x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&& !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u, x]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*Simplify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[n] && !FunctionOfQ[(a + b*x)^(m + 1), x] && !FunctionOfExponentialQ[(c + d*x)^n, x] && (LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])
```

#### Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```



Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(\sqrt{x})}{x^5} dx &= -\frac{\cos^{-1}(\sqrt{x})}{4x^4} - \frac{1}{4} \int \frac{1}{2\sqrt{1-xx^{9/2}}} dx \\
&= -\frac{\cos^{-1}(\sqrt{x})}{4x^4} - \frac{1}{8} \int \frac{1}{\sqrt{1-xx^{9/2}}} dx \\
&= \frac{\sqrt{1-x}}{28x^{7/2}} - \frac{\cos^{-1}(\sqrt{x})}{4x^4} - \frac{3}{28} \int \frac{1}{\sqrt{1-xx^{7/2}}} dx \\
&= \frac{\sqrt{1-x}}{28x^{7/2}} + \frac{3\sqrt{1-x}}{70x^{5/2}} - \frac{\cos^{-1}(\sqrt{x})}{4x^4} - \frac{3}{35} \int \frac{1}{\sqrt{1-xx^{5/2}}} dx \\
&= \frac{\sqrt{1-x}}{28x^{7/2}} + \frac{3\sqrt{1-x}}{70x^{5/2}} + \frac{2\sqrt{1-x}}{35x^{3/2}} - \frac{\cos^{-1}(\sqrt{x})}{4x^4} - \frac{2}{35} \int \frac{1}{\sqrt{1-xx^{3/2}}} dx \\
&= \frac{\sqrt{1-x}}{28x^{7/2}} + \frac{3\sqrt{1-x}}{70x^{5/2}} + \frac{2\sqrt{1-x}}{35x^{3/2}} + \frac{4\sqrt{1-x}}{35\sqrt{x}} - \frac{\cos^{-1}(\sqrt{x})}{4x^4}
\end{aligned}$$

**Mathematica [A]** time = 0.0436597, size = 42, normalized size = 0.49

$$\frac{\sqrt{-(x-1)x}(16x^3 + 8x^2 + 6x + 5) - 35 \cos^{-1}(\sqrt{x})}{140x^4}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[Sqrt[x]]/x^5,x]

[Out] (Sqrt[-((-1 + x)\*x)]\*(5 + 6\*x + 8\*x^2 + 16\*x^3) - 35\*ArcCos[Sqrt[x]])/(140\*x^4)

**Maple [A]** time = 0.003, size = 59, normalized size = 0.7

$$-\frac{1}{4x^4} \arccos(\sqrt{x}) + \frac{1}{28} \sqrt{1-xx}^{-\frac{7}{2}} + \frac{3}{70} \sqrt{1-xx}^{-\frac{5}{2}} + \frac{2}{35} \sqrt{1-xx}^{-\frac{3}{2}} + \frac{4}{35} \sqrt{1-x} \frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(x^(1/2))/x^5,x)

[Out] -1/4\*arccos(x^(1/2))/x^4+1/28\*(1-x)^(1/2)/x^(7/2)+3/70\*(1-x)^(1/2)/x^(5/2)+2/35\*(1-x)^(1/2)/x^(3/2)+4/35\*(1-x)^(1/2)/x^(1/2)

**Maxima [A]** time = 1.49298, size = 78, normalized size = 0.91

$$\frac{4\sqrt{-x+1}}{35\sqrt{x}} + \frac{2\sqrt{-x+1}}{35x^{\frac{3}{2}}} + \frac{3\sqrt{-x+1}}{70x^{\frac{5}{2}}} + \frac{\sqrt{-x+1}}{28x^{\frac{7}{2}}} - \frac{\arccos(\sqrt{x})}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x^(1/2))/x^5,x, algorithm="maxima")

[Out]  $4/35\sqrt{-x + 1}/\sqrt{x} + 2/35\sqrt{-x + 1}/x^{(3/2)} + 3/70\sqrt{-x + 1}/x^{(5/2)} + 1/28\sqrt{-x + 1}/x^{(7/2)} - 1/4\arccos(\sqrt{x})/x^4$

**Fricas [A]** time = 2.32641, size = 111, normalized size = 1.29

$$\frac{(16x^3 + 8x^2 + 6x + 5)\sqrt{x}\sqrt{-x + 1} - 35\arccos(\sqrt{x})}{140x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(x^(1/2))/x^5,x, algorithm="fricas")`

[Out]  $1/140*((16*x^3 + 8*x^2 + 6*x + 5)*\sqrt{x}*\sqrt{-x + 1} - 35*\arccos(\sqrt{x}))/x^4$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acos(x**(1/2))/x**5,x)`

[Out] Timed out

**Giac [B]** time = 1.28499, size = 186, normalized size = 2.16

$$\frac{(\sqrt{-x + 1} - 1)^7}{3584x^{\frac{7}{2}}} + \frac{7(\sqrt{-x + 1} - 1)^5}{2560x^{\frac{5}{2}}} + \frac{7(\sqrt{-x + 1} - 1)^3}{512x^{\frac{3}{2}}} + \frac{35(\sqrt{-x + 1} - 1)}{512\sqrt{x}} - \frac{\left(\frac{1225(\sqrt{-x+1}-1)^6}{x^3} + \frac{245(\sqrt{-x+1}-1)^4}{x^2} + \frac{49(\sqrt{-x+1}-1)^2}{x}\right)}{17920(\sqrt{-x+1}-1)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(x^(1/2))/x^5,x, algorithm="giac")`

[Out]  $1/3584*(\sqrt{-x + 1} - 1)^7/x^{(7/2)} + 7/2560*(\sqrt{-x + 1} - 1)^5/x^{(5/2)} + 7/512*(\sqrt{-x + 1} - 1)^3/x^{(3/2)} + 35/512*(\sqrt{-x + 1} - 1)/\sqrt{x} - 1/17920*(1225*(\sqrt{-x + 1} - 1)^6/x^3 + 245*(\sqrt{-x + 1} - 1)^4/x^2 + 49*(\sqrt{-x + 1} - 1)^2/x + 5)*x^{(7/2)}/(\sqrt{-x + 1} - 1)^7 - 1/4*\arccos(\sqrt{x}))/x^4$

$$3.68 \quad \int \frac{\cos^{-1}(\sqrt{x})}{\sqrt{x}} dx$$

**Optimal.** Leaf size=25

$$2\sqrt{x} \cos^{-1}(\sqrt{x}) - 2\sqrt{1-x}$$

[Out] -2\*Sqrt[1 - x] + 2\*Sqrt[x]\*ArcCos[Sqrt[x]]

**Rubi [A]** time = 0.0187569, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {6715, 4620, 261}

$$2\sqrt{x} \cos^{-1}(\sqrt{x}) - 2\sqrt{1-x}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[Sqrt[x]]/Sqrt[x], x]

[Out] -2\*Sqrt[1 - x] + 2\*Sqrt[x]\*ArcCos[Sqrt[x]]

#### Rule 6715

Int[(u\_)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

#### Rule 4620

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCos[c\*x])^n, x] + Dist[b\*c\*n, Int[(x\*(a + b\*ArcCos[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 261

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{\cos^{-1}(\sqrt{x})}{\sqrt{x}} dx &= 2 \operatorname{Subst} \left( \int \cos^{-1}(x) dx, x, \sqrt{x} \right) \\ &= 2\sqrt{x} \cos^{-1}(\sqrt{x}) + 2 \operatorname{Subst} \left( \int \frac{x}{\sqrt{1-x^2}} dx, x, \sqrt{x} \right) \\ &= -2\sqrt{1-x} + 2\sqrt{x} \cos^{-1}(\sqrt{x}) \end{aligned}$$

**Mathematica [A]** time = 0.0078178, size = 25, normalized size = 1.

$$2\sqrt{x} \cos^{-1}(\sqrt{x}) - 2\sqrt{1-x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[Sqrt[x]]/Sqrt[x],x]

[Out] -2\*Sqrt[1 - x] + 2\*Sqrt[x]\*ArcCos[Sqrt[x]]

**Maple [A]** time = 0.003, size = 20, normalized size = 0.8

$$-2\sqrt{1-x} + 2\arccos(\sqrt{x})\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(x^(1/2))/x^(1/2),x)

[Out] -2\*(1-x)^(1/2)+2\*arccos(x^(1/2))\*x^(1/2)

**Maxima [A]** time = 1.48198, size = 26, normalized size = 1.04

$$2\sqrt{x}\arccos(\sqrt{x}) - 2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x^(1/2))/x^(1/2),x, algorithm="maxima")

[Out] 2\*sqrt(x)\*arccos(sqrt(x)) - 2\*sqrt(-x + 1)

**Fricas [A]** time = 2.53294, size = 59, normalized size = 2.36

$$2\sqrt{x}\arccos(\sqrt{x}) - 2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x^(1/2))/x^(1/2),x, algorithm="fricas")

[Out] 2\*sqrt(x)\*arccos(sqrt(x)) - 2\*sqrt(-x + 1)

**Sympy [A]** time = 0.365094, size = 20, normalized size = 0.8

$$2\sqrt{x}\arccos(\sqrt{x}) - 2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(x\*\*(1/2))/x\*\*(1/2),x)

[Out] 2\*sqrt(x)\*acos(sqrt(x)) - 2\*sqrt(1 - x)

**Giac [A]** time = 1.2737, size = 26, normalized size = 1.04

$$2\sqrt{x}\arccos(\sqrt{x}) - 2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*arccos(sqrt(x)) - 2*sqrt(-x + 1)
```

$$3.69 \quad \int \frac{\cos^{-1}(ax^n)}{x} dx$$

**Optimal.** Leaf size=68

$$-\frac{i \operatorname{PolyLog}\left(2, -e^{2i \cos^{-1}(ax^n)}\right)}{2n} - \frac{i \cos^{-1}(ax^n)^2}{2n} + \frac{\cos^{-1}(ax^n) \log\left(1 + e^{2i \cos^{-1}(ax^n)}\right)}{n}$$

[Out]  $((-I/2)*\operatorname{ArcCos}[a*x^n]^2)/n + (\operatorname{ArcCos}[a*x^n]*\operatorname{Log}[1 + E^{((2*I)*\operatorname{ArcCos}[a*x^n])}])/n - ((I/2)*\operatorname{PolyLog}[2, -E^{((2*I)*\operatorname{ArcCos}[a*x^n])}])/n$

**Rubi [A]** time = 0.0614399, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4831, 3719, 2190, 2279, 2391}

$$-\frac{i \operatorname{PolyLog}\left(2, -e^{2i \cos^{-1}(ax^n)}\right)}{2n} - \frac{i \cos^{-1}(ax^n)^2}{2n} + \frac{\cos^{-1}(ax^n) \log\left(1 + e^{2i \cos^{-1}(ax^n)}\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a\*x^n]/x,x]

[Out]  $((-I/2)*\operatorname{ArcCos}[a*x^n]^2)/n + (\operatorname{ArcCos}[a*x^n]*\operatorname{Log}[1 + E^{((2*I)*\operatorname{ArcCos}[a*x^n])}])/n - ((I/2)*\operatorname{PolyLog}[2, -E^{((2*I)*\operatorname{ArcCos}[a*x^n])}])/n$

#### Rule 4831

Int[ArcCos[(a\_.)\*(x\_)^(p\_)]^(n\_.)/(x\_), x\_Symbol] :> -Dist[p^(-1), Subst[Int[x^n\*Tan[x], x], x, ArcCos[a\*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]

#### Rule 3719

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))\*tan[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_)^(m\_.)))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_.) + (b\_.)\*((F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_)))^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(ax^n)}{x} dx &= -\frac{\text{Subst}\left(\int x \tan(x) dx, x, \cos^{-1}(ax^n)\right)}{n} \\
&= -\frac{i \cos^{-1}(ax^n)^2}{2n} + \frac{(2i) \text{Subst}\left(\int \frac{e^{2ix}}{1+e^{2ix}} dx, x, \cos^{-1}(ax^n)\right)}{n} \\
&= -\frac{i \cos^{-1}(ax^n)^2}{2n} + \frac{\cos^{-1}(ax^n) \log\left(1 + e^{2i \cos^{-1}(ax^n)}\right)}{n} - \frac{\text{Subst}\left(\int \log(1 + e^{2ix}) dx, x, \cos^{-1}(ax^n)\right)}{n} \\
&= -\frac{i \cos^{-1}(ax^n)^2}{2n} + \frac{\cos^{-1}(ax^n) \log\left(1 + e^{2i \cos^{-1}(ax^n)}\right)}{n} + \frac{i \text{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^{2i \cos^{-1}(ax^n)}\right)}{2n} \\
&= -\frac{i \cos^{-1}(ax^n)^2}{2n} + \frac{\cos^{-1}(ax^n) \log\left(1 + e^{2i \cos^{-1}(ax^n)}\right)}{n} - \frac{i \text{Li}_2\left(-e^{2i \cos^{-1}(ax^n)}\right)}{2n}
\end{aligned}$$

**Mathematica [B]** time = 0.137089, size = 141, normalized size = 2.07

$$\frac{a \left( \text{PolyLog}\left(2, e^{-2 \sinh^{-1}\left(\sqrt{-a^2} x^n\right)}\right) + 2n \log(x) \log\left(\sqrt{-a^2} x^n + \sqrt{1 - a^2 x^{2n}}\right) - \sinh^{-1}\left(\sqrt{-a^2} x^n\right)^2 - 2 \sinh^{-1}\left(\sqrt{-a^2} x^n\right) \log\left(\sqrt{-a^2} x^n + \sqrt{1 - a^2 x^{2n}}\right) \right)}{2\sqrt{-a^2} n}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCos[a\*x^n]/x,x]

[Out] ArcCos[a\*x^n]\*Log[x] + (a\*(-ArcSinh[Sqrt[-a^2]\*x^n]^2 - 2\*ArcSinh[Sqrt[-a^2]\*x^n]\*Log[1 - E^(-2\*ArcSinh[Sqrt[-a^2]\*x^n]])] + 2\*n\*Log[x]\*Log[Sqrt[-a^2]\*x^n + Sqrt[1 - a^2\*x^(2\*n)]] + PolyLog[2, E^(-2\*ArcSinh[Sqrt[-a^2]\*x^n]])])/(2\*Sqrt[-a^2]\*n)

**Maple [A]** time = 0.04, size = 89, normalized size = 1.3

$$-\frac{i}{2} \frac{(\arccos(ax^n))^2}{n} + \frac{\arccos(ax^n)}{n} \ln\left(1 + \left(ax^n + i\sqrt{1 - a^2(x^n)^2}\right)^2\right) - \frac{i}{2} \text{polylog}\left(2, -\left(ax^n + i\sqrt{1 - a^2(x^n)^2}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(a\*x^n)/x,x)

[Out] -1/2\*I\*arccos(a\*x^n)^2/n+arccos(a\*x^n)\*ln(1+(a\*x^n+I\*(1-a^2\*(x^n)^2)^(1/2))^2)/n-1/2\*I\*polylog(2,-(a\*x^n+I\*(1-a^2\*(x^n)^2)^(1/2))^2)/n

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-an \int \frac{\sqrt{ax^n + 1} \sqrt{-ax^n + 1} x^n \log(x)}{a^2 x x^{2n} - x} dx + \arctan\left(\sqrt{ax^n + 1} \sqrt{-ax^n + 1}, ax^n\right) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a\*x^n)/x,x, algorithm="maxima")

[Out]  $-a^n \int \frac{\sqrt{ax^n + 1} \sqrt{-ax^n + 1} x^n \log(x)}{(a^2 x x^{2n} - x), x} + \operatorname{arctan2}(\sqrt{ax^n + 1} \sqrt{-ax^n + 1}, ax^n) \log(x)$

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(a*x^n)/x,x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acos(a*x**n)/x,x)`

[Out] `Integral(acos(a*x**n)/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(a*x^n)/x,x, algorithm="giac")`

[Out] `integrate(arccos(a*x^n)/x, x)`



$$3.70 \quad \int \frac{\cos^{-1}(ax^5)}{x} dx$$

**Optimal.** Leaf size=62

$$-\frac{1}{10}i\text{PolyLog}\left(2, -e^{2i\cos^{-1}(ax^5)}\right) - \frac{1}{10}i\cos^{-1}(ax^5)^2 + \frac{1}{5}\cos^{-1}(ax^5)\log\left(1 + e^{2i\cos^{-1}(ax^5)}\right)$$

[Out]  $(-I/10)*\text{ArcCos}[a*x^5]^2 + (\text{ArcCos}[a*x^5]*\text{Log}[1 + E^{((2*I)*\text{ArcCos}[a*x^5])}])/5 - (I/10)*\text{PolyLog}[2, -E^{((2*I)*\text{ArcCos}[a*x^5])}]$

**Rubi [A]** time = 0.0574571, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4831, 3719, 2190, 2279, 2391}

$$-\frac{1}{10}i\text{PolyLog}\left(2, -e^{2i\cos^{-1}(ax^5)}\right) - \frac{1}{10}i\cos^{-1}(ax^5)^2 + \frac{1}{5}\cos^{-1}(ax^5)\log\left(1 + e^{2i\cos^{-1}(ax^5)}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a\*x^5]/x,x]

[Out]  $(-I/10)*\text{ArcCos}[a*x^5]^2 + (\text{ArcCos}[a*x^5]*\text{Log}[1 + E^{((2*I)*\text{ArcCos}[a*x^5])}])/5 - (I/10)*\text{PolyLog}[2, -E^{((2*I)*\text{ArcCos}[a*x^5])}]$

#### Rule 4831

Int[ArcCos[(a\_.)\*(x\_)^(p\_)]^(n\_.)/(x\_), x\_Symbol] :> -Dist[p^(-1), Subst[Int[x^n\*Tan[x], x], x, ArcCos[a\*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]

#### Rule 3719

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

#### Rule 2190

Int[((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(ax^5)}{x} dx &= -\left(\frac{1}{5} \text{Subst}\left(\int x \tan(x) dx, x, \cos^{-1}(ax^5)\right)\right) \\
&= -\frac{1}{10}i \cos^{-1}(ax^5)^2 + \frac{2}{5}i \text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}} dx, x, \cos^{-1}(ax^5)\right) \\
&= -\frac{1}{10}i \cos^{-1}(ax^5)^2 + \frac{1}{5} \cos^{-1}(ax^5) \log\left(1+e^{2i\cos^{-1}(ax^5)}\right) - \frac{1}{5} \text{Subst}\left(\int \log(1+e^{2ix}) dx, x, \cos^{-1}(ax^5)\right) \\
&= -\frac{1}{10}i \cos^{-1}(ax^5)^2 + \frac{1}{5} \cos^{-1}(ax^5) \log\left(1+e^{2i\cos^{-1}(ax^5)}\right) + \frac{1}{10}i \text{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^{2i\cos^{-1}(ax^5)}\right) \\
&= -\frac{1}{10}i \cos^{-1}(ax^5)^2 + \frac{1}{5} \cos^{-1}(ax^5) \log\left(1+e^{2i\cos^{-1}(ax^5)}\right) - \frac{1}{10}i \text{Li}_2\left(-e^{2i\cos^{-1}(ax^5)}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0313788, size = 56, normalized size = 0.9

$$-\frac{1}{10}i \left( \text{PolyLog}\left(2, -e^{2i\cos^{-1}(ax^5)}\right) + \cos^{-1}(ax^5) \left( \cos^{-1}(ax^5) + 2i \log\left(1+e^{2i\cos^{-1}(ax^5)}\right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[a\*x^5]/x,x]

[Out] (-I/10)\*(ArcCos[a\*x^5]\*(ArcCos[a\*x^5] + (2\*I)\*Log[1 + E^((2\*I)\*ArcCos[a\*x^5])]) + PolyLog[2, -E^((2\*I)\*ArcCos[a\*x^5])])

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{\arccos(ax^5)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(a\*x^5)/x,x)

[Out] int(arccos(a\*x^5)/x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax^5)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a\*x^5)/x,x, algorithm="maxima")

[Out] integrate(arccos(a\*x^5)/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arccos(ax^5)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x^5)/x, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax^5)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x**5)/x,x)
```

```
[Out] Integral(acos(a*x**5)/x, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax^5)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x^5)/x, x)
```

### 3.71 $\int x^3 \cos^{-1}(a + bx^4) dx$

**Optimal.** Leaf size=47

$$\frac{(a + bx^4) \cos^{-1}(a + bx^4)}{4b} - \frac{\sqrt{1 - (a + bx^4)^2}}{4b}$$

[Out]  $-\text{Sqrt}[1 - (a + b*x^4)^2]/(4*b) + ((a + b*x^4)*\text{ArcCos}[a + b*x^4])/(4*b)$

**Rubi [A]** time = 0.0514806, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6715, 4804, 4620, 261}

$$\frac{(a + bx^4) \cos^{-1}(a + bx^4)}{4b} - \frac{\sqrt{1 - (a + bx^4)^2}}{4b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*\text{ArcCos}[a + b*x^4], x]$

[Out]  $-\text{Sqrt}[1 - (a + b*x^4)^2]/(4*b) + ((a + b*x^4)*\text{ArcCos}[a + b*x^4])/(4*b)$

#### Rule 6715

$\text{Int}[(u_)*(x_)^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/(m + 1), \text{Subst}[\text{Int}[\text{SubstFor}[x^{(m + 1)}, u, x], x, x^{(m + 1)}], x] /; \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{FunctionOfQ}[x^{(m + 1)}, u, x]$

#### Rule 4804

$\text{Int}[(a_.) + \text{ArcCos}[(c_.) + (d_.)*(x_)]*(b_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Subst}[\text{Int}[(a + b*\text{ArcCos}[x])^n, x], x, c + d*x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x]$

#### Rule 4620

$\text{Int}[(a_.) + \text{ArcCos}[(c_.)*(x_)]*(b_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcCos}[c*x])^n, x] + \text{Dist}[b*c*n, \text{Int}[(x*(a + b*\text{ArcCos}[c*x])^{(n - 1)})/\text{Sqrt}[1 - c^2*x^2], x], x] /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{GtQ}[n, 0]$

#### Rule 261

$\text{Int}[(x_)^{(m_.)*((a_.) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x^n)^{(p + 1)}/(b*n*(p + 1)), x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \ \&\& \ \text{EqQ}[m, n - 1] \ \&\& \ \text{NeQ}[p, -1]$

#### Rubi steps

$$\begin{aligned}
\int x^3 \cos^{-1}(a + bx^4) dx &= \frac{1}{4} \text{Subst} \left( \int \cos^{-1}(a + bx) dx, x, x^4 \right) \\
&= \frac{\text{Subst} \left( \int \cos^{-1}(x) dx, x, a + bx^4 \right)}{4b} \\
&= \frac{(a + bx^4) \cos^{-1}(a + bx^4)}{4b} + \frac{\text{Subst} \left( \int \frac{x}{\sqrt{1-x^2}} dx, x, a + bx^4 \right)}{4b} \\
&= -\frac{\sqrt{1 - (a + bx^4)^2}}{4b} + \frac{(a + bx^4) \cos^{-1}(a + bx^4)}{4b}
\end{aligned}$$

**Mathematica [A]** time = 0.0276867, size = 43, normalized size = 0.91

$$\frac{(a + bx^4) \cos^{-1}(a + bx^4) - \sqrt{1 - (a + bx^4)^2}}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCos[a + b\*x^4], x]

[Out] (-Sqrt[1 - (a + b\*x^4)^2] + (a + b\*x^4)\*ArcCos[a + b\*x^4])/(4\*b)

**Maple [A]** time = 0.003, size = 40, normalized size = 0.9

$$\frac{1}{4b} \left( (bx^4 + a) \arccos(bx^4 + a) - \sqrt{1 - (bx^4 + a)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arccos(b\*x^4+a), x)

[Out] 1/4/b\*((b\*x^4+a)\*arccos(b\*x^4+a)-(1-(b\*x^4+a)^2)^(1/2))

**Maxima [A]** time = 1.61154, size = 53, normalized size = 1.13

$$\frac{(bx^4 + a) \arccos(bx^4 + a) - \sqrt{-(bx^4 + a)^2 + 1}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccos(b\*x^4+a), x, algorithm="maxima")

[Out] 1/4\*((b\*x^4 + a)\*arccos(b\*x^4 + a) - sqrt(-(b\*x^4 + a)^2 + 1))/b

**Fricas [A]** time = 2.43861, size = 105, normalized size = 2.23

$$\frac{(bx^4 + a) \arccos(bx^4 + a) - \sqrt{-b^2x^8 - 2abx^4 - a^2 + 1}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccos(b\*x^4+a),x, algorithm="fricas")

[Out] 1/4\*((b\*x^4 + a)\*arccos(b\*x^4 + a) - sqrt(-b^2\*x^8 - 2\*a\*b\*x^4 - a^2 + 1))/b

**Sympy [A]** time = 1.14307, size = 61, normalized size = 1.3

$$\begin{cases} \frac{a \arccos(ax^4) + x^4 \arccos(ax^4) - \frac{\sqrt{-a^2 - 2abx^4 - b^2x^8 + 1}}{4b}}{4b} & \text{for } b \neq 0 \\ \frac{x^4 \arccos(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*acos(b\*x\*\*4+a),x)

[Out] Piecewise((a\*acos(a + b\*x\*\*4)/(4\*b) + x\*\*4\*acos(a + b\*x\*\*4)/4 - sqrt(-a\*\*2 - 2\*a\*b\*x\*\*4 - b\*\*2\*x\*\*8 + 1)/(4\*b), Ne(b, 0)), (x\*\*4\*acos(a)/4, True))

**Giac [A]** time = 1.26132, size = 53, normalized size = 1.13

$$\frac{(bx^4 + a) \arccos(bx^4 + a) - \sqrt{-(bx^4 + a)^2 + 1}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccos(b\*x^4+a),x, algorithm="giac")

[Out] 1/4\*((b\*x^4 + a)\*arccos(b\*x^4 + a) - sqrt(-(b\*x^4 + a)^2 + 1))/b

### 3.72 $\int x^{-1+n} \cos^{-1}(a + bx^n) dx$

**Optimal.** Leaf size=48

$$\frac{(a + bx^n) \cos^{-1}(a + bx^n)}{bn} - \frac{\sqrt{1 - (a + bx^n)^2}}{bn}$$

[Out]  $-(\text{Sqrt}[1 - (a + b*x^n)^2]/(b*n)) + ((a + b*x^n)*\text{ArcCos}[a + b*x^n])/(b*n)$

**Rubi [A]** time = 0.0544105, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6715, 4804, 4620, 261}

$$\frac{(a + bx^n) \cos^{-1}(a + bx^n)}{bn} - \frac{\sqrt{1 - (a + bx^n)^2}}{bn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(-1 + n)} * \text{ArcCos}[a + b*x^n], x]$

[Out]  $-(\text{Sqrt}[1 - (a + b*x^n)^2]/(b*n)) + ((a + b*x^n)*\text{ArcCos}[a + b*x^n])/(b*n)$

#### Rule 6715

$\text{Int}[(u_*) * (x_)^{(m_*)}, x\_Symbol] := \text{Dist}[1/(m + 1), \text{Subst}[\text{Int}[\text{SubstFor}[x^{(m + 1)}, u, x], x], x, x^{(m + 1)}], x] /;$  FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^{(m + 1)}, u, x]

#### Rule 4804

$\text{Int}[(a_*) + \text{ArcCos}[(c_*) + (d_*) * (x_*)] * (b_*)^{(n_*)}, x\_Symbol] := \text{Dist}[1/d, \text{Subst}[\text{Int}[(a + b * \text{ArcCos}[x])^n, x], x, c + d * x], x] /;$  FreeQ[{a, b, c, d, n}, x]

#### Rule 4620

$\text{Int}[(a_*) + \text{ArcCos}[(c_*) * (x_*)] * (b_*)^{(n_*)}, x\_Symbol] := \text{Simp}[x * (a + b * \text{ArcCos}[c * x])^n, x] + \text{Dist}[b * c * n, \text{Int}[(x * (a + b * \text{ArcCos}[c * x])^{(n - 1)}) / \text{Sqrt}[1 - c^2 * x^2], x], x] /;$  FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 261

$\text{Int}[(x_)^{(m_*)} * ((a_*) + (b_*) * (x_)^{(n_*)})^{(p_*)}, x\_Symbol] := \text{Simp}[(a + b * x^n)^{(p + 1)} / (b * n * (p + 1)), x] /;$  FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int x^{-1+n} \cos^{-1}(a + bx^n) dx &= \frac{\text{Subst}\left(\int \cos^{-1}(a + bx) dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \cos^{-1}(x) dx, x, a + bx^n\right)}{bn} \\
&= \frac{(a + bx^n) \cos^{-1}(a + bx^n)}{bn} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}} dx, x, a + bx^n\right)}{bn} \\
&= -\frac{\sqrt{1 - (a + bx^n)^2}}{bn} + \frac{(a + bx^n) \cos^{-1}(a + bx^n)}{bn}
\end{aligned}$$

**Mathematica [A]** time = 0.0413817, size = 43, normalized size = 0.9

$$\frac{(a + bx^n) \cos^{-1}(a + bx^n) - \sqrt{1 - (a + bx^n)^2}}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + n)\*ArcCos[a + b\*x^n], x]

[Out] (-Sqrt[1 - (a + b\*x^n)^2] + (a + b\*x^n)\*ArcCos[a + b\*x^n])/(b\*n)

**Maple [F]** time = 0.03, size = 0, normalized size = 0.

$$\int x^{n-1} \arccos(a + bx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n-1)\*arccos(a+b\*x^n), x)

[Out] int(x^(n-1)\*arccos(a+b\*x^n), x)

**Maxima [A]** time = 1.60569, size = 55, normalized size = 1.15

$$\frac{(bx^n + a) \arccos(bx^n + a) - \sqrt{-(bx^n + a)^2 + 1}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)\*arccos(a+b\*x^n), x, algorithm="maxima")

[Out] ((b\*x^n + a)\*arccos(b\*x^n + a) - sqrt(-(b\*x^n + a)^2 + 1))/(b\*n)

**Fricas [A]** time = 2.64617, size = 132, normalized size = 2.75

$$\frac{bx^n \arccos(bx^n + a) + a \arccos(bx^n + a) - \sqrt{-b^2x^{2n} - 2abx^n - a^2 + 1}}{bn}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1+n)</sup>\*arccos(a+b\*x<sup>n</sup>),x, algorithm="fricas")

[Out] (b\*x<sup>n</sup>\*arccos(b\*x<sup>n</sup> + a) + a\*arccos(b\*x<sup>n</sup> + a) - sqrt(-b<sup>2</sup>\*x<sup>(2\*n)</sup> - 2\*a\*b\*x<sup>n</sup> - a<sup>2</sup> + 1))/(b\*n)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1+n)</sup>\*acos(a+b\*x<sup>n</sup>),x)

[Out] Timed out

**Giac [A]** time = 1.31185, size = 55, normalized size = 1.15

$$\frac{(bx^n + a) \arccos(bx^n + a) - \sqrt{-(bx^n + a)^2 + 1}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1+n)</sup>\*arccos(a+b\*x<sup>n</sup>),x, algorithm="giac")

[Out] ((b\*x<sup>n</sup> + a)\*arccos(b\*x<sup>n</sup> + a) - sqrt(-(b\*x<sup>n</sup> + a)<sup>2</sup> + 1))/(b\*n)

### 3.73 $\int (a + b \cos^{-1}(1 + dx^2))^4 dx$

**Optimal.** Leaf size=127

$$\frac{192b^3\sqrt{-d^2x^4 - 2dx^2}(a + b \cos^{-1}(dx^2 + 1))}{dx} - 48b^2x(a + b \cos^{-1}(dx^2 + 1))^2 - \frac{8b\sqrt{-d^2x^4 - 2dx^2}(a + b \cos^{-1}(dx^2 + 1))}{dx}$$

[Out] 384\*b^4\*x + (192\*b^3\*Sqrt[-2\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[1 + d\*x^2]))/(d\*x) - 48\*b^2\*x\*(a + b\*ArcCos[1 + d\*x^2])^2 - (8\*b\*Sqrt[-2\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[1 + d\*x^2])^3)/(d\*x) + x\*(a + b\*ArcCos[1 + d\*x^2])^4

**Rubi [A]** time = 0.0287647, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}}$  = 0.143, Rules used = {4815, 8}

$$\frac{192b^3\sqrt{-d^2x^4 - 2dx^2}(a + b \cos^{-1}(dx^2 + 1))}{dx} - 48b^2x(a + b \cos^{-1}(dx^2 + 1))^2 - \frac{8b\sqrt{-d^2x^4 - 2dx^2}(a + b \cos^{-1}(dx^2 + 1))}{dx}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[1 + d\*x^2])^4, x]

[Out] 384\*b^4\*x + (192\*b^3\*Sqrt[-2\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[1 + d\*x^2]))/(d\*x) - 48\*b^2\*x\*(a + b\*ArcCos[1 + d\*x^2])^2 - (8\*b\*Sqrt[-2\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[1 + d\*x^2])^3)/(d\*x) + x\*(a + b\*ArcCos[1 + d\*x^2])^4

#### Rule 4815

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_)^2]\*(b\_.))^n, x\_Symbol] :> Simp[x\*(a + b\*ArcCos[c + d\*x^2])^n, x] + (-Dist[4\*b^2\*n\*(n - 1), Int[(a + b\*ArcCos[c + d\*x^2])^(n - 2), x], x] - Simp[(2\*b\*n\*Sqrt[-2\*c\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[c + d\*x^2])^(n - 1))/(d\*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]

#### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned} \int (a + b \cos^{-1}(1 + dx^2))^4 dx &= -\frac{8b\sqrt{-2dx^2 - d^2x^4}(a + b \cos^{-1}(1 + dx^2))^3}{dx} + x(a + b \cos^{-1}(1 + dx^2))^4 - (48b^2) \int (a + b \cos^{-1}(1 + dx^2))^2 dx \\ &= \frac{192b^3\sqrt{-2dx^2 - d^2x^4}(a + b \cos^{-1}(1 + dx^2))}{dx} - 48b^2x(a + b \cos^{-1}(1 + dx^2))^2 - \frac{8b\sqrt{-2dx^2 - d^2x^4}(a + b \cos^{-1}(1 + dx^2))^3}{dx} \\ &= 384b^4x + \frac{192b^3\sqrt{-2dx^2 - d^2x^4}(a + b \cos^{-1}(1 + dx^2))}{dx} - 48b^2x(a + b \cos^{-1}(1 + dx^2))^2 \end{aligned}$$

**Mathematica [A]** time = 0.232636, size = 249, normalized size = 1.96

$$dx^2(-48a^2b^2 + a^4 + 384b^4) - 8ab(a^2 - 24b^2)\sqrt{-dx^2(dx^2 + 2)} + 6b^2\cos^{-1}(dx^2 + 1)^2(a^2dx^2 - 4ab\sqrt{-dx^2(dx^2 + 2)} - 8$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^4,x]
```

```
[Out] ((a^4 - 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*a*b*(a^2 - 24*b^2)*Sqrt[-(d*x^2*(2 + d*x^2))] + 4*b*(a^3*d*x^2 - 24*a*b^2*d*x^2 - 6*a^2*b*Sqrt[-(d*x^2*(2 + d*x^2))] + 48*b^3*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^2] + 6*b^2*(a^2*d*x^2 - 8*b^2*d*x^2 - 4*a*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^2]^2 + 4*b^3*(a*d*x^2 - 2*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^2]^3 + b^4*d*x^2*ArcCos[1 + d*x^2]^4)/(d*x)
```

**Maple [F]** time = 0.121, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 + 1))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2+1))^4,x)
```

```
[Out] int((a+b*arccos(d*x^2+1))^4,x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 2.60548, size = 474, normalized size = 3.73

$$b^4 dx^2 \arccos(dx^2 + 1)^4 + 4 ab^3 dx^2 \arccos(dx^2 + 1)^3 + 6(a^2 b^2 - 8 b^4) dx^2 \arccos(dx^2 + 1)^2 + 4(a^3 b - 24 ab^3) dx^2 \arccos(dx^2 + 1) + (a^4 - 48 a^2 b^2 + 384 b^4) d x^2 - 8 (b^4 \arccos(dx^2 + 1))^2 + 3 a^3 b - 24 a^2 b^2 - 8 b^4) a \arccos(dx^2 + 1) * \sqrt{-d^2 x^4 - 2 d x^2} / (d x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x^2*arccos(d*x^2 + 1)^4 + 4*a*b^3*d*x^2*arccos(d*x^2 + 1)^3 + 6*(a^2*b^2 - 8*b^4)*d*x^2*arccos(d*x^2 + 1)^2 + 4*(a^3*b - 24*a*b^3)*d*x^2*arccos(d*x^2 + 1) + (a^4 - 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*(b^4*arccos(d*x^2 + 1))^2 + 3*a^3*b - 24*a^2*b^2 - 8*b^4)*a*arccos(d*x^2 + 1)*sqrt(-d^2*x^4 - 2*d*x^2))/(d*x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 + 1))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acos(d\*x\*\*2+1))\*\*4,x)

[Out] Integral((a + b\*acos(d\*x\*\*2 + 1))\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arccos(dx^2 + 1) + a)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2+1))^4,x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 + 1) + a)^4, x)

### 3.74 $\int (a + b \cos^{-1}(1 + dx^2))^3 dx$

**Optimal.** Leaf size=110

$$-24ab^2x - \frac{6b\sqrt{-d^2x^4 - 2dx^2}(a + b \cos^{-1}(dx^2 + 1))^2}{dx} + x(a + b \cos^{-1}(dx^2 + 1))^3 + \frac{48b^3\sqrt{-d^2x^4 - 2dx^2}}{dx} - 24b^3x \cos^{-1}(dx^2 + 1)$$

[Out]  $-24*a*b^2*x + (48*b^3*\text{Sqrt}[-2*d*x^2 - d^2*x^4])/(d*x) - 24*b^3*x*\text{ArcCos}[1 + d*x^2] - (6*b*\text{Sqrt}[-2*d*x^2 - d^2*x^4]*(a + b*\text{ArcCos}[1 + d*x^2])^2)/(d*x) + x*(a + b*\text{ArcCos}[1 + d*x^2])^3$

**Rubi [A]** time = 0.0571493, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {4815, 4841, 12, 1588}

$$-24ab^2x - \frac{6b\sqrt{-d^2x^4 - 2dx^2}(a + b \cos^{-1}(dx^2 + 1))^2}{dx} + x(a + b \cos^{-1}(dx^2 + 1))^3 + \frac{48b^3\sqrt{-d^2x^4 - 2dx^2}}{dx} - 24b^3x \cos^{-1}(dx^2 + 1)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcCos}[1 + d*x^2])^3, x]$

[Out]  $-24*a*b^2*x + (48*b^3*\text{Sqrt}[-2*d*x^2 - d^2*x^4])/(d*x) - 24*b^3*x*\text{ArcCos}[1 + d*x^2] - (6*b*\text{Sqrt}[-2*d*x^2 - d^2*x^4]*(a + b*\text{ArcCos}[1 + d*x^2])^2)/(d*x) + x*(a + b*\text{ArcCos}[1 + d*x^2])^3$

#### Rule 4815

$\text{Int}[(a + b*\text{ArcCos}[c + d*x^2])^n, x] + (-\text{Dist}[4*b^2*n*(n - 1), \text{Int}[(a + b*\text{ArcCos}[c + d*x^2])^{n - 2}, x], x] - \text{Simp}[(2*b*n*\text{Sqrt}[-2*c*d*x^2 - d^2*x^4]*(a + b*\text{ArcCos}[c + d*x^2])^{n - 1})/(d*x), x]) /;$  FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]

#### Rule 4841

$\text{Int}[\text{ArcCos}[u], x\_Symbol] := \text{Simp}[x*\text{ArcCos}[u], x] + \text{Int}[\text{SimplifyIntegrand}[(x*D[u, x])/Sqrt[1 - u^2], x], x] /;$  InverseFunctionFreeQ[u, x] && !FunctionOfExponentialQ[u, x]

#### Rule 12

$\text{Int}[(a_*)*(u_), x\_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 1588

$\text{Int}[(Pp_)*(Qq_)^{(m_.)}, x\_Symbol] := \text{With}[\{p = \text{Expon}[Pp, x], q = \text{Expon}[Qq, x]\}, \text{Simp}[(\text{Coeff}[Pp, x, p]*x^{(p - q + 1)}*Qq^{(m + 1)})/((p + m*q + 1)*\text{Coeff}[Qq, x, q]), x] /;$  NeQ[p + m\*q + 1, 0] && EqQ[(p + m\*q + 1)\*Coeff[Qq, x, q]\*Pp, Coeff[Pp, x, p]\*x^{(p - q)}\*((p - q + 1)\*Qq + (m + 1)\*x\*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int (a + b \cos^{-1}(1 + dx^2))^3 dx &= -\frac{6b\sqrt{-2dx^2 - d^2x^4} (a + b \cos^{-1}(1 + dx^2))^2}{dx} + x (a + b \cos^{-1}(1 + dx^2))^3 - (24b^2) \int (a + b \cos^{-1}(1 + dx^2))^2 dx \\
&= -24ab^2x - \frac{6b\sqrt{-2dx^2 - d^2x^4} (a + b \cos^{-1}(1 + dx^2))^2}{dx} + x (a + b \cos^{-1}(1 + dx^2))^3 - (24b^2) \int (a + b \cos^{-1}(1 + dx^2)) dx \\
&= -24ab^2x - 24b^3x \cos^{-1}(1 + dx^2) - \frac{6b\sqrt{-2dx^2 - d^2x^4} (a + b \cos^{-1}(1 + dx^2))^2}{dx} + x (a + b \cos^{-1}(1 + dx^2))^3 - (24b^2) \int (a + b \cos^{-1}(1 + dx^2)) dx \\
&= -24ab^2x - 24b^3x \cos^{-1}(1 + dx^2) - \frac{6b\sqrt{-2dx^2 - d^2x^4} (a + b \cos^{-1}(1 + dx^2))^2}{dx} + x (a + b \cos^{-1}(1 + dx^2))^3 - (24b^2) \int (a + b \cos^{-1}(1 + dx^2)) dx \\
&= -24ab^2x + \frac{48b^3\sqrt{-2dx^2 - d^2x^4}}{dx} - 24b^3x \cos^{-1}(1 + dx^2) - \frac{6b\sqrt{-2dx^2 - d^2x^4} (a + b \cos^{-1}(1 + dx^2))^2}{dx} + x (a + b \cos^{-1}(1 + dx^2))^3 - (24b^2) \int (a + b \cos^{-1}(1 + dx^2)) dx
\end{aligned}$$

**Mathematica [A]** time = 0.125622, size = 162, normalized size = 1.47

$$\frac{adx^2(a^2 - 24b^2) - 6b(a^2 - 8b^2)\sqrt{-dx^2(dx^2 + 2)} + 3b \cos^{-1}(dx^2 + 1)(a^2dx^2 - 4ab\sqrt{-dx^2(dx^2 + 2)} - 8b^2dx^2) + 3b^2 \cos^{-1}(dx^2 + 1)}{dx}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCos[1 + d\*x^2])^3, x]

[Out] (a\*(a^2 - 24\*b^2)\*d\*x^2 - 6\*b\*(a^2 - 8\*b^2)\*Sqrt[-(d\*x^2\*(2 + d\*x^2))] + 3\*b\*(a^2\*d\*x^2 - 8\*b^2\*d\*x^2 - 4\*a\*b\*Sqrt[-(d\*x^2\*(2 + d\*x^2))])\*ArcCos[1 + d\*x^2] + 3\*b^2\*(a\*d\*x^2 - 2\*b\*Sqrt[-(d\*x^2\*(2 + d\*x^2))])\*ArcCos[1 + d\*x^2]^2 + b^3\*d\*x^2\*ArcCos[1 + d\*x^2]^3)/(d\*x)

**Maple [F]** time = 0.116, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 + 1))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos(d\*x^2+1))^3,x)

[Out] int((a+b\*arccos(d\*x^2+1))^3,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2+1))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.36219, size = 328, normalized size = 2.98

$$\frac{b^3 dx^2 \arccos(dx^2 + 1)^3 + 3 ab^2 dx^2 \arccos(dx^2 + 1)^2 + 3(a^2 b - 8 b^3) dx^2 \arccos(dx^2 + 1) + (a^3 - 24 ab^2) dx^2 - 6 \sqrt{\dots}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2+1))^3,x, algorithm="fricas")

[Out] (b^3\*d\*x^2\*arccos(d\*x^2 + 1)^3 + 3\*a\*b^2\*d\*x^2\*arccos(d\*x^2 + 1)^2 + 3\*(a^2\*b - 8\*b^3)\*d\*x^2\*arccos(d\*x^2 + 1) + (a^3 - 24\*a\*b^2)\*d\*x^2 - 6\*sqrt(-d^2\*x^4 - 2\*d\*x^2)\* (b^3\*arccos(d\*x^2 + 1)^2 + 2\*a\*b^2\*arccos(d\*x^2 + 1) + a^2\*b - 8\*b^3))/(d\*x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 + 1))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acos(d\*x\*\*2+1))\*\*3,x)

[Out] Integral((a + b\*acos(d\*x\*\*2 + 1))\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arccos(dx^2 + 1) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2+1))^3,x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 + 1) + a)^3, x)

### 3.75 $\int (a + b \cos^{-1}(1 + dx^2))^2 dx$

**Optimal.** Leaf size=63

$$-\frac{4b\sqrt{-d^2x^4 - 2dx^2}(a + b \cos^{-1}(dx^2 + 1))}{dx} + x(a + b \cos^{-1}(dx^2 + 1))^2 - 8b^2x$$

[Out]  $-8*b^2*x - (4*b*sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2]))/(d*x) + x*(a + b*ArcCos[1 + d*x^2])^2$

**Rubi [A]** time = 0.0115554, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {4815, 8}

$$-\frac{4b\sqrt{-d^2x^4 - 2dx^2}(a + b \cos^{-1}(dx^2 + 1))}{dx} + x(a + b \cos^{-1}(dx^2 + 1))^2 - 8b^2x$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[1 + d\*x^2])^2, x]

[Out]  $-8*b^2*x - (4*b*sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2]))/(d*x) + x*(a + b*ArcCos[1 + d*x^2])^2$

#### Rule 4815

Int[((a\_.) + ArcCos[(c\_) + (d\_.)\*(x\_)^2]\*(b\_.))^n, x\_Symbol] :> Simp[x\*(a + b\*ArcCos[c + d\*x^2])^n, x] + (-Dist[4\*b^2\*n\*(n - 1), Int[(a + b\*ArcCos[c + d\*x^2])^(n - 2), x], x] - Simp[(2\*b\*n\*Sqrt[-2\*c\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[c + d\*x^2])^(n - 1))/(d\*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]

#### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned} \int (a + b \cos^{-1}(1 + dx^2))^2 dx &= -\frac{4b\sqrt{-2dx^2 - d^2x^4}(a + b \cos^{-1}(1 + dx^2))}{dx} + x(a + b \cos^{-1}(1 + dx^2))^2 - (8b^2) \int 1 dx \\ &= -8b^2x - \frac{4b\sqrt{-2dx^2 - d^2x^4}(a + b \cos^{-1}(1 + dx^2))}{dx} + x(a + b \cos^{-1}(1 + dx^2))^2 \end{aligned}$$

**Mathematica [A]** time = 0.0657708, size = 98, normalized size = 1.56

$$x(a^2 - 8b^2) - \frac{4ab\sqrt{-dx^2(dx^2 + 2)}}{dx} + \frac{2b \cos^{-1}(dx^2 + 1)(adx^2 - 2b\sqrt{-dx^2(dx^2 + 2)})}{dx} + b^2x \cos^{-1}(dx^2 + 1)^2$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCos[1 + d\*x^2])^2, x]



[Out]  $(a^2 - 8b^2)x - (4ab\sqrt{-(dx^2(2 + dx^2))})/(dx) + (2b(a^2dx^2 - 2b\sqrt{-(dx^2(2 + dx^2))})\text{ArcCos}[1 + dx^2])/(dx) + b^2x\text{ArcCos}[1 + dx^2]^2$

**Maple [F]** time = 0.117, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 + 1))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccos(dx^2+1))^2,x)`

[Out] `int((a+b*arccos(dx^2+1))^2,x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccos(dx^2+1))^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.50039, size = 205, normalized size = 3.25

$$\frac{b^2 dx^2 \arccos(dx^2 + 1)^2 + 2 ab dx^2 \arccos(dx^2 + 1) + (a^2 - 8b^2) dx^2 - 4 \sqrt{-d^2 x^4 - 2 dx^2} (b^2 \arccos(dx^2 + 1) + ab)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccos(dx^2+1))^2,x, algorithm="fricas")`

[Out]  $(b^2 dx^2 \arccos(dx^2 + 1)^2 + 2ab dx^2 \arccos(dx^2 + 1) + (a^2 - 8b^2) dx^2 - 4 \sqrt{-d^2 x^4 - 2 dx^2} (b^2 \arccos(dx^2 + 1) + ab))/(dx)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 + 1))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccos(dx**2+1))**2,x)`

[Out] `Integral((a + b*arccos(dx**2 + 1))**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arccos(dx^2 + 1) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^2, x)
```

### 3.76 $\int (a + b \cos^{-1}(1 + dx^2)) dx$

**Optimal.** Leaf size=43

$$ax - \frac{2b\sqrt{-d^2x^4 - 2dx^2}}{dx} + bx \cos^{-1}(dx^2 + 1)$$

[Out] a\*x - (2\*b\*Sqrt[-2\*d\*x^2 - d^2\*x^4])/(d\*x) + b\*x\*ArcCos[1 + d\*x^2]

**Rubi [A]** time = 0.0371819, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {4841, 12, 1588}

$$ax - \frac{2b\sqrt{-d^2x^4 - 2dx^2}}{dx} + bx \cos^{-1}(dx^2 + 1)$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcCos[1 + d\*x^2], x]

[Out] a\*x - (2\*b\*Sqrt[-2\*d\*x^2 - d^2\*x^4])/(d\*x) + b\*x\*ArcCos[1 + d\*x^2]

#### Rule 4841

Int[ArcCos[u\_], x\_Symbol] := Simp[x\*ArcCos[u], x] + Int[SimplifyIntegrand[(x\*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] && !FunctionOfExponentialQ[u, x]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 1588

Int[(Pp\_)\*(Qq\_)^(m\_.), x\_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]\*x^(p - q + 1)\*Qq^(m + 1))/((p + m\*q + 1)\*Coeff[Qq, x, q]), x] /; NeQ[p + m\*q + 1, 0] && EqQ[(p + m\*q + 1)\*Coeff[Qq, x, q]\*Pp, Coeff[Pp, x, p]\*x^(p - q)\*((p - q + 1)\*Qq + (m + 1)\*x\*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int (a + b \cos^{-1}(1 + dx^2)) dx &= ax + b \int \cos^{-1}(1 + dx^2) dx \\ &= ax + bx \cos^{-1}(1 + dx^2) + b \int \frac{2dx^2}{\sqrt{-2dx^2 - d^2x^4}} dx \\ &= ax + bx \cos^{-1}(1 + dx^2) + (2bd) \int \frac{x^2}{\sqrt{-2dx^2 - d^2x^4}} dx \\ &= ax - \frac{2b\sqrt{-2dx^2 - d^2x^4}}{dx} + bx \cos^{-1}(1 + dx^2) \end{aligned}$$

**Mathematica [A]** time = 0.0272371, size = 41, normalized size = 0.95

$$ax - \frac{2b\sqrt{-dx^2(dx^2+2)}}{dx} + bx \cos^{-1}(dx^2+1)$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcCos[1 + d\*x^2], x]

[Out] a\*x - (2\*b\*Sqrt[-(d\*x^2\*(2 + d\*x^2))])/(d\*x) + b\*x\*ArcCos[1 + d\*x^2]

**Maple [A]** time = 0.005, size = 45, normalized size = 1.1

$$ax + b \left( x \arccos(dx^2 + 1) + 2 \frac{x(dx^2 + 2)}{\sqrt{-d^2x^4 - 2dx^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b\*arccos(d\*x^2+1), x)

[Out] a\*x+b\*(x\*arccos(d\*x^2+1)+2/(-d^2\*x^4-2\*d\*x^2)^(1/2)\*x\*(d\*x^2+2))

**Maxima [A]** time = 1.56553, size = 61, normalized size = 1.42

$$\left( x \arccos(dx^2 + 1) + \frac{2(d^{3/2}x^2 + 2\sqrt{d})}{\sqrt{-dx^2 - 2d}} \right) b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccos(d\*x^2+1), x, algorithm="maxima")

[Out] (x\*arccos(d\*x^2 + 1) + 2\*(d^(3/2)\*x^2 + 2\*sqrt(d))/(sqrt(-d\*x^2 - 2)\*d))\*b + a\*x

**Fricas [A]** time = 2.30336, size = 103, normalized size = 2.4

$$\frac{bdx^2 \arccos(dx^2 + 1) + adx^2 - 2\sqrt{-d^2x^4 - 2dx^2}b}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccos(d\*x^2+1), x, algorithm="fricas")

[Out] (b\*d\*x^2\*arccos(d\*x^2 + 1) + a\*d\*x^2 - 2\*sqrt(-d^2\*x^4 - 2\*d\*x^2)\*b)/(d\*x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 + 1)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*acos(d\*x\*\*2+1),x)

[Out] Integral(a + b\*acos(d\*x\*\*2 + 1), x)

---

**Giac [A]** time = 1.34852, size = 78, normalized size = 1.81

$$\left(2d\left(\frac{\sqrt{2}\sqrt{-d}\operatorname{sgn}(x)}{d^2} - \frac{\sqrt{-d^2x^2 - 2d}}{d^2\operatorname{sgn}(x)}\right) + x \arccos(dx^2 + 1)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccos(d\*x^2+1),x, algorithm="giac")

[Out] (2\*d\*(sqrt(2)\*sqrt(-d)\*sgn(x)/d^2 - sqrt(-d^2\*x^2 - 2\*d)/(d^2\*sgn(x))) + x\*arccos(d\*x^2 + 1))\*b + a\*x

$$3.77 \quad \int \frac{1}{a+b \cos^{-1}(1+dx^2)} dx$$

**Optimal.** Leaf size=99

$$\frac{x \cos\left(\frac{a}{2b}\right) \operatorname{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2b}\sqrt{-dx^2}} + \frac{x \sin\left(\frac{a}{2b}\right) \operatorname{Si}\left(\frac{a+b \cos^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2b}\sqrt{-dx^2}}$$

[Out] (x\*Cos[a/(2\*b)]\*CosIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)]/(Sqrt[2]\*b\*Sqrt[-(d\*x^2)])) + (x\*Sin[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)]/(Sqrt[2]\*b\*Sqrt[-(d\*x^2)]))

**Rubi [A]** time = 0.0311988, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {4817}

$$\frac{x \cos\left(\frac{a}{2b}\right) \operatorname{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2b}\sqrt{-dx^2}} + \frac{x \sin\left(\frac{a}{2b}\right) \operatorname{Si}\left(\frac{a+b \cos^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2b}\sqrt{-dx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[1 + d\*x^2])^(-1), x]

[Out] (x\*Cos[a/(2\*b)]\*CosIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)]/(Sqrt[2]\*b\*Sqrt[-(d\*x^2)])) + (x\*Sin[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)]/(Sqrt[2]\*b\*Sqrt[-(d\*x^2)]))

#### Rule 4817

Int[((a\_.) + ArcCos[1 + (d\_.)\*(x\_)^2]\*(b\_.))^(-1), x\_Symbol] :> Simp[(x\*Cos[a/(2\*b)]\*CosIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)]/(Sqrt[2]\*b\*Sqrt[-(d\*x^2)]), x] + Simp[(x\*Sin[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)]/(Sqrt[2]\*b\*Sqrt[-(d\*x^2)]), x] /; FreeQ[{a, b, d}, x]

#### Rubi steps

$$\int \frac{1}{a+b \cos^{-1}(1+dx^2)} dx = \frac{x \cos\left(\frac{a}{2b}\right) \operatorname{Ci}\left(\frac{a+b \cos^{-1}(1+dx^2)}{2b}\right)}{\sqrt{2b}\sqrt{-dx^2}} + \frac{x \sin\left(\frac{a}{2b}\right) \operatorname{Si}\left(\frac{a+b \cos^{-1}(1+dx^2)}{2b}\right)}{\sqrt{2b}\sqrt{-dx^2}}$$

**Mathematica [A]** time = 0.103861, size = 85, normalized size = 0.86

$$\frac{\sin\left(\frac{1}{2} \cos^{-1}(dx^2+1)\right) \left( \cos\left(\frac{a}{2b}\right) \operatorname{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2+1)}{2b}\right) + \sin\left(\frac{a}{2b}\right) \operatorname{Si}\left(\frac{a+b \cos^{-1}(dx^2+1)}{2b}\right) \right)}{bdx}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCos[1 + d\*x^2])^(-1), x]

[Out]  $-\left(\frac{\sin\left(\arccos\left(\frac{1+dx^2}{2}\right)\right)\cos\left(\frac{a}{2b}\right)\cos\int\left(\frac{a+b\arccos\left(\frac{1+dx^2}{2}\right)}{2b}\right)+\sin\left(\frac{a}{2b}\right)\sin\int\left(\frac{a+b\arccos\left(\frac{1+dx^2}{2}\right)}{2b}\right)}{b dx}\right)$

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 + 1))^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*arccos(d*x^2+1)),x)`

[Out] `int(1/(a+b*arccos(d*x^2+1)),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \arccos(dx^2 + 1) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*arccos(d*x^2+1)),x, algorithm="maxima")`

[Out] `integrate(1/(b*arccos(d*x^2 + 1) + a), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b \arccos(dx^2 + 1) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*arccos(d*x^2+1)),x, algorithm="fricas")`

[Out] `integral(1/(b*arccos(d*x^2 + 1) + a), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b \arccos(dx^2 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*arccos(d*x**2+1)),x)`

[Out] `Integral(1/(a + b*arccos(d*x**2 + 1)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \arccos(dx^2 + 1) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1)),x, algorithm="giac")

[Out] integrate(1/(b\*arccos(d\*x^2 + 1) + a), x)



$$3.78 \quad \int \frac{1}{(a+b \cos^{-1}(1+dx^2))^2} dx$$

**Optimal.** Leaf size=151

$$\frac{x \sin\left(\frac{a}{2b}\right) \text{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2+1)}{2b}\right)}{2\sqrt{2}b^2\sqrt{-dx^2}} - \frac{x \cos\left(\frac{a}{2b}\right) \text{Si}\left(\frac{a+b \cos^{-1}(dx^2+1)}{2b}\right)}{2\sqrt{2}b^2\sqrt{-dx^2}} + \frac{\sqrt{-d^2x^4 - 2dx^2}}{2bdx(a+b \cos^{-1}(dx^2+1))}$$

[Out] Sqrt[-2\*d\*x^2 - d^2\*x^4]/(2\*b\*d\*x\*(a + b\*ArcCos[1 + d\*x^2])) + (x\*CosIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)]\*Sin[a/(2\*b)])/(2\*Sqrt[2]\*b^2\*Sqrt[-(d\*x^2)]) - (x\*Cos[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)])/(2\*Sqrt[2]\*b^2\*Sqrt[-(d\*x^2)])

**Rubi [A]** time = 0.0203403, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {4826}

$$\frac{x \sin\left(\frac{a}{2b}\right) \text{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2+1)}{2b}\right)}{2\sqrt{2}b^2\sqrt{-dx^2}} - \frac{x \cos\left(\frac{a}{2b}\right) \text{Si}\left(\frac{a+b \cos^{-1}(dx^2+1)}{2b}\right)}{2\sqrt{2}b^2\sqrt{-dx^2}} + \frac{\sqrt{-d^2x^4 - 2dx^2}}{2bdx(a+b \cos^{-1}(dx^2+1))}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[1 + d\*x^2])^(-2), x]

[Out] Sqrt[-2\*d\*x^2 - d^2\*x^4]/(2\*b\*d\*x\*(a + b\*ArcCos[1 + d\*x^2])) + (x\*CosIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)]\*Sin[a/(2\*b)])/(2\*Sqrt[2]\*b^2\*Sqrt[-(d\*x^2)]) - (x\*Cos[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)])/(2\*Sqrt[2]\*b^2\*Sqrt[-(d\*x^2)])

**Rule 4826**

Int[((a\_.) + ArcCos[1 + (d\_.)\*(x\_)^2]\*(b\_.))^(-2), x\_Symbol] :> Simp[Sqrt[-2\*d\*x^2 - d^2\*x^4]/(2\*b\*d\*x\*(a + b\*ArcCos[1 + d\*x^2])), x] + (Simp[(x\*Sin[a/(2\*b)]\*CosIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)])/(2\*Sqrt[2]\*b^2\*Sqrt[-(d)\*x^2]), x] - Simp[(x\*Cos[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)])/(2\*Sqrt[2]\*b^2\*Sqrt[-(d)\*x^2]), x]) /; FreeQ[{a, b, d}, x]

**Rubi steps**

$$\int \frac{1}{(a+b \cos^{-1}(1+dx^2))^2} dx = \frac{\sqrt{-2dx^2 - d^2x^4}}{2bdx(a+b \cos^{-1}(1+dx^2))} + \frac{x \text{Ci}\left(\frac{a+b \cos^{-1}(1+dx^2)}{2b}\right) \sin\left(\frac{a}{2b}\right)}{2\sqrt{2}b^2\sqrt{-dx^2}} - \frac{x \cos\left(\frac{a}{2b}\right) \text{Si}\left(\frac{a+b \cos^{-1}(1+dx^2)}{2b}\right)}{2\sqrt{2}b^2\sqrt{-dx^2}}$$

**Mathematica [A]** time = 0.332094, size = 133, normalized size = 0.88

$$\frac{\sqrt{-dx^2}(dx^2+2) \left( \frac{b}{a+b \cos^{-1}(dx^2+1)} - \frac{\cos\left(\frac{1}{2} \cos^{-1}(dx^2+1)\right) \left( \sin\left(\frac{a}{2b}\right) \text{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2+1)}{2b}\right) - \cos\left(\frac{a}{2b}\right) \text{Si}\left(\frac{a+b \cos^{-1}(dx^2+1)}{2b}\right) \right)}{dx^2+2} \right)}{2b^2 dx}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCos[1 + d\*x^2])^(-2),x]

[Out] (Sqrt[-(d\*x^2\*(2 + d\*x^2))]\*(b/(a + b\*ArcCos[1 + d\*x^2]) - (Cos[ArcCos[1 + d\*x^2]/2]\*(CosIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)]\*Sin[a/(2\*b)] - Cos[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)])))/(2 + d\*x^2)))/(2\*b^2\*d\*x)

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 + 1))^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arccos(d\*x^2+1))^2,x)

[Out] int(1/(a+b\*arccos(d\*x^2+1))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(b^2 d \arctan(\sqrt{-dx^2 - 2}\sqrt{dx}, dx^2 + 1) + abd)\sqrt{d} \int \frac{\sqrt{-dx^2 - 2x}}{abdx^2 + 2ab + (b^2 dx^2 + 2b^2) \arctan(\sqrt{-dx^2 - 2}\sqrt{dx}, dx^2 + 1)} dx - \sqrt{-dx^2 - 2}\sqrt{d}}{2(b^2 d \arctan(\sqrt{-dx^2 - 2}\sqrt{dx}, dx^2 + 1) + abd)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^2,x, algorithm="maxima")

[Out] -1/2\*(2\*(b^2\*d\*arctan2(sqrt(-d\*x^2 - 2)\*sqrt(d)\*x, d\*x^2 + 1) + a\*b\*d)\*sqrt(d)\*integrate(1/2\*sqrt(-d\*x^2 - 2)\*x/(a\*b\*d\*x^2 + 2\*a\*b + (b^2\*d\*x^2 + 2\*b^2)\*arctan2(sqrt(-d\*x^2 - 2)\*sqrt(d)\*x, d\*x^2 + 1)), x) - sqrt(-d\*x^2 - 2)\*sqrt(d))/(b^2\*d\*arctan2(sqrt(-d\*x^2 - 2)\*sqrt(d)\*x, d\*x^2 + 1) + a\*b\*d)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2 \arccos(dx^2 + 1)^2 + 2ab \arccos(dx^2 + 1) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*arccos(d\*x^2 + 1)^2 + 2\*a\*b\*arccos(d\*x^2 + 1) + a^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \arccos(dx^2 + 1))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*acos(d\*x\*\*2+1))\*\*2,x)

[Out] Integral((a + b\*acos(d\*x\*\*2 + 1))\*\*(-2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(dx^2 + 1) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^2,x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 + 1) + a)^(-2), x)

$$3.79 \quad \int \frac{1}{(a+b \cos^{-1}(1+dx^2))^3} dx$$

**Optimal.** Leaf size=173

$$\frac{x \cos\left(\frac{a}{2b}\right) \operatorname{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2+1)}{2b}\right)}{8\sqrt{2}b^3\sqrt{-dx^2}} - \frac{x \sin\left(\frac{a}{2b}\right) \operatorname{Si}\left(\frac{a+b \cos^{-1}(dx^2+1)}{2b}\right)}{8\sqrt{2}b^3\sqrt{-dx^2}} + \frac{x}{8b^2(a+b \cos^{-1}(dx^2+1))} + \frac{\sqrt{-d^2}}{4bdx(a+b \cos^{-1}(dx^2+1))}$$

[Out] Sqrt[-2\*d\*x^2 - d^2\*x^4]/(4\*b\*d\*x\*(a + b\*ArcCos[1 + d\*x^2])^2) + x/(8\*b^2\*(a + b\*ArcCos[1 + d\*x^2])) - (x\*Cos[a/(2\*b)]\*CosIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)])/(8\*Sqrt[2]\*b^3\*Sqrt[-(d\*x^2)]) - (x\*Sin[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)])/(8\*Sqrt[2]\*b^3\*Sqrt[-(d\*x^2)])

**Rubi [A]** time = 0.0362387, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {4829, 4817}

$$\frac{x \cos\left(\frac{a}{2b}\right) \operatorname{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2+1)}{2b}\right)}{8\sqrt{2}b^3\sqrt{-dx^2}} - \frac{x \sin\left(\frac{a}{2b}\right) \operatorname{Si}\left(\frac{a+b \cos^{-1}(dx^2+1)}{2b}\right)}{8\sqrt{2}b^3\sqrt{-dx^2}} + \frac{x}{8b^2(a+b \cos^{-1}(dx^2+1))} + \frac{\sqrt{-d^2}}{4bdx(a+b \cos^{-1}(dx^2+1))}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[1 + d\*x^2])^(-3), x]

[Out] Sqrt[-2\*d\*x^2 - d^2\*x^4]/(4\*b\*d\*x\*(a + b\*ArcCos[1 + d\*x^2])^2) + x/(8\*b^2\*(a + b\*ArcCos[1 + d\*x^2])) - (x\*Cos[a/(2\*b)]\*CosIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)])/(8\*Sqrt[2]\*b^3\*Sqrt[-(d\*x^2)]) - (x\*Sin[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)])/(8\*Sqrt[2]\*b^3\*Sqrt[-(d\*x^2)])

#### Rule 4829

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_)^2]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(x\*(a + b\*ArcCos[c + d\*x^2])^(n + 2))/(4\*b^2\*(n + 1)\*(n + 2)), x] + (-Dist[1/(4\*b^2\*(n + 1)\*(n + 2)), Int[(a + b\*ArcCos[c + d\*x^2])^(n + 2), x], x] - Simp[(Sqrt[-2\*c\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[c + d\*x^2])^(n + 1))/(2\*b\*d\*(n + 1)\*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]

#### Rule 4817

Int[((a\_.) + ArcCos[1 + (d\_.)\*(x\_)^2]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(x\*Cos[a/(2\*b)]\*CosIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)])/(Sqrt[2]\*b\*Sqrt[-(d\*x^2)]), x] + Simp[(x\*Sin[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)])/(Sqrt[2]\*b\*Sqrt[-(d\*x^2)]), x] /; FreeQ[{a, b, d}, x]

#### Rubi steps

$$\int \frac{1}{(a + b \cos^{-1}(1 + dx^2))^3} dx = \frac{\sqrt{-2dx^2 - d^2x^4}}{4bdx(a + b \cos^{-1}(1 + dx^2))^2} + \frac{x}{8b^2(a + b \cos^{-1}(1 + dx^2))} - \frac{\int \frac{1}{a + b \cos^{-1}(1 + dx^2)} dx}{8b^2}$$

$$= \frac{\sqrt{-2dx^2 - d^2x^4}}{4bdx(a + b \cos^{-1}(1 + dx^2))^2} + \frac{x}{8b^2(a + b \cos^{-1}(1 + dx^2))} - \frac{x \cos\left(\frac{a}{2b}\right) \text{Ci}\left(\frac{a + b \cos^{-1}(1 + dx^2)}{2}\right)}{8\sqrt{2}b^3\sqrt{-dx^2}}$$

**Mathematica [A]** time = 0.246495, size = 147, normalized size = 0.85

$$\frac{2b^2\sqrt{-dx^2(dx^2+2)}}{d(a+b\cos^{-1}(dx^2+1))^2} + \frac{\sin\left(\frac{1}{2}\cos^{-1}(dx^2+1)\right)\left(\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}(dx^2+1)}{2b}\right) + \sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(dx^2+1)}{2b}\right)\right)}{d} + \frac{bx^2}{a+b\cos^{-1}(dx^2+1)}$$


---


$$8b^3x$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCos[1 + d\*x^2])^(-3), x]

[Out] ((2\*b^2\*Sqrt[-(d\*x^2\*(2 + d\*x^2))])/(d\*(a + b\*ArcCos[1 + d\*x^2])^2) + (b\*x^2)/(a + b\*ArcCos[1 + d\*x^2]) + (Sin[ArcCos[1 + d\*x^2]/2]\*(Cos[a/(2\*b)]\*CosIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)] + Sin[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[1 + d\*x^2])/(2\*b)]))/d)/(8\*b^3\*x)

**Maple [F]** time = 0.067, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 + 1))^{-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arccos(d\*x^2+1))^3, x)

[Out] int(1/(a+b\*arccos(d\*x^2+1))^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bdx \arctan\left(\sqrt{-dx^2 - 2}\sqrt{dx}, dx^2 + 1\right) + adx + 2\sqrt{-dx^2 - 2}b\sqrt{d} - \left(b^4d \arctan\left(\sqrt{-dx^2 - 2}\sqrt{dx}, dx^2 + 1\right)^2 + 2ab^3d \arctan\left(\sqrt{-dx^2 - 2}\sqrt{dx}, dx^2 + 1\right)\right)}{8\left(b^4d \arctan\left(\sqrt{-dx^2 - 2}\sqrt{dx}, dx^2 + 1\right)^2 + 2ab^3d \arctan\left(\sqrt{-dx^2 - 2}\sqrt{dx}, dx^2 + 1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^3, x, algorithm="maxima")

[Out] 1/8\*(b\*d\*x\*arctan2(sqrt(-d\*x^2 - 2)\*sqrt(d)\*x, d\*x^2 + 1) + a\*d\*x + 2\*sqrt(-d\*x^2 - 2)\*b\*sqrt(d) - 8\*(b^4\*d\*arctan2(sqrt(-d\*x^2 - 2)\*sqrt(d)\*x, d\*x^2 + 1)^2 + 2\*a\*b^3\*d\*arctan2(sqrt(-d\*x^2 - 2)\*sqrt(d)\*x, d\*x^2 + 1) + a^2\*b^2\*d)\*integrate(1/8/(b^3\*arctan2(sqrt(-d\*x^2 - 2)\*sqrt(d)\*x, d\*x^2 + 1) + a\*b^2), x))/(b^4\*d\*arctan2(sqrt(-d\*x^2 - 2)\*sqrt(d)\*x, d\*x^2 + 1)^2 + 2\*a\*b^3\*

$d \cdot \arctan 2(\sqrt{-d \cdot x^2 - 2} \cdot \sqrt{d} \cdot x, d \cdot x^2 + 1) + a^2 \cdot b^2 \cdot d$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{b^3 \arccos(dx^2 + 1)^3 + 3ab^2 \arccos(dx^2 + 1)^2 + 3a^2b \arccos(dx^2 + 1) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^3,x, algorithm="fricas")

[Out] integral(1/(b^3\*arccos(d\*x^2 + 1)^3 + 3\*a\*b^2\*arccos(d\*x^2 + 1)^2 + 3\*a^2\*b\*arccos(d\*x^2 + 1) + a^3), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \arccos(dx^2 + 1))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*acos(d\*x\*\*2+1))\*\*3,x)

[Out] Integral((a + b\*acos(d\*x\*\*2 + 1))\*\*(-3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(dx^2 + 1) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^3,x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 + 1) + a)^(-3), x)

### 3.80 $\int \left(a + b \cos^{-1}(-1 + dx^2)\right)^4 dx$

**Optimal.** Leaf size=127

$$\frac{192b^3\sqrt{2dx^2 - d^2x^4}(a + b \cos^{-1}(dx^2 - 1))}{dx} - 48b^2x(a + b \cos^{-1}(dx^2 - 1))^2 - \frac{8b\sqrt{2dx^2 - d^2x^4}(a + b \cos^{-1}(dx^2 - 1))}{dx}$$

[Out] 384\*b^4\*x + (192\*b^3\*Sqrt[2\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[-1 + d\*x^2]))/(d\*x) - 48\*b^2\*x\*(a + b\*ArcCos[-1 + d\*x^2])^2 - (8\*b\*Sqrt[2\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[-1 + d\*x^2])^3)/(d\*x) + x\*(a + b\*ArcCos[-1 + d\*x^2])^4

**Rubi [A]** time = 0.0283064, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {4815, 8}

$$\frac{192b^3\sqrt{2dx^2 - d^2x^4}(a + b \cos^{-1}(dx^2 - 1))}{dx} - 48b^2x(a + b \cos^{-1}(dx^2 - 1))^2 - \frac{8b\sqrt{2dx^2 - d^2x^4}(a + b \cos^{-1}(dx^2 - 1))}{dx}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[-1 + d\*x^2])^4, x]

[Out] 384\*b^4\*x + (192\*b^3\*Sqrt[2\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[-1 + d\*x^2]))/(d\*x) - 48\*b^2\*x\*(a + b\*ArcCos[-1 + d\*x^2])^2 - (8\*b\*Sqrt[2\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[-1 + d\*x^2])^3)/(d\*x) + x\*(a + b\*ArcCos[-1 + d\*x^2])^4

#### Rule 4815

Int[((a\_) + ArcCos[(c\_) + (d\_.)\*(x\_)^2]\*(b\_.))^n, x\_Symbol] := Simp[x\*(a + b\*ArcCos[c + d\*x^2])^n, x] + (-Dist[4\*b^2\*n\*(n - 1), Int[(a + b\*ArcCos[c + d\*x^2])^(n - 2), x], x] - Simp[(2\*b\*n\*Sqrt[-2\*c\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[c + d\*x^2])^(n - 1))/(d\*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned} \int \left(a + b \cos^{-1}(-1 + dx^2)\right)^4 dx &= -\frac{8b\sqrt{2dx^2 - d^2x^4}(a + b \cos^{-1}(-1 + dx^2))^3}{dx} + x(a + b \cos^{-1}(-1 + dx^2))^4 - (48b^2) \\ &= \frac{192b^3\sqrt{2dx^2 - d^2x^4}(a + b \cos^{-1}(-1 + dx^2))}{dx} - 48b^2x(a + b \cos^{-1}(-1 + dx^2))^2 - 8 \\ &= 384b^4x + \frac{192b^3\sqrt{2dx^2 - d^2x^4}(a + b \cos^{-1}(-1 + dx^2))}{dx} - 48b^2x(a + b \cos^{-1}(-1 + dx^2))^2 - 8 \end{aligned}$$

**Mathematica [A]** time = 0.239112, size = 249, normalized size = 1.96

$$dx^2(-48a^2b^2 + a^4 + 384b^4) - 8ab(a^2 - 24b^2)\sqrt{dx^2(2 - dx^2)} + 6b^2 \cos^{-1}(dx^2 - 1)^2(a^2 dx^2 - 4ab\sqrt{-dx^2(dx^2 - 2)} -$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCos[-1 + d\*x^2])^4,x]

[Out] ((a^4 - 48\*a^2\*b^2 + 384\*b^4)\*d\*x^2 - 8\*a\*b\*(a^2 - 24\*b^2)\*Sqrt[d\*x^2\*(2 - d\*x^2)] + 4\*b\*(a^3\*d\*x^2 - 24\*a\*b^2\*d\*x^2 - 6\*a^2\*b\*Sqrt[-(d\*x^2\*(-2 + d\*x^2))]) + 48\*b^3\*Sqrt[-(d\*x^2\*(-2 + d\*x^2))])\*ArcCos[-1 + d\*x^2] + 6\*b^2\*(a^2\*d\*x^2 - 8\*b^2\*d\*x^2 - 4\*a\*b\*Sqrt[-(d\*x^2\*(-2 + d\*x^2))])\*ArcCos[-1 + d\*x^2]^2 + 4\*b^3\*(a\*d\*x^2 - 2\*b\*Sqrt[-(d\*x^2\*(-2 + d\*x^2))])\*ArcCos[-1 + d\*x^2]^3 + b^4\*d\*x^2\*ArcCos[-1 + d\*x^2]^4)/(d\*x)

**Maple [F]** time = 0.117, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 - 1))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos(d\*x^2-1))^4,x)

[Out] int((a+b\*arccos(d\*x^2-1))^4,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2-1))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.65725, size = 474, normalized size = 3.73

$$b^4 dx^2 \arccos(dx^2 - 1)^4 + 4ab^3 dx^2 \arccos(dx^2 - 1)^3 + 6(a^2 b^2 - 8b^4) dx^2 \arccos(dx^2 - 1)^2 + 4(a^3 b - 24ab^3) dx^2 \arccos(dx^2 - 1) + (a^4 - 48a^2 b^2 + 384b^4) dx^2 - 8(b^4 \arccos(dx^2 - 1))^3 + 3a^3 b - 24a^2 b^3 + 3(a^2 b^2 - 8b^4) a \arccos(dx^2 - 1) \sqrt{-d^2 x^4 + 2d x^2} / (d x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2-1))^4,x, algorithm="fricas")

[Out] (b^4\*d\*x^2\*arccos(d\*x^2 - 1)^4 + 4\*a\*b^3\*d\*x^2\*arccos(d\*x^2 - 1)^3 + 6\*(a^2\*b^2 - 8\*b^4)\*d\*x^2\*arccos(d\*x^2 - 1)^2 + 4\*(a^3\*b - 24\*a\*b^3)\*d\*x^2\*arccos(d\*x^2 - 1) + (a^4 - 48\*a^2\*b^2 + 384\*b^4)\*d\*x^2 - 8\*(b^4\*arccos(d\*x^2 - 1))^3 + 3\*a^3\*b - 24\*a^2\*b^3 + 3\*(a^2\*b^2 - 8\*b^4)\*a\*arccos(d\*x^2 - 1))\*sqrt(-d^2\*x^4 + 2\*d\*x^2)/(d\*x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 - 1))^4 dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acos(d\*x\*\*2-1))\*\*4,x)

[Out] Integral((a + b\*acos(d\*x\*\*2 - 1))\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arccos(dx^2 - 1) + a)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2-1))^4,x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 - 1) + a)^4, x)

### 3.81 $\int (a + b \cos^{-1}(-1 + dx^2))^3 dx$

**Optimal.** Leaf size=110

$$-24ab^2x - \frac{6b\sqrt{2dx^2 - d^2x^4}(a + b \cos^{-1}(dx^2 - 1))^2}{dx} + x(a + b \cos^{-1}(dx^2 - 1))^3 + \frac{48b^3\sqrt{2dx^2 - d^2x^4}}{dx} - 24b^3x \cos^{-1}(dx^2 - 1)$$

[Out] -24\*a\*b^2\*x + (48\*b^3\*Sqrt[2\*d\*x^2 - d^2\*x^4])/(d\*x) - 24\*b^3\*x\*ArcCos[-1 + d\*x^2] - (6\*b\*Sqrt[2\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[-1 + d\*x^2])^2)/(d\*x) + x\*(a + b\*ArcCos[-1 + d\*x^2])^3

**Rubi [A]** time = 0.055042, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {4815, 4841, 12, 1588}

$$-24ab^2x - \frac{6b\sqrt{2dx^2 - d^2x^4}(a + b \cos^{-1}(dx^2 - 1))^2}{dx} + x(a + b \cos^{-1}(dx^2 - 1))^3 + \frac{48b^3\sqrt{2dx^2 - d^2x^4}}{dx} - 24b^3x \cos^{-1}(dx^2 - 1)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[-1 + d\*x^2])^3,x]

[Out] -24\*a\*b^2\*x + (48\*b^3\*Sqrt[2\*d\*x^2 - d^2\*x^4])/(d\*x) - 24\*b^3\*x\*ArcCos[-1 + d\*x^2] - (6\*b\*Sqrt[2\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[-1 + d\*x^2])^2)/(d\*x) + x\*(a + b\*ArcCos[-1 + d\*x^2])^3

#### Rule 4815

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^n, x_Symbol] :> Simp[x*(a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcCos[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

#### Rule 4841

```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[(x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] && !FunctionOfExponentialQ[u, x]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

#### Rubi steps

$$\begin{aligned}
\int (a + b \cos^{-1}(-1 + dx^2))^3 dx &= -\frac{6b\sqrt{2dx^2 - d^2x^4} (a + b \cos^{-1}(-1 + dx^2))^2}{dx} + x(a + b \cos^{-1}(-1 + dx^2))^3 - (24b^2) \\
&= -24ab^2x - \frac{6b\sqrt{2dx^2 - d^2x^4} (a + b \cos^{-1}(-1 + dx^2))^2}{dx} + x(a + b \cos^{-1}(-1 + dx^2))^3 \\
&= -24ab^2x - 24b^3x \cos^{-1}(-1 + dx^2) - \frac{6b\sqrt{2dx^2 - d^2x^4} (a + b \cos^{-1}(-1 + dx^2))^2}{dx} + x(a + b \cos^{-1}(-1 + dx^2))^3 \\
&= -24ab^2x - 24b^3x \cos^{-1}(-1 + dx^2) - \frac{6b\sqrt{2dx^2 - d^2x^4} (a + b \cos^{-1}(-1 + dx^2))^2}{dx} + x(a + b \cos^{-1}(-1 + dx^2))^3 \\
&= -24ab^2x + \frac{48b^3\sqrt{2dx^2 - d^2x^4}}{dx} - 24b^3x \cos^{-1}(-1 + dx^2) - \frac{6b\sqrt{2dx^2 - d^2x^4} (a + b \cos^{-1}(-1 + dx^2))^2}{dx} + x(a + b \cos^{-1}(-1 + dx^2))^3
\end{aligned}$$

**Mathematica [A]** time = 0.135227, size = 162, normalized size = 1.47

$$\frac{adx^2(a^2 - 24b^2) - 6b(a^2 - 8b^2)\sqrt{dx^2(2 - dx^2)} + 3b \cos^{-1}(dx^2 - 1)(a^2dx^2 - 4ab\sqrt{-dx^2(dx^2 - 2)} - 8b^2dx^2) + 3b^2dx^2}{dx}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCos[-1 + d\*x^2])^3,x]

[Out] (a\*(a^2 - 24\*b^2)\*d\*x^2 - 6\*b\*(a^2 - 8\*b^2)\*Sqrt[d\*x^2\*(2 - d\*x^2)] + 3\*b\*(a^2\*d\*x^2 - 8\*b^2\*d\*x^2 - 4\*a\*b\*Sqrt[-(d\*x^2\*(-2 + d\*x^2))])\*ArcCos[-1 + d\*x^2] + 3\*b^2\*(a\*d\*x^2 - 2\*b\*Sqrt[-(d\*x^2\*(-2 + d\*x^2))])\*ArcCos[-1 + d\*x^2]^2 + b^3\*d\*x^2\*ArcCos[-1 + d\*x^2]^3)/(d\*x)

**Maple [F]** time = 0.119, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 - 1))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos(d\*x^2-1))^3,x)

[Out] int((a+b\*arccos(d\*x^2-1))^3,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2-1))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.5729, size = 328, normalized size = 2.98

$$\frac{b^3 dx^2 \arccos(dx^2 - 1)^3 + 3ab^2 dx^2 \arccos(dx^2 - 1)^2 + 3(a^2b - 8b^3) dx^2 \arccos(dx^2 - 1) + (a^3 - 24ab^2) dx^2 - 6\sqrt{-d^2x^2}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2-1))^3,x, algorithm="fricas")

[Out] (b^3\*d\*x^2\*arccos(d\*x^2 - 1)^3 + 3\*a\*b^2\*d\*x^2\*arccos(d\*x^2 - 1)^2 + 3\*(a^2\*b - 8\*b^3)\*d\*x^2\*arccos(d\*x^2 - 1) + (a^3 - 24\*a\*b^2)\*d\*x^2 - 6\*sqrt(-d^2\*x^4 + 2\*d\*x^2)\*(b^3\*arccos(d\*x^2 - 1)^2 + 2\*a\*b^2\*arccos(d\*x^2 - 1) + a^2\*b - 8\*b^3))/(d\*x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 - 1))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acos(d\*x\*\*2-1))\*\*3,x)

[Out] Integral((a + b\*acos(d\*x\*\*2 - 1))\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arccos(dx^2 - 1) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2-1))^3,x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 - 1) + a)^3, x)

$$3.82 \quad \int \left( a + b \cos^{-1}(-1 + dx^2) \right)^2 dx$$

**Optimal.** Leaf size=63

$$-\frac{4b\sqrt{2dx^2 - d^2x^4}(a + b \cos^{-1}(dx^2 - 1))}{dx} + x(a + b \cos^{-1}(dx^2 - 1))^2 - 8b^2x$$

[Out]  $-8*b^2*x - (4*b*sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2]))/(d*x) + x*(a + b*ArcCos[-1 + d*x^2])^2$

**Rubi [A]** time = 0.0115222, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {4815, 8}

$$-\frac{4b\sqrt{2dx^2 - d^2x^4}(a + b \cos^{-1}(dx^2 - 1))}{dx} + x(a + b \cos^{-1}(dx^2 - 1))^2 - 8b^2x$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[-1 + d\*x^2])^2, x]

[Out]  $-8*b^2*x - (4*b*sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2]))/(d*x) + x*(a + b*ArcCos[-1 + d*x^2])^2$

#### Rule 4815

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_)^2]\*(b\_.))^n, x\_Symbol] := Simp[x\*(a + b\*ArcCos[c + d\*x^2])^n, x] + (-Dist[4\*b^2\*n\*(n - 1), Int[(a + b\*ArcCos[c + d\*x^2])^(n - 2), x], x] - Simp[(2\*b\*n\*sqrt[-2\*c\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[c + d\*x^2])^(n - 1))/(d\*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned} \int \left( a + b \cos^{-1}(-1 + dx^2) \right)^2 dx &= -\frac{4b\sqrt{2dx^2 - d^2x^4}(a + b \cos^{-1}(-1 + dx^2))}{dx} + x(a + b \cos^{-1}(-1 + dx^2))^2 - (8b^2) \int \\ &= -8b^2x - \frac{4b\sqrt{2dx^2 - d^2x^4}(a + b \cos^{-1}(-1 + dx^2))}{dx} + x(a + b \cos^{-1}(-1 + dx^2))^2 \end{aligned}$$

**Mathematica [A]** time = 0.0658086, size = 98, normalized size = 1.56

$$x(a^2 - 8b^2) - \frac{4ab\sqrt{-dx^2(dx^2 - 2)}}{dx} + \frac{2b \cos^{-1}(dx^2 - 1)(adx^2 - 2b\sqrt{-dx^2(dx^2 - 2)})}{dx} + b^2x \cos^{-1}(dx^2 - 1)^2$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCos[-1 + d\*x^2])^2, x]

[Out]  $(a^2 - 8b^2)x - (4ab\sqrt{-(dx^2(-2 + dx^2))})/(dx) + (2b(a dx^2 - 2b\sqrt{-(dx^2(-2 + dx^2))})\text{ArcCos}[-1 + dx^2])/(dx) + b^2x\text{ArcCos}[-1 + dx^2]^2$

**Maple [F]** time = 0.118, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 - 1))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccos(d*x^2-1))^2,x)`

[Out] `int((a+b*arccos(d*x^2-1))^2,x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccos(d*x^2-1))^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.50507, size = 205, normalized size = 3.25

$$\frac{b^2 dx^2 \arccos(dx^2 - 1)^2 + 2 ab dx^2 \arccos(dx^2 - 1) + (a^2 - 8b^2) dx^2 - 4 \sqrt{-d^2 x^4 + 2 dx^2} (b^2 \arccos(dx^2 - 1) + ab)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccos(d*x^2-1))^2,x, algorithm="fricas")`

[Out]  $(b^2 dx^2 \arccos(dx^2 - 1)^2 + 2ab dx^2 \arccos(dx^2 - 1) + (a^2 - 8b^2) dx^2 - 4\sqrt{-d^2 x^4 + 2 dx^2} (b^2 \arccos(dx^2 - 1) + ab))/(dx)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 - 1))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccos(d*x**2-1))**2,x)`

[Out] `Integral((a + b*arccos(d*x**2 - 1))**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arccos(dx^2 - 1) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^2, x)
```

### 3.83 $\int (a + b \cos^{-1}(-1 + dx^2)) dx$

**Optimal.** Leaf size=43

$$ax - \frac{2b\sqrt{2dx^2 - d^2x^4}}{dx} + bx \cos^{-1}(dx^2 - 1)$$

[Out] a\*x - (2\*b\*Sqrt[2\*d\*x^2 - d^2\*x^4])/(d\*x) + b\*x\*ArcCos[-1 + d\*x^2]

**Rubi [A]** time = 0.0357681, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {4841, 12, 1588}

$$ax - \frac{2b\sqrt{2dx^2 - d^2x^4}}{dx} + bx \cos^{-1}(dx^2 - 1)$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcCos[-1 + d\*x^2], x]

[Out] a\*x - (2\*b\*Sqrt[2\*d\*x^2 - d^2\*x^4])/(d\*x) + b\*x\*ArcCos[-1 + d\*x^2]

#### Rule 4841

Int[ArcCos[u\_], x\_Symbol] := Simp[x\*ArcCos[u], x] + Int[SimplifyIntegrand[(x\*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] && !FunctionOfExponentialQ[u, x]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 1588

Int[(Pp\_)\*(Qq\_)^(m\_.), x\_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]\*x^(p - q + 1)\*Qq^(m + 1))/((p + m\*q + 1)\*Coeff[Qq, x, q]), x] /; NeQ[p + m\*q + 1, 0] && EqQ[(p + m\*q + 1)\*Coeff[Qq, x, q]\*Pp, Coeff[Pp, x, p]\*x^(p - q)\*((p - q + 1)\*Qq + (m + 1)\*x\*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int (a + b \cos^{-1}(-1 + dx^2)) dx &= ax + b \int \cos^{-1}(-1 + dx^2) dx \\ &= ax + bx \cos^{-1}(-1 + dx^2) + b \int \frac{2dx^2}{\sqrt{2dx^2 - d^2x^4}} dx \\ &= ax + bx \cos^{-1}(-1 + dx^2) + (2bd) \int \frac{x^2}{\sqrt{2dx^2 - d^2x^4}} dx \\ &= ax - \frac{2b\sqrt{2dx^2 - d^2x^4}}{dx} + bx \cos^{-1}(-1 + dx^2) \end{aligned}$$



**Mathematica [A]** time = 0.0286509, size = 41, normalized size = 0.95

$$ax - \frac{2b\sqrt{-dx^2(dx^2-2)}}{dx} + bx \cos^{-1}(dx^2-1)$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcCos[-1 + d\*x^2], x]

[Out] a\*x - (2\*b\*Sqrt[-(d\*x^2\*(-2 + d\*x^2))])/(d\*x) + b\*x\*ArcCos[-1 + d\*x^2]

**Maple [A]** time = 0.004, size = 45, normalized size = 1.1

$$ax + b \left( x \arccos(dx^2 - 1) + 2 \frac{x(dx^2 - 2)}{\sqrt{-d^2x^4 + 2dx^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b\*arccos(d\*x^2-1), x)

[Out] a\*x+b\*(x\*arccos(d\*x^2-1)+2/(-d^2\*x^4+2\*d\*x^2)^(1/2)\*x\*(d\*x^2-2))

**Maxima [A]** time = 1.52665, size = 61, normalized size = 1.42

$$\left( x \arccos(dx^2 - 1) + \frac{2(d^{\frac{3}{2}}x^2 - 2\sqrt{d})}{\sqrt{-dx^2 + 2d}} \right) b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccos(d\*x^2-1), x, algorithm="maxima")

[Out] (x\*arccos(d\*x^2 - 1) + 2\*(d^(3/2)\*x^2 - 2\*sqrt(d))/(sqrt(-d\*x^2 + 2)\*d))\*b + a\*x

**Fricas [A]** time = 2.68217, size = 103, normalized size = 2.4

$$\frac{bdx^2 \arccos(dx^2 - 1) + adx^2 - 2\sqrt{-d^2x^4 + 2dx^2}b}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccos(d\*x^2-1), x, algorithm="fricas")

[Out] (b\*d\*x^2\*arccos(d\*x^2 - 1) + a\*d\*x^2 - 2\*sqrt(-d^2\*x^4 + 2\*d\*x^2)\*b)/(d\*x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 - 1)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*acos(d\*x\*\*2-1),x)

[Out] Integral(a + b\*acos(d\*x\*\*2 - 1), x)

**Giac [A]** time = 1.3, size = 72, normalized size = 1.67

$$\left(2d\left(\frac{\sqrt{2}\operatorname{sgn}(x)}{d^{\frac{3}{2}}}-\frac{\sqrt{-d^2x^2+2d}}{d^2\operatorname{sgn}(x)}\right)+x\arccos(dx^2-1)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccos(d\*x^2-1),x, algorithm="giac")

[Out] (2\*d\*(sqrt(2)\*sgn(x)/d^(3/2) - sqrt(-d^2\*x^2 + 2\*d)/(d^2\*sgn(x))) + x\*arccos(d\*x^2 - 1))\*b + a\*x

$$3.84 \quad \int \frac{1}{a+b \cos^{-1}(-1+dx^2)} dx$$

**Optimal.** Leaf size=98

$$\frac{x \sin\left(\frac{a}{2b}\right) \operatorname{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right)}{\sqrt{2b}\sqrt{dx^2}} - \frac{x \cos\left(\frac{a}{2b}\right) \operatorname{Si}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right)}{\sqrt{2b}\sqrt{dx^2}}$$

[Out] (x\*CosIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)]\*Sin[a/(2\*b)])/(Sqrt[2]\*b\*Sqrt[d\*x^2]) - (x\*Cos[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)])/(Sqrt[2]\*b\*Sqrt[d\*x^2])

**Rubi [A]** time = 0.0133217, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {4818}

$$\frac{x \sin\left(\frac{a}{2b}\right) \operatorname{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right)}{\sqrt{2b}\sqrt{dx^2}} - \frac{x \cos\left(\frac{a}{2b}\right) \operatorname{Si}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right)}{\sqrt{2b}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[-1 + d\*x^2])^(-1), x]

[Out] (x\*CosIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)]\*Sin[a/(2\*b)])/(Sqrt[2]\*b\*Sqrt[d\*x^2]) - (x\*Cos[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)])/(Sqrt[2]\*b\*Sqrt[d\*x^2])

#### Rule 4818

Int[((a\_.) + ArcCos[-1 + (d\_.)\*(x\_)^2]\*(b\_.))^-1, x\_Symbol] := Simp[(x\*Sin[a/(2\*b)]\*CosIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)])/(Sqrt[2]\*b\*Sqrt[d\*x^2]), x] - Simp[(x\*Cos[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)])/(Sqrt[2]\*b\*Sqrt[d\*x^2]), x] /; FreeQ[{a, b, d}, x]

#### Rubi steps

$$\int \frac{1}{a+b \cos^{-1}(-1+dx^2)} dx = \frac{x \operatorname{Ci}\left(\frac{a+b \cos^{-1}(-1+dx^2)}{2b}\right) \sin\left(\frac{a}{2b}\right)}{\sqrt{2b}\sqrt{dx^2}} - \frac{x \cos\left(\frac{a}{2b}\right) \operatorname{Si}\left(\frac{a+b \cos^{-1}(-1+dx^2)}{2b}\right)}{\sqrt{2b}\sqrt{dx^2}}$$

**Mathematica [A]** time = 0.0998995, size = 85, normalized size = 0.87

$$\frac{\cos\left(\frac{1}{2} \cos^{-1}(dx^2-1)\right) \left( \sin\left(\frac{a}{2b}\right) \operatorname{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right) - \cos\left(\frac{a}{2b}\right) \operatorname{Si}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right) \right)}{bdx}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCos[-1 + d\*x^2])^(-1), x]

[Out]  $(\text{Cos}[\text{ArcCos}[-1 + d*x^2]/2] * (\text{CosIntegral}[(a + b*\text{ArcCos}[-1 + d*x^2])/(2*b)] * \text{Sin}[a/(2*b)] - \text{Cos}[a/(2*b)] * \text{SinIntegral}[(a + b*\text{ArcCos}[-1 + d*x^2])/(2*b)])) / (b*d*x)$

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 - 1))^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*arccos(d*x^2-1)),x)`

[Out] `int(1/(a+b*arccos(d*x^2-1)),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \arccos(dx^2 - 1) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*arccos(d*x^2-1)),x, algorithm="maxima")`

[Out] `integrate(1/(b*arccos(d*x^2 - 1) + a), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b \arccos(dx^2 - 1) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*arccos(d*x^2-1)),x, algorithm="fricas")`

[Out] `integral(1/(b*arccos(d*x^2 - 1) + a), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b \arccos(dx^2 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*arccos(d*x**2-1)),x)`

[Out] `Integral(1/(a + b*arccos(d*x**2 - 1)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \arccos(dx^2 - 1) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arccos(d*x^2 - 1) + a), x)
```

$$3.85 \quad \int \frac{1}{(a+b \cos^{-1}(-1+dx^2))^2} dx$$

**Optimal.** Leaf size=149

$$\frac{x \cos\left(\frac{a}{2b}\right) \operatorname{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}} - \frac{x \sin\left(\frac{a}{2b}\right) \operatorname{Si}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}} + \frac{\sqrt{2dx^2-d^2x^4}}{2bdx(a+b \cos^{-1}(dx^2-1))}$$

[Out] Sqrt[2\*d\*x^2 - d^2\*x^4]/(2\*b\*d\*x\*(a + b\*ArcCos[-1 + d\*x^2])) - (x\*Cos[a/(2\*b)]\*CosIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)])/(2\*Sqrt[2]\*b^2\*Sqrt[d\*x^2]) - (x\*Sin[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)])/(2\*Sqrt[2]\*b^2\*Sqrt[d\*x^2])

**Rubi [A]** time = 0.0176495, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {4827}

$$\frac{x \cos\left(\frac{a}{2b}\right) \operatorname{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}} - \frac{x \sin\left(\frac{a}{2b}\right) \operatorname{Si}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}} + \frac{\sqrt{2dx^2-d^2x^4}}{2bdx(a+b \cos^{-1}(dx^2-1))}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[-1 + d\*x^2])^(-2), x]

[Out] Sqrt[2\*d\*x^2 - d^2\*x^4]/(2\*b\*d\*x\*(a + b\*ArcCos[-1 + d\*x^2])) - (x\*Cos[a/(2\*b)]\*CosIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)])/(2\*Sqrt[2]\*b^2\*Sqrt[d\*x^2]) - (x\*Sin[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)])/(2\*Sqrt[2]\*b^2\*Sqrt[d\*x^2])

**Rule 4827**

Int[((a\_.) + ArcCos[-1 + (d\_.)\*(x\_)^2]\*(b\_.))^(-2), x\_Symbol] :> Simp[Sqrt[2\*d\*x^2 - d^2\*x^4]/(2\*b\*d\*x\*(a + b\*ArcCos[-1 + d\*x^2])), x] + (-Simp[(x\*Cos[a/(2\*b)]\*CosIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)])/(2\*Sqrt[2]\*b^2\*Sqrt[d\*x^2]), x] - Simp[(x\*Sin[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)])/(2\*Sqrt[2]\*b^2\*Sqrt[d\*x^2]), x]) /; FreeQ[{a, b, d}, x]

**Rubi steps**

$$\int \frac{1}{(a+b \cos^{-1}(-1+dx^2))^2} dx = \frac{\sqrt{2dx^2-d^2x^4}}{2bdx(a+b \cos^{-1}(-1+dx^2))} - \frac{x \cos\left(\frac{a}{2b}\right) \operatorname{Ci}\left(\frac{a+b \cos^{-1}(-1+dx^2)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}} - \frac{x \sin\left(\frac{a}{2b}\right) \operatorname{Si}\left(\frac{a+b \cos^{-1}(-1+dx^2)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}$$

**Mathematica [A]** time = 0.335094, size = 131, normalized size = 0.88

$$\frac{\sqrt{-dx^2(dx^2-2)} \left( \frac{\sin\left(\frac{1}{2} \cos^{-1}(dx^2-1)\right) \left( \cos\left(\frac{a}{2b}\right) \operatorname{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right) + \sin\left(\frac{a}{2b}\right) \operatorname{Si}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right) \right)}{dx^2-2} + \frac{b}{a+b \cos^{-1}(dx^2-1)} \right)}{2b^2 dx}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCos[-1 + d\*x^2])^(-2), x]

[Out] (Sqrt[-(d\*x^2\*(-2 + d\*x^2))]\*(b/(a + b\*ArcCos[-1 + d\*x^2]) + (Sin[ArcCos[-1 + d\*x^2]/2]\*(Cos[a/(2\*b)]\*CosIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)] + Sin[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)])))/(-2 + d\*x^2)))/(2\*b^2\*d\*x)

**Maple [F]** time = 0.065, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 - 1))^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arccos(d\*x^2-1))^2,x)

[Out] int(1/(a+b\*arccos(d\*x^2-1))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(b^2 d \arctan(\sqrt{-dx^2 + 2}\sqrt{dx}, dx^2 - 1) + abd)\sqrt{d} \int \frac{\sqrt{-dx^2 + 2}x}{abd^2 - 2ab + (b^2 dx^2 - 2b^2) \arctan(\sqrt{-dx^2 + 2}\sqrt{dx}, dx^2 - 1)} dx - \sqrt{-dx^2 + 2}\sqrt{d}}{2(b^2 d \arctan(\sqrt{-dx^2 + 2}\sqrt{dx}, dx^2 - 1) + abd)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^2,x, algorithm="maxima")

[Out] -1/2\*(2\*(b^2\*d\*arctan2(sqrt(-d\*x^2 + 2)\*sqrt(d)\*x, d\*x^2 - 1) + a\*b\*d)\*sqrt(d)\*integrate(1/2\*sqrt(-d\*x^2 + 2)\*x/(a\*b\*d\*x^2 - 2\*a\*b + (b^2\*d\*x^2 - 2\*b^2)\*arctan2(sqrt(-d\*x^2 + 2)\*sqrt(d)\*x, d\*x^2 - 1)), x) - sqrt(-d\*x^2 + 2)\*sqrt(d))/(b^2\*d\*arctan2(sqrt(-d\*x^2 + 2)\*sqrt(d)\*x, d\*x^2 - 1) + a\*b\*d)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2 \arccos(dx^2 - 1)^2 + 2ab \arccos(dx^2 - 1) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*arccos(d\*x^2 - 1)^2 + 2\*a\*b\*arccos(d\*x^2 - 1) + a^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \arccos(dx^2 - 1))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*acos(d\*x\*\*2-1))\*\*2,x)

[Out] Integral((a + b\*acos(d\*x\*\*2 - 1))\*\*(-2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(dx^2 - 1) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^2,x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 - 1) + a)^(-2), x)



$$3.86 \quad \int \frac{1}{(a+b \cos^{-1}(-1+dx^2))^3} dx$$

**Optimal.** Leaf size=171

$$\frac{x \sin\left(\frac{a}{2b}\right) \operatorname{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}} + \frac{x \cos\left(\frac{a}{2b}\right) \operatorname{Si}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}} + \frac{x}{8b^2(a+b \cos^{-1}(dx^2-1))} + \frac{\sqrt{dx^2-1}}{4bdx(a+b \cos^{-1}(dx^2-1))}$$

[Out] Sqrt[2\*d\*x^2 - d^2\*x^4]/(4\*b\*d\*x\*(a + b\*ArcCos[-1 + d\*x^2])^2) + x/(8\*b^2\*(a + b\*ArcCos[-1 + d\*x^2])) - (x\*CosIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)]\*Sin[a/(2\*b)])/(8\*Sqrt[2]\*b^3\*Sqrt[d\*x^2]) + (x\*Cos[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)])/(8\*Sqrt[2]\*b^3\*Sqrt[d\*x^2])

**Rubi [A]** time = 0.0328004, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {4829, 4818}

$$\frac{x \sin\left(\frac{a}{2b}\right) \operatorname{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}} + \frac{x \cos\left(\frac{a}{2b}\right) \operatorname{Si}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}} + \frac{x}{8b^2(a+b \cos^{-1}(dx^2-1))} + \frac{\sqrt{dx^2-1}}{4bdx(a+b \cos^{-1}(dx^2-1))}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[-1 + d\*x^2])^(-3), x]

[Out] Sqrt[2\*d\*x^2 - d^2\*x^4]/(4\*b\*d\*x\*(a + b\*ArcCos[-1 + d\*x^2])^2) + x/(8\*b^2\*(a + b\*ArcCos[-1 + d\*x^2])) - (x\*CosIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)]\*Sin[a/(2\*b)])/(8\*Sqrt[2]\*b^3\*Sqrt[d\*x^2]) + (x\*Cos[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)])/(8\*Sqrt[2]\*b^3\*Sqrt[d\*x^2])

#### Rule 4829

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_)^2]\*(b\_.))^ (n\_), x\_Symbol] := Simp[(x\*(a + b\*ArcCos[c + d\*x^2])^(n + 2))/(4\*b^2\*(n + 1)\*(n + 2)), x] + (-Dist[1/(4\*b^2\*(n + 1)\*(n + 2)), Int[(a + b\*ArcCos[c + d\*x^2])^(n + 2), x], x] - Simp[(Sqrt[-2\*c\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[c + d\*x^2])^(n + 1))/(2\*b\*d\*(n + 1)\*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]

#### Rule 4818

Int[((a\_.) + ArcCos[-1 + (d\_.)\*(x\_)^2]\*(b\_.))^ (-1), x\_Symbol] := Simp[(x\*Sin[a/(2\*b)]\*CosIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)])/(Sqrt[2]\*b\*Sqrt[d\*x^2]), x] - Simp[(x\*Cos[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)])/(Sqrt[2]\*b\*Sqrt[d\*x^2]), x] /; FreeQ[{a, b, d}, x]

#### Rubi steps

$$\int \frac{1}{(a + b \cos^{-1}(-1 + dx^2))^3} dx = \frac{\sqrt{2dx^2 - d^2x^4}}{4bdx(a + b \cos^{-1}(-1 + dx^2))^2} + \frac{x}{8b^2(a + b \cos^{-1}(-1 + dx^2))} - \frac{\int \frac{1}{a+b \cos^{-1}(-1+dx^2)} dx}{8b^2}$$

$$= \frac{\sqrt{2dx^2 - d^2x^4}}{4bdx(a + b \cos^{-1}(-1 + dx^2))^2} + \frac{x}{8b^2(a + b \cos^{-1}(-1 + dx^2))} - \frac{x \operatorname{Ci}\left(\frac{a+b \cos^{-1}(-1+dx^2)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx}}$$

**Mathematica [A]** time = 0.212646, size = 149, normalized size = 0.87

$$\frac{\frac{2b^2\sqrt{-dx^2(dx^2-2)}}{d(a+b \cos^{-1}(dx^2-1))^2} - \frac{\cos\left(\frac{1}{2} \cos^{-1}(dx^2-1)\right)\left(\sin\left(\frac{a}{2b}\right)\operatorname{CosIntegral}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right) - \cos\left(\frac{a}{2b}\right)\operatorname{Si}\left(\frac{a+b \cos^{-1}(dx^2-1)}{2b}\right)\right)}{d}}{8b^3x} + \frac{bx^2}{a+b \cos^{-1}(dx^2-1)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCos[-1 + d\*x^2])^(-3), x]

[Out] ((2\*b^2\*Sqrt[-(d\*x^2\*(-2 + d\*x^2))])/(d\*(a + b\*ArcCos[-1 + d\*x^2])^2) + (b\*x^2)/(a + b\*ArcCos[-1 + d\*x^2]) - (Cos[ArcCos[-1 + d\*x^2]/2]\*(CosIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)]\*Sin[a/(2\*b)] - Cos[a/(2\*b)]\*SinIntegral[(a + b\*ArcCos[-1 + d\*x^2])/(2\*b)]))/d)/(8\*b^3\*x)

**Maple [F]** time = 0.068, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 - 1))^{-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arccos(d\*x^2-1))^3, x)

[Out] int(1/(a+b\*arccos(d\*x^2-1))^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bdx \arctan\left(\sqrt{-dx^2 + 2}\sqrt{dx}, dx^2 - 1\right) + adx + 2\sqrt{-dx^2 + 2}b\sqrt{d} - \left(b^4d \arctan\left(\sqrt{-dx^2 + 2}\sqrt{dx}, dx^2 - 1\right)^2 + 2ab^3d \arctan\left(\sqrt{-dx^2 + 2}\sqrt{dx}, dx^2 - 1\right)\right)}{8\left(b^4d \arctan\left(\sqrt{-dx^2 + 2}\sqrt{dx}, dx^2 - 1\right)^2 + 2ab^3d \arctan\left(\sqrt{-dx^2 + 2}\sqrt{dx}, dx^2 - 1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^3, x, algorithm="maxima")

[Out] 1/8\*(b\*d\*x\*arctan2(sqrt(-d\*x^2 + 2)\*sqrt(d)\*x, d\*x^2 - 1) + a\*d\*x + 2\*sqrt(-d\*x^2 + 2)\*b\*sqrt(d) - 8\*(b^4\*d\*arctan2(sqrt(-d\*x^2 + 2)\*sqrt(d)\*x, d\*x^2 - 1)^2 + 2\*a\*b^3\*d\*arctan2(sqrt(-d\*x^2 + 2)\*sqrt(d)\*x, d\*x^2 - 1) + a^2\*b^2\*d)\*integrate(1/8/(b^3\*arctan2(sqrt(-d\*x^2 + 2)\*sqrt(d)\*x, d\*x^2 - 1) + a\*b^2), x))/(b^4\*d\*arctan2(sqrt(-d\*x^2 + 2)\*sqrt(d)\*x, d\*x^2 - 1)^2 + 2\*a\*b^3\*

$d \cdot \arctan_2(\sqrt{-d \cdot x^2 + 2}) \cdot \sqrt{d} \cdot x, d \cdot x^2 - 1) + a^2 \cdot b^2 \cdot d$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{b^3 \arccos(dx^2 - 1)^3 + 3ab^2 \arccos(dx^2 - 1)^2 + 3a^2b \arccos(dx^2 - 1) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^3,x, algorithm="fricas")

[Out] integral(1/(b^3\*arccos(d\*x^2 - 1)^3 + 3\*a\*b^2\*arccos(d\*x^2 - 1)^2 + 3\*a^2\*b\*arccos(d\*x^2 - 1) + a^3), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \arccos(dx^2 - 1))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*acos(d\*x\*\*2-1))\*\*3,x)

[Out] Integral((a + b\*acos(d\*x\*\*2 - 1))\*\*(-3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(dx^2 - 1) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^3,x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 - 1) + a)^(-3), x)

### 3.87 $\int \left(a + b \cos^{-1} \left(1 + dx^2\right)\right)^{5/2} dx$

**Optimal.** Leaf size=249

$$\frac{30b^2 \sin^2\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) \sqrt{a + b \cos^{-1}(dx^2 + 1)}}{dx} - \frac{5b\sqrt{-d^2x^4 - 2dx^2} (a + b \cos^{-1}(dx^2 + 1))^{3/2}}{dx} + \frac{30\sqrt{\pi} \sin\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right)}{dx}$$

```
[Out] (-5*b*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2])^(3/2))/(d*x) + x*(
a + b*ArcCos[1 + d*x^2])^(5/2) - (30*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b
^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/
(b^(-1))^(5/2)*d*x) + (30*Sqrt[Pi]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos
[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/((b^(-1))^(5
/2)*d*x) + (30*b^2*Sqrt[a + b*ArcCos[1 + d*x^2]]*Sin[ArcCos[1 + d*x^2]/2]^2
)/(d*x)
```

**Rubi [A]** time = 0.0949448, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {4815, 4812}

$$\frac{30b^2 \sin^2\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) \sqrt{a + b \cos^{-1}(dx^2 + 1)}}{dx} - \frac{5b\sqrt{-d^2x^4 - 2dx^2} (a + b \cos^{-1}(dx^2 + 1))^{3/2}}{dx} + \frac{30\sqrt{\pi} \sin\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(5/2), x]
```

```
[Out] (-5*b*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2])^(3/2))/(d*x) + x*(
a + b*ArcCos[1 + d*x^2])^(5/2) - (30*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b
^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/
(b^(-1))^(5/2)*d*x) + (30*Sqrt[Pi]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos
[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/((b^(-1))^(5
/2)*d*x) + (30*b^2*Sqrt[a + b*ArcCos[1 + d*x^2]]*Sin[ArcCos[1 + d*x^2]/2]^2
)/(d*x)
```

#### Rule 4815

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] := Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcCos[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

#### Rule 4812

```
Int[Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] := Simp[(-2*Sqr
t[a + b*ArcCos[1 + d*x^2]]*Sin[ArcCos[1 + d*x^2]/2]^2)/(d*x), x] + (-Simp[(
2*Sqrt[Pi]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sq
rt[a + b*ArcCos[1 + d*x^2]]]/(Sqrt[1/b]*d*x), x] + Simp[(2*Sqrt[Pi]*Cos[a/
(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1
+ d*x^2]]]/(Sqrt[1/b]*d*x), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int (a + b \cos^{-1}(1 + dx^2))^{5/2} dx = -\frac{5b\sqrt{-2dx^2 - d^2x^4} (a + b \cos^{-1}(1 + dx^2))^{3/2}}{dx} + x (a + b \cos^{-1}(1 + dx^2))^{5/2} - (15b^2 \dots)$$

$$= -\frac{5b\sqrt{-2dx^2 - d^2x^4} (a + b \cos^{-1}(1 + dx^2))^{3/2}}{dx} + x (a + b \cos^{-1}(1 + dx^2))^{5/2} - \frac{30\sqrt{\dots}}{\dots}$$

**Mathematica [A]** time = 2.61709, size = 256, normalized size = 1.03

$$2 \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) \left( \sqrt{a + b \cos^{-1}(dx^2 + 1)} \left( (a^2 - 15b^2) \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) + 5ab \cos\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCos[1 + d\*x^2])^(5/2), x]

[Out] (-2\*Sin[ArcCos[1 + d\*x^2]/2]\*((15\*Sqrt[Pi]\*Cos[a/(2\*b)]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]])/(b^(-1))^(5/2) - (15\*Sqrt[Pi]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)])/b^(-1))^(5/2) + Sqrt[a + b\*ArcCos[1 + d\*x^2]]\*(5\*a\*b\*Cos[ArcCos[1 + d\*x^2]/2] + (a^2 - 15\*b^2)\*Sin[ArcCos[1 + d\*x^2]/2] + b^2\*ArcCos[1 + d\*x^2]^2\*Sin[ArcCos[1 + d\*x^2]/2] + b\*ArcCos[1 + d\*x^2]\*(5\*b\*Cos[ArcCos[1 + d\*x^2]/2] + 2\*a\*Sin[ArcCos[1 + d\*x^2]/2]))) / (d\*x)

**Maple [F]** time = 0.076, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 + 1))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos(d\*x^2+1))^(5/2), x)

[Out] int((a+b\*arccos(d\*x^2+1))^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arccos(dx^2 + 1) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2+1))^(5/2), x, algorithm="maxima")

[Out] integrate((b\*arccos(d\*x^2 + 1) + a)^(5/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2+1))^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acos(d\*x\*\*2+1))\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arccos(dx^2 + 1) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2+1))^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 + 1) + a)^(5/2), x)

$$3.88 \quad \int \left( a + b \cos^{-1} \left( 1 + dx^2 \right) \right)^{3/2} dx$$

**Optimal.** Leaf size=207

$$\frac{3b\sqrt{-d^2x^4 - 2dx^2}\sqrt{a + b \cos^{-1}(dx^2 + 1)}}{dx} + \frac{6\sqrt{\pi} \cos\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{3/2} dx} +$$

```
[Out] (-3*b*Sqrt[-2*d*x^2 - d^2*x^4]*Sqrt[a + b*ArcCos[1 + d*x^2]])/(d*x) + x*(a + b*ArcCos[1 + d*x^2])^(3/2) + (6*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/((b^(-1))^(3/2)*d*x) + (6*Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/((b^(-1))^(3/2)*d*x)
```

**Rubi [A]** time = 0.0663642, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {4815, 4820}

$$\frac{3b\sqrt{-d^2x^4 - 2dx^2}\sqrt{a + b \cos^{-1}(dx^2 + 1)}}{dx} + \frac{6\sqrt{\pi} \cos\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{3/2} dx} +$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(3/2), x]
```

```
[Out] (-3*b*Sqrt[-2*d*x^2 - d^2*x^4]*Sqrt[a + b*ArcCos[1 + d*x^2]])/(d*x) + x*(a + b*ArcCos[1 + d*x^2])^(3/2) + (6*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/((b^(-1))^(3/2)*d*x) + (6*Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/((b^(-1))^(3/2)*d*x)
```

#### Rule 4815

```
Int[((a_.) + ArcCos[(c_.) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] := Simp[x*(a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcCos[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

#### Rule 4820

```
Int[1/Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] := Simp[(-2*Sqrt[Pi/b]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]]]/(d*x), x] - Simp[(2*Sqrt[Pi/b]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]]]/(d*x), x] /; FreeQ[{a, b, d}, x]
```

#### Rubi steps

$$\int (a + b \cos^{-1}(1 + dx^2))^{3/2} dx = -\frac{3b\sqrt{-2dx^2 - d^2x^4}\sqrt{a + b \cos^{-1}(1 + dx^2)}}{dx} + x(a + b \cos^{-1}(1 + dx^2))^{3/2} - (3b^2) \int \frac{1}{\sqrt{\dots}}$$

$$= -\frac{3b\sqrt{-2dx^2 - d^2x^4}\sqrt{a + b \cos^{-1}(1 + dx^2)}}{dx} + x(a + b \cos^{-1}(1 + dx^2))^{3/2} + \frac{6\sqrt{\pi} \cos(\dots)}{\dots}$$

**Mathematica [A]** time = 0.713281, size = 200, normalized size = 0.97

$$2 \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) \left( -3\sqrt{\pi} \cos\left(\frac{a}{2b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right) - 3\sqrt{\pi} \sin\left(\frac{a}{2b}\right) S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right) + \left(\frac{1}{b}\right)^{3/2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCos[1 + d\*x^2])^(3/2), x]

[Out] (-2\*Sin[ArcCos[1 + d\*x^2]/2]\*(-3\*Sqrt[Pi]\*Cos[a/(2\*b)]\*FresnelC[(Sqrt[b^(-1)])\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]]/Sqrt[Pi]] - 3\*Sqrt[Pi]\*FresnelS[(Sqrt[b^(-1)])\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]]/Sqrt[Pi]]\*Sin[a/(2\*b)] + (b^(-1))^(3/2)\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]\*(3\*b\*Cos[ArcCos[1 + d\*x^2]/2] + a\*Sin[ArcCos[1 + d\*x^2]/2] + b\*ArcCos[1 + d\*x^2]\*Sin[ArcCos[1 + d\*x^2]/2]))/(b^(-1))^(3/2)\*d\*x)

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 + 1))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos(d\*x^2+1))^(3/2), x)

[Out] int((a+b\*arccos(d\*x^2+1))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arccos(dx^2 + 1) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2+1))^(3/2), x, algorithm="maxima")

[Out] integrate((b\*arccos(d\*x^2 + 1) + a)^(3/2), x)



**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2+1))^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 + 1))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acos(d\*x\*\*2+1))\*\*(3/2),x)

[Out] Integral((a + b\*acos(d\*x\*\*2 + 1))\*\*(3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arccos(dx^2 + 1) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2+1))^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 + 1) + a)^(3/2), x)

### 3.89 $\int \sqrt{a + b \cos^{-1}(1 + dx^2)} dx$

**Optimal.** Leaf size=184

$$\frac{2\sqrt{\pi} \sin\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) \operatorname{FresnelC}\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}} dx} + \frac{2\sqrt{\pi} \cos\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) S\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}} dx}$$

[Out] (2\*Sqrt[Pi]\*Cos[a/(2\*b)]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[ArcCos[1 + d\*x^2]/2])/(Sqrt[b^(-1)]\*d\*x) - (2\*Sqrt[Pi]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2])/(Sqrt[b^(-1)]\*d\*x) - (2\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]\*Sin[ArcCos[1 + d\*x^2]/2]^2)/(d\*x)

**Rubi [A]** time = 0.0215428, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {4812}

$$\frac{2\sqrt{\pi} \sin\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) \operatorname{FresnelC}\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}} dx} + \frac{2\sqrt{\pi} \cos\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) S\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}} dx}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*ArcCos[1 + d\*x^2]], x]

[Out] (2\*Sqrt[Pi]\*Cos[a/(2\*b)]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[ArcCos[1 + d\*x^2]/2])/(Sqrt[b^(-1)]\*d\*x) - (2\*Sqrt[Pi]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2])/(Sqrt[b^(-1)]\*d\*x) - (2\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]\*Sin[ArcCos[1 + d\*x^2]/2]^2)/(d\*x)

#### Rule 4812

Int[Sqrt[(a\_.) + ArcCos[1 + (d\_.)\*(x\_)^2]\*(b\_.)], x\_Symbol] :> Simp[(-2\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]\*Sin[ArcCos[1 + d\*x^2]/2]^2)/(d\*x), x] + (-Simp[(2\*Sqrt[Pi]\*Sin[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2]\*FresnelC[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]])/(Sqrt[1/b]\*d\*x), x] + Simp[(2\*Sqrt[Pi]\*Cos[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2]\*FresnelS[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]])/(Sqrt[1/b]\*d\*x), x]) /; FreeQ[{a, b, d}, x]

#### Rubi steps

$$\int \sqrt{a + b \cos^{-1}(1 + dx^2)} dx = \frac{2\sqrt{\pi} \cos\left(\frac{a}{2b}\right) S\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(1+dx^2)}}{\sqrt{\pi}}\right) \sin\left(\frac{1}{2} \cos^{-1}(1 + dx^2)\right)}{\sqrt{\frac{1}{b}} dx} - \frac{2\sqrt{\pi} C\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(1+dx^2)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}} dx}$$

**Mathematica [A]** time = 0.0848811, size = 157, normalized size = 0.85

$$\frac{2 \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) \left( \sqrt{\pi} \sin\left(\frac{a}{2b}\right) \operatorname{FresnelC}\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right) - \sqrt{\pi} \cos\left(\frac{a}{2b}\right) S\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right) \right) + \sqrt{\frac{1}{b}} \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right)}{\sqrt{\frac{1}{b}} dx}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*ArcCos[1 + d\*x^2]],x]

[Out]  $(-2*\text{Sin}[\text{ArcCos}[1 + d*x^2]/2]*(-(\text{Sqrt}[\text{Pi}]*\text{Cos}[a/(2*b)]*\text{FresnelS}[(\text{Sqrt}[b^{(-1)}]*\text{Sqrt}[a + b*\text{ArcCos}[1 + d*x^2]])/\text{Sqrt}[\text{Pi}]]) + \text{Sqrt}[\text{Pi}]*\text{FresnelC}[(\text{Sqrt}[b^{(-1)}]*\text{Sqrt}[a + b*\text{ArcCos}[1 + d*x^2]])/\text{Sqrt}[\text{Pi}]]*\text{Sin}[a/(2*b)] + \text{Sqrt}[b^{(-1)}]*\text{Sqrt}[a + b*\text{ArcCos}[1 + d*x^2]]*\text{Sin}[\text{ArcCos}[1 + d*x^2]/2])))/(\text{Sqrt}[b^{(-1)}]*d*x)$

**Maple [F]** time = 0.063, size = 0, normalized size = 0.

$$\int \sqrt{a + b \arccos(dx^2 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos(d\*x^2+1))^(1/2),x)

[Out] int((a+b\*arccos(d\*x^2+1))^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \arccos(dx^2 + 1) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2+1))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b\*arccos(d\*x^2 + 1) + a), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2+1))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \arccos(dx^2 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x\*\*2+1))\*\*(1/2),x)

[Out] Integral(sqrt(a + b\*acos(d\*x\*\*2 + 1)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \arccos(dx^2 + 1) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2+1))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b\*arccos(d\*x^2 + 1) + a), x)

$$3.90 \quad \int \frac{1}{\sqrt{a+b \cos^{-1}(1+dx^2)}} dx$$

**Optimal.** Leaf size=145

$$\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}} \cos\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2+1)\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)}{dx} - \frac{2\sqrt{\pi}\sqrt{\frac{1}{b}} \sin\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2+1)\right)}{dx}$$

[Out] (-2\*Sqrt[b^(-1)]\*Sqrt[Pi]\*Cos[a/(2\*b)]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[ArcCos[1 + d\*x^2]/2])/(d\*x) - (2\*Sqrt[b^(-1)]\*Sqrt[Pi]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2])/(d\*x)

**Rubi [A]** time = 0.0175331, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {4820}

$$\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}} \cos\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2+1)\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)}{dx} - \frac{2\sqrt{\pi}\sqrt{\frac{1}{b}} \sin\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2+1)\right)}{dx}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b\*ArcCos[1 + d\*x^2]], x]

[Out] (-2\*Sqrt[b^(-1)]\*Sqrt[Pi]\*Cos[a/(2\*b)]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[ArcCos[1 + d\*x^2]/2])/(d\*x) - (2\*Sqrt[b^(-1)]\*Sqrt[Pi]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2])/(d\*x)

**Rule 4820**

Int[1/Sqrt[(a\_.) + ArcCos[1 + (d\_.)\*(x\_)^2]\*(b\_.)], x\_Symbol] := Simp[(-2\*Sqrt[Pi/b]\*Cos[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2]\*FresnelC[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]])/(d\*x), x] - Simp[(2\*Sqrt[Pi/b]\*Sin[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2]\*FresnelS[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]])/(d\*x), x] /; FreeQ[{a, b, d}, x]

**Rubi steps**

$$\int \frac{1}{\sqrt{a+b \cos^{-1}(1+dx^2)}} dx = -\frac{2\sqrt{\frac{1}{b}}\sqrt{\pi} \cos\left(\frac{a}{2b}\right) C\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(1+dx^2)}}{\sqrt{\pi}}\right) \sin\left(\frac{1}{2} \cos^{-1}(1+dx^2)\right)}{dx} - \frac{2\sqrt{\frac{1}{b}}\sqrt{\pi} S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(1+dx^2)}}{\sqrt{\pi}}\right) \sin\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(1+dx^2)\right)}{dx}$$

**Mathematica [A]** time = 0.161629, size = 114, normalized size = 0.79

$$\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}} \sin\left(\frac{1}{2} \cos^{-1}(dx^2+1)\right) \left( \cos\left(\frac{a}{2b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right) + \sin\left(\frac{a}{2b}\right) S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right) \right)}{dx}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b\*ArcCos[1 + d\*x^2]],x]

[Out]  $(-2\sqrt{b^{-1}})\sqrt{\pi}(\cos[a/(2b)]\text{FresnelC}[(\sqrt{b^{-1}})\sqrt{a + b\text{ArcCos}[1 + d x^2]})/\sqrt{\pi}] + \text{FresnelS}[(\sqrt{b^{-1}})\sqrt{a + b\text{ArcCos}[1 + d x^2]})/\sqrt{\pi}]\sin[a/(2b)])\sin[\text{ArcCos}[1 + d x^2]/2]/(d x)$

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \arccos(dx^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arccos(d\*x^2+1))^(1/2),x)

[Out] int(1/(a+b\*arccos(d\*x^2+1))^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \arccos(dx^2 + 1) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b\*arccos(d\*x^2 + 1) + a), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \arccos(dx^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x\*\*2+1))\*\*(1/2),x)

[Out] Integral(1/sqrt(a + b\*acos(d\*x\*\*2 + 1)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \arccos(dx^2 + 1) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(b\*arccos(d\*x^2 + 1) + a), x)

$$3.91 \quad \int \frac{1}{(a+b \cos^{-1}(1+dx^2))^{3/2}} dx$$

**Optimal.** Leaf size=190

$$\frac{\sqrt{-d^2x^4 - 2dx^2}}{bdx\sqrt{a + b \cos^{-1}(dx^2 + 1)}} - \frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2} \sin\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)}{dx} + \frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2} \cos\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) \text{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)}{dx}$$

[Out] Sqrt[-2\*d\*x^2 - d^2\*x^4]/(b\*d\*x\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]) + (2\*(b^(-1))^(3/2)\*Sqrt[Pi]\*Cos[a/(2\*b)]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[ArcCos[1 + d\*x^2]/2])/(d\*x) - (2\*(b^(-1))^(3/2)\*Sqrt[Pi]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2])/(d\*x)

**Rubi [A]** time = 0.0293162, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {4823}

$$\frac{\sqrt{-d^2x^4 - 2dx^2}}{bdx\sqrt{a + b \cos^{-1}(dx^2 + 1)}} - \frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2} \sin\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)}{dx} + \frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2} \cos\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) \text{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)}{dx}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[1 + d\*x^2])^(-3/2), x]

[Out] Sqrt[-2\*d\*x^2 - d^2\*x^4]/(b\*d\*x\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]) + (2\*(b^(-1))^(3/2)\*Sqrt[Pi]\*Cos[a/(2\*b)]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[ArcCos[1 + d\*x^2]/2])/(d\*x) - (2\*(b^(-1))^(3/2)\*Sqrt[Pi]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2])/(d\*x)

### Rule 4823

Int[((a\_.) + ArcCos[1 + (d\_.)\*(x\_)^2]\*(b\_.))^(3/2), x\_Symbol] := Simp[Sqrt[-2\*d\*x^2 - d^2\*x^4]/(b\*d\*x\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]), x] + (-Simp[(2\*(1/b)^(3/2)\*Sqrt[Pi]\*Sin[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2]\*FresnelC[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]])/(d\*x), x] + Simp[(2\*(1/b)^(3/2)\*Sqrt[Pi]\*Cos[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2]\*FresnelS[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]])/(d\*x), x]) /; FreeQ[{a, b, d}, x]

### Rubi steps

$$\int \frac{1}{(a + b \cos^{-1}(1 + dx^2))^{3/2}} dx = \frac{\sqrt{-2dx^2 - d^2x^4}}{bdx\sqrt{a + b \cos^{-1}(1 + dx^2)}} + \frac{2\left(\frac{1}{b}\right)^{3/2} \sqrt{\pi} \cos\left(\frac{a}{2b}\right) \text{S}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(1+dx^2)}}{\sqrt{\pi}}\right) \sin\left(\frac{1}{2} \cos^{-1}(1 + dx^2)\right)}{dx}$$



**Mathematica [A]** time = 0.388768, size = 177, normalized size = 0.93

$$\frac{-2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}(dx^2+1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)+2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}(dx^2+1)\right)}{bdx}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCos[1 + d\*x^2])^(-3/2), x]

[Out] (Sqrt[-(d\*x^2\*(2 + d\*x^2))]/Sqrt[a + b\*ArcCos[1 + d\*x^2]] + 2\*Sqrt[b^(-1)]\*Sqrt[Pi]\*Cos[a/(2\*b)]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[ArcCos[1 + d\*x^2]/2] - 2\*Sqrt[b^(-1)]\*Sqrt[Pi]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2])/(b\*d\*x)

**Maple [F]** time = 0.067, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 + 1))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arccos(d\*x^2+1))^(3/2), x)

[Out] int(1/(a+b\*arccos(d\*x^2+1))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(dx^2 + 1) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^(3/2), x, algorithm="maxima")

[Out] integrate((b\*arccos(d\*x^2 + 1) + a)^(-3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^(3/2), x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \arccos(dx^2 + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*acos(d\*x\*\*2+1))\*\*(3/2),x)

[Out] Integral((a + b\*acos(d\*x\*\*2 + 1))\*\*(-3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(dx^2 + 1) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 + 1) + a)^(-3/2), x)

$$3.92 \quad \int \frac{1}{(a+b \cos^{-1}(1+dx^2))^{5/2}} dx$$

**Optimal.** Leaf size=221

$$\frac{x}{3b^2 \sqrt{a+b \cos^{-1}(dx^2+1)}} + \frac{\sqrt{-d^2x^4-2dx^2}}{3bdx(a+b \cos^{-1}(dx^2+1))^{3/2}} + \frac{2\sqrt{\pi} \left(\frac{1}{b}\right)^{5/2} \cos\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2+1)\right) \text{FresnelC}}{3dx}$$

```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(3*b*d*x*(a + b*ArcCos[1 + d*x^2])^(3/2)) + x/(3*b
^2*Sqrt[a + b*ArcCos[1 + d*x^2]]) + (2*(b^(-1))^(5/2)*Sqrt[Pi]*Cos[a/(2*b)]
*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos
[1 + d*x^2]/2])/(3*d*x) + (2*(b^(-1))^(5/2)*Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)]
*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2
]/2])/(3*d*x)
```

**Rubi [A]** time = 0.0500054, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {4829, 4820}

$$\frac{x}{3b^2 \sqrt{a+b \cos^{-1}(dx^2+1)}} + \frac{\sqrt{-d^2x^4-2dx^2}}{3bdx(a+b \cos^{-1}(dx^2+1))^{3/2}} + \frac{2\sqrt{\pi} \left(\frac{1}{b}\right)^{5/2} \cos\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2} \cos^{-1}(dx^2+1)\right) \text{FresnelC}}{3dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(-5/2), x]
```

```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(3*b*d*x*(a + b*ArcCos[1 + d*x^2])^(3/2)) + x/(3*b
^2*Sqrt[a + b*ArcCos[1 + d*x^2]]) + (2*(b^(-1))^(5/2)*Sqrt[Pi]*Cos[a/(2*b)]
*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos
[1 + d*x^2]/2])/(3*d*x) + (2*(b^(-1))^(5/2)*Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)]
*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2
]/2])/(3*d*x)
```

#### Rule 4829

```
Int[((a_.) + ArcCos[(c_.) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] := Simp[(x*
(a + b*ArcCos[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCos[c + d*x^2])^(n + 2), x], x] - Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcCos[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

#### Rule 4820

```
Int[1/Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] := Simp[(-2*S
qrt[Pi/b]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sqr
t[a + b*ArcCos[1 + d*x^2]]])/(d*x), x] - Simp[(2*Sqrt[Pi/b]*Sin[a/(2*b)]*Si
n[ArcCos[1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2
]]])/(d*x), x] /; FreeQ[{a, b, d}, x]
```

#### Rubi steps

$$\int \frac{1}{(a + b \cos^{-1}(1 + dx^2))^{5/2}} dx = \frac{\sqrt{-2dx^2 - d^2x^4}}{3bdx(a + b \cos^{-1}(1 + dx^2))^{3/2}} + \frac{x}{3b^2\sqrt{a + b \cos^{-1}(1 + dx^2)}} - \frac{\int \frac{1}{\sqrt{a + b \cos^{-1}(1 + dx^2)}} dx}{3b^2}$$

$$= \frac{\sqrt{-2dx^2 - d^2x^4}}{3bdx(a + b \cos^{-1}(1 + dx^2))^{3/2}} + \frac{x}{3b^2\sqrt{a + b \cos^{-1}(1 + dx^2)}} + \frac{2\left(\frac{1}{b}\right)^{5/2} \sqrt{\pi} \cos\left(\frac{a}{2b}\right) C}{3b^2}$$

**Mathematica [A]** time = 0.72083, size = 234, normalized size = 1.06

$$2 \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) \left( \sqrt{\pi} \sqrt{\frac{1}{b}} \cos\left(\frac{a}{2b}\right) (a + b \cos^{-1}(dx^2 + 1))^{3/2} \operatorname{FresnelC}\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a + b \cos^{-1}(dx^2 + 1)}}{\sqrt{\pi}}\right) + \sqrt{\pi} \sqrt{\frac{1}{b}} \sin\left(\frac{a}{2b}\right) (a + b \cos^{-1}(dx^2 + 1))^{3/2} \operatorname{FresnelS}\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a + b \cos^{-1}(dx^2 + 1)}}{\sqrt{\pi}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCos[1 + d\*x^2])^(-5/2), x]

[Out] (2\*Sin[ArcCos[1 + d\*x^2]/2]\*(b\*Cos[ArcCos[1 + d\*x^2]/2] + Sqrt[b^(-1)]\*Sqrt[Pi]\*(a + b\*ArcCos[1 + d\*x^2])^(3/2)\*Cos[a/(2\*b)]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]] + Sqrt[b^(-1)]\*Sqrt[Pi]\*(a + b\*ArcCos[1 + d\*x^2])^(3/2)\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)] - a\*Sin[ArcCos[1 + d\*x^2]/2] - b\*ArcCos[1 + d\*x^2]\*Sin[ArcCos[1 + d\*x^2]/2])/(3\*b^2\*d\*x\*(a + b\*ArcCos[1 + d\*x^2])^(3/2))

**Maple [F]** time = 0.067, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 + 1))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arccos(d\*x^2+1))^(5/2), x)

[Out] int(1/(a+b\*arccos(d\*x^2+1))^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(dx^2 + 1) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^(5/2), x, algorithm="maxima")

[Out] integrate((b\*arccos(d\*x^2 + 1) + a)^(-5/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \arccos(dx^2 + 1))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*acos(d\*x\*\*2+1))\*\*(5/2),x)

[Out] Integral((a + b\*acos(d\*x\*\*2 + 1))\*\*(-5/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(dx^2 + 1) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 + 1) + a)^(-5/2), x)

### 3.93 $\int \frac{1}{(a+b \cos^{-1}(1+dx^2))^{7/2}} dx$

**Optimal.** Leaf size=269

$$-\frac{\sqrt{-d^2x^4 - 2dx^2}}{15b^3dx\sqrt{a + b \cos^{-1}(dx^2 + 1)}} + \frac{x}{15b^2(a + b \cos^{-1}(dx^2 + 1))^{3/2}} + \frac{\sqrt{-d^2x^4 - 2dx^2}}{5bdx(a + b \cos^{-1}(dx^2 + 1))^{5/2}} + \frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{7/2} \sin\left(\frac{a}{2b}\right)}{15b^3dx\sqrt{a + b \cos^{-1}(dx^2 + 1)}}$$

[Out] Sqrt[-2\*d\*x^2 - d^2\*x^4]/(5\*b\*d\*x\*(a + b\*ArcCos[1 + d\*x^2])^(5/2)) + x/(15\*b^2\*(a + b\*ArcCos[1 + d\*x^2])^(3/2)) - Sqrt[-2\*d\*x^2 - d^2\*x^4]/(15\*b^3\*d\*x\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]) - (2\*(b^(-1))^(7/2)\*Sqrt[Pi]\*Cos[a/(2\*b)]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[ArcCos[1 + d\*x^2]/2])/(15\*d\*x) + (2\*(b^(-1))^(7/2)\*Sqrt[Pi]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2])/(15\*d\*x)

**Rubi [A]** time = 0.060164, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {4829, 4823}

$$-\frac{\sqrt{-d^2x^4 - 2dx^2}}{15b^3dx\sqrt{a + b \cos^{-1}(dx^2 + 1)}} + \frac{x}{15b^2(a + b \cos^{-1}(dx^2 + 1))^{3/2}} + \frac{\sqrt{-d^2x^4 - 2dx^2}}{5bdx(a + b \cos^{-1}(dx^2 + 1))^{5/2}} + \frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{7/2} \sin\left(\frac{a}{2b}\right)}{15b^3dx\sqrt{a + b \cos^{-1}(dx^2 + 1)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[1 + d\*x^2])^(-7/2), x]

[Out] Sqrt[-2\*d\*x^2 - d^2\*x^4]/(5\*b\*d\*x\*(a + b\*ArcCos[1 + d\*x^2])^(5/2)) + x/(15\*b^2\*(a + b\*ArcCos[1 + d\*x^2])^(3/2)) - Sqrt[-2\*d\*x^2 - d^2\*x^4]/(15\*b^3\*d\*x\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]) - (2\*(b^(-1))^(7/2)\*Sqrt[Pi]\*Cos[a/(2\*b)]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[ArcCos[1 + d\*x^2]/2])/(15\*d\*x) + (2\*(b^(-1))^(7/2)\*Sqrt[Pi]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2])/(15\*d\*x)

#### Rule 4829

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_)^2]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(x\*(a + b\*ArcCos[c + d\*x^2])^(n + 2))/(4\*b^2\*(n + 1)\*(n + 2)), x] + (-Dist[1/(4\*b^2\*(n + 1)\*(n + 2)), Int[(a + b\*ArcCos[c + d\*x^2])^(n + 2), x], x] - Simp[(Sqrt[-2\*c\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[c + d\*x^2])^(n + 1))/(2\*b\*d\*(n + 1)\*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]

#### Rule 4823

Int[((a\_.) + ArcCos[1 + (d\_.)\*(x\_)^2]\*(b\_.))^(n\_), x\_Symbol] :> Simp[Sqrt[-2\*d\*x^2 - d^2\*x^4]/(b\*d\*x\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]), x] + (-Simp[(2\*(1/b)^(3/2)\*Sqrt[Pi]\*Sin[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2]\*FresnelC[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]])/(d\*x), x] + Simp[(2\*(1/b)^(3/2)\*Sqrt[Pi]\*Cos[a/(2\*b)]\*Sin[ArcCos[1 + d\*x^2]/2]\*FresnelS[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]]])/(d\*x), x]) /; FreeQ[{a, b, d}, x]

Rubi steps

$$\int \frac{1}{(a + b \cos^{-1}(1 + dx^2))^{7/2}} dx = \frac{\sqrt{-2dx^2 - d^2x^4}}{5bdx(a + b \cos^{-1}(1 + dx^2))^{5/2}} + \frac{x}{15b^2(a + b \cos^{-1}(1 + dx^2))^{3/2}} - \frac{\int \frac{1}{(a + b \cos^{-1}(1 + dx^2))^{5/2}} dx}{15b^2}$$

$$= \frac{\sqrt{-2dx^2 - d^2x^4}}{5bdx(a + b \cos^{-1}(1 + dx^2))^{5/2}} + \frac{x}{15b^2(a + b \cos^{-1}(1 + dx^2))^{3/2}} - \frac{\sqrt{-2dx^2}}{15b^3dx\sqrt{a + b}}$$

**Mathematica [A]** time = 0.546504, size = 308, normalized size = 1.14

$$2 \sin\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) \left( a^2 \cos\left(\frac{1}{2} \cos^{-1}(dx^2 + 1)\right) - \sqrt{\pi} \sqrt{\frac{1}{b}} \sin\left(\frac{a}{2b}\right) (a + b \cos^{-1}(dx^2 + 1))^{5/2} \operatorname{FresnelC}\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b}}{\dots}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCos[1 + d\*x^2])^(-7/2), x]

[Out] (-2\*Sin[ArcCos[1 + d\*x^2]/2]\*(a^2\*Cos[ArcCos[1 + d\*x^2]/2] - 3\*b^2\*Cos[ArcCos[1 + d\*x^2]/2] + 2\*a\*b\*ArcCos[1 + d\*x^2]\*Cos[ArcCos[1 + d\*x^2]/2] + b^2\*ArcCos[1 + d\*x^2]^2\*Cos[ArcCos[1 + d\*x^2]/2] + Sqrt[b^(-1)]\*Sqrt[Pi]\*(a + b\*ArcCos[1 + d\*x^2])^(5/2)\*Cos[a/(2\*b)]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]] - Sqrt[b^(-1)]\*Sqrt[Pi]\*(a + b\*ArcCos[1 + d\*x^2])^(5/2)\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)] + a\*b\*Sin[ArcCos[1 + d\*x^2]/2] + b^2\*ArcCos[1 + d\*x^2]\*Sin[ArcCos[1 + d\*x^2]/2]))/(15\*b^3\*d\*x\*(a + b\*ArcCos[1 + d\*x^2])^(5/2))

**Maple [F]** time = 0.068, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 + 1))^{-7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arccos(d\*x^2+1))^(7/2), x)

[Out] int(1/(a+b\*arccos(d\*x^2+1))^(7/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(dx^2 + 1) + a)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^(7/2), x, algorithm="maxima")

[Out] integrate((b\*arccos(d\*x^2 + 1) + a)^(-7/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^(7/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*acos(d\*x\*\*2+1))\*\*(7/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(dx^2 + 1) + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2+1))^(7/2),x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 + 1) + a)^(-7/2), x)



### 3.94 $\int \left( a + b \cos^{-1} \left( -1 + dx^2 \right) \right)^{5/2} dx$

**Optimal.** Leaf size=249

$$\frac{30b^2 \cos^2 \left( \frac{1}{2} \cos^{-1} (dx^2 - 1) \right) \sqrt{a + b \cos^{-1} (dx^2 - 1)}}{dx} - \frac{5b\sqrt{2dx^2 - d^2x^4} (a + b \cos^{-1} (dx^2 - 1))^{3/2}}{dx} + \frac{30\sqrt{\pi} \cos \left( \frac{a}{2b} \right)}{dx}$$

```
[Out] (-5*b*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2])^(3/2))/(d*x) + x*(
a + b*ArcCos[-1 + d*x^2])^(5/2) - (30*b^2*Sqrt[a + b*ArcCos[-1 + d*x^2]]*Co
s[ArcCos[-1 + d*x^2]/2]^2)/(d*x) + (30*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1
+ d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]
])/((b^(-1))^(5/2)*d*x) + (30*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(
Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/((b^(-
1))^(5/2)*d*x)
```

**Rubi [A]** time = 0.0552374, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {4815, 4813}

$$\frac{30b^2 \cos^2 \left( \frac{1}{2} \cos^{-1} (dx^2 - 1) \right) \sqrt{a + b \cos^{-1} (dx^2 - 1)}}{dx} - \frac{5b\sqrt{2dx^2 - d^2x^4} (a + b \cos^{-1} (dx^2 - 1))^{3/2}}{dx} + \frac{30\sqrt{\pi} \cos \left( \frac{a}{2b} \right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(5/2), x]
```

```
[Out] (-5*b*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2])^(3/2))/(d*x) + x*(
a + b*ArcCos[-1 + d*x^2])^(5/2) - (30*b^2*Sqrt[a + b*ArcCos[-1 + d*x^2]]*Co
s[ArcCos[-1 + d*x^2]/2]^2)/(d*x) + (30*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1
+ d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]
])/((b^(-1))^(5/2)*d*x) + (30*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(
Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/((b^(-
1))^(5/2)*d*x)
```

#### Rule 4815

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] := Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcCos[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

#### Rule 4813

```
Int[Sqrt[(a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] := Simp[(2*Sqr
t[a + b*ArcCos[-1 + d*x^2]]*Cos[(1/2)*ArcCos[-1 + d*x^2]]^2)/(d*x), x] + (-
Simp[(2*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[Sqrt[1/(Pi
*b)]*Sqrt[a + b*ArcCos[-1 + d*x^2]]]/(Sqrt[1/b]*d*x), x] - Simp[(2*Sqrt[Pi]
]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b
*ArcCos[-1 + d*x^2]]]/(Sqrt[1/b]*d*x), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int (a + b \cos^{-1}(-1 + dx^2))^{5/2} dx = -\frac{5b\sqrt{2dx^2 - d^2x^4} (a + b \cos^{-1}(-1 + dx^2))^{3/2}}{dx} + x (a + b \cos^{-1}(-1 + dx^2))^{5/2} - (15b^2) \\ = -\frac{5b\sqrt{2dx^2 - d^2x^4} (a + b \cos^{-1}(-1 + dx^2))^{3/2}}{dx} + x (a + b \cos^{-1}(-1 + dx^2))^{5/2} - \frac{30b^2}{dx}$$

**Mathematica [A]** time = 2.14601, size = 256, normalized size = 1.03

$$2 \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) \left[ \sqrt{a + b \cos^{-1}(dx^2 - 1)} \left( (a^2 - 15b^2) \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) + b \cos^{-1}(dx^2 - 1) \right) \left( 2a \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) \right) \right]$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCos[-1 + d\*x^2])^(5/2), x]

[Out] (2\*Cos[ArcCos[-1 + d\*x^2]/2]\*((15\*Sqrt[Pi]\*Cos[a/(2\*b)]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]])/(b^(-1))^(5/2) + (15\*Sqrt[Pi]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)])/(b^(-1))^(5/2) + Sqrt[a + b\*ArcCos[-1 + d\*x^2]]\*((a^2 - 15\*b^2)\*Cos[ArcCos[-1 + d\*x^2]/2] + b^2\*ArcCos[-1 + d\*x^2]^2\*Cos[ArcCos[-1 + d\*x^2]/2] - 5\*a\*b\*Sin[ArcCos[-1 + d\*x^2]/2] + b\*ArcCos[-1 + d\*x^2]\*(2\*a\*Cos[ArcCos[-1 + d\*x^2]/2] - 5\*b\*Sin[ArcCos[-1 + d\*x^2]/2]))) / (d\*x)

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 - 1))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos(d\*x^2-1))^(5/2), x)

[Out] int((a+b\*arccos(d\*x^2-1))^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arccos(dx^2 - 1) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2-1))^(5/2), x, algorithm="maxima")

[Out] integrate((b\*arccos(d\*x^2 - 1) + a)^(5/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2-1))^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acos(d\*x\*\*2-1))\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arccos(dx^2 - 1) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2-1))^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 - 1) + a)^(5/2), x)

### 3.95 $\int \left( a + b \cos^{-1}(-1 + dx^2) \right)^{3/2} dx$

**Optimal.** Leaf size=207

$$\frac{3b\sqrt{2dx^2 - d^2x^4}\sqrt{a + b \cos^{-1}(dx^2 - 1)}}{dx} - \frac{6\sqrt{\pi} \sin\left(\frac{a}{2b}\right) \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{3/2} dx} + \frac{6\sqrt{\pi}}{\dots}$$

[Out]  $(-3*b*\text{Sqrt}[2*d*x^2 - d^2*x^4]*\text{Sqrt}[a + b*\text{ArcCos}[-1 + d*x^2]])/(d*x) + x*(a + b*\text{ArcCos}[-1 + d*x^2])^{3/2} + (6*\text{Sqrt}[\text{Pi}]*\text{Cos}[a/(2*b)]*\text{Cos}[\text{ArcCos}[-1 + d*x^2]/2]*\text{FresnelS}[(\text{Sqrt}[b^{(-1)}]*\text{Sqrt}[a + b*\text{ArcCos}[-1 + d*x^2]])/\text{Sqrt}[\text{Pi}]])/(b^{(-1)})^{3/2}*d*x - (6*\text{Sqrt}[\text{Pi}]*\text{Cos}[\text{ArcCos}[-1 + d*x^2]/2]*\text{FresnelC}[(\text{Sqrt}[b^{(-1)}]*\text{Sqrt}[a + b*\text{ArcCos}[-1 + d*x^2]])/\text{Sqrt}[\text{Pi}]]*\text{Sin}[a/(2*b)])/((b^{(-1)})^{3/2}*d*x)$

**Rubi [A]** time = 0.0440606, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {4815, 4821}

$$\frac{3b\sqrt{2dx^2 - d^2x^4}\sqrt{a + b \cos^{-1}(dx^2 - 1)}}{dx} - \frac{6\sqrt{\pi} \sin\left(\frac{a}{2b}\right) \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{3/2} dx} + \frac{6\sqrt{\pi}}{\dots}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcCos}[-1 + d*x^2])^{3/2}, x]$

[Out]  $(-3*b*\text{Sqrt}[2*d*x^2 - d^2*x^4]*\text{Sqrt}[a + b*\text{ArcCos}[-1 + d*x^2]])/(d*x) + x*(a + b*\text{ArcCos}[-1 + d*x^2])^{3/2} + (6*\text{Sqrt}[\text{Pi}]*\text{Cos}[a/(2*b)]*\text{Cos}[\text{ArcCos}[-1 + d*x^2]/2]*\text{FresnelS}[(\text{Sqrt}[b^{(-1)}]*\text{Sqrt}[a + b*\text{ArcCos}[-1 + d*x^2]])/\text{Sqrt}[\text{Pi}]])/(b^{(-1)})^{3/2}*d*x - (6*\text{Sqrt}[\text{Pi}]*\text{Cos}[\text{ArcCos}[-1 + d*x^2]/2]*\text{FresnelC}[(\text{Sqrt}[b^{(-1)}]*\text{Sqrt}[a + b*\text{ArcCos}[-1 + d*x^2]])/\text{Sqrt}[\text{Pi}]]*\text{Sin}[a/(2*b)])/((b^{(-1)})^{3/2}*d*x)$

#### Rule 4815

$\text{Int}[(a_. + \text{ArcCos}[(c_) + (d_.)*(x_)^2]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcCos}[c + d*x^2])^n, x] + (-\text{Dist}[4*b^2*n*(n - 1), \text{Int}[(a + b*\text{ArcCos}[c + d*x^2])^{(n - 2)}, x], x] - \text{Simp}[(2*b*n*\text{Sqrt}[-2*c*d*x^2 - d^2*x^4]*(a + b*\text{ArcCos}[c + d*x^2])^{(n - 1)})/(d*x), x]) /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{EqQ}[c^2, 1] \&\& \text{GtQ}[n, 1]$

#### Rule 4821

$\text{Int}[1/\text{Sqrt}[(a_. + \text{ArcCos}[-1 + (d_.)*(x_)^2]*(b_.)]], x\_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[\text{Pi}/b]*\text{Sin}[a/(2*b)]*\text{Cos}[\text{ArcCos}[-1 + d*x^2]/2]*\text{FresnelC}[\text{Sqrt}[1/(\text{Pi}*b)]*\text{Sqrt}[a + b*\text{ArcCos}[-1 + d*x^2]]])/(d*x), x] - \text{Simp}[(2*\text{Sqrt}[\text{Pi}/b]*\text{Cos}[a/(2*b)]*\text{Cos}[\text{ArcCos}[-1 + d*x^2]/2]*\text{FresnelS}[\text{Sqrt}[1/(\text{Pi}*b)]*\text{Sqrt}[a + b*\text{ArcCos}[-1 + d*x^2]]])/(d*x), x] /; \text{FreeQ}\{a, b, d\}, x]$

#### Rubi steps

$$\int (a + b \cos^{-1}(-1 + dx^2))^{3/2} dx = -\frac{3b\sqrt{2dx^2 - d^2x^4}\sqrt{a + b \cos^{-1}(-1 + dx^2)}}{dx} + x(a + b \cos^{-1}(-1 + dx^2))^{3/2} - (3b^2)^{3/2}$$

$$= -\frac{3b\sqrt{2dx^2 - d^2x^4}\sqrt{a + b \cos^{-1}(-1 + dx^2)}}{dx} + x(a + b \cos^{-1}(-1 + dx^2))^{3/2} + \frac{6\sqrt{\pi}}{b}$$

**Mathematica [A]** time = 0.602761, size = 200, normalized size = 0.97

$$2 \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) \left( -3\sqrt{\pi} \sin\left(\frac{a}{2b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right) + 3\sqrt{\pi} \cos\left(\frac{a}{2b}\right) S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right) \right) + \left(\frac{1}{b}\right)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCos[-1 + d\*x^2])^(3/2), x]

[Out] (2\*Cos[ArcCos[-1 + d\*x^2]/2]\*(3\*Sqrt[Pi]\*Cos[a/(2\*b)]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]] - 3\*Sqrt[Pi]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)] + (b^(-1))^(3/2)\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]\*(a\*Cos[ArcCos[-1 + d\*x^2]/2] + b\*ArcCos[-1 + d\*x^2]\*Cos[ArcCos[-1 + d\*x^2]/2] - 3\*b\*Sin[ArcCos[-1 + d\*x^2]/2])))/(b^(-1))^(3/2)\*d\*x)

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 - 1))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos(d\*x^2-1))^(3/2), x)

[Out] int((a+b\*arccos(d\*x^2-1))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arccos(dx^2 - 1) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2-1))^(3/2), x, algorithm="maxima")

[Out] integrate((b\*arccos(d\*x^2 - 1) + a)^(3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2-1))^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 - 1))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acos(d\*x\*\*2-1))\*\*(3/2),x)

[Out] Integral((a + b\*acos(d\*x\*\*2 - 1))\*\*(3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arccos(dx^2 - 1) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2-1))^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 - 1) + a)^(3/2), x)

### 3.96 $\int \sqrt{a + b \cos^{-1}(-1 + dx^2)} dx$

**Optimal.** Leaf size=184

$$\frac{2\sqrt{\pi} \cos\left(\frac{a}{2b}\right) \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}} dx} - \frac{2\sqrt{\pi} \sin\left(\frac{a}{2b}\right) \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) S\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}} dx}$$

[Out] (2\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]\*Cos[ArcCos[-1 + d\*x^2]/2]^2)/(d\*x) - (2\*Sqrt[Pi]\*Cos[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]])/(Sqrt[b^(-1)]\*d\*x) - (2\*Sqrt[Pi]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)])/(Sqrt[b^(-1)]\*d\*x)

**Rubi [A]** time = 0.0202655, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {4813}

$$\frac{2\sqrt{\pi} \cos\left(\frac{a}{2b}\right) \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}} dx} - \frac{2\sqrt{\pi} \sin\left(\frac{a}{2b}\right) \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) S\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}} dx}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*ArcCos[-1 + d\*x^2]], x]

[Out] (2\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]\*Cos[ArcCos[-1 + d\*x^2]/2]^2)/(d\*x) - (2\*Sqrt[Pi]\*Cos[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]])/(Sqrt[b^(-1)]\*d\*x) - (2\*Sqrt[Pi]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)])/(Sqrt[b^(-1)]\*d\*x)

#### Rule 4813

Int[Sqrt[(a\_.) + ArcCos[-1 + (d\_.)\*(x\_)^2]\*(b\_.)], x\_Symbol] :> Simp[(2\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]\*Cos[(1/2)\*ArcCos[-1 + d\*x^2]]^2)/(d\*x), x] + (-Simp[(2\*Sqrt[Pi]\*Cos[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelC[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]])/(Sqrt[1/b]\*d\*x), x] - Simp[(2\*Sqrt[Pi]\*Sin[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelS[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]])/(Sqrt[1/b]\*d\*x), x]) /; FreeQ[{a, b, d}, x]

#### Rubi steps

$$\int \sqrt{a + b \cos^{-1}(-1 + dx^2)} dx = \frac{2\sqrt{a + b \cos^{-1}(-1 + dx^2)} \cos^2\left(\frac{1}{2} \cos^{-1}(-1 + dx^2)\right)}{dx} - \frac{2\sqrt{\pi} \cos\left(\frac{a}{2b}\right) \cos\left(\frac{1}{2} \cos^{-1}(-1 + dx^2)\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}} dx} - \frac{2\sqrt{\pi} \sin\left(\frac{a}{2b}\right) \cos\left(\frac{1}{2} \cos^{-1}(-1 + dx^2)\right) S\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}} dx}$$

**Mathematica [A]** time = 0.112897, size = 157, normalized size = 0.85

$$\frac{2 \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) \left( \sqrt{\pi} \cos\left(\frac{a}{2b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right) + \sqrt{\pi} \sin\left(\frac{a}{2b}\right) S\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right) - \sqrt{\frac{1}{b}} \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) \right)}{\sqrt{\frac{1}{b}} dx}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*ArcCos[-1 + d\*x^2]],x]

[Out]  $(-2*\text{Cos}[\text{ArcCos}[-1 + d*x^2]/2]*(-(\text{Sqrt}[b^{(-1)}]*\text{Sqrt}[a + b*\text{ArcCos}[-1 + d*x^2]]*\text{Cos}[\text{ArcCos}[-1 + d*x^2]/2]) + \text{Sqrt}[\text{Pi}]*\text{Cos}[a/(2*b)]*\text{FresnelC}[(\text{Sqrt}[b^{(-1)}]*\text{Sqrt}[a + b*\text{ArcCos}[-1 + d*x^2]])/\text{Sqrt}[\text{Pi}]] + \text{Sqrt}[\text{Pi}]*\text{FresnelS}[(\text{Sqrt}[b^{(-1)}]*\text{Sqrt}[a + b*\text{ArcCos}[-1 + d*x^2]])/\text{Sqrt}[\text{Pi}]]*\text{Sin}[a/(2*b)]))/(\text{Sqrt}[b^{(-1)}]*d*x)$

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int \sqrt{a + b \arccos(dx^2 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos(d\*x^2-1))^(1/2),x)

[Out] int((a+b\*arccos(d\*x^2-1))^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \arccos(dx^2 - 1) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2-1))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b\*arccos(d\*x^2 - 1) + a), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2-1))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \arccos(dx^2 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x\*\*2-1))\*\*(1/2),x)



[Out] Integral(sqrt(a + b\*acos(d\*x\*\*2 - 1)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \arccos(dx^2 - 1) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos(d\*x^2-1))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b\*arccos(d\*x^2 - 1) + a), x)

$$3.97 \quad \int \frac{1}{\sqrt{a+b \cos^{-1}(-1+dx^2)}} dx$$

**Optimal.** Leaf size=145

$$\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}(dx^2-1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{dx} - \frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}(dx^2-1)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{dx}$$

[Out] (-2\*Sqrt[b^(-1)]\*Sqrt[Pi]\*Cos[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]/(d\*x) + (2\*Sqrt[b^(-1)]\*Sqrt[Pi]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)])/(d\*x)

**Rubi [A]** time = 0.0172358, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {4821}

$$\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}(dx^2-1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{dx} - \frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}(dx^2-1)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{dx}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b\*ArcCos[-1 + d\*x^2]], x]

[Out] (-2\*Sqrt[b^(-1)]\*Sqrt[Pi]\*Cos[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]/(d\*x) + (2\*Sqrt[b^(-1)]\*Sqrt[Pi]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)])/(d\*x)

**Rule 4821**

Int[1/Sqrt[(a\_.) + ArcCos[-1 + (d\_.)\*(x\_)^2]\*(b\_.)], x\_Symbol] :> Simp[(2\*Sqrt[Pi/b]\*Sin[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelC[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]]/(d\*x), x] - Simp[(2\*Sqrt[Pi/b]\*Cos[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelS[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]]/(d\*x), x] /; FreeQ[{a, b, d}, x]

**Rubi steps**

$$\int \frac{1}{\sqrt{a+b \cos^{-1}(-1+dx^2)}} dx = -\frac{2\sqrt{\frac{1}{b}}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}(-1+dx^2)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(-1+dx^2)}}{\sqrt{\pi}}\right)}{dx} + \frac{2\sqrt{\frac{1}{b}}\sqrt{\pi}\cos\left(\frac{1}{2}\cos^{-1}(-1+dx^2)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(-1+dx^2)}}{\sqrt{\pi}}\right)}{dx}$$

**Mathematica [A]** time = 0.15789, size = 115, normalized size = 0.79

$$\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{1}{2}\cos^{-1}(dx^2-1)\right)\left(\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right) - \sin\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)\right)}{dx}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b\*ArcCos[-1 + d\*x^2]],x]

[Out]  $(-2\sqrt{b^{(-1)}}*\sqrt{\pi}*\cos[\text{ArcCos}[-1 + d*x^2]/2]*(\cos[a/(2*b)])*\text{FresnelS}[\sqrt{b^{(-1)}}*\sqrt{a + b*\text{ArcCos}[-1 + d*x^2]}/\sqrt{\pi}] - \text{FresnelC}[(\sqrt{b^{(-1)}}*\sqrt{a + b*\text{ArcCos}[-1 + d*x^2]}/\sqrt{\pi})*\sin[a/(2*b)]])/d*x$

**Maple [F]** time = 0.07, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \arccos(dx^2 - 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arccos(d\*x^2-1))^(1/2),x)

[Out] int(1/(a+b\*arccos(d\*x^2-1))^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \arccos(dx^2 - 1) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b\*arccos(d\*x^2 - 1) + a), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \arccos(dx^2 - 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x\*\*2-1))\*\*(1/2),x)

[Out] Integral(1/sqrt(a + b\*acos(d\*x\*\*2 - 1)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \arccos(dx^2 - 1) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(b\*arccos(d\*x^2 - 1) + a), x)

$$3.98 \quad \int \frac{1}{(a+b \cos^{-1}(-1+dx^2))^{3/2}} dx$$

**Optimal.** Leaf size=190

$$\frac{\sqrt{2dx^2 - d^2x^4}}{bdx\sqrt{a + b \cos^{-1}(dx^2 - 1)}} - \frac{2\sqrt{\pi} \left(\frac{1}{b}\right)^{3/2} \cos\left(\frac{a}{2b}\right) \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{dx} - \frac{2\sqrt{\pi} \left(\frac{1}{b}\right)^3}{dx}$$

[Out] Sqrt[2\*d\*x^2 - d^2\*x^4]/(b\*d\*x\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]) - (2\*(b^(-1))^(3/2)\*Sqrt[Pi]\*Cos[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]])/(d\*x) - (2\*(b^(-1))^(3/2)\*Sqrt[Pi]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)])/(d\*x)

**Rubi [A]** time = 0.0239368, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {4824}

$$\frac{\sqrt{2dx^2 - d^2x^4}}{bdx\sqrt{a + b \cos^{-1}(dx^2 - 1)}} - \frac{2\sqrt{\pi} \left(\frac{1}{b}\right)^{3/2} \cos\left(\frac{a}{2b}\right) \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{dx} - \frac{2\sqrt{\pi} \left(\frac{1}{b}\right)^3}{dx}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[-1 + d\*x^2])^(-3/2), x]

[Out] Sqrt[2\*d\*x^2 - d^2\*x^4]/(b\*d\*x\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]) - (2\*(b^(-1))^(3/2)\*Sqrt[Pi]\*Cos[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]])/(d\*x) - (2\*(b^(-1))^(3/2)\*Sqrt[Pi]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)])/(d\*x)

#### Rule 4824

Int[((a\_.) + ArcCos[-1 + (d\_.)\*(x\_)^2]\*(b\_.))^(3/2), x\_Symbol] := Simp[Sqrt[2\*d\*x^2 - d^2\*x^4]/(b\*d\*x\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]), x] + (-Simp[(2\*(1/b)^(3/2)\*Sqrt[Pi]\*Cos[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelC[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]])/(d\*x), x] - Simp[(2\*(1/b)^(3/2)\*Sqrt[Pi]\*Sin[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelS[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]])/(d\*x), x]) /; FreeQ[{a, b, d}, x]

#### Rubi steps

$$\int \frac{1}{(a+b \cos^{-1}(-1+dx^2))^{3/2}} dx = \frac{\sqrt{2dx^2 - d^2x^4}}{bdx\sqrt{a + b \cos^{-1}(-1 + dx^2)}} - \frac{2\left(\frac{1}{b}\right)^{3/2} \sqrt{\pi} \cos\left(\frac{a}{2b}\right) \cos\left(\frac{1}{2} \cos^{-1}(-1 + dx^2)\right) C\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b \cos^{-1}(-1+dx^2)}}{\sqrt{\pi}}\right)}{dx}$$

**Mathematica [A]** time = 0.320724, size = 161, normalized size = 0.85

$$\frac{2 \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) \left( \sqrt{\pi} \left(-\sqrt{\frac{1}{b}}\right) \cos\left(\frac{a}{2b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right) - \sqrt{\pi} \sqrt{\frac{1}{b}} \sin\left(\frac{a}{2b}\right) \text{S}\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a+b \cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right) \right)}{bdx}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCos[-1 + d\*x^2])^(-3/2), x]

[Out] (2\*Cos[ArcCos[-1 + d\*x^2]/2]\*(-(Sqrt[b^(-1)]\*Sqrt[Pi]\*Cos[a/(2\*b)]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]) - Sqrt[b^(-1)]\*Sqrt[Pi]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)] + Sin[ArcCos[-1 + d\*x^2]/2]/Sqrt[a + b\*ArcCos[-1 + d\*x^2]]))/(b\*d\*x)

**Maple [F]** time = 0.064, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 - 1))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arccos(d\*x^2-1))^(3/2), x)

[Out] int(1/(a+b\*arccos(d\*x^2-1))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(dx^2 - 1) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^(3/2), x, algorithm="maxima")

[Out] integrate((b\*arccos(d\*x^2 - 1) + a)^(-3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^(3/2), x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \arccos(dx^2 - 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*acos(d\*x\*\*2-1))\*\*(3/2),x)

[Out] Integral((a + b\*acos(d\*x\*\*2 - 1))\*\*(-3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(dx^2 - 1) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 - 1) + a)^(-3/2), x)

$$3.99 \quad \int \frac{1}{(a+b \cos^{-1}(-1+dx^2))^{5/2}} dx$$

**Optimal.** Leaf size=221

$$\frac{x}{3b^2 \sqrt{a+b \cos^{-1}(dx^2-1)}} + \frac{\sqrt{2dx^2-d^2x^4}}{3bdx(a+b \cos^{-1}(dx^2-1))^{3/2}} - \frac{2\sqrt{\pi} \left(\frac{1}{b}\right)^{5/2} \sin\left(\frac{a}{2b}\right) \cos\left(\frac{1}{2} \cos^{-1}(dx^2-1)\right) \text{FresnelC}\left(\frac{\sqrt{2dx^2-d^2x^4}}{2b}\right)}{3dx}$$

[Out] Sqrt[2\*d\*x^2 - d^2\*x^4]/(3\*b\*d\*x\*(a + b\*ArcCos[-1 + d\*x^2])^(3/2)) + x/(3\*b^2\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]) + (2\*(b^(-1))^(5/2)\*Sqrt[Pi]\*Cos[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]])/(3\*d\*x) - (2\*(b^(-1))^(5/2)\*Sqrt[Pi]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)])/(3\*d\*x)

**Rubi [A]** time = 0.0434584, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {4829, 4821}

$$\frac{x}{3b^2 \sqrt{a+b \cos^{-1}(dx^2-1)}} + \frac{\sqrt{2dx^2-d^2x^4}}{3bdx(a+b \cos^{-1}(dx^2-1))^{3/2}} - \frac{2\sqrt{\pi} \left(\frac{1}{b}\right)^{5/2} \sin\left(\frac{a}{2b}\right) \cos\left(\frac{1}{2} \cos^{-1}(dx^2-1)\right) \text{FresnelC}\left(\frac{\sqrt{2dx^2-d^2x^4}}{2b}\right)}{3dx}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[-1 + d\*x^2])^(-5/2), x]

[Out] Sqrt[2\*d\*x^2 - d^2\*x^4]/(3\*b\*d\*x\*(a + b\*ArcCos[-1 + d\*x^2])^(3/2)) + x/(3\*b^2\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]) + (2\*(b^(-1))^(5/2)\*Sqrt[Pi]\*Cos[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]])/(3\*d\*x) - (2\*(b^(-1))^(5/2)\*Sqrt[Pi]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)])/(3\*d\*x)

#### Rule 4829

Int[((a\_.) + ArcCos[(c\_) + (d\_.)\*(x\_)^2]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(x\*(a + b\*ArcCos[c + d\*x^2])^(n + 2))/(4\*b^2\*(n + 1)\*(n + 2)), x] + (-Dist[1/(4\*b^2\*(n + 1)\*(n + 2)), Int[(a + b\*ArcCos[c + d\*x^2])^(n + 2), x], x] - Simp[(Sqrt[-2\*c\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[c + d\*x^2])^(n + 1))/(2\*b\*d\*(n + 1)\*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]

#### Rule 4821

Int[1/Sqrt[(a\_.) + ArcCos[-1 + (d\_.)\*(x\_)^2]\*(b\_.)], x\_Symbol] :> Simp[(2\*Sqrt[Pi/b]\*Sin[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelC[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]])/(d\*x), x] - Simp[(2\*Sqrt[Pi/b]\*Cos[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelS[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]])/(d\*x), x] /; FreeQ[{a, b, d}, x]

#### Rubi steps



$$\int \frac{1}{(a + b \cos^{-1}(-1 + dx^2))^{5/2}} dx = \frac{\sqrt{2dx^2 - d^2x^4}}{3bdx(a + b \cos^{-1}(-1 + dx^2))^{3/2}} + \frac{x}{3b^2 \sqrt{a + b \cos^{-1}(-1 + dx^2)}} - \frac{\int \frac{1}{\sqrt{a + b \cos^{-1}(-1 + dx^2)}} dx}{3b^2}$$

$$= \frac{\sqrt{2dx^2 - d^2x^4}}{3bdx(a + b \cos^{-1}(-1 + dx^2))^{3/2}} + \frac{x}{3b^2 \sqrt{a + b \cos^{-1}(-1 + dx^2)}} + \frac{2\left(\frac{1}{b}\right)^{5/2} \sqrt{\pi} \cos\left(\frac{a}{2b}\right)}{3b^2}$$

**Mathematica [A]** time = 0.585468, size = 233, normalized size = 1.05

$$2 \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) \left( -\sqrt{\pi} \sqrt{\frac{1}{b}} \sin\left(\frac{a}{2b}\right) (a + b \cos^{-1}(dx^2 - 1))^{3/2} \operatorname{FresnelC}\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a + b \cos^{-1}(dx^2 - 1)}}{\sqrt{\pi}}\right) + \sqrt{\pi} \sqrt{\frac{1}{b}} \cos\left(\frac{a}{2b}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCos[-1 + d\*x^2])^(-5/2), x]

[Out] (2\*Cos[ArcCos[-1 + d\*x^2]/2]\*(a\*Cos[ArcCos[-1 + d\*x^2]/2] + b\*ArcCos[-1 + d\*x^2]\*Cos[ArcCos[-1 + d\*x^2]/2] + Sqrt[b^(-1)]\*Sqrt[Pi]\*(a + b\*ArcCos[-1 + d\*x^2])^(3/2)\*Cos[a/(2\*b)]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]] - Sqrt[b^(-1)]\*Sqrt[Pi]\*(a + b\*ArcCos[-1 + d\*x^2])^(3/2)\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)] + b\*Sin[ArcCos[-1 + d\*x^2]/2]))/(3\*b^2\*d\*x\*(a + b\*ArcCos[-1 + d\*x^2])^(3/2))

**Maple [F]** time = 0.065, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 - 1))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arccos(d\*x^2-1))^(5/2), x)

[Out] int(1/(a+b\*arccos(d\*x^2-1))^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(dx^2 - 1) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^(5/2), x, algorithm="maxima")

[Out] integrate((b\*arccos(d\*x^2 - 1) + a)^(-5/2), x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \arccos(dx^2 - 1))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*acos(d\*x\*\*2-1))\*\*(5/2),x)

[Out] Integral((a + b\*acos(d\*x\*\*2 - 1))\*\*(-5/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(dx^2 - 1) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 - 1) + a)^(-5/2), x)

$$3.100 \quad \int \frac{1}{(a+b \cos^{-1}(-1+dx^2))^{7/2}} dx$$

**Optimal.** Leaf size=269

$$\frac{\sqrt{2dx^2 - d^2x^4}}{15b^3dx\sqrt{a + b \cos^{-1}(dx^2 - 1)}} + \frac{x}{15b^2(a + b \cos^{-1}(dx^2 - 1))^{3/2}} + \frac{\sqrt{2dx^2 - d^2x^4}}{5bdx(a + b \cos^{-1}(dx^2 - 1))^{5/2}} + \frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{7/2} \cos}{1}$$

[Out] Sqrt[2\*d\*x^2 - d^2\*x^4]/(5\*b\*d\*x\*(a + b\*ArcCos[-1 + d\*x^2])^(5/2)) + x/(15\*b^2\*(a + b\*ArcCos[-1 + d\*x^2])^(3/2)) - Sqrt[2\*d\*x^2 - d^2\*x^4]/(15\*b^3\*d\*x\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]) + (2\*(b^(-1))^(7/2)\*Sqrt[Pi]\*Cos[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]])/(15\*d\*x) + (2\*(b^(-1))^(7/2)\*Sqrt[Pi]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)])/(15\*d\*x)

**Rubi [A]** time = 0.0500292, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {4829, 4824}

$$\frac{\sqrt{2dx^2 - d^2x^4}}{15b^3dx\sqrt{a + b \cos^{-1}(dx^2 - 1)}} + \frac{x}{15b^2(a + b \cos^{-1}(dx^2 - 1))^{3/2}} + \frac{\sqrt{2dx^2 - d^2x^4}}{5bdx(a + b \cos^{-1}(dx^2 - 1))^{5/2}} + \frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{7/2} \cos}{1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[-1 + d\*x^2])^(-7/2), x]

[Out] Sqrt[2\*d\*x^2 - d^2\*x^4]/(5\*b\*d\*x\*(a + b\*ArcCos[-1 + d\*x^2])^(5/2)) + x/(15\*b^2\*(a + b\*ArcCos[-1 + d\*x^2])^(3/2)) - Sqrt[2\*d\*x^2 - d^2\*x^4]/(15\*b^3\*d\*x\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]) + (2\*(b^(-1))^(7/2)\*Sqrt[Pi]\*Cos[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]])/(15\*d\*x) + (2\*(b^(-1))^(7/2)\*Sqrt[Pi]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)])/(15\*d\*x)

#### Rule 4829

Int[((a\_.) + ArcCos[(c\_.) + (d\_.)\*(x\_)^2]\*(b\_.))^(n\_), x\_Symbol] := Simp[(x\*(a + b\*ArcCos[c + d\*x^2])^(n + 2))/(4\*b^2\*(n + 1)\*(n + 2)), x] + (-Dist[1/(4\*b^2\*(n + 1)\*(n + 2)), Int[(a + b\*ArcCos[c + d\*x^2])^(n + 2), x], x] - Simp[(Sqrt[-2\*c\*d\*x^2 - d^2\*x^4]\*(a + b\*ArcCos[c + d\*x^2])^(n + 1))/(2\*b\*d\*(n + 1)\*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]

#### Rule 4824

Int[((a\_.) + ArcCos[-1 + (d\_.)\*(x\_)^2]\*(b\_.))^(n\_), x\_Symbol] := Simp[Sqrt[2\*d\*x^2 - d^2\*x^4]/(b\*d\*x\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]), x] + (-Simp[(2\*(1/b)^(3/2)\*Sqrt[Pi]\*Cos[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelC[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]]/(d\*x), x] - Simp[(2\*(1/b)^(3/2)\*Sqrt[Pi]\*Sin[a/(2\*b)]\*Cos[ArcCos[-1 + d\*x^2]/2]\*FresnelS[Sqrt[1/(Pi\*b)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]]]/(d\*x), x]) /; FreeQ[{a, b, d}, x]

Rubi steps

$$\int \frac{1}{(a + b \cos^{-1}(-1 + dx^2))^{7/2}} dx = \frac{\sqrt{2dx^2 - d^2x^4}}{5bdx(a + b \cos^{-1}(-1 + dx^2))^{5/2}} + \frac{x}{15b^2(a + b \cos^{-1}(-1 + dx^2))^{3/2}} - \frac{\int \frac{1}{(a + b \cos^{-1}(-1 + dx^2))^{5/2}} dx}{15b^2}$$

$$= \frac{\sqrt{2dx^2 - d^2x^4}}{5bdx(a + b \cos^{-1}(-1 + dx^2))^{5/2}} + \frac{x}{15b^2(a + b \cos^{-1}(-1 + dx^2))^{3/2}} - \frac{\sqrt{2dx^2 - d^2x^4}}{15b^3dx\sqrt{a + b \cos^{-1}(-1 + dx^2)}}$$

**Mathematica [A]** time = 0.553429, size = 309, normalized size = 1.15

$$2 \cos\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) \left( -a^2 \sin\left(\frac{1}{2} \cos^{-1}(dx^2 - 1)\right) + \sqrt{\pi} \sqrt{\frac{1}{b}} \cos\left(\frac{a}{2b}\right) (a + b \cos^{-1}(dx^2 - 1))^{5/2} \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}} \sqrt{a + b \cos^{-1}(dx^2 - 1)}}{\sqrt{\pi}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCos[-1 + d\*x^2])^(-7/2), x]

[Out] (2\*Cos[ArcCos[-1 + d\*x^2]/2]\*(a\*b\*Cos[ArcCos[-1 + d\*x^2]/2] + b^2\*ArcCos[-1 + d\*x^2]\*Cos[ArcCos[-1 + d\*x^2]/2] + Sqrt[b^(-1)]\*Sqrt[Pi]\*(a + b\*ArcCos[-1 + d\*x^2])^(5/2)\*Cos[a/(2\*b)]\*FresnelC[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]] + Sqrt[b^(-1)]\*Sqrt[Pi]\*(a + b\*ArcCos[-1 + d\*x^2])^(5/2)\*FresnelS[(Sqrt[b^(-1)]\*Sqrt[a + b\*ArcCos[-1 + d\*x^2]])/Sqrt[Pi]]\*Sin[a/(2\*b)] - a^2\*Sin[ArcCos[-1 + d\*x^2]/2] + 3\*b^2\*Sin[ArcCos[-1 + d\*x^2]/2] - 2\*a\*b\*ArcCos[-1 + d\*x^2]\*Sin[ArcCos[-1 + d\*x^2]/2] - b^2\*ArcCos[-1 + d\*x^2]^2\*Sin[ArcCos[-1 + d\*x^2]/2]))/(15\*b^3\*d\*x\*(a + b\*ArcCos[-1 + d\*x^2])^(5/2))

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int (a + b \arccos(dx^2 - 1))^{-7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arccos(d\*x^2-1))^(7/2), x)

[Out] int(1/(a+b\*arccos(d\*x^2-1))^(7/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(dx^2 - 1) + a)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^(7/2), x, algorithm="maxima")

[Out] integrate((b\*arccos(d\*x^2 - 1) + a)^(-7/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^(7/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*acos(d\*x\*\*2-1))\*\*(7/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(dx^2 - 1) + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccos(d\*x^2-1))^(7/2),x, algorithm="giac")

[Out] integrate((b\*arccos(d\*x^2 - 1) + a)^(-7/2), x)

$$\mathbf{3.101} \quad \int \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

**Optimal.** Leaf size=42

$$\text{Unintegrable}\left(\frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}, x\right)$$

[Out] Unintegrable[(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^n/(1 - c^2\*x^2), x]

**Rubi [A]** time = 0.0437305, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^n/(1 - c^2\*x^2), x]

[Out] Defer[Int][(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^n/(1 - c^2\*x^2), x]

Rubi steps

$$\int \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx = \int \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

**Mathematica [A]** time = 0.0972344, size = 0, normalized size = 0.

$$\int \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^n/(1 - c^2\*x^2), x]

[Out] Integrate[(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^n/(1 - c^2\*x^2), x]

**Maple [A]** time = 0.647, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2 + 1} \left(a + b \arccos\left(\sqrt{-cx + 1} \frac{1}{\sqrt{cx + 1}}\right)\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)`

[Out] `int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(b \arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^n}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorithm="maxima")`

[Out] `-integrate((b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b \arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^n}{c^2x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorithm="fricas")`

[Out] `integral(-(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b \arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^n}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorithm="giac")`

```
[Out] integrate(-(b*arccos(sqrt(-c*x + 1))/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```



$$3.102 \quad \int \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2} dx$$

**Optimal.** Leaf size=279

$$\frac{3b^2 \text{PolyLog}\left(3, -e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{2c} + \frac{3ib \text{PolyLog}\left(2, -e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2c}$$

[Out] ((I/4)\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^4)/(b\*c) - ((a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^3\*Log[1 + E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/c + (((3\*I)/2)\*b\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2\*PolyLog[2, -E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/c - (3\*b^2\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])\*PolyLog[3, -E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/((2\*c) - (((3\*I)/4)\*b^3\*PolyLog[4, -E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/c

**Rubi [A]** time = 0.207158, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {6681, 4626, 3719, 2190, 2531, 6609, 2282, 6589}

$$\frac{3b^2 \text{PolyLog}\left(3, -e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{2c} + \frac{3ib \text{PolyLog}\left(2, -e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^3/(1 - c^2\*x^2), x]

[Out] ((I/4)\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^4)/(b\*c) - ((a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^3\*Log[1 + E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/c + (((3\*I)/2)\*b\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2\*PolyLog[2, -E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/c - (3\*b^2\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])\*PolyLog[3, -E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/((2\*c) - (((3\*I)/4)\*b^3\*PolyLog[4, -E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/c

#### Rule 6681

Int[((a\_.) + (b\_.)\*(F\_)[((c\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_)])/Sqrt[(f\_.) + (g\_.)\*(x\_)])^(n\_.)/((A\_.) + (C\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + b\*F[c\*x])^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[e\*f + d\*g, 0] && IGtQ[n, 0]

#### Rule 4626

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.)^(n\_.)/(x\_), x\_Symbol] :> -Subst[Int[(a + b\*x)^n/Cot[x], x], x, ArcCos[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

#### Rule 3719

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))\*tan[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[(c + d\*x)^m\*E^(2\*I\*(e

+ f\*x)))/(1 + E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(n\_)]\*((f\_) + (g\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n])]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

#### Rule 6609

Int[((e\_) + (f\_)\*(x\_))^(m\_)\*PolyLog[n\_, (d\_)\*((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(p\_)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\* (F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2} dx &= -\frac{\text{Subst}\left(\int \frac{(a+b \cos^{-1}(x))^3}{x} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\
&= \frac{\text{Subst}\left(\int (a+bx)^3 \tan(x) dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c} \\
&= \frac{i\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{(2i) \text{Subst}\left(\int \frac{e^{2ix}(a+bx)^3}{1+e^{2ix}} dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c} \\
&= \frac{i\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \log\left(1 + e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{(3b) \text{Subst}\left(\int \frac{e^{2ix}(a+bx)^3}{1+e^{2ix}} dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c} \\
&= \frac{i\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \log\left(1 + e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{3ib\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{c} \\
&= \frac{i\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \log\left(1 + e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{3ib\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{c} \\
&= \frac{i\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \log\left(1 + e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{3ib\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{c} \\
&= \frac{i\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \log\left(1 + e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{3ib\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{c}
\end{aligned}$$

**Mathematica [F]** time = 0.296082, size = 0, normalized size = 0.

$$\int \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^3/(1 - c^2\*x^2), x]

[Out] Integrate[(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^3/(1 - c^2\*x^2), x]

**Maple [B]** time = 0.974, size = 707, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^3/(-c^2\*x^2+1), x)

[Out]  $\frac{1}{2}a^3/c \ln(c*x+1) - \frac{1}{2}a^3/c \ln(c*x-1) + \frac{1}{4}I*b^3/c \arccos((-c*x+1)^{1/2}/(c*x+1)^{1/2})^4 - \frac{b^3}{c} \arccos((-c*x+1)^{1/2}/(c*x+1)^{1/2})^3 \ln(1 + ((-c*x+1)^{1/2}/(c*x+1)^{1/2}) + I*(1 - (-c*x+1)/(c*x+1))^{1/2})^2 + \frac{3}{2}I*b^3/c \arccos((-c*x+1)^{1/2}/(c*x+1)^{1/2})^2 \text{polylog}(2, -((-c*x+1)^{1/2}/(c*x+1)^{1/2}) + I*(1 - (-c*x+1)/(c*x+1))^{1/2})^2 - \frac{3}{2}I*b^3/c \arccos((-c*x+1)^{1/2}/(c*x+1)^{1/2})$

```
*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)-
3/4*I*b^3*polylog(4,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(
1/2))^2)/c+I*a*b^2/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3-3*a*b^2/c*arcco
s((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-
c*x+1)/(c*x+1))^(1/2))^2)+3*I*a*b^2/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))
*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)-
3/2*a*b^2/c*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))
^(1/2))^2)+3/2*I*a^2*b/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2-3*a^2*b/c*a
rccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1
-(-c*x+1)/(c*x+1))^(1/2))^2)+3/2*I*a^2*b/c*polylog(2,-((-c*x+1)^(1/2)/(c*x+
1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} a^3 \left( \frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c} \right) - \int \frac{b^3 \arctan(\sqrt{2}\sqrt{c}\sqrt{x}, \sqrt{-cx+1})^3 + 3ab^2 \arctan(\sqrt{2}\sqrt{c}\sqrt{x}, \sqrt{-cx+1})^2 + 3a^2b \arctan(\sqrt{2}\sqrt{c}\sqrt{x}, \sqrt{-cx+1})}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) - integrate((b^3*arctan2(sqrt(2)*
sqrt(c)*sqrt(x), sqrt(-c*x + 1))^3 + 3*a*b^2*arctan2(sqrt(2)*sqrt(c)*sqrt(x)
), sqrt(-c*x + 1))^2 + 3*a^2*b*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x +
1)))/(c^2*x^2 - 1), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{b^3 \arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3 + 3ab^2 \arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 3a^2b \arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a^3}{c^2x^2 - 1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^3*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arccos(sqrt
(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1))
+ a^3)/(c^2*x^2 - 1), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

[Out] Timed out

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^3/(-c^2\*x^2+1),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.103 \quad \int \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2} dx$$

**Optimal.** Leaf size=207

$$\frac{ib \operatorname{PolyLog}\left(2, -e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right) \left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c} - \frac{b^2 \operatorname{PolyLog}\left(3, -e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c} + \frac{i \left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc} - \log$$

[Out] ((I/3)\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^3)/(b\*c) - ((a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2\*Log[1 + E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/c + (I\*b\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])\*PolyLog[2, -E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/c - (b^2\*PolyLog[3, -E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/(2\*c)

**Rubi [A]** time = 0.174358, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$ , Rules used = {6681, 4626, 3719, 2190, 2531, 2282, 6589}

$$\frac{ib \operatorname{PolyLog}\left(2, -e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right) \left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c} - \frac{b^2 \operatorname{PolyLog}\left(3, -e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c} + \frac{i \left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc} - \log$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2/(1 - c^2\*x^2), x]

[Out] ((I/3)\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^3)/(b\*c) - ((a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2\*Log[1 + E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/c + (I\*b\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])\*PolyLog[2, -E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/c - (b^2\*PolyLog[3, -E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/(2\*c)

#### Rule 6681

Int[((a\_.) + (b\_.)\*(F\_)[((c\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_)])/Sqrt[(f\_.) + (g\_.)\*(x\_)])^(n\_.)/((A\_.) + (C\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + b\*F[c\*x])^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[e\*f + d\*g, 0] && IGtQ[n, 0]

#### Rule 4626

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.)^(n\_.)/(x\_), x\_Symbol] := -Subst[Int[(a + b\*x)^n/Cot[x], x], x, ArcCos[c\*x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

#### Rule 3719

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))\*tan[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1 - c^2x^2} dx = \frac{\text{Subst}\left(\int \frac{(a+b \cos^{-1}(x))^2}{x} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= \frac{\text{Subst}\left(\int (a+bx)^2 \tan(x) dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{i\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{(2i) \text{Subst}\left(\int \frac{e^{2ix}(a+bx)^2}{1+e^{2ix}} dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{i\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \log\left(1 + e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{(2b) \text{Subst}\left(\int \frac{e^{2ix}(a+bx)^2}{1+e^{2ix}} dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{i\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \log\left(1 + e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{ib\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{c}$$

$$= \frac{i\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \log\left(1 + e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{ib\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{c}$$

$$= \frac{i\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \log\left(1 + e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{ib\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{c}$$

**Mathematica [F]** time = 0.695027, size = 0, normalized size = 0.

$$\int \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1 - c^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2/(1 - c^2\*x^2), x]

[Out] Integrate[(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2/(1 - c^2\*x^2), x]

**Maple [A]** time = 0.007, size = 401, normalized size = 1.9

$$\frac{a^2 \ln(cx+1)}{2c} - \frac{a^2 \ln(cx-1)}{2c} + \frac{i}{3} \frac{b^2}{c} \left( \arccos\left(\sqrt{-cx+1} \frac{1}{\sqrt{cx+1}}\right) \right)^3 - \frac{b^2}{c} \left( \arccos\left(\sqrt{-cx+1} \frac{1}{\sqrt{cx+1}}\right) \right)^2 \ln\left(1 + \left(\sqrt{-cx+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2/(-c^2\*x^2+1), x)

[Out] 1/2\*a^2/c\*ln(c\*x+1)-1/2\*a^2/c\*ln(c\*x-1)+1/3\*I\*b^2/c\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))^3-b^2/c\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))^2\*ln(1+((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)+I\*(1-(-c\*x+1)/(c\*x+1))^(1/2))^2)+I\*b^2/c\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))\*polylog(2,-((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)+I\*(1-(-c\*x+1)/(c\*x+1))^(1/2))^2)-1/2\*b^2\*polylog(3,-((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)+I\*(1-(-c\*x+1)/(c\*x+1))^(1/2))^2)/c+I\*a\*b/c\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))^2-2\*a\*b/c\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))\*ln(1+((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)+I\*(1-(-c\*x+1)/(c\*x+1))^(1/2))^2)+I\*a\*b/c\*polylog(2,-((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)+I\*(1-(-c\*x+1)/(c\*x+1))^(1/2))^2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} a^2 \left( \frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c} \right) - \int \frac{b^2 \arctan\left(\sqrt{2}\sqrt{c}\sqrt{x}, \sqrt{-cx+1}\right)^2 + 2ab \arctan\left(\sqrt{2}\sqrt{c}\sqrt{x}, \sqrt{-cx+1}\right)}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2/(-c^2\*x^2+1), x, algorithm="maxima")

[Out] 1/2\*a^2\*(log(c\*x + 1)/c - log(c\*x - 1)/c) - integrate((b^2\*arctan2(sqrt(2)\*sqrt(c)\*sqrt(x), sqrt(-c\*x + 1))^2 + 2\*a\*b\*arctan2(sqrt(2)\*sqrt(c)\*sqrt(x), sqrt(-c\*x + 1)))/(c^2\*x^2 - 1), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^2 \arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 2ab \arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a^2}{c^2x^2 - 1}, x\right)$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^2*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arccos(sqrt(-
c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.104 \quad \int \frac{a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2} dx$$

**Optimal.** Leaf size=141

$$\frac{ibPolyLog\left(2, -e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} + \frac{i\left(a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\log\left(1+e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)\left(a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

[Out] ((I/2)\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2)/(b\*c) - ((a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])\*Log[1 + E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/c + ((I/2)\*b\*PolyLog[2, -E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/c

**Rubi [A]** time = 0.104312, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$ , Rules used = {206, 6681, 4626, 3719, 2190, 2279, 2391}

$$\frac{ibPolyLog\left(2, -e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} + \frac{i\left(a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\log\left(1+e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)\left(a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])/(1 - c^2\*x^2), x]

[Out] ((I/2)\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2)/(b\*c) - ((a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])\*Log[1 + E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/c + ((I/2)\*b\*PolyLog[2, -E^((2\*I)\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])])/c

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 6681

Int[((a\_) + (b\_)\*(F\_)[((c\_)\*Sqrt[(d\_) + (e\_)\*(x\_)])/Sqrt[(f\_) + (g\_)\*(x\_)])^(n\_)]/(A\_) + (C\_)\*(x\_)^2, x\_Symbol] := Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + b\*F[c\*x])^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[e\*f + d\*g, 0] && IGtQ[n, 0]

### Rule 4626

Int[((a\_) + ArcCos[(c\_)\*(x\_)])\*(b\_)^(n\_)]/(x\_), x\_Symbol] := -Subst[Int[(a + b\*x)^n/Cot[x], x], x, ArcCos[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

### Rule 3719

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*tan[(e\_) + (f\_)\*(x\_)], x\_Symbol] := Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[(c + d\*x)^m\*E^(2\*I\*(e

+ f\*x)))/(1 + E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned} \int \frac{a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1 - c^2x^2} dx &= -\frac{\text{Subst}\left(\int \frac{a+b \cos^{-1}(x)}{x} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\ &= \frac{\text{Subst}\left(\int (a + bx) \tan(x) dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c} \\ &= \frac{i\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{(2i) \text{Subst}\left(\int \frac{e^{2ix(a+bx)}}{1+e^{2ix}} dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c} \\ &= \frac{i\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right) \log\left(1 + e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b \text{Subst}\left(\int \log\left(1 + e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right) dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c} \\ &= \frac{i\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right) \log\left(1 + e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{(ib) \text{Subst}\left(\int \log\left(1 + e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right) dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c} \\ &= \frac{i\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right) \log\left(1 + e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{ib \text{Li}_2\left(-e^{2i \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} \end{aligned}$$

**Mathematica [F]** time = 0.372158, size = 0, normalized size = 0.

$$\int \frac{a + b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1 - c^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])/(1 - c^2\*x^2), x]

[Out] Integrate[(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])/(1 - c^2\*x^2), x]

---

**Maple [A]** time = 0.005, size = 171, normalized size = 1.2

$$\frac{a \ln(cx+1)}{2c} - \frac{a \ln(cx-1)}{2c} + \frac{i}{2} \frac{b}{c} \left( \arccos \left( \sqrt{-cx+1} \frac{1}{\sqrt{cx+1}} \right) \right)^2 - \frac{b}{c} \arccos \left( \sqrt{-cx+1} \frac{1}{\sqrt{cx+1}} \right) \ln \left( 1 + \left( \sqrt{-cx+1} \frac{1}{\sqrt{cx+1}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))/(-c^2\*x^2+1),x)

[Out] 1/2\*a/c\*ln(c\*x+1)-1/2\*a/c\*ln(c\*x-1)+1/2\*I\*b/c\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))^2-b/c\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))\*ln(1+((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))+I\*(1-(-c\*x+1)/(c\*x+1))^(1/2))^2+1/2\*I\*b\*polylog(2,-((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)+I\*(1-(-c\*x+1)/(c\*x+1))^(1/2))^2)/c

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} a \left( \frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c} \right) - b \int \frac{\arctan(\sqrt{2}\sqrt{c}\sqrt{x}, \sqrt{-cx+1})}{c^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))/(-c^2\*x^2+1),x, algorithm="maxima")

[Out] 1/2\*a\*(log(c\*x + 1)/c - log(c\*x - 1)/c) - b\*integrate(arctan2(sqrt(2)\*sqrt(c)\*sqrt(x), sqrt(-c\*x + 1))/(c^2\*x^2 - 1), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{b \arccos \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a}{c^2x^2 - 1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))/(-c^2\*x^2+1),x, algorithm="fricas")

[Out] integral(-(b\*arccos(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)/(c^2\*x^2 - 1), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acos((-c\*x+1)\*\*(1/2)/(c\*x+1)\*\*(1/2)))/(-c\*\*2\*x\*\*2+1),x)

[Out] Timed out

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))/(-c^2\*x^2+1),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.105 \quad \int \frac{1}{(1-c^2x^2)\left(a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$$

**Optimal.** Leaf size=42

$$\text{Unintegrable}\left(\frac{1}{(1-c^2x^2)\left(a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}, x\right)$$

[Out] Unintegrable[1/((1 - c^2\*x^2)\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])), x]

**Rubi [A]** time = 0.0420346, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((1 - c^2\*x^2)\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])), x]

[Out] Defer[Int][1/((1 - c^2\*x^2)\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])), x]

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx = \int \frac{1}{(1-c^2x^2)\left(a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$$

**Mathematica [A]** time = 0.100509, size = 0, normalized size = 0.

$$\int \frac{1}{(1-c^2x^2)\left(a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((1 - c^2\*x^2)\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])), x]

[Out] Integrate[1/((1 - c^2\*x^2)\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])), x]

**Maple [A]** time = 0.344, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2 + 1} \left( a + b \arccos \left( \sqrt{-cx + 1} \frac{1}{\sqrt{cx + 1}} \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-c^2\*x^2+1)/(a+b\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))),x)

[Out] int(1/(-c^2\*x^2+1)/(a+b\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(c^2x^2 - 1)\left(b \arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))),x, algorithm="maxima")

[Out] -integrate(1/((c^2\*x^2 - 1)\*(b\*arccos(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{ac^2x^2 + (bc^2x^2 - b) \arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))),x, algorithm="fricas")

[Out] integral(-1/(a\*c^2\*x^2 + (b\*c^2\*x^2 - b)\*arccos(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) - a), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c\*\*2\*x\*\*2+1)/(a+b\*acos((-c\*x+1)\*\*(1/2)/(c\*x+1)\*\*(1/2))),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(c^2x^2 - 1)\left(b \arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo  
rithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),  
x)
```



$$3.106 \quad \int \frac{1}{(1-c^2x^2)\left(a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2} dx$$

**Optimal.** Leaf size=42

$$\text{Unintegrable}\left(\frac{1}{(1-c^2x^2)\left(a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}, x\right)$$

[Out] Unintegrable[1/((1 - c^2\*x^2)\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2), x]

**Rubi [A]** time = 0.040541, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((1 - c^2\*x^2)\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2), x]

[Out] Defer[Int][1/((1 - c^2\*x^2)\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2), x]

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2} dx = \int \frac{1}{(1-c^2x^2)\left(a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2} dx$$

**Mathematica [A]** time = 1.6925, size = 0, normalized size = 0.

$$\int \frac{1}{(1-c^2x^2)\left(a+b \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((1 - c^2\*x^2)\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2), x]

[Out] Integrate[1/((1 - c^2\*x^2)\*(a + b\*ArcCos[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2), x]

**Maple [A]** time = 0.342, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2 + 1} \left( a + b \arccos \left( \sqrt{-cx + 1} \frac{1}{\sqrt{cx + 1}} \right) \right)^{-2} dx$$



[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(c^2x^2 - 1)\left(b \arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*arccos((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2,x, algorithm="giac")

[Out] integrate(-1/((c^2\*x^2 - 1)\*(b\*arccos(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)^2), x)

### 3.107 $\int \cos^{-1}(ce^{a+bx}) dx$

**Optimal.** Leaf size=84

$$-\frac{i \operatorname{PolyLog}\left(2, -e^{2i \cos^{-1}(ce^{a+bx})}\right)}{2b} - \frac{i \cos^{-1}(ce^{a+bx})^2}{2b} + \frac{\cos^{-1}(ce^{a+bx}) \log\left(1 + e^{2i \cos^{-1}(ce^{a+bx})}\right)}{b}$$

[Out]  $((-I/2)*\operatorname{ArcCos}[c*E^{(a + b*x)}]^2)/b + (\operatorname{ArcCos}[c*E^{(a + b*x)}]*\operatorname{Log}[1 + E^{((2*I)*\operatorname{ArcCos}[c*E^{(a + b*x)})}]])/b - ((I/2)*\operatorname{PolyLog}[2, -E^{((2*I)*\operatorname{ArcCos}[c*E^{(a + b*x)})}]])/b$

**Rubi [A]** time = 0.0663046, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {2282, 4626, 3719, 2190, 2279, 2391}

$$-\frac{i \operatorname{PolyLog}\left(2, -e^{2i \cos^{-1}(ce^{a+bx})}\right)}{2b} - \frac{i \cos^{-1}(ce^{a+bx})^2}{2b} + \frac{\cos^{-1}(ce^{a+bx}) \log\left(1 + e^{2i \cos^{-1}(ce^{a+bx})}\right)}{b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{ArcCos}[c*E^{(a + b*x)}], x]$

[Out]  $((-I/2)*\operatorname{ArcCos}[c*E^{(a + b*x)}]^2)/b + (\operatorname{ArcCos}[c*E^{(a + b*x)}]*\operatorname{Log}[1 + E^{((2*I)*\operatorname{ArcCos}[c*E^{(a + b*x)})}]])/b - ((I/2)*\operatorname{PolyLog}[2, -E^{((2*I)*\operatorname{ArcCos}[c*E^{(a + b*x)})}]])/b$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.)^(n_.)/(x_), x_Symbol] := -Subst[Int[(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

#### Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^(n)], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{-1}(ce^{a+bx}) dx &= \frac{\text{Subst}\left(\int \frac{\cos^{-1}(cx)}{x} dx, x, e^{a+bx}\right)}{b} \\
&= -\frac{\text{Subst}\left(\int x \tan(x) dx, x, \cos^{-1}(ce^{a+bx})\right)}{b} \\
&= -\frac{i \cos^{-1}(ce^{a+bx})^2}{2b} + \frac{(2i) \text{Subst}\left(\int \frac{e^{2ix}}{1+e^{2ix}} dx, x, \cos^{-1}(ce^{a+bx})\right)}{b} \\
&= -\frac{i \cos^{-1}(ce^{a+bx})^2}{2b} + \frac{\cos^{-1}(ce^{a+bx}) \log\left(1 + e^{2i \cos^{-1}(ce^{a+bx})}\right)}{b} - \frac{\text{Subst}\left(\int \log(1 + e^{2ix}) dx, x, \cos^{-1}(ce^{a+bx})\right)}{b} \\
&= -\frac{i \cos^{-1}(ce^{a+bx})^2}{2b} + \frac{\cos^{-1}(ce^{a+bx}) \log\left(1 + e^{2i \cos^{-1}(ce^{a+bx})}\right)}{b} + \frac{i \text{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^{2i \cos^{-1}(ce^{a+bx})}\right)}{2b} \\
&= -\frac{i \cos^{-1}(ce^{a+bx})^2}{2b} + \frac{\cos^{-1}(ce^{a+bx}) \log\left(1 + e^{2i \cos^{-1}(ce^{a+bx})}\right)}{b} - \frac{i \text{Li}_2\left(-e^{2i \cos^{-1}(ce^{a+bx})}\right)}{2b}
\end{aligned}$$

**Mathematica [F]** time = 0.947438, size = 0, normalized size = 0.

$$\int \cos^{-1}(ce^{a+bx}) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCos[c*E^(a + b*x)], x]
```

```
[Out] Integrate[ArcCos[c*E^(a + b*x)], x]
```

**Maple [A]** time = 0.004, size = 107, normalized size = 1.3

$$\frac{-\frac{i}{2} \left(\arccos(ce^{bx+a})\right)^2}{b} + \frac{\arccos(ce^{bx+a})}{b} \ln\left(1 + \left(ce^{bx+a} + i\sqrt{1-c^2(e^{bx+a})^2}\right)^2\right) - \frac{i}{b} \text{polylog}\left(2, -\left(ce^{bx+a} + i\sqrt{1-c^2(e^{bx+a})^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(c*exp(b*x+a)), x)
```

```
[Out] -1/2*I*arccos(c*exp(b*x+a))^2/b+arccos(c*exp(b*x+a))*ln(1+(c*exp(b*x+a)+I*(
1-c^2*exp(b*x+a)^2)^(1/2))^2)/b-1/2*I*polylog(2,-(c*exp(b*x+a)+I*(1-c^2*exp
(b*x+a)^2)^(1/2))^2)/b
```

---

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(c\*exp(b\*x+a)),x, algorithm="maxima")

[Out] Timed out

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(c\*exp(b\*x+a)),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \arccos(ce^{a+bx}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(c\*exp(b\*x+a)),x)

[Out] Integral(acos(c\*exp(a + b\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \arccos(ce^{(bx+a)}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(c\*exp(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccos(c\*e^(b\*x + a)), x)

### 3.108 $\int e^{\cos^{-1}(ax)} x^3 dx$

**Optimal.** Leaf size=81

$$\frac{e^{\cos^{-1}(ax)} \cos(2 \cos^{-1}(ax))}{10a^4} + \frac{e^{\cos^{-1}(ax)} \cos(4 \cos^{-1}(ax))}{34a^4} - \frac{e^{\cos^{-1}(ax)} \sin(2 \cos^{-1}(ax))}{20a^4} - \frac{e^{\cos^{-1}(ax)} \sin(4 \cos^{-1}(ax))}{136a^4}$$

[Out] (E^ArcCos[a\*x]\*Cos[2\*ArcCos[a\*x]])/(10\*a^4) + (E^ArcCos[a\*x]\*Cos[4\*ArcCos[a\*x]])/(34\*a^4) - (E^ArcCos[a\*x]\*Sin[2\*ArcCos[a\*x]])/(20\*a^4) - (E^ArcCos[a\*x]\*Sin[4\*ArcCos[a\*x]])/(136\*a^4)

**Rubi [A]** time = 0.0643541, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4837, 12, 4469, 4432}

$$\frac{e^{\cos^{-1}(ax)} \cos(2 \cos^{-1}(ax))}{10a^4} + \frac{e^{\cos^{-1}(ax)} \cos(4 \cos^{-1}(ax))}{34a^4} - \frac{e^{\cos^{-1}(ax)} \sin(2 \cos^{-1}(ax))}{20a^4} - \frac{e^{\cos^{-1}(ax)} \sin(4 \cos^{-1}(ax))}{136a^4}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCos[a\*x]\*x^3,x]

[Out] (E^ArcCos[a\*x]\*Cos[2\*ArcCos[a\*x]])/(10\*a^4) + (E^ArcCos[a\*x]\*Cos[4\*ArcCos[a\*x]])/(34\*a^4) - (E^ArcCos[a\*x]\*Sin[2\*ArcCos[a\*x]])/(20\*a^4) - (E^ArcCos[a\*x]\*Sin[4\*ArcCos[a\*x]])/(136\*a^4)

#### Rule 4837

Int[(u\_)\*(f\_)^(ArcCos[(a\_.) + (b\_.)\*(x\_)]^(n\_)\*(c\_.)), x\_Symbol] :> -Dist[b^(-1), Subst[Int[(u /. x -> -(a/b) + Cos[x]/b)\*f^(c\*x^n)\*Sin[x], x], x, ArcCos[a + b\*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 4469

Int[Cos[(f\_.) + (g\_.)\*(x\_)]^(n\_)\*(F\_)^(c\_)\*((a\_.) + (b\_.)\*(x\_))\*Sin[(d\_.) + (e\_.)\*(x\_)]^(m\_), x\_Symbol] :> Int[ExpandTrigReduce[F^(c\*(a + b\*x)), Sin[d + e\*x]^m\*Cos[f + g\*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0]

#### Rule 4432

Int[(F\_)^(c\_)\*((a\_.) + (b\_.)\*(x\_))\*Sin[(d\_.) + (e\_.)\*(x\_)], x\_Symbol] :> Simp[(b\*c\*Log[F]\*F^(c\*(a + b\*x))\*Sin[d + e\*x]/(e^2 + b^2\*c^2\*Log[F]^2), x] - Simp[(e\*F^(c\*(a + b\*x))\*Cos[d + e\*x]/(e^2 + b^2\*c^2\*Log[F]^2), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2\*c^2\*Log[F]^2, 0]

#### Rubi steps

$$\begin{aligned}
\int e^{\cos^{-1}(ax)} x^3 dx &= -\frac{\text{Subst}\left(\int \frac{e^x \cos^3(x) \sin(x)}{a^3} dx, x, \cos^{-1}(ax)\right)}{a} \\
&= -\frac{\text{Subst}\left(\int e^x \cos^3(x) \sin(x) dx, x, \cos^{-1}(ax)\right)}{a^4} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{4}e^x \sin(2x) + \frac{1}{8}e^x \sin(4x)\right) dx, x, \cos^{-1}(ax)\right)}{a^4} \\
&= -\frac{\text{Subst}\left(\int e^x \sin(4x) dx, x, \cos^{-1}(ax)\right)}{8a^4} - \frac{\text{Subst}\left(\int e^x \sin(2x) dx, x, \cos^{-1}(ax)\right)}{4a^4} \\
&= \frac{e^{\cos^{-1}(ax)} \cos\left(2 \cos^{-1}(ax)\right)}{10a^4} + \frac{e^{\cos^{-1}(ax)} \cos\left(4 \cos^{-1}(ax)\right)}{34a^4} - \frac{e^{\cos^{-1}(ax)} \sin\left(2 \cos^{-1}(ax)\right)}{20a^4} - \frac{e^{\cos^{-1}(ax)} \sin\left(4 \cos^{-1}(ax)\right)}{136a^4}
\end{aligned}$$

**Mathematica [A]** time = 0.139171, size = 50, normalized size = 0.62

$$\frac{e^{\cos^{-1}(ax)} \left(-68 \cos\left(2 \cos^{-1}(ax)\right) - 20 \cos\left(4 \cos^{-1}(ax)\right) + 34 \sin\left(2 \cos^{-1}(ax)\right) + 5 \sin\left(4 \cos^{-1}(ax)\right)\right)}{680a^4}$$

Antiderivative was successfully verified.

[In] Integrate[E^ArcCos[a\*x]\*x^3,x]

[Out] -(E^ArcCos[a\*x]\*(-68\*Cos[2\*ArcCos[a\*x]] - 20\*Cos[4\*ArcCos[a\*x]] + 34\*Sin[2\*ArcCos[a\*x]] + 5\*Sin[4\*ArcCos[a\*x]]))/(680\*a^4)

**Maple [F]** time = 0.01, size = 0, normalized size = 0.

$$\int e^{\arccos(ax)} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(arccos(a\*x))\*x^3,x)

[Out] int(exp(arccos(a\*x))\*x^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 e^{\arccos(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arccos(a\*x))\*x^3,x, algorithm="maxima")

[Out] integrate(x^3\*e^(arccos(a\*x)), x)

**Fricas [A]** time = 2.61862, size = 130, normalized size = 1.6

$$\frac{\left(20 a^4 x^4 - 3 a^2 x^2 - \left(5 a^3 x^3 + 6 a x\right) \sqrt{-a^2 x^2 + 1} - 6\right) e^{\arccos(ax)}}{85 a^4}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arccos(a\*x))\*x^3,x, algorithm="fricas")

[Out]  $\frac{1}{85}*(20*a^4*x^4 - 3*a^2*x^2 - (5*a^3*x^3 + 6*a*x)*\sqrt{-a^2*x^2 + 1} - 6)*e^{\arccos(a*x)}/a^4$

**Sympy [A]** time = 4.83622, size = 105, normalized size = 1.3

$$\begin{cases} \frac{4x^4 e^{\arccos(ax)}}{17} - \frac{x^3 \sqrt{-a^2x^2+1} e^{\arccos(ax)}}{17a} - \frac{3x^2 e^{\arccos(ax)}}{85a^2} - \frac{6x \sqrt{-a^2x^2+1} e^{\arccos(ax)}}{85a^3} - \frac{6e^{\arccos(ax)}}{85a^4} & \text{for } a \neq 0 \\ \frac{x^4 e^{\frac{\pi}{2}}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(acos(a\*x))\*x\*\*3,x)

[Out] Piecewise((4\*x\*\*4\*exp(acos(a\*x))/17 - x\*\*3\*sqrt(-a\*\*2\*x\*\*2 + 1)\*exp(acos(a\*x))/(17\*a) - 3\*x\*\*2\*exp(acos(a\*x))/(85\*a\*\*2) - 6\*x\*sqrt(-a\*\*2\*x\*\*2 + 1)\*exp(acos(a\*x))/(85\*a\*\*3) - 6\*exp(acos(a\*x))/(85\*a\*\*4), Ne(a, 0)), (x\*\*4\*exp(pi/2)/4, True))

**Giac [A]** time = 1.32475, size = 111, normalized size = 1.37

$$\frac{4}{17} x^4 e^{\arccos(ax)} - \frac{\sqrt{-a^2x^2+1} x^3 e^{\arccos(ax)}}{17a} - \frac{3x^2 e^{\arccos(ax)}}{85a^2} - \frac{6\sqrt{-a^2x^2+1} x e^{\arccos(ax)}}{85a^3} - \frac{6e^{\arccos(ax)}}{85a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arccos(a\*x))\*x^3,x, algorithm="giac")

[Out]  $\frac{4}{17}*x^4*e^{\arccos(a*x)} - \frac{1}{17}*\sqrt{-a^2*x^2 + 1}*x^3*e^{\arccos(a*x)}/a - \frac{3}{85}*x^2*e^{\arccos(a*x)}/a^2 - \frac{6}{85}*\sqrt{-a^2*x^2 + 1}*x*e^{\arccos(a*x)}/a^3 - \frac{6}{85}*e^{\arccos(a*x)}/a^4$

### 3.109 $\int e^{\cos^{-1}(ax)} x^2 dx$

**Optimal.** Leaf size=82

$$-\frac{\sqrt{1-a^2x^2}e^{\cos^{-1}(ax)}}{8a^3} + \frac{xe^{\cos^{-1}(ax)}}{8a^2} + \frac{3e^{\cos^{-1}(ax)}\cos(3\cos^{-1}(ax))}{40a^3} - \frac{e^{\cos^{-1}(ax)}\sin(3\cos^{-1}(ax))}{40a^3}$$

[Out] (E^ArcCos[a\*x]\*x)/(8\*a^2) - (E^ArcCos[a\*x]\*Sqrt[1 - a^2\*x^2])/(8\*a^3) + (3\*E^ArcCos[a\*x]\*Cos[3\*ArcCos[a\*x]])/(40\*a^3) - (E^ArcCos[a\*x]\*Sin[3\*ArcCos[a\*x]])/(40\*a^3)

**Rubi [A]** time = 0.0613399, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4837, 12, 4469, 4432}

$$-\frac{\sqrt{1-a^2x^2}e^{\cos^{-1}(ax)}}{8a^3} + \frac{xe^{\cos^{-1}(ax)}}{8a^2} + \frac{3e^{\cos^{-1}(ax)}\cos(3\cos^{-1}(ax))}{40a^3} - \frac{e^{\cos^{-1}(ax)}\sin(3\cos^{-1}(ax))}{40a^3}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCos[a\*x]\*x^2, x]

[Out] (E^ArcCos[a\*x]\*x)/(8\*a^2) - (E^ArcCos[a\*x]\*Sqrt[1 - a^2\*x^2])/(8\*a^3) + (3\*E^ArcCos[a\*x]\*Cos[3\*ArcCos[a\*x]])/(40\*a^3) - (E^ArcCos[a\*x]\*Sin[3\*ArcCos[a\*x]])/(40\*a^3)

#### Rule 4837

Int[(u\_)\*(f\_)^(ArcCos[(a\_) + (b\_)\*(x\_)]^(n\_)\*(c\_.)), x\_Symbol] :> -Dist[b^(-1), Subst[Int[(u /. x -> -(a/b) + Cos[x]/b)\*f^(c\*x^n)\*Sin[x], x], x, ArcCos[a + b\*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 4469

Int[Cos[(f\_) + (g\_)\*(x\_)]^(n\_)\*(F\_)^(c\_)\*((a\_) + (b\_)\*(x\_))\*Sin[(d\_) + (e\_)\*(x\_)]^(m\_), x\_Symbol] :> Int[ExpandTrigReduce[F^(c\*(a + b\*x)), Sin[d + e\*x]^m\*Cos[f + g\*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0]

#### Rule 4432

Int[(F\_)^(c\_)\*((a\_) + (b\_)\*(x\_))\*Sin[(d\_) + (e\_)\*(x\_)], x\_Symbol] :> Simp[(b\*c\*Log[F]\*F^(c\*(a + b\*x))\*Sin[d + e\*x])/(e^2 + b^2\*c^2\*Log[F]^2), x] - Simp[(e\*F^(c\*(a + b\*x))\*Cos[d + e\*x])/(e^2 + b^2\*c^2\*Log[F]^2), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2\*c^2\*Log[F]^2, 0]

#### Rubi steps

$$\begin{aligned}
\int e^{\cos^{-1}(ax)} x^2 dx &= -\frac{\text{Subst}\left(\int \frac{e^x \cos^2(x) \sin(x)}{a^2} dx, x, \cos^{-1}(ax)\right)}{a} \\
&= -\frac{\text{Subst}\left(\int e^x \cos^2(x) \sin(x) dx, x, \cos^{-1}(ax)\right)}{a^3} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{4}e^x \sin(x) + \frac{1}{4}e^x \sin(3x)\right) dx, x, \cos^{-1}(ax)\right)}{a^3} \\
&= -\frac{\text{Subst}\left(\int e^x \sin(x) dx, x, \cos^{-1}(ax)\right)}{4a^3} - \frac{\text{Subst}\left(\int e^x \sin(3x) dx, x, \cos^{-1}(ax)\right)}{4a^3} \\
&= \frac{e^{\cos^{-1}(ax)} x}{8a^2} - \frac{e^{\cos^{-1}(ax)} \sqrt{1-a^2x^2}}{8a^3} + \frac{3e^{\cos^{-1}(ax)} \cos\left(3\cos^{-1}(ax)\right)}{40a^3} - \frac{e^{\cos^{-1}(ax)} \sin\left(3\cos^{-1}(ax)\right)}{40a^3}
\end{aligned}$$

**Mathematica [A]** time = 0.106531, size = 50, normalized size = 0.61

$$-\frac{e^{\cos^{-1}(ax)} \left(5\sqrt{1-a^2x^2} - 5ax - 3\cos\left(3\cos^{-1}(ax)\right) + \sin\left(3\cos^{-1}(ax)\right)\right)}{40a^3}$$

Antiderivative was successfully verified.

[In] Integrate[E^ArcCos[a\*x]\*x^2,x]

[Out] -(E^ArcCos[a\*x]\*(-5\*a\*x + 5\*Sqrt[1 - a^2\*x^2] - 3\*Cos[3\*ArcCos[a\*x]] + Sin[3\*ArcCos[a\*x]]))/(40\*a^3)

**Maple [F]** time = 0.009, size = 0, normalized size = 0.

$$\int e^{\arccos(ax)} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(arccos(a\*x))\*x^2,x)

[Out] int(exp(arccos(a\*x))\*x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 e^{(\arccos(ax))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arccos(a\*x))\*x^2,x, algorithm="maxima")

[Out] integrate(x^2\*e^(arccos(a\*x)), x)

**Fricas [A]** time = 2.5661, size = 107, normalized size = 1.3

$$\frac{\left(3a^3x^3 - ax - (a^2x^2 + 1)\sqrt{-a^2x^2 + 1}\right)e^{(\arccos(ax))}}{10a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arccos(a\*x))\*x^2,x, algorithm="fricas")

[Out]  $\frac{1}{10}*(3*a^3*x^3 - a*x - (a^2*x^2 + 1)*\sqrt{-a^2*x^2 + 1})*e^{\arccos(a*x)}/a^3$

**Sympy [A]** time = 1.7158, size = 85, normalized size = 1.04

$$\begin{cases} \frac{3x^3 e^{\arccos(ax)}}{10} - \frac{x^2 \sqrt{-a^2 x^2 + 1} e^{\arccos(ax)}}{10a} - \frac{x e^{\arccos(ax)}}{10a^2} - \frac{\sqrt{-a^2 x^2 + 1} e^{\arccos(ax)}}{10a^3} & \text{for } a \neq 0 \\ \frac{x^3 e^{\frac{\pi}{2}}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(acos(a\*x))\*x\*\*2,x)

[Out] Piecewise(((3\*x\*\*3\*exp(acos(a\*x))/10 - x\*\*2\*sqrt(-a\*\*2\*x\*\*2 + 1)\*exp(acos(a\*x)))/(10\*a) - x\*exp(acos(a\*x))/(10\*a\*\*2) - sqrt(-a\*\*2\*x\*\*2 + 1)\*exp(acos(a\*x)))/(10\*a\*\*3), Ne(a, 0)), (x\*\*3\*exp(pi/2)/3, True))

**Giac [A]** time = 1.37588, size = 93, normalized size = 1.13

$$\frac{3}{10} x^3 e^{\arccos(ax)} - \frac{\sqrt{-a^2 x^2 + 1} x^2 e^{\arccos(ax)}}{10 a} - \frac{x e^{\arccos(ax)}}{10 a^2} - \frac{\sqrt{-a^2 x^2 + 1} e^{\arccos(ax)}}{10 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arccos(a\*x))\*x^2,x, algorithm="giac")

[Out]  $\frac{3}{10}*x^3*e^{\arccos(a*x)} - \frac{1}{10}*\sqrt{-a^2*x^2 + 1}*x^2*e^{\arccos(a*x)}/a - \frac{1}{10}*x*e^{\arccos(a*x)}/a^2 - \frac{1}{10}*\sqrt{-a^2*x^2 + 1}*e^{\arccos(a*x)}/a^3$

### 3.110 $\int e^{\cos^{-1}(ax)} x dx$

**Optimal.** Leaf size=41

$$\frac{e^{\cos^{-1}(ax)} \cos(2 \cos^{-1}(ax))}{5a^2} - \frac{e^{\cos^{-1}(ax)} \sin(2 \cos^{-1}(ax))}{10a^2}$$

[Out] (E^ArcCos[a\*x]\*Cos[2\*ArcCos[a\*x]])/(5\*a^2) - (E^ArcCos[a\*x]\*Sin[2\*ArcCos[a\*x]])/(10\*a^2)

**Rubi [A]** time = 0.0340067, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4837, 12, 4469, 4432}

$$\frac{e^{\cos^{-1}(ax)} \cos(2 \cos^{-1}(ax))}{5a^2} - \frac{e^{\cos^{-1}(ax)} \sin(2 \cos^{-1}(ax))}{10a^2}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCos[a\*x]\*x,x]

[Out] (E^ArcCos[a\*x]\*Cos[2\*ArcCos[a\*x]])/(5\*a^2) - (E^ArcCos[a\*x]\*Sin[2\*ArcCos[a\*x]])/(10\*a^2)

#### Rule 4837

Int[(u\_)\*(f\_)^(ArcCos[(a\_.) + (b\_.)\*(x\_)]^(n\_)\*(c\_.)), x\_Symbol] := -Dist[b^(-1), Subst[Int[(u /. x -> -(a/b) + Cos[x]/b)\*f^(c\*x^n)\*Sin[x], x], x, ArcCos[a + b\*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 4469

Int[Cos[(f\_.) + (g\_.)\*(x\_)]^(n\_)\*(F\_)^(c\_)\*((a\_.) + (b\_.)\*(x\_))\*Sin[(d\_.) + (e\_.)\*(x\_)]^(m\_), x\_Symbol] := Int[ExpandTrigReduce[F^(c\*(a + b\*x)), Sin[d + e\*x]^m \* Cos[f + g\*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0]

#### Rule 4432

Int[(F\_)^(c\_)\*((a\_.) + (b\_.)\*(x\_))\*Sin[(d\_.) + (e\_.)\*(x\_)], x\_Symbol] := Simp[(b\*c\*Log[F]\*F^(c\*(a + b\*x))\*Sin[d + e\*x]/(e^2 + b^2\*c^2\*Log[F]^2), x] - Simp[(e\*F^(c\*(a + b\*x))\*Cos[d + e\*x]/(e^2 + b^2\*c^2\*Log[F]^2), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2\*c^2\*Log[F]^2, 0]

#### Rubi steps

$$\begin{aligned}
\int e^{\cos^{-1}(ax)} x dx &= -\frac{\text{Subst}\left(\int \frac{e^x \cos(x) \sin(x)}{a} dx, x, \cos^{-1}(ax)\right)}{a} \\
&= -\frac{\text{Subst}\left(\int e^x \cos(x) \sin(x) dx, x, \cos^{-1}(ax)\right)}{a^2} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{2} e^x \sin(2x) dx, x, \cos^{-1}(ax)\right)}{a^2} \\
&= -\frac{\text{Subst}\left(\int e^x \sin(2x) dx, x, \cos^{-1}(ax)\right)}{2a^2} \\
&= \frac{e^{\cos^{-1}(ax)} \cos\left(2 \cos^{-1}(ax)\right)}{5a^2} - \frac{e^{\cos^{-1}(ax)} \sin\left(2 \cos^{-1}(ax)\right)}{10a^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0366191, size = 30, normalized size = 0.73

$$-\frac{e^{\cos^{-1}(ax)} \left( \sin\left(2 \cos^{-1}(ax)\right) - 2 \cos\left(2 \cos^{-1}(ax)\right) \right)}{10a^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^ArcCos[a\*x]\*x,x]

[Out] -(E^ArcCos[a\*x]\*(-2\*Cos[2\*ArcCos[a\*x]] + Sin[2\*ArcCos[a\*x]]))/(10\*a^2)

**Maple [F]** time = 0.007, size = 0, normalized size = 0.

$$\int e^{\arccos(ax)} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(arccos(a\*x))\*x,x)

[Out] int(exp(arccos(a\*x))\*x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x e^{\arccos(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arccos(a\*x))\*x,x, algorithm="maxima")

[Out] integrate(x\*e^(arccos(a\*x)), x)

**Fricas [A]** time = 2.63661, size = 89, normalized size = 2.17

$$\frac{\left(2a^2x^2 - \sqrt{-a^2x^2 + 1}ax - 1\right)e^{\arccos(ax)}}{5a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arccos(a\*x))\*x,x, algorithm="fricas")

[Out]  $1/5*(2*a^2*x^2 - \sqrt{-a^2*x^2 + 1})*a*x - 1)*e^{(\arccos(a*x))}/a^2$

**Sympy [A]** time = 0.539324, size = 58, normalized size = 1.41

$$\begin{cases} \frac{2x^2 e^{\arccos(ax)}}{5} - \frac{x\sqrt{-a^2x^2+1}e^{\arccos(ax)}}{5a} - \frac{e^{\arccos(ax)}}{5a^2} & \text{for } a \neq 0 \\ \frac{x^2 e^{\frac{\pi}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(acos(a\*x))\*x,x)

[Out] Piecewise((2\*x\*\*2\*exp(acos(a\*x))/5 - x\*sqrt(-a\*\*2\*x\*\*2 + 1)\*exp(acos(a\*x))/(5\*a) - exp(acos(a\*x))/(5\*a\*\*2), Ne(a, 0)), (x\*\*2\*exp(pi/2)/2, True))

**Giac [A]** time = 1.23812, size = 59, normalized size = 1.44

$$\frac{2}{5}x^2e^{(\arccos(ax))} - \frac{\sqrt{-a^2x^2 + 1}xe^{(\arccos(ax))}}{5a} - \frac{e^{(\arccos(ax))}}{5a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arccos(a\*x))\*x,x, algorithm="giac")

[Out]  $2/5*x^2*e^{(\arccos(a*x))} - 1/5*\sqrt{-a^2*x^2 + 1}*x*e^{(\arccos(a*x))}/a - 1/5*e^{(\arccos(a*x))}/a^2$

### 3.111 $\int e^{\cos^{-1}(ax)} dx$

**Optimal.** Leaf size=39

$$\frac{1}{2}xe^{\cos^{-1}(ax)} - \frac{\sqrt{1-a^2x^2}e^{\cos^{-1}(ax)}}{2a}$$

[Out]  $(E^{\text{ArcCos}[a*x]*x}/2 - (E^{\text{ArcCos}[a*x]*\text{Sqrt}[1 - a^2*x^2]))/(2*a)$

**Rubi [A]** time = 0.0140558, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {4837, 4432}

$$\frac{1}{2}xe^{\cos^{-1}(ax)} - \frac{\sqrt{1-a^2x^2}e^{\cos^{-1}(ax)}}{2a}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCos[a\*x], x]

[Out]  $(E^{\text{ArcCos}[a*x]*x}/2 - (E^{\text{ArcCos}[a*x]*\text{Sqrt}[1 - a^2*x^2]))/(2*a)$

#### Rule 4837

Int[(u\_)\*(f\_)^(ArcCos[(a\_.) + (b\_.)\*(x\_)]^(n\_)\*(c\_.)), x\_Symbol] :> -Dist[b^(-1), Subst[Int[(u / . x -> -(a/b) + Cos[x]/b)\*f^(c\*x^n)\*Sin[x], x], x, ArcCos[a + b\*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]

#### Rule 4432

Int[(F\_)^(c\_)\*((a\_.) + (b\_.)\*(x\_))\*Sin[(d\_.) + (e\_.)\*(x\_)], x\_Symbol] :> Simp[(b\*c\*Log[F]\*F^(c\*(a + b\*x))\*Sin[d + e\*x])/(e^2 + b^2\*c^2\*Log[F]^2), x] - Simp[(e\*F^(c\*(a + b\*x))\*Cos[d + e\*x])/(e^2 + b^2\*c^2\*Log[F]^2), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2\*c^2\*Log[F]^2, 0]

#### Rubi steps

$$\begin{aligned} \int e^{\cos^{-1}(ax)} dx &= -\frac{\text{Subst}\left(\int e^x \sin(x) dx, x, \cos^{-1}(ax)\right)}{a} \\ &= \frac{1}{2}e^{\cos^{-1}(ax)}x - \frac{e^{\cos^{-1}(ax)}\sqrt{1-a^2x^2}}{2a} \end{aligned}$$

**Mathematica [A]** time = 0.0326375, size = 32, normalized size = 0.82

$$-\frac{\left(\sqrt{1-a^2x^2} - ax\right)e^{\cos^{-1}(ax)}}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[E^ArcCos[a\*x], x]

[Out]  $-(E^{\text{ArcCos}[a*x]*(-a*x + \text{Sqrt}[1 - a^2*x^2])})/(2*a)$



---

**Maple [F]** time = 0.004, size = 0, normalized size = 0.

$$\int e^{\arccos(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(arccos(a\*x)), x)

[Out] int(exp(arccos(a\*x)), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int e^{(\arccos(ax))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arccos(a\*x)), x, algorithm="maxima")

[Out] integrate(e^(arccos(a\*x)), x)

---

**Fricas [A]** time = 2.59955, size = 68, normalized size = 1.74

$$\frac{(ax - \sqrt{-a^2x^2 + 1})e^{\arccos(ax)}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arccos(a\*x)), x, algorithm="fricas")

[Out] 1/2\*(a\*x - sqrt(-a^2\*x^2 + 1))\*e^(arccos(a\*x))/a

---

**Sympy [A]** time = 0.201251, size = 37, normalized size = 0.95

$$\begin{cases} \frac{x e^{\arccos(ax)}}{\pi^2} - \frac{\sqrt{-a^2x^2+1} e^{\arccos(ax)}}{2a} & \text{for } a \neq 0 \\ x e^{\frac{\pi}{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(acos(a\*x)), x)

[Out] Piecewise((x\*exp(acos(a\*x))/2 - sqrt(-a\*\*2\*x\*\*2 + 1)\*exp(acos(a\*x))/(2\*a), Ne(a, 0)), (x\*exp(pi/2), True))

---

**Giac [A]** time = 1.27696, size = 42, normalized size = 1.08

$$\frac{1}{2} x e^{\arccos(ax)} - \frac{\sqrt{-a^2x^2 + 1} e^{\arccos(ax)}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x)),x, algorithm="giac")
```

```
[Out] 1/2*x*e^(arccos(a*x)) - 1/2*sqrt(-a^2*x^2 + 1)*e^(arccos(a*x))/a
```

$$3.112 \quad \int \frac{e^{\cos^{-1}(ax)}}{x} dx$$

**Optimal.** Leaf size=45

$$ie^{\cos^{-1}(ax)} - 2ie^{\cos^{-1}(ax)} \text{Hypergeometric2F1}\left(-\frac{i}{2}, 1, 1 - \frac{i}{2}, -e^{2i\cos^{-1}(ax)}\right)$$

[Out] I\*E^ArcCos[a\*x] - (2\*I)\*E^ArcCos[a\*x]\*Hypergeometric2F1[-I/2, 1, 1 - I/2, -E^((2\*I)\*ArcCos[a\*x])]

**Rubi [A]** time = 0.0543512, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4837, 12, 4442, 2194, 2251}

$$ie^{\cos^{-1}(ax)} - 2ie^{\cos^{-1}(ax)} {}_2F_1\left(-\frac{i}{2}, 1; 1 - \frac{i}{2}; -e^{2i\cos^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

[In] Int[E^ArcCos[a\*x]/x, x]

[Out] I\*E^ArcCos[a\*x] - (2\*I)\*E^ArcCos[a\*x]\*Hypergeometric2F1[-I/2, 1, 1 - I/2, -E^((2\*I)\*ArcCos[a\*x])]

#### Rule 4837

Int[(u\_)\*(f\_)^(ArcCos[(a\_) + (b\_)\*(x\_)])^(n\_)\*(c\_), x\_Symbol] := -Dist[b^(-1), Subst[Int[(u /. x -> -(a/b) + Cos[x]/b)\*f^(c\*x^n)\*Sin[x], x], x, ArcCos[a + b\*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 4442

Int[(F\_)^((c\_)\*((a\_) + (b\_)\*(x\_)))\*Tan[(d\_) + (e\_)\*(x\_)]^(n\_), x\_Symbol] := Dist[I^n, Int[ExpandIntegrand[(F^(c\*(a + b\*x)))\*(1 - E^(2\*I\*(d + e\*x)))^n]/(1 + E^(2\*I\*(d + e\*x)))^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]

#### Rule 2194

Int[((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rule 2251

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] := Simp[(a^p\*G^(h\*(f + g\*x))\*Hypergeometric2F1[-p, (g\*h\*Log[G])/(d\*e\*Log[F]), (g\*h\*Log[G])/(d\*e\*Log[F]) + 1, Simplify[-((b\*F^(e\*(c + d\*x)))/a])])]/(g\*h\*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{e^{\cos^{-1}(ax)}}{x} dx &= -\frac{\text{Subst}\left(\int ae^x \tan(x) dx, x, \cos^{-1}(ax)\right)}{a} \\
&= -\text{Subst}\left(\int e^x \tan(x) dx, x, \cos^{-1}(ax)\right) \\
&= -\left(i \text{Subst}\left(\int \left(-e^x + \frac{2e^x}{1+e^{2ix}}\right) dx, x, \cos^{-1}(ax)\right)\right) \\
&= i \text{Subst}\left(\int e^x dx, x, \cos^{-1}(ax)\right) - 2i \text{Subst}\left(\int \frac{e^x}{1+e^{2ix}} dx, x, \cos^{-1}(ax)\right) \\
&= ie^{\cos^{-1}(ax)} - 2ie^{\cos^{-1}(ax)} {}_2F_1\left(-\frac{i}{2}, 1; 1 - \frac{i}{2}; -e^{2i\cos^{-1}(ax)}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0559199, size = 79, normalized size = 1.76

$$i\left(\left(\frac{1}{5} - \frac{2i}{5}\right)e^{(1+2i)\cos^{-1}(ax)}\text{Hypergeometric2F1}\left(1, 1 - \frac{i}{2}, 2 - \frac{i}{2}, -e^{2i\cos^{-1}(ax)}\right) - e^{\cos^{-1}(ax)}\text{Hypergeometric2F1}\left(-\frac{i}{2}, 1, 1\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcCos[a\*x]/x,x]

[Out] I\*(-(E^ArcCos[a\*x]\*Hypergeometric2F1[-I/2, 1, 1 - I/2, -E^((2\*I)\*ArcCos[a\*x])]) + (1/5 - (2\*I)/5)\*E^((1 + 2\*I)\*ArcCos[a\*x])\*Hypergeometric2F1[1, 1 - I/2, 2 - I/2, -E^((2\*I)\*ArcCos[a\*x])])

**Maple [F]** time = 0.007, size = 0, normalized size = 0.

$$\int \frac{e^{\arccos(ax)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(arccos(a\*x))/x,x)

[Out] int(exp(arccos(a\*x))/x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\arccos(ax)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arccos(a\*x))/x,x, algorithm="maxima")

[Out] integrate(e^(arccos(a\*x))/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{\arccos(ax)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))/x,x, algorithm="fricas")
```

```
[Out] integral(e^(arccos(a*x))/x, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\arccos(ax)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acos(a*x))/x,x)
```

```
[Out] Integral(exp(acos(a*x))/x, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\arccos(ax)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))/x,x, algorithm="giac")
```

```
[Out] integrate(e^(arccos(a*x))/x, x)
```

$$3.113 \quad \int \frac{e^{\cos^{-1}(ax)}}{x^2} dx$$

**Optimal.** Leaf size=87

$$(1+i)ae^{(1+i)\cos^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2}, 1, \frac{3}{2}-\frac{i}{2}, -e^{2i\cos^{-1}(ax)}\right) - (2+2i)ae^{(1+i)\cos^{-1}(ax)}\text{Hypergeometric2F1}$$

[Out] (1 + I)\*a\*E^((1 + I)\*ArcCos[a\*x])\*Hypergeometric2F1[1/2 - I/2, 1, 3/2 - I/2, -E^((2\*I)\*ArcCos[a\*x])] - (2 + 2\*I)\*a\*E^((1 + I)\*ArcCos[a\*x])\*Hypergeometric2F1[1/2 - I/2, 2, 3/2 - I/2, -E^((2\*I)\*ArcCos[a\*x])]

**Rubi [A]** time = 0.105852, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4837, 12, 4471, 2251}

$$(1+i)ae^{(1+i)\cos^{-1}(ax)}{}_2F_1\left(\frac{1}{2}-\frac{i}{2}, 1; \frac{3}{2}-\frac{i}{2}; -e^{2i\cos^{-1}(ax)}\right) - (2+2i)ae^{(1+i)\cos^{-1}(ax)}{}_2F_1\left(\frac{1}{2}-\frac{i}{2}, 2; \frac{3}{2}-\frac{i}{2}; -e^{2i\cos^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

[In] Int[E^ArcCos[a\*x]/x^2,x]

[Out] (1 + I)\*a\*E^((1 + I)\*ArcCos[a\*x])\*Hypergeometric2F1[1/2 - I/2, 1, 3/2 - I/2, -E^((2\*I)\*ArcCos[a\*x])] - (2 + 2\*I)\*a\*E^((1 + I)\*ArcCos[a\*x])\*Hypergeometric2F1[1/2 - I/2, 2, 3/2 - I/2, -E^((2\*I)\*ArcCos[a\*x])]

#### Rule 4837

Int[(u\_)\*(f\_)^(ArcCos[(a\_.) + (b\_.)\*(x\_)]^(n\_)\*(c\_.)), x\_Symbol] :> -Dist[b^(-1), Subst[Int[(u / . x -> -(a/b) + Cos[x]/b)\*f^(c\*x^n)\*Sin[x], x], x, ArcCos[a + b\*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 4471

Int[(F\_)^(c\_)\*((a\_.) + (b\_.)\*(x\_))\*(G\_)[(d\_.) + (e\_.)\*(x\_)]^(m\_)\*(H\_)[(d\_.) + (e\_.)\*(x\_)]^(n\_), x\_Symbol] :> Int[ExpandTrigToExp[F^(c\*(a + b\*x)), G[d + e\*x]^m\*H[d + e\*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[m, 0] && IGtQ[n, 0] && TrigQ[G] && TrigQ[H]

#### Rule 2251

Int[((a\_.) + (b\_.)\*(F\_)^(e\_)\*((c\_.) + (d\_.)\*(x\_)))^(p\_)\*(G\_)^(h\_)\*((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Simp[(a^p\*G^(h\*(f + g\*x))\*Hypergeometric2F1[-p, (g\*h\*Log[G])/(d\*e\*Log[F]), (g\*h\*Log[G])/(d\*e\*Log[F]) + 1, Simplify[-((b\*F^(e\*(c + d\*x)))/a])]/(g\*h\*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{e^{\cos^{-1}(ax)}}{x^2} dx &= -\frac{\text{Subst}\left(\int a^2 e^x \sec(x) \tan(x) dx, x, \cos^{-1}(ax)\right)}{a} \\
&= -\left(a \text{Subst}\left(\int e^x \sec(x) \tan(x) dx, x, \cos^{-1}(ax)\right)\right) \\
&= -\left(a \text{Subst}\left(\int \left(\frac{4ie^{(1+i)x}}{(1+e^{2ix})^2} - \frac{2ie^{(1+i)x}}{1+e^{2ix}}\right) dx, x, \cos^{-1}(ax)\right)\right) \\
&= (2ia) \text{Subst}\left(\int \frac{e^{(1+i)x}}{1+e^{2ix}} dx, x, \cos^{-1}(ax)\right) - (4ia) \text{Subst}\left(\int \frac{e^{(1+i)x}}{(1+e^{2ix})^2} dx, x, \cos^{-1}(ax)\right) \\
&= (1+i)ae^{(1+i)\cos^{-1}(ax)} {}_2F_1\left(\frac{1}{2} - \frac{i}{2}, 1; \frac{3}{2} - \frac{i}{2}; -e^{2i\cos^{-1}(ax)}\right) - (2+2i)ae^{(1+i)\cos^{-1}(ax)} {}_2F_1\left(\frac{1}{2} - \frac{i}{2}, 2; \frac{3}{2} - \frac{i}{2}; -e^{2i\cos^{-1}(ax)}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0618076, size = 55, normalized size = 0.63

$$-\frac{e^{\cos^{-1}(ax)}}{x} + (1-i)ae^{(1+i)\cos^{-1}(ax)} \text{Hypergeometric2F1}\left(\frac{1}{2} - \frac{i}{2}, 1, \frac{3}{2} - \frac{i}{2}, -e^{2i\cos^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcCos[a\*x]/x^2,x]

[Out] -(E^ArcCos[a\*x]/x) + (1 - I)\*a\*E^((1 + I)\*ArcCos[a\*x])\*Hypergeometric2F1[1/2 - I/2, 1, 3/2 - I/2, -E^((2\*I)\*ArcCos[a\*x])]

**Maple [F]** time = 0.007, size = 0, normalized size = 0.

$$\int \frac{e^{\arccos(ax)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(arccos(a\*x))/x^2,x)

[Out] int(exp(arccos(a\*x))/x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\arccos(ax)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arccos(a\*x))/x^2,x, algorithm="maxima")

[Out] integrate(e^(arccos(a\*x))/x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{\arccos(ax)}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arccos(a\*x))/x^2,x, algorithm="fricas")

[Out] integral(e^(arccos(a\*x))/x^2, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\arccos(ax)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(acos(a\*x))/x\*\*2,x)

[Out] Integral(exp(acos(a\*x))/x\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\arccos(ax)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arccos(a\*x))/x^2,x, algorithm="giac")

[Out] integrate(e^(arccos(a\*x))/x^2, x)



### 3.114 $\int \cos^{-1}\left(\frac{c}{a+bx}\right) dx$

**Optimal.** Leaf size=48

$$\frac{(a+bx)\sec^{-1}\left(\frac{a}{c} + \frac{bx}{c}\right)}{b} - \frac{c \tanh^{-1}\left(\sqrt{1 - \frac{c^2}{(a+bx)^2}}\right)}{b}$$

[Out] ((a + b\*x)\*ArcSec[a/c + (b\*x)/c])/b - (c\*ArcTanh[Sqrt[1 - c^2/(a + b\*x)^2]])/b

**Rubi [A]** time = 0.0328311, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {4833, 5250, 372, 266, 63, 206}

$$\frac{(a+bx)\sec^{-1}\left(\frac{a}{c} + \frac{bx}{c}\right)}{b} - \frac{c \tanh^{-1}\left(\sqrt{1 - \frac{c^2}{(a+bx)^2}}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[c/(a + b\*x)],x]

[Out] ((a + b\*x)\*ArcSec[a/c + (b\*x)/c])/b - (c\*ArcTanh[Sqrt[1 - c^2/(a + b\*x)^2]])/b

#### Rule 4833

Int[ArcCos[(c\_.)/((a\_.) + (b\_.)\*(x\_)^(n\_.))]^(m\_.)\*(u\_.), x\_Symbol] := Int[u\*ArcSec[a/c + (b\*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]

#### Rule 5250

Int[ArcSec[(c\_) + (d\_.)\*(x\_)], x\_Symbol] := Simp[((c + d\*x)\*ArcSec[c + d\*x])/d, x] - Int[1/((c + d\*x)\*Sqrt[1 - 1/(c + d\*x)^2]), x] /; FreeQ[{c, d}, x]

#### Rule 372

Int[(u\_)^(m\_.)\*((a\_) + (b\_.)\*(v\_)^(n\_.))^(p\_.), x\_Symbol] := Dist[u^m/(Coefficient[v, x, 1]\*v^m), Subst[Int[x^m\*(a + b\*x^n)^p, x], x, v], x] /; FreeQ[{a, b, m, n, p}, x] && LinearPairQ[u, v, x]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \cos^{-1}\left(\frac{c}{a+bx}\right) dx &= \int \sec^{-1}\left(\frac{a}{c} + \frac{bx}{c}\right) dx \\
 &= \frac{(a+bx) \sec^{-1}\left(\frac{a}{c} + \frac{bx}{c}\right)}{b} - \int \frac{1}{\left(\frac{a}{c} + \frac{bx}{c}\right) \sqrt{1 - \frac{1}{\left(\frac{a}{c} + \frac{bx}{c}\right)^2}}} dx \\
 &= \frac{(a+bx) \sec^{-1}\left(\frac{a}{c} + \frac{bx}{c}\right)}{b} - \frac{c \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^2}}} dx, x, \frac{a}{c} + \frac{bx}{c}\right)}{b} \\
 &= \frac{(a+bx) \sec^{-1}\left(\frac{a}{c} + \frac{bx}{c}\right)}{b} + \frac{c \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-xx}} dx, x, \frac{1}{\left(\frac{a}{c} + \frac{bx}{c}\right)^2}\right)}{2b} \\
 &= \frac{(a+bx) \sec^{-1}\left(\frac{a}{c} + \frac{bx}{c}\right)}{b} - \frac{c \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{1 - \frac{c^2}{(a+bx)^2}}\right)}{b} \\
 &= \frac{(a+bx) \sec^{-1}\left(\frac{a}{c} + \frac{bx}{c}\right)}{b} - \frac{c \tanh^{-1}\left(\sqrt{1 - \frac{c^2}{(a+bx)^2}}\right)}{b}
 \end{aligned}$$

**Mathematica [B]** time = 0.162596, size = 141, normalized size = 2.94

$$x \cos^{-1}\left(\frac{c}{a+bx}\right) - \frac{(a+bx) \sqrt{\frac{a^2+2abx+b^2x^2-c^2}{(a+bx)^2}} \left( c \tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2+2abx+b^2x^2-c^2}}\right) - a \tan^{-1}\left(\frac{\sqrt{(a+bx)^2-c^2}}{c}\right) \right)}{b \sqrt{a^2+2abx+b^2x^2-c^2}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[c/(a + b\*x)], x]

[Out] x\*ArcCos[c/(a + b\*x)] - ((a + b\*x)\*Sqrt[(a^2 - c^2 + 2\*a\*b\*x + b^2\*x^2)/(a + b\*x)^2]\*(-(a\*ArcTan[Sqrt[-c^2 + (a + b\*x)^2]/c]) + c\*ArcTanh[(a + b\*x)/Sqrt[a^2 - c^2 + 2\*a\*b\*x + b^2\*x^2]]))/(b\*Sqrt[a^2 - c^2 + 2\*a\*b\*x + b^2\*x^2])

**Maple [A]** time = 0.008, size = 45, normalized size = 0.9

$$-\frac{c}{b} \left( -\frac{bx+a}{c} \arccos\left(\frac{c}{bx+a}\right) + \operatorname{Artanh}\left(\frac{1}{\sqrt{1 - \frac{c^2}{(bx+a)^2}}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccos(c/(b*x+a)),x)`

[Out]  $-1/b*c*(-1/c*(b*x+a)*\arccos(c/(b*x+a))+\operatorname{arctanh}(1/(1-c^2/(b*x+a)^2)^{(1/2)}))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$x \arctan\left(\sqrt{bx+a+c}\sqrt{bx+a-c}, c\right) - \int \frac{(b^2cx^2 + abcx)e^{\left(\frac{1}{2}\log(bx+a+c) + \frac{1}{2}\log(bx+a-c)\right)}}{b^2c^2x^2 + 2abc^2x + a^2c^2 - c^4 + (b^2x^2 + 2abx + a^2 - c^2)e^{(\log(bx+a+c)+\log(bx+a-c))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(c/(b*x+a)),x, algorithm="maxima")`

[Out]  $x*\arctan2(\operatorname{sqrt}(b*x + a + c)*\operatorname{sqrt}(b*x + a - c), c) - \operatorname{integrate}((b^2*c*x^2 + a*b*c*x)*e^{(1/2*\log(b*x + a + c) + 1/2*\log(b*x + a - c))}/(b^2*c^2*x^2 + 2*a*b*c^2*x + a^2*c^2 - c^4 + (b^2*x^2 + 2*a*b*x + a^2 - c^2)*e^{(\log(b*x + a + c) + \log(b*x + a - c))}), x)$

**Fricas [B]** time = 2.52899, size = 305, normalized size = 6.35

$$\frac{bx \arccos\left(\frac{c}{bx+a}\right) + 2a \arctan\left(-\frac{bx-(bx+a)\sqrt{\frac{b^2x^2+2abx+a^2-c^2}{b^2x^2+2abx+a^2}}+a}{c}\right) + c \log\left(-bx + (bx+a)\sqrt{\frac{b^2x^2+2abx+a^2-c^2}{b^2x^2+2abx+a^2}} - a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(c/(b*x+a)),x, algorithm="fricas")`

[Out]  $(b*x*\arccos(c/(b*x + a)) + 2*a*\arctan(-(b*x - (b*x + a)*\operatorname{sqrt}((b^2*x^2 + 2*a*b*x + a^2 - c^2)/(b^2*x^2 + 2*a*b*x + a^2)) + a)/c) + c*\log(-b*x + (b*x + a)*\operatorname{sqrt}((b^2*x^2 + 2*a*b*x + a^2 - c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - a))/b$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acos}\left(\frac{c}{a+bx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acos(c/(b*x+a)),x)`

[Out] `Integral(acos(c/(a + b*x)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccos}\left(\frac{c}{bx+a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(c/(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccos(c/(b*x + a)), x)
```

$$3.115 \quad \int \frac{x}{\sqrt{1-x^2}\sqrt{\cos^{-1}(x)}} dx$$

**Optimal.** Leaf size=26

$$-\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(x)}\right)$$

[Out] -(Sqrt[2\*Pi]\*FresnelC[Sqrt[2/Pi]\*Sqrt[ArcCos[x]]])

**Rubi [A]** time = 0.0706919, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4724, 3304, 3352}

$$-\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(x)}\right)$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[1 - x^2]\*Sqrt[ArcCos[x]]), x]

[Out] -(Sqrt[2\*Pi]\*FresnelC[Sqrt[2/Pi]\*Sqrt[ArcCos[x]]])

#### Rule 4724

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Cos[x]^m\*Sin[x]^(2\*p + 1), x], x, ArcCos[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_)^2)], x\_Symbol] :> Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{1-x^2}\sqrt{\cos^{-1}(x)}} dx &= -\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}} dx, x, \cos^{-1}(x)\right) \\ &= -\left(2\text{Subst}\left(\int \cos(x^2) dx, x, \sqrt{\cos^{-1}(x)}\right)\right) \\ &= -\sqrt{2\pi}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(x)}\right) \end{aligned}$$

**Mathematica [C]** time = 0.0848823, size = 56, normalized size = 2.15

$$\frac{i\left(\sqrt{-i\cos^{-1}(x)}\Gamma\left(\frac{1}{2}, -i\cos^{-1}(x)\right) - \sqrt{i\cos^{-1}(x)}\Gamma\left(\frac{1}{2}, i\cos^{-1}(x)\right)\right)}{2\sqrt{\cos^{-1}(x)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[1 - x^2]\*Sqrt[ArcCos[x]]),x]

[Out] ((I/2)\*(Sqrt[(-I)\*ArcCos[x]]\*Gamma[1/2, (-I)\*ArcCos[x]] - Sqrt[I\*ArcCos[x]]\*Gamma[1/2, I\*ArcCos[x]]))/Sqrt[ArcCos[x]]

**Maple [A]** time = 0.106, size = 21, normalized size = 0.8

$$-\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\sqrt{\arccos(x)}\right)\sqrt{2}\sqrt{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x)

[Out] -FresnelC(2^(1/2)/Pi^(1/2)\*arccos(x)^(1/2))\*2^(1/2)\*Pi^(1/2)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(x-1)(x+1)}\sqrt{\arccos(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x\*\*2+1)\*\*(1/2)/acos(x)\*\*(1/2),x)

[Out] Integral(x/(sqrt(-(x - 1)\*(x + 1))\*sqrt(acos(x))), x)

**Giac [B]** time = 1.39764, size = 78, normalized size = 3.

$$\frac{\sqrt{2}\sqrt{\pi}i \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\arccos(x)}}{i-1}\right)}{2(i-1)} - \frac{\sqrt{2}\sqrt{\pi} \operatorname{erf}\left(-\frac{\sqrt{2}i\sqrt{\arccos(x)}}{i-1}\right)}{2(i-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x, algorithm="giac")

[Out] 1/2\*sqrt(2)\*sqrt(pi)\*i\*erf(sqrt(2)\*sqrt(arccos(x))/(i - 1))/(i - 1) - 1/2\*sqrt(2)\*sqrt(pi)\*erf(-sqrt(2)\*i\*sqrt(arccos(x))/(i - 1))/(i - 1)

$$3.116 \quad \int \frac{x}{\sqrt{1-x^2} \cos^{-1}(x)} dx$$

**Optimal.** Leaf size=5

$$-\text{CosIntegral}(\cos^{-1}(x))$$

[Out] -CosIntegral[ArcCos[x]]

**Rubi [A]** time = 0.0608316, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4724, 3302}

$$-\text{CosIntegral}(\cos^{-1}(x))$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[1 - x^2]\*ArcCos[x]),x]

[Out] -CosIntegral[ArcCos[x]]

#### Rule 4724

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := -Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Cos[x]^m\*Sin[x]^(2\*p + 1), x], x, ArcCos[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

#### Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rubi steps

$$\int \frac{x}{\sqrt{1-x^2} \cos^{-1}(x)} dx = -\text{Subst} \left( \int \frac{\cos(x)}{x} dx, x, \cos^{-1}(x) \right) = -\text{Ci}(\cos^{-1}(x))$$

**Mathematica [A]** time = 0.0400906, size = 5, normalized size = 1.

$$-\text{CosIntegral}(\cos^{-1}(x))$$

Antiderivative was successfully verified.

[In] Integrate[x/(Sqrt[1 - x^2]\*ArcCos[x]),x]

[Out] -CosIntegral[ArcCos[x]]



**Maple [A]** time = 0.067, size = 6, normalized size = 1.2

$$-\text{Ci}(\arccos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/arccos(x)/(-x^2+1)^(1/2),x)

[Out] -Ci(arccos(x))

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^2+1} \arccos(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccos(x)/(-x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(x/(sqrt(-x^2 + 1)\*arccos(x)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^2+1}}{(x^2-1)\arccos(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccos(x)/(-x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-x^2 + 1)\*x/((x^2 - 1)\*arccos(x)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(x-1)(x+1)} \arccos(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/acos(x)/(-x\*\*2+1)\*\*(1/2),x)

[Out] Integral(x/(sqrt(-(x - 1)\*(x + 1))\*acos(x)), x)

---

**Giac [A]** time = 1.37826, size = 7, normalized size = 1.4

$$-\text{Ci}(\arccos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccos(x)/(-x^2+1)^(1/2),x, algorithm="giac")

[Out] -cos\_integral(arccos(x))

$$3.117 \quad \int \frac{\cos^{-1}(\sqrt{1+bx^2})^n}{\sqrt{1+bx^2}} dx$$

**Optimal.** Leaf size=39

$$-\frac{\sqrt{-bx^2} \cos^{-1}(\sqrt{bx^2+1})^{n+1}}{b(n+1)x}$$

[Out] -((Sqrt[-(b\*x^2)]\*ArcCos[Sqrt[1 + b\*x^2]]^(1 + n))/(b\*(1 + n)\*x))

**Rubi [A]** time = 0.0653671, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {4835, 4642}

$$-\frac{\sqrt{-bx^2} \cos^{-1}(\sqrt{bx^2+1})^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[Sqrt[1 + b\*x^2]]^n/Sqrt[1 + b\*x^2], x]

[Out] -((Sqrt[-(b\*x^2)]\*ArcCos[Sqrt[1 + b\*x^2]]^(1 + n))/(b\*(1 + n)\*x))

#### Rule 4835

Int[ArcCos[Sqrt[1 + (b\_.)\*(x\_)^2]]^(n\_.)/Sqrt[1 + (b\_.)\*(x\_)^2], x\_Symbol] :> Dist[Sqrt[-(b\*x^2)]/(b\*x), Subst[Int[ArcCos[x]^n/Sqrt[1 - x^2], x], x, Sqrt[1 + b\*x^2]], x] /; FreeQ[{b, n}, x]

#### Rule 4642

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.)^(n\_.)/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] :> -Simp[(a + b\*ArcCos[c\*x])^(n + 1)/(b\*c\*Sqrt[d]\*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{\cos^{-1}(\sqrt{1+bx^2})^n}{\sqrt{1+bx^2}} dx &= \frac{\sqrt{-bx^2} \text{Subst}\left(\int \frac{\cos^{-1}(x)^n}{\sqrt{1-x^2}} dx, x, \sqrt{1+bx^2}\right)}{bx} \\ &= -\frac{\sqrt{-bx^2} \cos^{-1}(\sqrt{1+bx^2})^{1+n}}{b(1+n)x} \end{aligned}$$

**Mathematica [A]** time = 0.0458177, size = 39, normalized size = 1.

$$-\frac{\sqrt{-bx^2} \cos^{-1}(\sqrt{bx^2+1})^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[Sqrt[1 + b\*x^2]]^n/Sqrt[1 + b\*x^2], x]

[Out] -((Sqrt[-(b\*x^2)]\*ArcCos[Sqrt[1 + b\*x^2]]^(1 + n))/(b\*(1 + n)\*x))

**Maple [F]** time = 0.193, size = 0, normalized size = 0.

$$\int \left( \arccos \left( \sqrt{bx^2 + 1} \right) \right)^n \frac{1}{\sqrt{bx^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos((b\*x^2+1)^(1/2))^n/(b\*x^2+1)^(1/2), x)

[Out] int(arccos((b\*x^2+1)^(1/2))^n/(b\*x^2+1)^(1/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos((b\*x^2+1)^(1/2))^n/(b\*x^2+1)^(1/2), x, algorithm="maxima")

[Out] Exception raised: RuntimeError

**Fricas [A]** time = 2.99523, size = 107, normalized size = 2.74

$$\frac{\sqrt{-bx^2} \arccos \left( \sqrt{bx^2 + 1} \right)^n \arccos \left( \sqrt{bx^2 + 1} \right)}{(bn + b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos((b\*x^2+1)^(1/2))^n/(b\*x^2+1)^(1/2), x, algorithm="fricas")

[Out] -sqrt(-b\*x^2)\*arccos(sqrt(b\*x^2 + 1))^n\*arccos(sqrt(b\*x^2 + 1))/((b\*n + b)\*x)

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos((b\*x\*\*2+1)\*\*(1/2))\*\*n/(b\*x\*\*2+1)\*\*(1/2), x)

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(\sqrt{bx^2 + 1})^n}{\sqrt{bx^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccos(sqrt(b*x^2 + 1))^n/sqrt(b*x^2 + 1), x)
```

$$3.118 \quad \int \frac{1}{\sqrt{1+bx^2} \cos^{-1}(\sqrt{1+bx^2})} dx$$

**Optimal.** Leaf size=31

$$-\frac{\sqrt{-bx^2} \log\left(\cos^{-1}\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

[Out]  $-\left(\text{Sqrt}[-(b*x^2)]*\text{Log}[\text{ArcCos}[\text{Sqrt}[1 + b*x^2]]]\right)/(b*x)$

**Rubi [A]** time = 0.0608216, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {4835, 4640}

$$-\frac{\sqrt{-bx^2} \log\left(\cos^{-1}\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(\text{Sqrt}[1 + b*x^2]*\text{ArcCos}[\text{Sqrt}[1 + b*x^2]]), x]$

[Out]  $-\left(\text{Sqrt}[-(b*x^2)]*\text{Log}[\text{ArcCos}[\text{Sqrt}[1 + b*x^2]]]\right)/(b*x)$

**Rule 4835**

$\text{Int}[\text{ArcCos}[\text{Sqrt}[1 + (b_.)*(x_)^2]]^{(n_.)}/\text{Sqrt}[1 + (b_.)*(x_)^2], x\_Symbol]$   
 $:\> \text{Dist}[\text{Sqrt}[-(b*x^2)]/(b*x), \text{Subst}[\text{Int}[\text{ArcCos}[x]^n/\text{Sqrt}[1 - x^2], x], x, \text{Sqrt}[1 + b*x^2]], x] /;$   $\text{FreeQ}\{b, n\}, x]$

**Rule 4640**

$\text{Int}[1/(((a_.) + \text{ArcCos}[(c_.)*(x_)])*(b_.))*\text{Sqrt}[(d_) + (e_.)*(x_)^2]), x\_Symbol]$   
 $:\> -\text{Simp}[\text{Log}[a + b*\text{ArcCos}[c*x]]/(b*c*\text{Sqrt}[d]), x] /;$   $\text{FreeQ}\{a, b, c, d, e\}, x]$  &&  $\text{EqQ}[c^2*d + e, 0]$  &&  $\text{GtQ}[d, 0]$

**Rubi steps**

$$\int \frac{1}{\sqrt{1+bx^2} \cos^{-1}(\sqrt{1+bx^2})} dx = \frac{\sqrt{-bx^2} \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2} \cos^{-1}(x)} dx, x, \sqrt{1+bx^2}\right)}{bx}$$

$$= -\frac{\sqrt{-bx^2} \log\left(\cos^{-1}\left(\sqrt{1+bx^2}\right)\right)}{bx}$$

**Mathematica [A]** time = 0.0239597, size = 25, normalized size = 0.81

$$\frac{x \log\left(\cos^{-1}\left(\sqrt{bx^2+1}\right)\right)}{\sqrt{-bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[1 + b\*x^2]\*ArcCos[Sqrt[1 + b\*x^2]]),x]

[Out] (x\*Log[ArcCos[Sqrt[1 + b\*x^2]]])/Sqrt[-(b\*x^2)]

**Maple [F]** time = 0.184, size = 0, normalized size = 0.

$$\int \left( \arccos\left(\sqrt{bx^2 + 1}\right) \right)^{-1} \frac{1}{\sqrt{bx^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arccos((b\*x^2+1)^(1/2))/(b\*x^2+1)^(1/2),x)

[Out] int(1/arccos((b\*x^2+1)^(1/2))/(b\*x^2+1)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + 1} \arccos\left(\sqrt{bx^2 + 1}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos((b\*x^2+1)^(1/2))/(b\*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b\*x^2 + 1)\*arccos(sqrt(b\*x^2 + 1))), x)

**Fricas [A]** time = 2.57099, size = 68, normalized size = 2.19

$$-\frac{\sqrt{-bx^2} \log\left(\arccos\left(\sqrt{bx^2 + 1}\right)\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos((b\*x^2+1)^(1/2))/(b\*x^2+1)^(1/2),x, algorithm="fricas")

[Out] -sqrt(-b\*x^2)\*log(arccos(sqrt(b\*x^2 + 1)))/(b\*x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + 1} \operatorname{acos}\left(\sqrt{bx^2 + 1}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/acos((b\*x\*\*2+1)\*\*(1/2))/(b\*x\*\*2+1)\*\*(1/2),x)

[Out] Integral(1/(sqrt(b\*x\*\*2 + 1)\*acos(sqrt(b\*x\*\*2 + 1))), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + 1} \arccos(\sqrt{bx^2 + 1})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + 1)*arccos(sqrt(b*x^2 + 1))), x)
```





# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*   is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*   antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25       "C"],
26     If[FreeQ[result,Integrate] && FreeQ[result,Int],
27       "C",
28       "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```



```

101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 # if leaf size is "too large". Set at 500,000
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 # see problem 156, file Apostol_Problems
11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
14     debug:=false;
15
16     leaf_count_result:=leafcount(result);
17     #do NOT call ExpnType() if leaf size is too large. Recursion problem
18     if leaf_count_result > 500000 then
19         return "B";
20     fi;
21
22     leaf_count_optimal:=leafcount(optimal);
23
24     ExpnType_result:=ExpnType(result);
25     ExpnType_optimal:=ExpnType(optimal);
26
27     if debug then
28         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
29             ExpnType_optimal);
30     fi;
31
32 # If result and optimal are mathematical expressions,
33 # GradeAntiderivative[result,optimal] returns
34 # "F" if the result fails to integrate an expression that
35 # is integrable
36 # "C" if result involves higher level functions than necessary
37 # "B" if result is more than twice the size of the optimal
38 # antiderivative
39 # "A" if result can be considered optimal
40
41 #This check below actually is not needed, since I only
42 #call this grading only for passed integrals. i.e. I check
43 #for "F" before calling this. But no harm of keeping it here.
44 #just in case.
45
46 if not type(result,freeof('int')) then
47     return "F";
48 end if;
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then
56             if debug then

```

```

57         print("both result and optimal complex");
58         fi;
59         #both result and optimal complex
60         if leaf_count_result<=2*leaf_count_optimal then
61             return "A";
62         else
63             return "B";
64         end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do
not as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103     return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)

```

```

119   if type(expn,'atomic') then
120       1
121   elif type(expn,'list') then
122       apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124       if type(op(1,expn),'rational') then
125           1
126       else
127           max(2,ExpnType(op(1,expn)))
128       end if
129   elif type(expn,'^^') then
130       if type(op(2,expn),'integer') then
131           ExpnType(op(1,expn))
132       elif type(op(2,expn),'rational') then
133           if type(op(1,expn),'rational') then
134               1
135           else
136               max(2,ExpnType(op(1,expn)))
137           end if
138       else
139           max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140       end if
141   elif type(expn,'+`') or type(expn,'*`') then
142       max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144       max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146       max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148       max(5,apply(max,map(ExpnType,[op(expn)])))
149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159     member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168     member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181     member(func,[AppellF1])

```

```

182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #     Port of original Maple grading function by
3 #     Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #     added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:
42         if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True

```

```

44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,``^`)
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn
72 )))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or
77 type(expn,``*`)
78     m1 = expnType(expn.args[0])
79     m2 = expnType(list(expn.args[1:]))
80     return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82     return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84     m1 = max(map(expnType, list(expn.args)))
85     return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88     m1 = max(map(expnType, list(expn.args)))
89     return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
90     elif is_appell_function(expn.func):
91     m1 = max(map(expnType, list(expn.args)))
92     return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
93     elif isinstance(expn,RootSum):
94     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
95 ,Apply[List,expn]],7]],
96     return max(7,m1)
97     elif str(expn).find("Integral") != -1:
98     m1 = max(map(expnType, list(expn.args)))
99     return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
100     else:
101     return 9
102
103 #main function
104 def grade_antiderivative(result,optimal):
105
106     leaf_count_result = leaf_count(result)

```

```

102 leaf_count_optimal = leaf_count(optimal)
103
104 expnType_result = expnType(result)
105 expnType_optimal = expnType(optimal)
106
107 if str(result).find("Integral") != -1:
108     return "F"
109
110 if expnType_result <= expnType_optimal:
111     if result.has(I):
112         if optimal.has(I): #both result and optimal complex
113             if leaf_count_result <= 2*leaf_count_optimal:
114                 return "A"
115             else:
116                 return "B"
117         else: #result contains complex but optimal is not
118             return "C"
119     else: # result do not contain complex, this assumes optimal do not as
well
120         if leaf_count_result <= 2*leaf_count_optimal:
121             return "A"
122         else:
123             return "B"
124 else:
125     return "C"

```

#### 4.0.4 SageMath grading function

```

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by
2 #     Albert Rich to use with Sagemath. This is used to
3 #     grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #     'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
(flatten(tree(anti))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34         #since this estimate of leaf count is bit lower than

```



```

35         #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow: #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func , " is special_function")
83         else:
84             print ("func ", func , " is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','
91     hypergeometric_U']
92
93 def is_appell_function(func):
94     return func.name() in ['hypergeometric'] #[appellf1] can't find this in
95     sagemath

```

```

95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
104             return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list: #isinstance(expn,list):
121         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
127     elif expn.operator() == operator.pow: #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
129             return expnType(expn.operands()[0]) #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
132                 return 1
133             else:
134                 return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141     elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

145     return max(4,m1)    #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
148         return max(5,m1)    #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
151         return max(6,m1)    #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153         #is checked before calling the grading function that is passed.
154         #but kept it here.
155         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
156         return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"

```