Computer algebra independent integration tests

5-Inverse-trig-functions/5.2-Inverse-cosine/5.2.5-Inverse-cosine-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [118]. This is test number [147].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (118)	% 0. (0)
Mathematica	% 95.76 (113)	% 4.24 (5)
Maple	% 66.1 (78)	% 33.9 (40)
Maxima	% 22.03 (26)	% 77.97 (92)
Fricas	% 40.68 (48)	% 59.32 (70)
Sympy	% 26.27 (31)	% 73.73 (87)
Giac	% 42.37 (50)	% 57.63 (68)

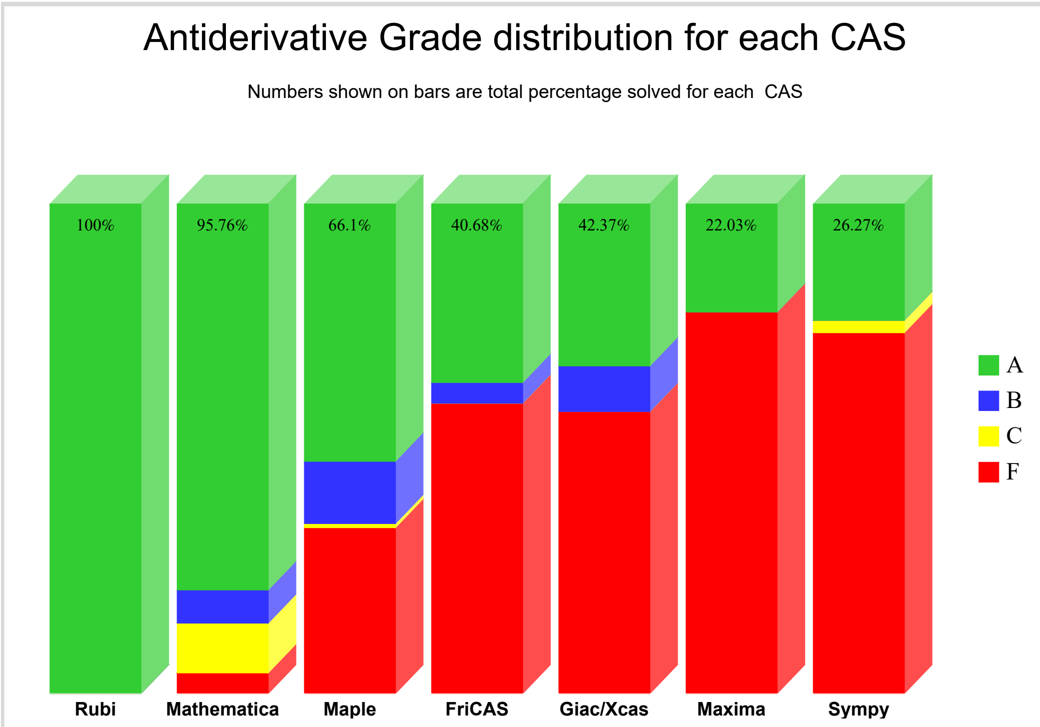
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

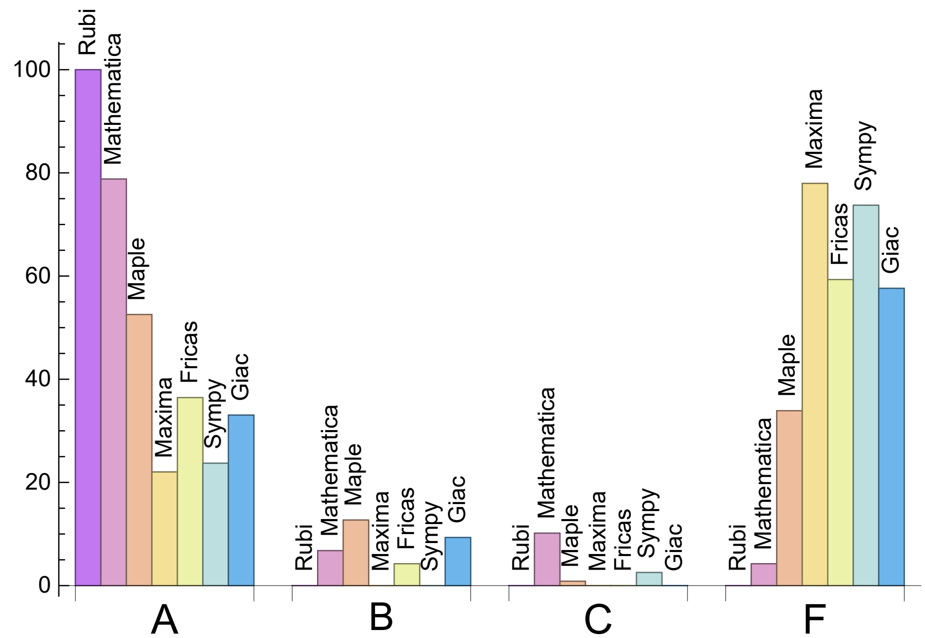
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	78.81	6.78	10.17	4.24
Maple	52.54	12.71	0.85	33.9
Maxima	22.03	0.	0.	77.97
Fricas	36.44	4.24	0.	59.32
Sympy	23.73	0.	2.54	73.73
Giac	33.05	9.32	0.	57.63

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.22	186.74	0.96	86.5	1.
Mathematica	0.93	262.63	1.12	85.	0.95
Maple	0.27	376.65	1.25	74.	1.16
Maxima	1.26	48.81	1.08	49.5	1.12
Fricas	2.31	191.46	2.74	104.5	2.18
Sympy	8.3	67.48	1.21	49.	1.28
Giac	1.24	106.2	1.6	71.	1.46

1.4 list of integrals that has no closed form antiderivative

{19, 23, 101, 105, 106}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {4, 5, 9, 13, 17, 18, 21, 28, 37, 38, 39, 40, 41, 42, 43, 44, 69, 77, 78, 79, 84, 85, 86, 112, 113, 115}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

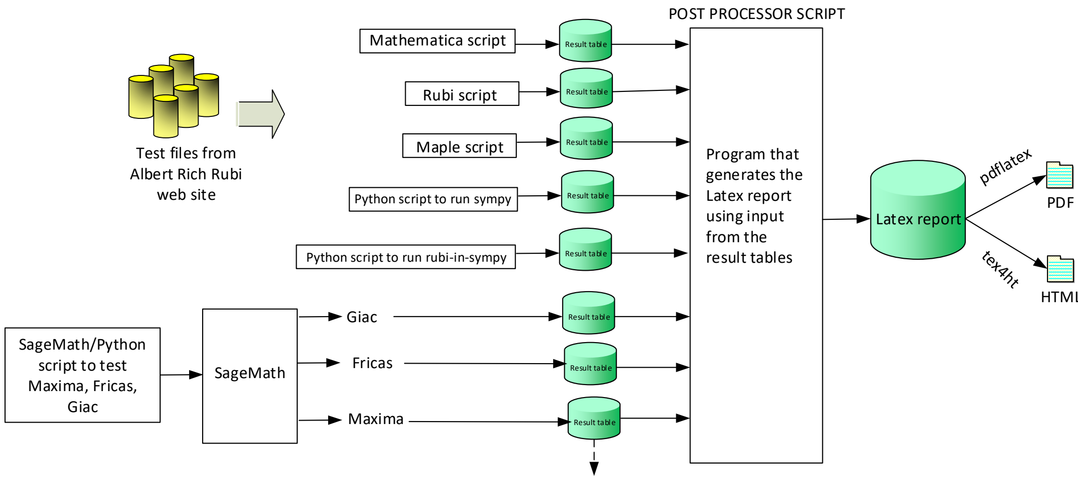
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 19, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 45, 46, 47, 49, 51, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 118 }

B grade: { 9, 13, 17, 18, 21, 55, 69, 114 }

C grade: { 37, 38, 39, 40, 41, 42, 43, 44, 48, 50, 52, 115 }

F grade: { 20, 102, 103, 104, 107}

2.1.3 Maple

A grade: { 4, 10, 11, 17, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 76, 83, 101, 103, 104, 105, 106, 107, 114, 115, 116 }

B grade: { 1, 2, 3, 6, 7, 8, 9, 12, 13, 14, 15, 16, 18, 52, 102 }

C grade: { 5 }

F grade: { 20, 21, 22, 51, 70, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 108, 109, 110, 111, 112, 113, 117, 118 }

2.1.4 Maxima

A grade: { 23, 27, 46, 47, 49, 53, 54, 55, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 71, 72, 76, 83, 101, 105, 106

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 48, 50, 51, 52, 56, 63, 69, 70, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

2.1.5 FriCAS

A grade: { 19, 23, 24, 25, 26, 27, 32, 33, 46, 47, 49, 53, 54, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 80, 81, 82, 83, 101, 105, 106, 108, 109, 110, 111, 117, 118 }

B grade: { 29, 30, 31, 55, 114 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 28, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 48, 50, 51, 52, 56, 63, 69, 70, 77, 78, 79, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 107, 112, 113, 115, 116 }

2.1.6 Sympy

A grade: { 23, 24, 25, 26, 27, 32, 33, 46, 47, 48, 49, 50, 52, 54, 55, 57, 59, 60, 61, 62, 65, 66, 68, 71, 108, 109, 110, 111 }

B grade: { }

C grade: { 53, 58, 64 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 56, 63, 67, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116, 117, 118 }

2.1.7 Giac

A grade: { 19, 23, 25, 26, 27, 29, 32, 33, 34, 35, 36, 43, 44, 46, 47, 49, 53, 54, 55, 57, 58, 59, 60, 61, 62, 64, 68, 71, 72, 76, 83, 101, 105, 106, 108, 109, 110, 111, 116 }

B grade: { 24, 30, 31, 37, 38, 39, 40, 65, 66, 67, 115 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 28, 41, 42, 45, 48, 50, 51, 52, 56, 63, 69, 70, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 107, 112, 113, 114, 117, 118 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	670	670	442	1285	0	0	0	0
normalized size	1	1.	0.66	1.92	0.	0.	0.	0.
time (sec)	N/A	0.705	1.204	0.848	0.	0.	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	450	450	320	912	0	0	0	0
normalized size	1	1.	0.71	2.03	0.	0.	0.	0.
time (sec)	N/A	0.53	1.314	0.533	0.	0.	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	219	491	0	0	0	0
normalized size	1	1.	0.92	2.06	0.	0.	0.	0.
time (sec)	N/A	0.251	1.474	0.481	0.	0.	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	725	725	1095	1209	0	0	0	0
normalized size	1	1.	1.51	1.67	0.	0.	0.	0.
time (sec)	N/A	1.859	3.38	0.437	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	851	851	1130	1573	0	0	0	0
normalized size	1	1.	1.33	1.85	0.	0.	0.	0.
time (sec)	N/A	2.694	9.482	0.476	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	959	959	910	1734	0	0	0	0
normalized size	1	1.	0.95	1.81	0.	0.	0.	0.
time (sec)	N/A	0.962	4.572	0.811	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	680	680	591	1252	0	0	0	0
normalized size	1	1.	0.87	1.84	0.	0.	0.	0.
time (sec)	N/A	0.735	2.143	0.579	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	337	698	0	0	0	0
normalized size	1	1.	0.91	1.89	0.	0.	0.	0.
time (sec)	N/A	0.335	1.566	0.49	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1064	1064	3034	2132	0	0	0	0
normalized size	1	1.	2.85	2.	0.	0.	0.	0.
time (sec)	N/A	2.28	12.504	0.398	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1281	1281	1144	2236	0	0	0	0
normalized size	1	1.	0.89	1.75	0.	0.	0.	0.
time (sec)	N/A	1.177	7.462	0.959	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	940	940	794	1633	0	0	0	0
normalized size	1	1.	0.84	1.74	0.	0.	0.	0.
time (sec)	N/A	0.959	4.516	0.702	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	517	517	526	931	0	0	0	0
normalized size	1	1.	1.02	1.8	0.	0.	0.	0.
time (sec)	N/A	0.403	2.901	0.602	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1637	1637	6216	4692	0	0	0	0
normalized size	1	1.	3.8	2.87	0.	0.	0.	0.
time (sec)	N/A	2.73	19.605	0.531	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	450	450	342	845	0	0	0	0
normalized size	1	1.	0.76	1.88	0.	0.	0.	0.
time (sec)	N/A	0.585	1.182	0.605	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	266	549	0	0	0	0
normalized size	1	1.	0.99	2.03	0.	0.	0.	0.
time (sec)	N/A	0.436	0.757	0.356	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	172	235	0	0	0	0
normalized size	1	1.	1.35	1.85	0.	0.	0.	0.
time (sec)	N/A	0.223	0.413	0.299	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	370	370	930	487	0	0	0	0
normalized size	1	1.	2.51	1.32	0.	0.	0.	0.
time (sec)	N/A	0.606	2.056	0.196	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	496	496	1108	1622	0	0	0	0
normalized size	1	1.	2.23	3.27	0.	0.	0.	0.
time (sec)	N/A	0.722	4.987	0.305	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	0.158	5.372	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	496	496	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.791	26.654	4.708	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	374	374	1248	0	0	0	0	0
normalized size	1	1.	3.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.611	5.566	3.134	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	246	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.338	0.022	0.015	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.2	0.199	2.447	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	104	235	0	219	255	327
normalized size	1	1.	0.76	1.72	0.	1.6	1.86	2.39
time (sec)	N/A	0.194	0.1	0.02	0.	2.479	1.757	1.316

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	83	161	0	173	170	211
normalized size	1	1.	0.88	1.71	0.	1.84	1.81	2.24
time (sec)	N/A	0.115	0.083	0.003	0.	2.421	0.82	1.306

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	69	78	0	135	104	119
normalized size	1	1.	0.86	0.98	0.	1.69	1.3	1.49
time (sec)	N/A	0.073	0.049	0.004	0.	2.551	0.361	1.322

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	47	33	43	92	46	43
normalized size	1	1.	1.31	0.92	1.19	2.56	1.28	1.19
time (sec)	N/A	0.017	0.037	0.002	1.41	2.507	0.174	1.281

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	228	199	0	0	0	0
normalized size	1	1.	1.29	1.12	0.	0.	0.	0.
time (sec)	N/A	0.276	0.192	0.104	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	79	77	0	857	0	107
normalized size	1	1.	1.25	1.22	0.	13.6	0.	1.7
time (sec)	N/A	0.076	0.055	0.003	0.	3.011	0.	1.316

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	126	118	0	1121	0	327
normalized size	1	1.	1.22	1.15	0.	10.88	0.	3.17
time (sec)	N/A	0.111	0.175	0.004	0.	2.947	0.	1.336

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	168	227	0	1328	0	752
normalized size	1	1.	1.17	1.58	0.	9.22	0.	5.22
time (sec)	N/A	0.179	0.184	0.005	0.	3.544	0.	1.356

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	74	71	0	170	109	105
normalized size	1	1.	0.9	0.87	0.	2.07	1.33	1.28
time (sec)	N/A	0.082	0.036	0.054	0.	2.559	0.693	1.43

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	49	48	0	130	63	70
normalized size	1	1.	1.04	1.02	0.	2.77	1.34	1.49
time (sec)	N/A	0.054	0.023	0.049	0.	2.336	0.283	1.342

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	0	0	16
normalized size	1	1.	1.	1.08	0.	0.	0.	1.33
time (sec)	N/A	0.022	0.026	0.046	0.	0.	0.	1.343

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	37	0	0	0	51
normalized size	1	1.	1.	0.92	0.	0.	0.	1.27
time (sec)	N/A	0.078	0.051	0.049	0.	0.	0.	1.33

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	53	0	0	0	77
normalized size	1	1.	1.	0.82	0.	0.	0.	1.18
time (sec)	N/A	0.081	0.05	0.051	0.	0.	0.	1.298

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	90	140	0	0	0	282
normalized size	1	1.	0.81	1.26	0.	0.	0.	2.54
time (sec)	N/A	0.147	0.049	0.092	0.	0.	0.	1.57

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	76	105	0	0	0	221
normalized size	1	1.	0.85	1.18	0.	0.	0.	2.48
time (sec)	N/A	0.092	0.035	0.082	0.	0.	0.	1.624

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	90	66	0	0	0	158
normalized size	1	1.	1.64	1.2	0.	0.	0.	2.87
time (sec)	N/A	0.081	0.039	0.073	0.	0.	0.	1.378

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	33	33	78	28	0	0	0	97
normalized size	1	1.	2.36	0.85	0.	0.	0.	2.94
time (sec)	N/A	0.03	0.032	0.055	0.	0.	0.	1.575

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	97	84	0	0	0	0
normalized size	1	1.	1.52	1.31	0.	0.	0.	0.
time (sec)	N/A	0.086	0.054	0.075	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	139	120	0	0	0	0
normalized size	1	1.	1.54	1.33	0.	0.	0.	0.
time (sec)	N/A	0.092	0.291	0.079	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	128	89	0	0	0	231
normalized size	1	1.	1.21	0.84	0.	0.	0.	2.18
time (sec)	N/A	0.13	0.105	0.071	0.	0.	0.	2.344

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	133	90	0	0	0	236
normalized size	1	1.	1.23	0.83	0.	0.	0.	2.19
time (sec)	N/A	0.116	0.102	0.064	0.	0.	0.	2.944

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	59	85	0	0	0	0
normalized size	1	1.	0.87	1.25	0.	0.	0.	0.
time (sec)	N/A	0.081	0.041	0.089	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	33	41	81	48	36
normalized size	1	1.	0.88	0.97	1.21	2.38	1.41	1.06
time (sec)	N/A	0.03	0.016	0.083	1.49	2.381	21.528	1.323

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	48	65	107	93	48	62
normalized size	1	1.	0.94	1.27	2.1	1.82	0.94	1.22
time (sec)	N/A	0.039	0.028	0.053	1.476	2.53	1.032	1.343

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	63	79	0	0	48	0
normalized size	1	1.	1.15	1.44	0.	0.	0.87	0.
time (sec)	N/A	0.028	0.155	0.007	0.	0.	1.427	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	42	68	32	42
normalized size	1	1.	1.	0.91	1.2	1.94	0.91	1.2
time (sec)	N/A	0.024	0.016	0.002	1.448	2.378	0.23	1.328

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	34	65	0	0	44	0
normalized size	1	1.	0.79	1.51	0.	0.	1.02	0.
time (sec)	N/A	0.032	0.005	0.007	0.	0.	1.111	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	56	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.031	0.066	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	40	57	0	0	44	0
normalized size	1	1.	1.38	1.97	0.	0.	1.52	0.
time (sec)	N/A	0.014	0.039	0.006	0.	0.	1.211	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	61	56	97	207	99	95
normalized size	1	1.	1.05	0.97	1.67	3.57	1.71	1.64
time (sec)	N/A	0.036	0.051	0.01	1.462	2.539	5.296	1.29

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	39	38	73	49	58
normalized size	1	1.	0.97	1.15	1.12	2.15	1.44	1.71
time (sec)	N/A	0.017	0.021	0.004	1.442	2.563	2.684	1.248

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	84	30	61	140	29	58
normalized size	1	1.	3.11	1.11	2.26	5.19	1.07	2.15
time (sec)	N/A	0.017	0.101	0.006	1.454	2.521	2.271	1.262

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	77	0	0	0	0
normalized size	1	1.	1.	1.28	0.	0.	0.	0.
time (sec)	N/A	0.055	0.021	0.055	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	32	42	69	26	42
normalized size	1	1.	1.	1.07	1.4	2.3	0.87	1.4
time (sec)	N/A	0.022	0.019	0.001	1.438	2.394	2.34	1.228

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	47	104	97	102	84
normalized size	1	1.	0.98	0.92	2.04	1.9	2.	1.65
time (sec)	N/A	0.033	0.024	0.006	1.465	2.47	5.305	1.28

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	47	55	66	104	102	88
normalized size	1	1.	0.84	0.98	1.18	1.86	1.82	1.57
time (sec)	N/A	0.038	0.031	0.006	1.445	2.462	6.366	1.314

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	46	53	70	115	73	70
normalized size	1	1.	0.59	0.68	0.9	1.47	0.94	0.9
time (sec)	N/A	0.028	0.038	0.004	1.436	2.632	10.162	1.128

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	41	41	54	99	58	54
normalized size	1	1.	0.68	0.68	0.9	1.65	0.97	0.9
time (sec)	N/A	0.019	0.029	0.003	1.465	2.585	3.901	1.277

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	38	26	34	78	29	34
normalized size	1	1.	1.03	0.7	0.92	2.11	0.78	0.92
time (sec)	N/A	0.011	0.015	0.002	1.524	2.612	0.337	1.193

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	59	0	0	0	0
normalized size	1	1.	0.96	1.05	0.	0.	0.	0.
time (sec)	N/A	0.056	0.026	0.002	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	22	28	59	44	54
normalized size	1	1.	0.89	0.81	1.04	2.19	1.63	2.
time (sec)	N/A	0.012	0.016	0.001	1.499	2.561	4.222	1.179

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	43	35	46	84	44	100
normalized size	1	1.	0.86	0.7	0.92	1.68	0.88	2.
time (sec)	N/A	0.018	0.021	0.004	1.473	2.628	24.914	1.281

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	37	47	62	97	60	143
normalized size	1	1.	0.54	0.69	0.91	1.43	0.88	2.1
time (sec)	N/A	0.022	0.038	0.003	1.476	2.631	150.038	1.312

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	42	59	78	111	0	186
normalized size	1	1.	0.49	0.69	0.91	1.29	0.	2.16
time (sec)	N/A	0.028	0.044	0.003	1.493	2.326	0.	1.285

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	59	20	26
normalized size	1	1.	1.	0.8	1.04	2.36	0.8	1.04
time (sec)	N/A	0.019	0.008	0.003	1.482	2.533	0.365	1.274

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	68	68	141	89	0	0	0	0
normalized size	1	1.	2.07	1.31	0.	0.	0.	0.
time (sec)	N/A	0.061	0.137	0.04	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	56	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.031	0.058	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	43	40	53	105	61	53
normalized size	1	1.	0.91	0.85	1.13	2.23	1.3	1.13
time (sec)	N/A	0.051	0.028	0.003	1.612	2.439	1.143	1.261

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	43	0	55	132	0	55
normalized size	1	1.	0.9	0.	1.15	2.75	0.	1.15
time (sec)	N/A	0.054	0.041	0.03	1.606	2.646	0.	1.312

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	249	0	0	474	0	0
normalized size	1	1.	1.96	0.	0.	3.73	0.	0.
time (sec)	N/A	0.029	0.233	0.121	0.	2.605	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	162	0	0	328	0	0
normalized size	1	1.	1.47	0.	0.	2.98	0.	0.
time (sec)	N/A	0.057	0.126	0.116	0.	2.362	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	98	0	0	205	0	0
normalized size	1	1.	1.56	0.	0.	3.25	0.	0.
time (sec)	N/A	0.012	0.066	0.117	0.	2.5	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	41	45	61	103	0	78
normalized size	1	1.	0.95	1.05	1.42	2.4	0.	1.81
time (sec)	N/A	0.037	0.027	0.005	1.566	2.303	0.	1.349

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	99	99	85	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.104	0.066	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	133	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.332	0.066	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	147	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.246	0.067	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	249	0	0	474	0	0
normalized size	1	1.	1.96	0.	0.	3.73	0.	0.
time (sec)	N/A	0.028	0.239	0.117	0.	2.657	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	162	0	0	328	0	0
normalized size	1	1.	1.47	0.	0.	2.98	0.	0.
time (sec)	N/A	0.055	0.135	0.119	0.	2.573	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	98	0	0	205	0	0
normalized size	1	1.	1.56	0.	0.	3.25	0.	0.
time (sec)	N/A	0.012	0.066	0.118	0.	2.505	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	41	45	61	103	0	72
normalized size	1	1.	0.95	1.05	1.42	2.4	0.	1.67
time (sec)	N/A	0.036	0.029	0.004	1.527	2.682	0.	1.3

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	85	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.1	0.066	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	131	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.335	0.065	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	149	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.213	0.068	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	256	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	2.617	0.076	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	200	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.713	0.066	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	157	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.085	0.063	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	114	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.162	0.069	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	177	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.389	0.067	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	234	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.721	0.067	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	308	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.547	0.068	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	256	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	2.146	0.066	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	200	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.603	0.066	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	157	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.113	0.066	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	115	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.158	0.07	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	161	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.321	0.064	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	233	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.585	0.065	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	309	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.553	0.066	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.097	0.647	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	279	279	0	707	0	0	0	0
normalized size	1	1.	0.	2.53	0.	0.	0.	0.
time (sec)	N/A	0.207	0.296	0.974	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	207	0	401	0	0	0	0
normalized size	1	1.	0.	1.94	0.	0.	0.	0.
time (sec)	N/A	0.174	0.695	0.007	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	0	171	0	0	0	0
normalized size	1	1.	0.	1.21	0.	0.	0.	0.
time (sec)	N/A	0.104	0.372	0.005	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.101	0.344	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	1.693	0.342	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F(-1)	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	0	107	0	0	0	0
normalized size	1	1.	0.	1.27	0.	0.	0.	0.
time (sec)	N/A	0.066	0.947	0.004	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	50	0	0	130	105	111
normalized size	1	1.	0.62	0.	0.	1.6	1.3	1.37
time (sec)	N/A	0.064	0.139	0.01	0.	2.619	4.836	1.325

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	50	0	0	107	85	93
normalized size	1	1.	0.61	0.	0.	1.3	1.04	1.13
time (sec)	N/A	0.061	0.107	0.009	0.	2.566	1.716	1.376

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	30	0	0	89	58	59
normalized size	1	1.	0.73	0.	0.	2.17	1.41	1.44
time (sec)	N/A	0.034	0.037	0.007	0.	2.637	0.539	1.238

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	32	0	0	68	37	42
normalized size	1	1.	0.82	0.	0.	1.74	0.95	1.08
time (sec)	N/A	0.014	0.033	0.004	0.	2.6	0.201	1.277

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	79	0	0	0	0	0
normalized size	1	1.	1.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.056	0.007	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	55	0	0	0	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.062	0.007	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	141	45	0	305	0	0
normalized size	1	1.	2.94	0.94	0.	6.35	0.	0.
time (sec)	N/A	0.033	0.163	0.008	0.	2.529	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	26	26	56	21	0	0	0	78
normalized size	1	1.	2.15	0.81	0.	0.	0.	3.
time (sec)	N/A	0.071	0.085	0.106	0.	0.	0.	1.398

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	0	0	0	7
normalized size	1	1.	1.	1.2	0.	0.	0.	1.4
time (sec)	N/A	0.061	0.04	0.067	0.	0.	0.	1.378

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	0	107	0	0
normalized size	1	1.	1.	0.	0.	2.74	0.	0.
time (sec)	N/A	0.065	0.046	0.193	0.	2.995	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	25	0	0	68	0	0
normalized size	1	1.	0.81	0.	0.	2.19	0.	0.
time (sec)	N/A	0.061	0.024	0.184	0.	2.571	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [50] had the largest ratio of [1.]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	16	12	1.	31	0.387
2	A	13	8	1.	31	0.258
3	A	8	6	1.	29	0.207
4	A	22	19	1.	31	0.613
5	A	35	22	1.	31	0.71
6	A	24	17	1.	31	0.548
7	A	20	12	1.	31	0.387
8	A	12	9	1.	29	0.31

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
9	A	29	23	1.	31	0.742
10	A	30	18	1.	31	0.581
11	A	26	15	1.	31	0.484
12	A	14	10	1.	29	0.345
13	A	37	28	1.	31	0.903
14	A	13	7	1.	31	0.226
15	A	9	7	1.	31	0.226
16	A	6	5	1.	29	0.172
17	A	10	7	1.	31	0.226
18	A	13	10	1.	31	0.323
19	A	0	0	0.	0	0.
20	A	13	9	1.	35	0.257
21	A	11	8	1.	33	0.242
22	A	9	7	1.	25	0.28
23	A	0	0	0.	0	0.
24	A	6	6	1.	10	0.6
25	A	5	5	1.	10	0.5
26	A	5	5	1.	8	0.625
27	A	3	3	1.	6	0.5
28	A	9	6	1.	10	0.6
29	A	4	4	1.	10	0.4
30	A	5	5	1.	10	0.5
31	A	6	6	1.	10	0.6
32	A	5	4	1.	8	0.5
33	A	4	4	1.	8	0.5
34	A	3	3	1.	8	0.375
35	A	4	4	1.	8	0.5
36	A	5	5	1.	8	0.625
37	A	7	6	1.	10	0.6
38	A	6	6	1.	10	0.6
39	A	5	5	1.	10	0.5
40	A	4	4	1.	10	0.4

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
41	A	5	5	1.	10	0.5
42	A	6	6	1.	10	0.6
43	A	7	7	1.	14	0.5
44	A	7	7	1.	15	0.467
45	A	7	7	1.	19	0.368
46	A	3	3	1.	14	0.214
47	A	5	5	1.	10	0.5
48	A	4	4	1.	10	0.4
49	A	3	3	1.	8	0.375
50	A	6	6	1.	6	1.
51	A	5	5	1.	10	0.5
52	A	3	3	1.	10	0.3
53	A	6	6	1.	10	0.6
54	A	3	3	1.	8	0.375
55	A	5	5	1.	6	0.833
56	A	5	5	1.	10	0.5
57	A	3	3	1.	10	0.3
58	A	5	5	1.	10	0.5
59	A	5	4	1.	10	0.4
60	A	8	6	1.	10	0.6
61	A	7	6	1.	8	0.75
62	A	6	6	1.	6	1.
63	A	5	5	1.	10	0.5
64	A	3	3	1.	10	0.3
65	A	4	4	1.	10	0.4
66	A	5	4	1.	10	0.4
67	A	6	4	1.	10	0.4
68	A	3	3	1.	12	0.25
69	A	5	5	1.	10	0.5
70	A	5	5	1.	10	0.5
71	A	4	4	1.	12	0.333
72	A	4	4	1.	14	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
73	A	3	2	1.	14	0.143
74	A	5	4	1.	14	0.286
75	A	2	2	1.	14	0.143
76	A	4	3	1.	12	0.25
77	A	1	1	1.	14	0.071
78	A	1	1	1.	14	0.071
79	A	2	2	1.	14	0.143
80	A	3	2	1.	14	0.143
81	A	5	4	1.	14	0.286
82	A	2	2	1.	14	0.143
83	A	4	3	1.	12	0.25
84	A	1	1	1.	14	0.071
85	A	1	1	1.	14	0.071
86	A	2	2	1.	14	0.143
87	A	2	2	1.	16	0.125
88	A	2	2	1.	16	0.125
89	A	1	1	1.	16	0.062
90	A	1	1	1.	16	0.062
91	A	1	1	1.	16	0.062
92	A	2	2	1.	16	0.125
93	A	2	2	1.	16	0.125
94	A	2	2	1.	16	0.125
95	A	2	2	1.	16	0.125
96	A	1	1	1.	16	0.062
97	A	1	1	1.	16	0.062
98	A	1	1	1.	16	0.062
99	A	2	2	1.	16	0.125
100	A	2	2	1.	16	0.125
101	A	0	0	0.	0	0.
102	A	8	8	1.	40	0.2
103	A	7	7	1.	40	0.175
104	A	6	7	1.	38	0.184

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
105	A	0	0	0.	0	0.
106	A	0	0	0.	0	0.
107	A	6	6	1.	10	0.6
108	A	6	4	1.	10	0.4
109	A	6	4	1.	10	0.4
110	A	5	4	1.	8	0.5
111	A	2	2	1.	6	0.333
112	A	6	5	1.	10	0.5
113	A	6	4	1.	10	0.4
114	A	6	6	1.	10	0.6
115	A	3	3	1.	19	0.158
116	A	2	2	1.	17	0.118
117	A	2	2	1.	26	0.077
118	A	2	2	1.	26	0.077

Chapter 3

Listing of integrals

3.1 $\int (f+gx)^3 \sqrt{d-c^2dx^2} \left(a+b\cos^{-1}(cx)\right) dx$

Optimal. Leaf size=670

$$-\frac{f^2g\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)}{c^2}+\frac{1}{2}f^3x\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)-\frac{f^3\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{4bc\sqrt{1-c^2x^2}}+$$

```
[Out] -((b*f^2*g*x*Sqrt[d - c^2*d*x^2])/(c*Sqrt[1 - c^2*x^2])) - (2*b*g^3*x*Sqrt[
d - c^2*d*x^2])/(15*c^3*Sqrt[1 - c^2*x^2]) + (b*c*f^3*x^2*Sqrt[d - c^2*d*x^
2])/(4*Sqrt[1 - c^2*x^2]) - (3*b*f*g^2*x^2*Sqrt[d - c^2*d*x^2])/(16*c*Sqrt[
1 - c^2*x^2]) + (b*c*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(3*Sqrt[1 - c^2*x^2]) -
 (b*g^3*x^3*Sqrt[d - c^2*d*x^2])/(45*c*Sqrt[1 - c^2*x^2]) + (3*b*c*f*g^2*x^
4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (b*c*g^3*x^5*Sqrt[d - c^2*d
*x^2])/(25*Sqrt[1 - c^2*x^2]) + (f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*
x]))/2 - (3*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(8*c^2) + (3*f
*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 - (f^2*g*(1 - c^2*x^2)*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/c^2 - (g^3*(1 - c^2*x^2)*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*c^4) + (g^3*(1 - c^2*x^2)^2*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^4) - (f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
os[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2]) - (3*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCos[c*x])^2)/(16*b*c^3*Sqrt[1 - c^2*x^2])
```

Rubi [A] time = 0.705186, antiderivative size = 670, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$

= 0.387, Rules used = {4778, 4764, 4648, 4642, 30, 4678, 4698, 4708, 266, 43, 4690, 12}

$$-\frac{f^2g\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)}{c^2}+\frac{1}{2}f^3x\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)-\frac{f^3\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{4bc\sqrt{1-c^2x^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

```
[Out] -((b*f^2*g*x*Sqrt[d - c^2*d*x^2])/(c*Sqrt[1 - c^2*x^2])) - (2*b*g^3*x*Sqrt[
d - c^2*d*x^2])/(15*c^3*Sqrt[1 - c^2*x^2]) + (b*c*f^3*x^2*Sqrt[d - c^2*d*x^
2])/(4*Sqrt[1 - c^2*x^2]) - (3*b*f*g^2*x^2*Sqrt[d - c^2*d*x^2])/(16*c*Sqrt[
1 - c^2*x^2]) + (b*c*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(3*Sqrt[1 - c^2*x^2]) -
 (b*g^3*x^3*Sqrt[d - c^2*d*x^2])/(45*c*Sqrt[1 - c^2*x^2]) + (3*b*c*f*g^2*x^
4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (b*c*g^3*x^5*Sqrt[d - c^2*d
*x^2])/(25*Sqrt[1 - c^2*x^2]) + (f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*
x]))/2 - (3*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(8*c^2) + (3*f
*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 - (f^2*g*(1 - c^2*x^2)*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/c^2 - (g^3*(1 - c^2*x^2)*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*c^4) + (g^3*(1 - c^2*x^2)^2*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^4) - (f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
os[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2]) - (3*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCos[c*x])^2)/(16*b*c^3*Sqrt[1 - c^2*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
```

```
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcC
os[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x
^2]), Int[((f*x)^m*(a + b*ArcCos[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] + Dist[
(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a
 + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && Eq
Q[c^2*d + e, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4690

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(1 - c^2*x^2)^p, x]}, Dist[d^p*(a + b*
ArcCos[c*x]), u, x] + Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^
2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Intege
rQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -
2^(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int (f+gx)^3\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)dx = \frac{\sqrt{d-c^2dx^2}\int (f+gx)^3\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int \left(f^3\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)+3f^2gx\sqrt{1-c^2x^2}\left(a+b\,\mathrm{c}\right.\right.}{}$$

$$= \frac{\left(f^3\sqrt{d-c^2dx^2}\right)\int \sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)dx}{\sqrt{1-c^2x^2}} + \frac{\left(3f^2g\sqrt{d-c^2dx^2}\right)}{}$$

$$= \frac{1}{2}f^3x\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{3}{4}fg^2x^3\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right.$$

$$= -\frac{bf^2gx\sqrt{d-c^2dx^2}}{c\sqrt{1-c^2x^2}}+\frac{bcf^3x^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}+\frac{bcf^2gx^3\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}}+\frac{3bcfg}{1}$$

$$= -\frac{bf^2gx\sqrt{d-c^2dx^2}}{c\sqrt{1-c^2x^2}}-\frac{2bg^3x\sqrt{d-c^2dx^2}}{15c^3\sqrt{1-c^2x^2}}+\frac{bcf^3x^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}-\frac{3bfg^2x^2}{16c\sqrt{}}$$

Mathematica [A] time = 1.20368, size = 442, normalized size = 0.66

$$\frac{240a\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}\left(6c^4x\left(20f^2gx+10f^3+15fg^2x^2+4g^3x^3\right)-c^2g\left(120f^2+45fgx+8g^2x^2\right)-16g^3\right)-3600ac}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

```
[Out] (240*a*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(-16*g^3 - c^2*g*(120*f^2 + 45
*f*g*x + 8*g^2*x^2) + 6*c^4*x*(10*f^3 + 20*f^2*g*x + 15*f*g^2*x^2 + 4*g^3*x
^3)) - 3600*a*c*Sqrt[d]*f*(4*c^2*f^2 + 3*g^2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x
*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 2400*b*c^2*f^2*g*Sqrt[d -
 c^2*d*x^2]*(9*c*x + 12*(1 - c^2*x^2)^(3/2)*ArcCos[c*x] - Cos[3*ArcCos[c*x]
]) + 3600*b*c^3*f^3*Sqrt[d - c^2*d*x^2]*(Cos[2*ArcCos[c*x]] + 2*ArcCos[c*x]
*(-ArcCos[c*x] + Sin[2*ArcCos[c*x]])) + 675*b*c*f*g^2*Sqrt[d - c^2*d*x^2]*(
-8*ArcCos[c*x]^2 + Cos[4*ArcCos[c*x]] + 4*ArcCos[c*x]*Sin[4*ArcCos[c*x]]) -
 8*b*g^3*Sqrt[d - c^2*d*x^2]*(16*c*x*(30 + 5*c^2*x^2 - 9*c^4*x^4) + 15*ArcC
os[c*x]*(30*Sqrt[1 - c^2*x^2] - 5*Sin[3*ArcCos[c*x]] - 3*Sin[5*ArcCos[c*x]]
)))/(28800*c^4*Sqrt[1 - c^2*x^2])
```

Maple [B] time = 0.848, size = 1285, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/8*b*(-d*(c^2*x^2-1))^(1/2)*f^3/c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)-1/5*a*g^3
*x^2*(-c^2*d*x^2+d)^(3/2)/c^2/d+3/8*a*f*g^2/c^2*x*(-c^2*d*x^2+d)^(1/2)-a*f^
2*g/c^2/d*(-c^2*d*x^2+d)^(3/2)+2/15*b*(-d*(c^2*x^2-1))^(1/2)*g^3/c^4/(c^2*x
^2-1)*arccos(c*x)-4/15*b*(-d*(c^2*x^2-1))^(1/2)*g^3/(c^2*x^2-1)*arccos(c*x)
*x^4-1/2*b*(-d*(c^2*x^2-1))^(1/2)*f^3/(c^2*x^2-1)*arccos(c*x)*x-2/15*a*g^3/
d/c^4*(-c^2*d*x^2+d)^(3/2)+3/16*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)
/c^3/(c^2*x^2-1)*arccos(c*x)^2*f*g^2+3/4*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*c^2
/(c^2*x^2-1)*arccos(c*x)*x^5+3/8*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/c^2/(c^2*x^
2-1)*arccos(c*x)*x+b*(-d*(c^2*x^2-1))^(1/2)*g*c^2/(c^2*x^2-1)*arccos(c*x)*x
^4*f^2-1/3*b*(-d*(c^2*x^2-1))^(1/2)*g*c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^3*
f^2+b*(-d*(c^2*x^2-1))^(1/2)*g/c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x*f^2-3/16*
b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^4+3/16*b*
(-d*(c^2*x^2-1))^(1/2)*f*g^2/c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^2-3/4*a*f*g
^2*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+3/8*a*f*g^2/c^2*d/(c^2*d)^(1/2)*arctan((c^2
*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/25*b*(-d*(c^2*x^2-1))^(1/2)*g^3*c/(c^2*
x^2-1)*(-c^2*x^2+1)^(1/2)*x^5+1/45*b*(-d*(c^2*x^2-1))^(1/2)*g^3/c/(c^2*x^2-
1)*(-c^2*x^2+1)^(1/2)*x^3+2/15*b*(-d*(c^2*x^2-1))^(1/2)*g^3/c^3/(c^2*x^2-1)
*(-c^2*x^2+1)^(1/2)*x-3/128*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/c^3/(c^2*x^2-1)*
(-c^2*x^2+1)^(1/2)-1/4*b*(-d*(c^2*x^2-1))^(1/2)*f^3*c/(c^2*x^2-1)*(-c^2*x^2
+1)^(1/2)*x^2-9/8*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/(c^2*x^2-1)*arccos(c*x)*x^
3-2*b*(-d*(c^2*x^2-1))^(1/2)*g/(c^2*x^2-1)*arccos(c*x)*x^2*f^2+1/2*a*f^3*x*
(-c^2*d*x^2+d)^(1/2)+1/4*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/(c^2
*x^2-1)*arccos(c*x)^2*f^3+1/5*b*(-d*(c^2*x^2-1))^(1/2)*g^3*c^2/(c^2*x^2-1)*
arccos(c*x)*x^6-1/15*b*(-d*(c^2*x^2-1))^(1/2)*g^3/c^2/(c^2*x^2-1)*arccos(c*
x)*x^2+b*(-d*(c^2*x^2-1))^(1/2)*g/c^2/(c^2*x^2-1)*arccos(c*x)*f^2+1/2*b*(-d
*(c^2*x^2-1))^(1/2)*f^3*c^2/(c^2*x^2-1)*arccos(c*x)*x^3+1/2*a*f^3*d/(c^2*d)
^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ag^3x^3 + 3\,afg^2x^2 + 3\,af^2gx + af^3 + \left(bg^3x^3 + 3\,bfg^2x^2 + 3\,bf^2gx + bf^3\right)\arccos\left(cx\right)\right)\sqrt{-c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3*
b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*acos(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c^2dx^2 + d}\left(gx + f\right)^3\left(b\arccos\left(cx\right) + a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(g*x + f)^3*(b*arccos(c*x) + a), x)
```

3.2 $\int (f+gx)^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)\,dx$

Optimal. Leaf size=450

$$\frac{1}{2}f^2x\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)-\frac{f^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{4bc\sqrt{1-c^2x^2}}-\frac{2fg\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)}{3c^2}+\frac{1}{4}$$

```
[Out] (-2*b*f*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[1 - c^2*x^2]) + (b*c*f^2*x^2*Sqr
t[d - c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) - (b*g^2*x^2*Sqrt[d - c^2*d*x^2])/(
16*c*Sqrt[1 - c^2*x^2]) + (2*b*c*f*g*x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt[1 - c
^2*x^2]) + (b*c*g^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (f^2*
x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (g^2*x*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCos[c*x]))/(8*c^2) + (g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*
x]))/4 - (2*f*g*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*c
^2) - (f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x
^2]) - (g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*c^3*Sqrt[1 - c
^2*x^2])
```

Rubi [A] time = 0.529749, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4778, 4764, 4648, 4642, 30, 4678, 4698, 4708}

$$\frac{1}{2}f^2x\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)-\frac{f^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{4bc\sqrt{1-c^2x^2}}-\frac{2fg\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)}{3c^2}+\frac{1}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```


```
[Out] (-2*b*f*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[1 - c^2*x^2]) + (b*c*f^2*x^2*Sqr
t[d - c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) - (b*g^2*x^2*Sqrt[d - c^2*d*x^2])/(
16*c*Sqrt[1 - c^2*x^2]) + (2*b*c*f*g*x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt[1 - c
^2*x^2]) + (b*c*g^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (f^2*
x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (g^2*x*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCos[c*x]))/(8*c^2) + (g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*
x]))/4 - (2*f*g*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*c
^2) - (f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x
^2]) - (g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*c^3*Sqrt[1 - c
^2*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcC
os[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x
^2]), Int[((f*x)^m*(a + b*ArcCos[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] + Dist[
(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a
 + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && Eq
Q[c^2*d + e, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int (f+gx)^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)dx = \frac{\sqrt{d-c^2dx^2}\int (f+gx)^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int \left(f^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)+2fgx\sqrt{1-c^2x^2}\left(a+b\cos\right.\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(f^2\sqrt{d-c^2dx^2}\right)\int \sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)dx}{\sqrt{1-c^2x^2}} + \frac{\left(2fg\sqrt{d-c^2dx^2}\right)\int x}{}$$

$$= \frac{1}{2}f^2x\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right) + \frac{1}{4}g^2x^3\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right) -$$

$$= -\frac{2bfgx\sqrt{d-c^2dx^2}}{3c\sqrt{1-c^2x^2}} + \frac{bcf^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}} + \frac{2bcfgx^3\sqrt{d-c^2dx^2}}{9\sqrt{1-c^2x^2}} + \frac{bcg^2x^4}{16\sqrt{}}$$

$$= -\frac{2bfgx\sqrt{d-c^2dx^2}}{3c\sqrt{1-c^2x^2}} + \frac{bcf^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}} - \frac{bg^2x^2\sqrt{d-c^2dx^2}}{16c\sqrt{1-c^2x^2}} + \frac{2bcfgx^3\sqrt{}}{9\sqrt{1-}}$$

Mathematica [A] time = 1.3145, size = 320, normalized size = 0.71

$$\frac{48ac\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}\left(12c^2f^2x+16fg\left(c^2x^2-1\right)+3g^2x\left(2c^2x^2-1\right)\right)-144a\sqrt{d}\sqrt{1-c^2x^2}\left(4c^2f^2+g^2\right)\tan^{-1}\left(\frac{cx\sqrt{d}}{\sqrt{d}(c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

```
[Out] (48*a*c*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(12*c^2*f^2*x + 16*f*g*(-1 +
c^2*x^2) + 3*g^2*x*(-1 + 2*c^2*x^2)) - 144*a*Sqrt[d]*(4*c^2*f^2 + g^2)*Sqrt
[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] -
64*b*c*f*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*(1 - c^2*x^2)^(3/2)*ArcCos[c*x]
- Cos[3*ArcCos[c*x]]) + 144*b*c^2*f^2*Sqrt[d - c^2*d*x^2]*(Cos[2*ArcCos[c*x
]] + 2*ArcCos[c*x]*(-ArcCos[c*x] + Sin[2*ArcCos[c*x]])) + 9*b*g^2*Sqrt[d -
c^2*d*x^2]*(-8*ArcCos[c*x]^2 + Cos[4*ArcCos[c*x]] + 4*ArcCos[c*x]*Sin[4*Arc
Cos[c*x]]))/(1152*c^3*Sqrt[1 - c^2*x^2])
```

Maple [B] time = 0.533, size = 912, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/4*a*g^2*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a*g^2/c^2*x*(-c^2*d*x^2+d)^(1/2
)+1/8*a*g^2/c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2)
)-2/3*a*f*g/c^2/d*(-c^2*d*x^2+d)^(3/2)+1/2*a*f^2*x*(-c^2*d*x^2+d)^(1/2)+1/2
*a*f^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-2/9*b*(
-d*(c^2*x^2-1))^(1/2)*f*g*c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^3+2/3*b*(-d*(c
^2*x^2-1))^(1/2)*f*g/c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x+1/4*b*(-d*(c^2*x^2-
1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/(c^2*x^2-1)*arccos(c*x)^2*f^2+1/16*b*(-d*(c^
2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2*g^2+1/4*b*
(-d*(c^2*x^2-1))^(1/2)*g^2*c^2/(c^2*x^2-1)*arccos(c*x)*x^5-3/8*b*(-d*(c^2*x
^2-1))^(1/2)*g^2/(c^2*x^2-1)*arccos(c*x)*x^3+1/8*b*(-d*(c^2*x^2-1))^(1/2)*g
^2/c^2/(c^2*x^2-1)*arccos(c*x)*x-1/128*b*(-d*(c^2*x^2-1))^(1/2)*g^2/c^3/(c^
2*x^2-1)*(-c^2*x^2+1)^(1/2)+2/3*b*(-d*(c^2*x^2-1))^(1/2)*f*g/c^2/(c^2*x^2-1
)*arccos(c*x)+1/2*b*(-d*(c^2*x^2-1))^(1/2)*f^2*c^2/(c^2*x^2-1)*arccos(c*x)*
x^3-1/2*b*(-d*(c^2*x^2-1))^(1/2)*f^2/(c^2*x^2-1)*arccos(c*x)*x+1/8*b*(-d*(c
```

```
^2*x^2-1))^(1/2)*f^2/c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)-1/16*b*(-d*(c^2*x^2-1
))^(1/2)*g^2*c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^4+1/16*b*(-d*(c^2*x^2-1))^(
1/2)*g^2/c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^2+2/3*b*(-d*(c^2*x^2-1))^(1/2)*
f*g*c^2/(c^2*x^2-1)*arccos(c*x)*x^4-4/3*b*(-d*(c^2*x^2-1))^(1/2)*f*g/(c^2*x
^2-1)*arccos(c*x)*x^2-1/4*b*(-d*(c^2*x^2-1))^(1/2)*f^2*c/(c^2*x^2-1)*(-c^2*
x^2+1)^(1/2)*x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-c^2dx^2+d}\left(ag^2x^2+2\,afgx+af^2+\left(bg^2x^2+2\,bfgx+bf^2\right)\arccos\left(cx\right)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 +
 2*b*f*g*x + b*f^2)*arccos(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*acos(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c^2dx^2 + d}(gx + f)^2 (b \arccos (cx) + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(g*x + f)^2*(b*arccos(c*x) + a), x)
```

3.3 $\int (f+gx)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)\,dx$

Optimal. Leaf size=238

$$\frac{1}{2}fx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)-\frac{f\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{4bc\sqrt{1-c^2x^2}}-\frac{g\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)}{3c^2}+\frac{bcfx^2\cdots}{4\sqrt{\cdots}}$$

```
[Out] -(b*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[1 - c^2*x^2]) + (b*c*f*x^2*Sqrt[d -
c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) + (b*c*g*x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt
[1 - c^2*x^2]) + (f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (g*(1 -
c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*c^2) - (f*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] time = 0.251246, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4778, 4764, 4648, 4642, 30, 4678}

$$\frac{1}{2}fx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)-\frac{f\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{4bc\sqrt{1-c^2x^2}}-\frac{g\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)}{3c^2}+\frac{bcfx^2\cdots}{4\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

```
[Out] -(b*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[1 - c^2*x^2]) + (b*c*f*x^2*Sqrt[d -
c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) + (b*c*g*x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt
[1 - c^2*x^2]) + (f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (g*(1 -
c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*c^2) - (f*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (f+gx)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)dx = \frac{\sqrt{d-c^2dx^2}\int (f+gx)\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int \left(f\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)+gx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx\right.\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(f\sqrt{d-c^2dx^2}\right)\int \sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)dx}{\sqrt{1-c^2x^2}} + \frac{\left(g\sqrt{d-c^2dx^2}\right)\int x\sqrt{1-}}{\sqrt{1}}$$

$$= \frac{1}{2}fx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right) - \frac{g\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx\right.}{3c^2}$$

$$= -\frac{bgx\sqrt{d-c^2dx^2}}{3c\sqrt{1-c^2x^2}} + \frac{bcfx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}} + \frac{bcgx^3\sqrt{d-c^2dx^2}}{9\sqrt{1-c^2x^2}} + \frac{1}{2}fx\sqrt{d-c^2dx}$$

Mathematica [A] time = 1.47429, size = 219, normalized size = 0.92

$$\frac{12a\sqrt{d-c^2dx^2}\left(3c^2fx+2g\left(c^2x^2-1\right)\right)-36ac\sqrt{d}f\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+\frac{9bcf\sqrt{d-c^2dx^2}\left(-2\cos^{-1}(cx)^2+\cos\left(2\cos^{-1}(cx)\right)+2\cos^{-1}(cx)\sin\right.}{\sqrt{1-c^2x^2}}}{72c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

```
[Out] (12*a*Sqrt[d - c^2*d*x^2]*(3*c^2*f*x + 2*g*(-1 + c^2*x^2)) - 36*a*c*Sqrt[d]
*f*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (2*b*g*Sqrt
[d - c^2*d*x^2]*(-9*c*x - 12*(1 - c^2*x^2)^(3/2)*ArcCos[c*x] + Cos[3*ArcCos
[c*x]]))/Sqrt[1 - c^2*x^2] + (9*b*c*f*Sqrt[d - c^2*d*x^2]*(-2*ArcCos[c*x]^2
 + Cos[2*ArcCos[c*x]] + 2*ArcCos[c*x]*Sin[2*ArcCos[c*x]]))/Sqrt[1 - c^2*x^2
])/(72*c^2)
```

Maple [B] time = 0.481, size = 491, normalized size = 2.1

$$-\frac{ag}{3c^2d}\left(-c^2dx^2+d\right)^{\frac{3}{2}}+\frac{afx}{2}\sqrt{-c^2dx^2+d}+\frac{afd}{2}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}+\frac{bg\arccos(cx)}{3c^2\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/3*a*g/c^2/d*(-c^2*d*x^2+d)^(3/2)+1/2*a*f*x*(-c^2*d*x^2+d)^(1/2)+1/2*a*f*
d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/3*b*(-d*(c^2
*x^2-1))^(1/2)*g/c^2/(c^2*x^2-1)*arccos(c*x)+1/3*b*(-d*(c^2*x^2-1))^(1/2)*g
*c^2/(c^2*x^2-1)*arccos(c*x)*x^4-2/3*b*(-d*(c^2*x^2-1))^(1/2)*g/(c^2*x^2-1)
*arccos(c*x)*x^2+1/2*b*(-d*(c^2*x^2-1))^(1/2)*f*c^2/(c^2*x^2-1)*arccos(c*x)
*x^3-1/2*b*(-d*(c^2*x^2-1))^(1/2)*f/(c^2*x^2-1)*arccos(c*x)*x+1/8*b*(-d*(c^
2*x^2-1))^(1/2)*f/c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)+1/4*b*(-d*(c^2*x^2-1))^(
1/2)*(-c^2*x^2+1)^(1/2)/c/(c^2*x^2-1)*arccos(c*x)^2*f-1/9*b*(-d*(c^2*x^2-1)
)^(1/2)*g*c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^3+1/3*b*(-d*(c^2*x^2-1))^(1/2)
*g/c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x-1/4*b*(-d*(c^2*x^2-1))^(1/2)*f*c/(c^2
*x^2-1)*(-c^2*x^2+1)^(1/2)*x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-c^2dx^2+d}\left(agx+af+\left(bgx+bf\right)\arccos\left(cx\right)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arccos(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acos(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c^2dx^2+d}\left(gx+f\right)\left(b\arccos\left(cx\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(g*x + f)*(b*arccos(c*x) + a), x)
```

3.4 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)}{f+gx}\,dx$

Optimal. Leaf size=725

$$-\frac{b\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}\text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{1-c^2x^2}}+\frac{b\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}\text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{g^2\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx}}{2}$$

```
[Out] (a*Sqrt[d - c^2*d*x^2])/g + (b*c*x*Sqrt[d - c^2*d*x^2])/(g*Sqrt[1 - c^2*x^2
]) + (b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])/g - (c*x*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCos[c*x])^2)/(2*b*g*Sqrt[1 - c^2*x^2]) + ((1 - (c^2*f^2)/g^2)*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(f + g*x)*Sqrt[1 - c^2*x^2]) - (S
qrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(f + g*x
)) - (a*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqrt[
c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])])/(g^2*Sqrt[1 - c^2*x^2]) - (I*b*Sqrt[c^2
*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(
c*f - Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[1 - c^2*x^2]) + (I*b*Sqrt[c^2*f^2 -
g^2]*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + S
qrt[c^2*f^2 - g^2])])/(g^2*Sqrt[1 - c^2*x^2]) - (b*Sqrt[c^2*f^2 - g^2]*Sqrt
[d - c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^
2]))])/(g^2*Sqrt[1 - c^2*x^2]) + (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]
*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqr
t[1 - c^2*x^2])
```

Rubi [A] time = 1.85891, antiderivative size = 725, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 19, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.613, Rules used = {4778, 4766, 683, 4758, 6742, 725, 204, 1654, 12, 4800, 4798, 4678, 8, 4774, 3321, 2264, 2190, 2279, 2391}

$$-\frac{b\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}\text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{1-c^2x^2}}+\frac{b\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}\text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{g^2\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

```
[Out] (a*Sqrt[d - c^2*d*x^2])/g + (b*c*x*Sqrt[d - c^2*d*x^2])/(g*Sqrt[1 - c^2*x^2
]) + (b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])/g - (c*x*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCos[c*x])^2)/(2*b*g*Sqrt[1 - c^2*x^2]) + ((1 - (c^2*f^2)/g^2)*Sqrt[d -
```

```
c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(f + g*x)*Sqrt[1 - c^2*x^2]) - (S
qrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(f + g*x
)) - (a*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqrt[
c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])])/(g^2*Sqrt[1 - c^2*x^2]) - (I*b*Sqrt[c^2
*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(
c*f - Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[1 - c^2*x^2]) + (I*b*Sqrt[c^2*f^2 -
g^2]*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + S
qrt[c^2*f^2 - g^2])])/(g^2*Sqrt[1 - c^2*x^2]) - (b*Sqrt[c^2*f^2 - g^2]*Sqrt
[d - c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^
2]))])/(g^2*Sqrt[1 - c^2*x^2]) + (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]
*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqr
t[1 - c^2*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4766

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_)*Sqrt[
(d_) + (e_.)*(x_)^2], x_Symbol] :> -Simp[((f + g*x)^m*(d + e*x^2)*(a + b*Ar
cCos[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] + Dist[1/(b*c*Sqrt[d]*(n + 1)
), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCos[
c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e,
0] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 4758

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(x
_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*x
 + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcCos[c*x])^n, u, x] + Dist[b*c*
n, Int[SimplifyIntegrand[(u*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2],
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ[p,
```

```
0] && EqQ[e*g - 2*d*h, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4800

```
Int[(ArcCos[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcCos[c*x])
^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && IGt
Q[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

Rule 4798

```
Int[ArcCos[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :
> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcCos[c*x]^n, RFx, x]}, Int[u, x
] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ[n
, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4774

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst
[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &&
 (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)}{f+gx}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc(f+gx)} + \frac{\sqrt{d-c^2dx^2}\int \frac{\left(-g-2c^2fx-c^2gx^2\right)\left(a+b\cos^{-1}(c\right.}{(f+gx)^2}}{2bc\sqrt{1-c^2x^2}}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc(f+gx)\sqrt{1-c^2x^2}} - \frac{\sqrt{1}}{}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc(f+gx)\sqrt{1-c^2x^2}} - \frac{\sqrt{1}}{}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc(f+gx)\sqrt{1-c^2x^2}} - \frac{\sqrt{1}}{}$$

$$= \frac{a\sqrt{d-c^2dx^2}}{g} - \frac{cx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{2bc(f+gx)\sqrt{1-c^2x^2}}$$

$$= \frac{a\sqrt{d-c^2dx^2}}{g} - \frac{cx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{2bc(f+gx)\sqrt{1-c^2x^2}}$$

$$= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\right.}{}$$

$$= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{2bg\sqrt{1-c^2x^2}}$$

$$= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{2bg\sqrt{1-c^2x^2}}$$

$$= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{2bg\sqrt{1-c^2x^2}}$$

$$= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{2bg\sqrt{1-c^2x^2}}$$

$$= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{2bg\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 3.37971, size = 1095, normalized size = 1.51

$$-2a\sqrt{d-c^2dx^2}g+2ac\sqrt{d}f\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)-2a\sqrt{d}\sqrt{g^2-c^2f^2}\log(f+gx)+2a\sqrt{d}\sqrt{g^2-c^2f^2}\log\left(d\left(fxc^2+g\right)+\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

```
[Out] -(-2*a*g*Sqrt[d - c^2*d*x^2] + 2*a*c*Sqrt[d]*f*ArcTan[(c*x*Sqrt[d - c^2*d*x
^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 2*a*Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Log[f +
 g*x] + 2*a*Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Log[d*(g + c^2*f*x) + Sqrt[d]*Sq
rt[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]] + b*Sqrt[d - c^2*d*x^2]*((-2*c*g*
x)/Sqrt[1 - c^2*x^2] - 2*g*ArcCos[c*x] + (c*f*ArcCos[c*x]^2)/Sqrt[1 - c^2*x
^2] + (2*(-(c*f) + g)*(c*f + g)*(2*ArcCos[c*x]*ArcTanh[((c*f + g)*Cot[ArcCo
s[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - 2*ArcCos[-((c*f)/g)]*ArcTanh[((-(c*f)
+ g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] - (2
*I)*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*
ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[Sqrt
[-(c^2*f^2) + g^2]/(Sqrt[2]*E^((I/2)*ArcCos[c*x])*Sqrt[g]*Sqrt[c*(f + g*x)]
)] + (ArcCos[-((c*f)/g)] + (2*I)*(ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sq
rt[-(c^2*f^2) + g^2]] - ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^
2*f^2) + g^2]]))*Log[(E^((I/2)*ArcCos[c*x])*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2
]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((-(c*f
) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*((-I)*c*
f + I*g + Sqrt[-(c^2*f^2) + g^2])*(-I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g +
Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - (ArcCos[-((c*f)/g)] + (2*I)*
ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*
f + g)*(I*c*f - I*g + Sqrt[-(c^2*f^2) + g^2])*(I + Tan[ArcCos[c*x]/2]))/(g*
(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] + I*(PolyLog[2, ((c
*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos
[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - Pol
yLog[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]
*Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/
2]))])))/(Sqrt[-(c^2*f^2) + g^2]*Sqrt[1 - c^2*x^2])))/(2*g^2)
```

Maple [A] time = 0.437, size = 1209, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x)
```

```
[Out] a/g*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)+a/g^2*
c^2*d*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*
(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+a/g^3*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln
((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)
*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))
*c^2*f^2-a/g*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^
2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-d*c^2*(x+f/g)^2+2*c^2*d*f/
g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))+1/2*b*(-d*(c^2*x^2-1))^(1/2)
*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^2*f*c/g^2+b*(-d*(c^2*x^2-1))^(1
/2)/(c^2*x^2-1)/g*arccos(c*x)*x^2*c^2-b*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/
g*(-c^2*x^2+1)^(1/2)*x*c-b*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/g*arccos(c*x)
+I*b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2
-1)/g^2*arccos(c*x)*ln((-(c*x+I*(-c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/
2))/(-c*f+(c^2*f^2-g^2)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1
/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^2*arccos(c*x)*ln(((c*x+I*(-c^2*x^2+1)^
(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))+b*(-d*(c^2*x^2
-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^2*dilog(-1/
(-c*f+(c^2*f^2-g^2)^(1/2))*(c*x+I*(-c^2*x^2+1)^(1/2))*g-1/(-c*f+(c^2*f^2-g^
2)^(1/2))*c*f+1/(-c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))-b*(-d*(c^2*
x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^2*dilog(
(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2))+1/(c*f+(c^2*f^2-g^2)
^(1/2))*c*f+1/(c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)}{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-d(cx-1)(cx+1)}\,(a+b\,\mathrm{acos}\,(cx))}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))*(-c**2*d*x**2+d)**(1/2)/(g*x+f),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))/(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(g*x + f), x)
```

3.5 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)}{(f+gx)^2}\,dx$

Optimal. Leaf size=851

$$\frac{bf^2\sqrt{d-c^2dx^2}\cos^{-1}(cx)^2c^3}{2g^2\left(c^2f^2-g^2\right)\sqrt{1-c^2x^2}}-\frac{af^2\sqrt{d-c^2dx^2}\sin^{-1}(cx)c^3}{g^2\left(c^2f^2-g^2\right)\sqrt{1-c^2x^2}}+\frac{af\sqrt{d-c^2dx^2}\tan^{-1}\left(\frac{fxc^2+g}{\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}\right)c^2}{g^2\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}+\frac{ibf\sqrt{d-c^2dx^2}\,\mathrm{c}}{g^2\sqrt{}}$$

```
[Out] -((a*Sqrt[d - c^2*d*x^2])/(g*(f + g*x))) - (b*Sqrt[d - c^2*d*x^2]*ArcCos[c*
x])/(g*(f + g*x)) + (b*c^3*f^2*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2)/(2*g^2*(c
^2*f^2 - g^2)*Sqrt[1 - c^2*x^2]) - ((g + c^2*f*x)^2*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCos[c*x])^2)/(2*b*c*(c^2*f^2 - g^2)*(f + g*x)^2*Sqrt[1 - c^2*x^2]) -
 (Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(f +
g*x)^2) - (a*c^3*f^2*Sqrt[d - c^2*d*x^2]*ArcSin[c*x])/(g^2*(c^2*f^2 - g^2)*
Sqrt[1 - c^2*x^2]) + (a*c^2*f*Sqrt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqr
t[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2
*x^2]) + (I*b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*
x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*
x^2]) - (I*b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x
])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x
^2]) - (b*c*Sqrt[d - c^2*d*x^2]*Log[f + g*x])/(g^2*Sqrt[1 - c^2*x^2]) + (b*
c^2*f*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^
2*f^2 - g^2]))])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2]) - (b*c^2*f*Sqr
t[d - c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g
^2]))])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])
```

Rubi [A] time = 2.69367, antiderivative size = 851, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 22, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.71, Rules used = {4778, 4766, 37, 4756, 12, 1651, 844, 216, 725, 204, 4800, 4798, 4642, 4774, 3324, 3321, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{bf^2\sqrt{d-c^2dx^2}\cos^{-1}(cx)^2c^3}{2g^2\left(c^2f^2-g^2\right)\sqrt{1-c^2x^2}}-\frac{af^2\sqrt{d-c^2dx^2}\sin^{-1}(cx)c^3}{g^2\left(c^2f^2-g^2\right)\sqrt{1-c^2x^2}}+\frac{af\sqrt{d-c^2dx^2}\tan^{-1}\left(\frac{fxc^2+g}{\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}\right)c^2}{g^2\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}+\frac{ibf\sqrt{d-c^2dx^2}\,\mathrm{c}}{g^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(f + g*x)^2,x]
```

```
[Out] -((a*Sqrt[d - c^2*d*x^2])/(g*(f + g*x))) - (b*Sqrt[d - c^2*d*x^2]*ArcCos[c*
x])/(g*(f + g*x)) + (b*c^3*f^2*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2)/(2*g^2*(c
```

```
^2*f^2 - g^2)*Sqrt[1 - c^2*x^2]) - ((g + c^2*f*x)^2*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCos[c*x])^2)/(2*b*c*(c^2*f^2 - g^2)*(f + g*x)^2*Sqrt[1 - c^2*x^2]) -
 (Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(f +
g*x)^2) - (a*c^3*f^2*Sqrt[d - c^2*d*x^2]*ArcSin[c*x])/(g^2*(c^2*f^2 - g^2)*
Sqrt[1 - c^2*x^2]) + (a*c^2*f*Sqrt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqr
t[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2
*x^2]) + (I*b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*
x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])]])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*
x^2]) - (I*b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x
])*g)/(c*f + Sqrt[c^2*f^2 - g^2])]])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x
^2]) - (b*c*Sqrt[d - c^2*d*x^2]*Log[f + g*x])/(g^2*Sqrt[1 - c^2*x^2]) + (b*
c^2*f*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^
2*f^2 - g^2]))]])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2]) - (b*c^2*f*Sqr
t[d - c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g
^2]))]])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4766

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_)*Sqrt[
(d_) + (e_.)*(x_)^2], x_Symbol] :> -Simp[((f + g*x)^m*(d + e*x^2)*(a + b*Ar
cCos[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] + Dist[1/(b*c*Sqrt[d]*(n + 1)
), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCos[
c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e,
0] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 4756

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_)*((f_.)
 + (g_.)*(x_))^(p_.), x_Symbol] :> With[{u = IntHide[(f + g*x)^p*(d + e*x)^
m, x]}, Dist[(a + b*ArcCos[c*x])^n, u, x] + Dist[b*c*n, Int[SimplifyIntegra
```

```
nd[(u*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[m, 0] && LtQ[
m + p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 4800

```
Int[(ArcCos[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcCos[c*x])
^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && IGt
Q[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

Rule 4798

```
Int[ArcCos[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :
> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcCos[c*x]^n, RFx, x]}, Int[u, x
] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ[n
, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4774

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst
[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &&
 (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)}{(f+gx)^2}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{(f+gx)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc(f+gx)^2} + \frac{\sqrt{d-c^2dx^2}\int \frac{\left(-2g-2c^2fx\right)\left(a+b\cos^{-1}(cx)\right)^2}{(f+gx)^3}}{2bc\sqrt{1-c^2x^2}} \cdots$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}\right.}{2bc(f+gx)^2} \cdots$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}\right.}{2bc(f+gx)^2} \cdots$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}\right.}{2bc(f+gx)^2} \cdots$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}\right.}{2bc(f+gx)^2} \cdots$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}\sqrt{d-c^2}\cdots}{2bc(f\cdots} \cdots$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}\sqrt{d-c^2}\cdots}{2bc(f\cdots} \cdots$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\cos^{-1}(cx)^2}{2g^2\left(c^2f^2-g^2\right)\sqrt{1-c^2x^2}} - \frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cos\cdots\right.}{2bc\left(c^2f^2-g^2\right)(f+gx)^2\sqrt{1-\cdots}} \cdots$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g(f+gx)} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\cos^{-1}(cx)^2}{2g^2\left(c^2f^2-g^2\right)\sqrt{1-c^2x^2}} - \frac{\left(g+c^2\cdots\right.}{2bc\cdots} \cdots$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g(f+gx)} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\cos^{-1}(cx)^2}{2g^2\left(c^2f^2-g^2\right)\sqrt{1-c^2x^2}} - \frac{\left(g+c^2\cdots\right.}{2bc\cdots} \cdots$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{g(f+gx)} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\cos^{-1}(cx)^2}{2g^2\left(c^2f^2-g^2\right)\sqrt{1-c^2x^2}} - \frac{\left(g+c^2\cdots\right.}{2bc\cdots} \cdots$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{\cdots} - \frac{b\sqrt{d-c^2dx^2}\cos^{-1}(cx)}{\cdots} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\cos^{-1}(cx)^2}{\cdots} - \frac{\left(g+c^2\cdots\right.}{\cdots} \cdots$$

Mathematica [A] time = 9.48203, size = 1130, normalized size = 1.33

$$\frac{a\sqrt{d}f\log(f+gx)c^2}{g^2\sqrt{g^2-c^2f^2}}-\frac{a\sqrt{d}f\log\left(dfxc^2+dg+\sqrt{d}\sqrt{g^2-c^2f^2}\sqrt{-d\left(c^2x^2-1\right)}\right)c^2}{g^2\sqrt{g^2-c^2f^2}}+\frac{a\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{-d(c^2x^2-1)}}{\sqrt{d}(c^2x^2-1)}\right)c}{g^2}-\frac{b\sqrt{d\left(\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(f + g*x)^2,x]
```

```
[Out] -((a*Sqrt[-(d*(-1 + c^2*x^2))])/(g*(f + g*x))) + (a*c*Sqrt[d]*ArcTan[(c*x*S
qrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/g^2 + (a*c^2*Sqrt[d]*f
*Log[f + g*x])/(g^2*Sqrt[-(c^2*f^2) + g^2]) - (a*c^2*Sqrt[d]*f*Log[d*g + c^
2*d*f*x + Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[-(d*(-1 + c^2*x^2))]])/(g^2*S
qrt[-(c^2*f^2) + g^2]) - (b*c*Sqrt[d*(1 - c^2*x^2)]*((2*g*ArcCos[c*x])/(c*f
 + c*g*x) - ArcCos[c*x]^2/Sqrt[1 - c^2*x^2] + (2*Log[1 + (g*x)/f])/Sqrt[1 -
 c^2*x^2] + (2*c*f*(2*ArcCos[c*x]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sq
rt[-(c^2*f^2) + g^2]] - 2*ArcCos[-((c*f)/g)]*ArcTanh[((-(c*f) + g)*Tan[ArcC
os[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[(
(c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*ArcTanh[((-(c
*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[Sqrt[-(c^2*f^2) +
 g^2]/(Sqrt[2]*E^((I/2)*ArcCos[c*x])*Sqrt[g]*Sqrt[c*f + c*g*x])] + (ArcCos[
-((c*f)/g)] + (2*I)*(ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2)
 + g^2]] - ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]
]))*Log[(E^((I/2)*ArcCos[c*x])*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqr
t[c*f + c*g*x])] - (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((-(c*f) + g)*Tan[Ar
cCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*((-I)*c*f + I*g + Sqr
t[-(c^2*f^2) + g^2])*(-I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^
2) + g^2]*Tan[ArcCos[c*x]/2]))] - (ArcCos[-((c*f)/g)] + (2*I)*ArcTanh[((-(c
*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(I*c*f
 - I*g + Sqrt[-(c^2*f^2) + g^2])*(I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sq
rt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-
(c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g
*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - PolyLog[2, ((c*f
 + I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c
*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))])))/(Sqr
t[-(c^2*f^2) + g^2]*Sqrt[1 - c^2*x^2])))/(2*g^2)
```

Maple [C] time = 0.476, size = 1573, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x)
```

```
[Out] a/d/(c^2*f^2-g^2)/(x+f/g)*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-
g^2)/g^2)^(3/2)-a/g*c^2*f/(c^2*f^2-g^2)*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/
g)-d*(c^2*f^2-g^2)/g^2)^(1/2)-a/g^2*c^4*f^2/(c^2*f^2-g^2)*d/(c^2*d)^(1/2)*a
rctan((c^2*d)^(1/2)*x/(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)
/g^2)^(1/2))-a/g^3*c^4*f^3/(c^2*f^2-g^2)*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln(
(-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*
(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))+
a/g*c^2*f/(c^2*f^2-g^2)*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^
2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-d*c^2*(x+f/g)^2
+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))+a*c^2/(c^2*f^2-g^
2)*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*x+a*c^2
/(c^2*f^2-g^2)*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-d*c^2*(x+f/g)^2+2*c
^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+b*(-1/2*(-d*(c^2*x^2-1))^(1/2)
*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^2*c/g^2-(-d*(c^2*x^2-1))^(1/2)*
(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*arccos(c*x)*(c^2*f*x+g+I*(-c^2*x^2+1)^
(1/2)*c*f)/(c^2*x^2-1)/g^2/(g*x+f)-c*(I*arccos(c*x)*ln((-(c*x+I*(-c^2*x^2+1
)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^
2)^(1/2)*c*f-I*arccos(c*x)*ln(((c*x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^
2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*c*f-2*Im(arccos(c*
x))*c^2*f^2+2*ln(exp(I*Re(arccos(c*x))))*c^2*f^2-ln((c*x+I*(-c^2*x^2+1)^(1/
2))^2*g+2*c*f*(c*x+I*(-c^2*x^2+1)^(1/2))+g)*c^2*f^2+dilog(-1/(-c*f+(c^2*f^2
-g^2)^(1/2))*(c*x+I*(-c^2*x^2+1)^(1/2))*g-1/(-c*f+(c^2*f^2-g^2)^(1/2))*c*f+
1/(-c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2)*c*f-d
ilog((c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2))+1/(c*f+(c^2*f^2
-g^2)^(1/2))*c*f+1/(c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))*(c^2*f^2-
g^2)^(1/2)*c*f+2*Im(arccos(c*x))*g^2-2*ln(exp(I*Re(arccos(c*x))))*g^2+ln((c
*x+I*(-c^2*x^2+1)^(1/2))^2*g+2*c*f*(c*x+I*(-c^2*x^2+1)^(1/2))+g)*g^2)*(-d*(
c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*f^2-g^2)/(c^2*x^2-1)/g^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)}{g^2x^2+2\,fgx+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(g^2*x^2 + 2*f*g*x + f^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\,\text{acos}\,(cx))}{\left(f+gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))*(-c**2*d*x**2+d)**(1/2)/(g*x+f)**2,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))/(f + g*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)}{\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(g*x + f)^2, x)
```

3.6 $\int (f+gx)^3 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cos^{-1}(cx)\right) dx$

Optimal. Leaf size=959

$$-\frac{bc^3dg^3\sqrt{d-c^2dx^2}x^7}{49\sqrt{1-c^2x^2}}-\frac{bc^3dfg^2\sqrt{d-c^2dx^2}x^6}{12\sqrt{1-c^2x^2}}+\frac{8bcdg^3\sqrt{d-c^2dx^2}x^5}{175\sqrt{1-c^2x^2}}-\frac{3bc^3df^2g\sqrt{d-c^2dx^2}x^5}{25\sqrt{1-c^2x^2}}-\frac{bc^3df^3\sqrt{d-c^2dx^2}x^4}{16\sqrt{1-c^2x^2}}$$

```
[Out] (-3*b*d*f^2*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[1 - c^2*x^2]) - (2*b*d*g^3*x
*Sqrt[d - c^2*d*x^2])/(35*c^3*Sqrt[1 - c^2*x^2]) + (5*b*c*d*f^3*x^2*Sqrt[d
- c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) - (3*b*d*f*g^2*x^2*Sqrt[d - c^2*d*x^2]
)/(32*c*Sqrt[1 - c^2*x^2]) + (2*b*c*d*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(5*Sqr
t[1 - c^2*x^2]) - (b*d*g^3*x^3*Sqrt[d - c^2*d*x^2])/(105*c*Sqrt[1 - c^2*x^2
]) - (b*c^3*d*f^3*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (7*b*c*
d*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(32*Sqrt[1 - c^2*x^2]) - (3*b*c^3*d*f^2*g*
x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[1 - c^2*x^2]) + (8*b*c*d*g^3*x^5*Sqrt[d -
 c^2*d*x^2])/(175*Sqrt[1 - c^2*x^2]) - (b*c^3*d*f*g^2*x^6*Sqrt[d - c^2*d*x^
2])/(12*Sqrt[1 - c^2*x^2]) - (b*c^3*d*g^3*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt
[1 - c^2*x^2]) + (3*d*f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 - (3
*d*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(16*c^2) + (3*d*f*g^2*x
^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 + (d*f^3*x*(1 - c^2*x^2)*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 + (d*f*g^2*x^3*(1 - c^2*x^2)*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (3*d*f^2*g*(1 - c^2*x^2)^2*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^2) - (d*g^3*(1 - c^2*x^2)^2*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^4) + (d*g^3*(1 - c^2*x^2)^3*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^4) - (3*d*f^3*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCos[c*x])^2)/(16*b*c*Sqrt[1 - c^2*x^2]) - (3*d*f*g^2*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCos[c*x])^2)/(32*b*c^3*Sqrt[1 - c^2*x^2])
```

Rubi [A] time = 0.96175, antiderivative size = 959, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 17, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.548, Rules used = {4778, 4764, 4650, 4648, 4642, 30, 14, 4678, 194, 4700, 4698, 4708, 266, 43, 4690, 12, 373}

$$-\frac{bc^3dg^3\sqrt{d-c^2dx^2}x^7}{49\sqrt{1-c^2x^2}}-\frac{bc^3dfg^2\sqrt{d-c^2dx^2}x^6}{12\sqrt{1-c^2x^2}}+\frac{8bcdg^3\sqrt{d-c^2dx^2}x^5}{175\sqrt{1-c^2x^2}}-\frac{3bc^3df^2g\sqrt{d-c^2dx^2}x^5}{25\sqrt{1-c^2x^2}}-\frac{bc^3df^3\sqrt{d-c^2dx^2}x^4}{16\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-3*b*d*f^2*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[1 - c^2*x^2]) - (2*b*d*g^3*x
*Sqrt[d - c^2*d*x^2])/(35*c^3*Sqrt[1 - c^2*x^2]) + (5*b*c*d*f^3*x^2*Sqrt[d
- c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) - (3*b*d*f*g^2*x^2*Sqrt[d - c^2*d*x^2]
)/(32*c*Sqrt[1 - c^2*x^2]) + (2*b*c*d*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(5*Sqr
t[1 - c^2*x^2]) - (b*d*g^3*x^3*Sqrt[d - c^2*d*x^2])/(105*c*Sqrt[1 - c^2*x^2
]) - (b*c^3*d*f^3*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (7*b*c*
d*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(32*Sqrt[1 - c^2*x^2]) - (3*b*c^3*d*f^2*g*
x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[1 - c^2*x^2]) + (8*b*c*d*g^3*x^5*Sqrt[d -
 c^2*d*x^2])/(175*Sqrt[1 - c^2*x^2]) - (b*c^3*d*f*g^2*x^6*Sqrt[d - c^2*d*x^
2])/(12*Sqrt[1 - c^2*x^2]) - (b*c^3*d*g^3*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt
[1 - c^2*x^2]) + (3*d*f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 - (3
*d*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(16*c^2) + (3*d*f*g^2*x
^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 + (d*f^3*x*(1 - c^2*x^2)*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 + (d*f*g^2*x^3*(1 - c^2*x^2)*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (3*d*f^2*g*(1 - c^2*x^2)^2*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^2) - (d*g^3*(1 - c^2*x^2)^2*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^4) + (d*g^3*(1 - c^2*x^2)^3*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^4) - (3*d*f^3*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCos[c*x])^2)/(16*b*c*Sqrt[1 - c^2*x^2]) - (3*d*f*g^2*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCos[c*x])^2)/(32*b*c^3*Sqrt[1 - c^2*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4650

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n)/(2*p + 1), x] + (D
ist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 - c^2*x
^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
```

```
GtQ[p, 0]
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4700

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcC
os[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x)^
m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart
[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 - c^2*x^2)^FracPart[p]), I
nt[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcC
os[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x
^2]), Int[((f*x)^m*(a + b*ArcCos[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] + Dist[
(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a
 + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && Eq
Q[c^2*d + e, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4690

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
```

```
, x_Symbol] :> With[{u = IntHide[x^m*(1 - c^2*x^2)^p, x]}, Dist[d^p*(a + b*
ArcCos[c*x]), u, x] + Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^
2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Intege
rQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -
2^(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (f+gx)^3\left(d-c^2dx^2\right)^{3/2}\left(a+b\cos^{-1}(cx)\right)dx = \frac{\left(d\sqrt{d-c^2dx^2}\right)\int (f+gx)^3\left(1-c^2x^2\right)^{3/2}\left(a+b\cos^{-1}(cx)\right)dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \left(f^3\left(1-c^2x^2\right)^{3/2}\left(a+b\cos^{-1}(cx)\right)+3f^2gx\left(1-c^2x^2\right)\right.}{}$$

$$= \frac{\left(df^3\sqrt{d-c^2dx^2}\right)\int \left(1-c^2x^2\right)^{3/2}\left(a+b\cos^{-1}(cx)\right)dx}{\sqrt{1-c^2x^2}} + \frac{\left(3df^2g\sqrt{d-c\right.}{}$$

$$= \frac{1}{4}df^3x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{1}{2}dfg^2x^3\left(1-c^2x^2\right)\sqrt{}$$

$$= \frac{3}{8}df^3x\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{3}{8}dfg^2x^3\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}\right.$$

$$= -\frac{3bdf^2gx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}}+\frac{5bcdf^3x^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}+\frac{2bcdf^2gx^3\sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}}$$

$$= -\frac{3bdf^2gx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}}-\frac{2bdg^3x\sqrt{d-c^2dx^2}}{35c^3\sqrt{1-c^2x^2}}+\frac{5bcdf^3x^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}-$$

Mathematica [A] time = 4.57178, size = 910, normalized size = 0.95

$$\frac{-88200bcdf\left(2c^2f^2+g^2\right)\sqrt{d-c^2dx^2}\cos^{-1}(cx)^2+140bd\sqrt{d-c^2dx^2}\left(6720f^2gx^2\sqrt{1-c^2x^2}c^4+1680f^3\sin\left(2\cos^{-1}(cx)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-88200*b*c*d*f*(2*c^2*f^2 + g^2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 17640
0*a*c*d^(3/2)*f*(2*c^2*f^2 + g^2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^
2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - d*Sqrt[d - c^2*d*x^2]*(352800*b*c^3*f
^2*g*x + 44100*b*c*g^3*x + 564480*a*c^2*f^2*g*Sqrt[1 - c^2*x^2] + 53760*a*g
^3*Sqrt[1 - c^2*x^2] - 588000*a*c^4*f^3*x*Sqrt[1 - c^2*x^2] + 176400*a*c^2*
f*g^2*x*Sqrt[1 - c^2*x^2] - 1128960*a*c^4*f^2*g*x^2*Sqrt[1 - c^2*x^2] + 268
80*a*c^2*g^3*x^2*Sqrt[1 - c^2*x^2] + 235200*a*c^6*f^3*x^3*Sqrt[1 - c^2*x^2]
 - 823200*a*c^4*f*g^2*x^3*Sqrt[1 - c^2*x^2] + 564480*a*c^6*f^2*g*x^4*Sqrt[1
 - c^2*x^2] - 215040*a*c^4*g^3*x^4*Sqrt[1 - c^2*x^2] + 470400*a*c^6*f*g^2*x
^5*Sqrt[1 - c^2*x^2] + 134400*a*c^6*g^3*x^6*Sqrt[1 - c^2*x^2] - 7350*b*c*f*
(16*c^2*f^2 + 3*g^2)*Cos[2*ArcCos[c*x]] - 4900*b*g*(12*c^2*f^2 + g^2)*Cos[3
*ArcCos[c*x]] + 7350*b*c^3*f^3*Cos[4*ArcCos[c*x]] - 11025*b*c*f*g^2*Cos[4*A
rcCos[c*x]] + 7056*b*c^2*f^2*g*Cos[5*ArcCos[c*x]] - 588*b*g^3*Cos[5*ArcCos[
c*x]] + 2450*b*c*f*g^2*Cos[6*ArcCos[c*x]] + 300*b*g^3*Cos[7*ArcCos[c*x]]) +
 140*b*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(-4200*c^2*f^2*g*Sqrt[1 - c^2*x^2]
 + 416*g^3*Sqrt[1 - c^2*x^2] + 6720*c^4*f^2*g*x^2*Sqrt[1 - c^2*x^2] - 1256*
c^2*g^3*x^2*Sqrt[1 - c^2*x^2] + 864*g^3*(1 - c^2*x^2)^(3/2)*Cos[2*ArcCos[c*
x]] + 120*g^3*(1 - c^2*x^2)^(3/2)*Cos[4*ArcCos[c*x]] + 1680*c^3*f^3*Sin[2*A
rcCos[c*x]] + 315*c*f*g^2*Sin[2*ArcCos[c*x]] - 420*c^2*f^2*g*Sin[3*ArcCos[c
*x]] + 140*g^3*Sin[3*ArcCos[c*x]] - 210*c^3*f^3*Sin[4*ArcCos[c*x]] + 315*c*
f*g^2*Sin[4*ArcCos[c*x]] - 252*c^2*f^2*g*Sin[5*ArcCos[c*x]] + 84*g^3*Sin[5*
ArcCos[c*x]] - 105*c*f*g^2*Sin[6*ArcCos[c*x]]))/(940800*c^4*Sqrt[1 - c^2*x^
2])
```

Maple [B] time = 0.811, size = 1734, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x)
```

```
[Out] 3/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/c^2/(c^2*x^2-1)*arccos(c*x)*f^2-1/4*b*(-d*
(c^2*x^2-1))^(1/2)*f^3*d*c^4/(c^2*x^2-1)*arccos(c*x)*x^5+7/8*b*(-d*(c^2*x^2
-1))^(1/2)*f^3*d*c^2/(c^2*x^2-1)*arccos(c*x)*x^3-17/16*b*(-d*(c^2*x^2-1))^(
1/2)*f*g^2*d/(c^2*x^2-1)*arccos(c*x)*x^3-9/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(
c^2*x^2-1)*arccos(c*x)*x^2*f^2+3/16*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(
1/2)/c/(c^2*x^2-1)*arccos(c*x)^2*f^3*d-1/7*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d*c
^4/(c^2*x^2-1)*arccos(c*x)*x^8+13/35*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d*c^2/(c^
2*x^2-1)*arccos(c*x)*x^6-1/35*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d/c^2/(c^2*x^2-1
)*arccos(c*x)*x^2+3/8*a*f^3*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a*f^3*d^2/(c^2*d)^
(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-2/35*a*g^3/d/c^4*(-c^2*d
*x^2+d)^(5/2)-9/35*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d/(c^2*x^2-1)*arccos(c*x)*x
^4-5/8*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d/(c^2*x^2-1)*arccos(c*x)*x+2/35*b*(-d*
(c^2*x^2-1))^(1/2)*g^3*d/c^4/(c^2*x^2-1)*arccos(c*x)-1/7*a*g^3*x^2*(-c^2*d*
x^2+d)^(5/2)/c^2/d+1/8*a*f*g^2/c^2*x*(-c^2*d*x^2+d)^(3/2)-3/5*a*f^2*g/c^2/d
*(-c^2*d*x^2+d)^(5/2)+1/4*a*f^3*x*(-c^2*d*x^2+d)^(3/2)-1/2*b*(-d*(c^2*x^2-1
))^(1/2)*f*g^2*d*c^4/(c^2*x^2-1)*arccos(c*x)*x^7+11/8*b*(-d*(c^2*x^2-1))^(1
/2)*f*g^2*d*c^2/(c^2*x^2-1)*arccos(c*x)*x^5+3/16*b*(-d*(c^2*x^2-1))^(1/2)*f
*g^2*d/c^2/(c^2*x^2-1)*arccos(c*x)*x-3/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d*c^4/(
c^2*x^2-1)*arccos(c*x)*x^6*f^2+9/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d*c^2/(c^2*x^
2-1)*arccos(c*x)*x^4*f^2+3/32*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c
^3/(c^2*x^2-1)*arccos(c*x)^2*f*d*g^2-7/32*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d*
c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^4+3/32*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d/
c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^2+3/25*b*(-d*(c^2*x^2-1))^(1/2)*g*d*c^3/
(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^5*f^2-2/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d*c/(
c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^3*f^2+3/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/c/(c
^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x*f^2+1/12*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d*c^
3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^6-1/2*a*f*g^2*x*(-c^2*d*x^2+d)^(5/2)/c^2
/d+3/16*a*f*g^2/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/49*b*(-d*(c^2*x^2-1))^(1/2)*
g^3*d*c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^7-8/175*b*(-d*(c^2*x^2-1))^(1/2)
*g^3*d*c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^5+1/105*b*(-d*(c^2*x^2-1))^(1/2)*
g^3*d/c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^3+2/35*b*(-d*(c^2*x^2-1))^(1/2)*g^
3*d/c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x+7/768*b*(-d*(c^2*x^2-1))^(1/2)*f*g
^2*d/c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)+1/16*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d
*c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^4-5/16*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d
*c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^2+3/16*a*f*g^2/c^2*d^2/(c^2*d)^(1/2)*ar
ctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+17/128*b*(-d*(c^2*x^2-1))^(1/2)*
f^3*d/c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(ac^2dg^3x^5+3\,ac^2dfg^2x^4-3\,adf^2gx-adf^3+\left(3\,ac^2df^2g-adg^3\right)x^3+\left(ac^2df^3-3\,adfg^2\right)x^2+\left(bc^2dg^3x^5+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral(-(a*c^2*d*g^3*x^5 + 3*a*c^2*d*f*g^2*x^4 - 3*a*d*f^2*g*x - a*d*f^3
+ (3*a*c^2*d*f^2*g - a*d*g^3)*x^3 + (a*c^2*d*f^3 - 3*a*d*f*g^2)*x^2 + (b*c^
2*d*g^3*x^5 + 3*b*c^2*d*f*g^2*x^4 - 3*b*d*f^2*g*x - b*d*f^3 + (3*b*c^2*d*f^
2*g - b*d*g^3)*x^3 + (b*c^2*d*f^3 - 3*b*d*f*g^2)*x^2)*arccos(c*x))*sqrt(-c^
2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-c^2dx^2+d\right)^{\frac{3}{2}}\left(gx+f\right)^3\left(b\arccos\left(cx\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="gi
ac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(g*x + f)^3*(b*arccos(c*x) + a), x)
```

3.7 $\int (f+gx)^2 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cos^{-1}(cx)\right) dx$

Optimal. Leaf size=680

$$\frac{3}{8}df^2x\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{1}{4}df^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)-\frac{3df^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx\right.}{16bc\sqrt{1-c^2x^2}}$$

```
[Out] (-2*b*d*f*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[1 - c^2*x^2]) + (5*b*c*d*f^2*x
^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) - (b*d*g^2*x^2*Sqrt[d - c^2*
d*x^2])/(32*c*Sqrt[1 - c^2*x^2]) + (4*b*c*d*f*g*x^3*Sqrt[d - c^2*d*x^2])/(1
5*Sqrt[1 - c^2*x^2]) - (b*c^3*d*f^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c
^2*x^2]) + (7*b*c*d*g^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) - (
2*b*c^3*d*f*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[1 - c^2*x^2]) - (b*c^3*d*g^
2*x^6*Sqrt[d - c^2*d*x^2])/(36*Sqrt[1 - c^2*x^2]) + (3*d*f^2*x*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCos[c*x]))/8 - (d*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
[c*x]))/(16*c^2) + (d*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 +
(d*f^2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 + (d*g^2*
x^3*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/6 - (2*d*f*g*(1
- c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^2) - (3*d*f^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*c*Sqrt[1 - c^2*x^2]) - (d*g^
2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(32*b*c^3*Sqrt[1 - c^2*x^2])
```

Rubi [A] time = 0.735067, antiderivative size = 680, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {4778, 4764, 4650, 4648, 4642, 30, 14, 4678, 194, 4700, 4698, 4708}

$$\frac{3}{8}df^2x\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{1}{4}df^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)-\frac{3df^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx\right.}{16bc\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-2*b*d*f*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[1 - c^2*x^2]) + (5*b*c*d*f^2*x
^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) - (b*d*g^2*x^2*Sqrt[d - c^2*
d*x^2])/(32*c*Sqrt[1 - c^2*x^2]) + (4*b*c*d*f*g*x^3*Sqrt[d - c^2*d*x^2])/(1
5*Sqrt[1 - c^2*x^2]) - (b*c^3*d*f^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c
^2*x^2]) + (7*b*c*d*g^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) - (
2*b*c^3*d*f*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[1 - c^2*x^2]) - (b*c^3*d*g^
2*x^6*Sqrt[d - c^2*d*x^2])/(36*Sqrt[1 - c^2*x^2]) + (3*d*f^2*x*Sqrt[d - c^2
```

```
*d*x^2]*(a + b*ArcCos[c*x]))/8 - (d*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
[c*x]))/(16*c^2) + (d*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 +
(d*f^2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 + (d*g^2*
x^3*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/6 - (2*d*f*g*(1
- c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^2) - (3*d*f^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*c*Sqrt[1 - c^2*x^2]) - (d*g^
2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(32*b*c^3*Sqrt[1 - c^2*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4650

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n)/(2*p + 1), x] + (D
ist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 - c^2*x
^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4700

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcC
os[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x)^
m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart
[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 - c^2*x^2)^FracPart[p]), I
nt[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcC
os[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x
```

```
^2]), Int[((f*x)^m*(a + b*ArcCos[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] + Dist[
(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a
 + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && Eq
Q[c^2*d + e, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int (f+gx)^2 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cos^{-1}(cx)\right) dx = \frac{\left(d\sqrt{d-c^2dx^2}\right)\int (f+gx)^2 \left(1-c^2x^2\right)^{3/2} \left(a+b\cos^{-1}(cx)\right) dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \left(f^2 \left(1-c^2x^2\right)^{3/2} \left(a+b\cos^{-1}(cx)\right)+2fgx\left(1-c^2x^2\right)^{3/}\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(df^2\sqrt{d-c^2dx^2}\right)\int \left(1-c^2x^2\right)^{3/2} \left(a+b\cos^{-1}(cx)\right) dx}{\sqrt{1-c^2x^2}} + \frac{\left(2dfg\sqrt{d-c^2}\right.}{}$$

$$= \frac{1}{4}df^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{1}{6}dg^2x^3\left(1-c^2x^2\right)\sqrt{d}$$

$$= \frac{3}{8}df^2x\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{1}{8}dg^2x^3\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(c\right.$$

$$= -\frac{2bdfgx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}}+\frac{5bcdf^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}+\frac{4bcdfgx^3\sqrt{d-c^2dx^2}}{15\sqrt{1-c^2x^2}}-$$

$$= -\frac{2bdfgx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}}+\frac{5bcdf^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}-\frac{bdg^2x^2\sqrt{d-c^2dx^2}}{32c\sqrt{1-c^2x^2}}+\frac{4b}{}$$

Mathematica [A] time = 2.14273, size = 591, normalized size = 0.87

$$\frac{-d\sqrt{d-c^2dx^2}\left(14400ac^5f^2x^3\sqrt{1-c^2x^2}-36000ac^3f^2x\sqrt{1-c^2x^2}+23040ac^5fgx^4\sqrt{1-c^2x^2}-46080ac^3fgx^2\sqrt{1-c^2x^2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-1800*b*d*(6*c^2*f^2 + g^2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 3600*a*d^(
3/2)*(6*c^2*f^2 + g^2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(
Sqrt[d]*(-1 + c^2*x^2))] - d*Sqrt[d - c^2*d*x^2]*(14400*b*c^2*f*g*x + 23040
*a*c*f*g*Sqrt[1 - c^2*x^2] - 36000*a*c^3*f^2*x*Sqrt[1 - c^2*x^2] + 3600*a*c
*g^2*x*Sqrt[1 - c^2*x^2] - 46080*a*c^3*f*g*x^2*Sqrt[1 - c^2*x^2] + 14400*a*
c^5*f^2*x^3*Sqrt[1 - c^2*x^2] - 16800*a*c^3*g^2*x^3*Sqrt[1 - c^2*x^2] + 230
40*a*c^5*f*g*x^4*Sqrt[1 - c^2*x^2] + 9600*a*c^5*g^2*x^5*Sqrt[1 - c^2*x^2] -
 450*b*(16*c^2*f^2 + g^2)*Cos[2*ArcCos[c*x]] - 2400*b*c*f*g*Cos[3*ArcCos[c*
x]] + 450*b*c^2*f^2*Cos[4*ArcCos[c*x]] - 225*b*g^2*Cos[4*ArcCos[c*x]] + 288
*b*c*f*g*Cos[5*ArcCos[c*x]] + 50*b*g^2*Cos[6*ArcCos[c*x]]) + 60*b*d*Sqrt[d
- c^2*d*x^2]*ArcCos[c*x]*(-400*c*f*g*Sqrt[1 - c^2*x^2] + 640*c^3*f*g*x^2*Sq
rt[1 - c^2*x^2] + 15*(16*c^2*f^2 + g^2)*Sin[2*ArcCos[c*x]] - 40*c*f*g*Sin[3
*ArcCos[c*x]] - 30*c^2*f^2*Sin[4*ArcCos[c*x]] + 15*g^2*Sin[4*ArcCos[c*x]] -
 24*c*f*g*Sin[5*ArcCos[c*x]] - 5*g^2*Sin[6*ArcCos[c*x]]))/(57600*c^3*Sqrt[1
 - c^2*x^2])
```

Maple [B] time = 0.579, size = 1252, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x)
```

```
[Out] 1/24*a*g^2/c^2*x*(-c^2*d*x^2+d)^(3/2)-5/8*b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x
^2-1)*arccos(c*x)*x*f^2+17/128*b*(-d*(c^2*x^2-1))^(1/2)*d/c/(c^2*x^2-1)*(-c
^2*x^2+1)^(1/2)*f^2-17/48*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d/(c^2*x^2-1)*arccos
(c*x)*x^3+7/2304*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d/c^3/(c^2*x^2-1)*(-c^2*x^2+1
)^(1/2)+3/8*a*f^2*d*x*(-c^2*d*x^2+d)^(1/2)-2/5*b*(-d*(c^2*x^2-1))^(1/2)*f*g
*d*c^4/(c^2*x^2-1)*arccos(c*x)*x^6+6/5*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d*c^2/(
c^2*x^2-1)*arccos(c*x)*x^4+2/25*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d*c^3/(c^2*x^2
```

```
-1)*(-c^2*x^2+1)^(1/2)*x^5-4/15*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d*c/(c^2*x^2-1
)*(-c^2*x^2+1)^(1/2)*x^3+2/5*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d/c/(c^2*x^2-1)*(
-c^2*x^2+1)^(1/2)*x+1/4*a*f^2*x*(-c^2*d*x^2+d)^(3/2)+1/36*b*(-d*(c^2*x^2-1)
)^(1/2)*g^2*d*c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^6-7/96*b*(-d*(c^2*x^2-1)
)^(1/2)*g^2*d*c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^4+1/32*b*(-d*(c^2*x^2-1))^
(1/2)*g^2*d/c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^2-6/5*b*(-d*(c^2*x^2-1))^(1/
2)*f*g*d/(c^2*x^2-1)*arccos(c*x)*x^2+1/16*b*(-d*(c^2*x^2-1))^(1/2)*d*c^3/(c
^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^4*f^2-5/16*b*(-d*(c^2*x^2-1))^(1/2)*d*c/(c^2
*x^2-1)*(-c^2*x^2+1)^(1/2)*x^2*f^2+3/16*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+
1)^(1/2)/c/(c^2*x^2-1)*arccos(c*x)^2*d*f^2+1/32*b*(-d*(c^2*x^2-1))^(1/2)*(-
c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2*d*g^2-1/6*b*(-d*(c^2*x^2-1))
^(1/2)*g^2*d*c^4/(c^2*x^2-1)*arccos(c*x)*x^7+11/24*b*(-d*(c^2*x^2-1))^(1/2)
*g^2*d*c^2/(c^2*x^2-1)*arccos(c*x)*x^5+1/16*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d/
c^2/(c^2*x^2-1)*arccos(c*x)*x+2/5*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d/c^2/(c^2*x
^2-1)*arccos(c*x)-1/4*b*(-d*(c^2*x^2-1))^(1/2)*d*c^4/(c^2*x^2-1)*arccos(c*x
)*x^5*f^2+7/8*b*(-d*(c^2*x^2-1))^(1/2)*d*c^2/(c^2*x^2-1)*arccos(c*x)*x^3*f^
2+3/8*a*f^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+
1/16*a*g^2/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/16*a*g^2/c^2*d^2/(c^2*d)^(1/2)*ar
ctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-2/5*a*f*g/c^2/d*(-c^2*d*x^2+d)^(
5/2)-1/6*a*g^2*x*(-c^2*d*x^2+d)^(5/2)/c^2/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\Big(-\big(ac^2dg^2x^4 + 2\,ac^2dfgx^3 - 2\,adfgx - adf^2 + \big(ac^2df^2 - adg^2\big)x^2 + \big(bc^2dg^2x^4 + 2\,bc^2dfgx^3 - 2\,bdfgx - bdf^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral(-(a*c^2*d*g^2*x^4 + 2*a*c^2*d*f*g*x^3 - 2*a*d*f*g*x - a*d*f^2 + (a
*c^2*d*f^2 - a*d*g^2)*x^2 + (b*c^2*d*g^2*x^4 + 2*b*c^2*d*f*g*x^3 - 2*b*d*f*
g*x - b*d*f^2 + (b*c^2*d*f^2 - b*d*g^2)*x^2)*arccos(c*x))*sqrt(-c^2*d*x^2 +
 d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}\left(gx+f\right)^2\left(b\arccos\left(cx\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="gi
ac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(g*x + f)^2*(b*arccos(c*x) + a), x)
```

3.8 $\int (f+gx)\left(d-c^2dx^2\right)^{3/2}\left(a+b\cos^{-1}(cx)\right)dx$

Optimal. Leaf size=370

$$\frac{3}{8}dfx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{1}{4}dfx\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)-\frac{3df\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{16bc\sqrt{1-c^2x^2}}-$$

```
[Out] -(b*d*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[1 - c^2*x^2]) + (5*b*c*d*f*x^2*Sqr
t[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (2*b*c*d*g*x^3*Sqrt[d - c^2*d*x^
2])/(15*Sqrt[1 - c^2*x^2]) - (b*c^3*d*f*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1
 - c^2*x^2]) - (b*c^3*d*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[1 - c^2*x^2]) +
 (3*d*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 + (d*f*x*(1 - c^2*x^2)
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 - (d*g*(1 - c^2*x^2)^2*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^2) - (3*d*f*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCos[c*x])^2)/(16*b*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] time = 0.334807, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {4778, 4764, 4650, 4648, 4642, 30, 14, 4678, 194}

$$\frac{3}{8}dfx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{1}{4}dfx\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)-\frac{3df\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)^2}{16bc\sqrt{1-c^2x^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] -(b*d*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[1 - c^2*x^2]) + (5*b*c*d*f*x^2*Sqr
t[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (2*b*c*d*g*x^3*Sqrt[d - c^2*d*x^
2])/(15*Sqrt[1 - c^2*x^2]) - (b*c^3*d*f*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1
 - c^2*x^2]) - (b*c^3*d*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[1 - c^2*x^2]) +
 (3*d*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 + (d*f*x*(1 - c^2*x^2)
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 - (d*g*(1 - c^2*x^2)^2*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^2) - (3*d*f*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCos[c*x])^2)/(16*b*c*Sqrt[1 - c^2*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
```

```
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4650

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n)/(2*p + 1), x] + (D
ist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 - c^2*x
^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (f+gx)\left(d-c^2dx^2\right)^{3/2}\left(a+b\cos^{-1}(cx)\right)dx = \frac{\left(d\sqrt{d-c^2dx^2}\right)\int (f+gx)\left(1-c^2x^2\right)^{3/2}\left(a+b\cos^{-1}(cx)\right)dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \left(f\left(1-c^2x^2\right)^{3/2}\left(a+b\cos^{-1}(cx)\right)+gx\left(1-c^2x^2\right)^{3/2}\left(a\cdots\right.\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(df\sqrt{d-c^2dx^2}\right)\int \left(1-c^2x^2\right)^{3/2}\left(a+b\cos^{-1}(cx)\right)dx}{\sqrt{1-c^2x^2}} + \frac{\left(dg\sqrt{d-c^2dx^2}\right)\cdots}{\cdots}$$

$$= \frac{1}{4}dfx\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right) - \frac{dg\left(1-c^2x^2\right)^2\sqrt{d-c^2d\cdots}}{5c^2\cdots}$$

$$= \frac{3}{8}dfx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right) + \frac{1}{4}dfx\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cdots\right.$$

$$= -\frac{bdgx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}} + \frac{5bcdfx^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}} + \frac{2bcdgx^3\sqrt{d-c^2dx^2}}{15\sqrt{1-c^2x^2}} - \frac{bc^3d\cdots}{16\cdots}$$

Mathematica [A] time = 1.56584, size = 337, normalized size = 0.91

$$\frac{-d\sqrt{d-c^2dx^2}\left(3\left(80a\sqrt{1-c^2x^2}\left(5c^2fx\left(2c^2x^2-5\right)+8g\left(c^2x^2-1\right)^2\right)+25bcf\cos\left(4\cos^{-1}(cx)\right)+400bcgx+8bg\cos\left(5\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-1800*b*c*d*f*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 3600*a*c*d^(3/2)*f*Sqrt[
1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - d
*Sqrt[d - c^2*d*x^2]*(-1200*b*c*f*Cos[2*ArcCos[c*x]] - 200*b*g*Cos[3*ArcCos
[c*x]] + 3*(400*b*c*g*x + 80*a*Sqrt[1 - c^2*x^2]*(8*g*(-1 + c^2*x^2)^2 + 5*
c^2*f*x*(-5 + 2*c^2*x^2)) + 25*b*c*f*Cos[4*ArcCos[c*x]] + 8*b*g*Cos[5*ArcCo
s[c*x]])) + 20*b*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(-100*g*Sqrt[1 - c^2*x^2
] + 160*c^2*g*x^2*Sqrt[1 - c^2*x^2] + 120*c*f*Sin[2*ArcCos[c*x]] - 10*g*Sin
[3*ArcCos[c*x]] - 15*c*f*Sin[4*ArcCos[c*x]] - 6*g*Sin[5*ArcCos[c*x]]))/(960
0*c^2*Sqrt[1 - c^2*x^2])
```

Maple [B] time = 0.49, size = 698, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x)
```

```
[Out] -1/5*a*g/c^2/d*(-c^2*d*x^2+d)^(5/2)+1/4*a*f*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*f*
d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a*f*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(
-c^2*d*x^2+d)^(1/2))+3/16*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/(c^
2*x^2-1)*arccos(c*x)^2*f*d+1/25*b*(-d*(c^2*x^2-1))^(1/2)*g*d*c^3/(c^2*x^2-1
)*(-c^2*x^2+1)^(1/2)*x^5-2/15*b*(-d*(c^2*x^2-1))^(1/2)*g*d*c/(c^2*x^2-1)*(-
c^2*x^2+1)^(1/2)*x^3+1/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/c/(c^2*x^2-1)*(-c^2*x
^2+1)^(1/2)*x+1/16*b*(-d*(c^2*x^2-1))^(1/2)*f*d*c^3/(c^2*x^2-1)*(-c^2*x^2+1
)^(1/2)*x^4-5/16*b*(-d*(c^2*x^2-1))^(1/2)*f*d*c/(c^2*x^2-1)*(-c^2*x^2+1)^(1
/2)*x^2+1/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/c^2/(c^2*x^2-1)*arccos(c*x)-1/5*b*
(-d*(c^2*x^2-1))^(1/2)*g*d*c^4/(c^2*x^2-1)*arccos(c*x)*x^6+3/5*b*(-d*(c^2*x
^2-1))^(1/2)*g*d*c^2/(c^2*x^2-1)*arccos(c*x)*x^4-3/5*b*(-d*(c^2*x^2-1))^(1/
2)*g*d/(c^2*x^2-1)*arccos(c*x)*x^2-1/4*b*(-d*(c^2*x^2-1))^(1/2)*f*d*c^4/(c^
2*x^2-1)*arccos(c*x)*x^5+7/8*b*(-d*(c^2*x^2-1))^(1/2)*f*d*c^2/(c^2*x^2-1)*a
```

```
rccos(c*x)*x^3-5/8*b*(-d*(c^2*x^2-1))^(1/2)*f*d/(c^2*x^2-1)*arccos(c*x)*x+1
7/128*b*(-d*(c^2*x^2-1))^(1/2)*f*d/c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(ac^2dgx^3 + ac^2dfx^2 - adgx - adf + \left(bc^2dgx^3 + bc^2dfx^2 - bdgx - bdf\right)\arccos\left(cx\right)\right)\sqrt{-c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fric
as")
```

```
[Out] integral(-(a*c^2*d*g*x^3 + a*c^2*d*f*x^2 - a*d*g*x - a*d*f + (b*c^2*d*g*x^3
 + b*c^2*d*f*x^2 - b*d*g*x - b*d*f)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}\left(gx+f\right)\left(b\arccos\left(cx\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="giac
")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(g*x + f)*(b*arccos(c*x) + a), x)
```

3.9 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cos^{-1}(cx)\right)}{f+gx}\,dx$

Optimal. Leaf size=1064

$$-\frac{bdx^3\sqrt{d-c^2dx^2}c^3}{9g\sqrt{1-c^2x^2}}+\frac{bdfx^2\sqrt{d-c^2dx^2}c^3}{4g^2\sqrt{1-c^2x^2}}+\frac{dfx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)c^2}{2g^2}+\frac{d(cf-g)(cf+g)x\sqrt{d-c^2dx^2}\left(a+b\right.}{2bg^3\sqrt{1-c^2x^2}}$$

```
[Out] -((a*d*(c*f - g)*(c*f + g)*Sqrt[d - c^2*d*x^2])/g^3) + (b*c*d*x*Sqrt[d - c^
2*d*x^2])/(3*g*Sqrt[1 - c^2*x^2]) - (b*c*d*(c*f - g)*(c*f + g)*x*Sqrt[d - c
^2*d*x^2])/(g^3*Sqrt[1 - c^2*x^2]) + (b*c^3*d*f*x^2*Sqrt[d - c^2*d*x^2])/(4
*g^2*Sqrt[1 - c^2*x^2]) - (b*c^3*d*x^3*Sqrt[d - c^2*d*x^2])/(9*g*Sqrt[1 - c
^2*x^2]) - (b*d*(c*f - g)*(c*f + g)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])/g^3 +
(c^2*d*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(2*g^2) + (d*(1 - c^2*x
^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*g) - (c*d*f*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCos[c*x])^2)/(4*b*g^2*Sqrt[1 - c^2*x^2]) + (c*d*(c*f - g)*(c
*f + g)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*g^3*Sqrt[1 - c^2*
x^2]) + (d*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*
b*c*g^4*(f + g*x)*Sqrt[1 - c^2*x^2]) + (d*(c*f - g)*(c*f + g)*Sqrt[1 - c^2*
x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*g^2*(f + g*x)) + (a*
d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqrt[c^2*
f^2 - g^2]*Sqrt[1 - c^2*x^2])])/(g^4*Sqrt[1 - c^2*x^2]) + (I*b*d*(c^2*f^2 -
 g^2)^(3/2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(
c*f - Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[1 - c^2*x^2]) - (I*b*d*(c^2*f^2 - g^
2)^(3/2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f
 + Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[1 - c^2*x^2]) + (b*d*(c^2*f^2 - g^2)^(3
/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*
f^2 - g^2]))])/(g^4*Sqrt[1 - c^2*x^2]) - (b*d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d
- c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])
)])/(g^4*Sqrt[1 - c^2*x^2])
```

Rubi [A] time = 2.28, antiderivative size = 1064, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 23, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.742, Rules used = {4778, 4768, 4648, 4642, 30, 4678, 4766, 683, 4758, 6742, 725, 204, 1654, 12, 4800, 4798, 8, 4774, 3321, 2264, 2190, 2279, 2391}

$$-\frac{bdx^3\sqrt{d-c^2dx^2}c^3}{9g\sqrt{1-c^2x^2}}+\frac{bdfx^2\sqrt{d-c^2dx^2}c^3}{4g^2\sqrt{1-c^2x^2}}+\frac{dfx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)c^2}{2g^2}+\frac{d(cf-g)(cf+g)x\sqrt{d-c^2dx^2}\left(a+b\right.}{2bg^3\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

```
[Out] -((a*d*(c*f - g)*(c*f + g)*Sqrt[d - c^2*d*x^2])/g^3) + (b*c*d*x*Sqrt[d - c^
2*d*x^2])/(3*g*Sqrt[1 - c^2*x^2]) - (b*c*d*(c*f - g)*(c*f + g)*x*Sqrt[d - c
^2*d*x^2])/(g^3*Sqrt[1 - c^2*x^2]) + (b*c^3*d*f*x^2*Sqrt[d - c^2*d*x^2])/(4
*g^2*Sqrt[1 - c^2*x^2]) - (b*c^3*d*x^3*Sqrt[d - c^2*d*x^2])/(9*g*Sqrt[1 - c
^2*x^2]) - (b*d*(c*f - g)*(c*f + g)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])/g^3 +
(c^2*d*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(2*g^2) + (d*(1 - c^2*x
^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*g) - (c*d*f*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCos[c*x])^2)/(4*b*g^2*Sqrt[1 - c^2*x^2]) + (c*d*(c*f - g)*(c
*f + g)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*g^3*Sqrt[1 - c^2*
x^2]) + (d*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*
b*c*g^4*(f + g*x)*Sqrt[1 - c^2*x^2]) + (d*(c*f - g)*(c*f + g)*Sqrt[1 - c^2*
x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*g^2*(f + g*x)) + (a*
d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqrt[c^2*
f^2 - g^2]*Sqrt[1 - c^2*x^2])])/(g^4*Sqrt[1 - c^2*x^2]) + (I*b*d*(c^2*f^2 -
 g^2)^(3/2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(
c*f - Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[1 - c^2*x^2]) - (I*b*d*(c^2*f^2 - g^
2)^(3/2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f
 + Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[1 - c^2*x^2]) + (b*d*(c^2*f^2 - g^2)^(3
/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*
f^2 - g^2]))])/(g^4*Sqrt[1 - c^2*x^2]) - (b*d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d
- c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])
)])/(g^4*Sqrt[1 - c^2*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(a
 + b*ArcCos[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IGtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
```

```
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4766

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_)*Sqrt[
(d_) + (e_.)*(x_)^2], x_Symbol] :> -Simp[((f + g*x)^m*(d + e*x^2)*(a + b*Ar
cCos[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] + Dist[1/(b*c*Sqrt[d]*(n + 1)
), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCos[
c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e,
0] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 4758

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(x
_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*x
 + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcCos[c*x])^n, u, x] + Dist[b*c*
n, Int[SimplifyIntegrand[(u*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2],
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ[p,
 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4800

```
Int[(ArcCos[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcCos[c*x])
^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && IGt
Q[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

Rule 4798

```
Int[ArcCos[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :
> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcCos[c*x]^n, RFx, x]}, Int[u, x
] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ[n
, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4774

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst
[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &&
 (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cos^{-1}(cx)\right)}{f+gx}\,dx = \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{\left(1-c^2x^2\right)^{3/2}\left(a+b\cos^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d\sqrt{d-c^2dx^2}\right)\int\left(\frac{c^2f\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{g^2} - \frac{c^2x\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{g} + \frac{\left(-c^2f^2+g^2\right)\sqrt{1-}}{g^2(}\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\right)\int \frac{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{1-c^2x^2}} + \frac{\left(c^2df\sqrt{d-c^2dx^2}\right)\int\sqrt{1-}}{g^2\sqrt{1-}}$$

$$= \frac{c^2dfx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)}{2g^2} + \frac{d\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)}{3g}$$

$$= \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bc^3dfx^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{1-c^2x^2}} - \frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9g\sqrt{1-c^2x^2}} + \frac{c^2dfx\sqrt{d-c^2dx^2}}{2}$$

$$= \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bc^3dfx^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{1-c^2x^2}} - \frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9g\sqrt{1-c^2x^2}} + \frac{c^2dfx\sqrt{d-c^2dx^2}}{2}$$

$$= \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bc^3dfx^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{1-c^2x^2}} - \frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9g\sqrt{1-c^2x^2}} + \frac{c^2dfx\sqrt{d-c^2dx^2}}{2}$$

$$= -\frac{ad(cf-g)(cf+g)\sqrt{d-c^2dx^2}}{g^3} + \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bc^3dfx^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{1-c^2x^2}} - \frac{bc^3}{9}$$

$$= -\frac{ad(cf-g)(cf+g)\sqrt{d-c^2dx^2}}{g^3} + \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bc^3dfx^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{1-c^2x^2}} - \frac{bc^3}{9}$$

$$= -\frac{ad(cf-g)(cf+g)\sqrt{d-c^2dx^2}}{g^3} + \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bc^3dfx^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{1-c^2x^2}} - \frac{bc^3}{9}$$

$$= -\frac{ad(cf-g)(cf+g)\sqrt{d-c^2dx^2}}{g^3} + \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bcd\left(1-\frac{c^2f^2}{g^2}\right)x\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}}$$

$$= -\frac{ad(cf-g)(cf+g)\sqrt{d-c^2dx^2}}{g^3} + \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bcd\left(1-\frac{c^2f^2}{g^2}\right)x\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}}$$

$$= -\frac{ad(cf-g)(cf+g)\sqrt{d-c^2dx^2}}{g^3} + \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bcd\left(1-\frac{c^2f^2}{g^2}\right)x\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}}$$

Mathematica [B] time = 12.5042, size = 3034, normalized size = 2.85

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*((a*d*(-3*c^2*f^2 + 4*g^2))/(3*g^3) + (a*c^2*d*f*
x)/(2*g^2) - (a*c^2*d*x^2)/(3*g)) + (a*c*d^(3/2)*f*(2*c^2*f^2 - 3*g^2)*ArcT
an[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/(2*g^4) + (a*
d^(3/2)*(-(c^2*f^2) + g^2)^(3/2)*Log[f + g*x])/g^4 - (a*d^(3/2)*(-(c^2*f^2)
 + g^2)^(3/2)*Log[d*g + c^2*d*f*x + Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[-(d
*(-1 + c^2*x^2))]])/g^4 - (b*d*Sqrt[d*(1 - c^2*x^2)]*((-2*c*g*x)/Sqrt[1 - c
^2*x^2] - 2*g*ArcCos[c*x] + (c*f*ArcCos[c*x]^2)/Sqrt[1 - c^2*x^2] + (2*(-(c
*f) + g)*(c*f + g)*(2*ArcCos[c*x]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sq
rt[-(c^2*f^2) + g^2]] - 2*ArcCos[-((c*f)/g)]*ArcTanh[((-(c*f) + g)*Tan[ArcC
os[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[(
(c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*ArcTanh[((-(c
*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[Sqrt[-(c^2*f^2) +
 g^2]/(Sqrt[2]*E^((I/2)*ArcCos[c*x])*Sqrt[g]*Sqrt[c*f + c*g*x])] + (ArcCos[
-((c*f)/g)] + (2*I)*(ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2)
 + g^2]] - ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]
]))*Log[(E^((I/2)*ArcCos[c*x])*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqr
t[c*f + c*g*x])] - (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((-(c*f) + g)*Tan[Ar
cCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*((-I)*c*f + I*g + Sqr
t[-(c^2*f^2) + g^2])*(-I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^
2) + g^2]*Tan[ArcCos[c*x]/2]))] - (ArcCos[-((c*f)/g)] + (2*I)*ArcTanh[((-(c
*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(I*c*f
 - I*g + Sqrt[-(c^2*f^2) + g^2])*(I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sq
rt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-
(c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g
*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - PolyLog[2, ((c*f
 + I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c
*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))]))))/(Sqr
t[-(c^2*f^2) + g^2]*Sqrt[1 - c^2*x^2])))/(2*g^2) + (b*d*Sqrt[d*(1 - c^2*x^2
)]*((9*(2*ArcCos[c*x]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2
) + g^2]] - 2*ArcCos[-((c*f)/g)]*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/
Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((c*f + g)*Co
t[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*ArcTanh[((-(c*f) + g)*Tan
[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[
2]*E^((I/2)*ArcCos[c*x])*Sqrt[g]*Sqrt[c*f + c*g*x])] + (ArcCos[-((c*f)/g)]
+ (2*I)*(ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - A
rcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(
```

```
(I/2)*ArcCos[c*x])*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*f + c*g*
x])] - (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2]
)/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*((-I)*c*f + I*g + Sqrt[-(c^2*f^2)
 + g^2])*(-I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Ta
n[ArcCos[c*x]/2]))] - (ArcCos[-((c*f)/g)] + (2*I)*ArcTanh[((-(c*f) + g)*Tan
[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(I*c*f - I*g + Sqr
t[-(c^2*f^2) + g^2])*(I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2
) + g^2]*Tan[ArcCos[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) +
g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g*(c*f + g +
Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(
c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g*
(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))])))/Sqrt[-(c^2*f^2)
+ g^2] + (18*c*g*(-4*c^2*f^2 + g^2)*x + 18*g*(-4*c^2*f^2 + g^2)*Sqrt[1 - c^
2*x^2]*ArcCos[c*x] + 18*c*f*(2*c^2*f^2 - g^2)*ArcCos[c*x]^2 + 9*c*f*g^2*Cos
[2*ArcCos[c*x]] - 2*g^3*Cos[3*ArcCos[c*x]] - (9*(8*c^4*f^4 - 8*c^2*f^2*g^2
+ g^4)*(2*ArcCos[c*x]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2
) + g^2]] - 2*ArcCos[-((c*f)/g)]*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/
Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((c*f + g)*Co
t[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*ArcTanh[((-(c*f) + g)*Tan
[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[
2]*E^((I/2)*ArcCos[c*x])*Sqrt[g]*Sqrt[c*f + c*g*x])] + (ArcCos[-((c*f)/g)]
+ (2*I)*(ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - A
rcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(
(I/2)*ArcCos[c*x])*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*f + c*g*
x])] - (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2]
)/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*((-I)*c*f + I*g + Sqrt[-(c^2*f^2)
 + g^2])*(-I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Ta
n[ArcCos[c*x]/2]))] - (ArcCos[-((c*f)/g)] + (2*I)*ArcTanh[((-(c*f) + g)*Tan
[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(I*c*f - I*g + Sqr
t[-(c^2*f^2) + g^2])*(I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2
) + g^2]*Tan[ArcCos[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) +
g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g*(c*f + g +
Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(
c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g*
(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))])))/Sqrt[-(c^2*f^2)
+ g^2] + 18*c*f*g^2*ArcCos[c*x]*Sin[2*ArcCos[c*x]] - 6*g^3*ArcCos[c*x]*Sin[
3*ArcCos[c*x]])/g^4))/(72*Sqrt[1 - c^2*x^2])
```

Maple [B] time = 0.398, size = 2132, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/(g*x+f),x)
```

```
[Out] a/g*d*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)+1/8*
b*(-d*(c^2*x^2-1))^(1/2)*f*d*c/(c^2*x^2-1)/g^2*(-c^2*x^2+1)^(1/2)+1/9*b*(-d
*(c^2*x^2-1))^(1/2)*d/(c^2*x^2-1)/g*(-c^2*x^2+1)^(1/2)*x^3*c^3-4/3*b*(-d*(c
^2*x^2-1))^(1/2)*d/(c^2*x^2-1)/g*(-c^2*x^2+1)^(1/2)*x*c+b*(-d*(c^2*x^2-1))^
(1/2)*d/(c^2*x^2-1)/g^3*arccos(c*x)*c^2*f^2-b*(c^2*f^2-g^2)^(3/2)*d*(-d*(c^
2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^4*dilog(-1/(-c*f+(c^2*f^2-
g^2)^(1/2))*(c*x+I*(-c^2*x^2+1)^(1/2))*g-1/(-c*f+(c^2*f^2-g^2)^(1/2))*c*f+1
/(-c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))+b*(c^2*f^2-g^2)^(3/2)*d*(-
d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^4*dilog((c*x+I*(-c^2*
x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2))+1/(c*f+(c^2*f^2-g^2)^(1/2))*c*f+1
/(c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))-1/3*b*(-d*(c^2*x^2-1))^(1/2
)*d/(c^2*x^2-1)/g*arccos(c*x)*x^4*c^4+5/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x
^2-1)/g*arccos(c*x)*x^2*c^2+1/3*a/g*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d
*(c^2*f^2-g^2)/g^2)^(3/2)+2*a/g^3*d^2/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d
*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-d*c
^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))*c^2*f
^2+1/2*a/g^2*c^2*d*f*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/
g^2)^(1/2)*x+3/2*a/g^2*c^2*d^2*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-d*c
^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))-a/g^4*d^2*c^4*
f^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f
/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))-a/g^5*d^2/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((
-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(
-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))*c
^4*f^4-a/g^3*d*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(
1/2)*c^2*f^2-4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x^2-1)/g*arccos(c*x)+I*b*(
c^2*f^2-g^2)^(3/2)*d*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/
g^4*arccos(c*x)*ln(((c*x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(
c*f+(c^2*f^2-g^2)^(1/2)))-I*b*(c^2*f^2-g^2)^(3/2)*d*(-d*(c^2*x^2-1))^(1/2)*
(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^4*arccos(c*x)*ln((-(c*x+I*(-c^2*x^2+1)^(1/
2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))-1/4*b*(-d*(c^2*x
^2-1))^(1/2)*f*d*c^3/(c^2*x^2-1)/g^2*(-c^2*x^2+1)^(1/2)*x^2+b*(-d*(c^2*x^2-
1))^(1/2)*d/(c^2*x^2-1)/g^3*(-c^2*x^2+1)^(1/2)*x*c^3*f^2-1/2*b*(-d*(c^2*x^2
-1))^(1/2)*f*d*c^2/(c^2*x^2-1)/g^2*arccos(c*x)*x-b*(-d*(c^2*x^2-1))^(1/2)*d
/(c^2*x^2-1)/g^3*arccos(c*x)*x^2*c^4*f^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2
*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^2*f^3*d*c^3/g^4+3/4*b*(-d*(c^2*x^2-1)
)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^2*f*d*c/g^2+1/2*b*(-d*(c
^2*x^2-1))^(1/2)*f*d*c^4/(c^2*x^2-1)/g^2*arccos(c*x)*x^3-a/g*d^2/(-d*(c^2*f
^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^
2*f^2-g^2)/g^2)^(1/2)*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)
/g^2)^(1/2))/(x+f/g))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\arccos\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccos(c*x))*sqrt(-c^2*d
*x^2 + d)/(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acos}\left(cx\right)\right)}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))/(g*x+f),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))/(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\arccos(cx)+a)}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="giac
")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccos(c*x) + a)/(g*x + f), x)
```

3.10 $\int (f+gx)^3 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cos^{-1}(cx)\right) dx$

Optimal. Leaf size=1281

result too large to display

```
[Out] (-3*b*d^2*f^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[1 - c^2*x^2]) - (2*b*d^2*g
^3*x*Sqrt[d - c^2*d*x^2])/(63*c^3*Sqrt[1 - c^2*x^2]) + (25*b*c*d^2*f^3*x^2*
Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) - (15*b*d^2*f*g^2*x^2*Sqrt[d -
c^2*d*x^2])/(256*c*Sqrt[1 - c^2*x^2]) + (3*b*c*d^2*f^2*g*x^3*Sqrt[d - c^2*d
*x^2])/(7*Sqrt[1 - c^2*x^2]) - (b*d^2*g^3*x^3*Sqrt[d - c^2*d*x^2])/(189*c*S
qrt[1 - c^2*x^2]) - (5*b*c^3*d^2*f^3*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 -
c^2*x^2]) + (59*b*c*d^2*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(256*Sqrt[1 - c^2*x^
2]) - (9*b*c^3*d^2*f^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[1 - c^2*x^2]) +
(b*c*d^2*g^3*x^5*Sqrt[d - c^2*d*x^2])/(21*Sqrt[1 - c^2*x^2]) - (17*b*c^3*d^
2*f*g^2*x^6*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) + (3*b*c^5*d^2*f^2*
g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[1 - c^2*x^2]) - (19*b*c^3*d^2*g^3*x^7*S
qrt[d - c^2*d*x^2])/(441*Sqrt[1 - c^2*x^2]) + (3*b*c^5*d^2*f*g^2*x^8*Sqrt[d
 - c^2*d*x^2])/(64*Sqrt[1 - c^2*x^2]) + (b*c^5*d^2*g^3*x^9*Sqrt[d - c^2*d*x
^2])/(81*Sqrt[1 - c^2*x^2]) - (b*d^2*f^3*(1 - c^2*x^2)^(5/2)*Sqrt[d - c^2*d
*x^2])/(36*c) + (5*d^2*f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 -
(15*d^2*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(128*c^2) + (15*d^
2*f*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/64 + (5*d^2*f^3*x*(1 -
 c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/24 + (5*d^2*f*g^2*x^3*(1
 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 + (d^2*f^3*x*(1 - c
^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/6 + (3*d^2*f*g^2*x^3*(1
- c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 - (3*d^2*f^2*g*(1 -
 c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^2) - (d^2*g^3*(1
- c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^4) + (d^2*g^3*(1
 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(9*c^4) - (5*d^2*f^3
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(32*b*c*Sqrt[1 - c^2*x^2]) - (1
5*d^2*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(256*b*c^3*Sqrt[1 -
c^2*x^2])
```

Rubi [A] time = 1.17746, antiderivative size = 1281, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 18, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.581, Rules used = {4778, 4764, 4650, 4648, 4642, 30, 14, 261, 4678, 194, 4700, 4698, 4708, 266, 43, 4690, 12, 373}

$$\frac{bc^5d^2g^3\sqrt{d-c^2dx^2}x^9}{81\sqrt{1-c^2x^2}}+\frac{3bc^5d^2fg^2\sqrt{d-c^2dx^2}x^8}{64\sqrt{1-c^2x^2}}-\frac{19bc^3d^2g^3\sqrt{d-c^2dx^2}x^7}{441\sqrt{1-c^2x^2}}+\frac{3bc^5d^2f^2g\sqrt{d-c^2dx^2}x^7}{49\sqrt{1-c^2x^2}}-\frac{17bc^3d^2fg^2\sqrt{d}}{96\sqrt{1-c}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-3*b*d^2*f^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[1 - c^2*x^2]) - (2*b*d^2*g
^3*x*Sqrt[d - c^2*d*x^2])/(63*c^3*Sqrt[1 - c^2*x^2]) + (25*b*c*d^2*f^3*x^2*
Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) - (15*b*d^2*f*g^2*x^2*Sqrt[d -
c^2*d*x^2])/(256*c*Sqrt[1 - c^2*x^2]) + (3*b*c*d^2*f^2*g*x^3*Sqrt[d - c^2*d
*x^2])/(7*Sqrt[1 - c^2*x^2]) - (b*d^2*g^3*x^3*Sqrt[d - c^2*d*x^2])/(189*c*S
qrt[1 - c^2*x^2]) - (5*b*c^3*d^2*f^3*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 -
c^2*x^2]) + (59*b*c*d^2*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(256*Sqrt[1 - c^2*x^
2]) - (9*b*c^3*d^2*f^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[1 - c^2*x^2]) +
(b*c*d^2*g^3*x^5*Sqrt[d - c^2*d*x^2])/(21*Sqrt[1 - c^2*x^2]) - (17*b*c^3*d^
2*f*g^2*x^6*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) + (3*b*c^5*d^2*f^2*
g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[1 - c^2*x^2]) - (19*b*c^3*d^2*g^3*x^7*S
qrt[d - c^2*d*x^2])/(441*Sqrt[1 - c^2*x^2]) + (3*b*c^5*d^2*f*g^2*x^8*Sqrt[d
 - c^2*d*x^2])/(64*Sqrt[1 - c^2*x^2]) + (b*c^5*d^2*g^3*x^9*Sqrt[d - c^2*d*x
^2])/(81*Sqrt[1 - c^2*x^2]) - (b*d^2*f^3*(1 - c^2*x^2)^(5/2)*Sqrt[d - c^2*d
*x^2])/(36*c) + (5*d^2*f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 -
(15*d^2*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(128*c^2) + (15*d^
2*f*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/64 + (5*d^2*f^3*x*(1 -
 c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/24 + (5*d^2*f*g^2*x^3*(1
 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 + (d^2*f^3*x*(1 - c
^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/6 + (3*d^2*f*g^2*x^3*(1
- c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 - (3*d^2*f^2*g*(1 -
 c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^2) - (d^2*g^3*(1
- c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^4) + (d^2*g^3*(1
 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(9*c^4) - (5*d^2*f^3
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(32*b*c*Sqrt[1 - c^2*x^2]) - (1
5*d^2*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(256*b*c^3*Sqrt[1 -
c^2*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
```

```
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4650

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n)/(2*p + 1), x] + (D
ist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 - c^2*x
^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4700

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcC
os[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x)^
m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart
[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 - c^2*x^2)^FracPart[p]), I
nt[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcC
os[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x
^2]), Int[((f*x)^m*(a + b*ArcCos[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] + Dist[
(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a
 + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && Eq
Q[c^2*d + e, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4690

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(1 - c^2*x^2)^p, x]}, Dist[d^p*(a + b*
ArcCos[c*x]), u, x] + Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^
2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Intege
rQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -
2^(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (f+gx)^3\left(d-c^2dx^2\right)^{5/2}\left(a+b\cos^{-1}(cx)\right)dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int (f+gx)^3\left(1-c^2x^2\right)^{5/2}\left(a+b\cos^{-1}(cx)\right)dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(f^3\left(1-c^2x^2\right)^{5/2}\left(a+b\cos^{-1}(cx)\right)+3f^2gx\left(1-c^2x\right.\right.}{}$$

$$= \frac{\left(d^2f^3\sqrt{d-c^2dx^2}\right)\int \left(1-c^2x^2\right)^{5/2}\left(a+b\cos^{-1}(cx)\right)dx}{\sqrt{1-c^2x^2}} + \frac{\left(3d^2f^2g\sqrt{d\right.}{}$$

$$= \frac{1}{6}d^2f^3x\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{3}{8}d^2fg^2x^3\left(1-c^2x\right.$$

$$= -\frac{bd^2f^3\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c}+\frac{5}{24}d^2f^3x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a\right.$$

$$= -\frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}+\frac{3bcd^2f^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{1-c^2x^2}}-\frac{9bc^3d^2f^2gx^5\sqrt{d-}}{35\sqrt{1-c^2x}}$$

$$= -\frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}-\frac{2bd^2g^3x\sqrt{d-c^2dx^2}}{63c^3\sqrt{1-c^2x^2}}+\frac{25bcd^2f^3x^2\sqrt{d-c^2d}}{96\sqrt{1-c^2x^2}}$$

$$= -\frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}-\frac{2bd^2g^3x\sqrt{d-c^2dx^2}}{63c^3\sqrt{1-c^2x^2}}+\frac{25bcd^2f^3x^2\sqrt{d-c^2d}}{96\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 7.46235, size = 1144, normalized size = 0.89

$$\frac{d^2\left(-3175200bcf\left(8c^2f^2+3g^2\right)\sqrt{d-c^2dx^2}\cos^{-1}(cx)^2+504b\sqrt{d-c^2dx^2}\left(503424f^2gx^2\sqrt{1-c^2x^2}c^4+75600f^3\sin\left(2c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (d^2*(-3175200*b*c*f*(8*c^2*f^2 + 3*g^2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2
- 6350400*a*c*Sqrt[d]*f*(8*c^2*f^2 + 3*g^2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*S
qrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d - c^2*d*x^2]*(-38102
400*b*c^3*f^2*g*x - 3810240*b*c*g^3*x - 69672960*a*c^2*f^2*g*Sqrt[1 - c^2*x
^2] - 5160960*a*g^3*Sqrt[1 - c^2*x^2] + 111767040*a*c^4*f^3*x*Sqrt[1 - c^2*
x^2] - 19051200*a*c^2*f*g^2*x*Sqrt[1 - c^2*x^2] + 209018880*a*c^4*f^2*g*x^2
```

```
*Sqrt[1 - c^2*x^2] - 2580480*a*c^2*g^3*x^2*Sqrt[1 - c^2*x^2] - 88058880*a*c
^6*f^3*x^3*Sqrt[1 - c^2*x^2] + 149869440*a*c^4*f*g^2*x^3*Sqrt[1 - c^2*x^2]
- 209018880*a*c^6*f^2*g*x^4*Sqrt[1 - c^2*x^2] + 38707200*a*c^4*g^3*x^4*Sqrt
[1 - c^2*x^2] + 27095040*a*c^8*f^3*x^5*Sqrt[1 - c^2*x^2] - 172730880*a*c^6*
f*g^2*x^5*Sqrt[1 - c^2*x^2] + 69672960*a*c^8*f^2*g*x^6*Sqrt[1 - c^2*x^2] -
49029120*a*c^6*g^3*x^6*Sqrt[1 - c^2*x^2] + 60963840*a*c^8*f*g^2*x^7*Sqrt[1
- c^2*x^2] + 18063360*a*c^8*g^3*x^8*Sqrt[1 - c^2*x^2] + 3810240*b*c*f*(5*c^
2*f^2 + g^2)*Cos[2*ArcCos[c*x]] + 282240*b*g*(27*c^2*f^2 + 2*g^2)*Cos[3*Arc
Cos[c*x]] - 1905120*b*c^3*f^3*Cos[4*ArcCos[c*x]] + 952560*b*c*f*g^2*Cos[4*A
rcCos[c*x]] - 1524096*b*c^2*f^2*g*Cos[5*ArcCos[c*x]] + 141120*b*c^3*f^3*Cos
[6*ArcCos[c*x]] - 423360*b*c*f*g^2*Cos[6*ArcCos[c*x]] + 155520*b*c^2*f^2*g*
Cos[7*ArcCos[c*x]] - 38880*b*g^3*Cos[7*ArcCos[c*x]] + 59535*b*c*f*g^2*Cos[8
*ArcCos[c*x]] + 7840*b*g^3*Cos[9*ArcCos[c*x]]) + 504*b*Sqrt[d - c^2*d*x^2]*
ArcCos[c*x]*(-261504*c^2*f^2*g*Sqrt[1 - c^2*x^2] + 62616*g^3*Sqrt[1 - c^2*x
^2] + 503424*c^4*f^2*g*x^2*Sqrt[1 - c^2*x^2] - 120576*c^2*g^3*x^2*Sqrt[1 -
c^2*x^2] - 41472*g*(3*c^2*f^2 - 2*g^2)*(1 - c^2*x^2)^(3/2)*Cos[2*ArcCos[c*x
]] - 5760*g*(3*c^2*f^2 - 2*g^2)*(1 - c^2*x^2)^(3/2)*Cos[4*ArcCos[c*x]] + 75
600*c^3*f^3*Sin[2*ArcCos[c*x]] + 15120*c*f*g^2*Sin[2*ArcCos[c*x]] - 40320*c
^2*f^2*g*Sin[3*ArcCos[c*x]] + 6720*g^3*Sin[3*ArcCos[c*x]] - 15120*c^3*f^3*S
in[4*ArcCos[c*x]] + 7560*c*f*g^2*Sin[4*ArcCos[c*x]] - 24192*c^2*f^2*g*Sin[5
*ArcCos[c*x]] + 6048*g^3*Sin[5*ArcCos[c*x]] + 1680*c^3*f^3*Sin[6*ArcCos[c*x
]] - 5040*c*f*g^2*Sin[6*ArcCos[c*x]] + 900*g^3*Sin[7*ArcCos[c*x]] + 945*c*f
*g^2*Sin[8*ArcCos[c*x]] + 140*g^3*Sin[9*ArcCos[c*x]])))/(162570240*c^4*Sqrt
[1 - c^2*x^2])
```

Maple [A] time = 0.959, size = 2236, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x)
```

```
[Out] 5/16*a*f^3*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a*f^3*d^3/(c^2*d)^(1/2)*arctan((
c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-23/16*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d
^2*c^4/(c^2*x^2-1)*arccos(c*x)*x^7+3/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2*c^6/(
c^2*x^2-1)*arccos(c*x)*x^8*f^2-12/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2*c^4/(c^2
*x^2-1)*arccos(c*x)*x^6*f^2+18/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2*c^2/(c^2*x^
2-1)*arccos(c*x)*x^4*f^2-2/63*a*g^3/d/c^4*(-c^2*d*x^2+d)^(7/2)+3/7*b*(-d*(c
^2*x^2-1))^(1/2)*g*d^2/c^2/(c^2*x^2-1)*arccos(c*x)*f^2+1/6*b*(-d*(c^2*x^2-1
))^(1/2)*f^3*d^2*c^6/(c^2*x^2-1)*arccos(c*x)*x^7-17/24*b*(-d*(c^2*x^2-1))^(
1/2)*f^3*d^2*c^4/(c^2*x^2-1)*arccos(c*x)*x^5+59/48*b*(-d*(c^2*x^2-1))^(1/2)
```

```
*f^3*d^2*c^2/(c^2*x^2-1)*arccos(c*x)*x^3-1/36*b*(-d*(c^2*x^2-1))^(1/2)*f^3*
d^2*c^5/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^6+13/96*b*(-d*(c^2*x^2-1))^(1/2)*f
^3*d^2*c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^4-11/32*b*(-d*(c^2*x^2-1))^(1/2
)*f^3*d^2*c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^2-1/81*b*(-d*(c^2*x^2-1))^(1/2
)*g^3*d^2*c^5/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^9+19/441*b*(-d*(c^2*x^2-1))^
(1/2)*g^3*d^2*c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^7-1/21*b*(-d*(c^2*x^2-1)
)^(1/2)*g^3*d^2*c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^5+1/189*b*(-d*(c^2*x^2-1
))^(1/2)*g^3*d^2/c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^3+2/63*b*(-d*(c^2*x^2-1
))^(1/2)*g^3*d^2/c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x+359/24576*b*(-d*(c^2*
x^2-1))^(1/2)*f*g^2*d^2/c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)-133/128*b*(-d*(c
^2*x^2-1))^(1/2)*f*g^2*d^2/(c^2*x^2-1)*arccos(c*x)*x^3-12/7*b*(-d*(c^2*x^2-
1))^(1/2)*g*d^2/(c^2*x^2-1)*arccos(c*x)*x^2*f^2+5/32*b*(-d*(c^2*x^2-1))^(1/
2)*(-c^2*x^2+1)^(1/2)/c/(c^2*x^2-1)*arccos(c*x)^2*f^3*d^2+1/9*b*(-d*(c^2*x^
2-1))^(1/2)*g^3*d^2*c^6/(c^2*x^2-1)*arccos(c*x)*x^10-26/63*b*(-d*(c^2*x^2-1
))^(1/2)*g^3*d^2*c^4/(c^2*x^2-1)*arccos(c*x)*x^8+34/63*b*(-d*(c^2*x^2-1))^(
1/2)*g^3*d^2*c^2/(c^2*x^2-1)*arccos(c*x)*x^6-1/63*b*(-d*(c^2*x^2-1))^(1/2)*
g^3*d^2/c^2/(c^2*x^2-1)*arccos(c*x)*x^2+3/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/
c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x*f^2-3/64*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*
d^2*c^5/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^8+17/96*b*(-d*(c^2*x^2-1))^(1/2)*f
*g^2*d^2*c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^6-3/49*b*(-d*(c^2*x^2-1))^(1/
2)*g*d^2*c^5/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^7*f^2-3/7*b*(-d*(c^2*x^2-1))^
(1/2)*g*d^2*c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^3*f^2-59/256*b*(-d*(c^2*x^2-
1))^(1/2)*f*g^2*d^2*c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^4+15/256*b*(-d*(c^2*
x^2-1))^(1/2)*f*g^2*d^2/c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^2+15/256*b*(-d*(
c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2*f*d^2*g^
2+3/8*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d^2*c^6/(c^2*x^2-1)*arccos(c*x)*x^9+12
7/64*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d^2*c^2/(c^2*x^2-1)*arccos(c*x)*x^5+15/
128*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d^2/c^2/(c^2*x^2-1)*arccos(c*x)*x+9/35*b
*(-d*(c^2*x^2-1))^(1/2)*g*d^2*c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^5*f^2+5/
24*a*f^3*d*x*(-c^2*d*x^2+d)^(3/2)-1/9*a*g^3*x^2*(-c^2*d*x^2+d)^(7/2)/c^2/d+
1/16*a*f*g^2/c^2*x*(-c^2*d*x^2+d)^(5/2)-3/7*a*f^2*g*(-c^2*d*x^2+d)^(7/2)/c^
2/d+1/6*a*f^3*x*(-c^2*d*x^2+d)^(5/2)-3/8*a*f*g^2*x*(-c^2*d*x^2+d)^(7/2)/c^2
/d+5/64*a*f*g^2/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+15/128*a*f*g^2/c^2*d^2*x*(-c^2
*d*x^2+d)^(1/2)+15/128*a*f*g^2/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x
/(-c^2*d*x^2+d)^(1/2))+299/2304*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d^2/c/(c^2*x^2
-1)*(-c^2*x^2+1)^(1/2)+2/63*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d^2/c^4/(c^2*x^2-1
)*arccos(c*x)-16/63*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d^2/(c^2*x^2-1)*arccos(c*x
)*x^4-11/16*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d^2/(c^2*x^2-1)*arccos(c*x)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ac^4d^2g^3x^7 + 3\,ac^4d^2fg^2x^6 + 3\,ad^2f^2gx + ad^2f^3 + \left(3\,ac^4d^2f^2g - 2\,ac^2d^2g^3\right)x^5 + \left(ac^4d^2f^3 - 6\,ac^2d^2fg^2\right)x^4 - \left(6\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^4*d^2*g^3*x^7 + 3*a*c^4*d^2*f*g^2*x^6 + 3*a*d^2*f^2*g*x + a*d
^2*f^3 + (3*a*c^4*d^2*f^2*g - 2*a*c^2*d^2*g^3)*x^5 + (a*c^4*d^2*f^3 - 6*a*c
^2*d^2*f*g^2)*x^4 - (6*a*c^2*d^2*f^2*g - a*d^2*g^3)*x^3 - (2*a*c^2*d^2*f^3
- 3*a*d^2*f*g^2)*x^2 + (b*c^4*d^2*g^3*x^7 + 3*b*c^4*d^2*f*g^2*x^6 + 3*b*d^2
*f^2*g*x + b*d^2*f^3 + (3*b*c^4*d^2*f^2*g - 2*b*c^2*d^2*g^3)*x^5 + (b*c^4*d
^2*f^3 - 6*b*c^2*d^2*f*g^2)*x^4 - (6*b*c^2*d^2*f^2*g - b*d^2*g^3)*x^3 - (2*
b*c^2*d^2*f^3 - 3*b*d^2*f*g^2)*x^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^2dx^2 + d\right)^{\frac{5}{2}}\left(gx + f\right)^3\left(b\arccos\left(cx\right) + a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="gi
ac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(g*x + f)^3*(b*arccos(c*x) + a), x)
```

3.11 $\int (f+gx)^2 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cos^{-1}(cx)\right)\,dx$

Optimal. Leaf size=940

$$\frac{bc^5d^2g^2\sqrt{d-c^2dx^2}x^8}{64\sqrt{1-c^2x^2}}+\frac{2bc^5d^2fg\sqrt{d-c^2dx^2}x^7}{49\sqrt{1-c^2x^2}}-\frac{17bc^3d^2g^2\sqrt{d-c^2dx^2}x^6}{288\sqrt{1-c^2x^2}}-\frac{6bc^3d^2fg\sqrt{d-c^2dx^2}x^5}{35\sqrt{1-c^2x^2}}-\frac{5bc^3d^2f^2\sqrt{d-c^2}}{96\sqrt{1-c^2x^2}}$$

```
[Out] (-2*b*d^2*f*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[1 - c^2*x^2]) + (25*b*c*d^2*
f^2*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) - (5*b*d^2*g^2*x^2*Sqrt
[d - c^2*d*x^2])/(256*c*Sqrt[1 - c^2*x^2]) + (2*b*c*d^2*f*g*x^3*Sqrt[d - c^
2*d*x^2])/(7*Sqrt[1 - c^2*x^2]) - (5*b*c^3*d^2*f^2*x^4*Sqrt[d - c^2*d*x^2])
/(96*Sqrt[1 - c^2*x^2]) + (59*b*c*d^2*g^2*x^4*Sqrt[d - c^2*d*x^2])/(768*Sqr
t[1 - c^2*x^2]) - (6*b*c^3*d^2*f*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[1 - c^
2*x^2]) - (17*b*c^3*d^2*g^2*x^6*Sqrt[d - c^2*d*x^2])/(288*Sqrt[1 - c^2*x^2]
) + (2*b*c^5*d^2*f*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[1 - c^2*x^2]) + (b*c
^5*d^2*g^2*x^8*Sqrt[d - c^2*d*x^2])/(64*Sqrt[1 - c^2*x^2]) - (b*d^2*f^2*(1
- c^2*x^2)^(5/2)*Sqrt[d - c^2*d*x^2])/(36*c) + (5*d^2*f^2*x*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCos[c*x]))/16 - (5*d^2*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
os[c*x]))/(128*c^2) + (5*d^2*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]
))/64 + (5*d^2*f^2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))
/24 + (5*d^2*g^2*x^3*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))
/48 + (d^2*f^2*x*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/6
 + (d^2*g^2*x^3*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8
- (2*d^2*f*g*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^
2) - (5*d^2*f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(32*b*c*Sqrt[1 -
 c^2*x^2]) - (5*d^2*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(256*b*c
^3*Sqrt[1 - c^2*x^2])
```

Rubi [A] time = 0.95887, antiderivative size = 940, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.484, Rules used = {4778, 4764, 4650, 4648, 4642, 30, 14, 261, 4678, 194, 4700, 4698, 4708, 266, 43}

$$\frac{bc^5d^2g^2\sqrt{d-c^2dx^2}x^8}{64\sqrt{1-c^2x^2}}+\frac{2bc^5d^2fg\sqrt{d-c^2dx^2}x^7}{49\sqrt{1-c^2x^2}}-\frac{17bc^3d^2g^2\sqrt{d-c^2dx^2}x^6}{288\sqrt{1-c^2x^2}}-\frac{6bc^3d^2fg\sqrt{d-c^2dx^2}x^5}{35\sqrt{1-c^2x^2}}-\frac{5bc^3d^2f^2\sqrt{d-c^2}}{96\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-2*b*d^2*f*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[1 - c^2*x^2]) + (25*b*c*d^2*
f^2*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) - (5*b*d^2*g^2*x^2*Sqrt
[d - c^2*d*x^2])/(256*c*Sqrt[1 - c^2*x^2]) + (2*b*c*d^2*f*g*x^3*Sqrt[d - c^
2*d*x^2])/(7*Sqrt[1 - c^2*x^2]) - (5*b*c^3*d^2*f^2*x^4*Sqrt[d - c^2*d*x^2])
/(96*Sqrt[1 - c^2*x^2]) + (59*b*c*d^2*g^2*x^4*Sqrt[d - c^2*d*x^2])/(768*Sqr
t[1 - c^2*x^2]) - (6*b*c^3*d^2*f*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[1 - c^
2*x^2]) - (17*b*c^3*d^2*g^2*x^6*Sqrt[d - c^2*d*x^2])/(288*Sqrt[1 - c^2*x^2]
) + (2*b*c^5*d^2*f*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[1 - c^2*x^2]) + (b*c
^5*d^2*g^2*x^8*Sqrt[d - c^2*d*x^2])/(64*Sqrt[1 - c^2*x^2]) - (b*d^2*f^2*(1
- c^2*x^2)^(5/2)*Sqrt[d - c^2*d*x^2])/(36*c) + (5*d^2*f^2*x*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCos[c*x]))/16 - (5*d^2*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
os[c*x]))/(128*c^2) + (5*d^2*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]
))/64 + (5*d^2*f^2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))
/24 + (5*d^2*g^2*x^3*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))
/48 + (d^2*f^2*x*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/6
 + (d^2*g^2*x^3*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8
- (2*d^2*f*g*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^
2) - (5*d^2*f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(32*b*c*Sqrt[1 -
 c^2*x^2]) - (5*d^2*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(256*b*c
^3*Sqrt[1 - c^2*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4650

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n)/(2*p + 1), x] + (D
ist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 - c^2*x
^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
```

```
 GtQ[p, 0]
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4700

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcC
os[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x)^
m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart
[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 - c^2*x^2)^FracPart[p]), I
nt[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcC
os[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x
^2]), Int[((f*x)^m*(a + b*ArcCos[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] + Dist[
(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a
 + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && Eq
Q[c^2*d + e, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (f+gx)^2 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cos^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int (f+gx)^2 \left(1-c^2x^2\right)^{5/2} \left(a+b\cos^{-1}(cx)\right) dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(f^2 \left(1-c^2x^2\right)^{5/2} \left(a+b\cos^{-1}(cx)\right)+2fgx\left(1-c^2x^2\right)^{5/2}\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d^2f^2\sqrt{d-c^2dx^2}\right)\int \left(1-c^2x^2\right)^{5/2} \left(a+b\cos^{-1}(cx)\right) dx}{\sqrt{1-c^2x^2}} + \frac{\left(2d^2fg\sqrt{d-c^2dx^2}\right.}{}$$

$$= \frac{1}{6}d^2f^2x\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right) + \frac{1}{8}d^2g^2x^3\left(1-c^2x^2\right)^2$$

$$= -\frac{bd^2f^2\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c} + \frac{5}{24}d^2f^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+\right.$$

$$= -\frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}} + \frac{2bcd^2fgx^3\sqrt{d-c^2dx^2}}{7\sqrt{1-c^2x^2}} - \frac{6bc^3d^2fgx^5\sqrt{d-c^2d}}{35\sqrt{1-c^2x^2}}$$

$$= -\frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}} + \frac{25bcd^2f^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}} + \frac{2bcd^2fgx^3\sqrt{d-c^2d}}{7\sqrt{1-c^2x^2}}$$

$$= -\frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}} + \frac{25bcd^2f^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}} - \frac{5bd^2g^2x^2\sqrt{d-c^2dx^2}}{256c\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 4.516, size = 794, normalized size = 0.84

$$\frac{d^2\left(\sqrt{d-c^2dx^2}\left(3010560ac^7f^2x^5\sqrt{1-c^2x^2} - 9784320ac^5f^2x^3\sqrt{1-c^2x^2} + 12418560ac^3f^2x\sqrt{1-c^2x^2} + 5160960ac^7fgx^6\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (d^2*(-352800*b*(8*c^2*f^2 + g^2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 70560
0*a*Sqrt[d]*(8*c^2*f^2 + g^2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*
```

```
x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d - c^2*d*x^2]*(-2822400*b*c^2*f*g*x
 - 5160960*a*c*f*g*Sqrt[1 - c^2*x^2] + 12418560*a*c^3*f^2*x*Sqrt[1 - c^2*x^
2] - 705600*a*c*g^2*x*Sqrt[1 - c^2*x^2] + 15482880*a*c^3*f*g*x^2*Sqrt[1 - c
^2*x^2] - 9784320*a*c^5*f^2*x^3*Sqrt[1 - c^2*x^2] + 5550720*a*c^3*g^2*x^3*S
qrt[1 - c^2*x^2] - 15482880*a*c^5*f*g*x^4*Sqrt[1 - c^2*x^2] + 3010560*a*c^7
*f^2*x^5*Sqrt[1 - c^2*x^2] - 6397440*a*c^5*g^2*x^5*Sqrt[1 - c^2*x^2] + 5160
960*a*c^7*f*g*x^6*Sqrt[1 - c^2*x^2] + 2257920*a*c^7*g^2*x^7*Sqrt[1 - c^2*x^
2] + 141120*b*(15*c^2*f^2 + g^2)*Cos[2*ArcCos[c*x]] + 564480*b*c*f*g*Cos[3*
ArcCos[c*x]] - 211680*b*c^2*f^2*Cos[4*ArcCos[c*x]] + 35280*b*g^2*Cos[4*ArcC
os[c*x]] - 112896*b*c*f*g*Cos[5*ArcCos[c*x]] + 15680*b*c^2*f^2*Cos[6*ArcCos
[c*x]] - 15680*b*g^2*Cos[6*ArcCos[c*x]] + 11520*b*c*f*g*Cos[7*ArcCos[c*x]]
+ 2205*b*g^2*Cos[8*ArcCos[c*x]]) + 168*b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(-
58112*c*f*g*Sqrt[1 - c^2*x^2] + 111872*c^3*f*g*x^2*Sqrt[1 - c^2*x^2] - 2764
8*c*f*g*(1 - c^2*x^2)^(3/2)*Cos[2*ArcCos[c*x]] - 3840*c*f*g*(1 - c^2*x^2)^(
3/2)*Cos[4*ArcCos[c*x]] + 25200*c^2*f^2*Sin[2*ArcCos[c*x]] + 1680*g^2*Sin[2
*ArcCos[c*x]] - 8960*c*f*g*Sin[3*ArcCos[c*x]] - 5040*c^2*f^2*Sin[4*ArcCos[c
*x]] + 840*g^2*Sin[4*ArcCos[c*x]] - 5376*c*f*g*Sin[5*ArcCos[c*x]] + 560*c^2
*f^2*Sin[6*ArcCos[c*x]] - 560*g^2*Sin[6*ArcCos[c*x]] + 105*g^2*Sin[8*ArcCos
[c*x]])))/(18063360*c^3*Sqrt[1 - c^2*x^2])
```

Maple [A] time = 0.702, size = 1633, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x)
```

```
[Out] -2/7*a*f*g*(-c^2*d*x^2+d)^(7/2)/c^2/d-1/8*a*g^2*x*(-c^2*d*x^2+d)^(7/2)/c^2/
d+5/192*a*g^2/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/128*a*g^2/c^2*d^2*x*(-c^2*d*x^
2+d)^(1/2)+5/128*a*g^2/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d
*x^2+d)^(1/2))+13/96*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^3/(c^2*x^2-1)*(-c^2*x^2
+1)^(1/2)*x^4*f^2-59/768*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c/(c^2*x^2-1)*(-c^2*x
^2+1)^(1/2)*x^4*g^2-11/32*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c/(c^2*x^2-1)*(-c^2*
x^2+1)^(1/2)*x^2*f^2+5/256*b*(-d*(c^2*x^2-1))^(1/2)*d^2/c/(c^2*x^2-1)*(-c^2
*x^2+1)^(1/2)*x^2*g^2-1/64*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d^2*c^5/(c^2*x^2-1)
*(-c^2*x^2+1)^(1/2)*x^8+17/288*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d^2*c^3/(c^2*x^
2-1)*(-c^2*x^2+1)^(1/2)*x^6-1/36*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^5/(c^2*x^2-
1)*(-c^2*x^2+1)^(1/2)*x^6*f^2+1/48*a*g^2/c^2*x*(-c^2*d*x^2+d)^(5/2)-133/384
*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d^2/(c^2*x^2-1)*arccos(c*x)*x^3-11/16*b*(-d*(
c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)*arccos(c*x)*x*f^2+1/6*a*f^2*x*(-c^2*d*x^2
+d)^(5/2)-2/49*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2*c^5/(c^2*x^2-1)*(-c^2*x^2+1
```

```
)^(1/2)*x^7+6/35*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2*c^3/(c^2*x^2-1)*(-c^2*x^2
+1)^(1/2)*x^5-2/7*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2*c/(c^2*x^2-1)*(-c^2*x^2+
1)^(1/2)*x^3+2/7*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2/c/(c^2*x^2-1)*(-c^2*x^2+1
)^(1/2)*x+2/7*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2*c^6/(c^2*x^2-1)*arccos(c*x)*
x^8-8/7*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2*c^4/(c^2*x^2-1)*arccos(c*x)*x^6+12
/7*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2*c^2/(c^2*x^2-1)*arccos(c*x)*x^4+5/128*b
*(-d*(c^2*x^2-1))^(1/2)*g^2*d^2/c^2/(c^2*x^2-1)*arccos(c*x)*x+2/7*b*(-d*(c^
2*x^2-1))^(1/2)*f*g*d^2/c^2/(c^2*x^2-1)*arccos(c*x)+1/6*b*(-d*(c^2*x^2-1))^
(1/2)*d^2*c^6/(c^2*x^2-1)*arccos(c*x)*x^7*f^2-17/24*b*(-d*(c^2*x^2-1))^(1/2
)*d^2*c^4/(c^2*x^2-1)*arccos(c*x)*x^5*f^2+59/48*b*(-d*(c^2*x^2-1))^(1/2)*d^
2*c^2/(c^2*x^2-1)*arccos(c*x)*x^3*f^2-8/7*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2/
(c^2*x^2-1)*arccos(c*x)*x^2+5/32*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2
)/c/(c^2*x^2-1)*arccos(c*x)^2*d^2*f^2+5/256*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*
x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2*d^2*g^2+1/8*b*(-d*(c^2*x^2-1))^(
1/2)*g^2*d^2*c^6/(c^2*x^2-1)*arccos(c*x)*x^9-23/48*b*(-d*(c^2*x^2-1))^(1/2)
*g^2*d^2*c^4/(c^2*x^2-1)*arccos(c*x)*x^7+127/192*b*(-d*(c^2*x^2-1))^(1/2)*g
^2*d^2*c^2/(c^2*x^2-1)*arccos(c*x)*x^5+5/16*a*f^2*d^2*x*(-c^2*d*x^2+d)^(1/2
)+5/16*a*f^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))
+5/24*a*f^2*d*x*(-c^2*d*x^2+d)^(3/2)+299/2304*b*(-d*(c^2*x^2-1))^(1/2)*d^2/
c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*f^2+359/73728*b*(-d*(c^2*x^2-1))^(1/2)*d^2
/c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*g^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\Big(\Big(ac^4d^2g^2x^6 + 2\,ac^4d^2fgx^5 - 4\,ac^2d^2fgx^3 + 2\,ad^2fgx + ad^2f^2 + \big(ac^4d^2f^2 - 2\,ac^2d^2g^2\big)x^4 - \big(2\,ac^2d^2f^2 - ad^2g^2\big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^4*d^2*g^2*x^6 + 2*a*c^4*d^2*f*g*x^5 - 4*a*c^2*d^2*f*g*x^3 + 2
*a*d^2*f*g*x + a*d^2*f^2 + (a*c^4*d^2*f^2 - 2*a*c^2*d^2*g^2)*x^4 - (2*a*c^2
*d^2*f^2 - a*d^2*g^2)*x^2 + (b*c^4*d^2*g^2*x^6 + 2*b*c^4*d^2*f*g*x^5 - 4*b*
c^2*d^2*f*g*x^3 + 2*b*d^2*f*g*x + b*d^2*f^2 + (b*c^4*d^2*f^2 - 2*b*c^2*d^2*
g^2)*x^4 - (2*b*c^2*d^2*f^2 - b*d^2*g^2)*x^2)*arccos(c*x))*sqrt(-c^2*d*x^2
+ d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}\left(gx+f\right)^2\left(b\arccos\left(cx\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="gi
ac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(g*x + f)^2*(b*arccos(c*x) + a), x)
```

3.12 $\int (f+gx)\left(d-c^2dx^2\right)^{5/2}\left(a+b\cos^{-1}(cx)\right)\,dx$

Optimal. Leaf size=517

$$\frac{1}{6}d^2fx\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{5}{16}d^2fx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{5}{24}d^2fx\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\,($$

```
[Out] -(b*d^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[1 - c^2*x^2]) + (25*b*c*d^2*f*x^
2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) + (b*c*d^2*g*x^3*Sqrt[d - c^2
*d*x^2])/(7*Sqrt[1 - c^2*x^2]) - (5*b*c^3*d^2*f*x^4*Sqrt[d - c^2*d*x^2])/(9
6*Sqrt[1 - c^2*x^2]) - (3*b*c^3*d^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[1 -
 c^2*x^2]) + (b*c^5*d^2*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[1 - c^2*x^2]) -
 (b*d^2*f*(1 - c^2*x^2)^(5/2)*Sqrt[d - c^2*d*x^2])/(36*c) + (5*d^2*f*x*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 + (5*d^2*f*x*(1 - c^2*x^2)*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCos[c*x]))/24 + (d^2*f*x*(1 - c^2*x^2)^2*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCos[c*x]))/6 - (d^2*g*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCos[c*x]))/(7*c^2) - (5*d^2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[
c*x])^2)/(32*b*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] time = 0.402964, antiderivative size = 517, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {4778, 4764, 4650, 4648, 4642, 30, 14, 261, 4678, 194}

$$\frac{1}{6}d^2fx\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{5}{16}d^2fx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{5}{24}d^2fx\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\,($$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```


```
[Out] -(b*d^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[1 - c^2*x^2]) + (25*b*c*d^2*f*x^
2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) + (b*c*d^2*g*x^3*Sqrt[d - c^2
*d*x^2])/(7*Sqrt[1 - c^2*x^2]) - (5*b*c^3*d^2*f*x^4*Sqrt[d - c^2*d*x^2])/(9
6*Sqrt[1 - c^2*x^2]) - (3*b*c^3*d^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[1 -
 c^2*x^2]) + (b*c^5*d^2*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[1 - c^2*x^2]) -
 (b*d^2*f*(1 - c^2*x^2)^(5/2)*Sqrt[d - c^2*d*x^2])/(36*c) + (5*d^2*f*x*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 + (5*d^2*f*x*(1 - c^2*x^2)*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCos[c*x]))/24 + (d^2*f*x*(1 - c^2*x^2)^2*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCos[c*x]))/6 - (d^2*g*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCos[c*x]))/(7*c^2) - (5*d^2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[
```

```
c*x])^2)/(32*b*c*Sqrt[1 - c^2*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4650

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n)/(2*p + 1), x] + (D
ist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 - c^2*x
^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (f+gx)\left(d-c^2dx^2\right)^{5/2}\left(a+b\cos^{-1}(cx)\right)\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int (f+gx)\left(1-c^2x^2\right)^{5/2}\left(a+b\cos^{-1}(cx)\right)\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(f\left(1-c^2x^2\right)^{5/2}\left(a+b\cos^{-1}(cx)\right)+gx\left(1-c^2x^2\right)^{5/2}\left(\right.\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d^2f\sqrt{d-c^2dx^2}\right)\int \left(1-c^2x^2\right)^{5/2}\left(a+b\cos^{-1}(cx)\right)\,dx}{\sqrt{1-c^2x^2}} + \frac{\left(d^2g\sqrt{d-c^2d\right.}{}$$

$$= \frac{1}{6}d^2fx\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right) - \frac{d^2g\left(1-c^2x^2\right)^3\sqrt{d-}}{}$$

$$= -\frac{bd^2f\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c} + \frac{5}{24}d^2fx\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\right.$$

$$= -\frac{bd^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}} + \frac{bcd^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{1-c^2x^2}} - \frac{3bc^3d^2gx^5\sqrt{d-c^2dx^2}}{35\sqrt{1-c^2x^2}} + \frac{b}{}$$

$$= -\frac{bd^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}} + \frac{25bcd^2fx^2\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}} + \frac{bcd^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{1-c^2x^2}} - \frac{5}{}$$

<u>Mathematica</u> [A] time = 2.90072, size = 526, normalized size = 1.02

$$\frac{d^2\left(\sqrt{d-c^2dx^2}\left(94080ac^6fx^5\sqrt{1-c^2x^2}-305760ac^4fx^3\sqrt{1-c^2x^2}+388080ac^2fx\sqrt{1-c^2x^2}+80640ac^6gx^6\sqrt{1-c^2x^2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (d^2*(-88200*b*c*f*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 176400*a*c*Sqrt[d]*f
*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2)
)] + Sqrt[d - c^2*d*x^2]*(-44100*b*c*g*x - 80640*a*g*Sqrt[1 - c^2*x^2] + 38
8080*a*c^2*f*x*Sqrt[1 - c^2*x^2] + 241920*a*c^2*g*x^2*Sqrt[1 - c^2*x^2] - 3
05760*a*c^4*f*x^3*Sqrt[1 - c^2*x^2] - 241920*a*c^4*g*x^4*Sqrt[1 - c^2*x^2]
+ 94080*a*c^6*f*x^5*Sqrt[1 - c^2*x^2] + 80640*a*c^6*g*x^6*Sqrt[1 - c^2*x^2]
 + 66150*b*c*f*Cos[2*ArcCos[c*x]] + 8820*b*g*Cos[3*ArcCos[c*x]] - 6615*b*c*
f*Cos[4*ArcCos[c*x]] - 1764*b*g*Cos[5*ArcCos[c*x]] + 490*b*c*f*Cos[6*ArcCos
[c*x]] + 180*b*g*Cos[7*ArcCos[c*x]]) + 84*b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]
*(-1816*g*Sqrt[1 - c^2*x^2] + 3496*c^2*g*x^2*Sqrt[1 - c^2*x^2] - 864*g*(1 -
```

```
 c^2*x^2)^(3/2)*Cos[2*ArcCos[c*x]] - 120*g*(1 - c^2*x^2)^(3/2)*Cos[4*ArcCos
[c*x]] + 1575*c*f*Sin[2*ArcCos[c*x]] - 280*g*Sin[3*ArcCos[c*x]] - 315*c*f*S
in[4*ArcCos[c*x]] - 168*g*Sin[5*ArcCos[c*x]] + 35*c*f*Sin[6*ArcCos[c*x]]))
/(564480*c^2*Sqrt[1 - c^2*x^2])
```

Maple [B] time = 0.602, size = 931, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x)
```

```
[Out] -1/7*a*g*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/6*a*f*x*(-c^2*d*x^2+d)^(5/2)+5/24*a*f
*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*f*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a*f*d^3/
(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+5/32*b*(-d*(c^2*
x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/(c^2*x^2-1)*arccos(c*x)^2*f*d^2-1/36*b*(
-d*(c^2*x^2-1))^(1/2)*f*d^2*c^5/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^6+13/96*b*
(-d*(c^2*x^2-1))^(1/2)*f*d^2*c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^4-11/32*b
*(-d*(c^2*x^2-1))^(1/2)*f*d^2*c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^2-1/49*b*(
-d*(c^2*x^2-1))^(1/2)*g*d^2*c^5/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^7+3/35*b*(
-d*(c^2*x^2-1))^(1/2)*g*d^2*c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^5-1/7*b*(-
d*(c^2*x^2-1))^(1/2)*g*d^2*c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^3+1/7*b*(-d*(
c^2*x^2-1))^(1/2)*g*d^2/c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x+1/7*b*(-d*(c^2*x
^2-1))^(1/2)*g*d^2/c^2/(c^2*x^2-1)*arccos(c*x)+1/7*b*(-d*(c^2*x^2-1))^(1/2)
*g*d^2*c^6/(c^2*x^2-1)*arccos(c*x)*x^8-4/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2*c
^4/(c^2*x^2-1)*arccos(c*x)*x^6+6/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2*c^2/(c^2*
x^2-1)*arccos(c*x)*x^4-4/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c^2*x^2-1)*arcco
s(c*x)*x^2+1/6*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2*c^6/(c^2*x^2-1)*arccos(c*x)*x
^7-17/24*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2*c^4/(c^2*x^2-1)*arccos(c*x)*x^5+59/
48*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2*c^2/(c^2*x^2-1)*arccos(c*x)*x^3-11/16*b*(
-d*(c^2*x^2-1))^(1/2)*f*d^2/(c^2*x^2-1)*arccos(c*x)*x+299/2304*b*(-d*(c^2*x
^2-1))^(1/2)*f*d^2/c/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\Big(\big(ac^4d^2gx^5 + ac^4d^2fx^4 - 2\,ac^2d^2gx^3 - 2\,ac^2d^2fx^2 + ad^2gx + ad^2f + \big(bc^4d^2gx^5 + bc^4d^2fx^4 - 2\,bc^2d^2gx^3 - 2\,bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fric
as")
```

```
[Out] integral((a*c^4*d^2*g*x^5 + a*c^4*d^2*f*x^4 - 2*a*c^2*d^2*g*x^3 - 2*a*c^2*d
^2*f*x^2 + a*d^2*g*x + a*d^2*f + (b*c^4*d^2*g*x^5 + b*c^4*d^2*f*x^4 - 2*b*c
^2*d^2*g*x^3 - 2*b*c^2*d^2*f*x^2 + b*d^2*g*x + b*d^2*f)*arccos(c*x))*sqrt(-
c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^2dx^2 + d\right)^{\frac{5}{2}} \left(gx + f\right)\left(b \arccos\left(cx\right) + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="giac
")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(g*x + f)*(b*arccos(c*x) + a), x)
```

3.13 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cos^{-1}(cx)\right)}{f+gx}\,dx$

Optimal. Leaf size=1637

result too large to display

```
[Out] (a*d^2*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2])/g^5 - (2*b*c*d^2*x*Sqrt[d - c
^2*d*x^2])/(15*g*Sqrt[1 - c^2*x^2]) - (b*c*d^2*(c^2*f^2 - 2*g^2)*x*Sqrt[d -
 c^2*d*x^2])/(3*g^3*Sqrt[1 - c^2*x^2]) + (b*c*d^2*(c^2*f^2 - g^2)^2*x*Sqrt[
d - c^2*d*x^2])/(g^5*Sqrt[1 - c^2*x^2]) + (b*c^3*d^2*f*x^2*Sqrt[d - c^2*d*x
^2])/(16*g^2*Sqrt[1 - c^2*x^2]) - (b*c^3*d^2*f*(c^2*f^2 - 2*g^2)*x^2*Sqrt[d
 - c^2*d*x^2])/(4*g^4*Sqrt[1 - c^2*x^2]) - (b*c^3*d^2*x^3*Sqrt[d - c^2*d*x^
2])/(45*g*Sqrt[1 - c^2*x^2]) + (b*c^3*d^2*(c^2*f^2 - 2*g^2)*x^3*Sqrt[d - c^
2*d*x^2])/(9*g^3*Sqrt[1 - c^2*x^2]) - (b*c^5*d^2*f*x^4*Sqrt[d - c^2*d*x^2])
/(16*g^2*Sqrt[1 - c^2*x^2]) + (b*c^5*d^2*x^5*Sqrt[d - c^2*d*x^2])/(25*g*Sqr
t[1 - c^2*x^2]) + (b*d^2*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])
/g^5 + (c^2*d^2*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(8*g^2) - (c^2
*d^2*f*(c^2*f^2 - 2*g^2)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(2*g^4)
 - (c^4*d^2*f*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(4*g^2) - (d^2*(
1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*g) - (d^2*(c^2*f^2
 - 2*g^2)*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*g^3) +
(d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*g) + (c*d^
2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*g^2*Sqrt[1 - c^2*x^2])
 + (c*d^2*f*(c^2*f^2 - 2*g^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4
*b*g^4*Sqrt[1 - c^2*x^2]) - (c*d^2*(c^2*f^2 - g^2)^2*x*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCos[c*x])^2)/(2*b*g^5*Sqrt[1 - c^2*x^2]) - (d^2*(c^2*f^2 - g^2)^3
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*g^6*(f + g*x)*Sqrt[1 - c
^2*x^2]) - (d^2*(c^2*f^2 - g^2)^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCos[c*x])^2)/(2*b*c*g^4*(f + g*x)) - (a*d^2*(c^2*f^2 - g^2)^(5/2)*Sq
rt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^
2])])/(g^6*Sqrt[1 - c^2*x^2]) - (I*b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2
*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2
])])/(g^6*Sqrt[1 - c^2*x^2]) + (I*b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*
d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]
)])/(g^6*Sqrt[1 - c^2*x^2]) - (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x
^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(g^6*
Sqrt[1 - c^2*x^2]) + (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*PolyL
og[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^6*Sqrt[1 -
c^2*x^2])
```

Rubi [A] time = 2.73, antiderivative size = 1637, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 28, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.903, Rules used = {4778, 4768, 4648, 4642, 30, 4678, 4698, 4708, 266, 43, 4690, 12, 4766, 683, 4758, 6742, 725, 204, 1654, 4800, 4798, 8, 4774, 3321, 2264, 2190, 2279, 2391}

$$\frac{bd^2x^5\sqrt{d-c^2dx^2}c^5}{25g\sqrt{1-c^2x^2}}-\frac{bd^2fx^4\sqrt{d-c^2dx^2}c^5}{16g^2\sqrt{1-c^2x^2}}-\frac{d^2fx^3\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)c^4}{4g^2}+\frac{bd^2\left(c^2f^2-2g^2\right)x^3\sqrt{d-c^2dx^2}c^3}{9g^3\sqrt{1-c^2x^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

```
[Out] (a*d^2*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2])/g^5 - (2*b*c*d^2*x*Sqrt[d - c
^2*d*x^2])/(15*g*Sqrt[1 - c^2*x^2]) - (b*c*d^2*(c^2*f^2 - 2*g^2)*x*Sqrt[d -
 c^2*d*x^2])/(3*g^3*Sqrt[1 - c^2*x^2]) + (b*c*d^2*(c^2*f^2 - g^2)^2*x*Sqrt[
d - c^2*d*x^2])/(g^5*Sqrt[1 - c^2*x^2]) + (b*c^3*d^2*f*x^2*Sqrt[d - c^2*d*x
^2])/(16*g^2*Sqrt[1 - c^2*x^2]) - (b*c^3*d^2*f*(c^2*f^2 - 2*g^2)*x^2*Sqrt[d
 - c^2*d*x^2])/(4*g^4*Sqrt[1 - c^2*x^2]) - (b*c^3*d^2*x^3*Sqrt[d - c^2*d*x^
2])/(45*g*Sqrt[1 - c^2*x^2]) + (b*c^3*d^2*(c^2*f^2 - 2*g^2)*x^3*Sqrt[d - c^
2*d*x^2])/(9*g^3*Sqrt[1 - c^2*x^2]) - (b*c^5*d^2*f*x^4*Sqrt[d - c^2*d*x^2])
/(16*g^2*Sqrt[1 - c^2*x^2]) + (b*c^5*d^2*x^5*Sqrt[d - c^2*d*x^2])/(25*g*Sqr
t[1 - c^2*x^2]) + (b*d^2*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])
/g^5 + (c^2*d^2*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(8*g^2) - (c^2
*d^2*f*(c^2*f^2 - 2*g^2)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(2*g^4)
 - (c^4*d^2*f*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(4*g^2) - (d^2*(
1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*g) - (d^2*(c^2*f^2
 - 2*g^2)*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*g^3) +
(d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*g) + (c*d^
2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*g^2*Sqrt[1 - c^2*x^2])
 + (c*d^2*f*(c^2*f^2 - 2*g^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4
*b*g^4*Sqrt[1 - c^2*x^2]) - (c*d^2*(c^2*f^2 - g^2)^2*x*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCos[c*x])^2)/(2*b*g^5*Sqrt[1 - c^2*x^2]) - (d^2*(c^2*f^2 - g^2)^3
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*g^6*(f + g*x)*Sqrt[1 - c
^2*x^2]) - (d^2*(c^2*f^2 - g^2)^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCos[c*x])^2)/(2*b*c*g^4*(f + g*x)) - (a*d^2*(c^2*f^2 - g^2)^(5/2)*Sq
rt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^
2])])/(g^6*Sqrt[1 - c^2*x^2]) - (I*b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2
*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2
])])/(g^6*Sqrt[1 - c^2*x^2]) + (I*b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*
d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]
)])/(g^6*Sqrt[1 - c^2*x^2]) - (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x
^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(g^6*
Sqrt[1 - c^2*x^2]) + (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*PolyL
og[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^6*Sqrt[1 -
```

```
c^2*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(a
 + b*ArcCos[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IGtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
```

```
, 0] && NeQ[p, -1]
```

Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcC
os[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x
^2]), Int[((f*x)^m*(a + b*ArcCos[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] + Dist[
(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a
 + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && Eq
Q[c^2*d + e, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4690

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(1 - c^2*x^2)^p, x]}, Dist[d^p*(a + b*
ArcCos[c*x]), u, x] + Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^
2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Intege
rQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -
2^(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4766

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_)*Sqrt[
(d_) + (e_.)*(x_)^2], x_Symbol] :> -Simp[((f + g*x)^m*(d + e*x^2)*(a + b*Ar
cCos[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] + Dist[1/(b*c*Sqrt[d]*(n + 1)
), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCos[
c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e,
0] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 4758

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(x
_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*x
 + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcCos[c*x])^n, u, x] + Dist[b*c*
n, Int[SimplifyIntegrand[(u*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2],
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ[p,
 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 4800

```
Int[(ArcCos[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcCos[c*x])
^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && IGt
Q[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

Rule 4798

```
Int[ArcCos[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :
> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcCos[c*x]^n, RFx, x]}, Int[u, x
] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ[n
, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4774

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst
[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &&
 (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
```

```
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cos^{-1}(cx)\right)}{f+gx}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(1-c^2x^2\right)^{5/2}\left(a+b\cos^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int\left(-\frac{c^2f\left(c^2f^2-2g^2\right)\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{g^4}+\frac{c^2\left(c^2f^2-2g^2\right)x\sqrt{1-c^2x^2}\left(a+b\cos^{-1}\right.}{g^3}\right.}{}$$

$$= -\frac{\left(c^4d^2f\sqrt{d-c^2dx^2}\right)\int x^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\,dx}{g^2\sqrt{1-c^2x^2}}+\frac{\left(c^4d^2\sqrt{d-c^2dx^2}\right)\int}{}$$

$$= -\frac{c^2d^2f\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)}{2g^4}-\frac{c^4d^2fx^3\sqrt{d-c^2dx^2}\left(a+b\right.}{4g^2}$$

$$= -\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{1-c^2x^2}}-\frac{bc^3d^2f\left(c^2f^2-2g^2\right)x^2\sqrt{d-c^2dx^2}}{4g^4\sqrt{1-c^2x^2}}+\frac{bc^3d^2\left(c\right.}{}$$

$$= -\frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}}-\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{1-c^2x^2}}+\frac{bc^3d^2fx^2\sqrt{d-c^2dx^2}}{16g^2\sqrt{1-c^2x^2}}-$$

$$= -\frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}}-\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{1-c^2x^2}}+\frac{bc^3d^2fx^2\sqrt{d-c^2dx^2}}{16g^2\sqrt{1-c^2x^2}}-$$

$$= \frac{ad^2\left(c^2f^2-g^2\right)^2\sqrt{d-c^2dx^2}}{g^5}-\frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}}-\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2d}}{3g^3\sqrt{1-c^2x^2}}$$

$$= \frac{ad^2\left(c^2f^2-g^2\right)^2\sqrt{d-c^2dx^2}}{g^5}-\frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}}-\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2d}}{3g^3\sqrt{1-c^2x^2}}$$

$$= \frac{ad^2\left(c^2f^2-g^2\right)^2\sqrt{d-c^2dx^2}}{g^5}-\frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}}-\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2d}}{3g^3\sqrt{1-c^2x^2}}$$

$$= \frac{ad^2\left(c^2f^2-g^2\right)^2\sqrt{d-c^2dx^2}}{g^5}-\frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}}-\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2d}}{3g^3\sqrt{1-c^2x^2}}$$

$$= \frac{ad^2\left(c^2f^2-g^2\right)^2\sqrt{d-c^2dx^2}}{g^5}-\frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}}-\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2d}}{3g^3\sqrt{1-c^2x^2}}$$

$$= \frac{ad^2\left(c^2f^2-g^2\right)^2\sqrt{d-c^2dx^2}}{g^5}-\frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}}-\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2d}}{3g^3\sqrt{1-c^2x^2}}$$

Mathematica [B] time = 19.6051, size = 6216, normalized size = 3.8

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.531, size = 4692, normalized size = 2.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/(g*x+f),x)
```

```
[Out] -5/4*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^2*
f^3*d^2*c^3/g^4-8/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g^3*arccos(c*x
)*x^2*c^4*f^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d^2*c^6/(c^2*x^2-1)/g^4*arcc
os(c*x)*x^3+1/2*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d^2*c^4/(c^2*x^2-1)/g^4*arccos
(c*x)*x+b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g^5*arccos(c*x)*x^2*c^6*f^
4+1/2*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^2
*f^5*d^2*c^5/g^6+1/5*a/g*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g
^2)/g^2)^(5/2)+1/3*a/g*d*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g
^2)/g^2)^(3/2)+a/g*d^2*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2
)/g^2)^(1/2)-a/g*d^3/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^
2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-d*c^2*(x+f/g)^2+2*c^
2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))+15/16*b*(-d*(c^2*x^2-1
))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^2*f*d^2*c/g^2-1/4*b*(-d
*(c^2*x^2-1))^(1/2)*f*d^2*c^6/(c^2*x^2-1)/g^2*arccos(c*x)*x^5+11/8*b*(-d*(c
^2*x^2-1))^(1/2)*f*d^2*c^4/(c^2*x^2-1)/g^2*arccos(c*x)*x^3-9/8*b*(-d*(c^2*x
^2-1))^(1/2)*f*d^2*c^2/(c^2*x^2-1)/g^2*arccos(c*x)*x+1/3*b*(-d*(c^2*x^2-1))
^(1/2)*d^2/(c^2*x^2-1)/g^3*arccos(c*x)*x^4*c^6*f^2+a/g^6*d^3*c^6*f^5/(c^2*d
)^(1/2)*arctan((c^2*d)^(1/2)*x/(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2
*f^2-g^2)/g^2)^(1/2))+a/g^7*d^3/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*
f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-d*c^2*(x+
f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))*c^6*f^6-3*a
/g^5*d^3/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/
g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f
/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))*c^4*f^4+3*a/g^3*d^3/(-d*(c^2*f^2-g
```

^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))*c^2*f^2+15/8*a/g^2*c^2*d^3*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))-1/2*a/g^4*d^2*c^4*f^3*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*x-5/2*a/g^4*d^3*c^4*f^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+1/4*a/g^2*c^2*d*f*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(3/2)*x+7/8*a/g^2*c^2*d^2*f*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*x-1/25*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g*(-c^2*x^2+1)^(1/2)*x^5*c^5+33/128*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2*c/(c^2*x^2-1)/g^2*(-c^2*x^2+1)^(1/2)+11/45*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g*(-c^2*x^2+1)^(1/2)*x^3*c^3-23/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g*(-c^2*x^2+1)^(1/2)*x*c-1/8*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d^2*c^3/(c^2*x^2-1)/g^4*(-c^2*x^2+1)^(1/2)-2*I*b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^4*arccos(c*x)*ln((-(c*x+I*(-c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*c^2*f^2+2*I*b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^4*arccos(c*x)*ln(((c*x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*c^2*f^2+34/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g*arccos(c*x)*x^2*c^2+7/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g^3*arccos(c*x)*c^2*f^2-b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g^5*arccos(c*x)*c^4*f^4+b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^2*dilog(-1/(-c*f+(c^2*f^2-g^2)^(1/2))*(c*x+I*(-c^2*x^2+1)^(1/2))*g-1/(-c*f+(c^2*f^2-g^2)^(1/2))*c*f+1/(-c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))-b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^2*dilog((c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2))+1/(c*f+(c^2*f^2-g^2)^(1/2))*c*f+1/(c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))+1/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g*arccos(c*x)*x^6*c^6-14/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g*arccos(c*x)*x^4*c^4-1/3*a/g^3*d*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(3/2)*c^2*f^2+a/g^5*d^2*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*c^4*f^4-2*a/g^3*d^2*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*c^2*f^2+I*b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^6*arccos(c*x)*ln((-(c*x+I*(-c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*c^4*f^4-I*b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^6*arccos(c*x)*ln(((c*x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*c^4*f^4+2*b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^4*dilog((c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2))+1/(c*f+(c^2*f^2-g^2)^(1/2))*c*f+1/(c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))*c^2*f^2+b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^6*dilog(-1/(-c*f+(c^2*f^2-g^2)^(1/2))*(c*x+I*(-c^2*x^2+1)^(1/2))*g-1/(-c*f+(c^2*f^2-g^2)^(1/2))*c*f+1/(-c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))*c^4*f^4-b*d^2*(-d*(c^2*x^

```
2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^6*dilog((c
*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2))+1/(c*f+(c^2*f^2-g^2)^(
1/2))*c*f+1/(c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))*c^4*f^4+I*b*d^2*
(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g
^2*arccos(c*x)*ln((-(c*x+I*(-c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(
-c*f+(c^2*f^2-g^2)^(1/2)))-2*b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/
2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^4*dilog(-1/(-c*f+(c^2*f^2-g^2)^(1/2))*(
c*x+I*(-c^2*x^2+1)^(1/2))*g-1/(-c*f+(c^2*f^2-g^2)^(1/2))*c*f+1/(-c*f+(c^2*f
^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))*c^2*f^2-I*b*d^2*(-d*(c^2*x^2-1))^(1/2)*
(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^2*arccos(c*x)*ln(((c*x
+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2))
)-23/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g*arccos(c*x)-9/16*b*(-d*(
c^2*x^2-1))^(1/2)*f*d^2*c^3/(c^2*x^2-1)/g^2*(-c^2*x^2+1)^(1/2)*x^2-1/9*b*(-
d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g^3*(-c^2*x^2+1)^(1/2)*x^3*c^5*f^2+7/3
*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g^3*(-c^2*x^2+1)^(1/2)*x*c^3*f^2+
1/4*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d^2*c^5/(c^2*x^2-1)/g^4*(-c^2*x^2+1)^(1/2)
*x^2-b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g^5*(-c^2*x^2+1)^(1/2)*x*c^5*
f^4+1/16*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2*c^5/(c^2*x^2-1)/g^2*(-c^2*x^2+1)^(1
/2)*x^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\arccos\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/(g*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\arccos{(cx)}+a)}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="giac
")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccos(c*x) + a)/(g*x + f), x)
```

3.14 $\int \frac{(f+gx)^3\left(a+b\cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=450

$$-\frac{3f^2g\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{f^3\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{3fg^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{3fg^2x\left(1-\right.}{2}$$

```
[Out] (-3*b*f^2*g*x*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2]) - (2*b*g^3*x*Sqrt[
1 - c^2*x^2])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (3*b*f*g^2*x^2*Sqrt[1 - c^2*x^2
])/(4*c*Sqrt[d - c^2*d*x^2]) - (b*g^3*x^3*Sqrt[1 - c^2*x^2])/(9*c*Sqrt[d -
c^2*d*x^2]) - (3*f^2*g*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/(c^2*Sqrt[d - c^2
*d*x^2]) - (2*g^3*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/(3*c^4*Sqrt[d - c^2*d*
x^2]) - (3*f*g^2*x*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/(2*c^2*Sqrt[d - c^2*d
*x^2]) - (g^3*x^2*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/(3*c^2*Sqrt[d - c^2*d*
x^2]) - (f^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*Sqrt[d - c^2*d
*x^2]) - (3*f*g^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c^3*Sqrt[d
- c^2*d*x^2])
```

Rubi [A] time = 0.585273, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4778, 4764, 4642, 4678, 8, 4708, 30}

$$-\frac{3f^2g\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{f^3\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{3fg^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{3fg^2x\left(1-\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```


```
[Out] (-3*b*f^2*g*x*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2]) - (2*b*g^3*x*Sqrt[
1 - c^2*x^2])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (3*b*f*g^2*x^2*Sqrt[1 - c^2*x^2
])/(4*c*Sqrt[d - c^2*d*x^2]) - (b*g^3*x^3*Sqrt[1 - c^2*x^2])/(9*c*Sqrt[d -
c^2*d*x^2]) - (3*f^2*g*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/(c^2*Sqrt[d - c^2
*d*x^2]) - (2*g^3*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/(3*c^4*Sqrt[d - c^2*d*
x^2]) - (3*f*g^2*x*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/(2*c^2*Sqrt[d - c^2*d
*x^2]) - (g^3*x^2*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/(3*c^2*Sqrt[d - c^2*d*
x^2]) - (f^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*Sqrt[d - c^2*d
*x^2]) - (3*f*g^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c^3*Sqrt[d
- c^2*d*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{(f+gx)^3\left(a+b\cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(f+gx)^3\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\sqrt{1-c^2x^2}\int \left(\frac{f^3\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}} + \frac{3f^2gx\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}} + \frac{3fg^2x^2\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}} + \frac{g^3x^3\left(a+b\cos^{-1}\right.}{\sqrt{1-c^2x^2}}\right.}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(f^3\sqrt{1-c^2x^2}\right)\int \frac{a+b\cos^{-1}(cx)}{\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}} + \frac{\left(3f^2g\sqrt{1-c^2x^2}\right)\int \frac{x\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}} + \frac{\left(3fg^2\sqrt{1\right.}{}$$

$$= -\frac{3f^2g\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} - \frac{3fg^2x\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}} - \frac{g^3x^2\left(1\right.}{}$$

$$= -\frac{3bf^2gx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}} - \frac{3bfg^2x^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}} - \frac{bg^3x^3\sqrt{1-c^2x^2}}{9c\sqrt{d-c^2dx^2}} - \frac{3f^2g\left(1-c^2x^2\right)\left(a+\right.}{c^2\sqrt{d-c^2d}}$$

$$= -\frac{3bf^2gx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}} - \frac{2bg^3x\sqrt{1-c^2x^2}}{3c^3\sqrt{d-c^2dx^2}} - \frac{3bfg^2x^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}} - \frac{bg^3x^3\sqrt{1-c^2x^2}}{9c\sqrt{d-c^2dx^2}} - \frac{3}{}$$

Mathematica [A] time = 1.18172, size = 342, normalized size = 0.76

$$\frac{\sqrt{d}g\left(c^2x^2-1\right)\left(12a\sqrt{1-c^2x^2}\left(c^2\left(18f^2+9fgx+2g^2x^2\right)+4g^2\right)+8bcx\left(c^2\left(27f^2+g^2x^2\right)+6g^2\right)+27bcfg\cos\left(2\cos^{-1}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (18*b*c*Sqrt[d]*f*(2*c^2*f^2 + 3*g^2)*(-1 + c^2*x^2)*ArcCos[c*x]^2 - 36*a*c
*f*(2*c^2*f^2 + 3*g^2)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sq
rt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d]*g*(-1 + c^2*x^2)*(8*
b*c*x*(6*g^2 + c^2*(27*f^2 + g^2*x^2)) + 12*a*Sqrt[1 - c^2*x^2]*(4*g^2 + c^
2*(18*f^2 + 9*f*g*x + 2*g^2*x^2)) + 27*b*c*f*g*Cos[2*ArcCos[c*x]]) + 6*b*Sq
```

```
rt[d]*g*(-1 + c^2*x^2)*ArcCos[c*x]*(4*Sqrt[1 - c^2*x^2]*(2*g^2 + c^2*(9*f^2
 + g^2*x^2)) + 9*c*f*g*Sin[2*ArcCos[c*x]]))/(72*c^4*Sqrt[d]*Sqrt[1 - c^2*x^
2]*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.605, size = 845, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/3*a*g^3*x^2/c^2/d*(-c^2*d*x^2+d)^(1/2)-2/3*a*g^3/d/c^4*(-c^2*d*x^2+d)^(1
/2)-3/2*a*f*g^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+3/2*a*f*g^2/c^2/(c^2*d)^(1/2)*
arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-3*a*f^2*g/c^2/d*(-c^2*d*x^2+d)
^(1/2)+a*f^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/2
*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d/(c^2*x^2-1)*arccos(c*x)^2*
f^3-1/3*b*(-d*(c^2*x^2-1))^(1/2)*g^3/d/(c^2*x^2-1)*arccos(c*x)*x^4-1/3*b*(-
d*(c^2*x^2-1))^(1/2)*g^3/c^2/d/(c^2*x^2-1)*arccos(c*x)*x^2+3*b*(-d*(c^2*x^2
-1))^(1/2)*g/c^2/d/(c^2*x^2-1)*arccos(c*x)*f^2+1/9*b*(-d*(c^2*x^2-1))^(1/2)
*g^3/c/d/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^3+2/3*b*(-d*(c^2*x^2-1))^(1/2)*g^
3/c^3/d/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x-3/2*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2
/d/(c^2*x^2-1)*arccos(c*x)*x^3+3/2*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/c^2/d/(c^
2*x^2-1)*arccos(c*x)*x-3/8*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/c^3/d/(c^2*x^2-1)
*(-c^2*x^2+1)^(1/2)-3*b*(-d*(c^2*x^2-1))^(1/2)*g/d/(c^2*x^2-1)*arccos(c*x)*
x^2*f^2+3/4*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*a
rccos(c*x)^2*f*g^2+3/4*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/c/d/(c^2*x^2-1)*(-c^2
*x^2+1)^(1/2)*x^2+3*b*(-d*(c^2*x^2-1))^(1/2)*g/c/d/(c^2*x^2-1)*(-c^2*x^2+1)
^(1/2)*x*f^2+2/3*b*(-d*(c^2*x^2-1))^(1/2)*g^3/c^4/d/(c^2*x^2-1)*arccos(c*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(ag^3x^3 + 3\,afg^2x^2 + 3\,af^2gx + af^3 + \left(bg^3x^3 + 3\,bfg^2x^2 + 3\,bf^2gx + bf^3\right)\arccos\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{c^2dx^2 - d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3
*b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/(c^2*
d*x^2 - d), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx + f\right)^3 \left(b \arccos\left(cx\right) + a\right)}{\sqrt{-c^2dx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((g*x + f)^3*(b*arccos(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

3.15 $\int \frac{(f+gx)^2\left(a+b\cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=270

$$-\frac{f^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{2fg\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{g^2x\left(1-c^2x^2\right)(a}{2c^2\sqrt{d-}}$$

```
[Out] (-2*b*f*g*x*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2]) - (b*g^2*x^2*Sqrt[1
- c^2*x^2])/(4*c*Sqrt[d - c^2*d*x^2]) - (2*f*g*(1 - c^2*x^2)*(a + b*ArcCos[
c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) - (g^2*x*(1 - c^2*x^2)*(a + b*ArcCos[c*x])
)/(2*c^2*Sqrt[d - c^2*d*x^2]) - (f^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^
2)/(2*b*c*Sqrt[d - c^2*d*x^2]) - (g^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])
^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.43615, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4778, 4764, 4642, 4678, 8, 4708, 30}

$$-\frac{f^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{2fg\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{g^2x\left(1-c^2x^2\right)(a}{2c^2\sqrt{d-}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-2*b*f*g*x*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2]) - (b*g^2*x^2*Sqrt[1
- c^2*x^2])/(4*c*Sqrt[d - c^2*d*x^2]) - (2*f*g*(1 - c^2*x^2)*(a + b*ArcCos[
c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) - (g^2*x*(1 - c^2*x^2)*(a + b*ArcCos[c*x])
)/(2*c^2*Sqrt[d - c^2*d*x^2]) - (f^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^
2)/(2*b*c*Sqrt[d - c^2*d*x^2]) - (g^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])
^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(f+gx)^2\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\sqrt{1-c^2x^2}\int \left(\frac{f^2\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}} + \frac{2fgx\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}} + \frac{g^2x^2\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\right)dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(f^2\sqrt{1-c^2x^2}\right)\int \frac{a+b\cos^{-1}(cx)}{\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}} + \frac{\left(2fg\sqrt{1-c^2x^2}\right)\int \frac{x\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}} + \frac{\left(g^2\sqrt{1-c^2x\right.}{}$$

$$= -\frac{2fg\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} - \frac{g^2x\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}} - \frac{f^2\sqrt{1-c^2x^2}\,(}{2bc\sqrt{}}$$

$$= -\frac{2bfgx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}} - \frac{bg^2x^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}} - \frac{2fg\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} - \frac{g^2x\left(1-c\right.}{2}$$

Mathematica [A] time = 0.757153, size = 266, normalized size = 0.99

$$\frac{\sqrt{d}g\left(c^2x^2-1\right)\left(4c\left(a\sqrt{1-c^2x^2}(4f+gx)+4bcfx\right)+bg\cos\left(2\cos^{-1}(cx)\right)\right)-4a\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}\left(2c^2f^2+g^2\right)\tan^{-1}}{8c^3\sqrt{d}\sqrt{1-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (2*b*Sqrt[d]*(2*c^2*f^2 + g^2)*(-1 + c^2*x^2)*ArcCos[c*x]^2 - 4*a*(2*c^2*f^
2 + g^2)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x
^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d]*g*(-1 + c^2*x^2)*(4*c*(4*b*c*f*x +
 a*(4*f + g*x)*Sqrt[1 - c^2*x^2]) + b*g*Cos[2*ArcCos[c*x]]) + 2*b*Sqrt[d]*g
*(-1 + c^2*x^2)*ArcCos[c*x]*(8*c*f*Sqrt[1 - c^2*x^2] + g*Sin[2*ArcCos[c*x]]
))/(8*c^3*Sqrt[d]*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.356, size = 549, normalized size = 2.

$$-\frac{ag^2x}{2c^2d}\sqrt{-c^2dx^2+d} + \frac{ag^2}{2c^2}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}} - 2\frac{afg\sqrt{-c^2dx^2+d}}{c^2d} + af^2\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/2*a*g^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a*g^2/c^2/(c^2*d)^(1/2)*arctan(
(c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-2*a*f*g/c^2/d*(-c^2*d*x^2+d)^(1/2)+a*
f^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/2*b*(-d*(c
^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d/(c^2*x^2-1)*arccos(c*x)^2*f^2+1/4*b
*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*arccos(c*x)^2*
g^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)*g^2/d/(c^2*x^2-1)*arccos(c*x)*x^3+1/2*b*(-
d*(c^2*x^2-1))^(1/2)*g^2/c^2/d/(c^2*x^2-1)*arccos(c*x)*x-1/8*b*(-d*(c^2*x^2
-1))^(1/2)*g^2/c^3/d/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)+2*b*(-d*(c^2*x^2-1))^(1
/2)*f*g/c^2/d/(c^2*x^2-1)*arccos(c*x)-2*b*(-d*(c^2*x^2-1))^(1/2)*f*g/d/(c^2
*x^2-1)*arccos(c*x)*x^2+1/4*b*(-d*(c^2*x^2-1))^(1/2)*g^2/c/d/(c^2*x^2-1)*(-
c^2*x^2+1)^(1/2)*x^2+2*b*(-d*(c^2*x^2-1))^(1/2)*f*g/c/d/(c^2*x^2-1)*(-c^2*x
^2+1)^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(ag^2x^2+2\,afgx+af^2+\left(bg^2x^2+2\,bfgx+bf^2\right)\arccos\left(cx\right)\right)}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2
+ 2*b*f*g*x + b*f^2)*arccos(c*x))/(c^2*d*x^2 - d), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx+f\right)^{2}\left(b\arccos\left(cx\right)+a\right)}{\sqrt{-c^{2}dx^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((g*x + f)^2*(b*arccos(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

3.16 $\int \frac{(f+gx)\left(a+b\cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=127

$$-\frac{f\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{g\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{bgx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}}$$

```
[Out] -((b*g*x*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2])) - (g*(1 - c^2*x^2)*(a
+ b*ArcCos[c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) - (f*Sqrt[1 - c^2*x^2]*(a + b*A
rcCos[c*x])^2)/(2*b*c*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.223334, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4778, 4764, 4642, 4678, 8}

$$-\frac{f\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{g\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{bgx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -((b*g*x*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2])) - (g*(1 - c^2*x^2)*(a
+ b*ArcCos[c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) - (f*Sqrt[1 - c^2*x^2]*(a + b*A
rcCos[c*x])^2)/(2*b*c*Sqrt[d - c^2*d*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4764

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
```

```
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(f+gx)\left(a+b\cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx &= \frac{\sqrt{1-c^2x^2}\int \frac{(f+gx)\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}} \\
&= \frac{\sqrt{1-c^2x^2}\int \left(\frac{f\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}+\frac{gx\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\right)dx}{\sqrt{d-c^2dx^2}} \\
&= \frac{\left(f\sqrt{1-c^2x^2}\right)\int \frac{a+b\cos^{-1}(cx)}{\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}}+\frac{\left(g\sqrt{1-c^2x^2}\right)\int \frac{x\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}} \\
&= -\frac{g\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{f\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{\left(bg\sqrt{1-c^2x^2}\right)\int 1}{c\sqrt{d-c^2dx^2}} \\
&= -\frac{bgx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}}-\frac{g\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{f\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.413451, size = 172, normalized size = 1.35

$$\frac{-2\sqrt{d}g\left(-ac^2x^2+a+bcx\sqrt{1-c^2x^2}\right)-2acf\sqrt{d-c^2dx^2}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(c^2x^2-1)}\right)-bc\sqrt{d}f\sqrt{1-c^2x^2}\cos^{-1}(cx)^2+2b\sqrt{d}g\left(c^2x^2\right.}{2c^2\sqrt{d}\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-2*Sqrt[d]*g*(a - a*c^2*x^2 + b*c*x*Sqrt[1 - c^2*x^2]) + 2*b*Sqrt[d]*g*(-1
 + c^2*x^2)*ArcCos[c*x] - b*c*Sqrt[d]*f*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2 - 2
*a*c*f*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 +
c^2*x^2))])/(2*c^2*Sqrt[d]*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.299, size = 235, normalized size = 1.9

$$-\frac{ag}{c^2d}\sqrt{-c^2dx^2+d}+af\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}+\frac{b(\arccos(cx))^2f}{2dc(c^2x^2-1)}\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}-\frac{bg\,\mathrm{arcco}}{d(c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a*g/c^2/d*(-c^2*d*x^2+d)^(1/2)+a*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-
c^2*d*x^2+d)^(1/2))+1/2*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d/(c^
2*x^2-1)*arccos(c*x)^2*f-b*(-d*(c^2*x^2-1))^(1/2)*g/d/(c^2*x^2-1)*arccos(c*
x)*x^2+b*(-d*(c^2*x^2-1))^(1/2)*g/c/d/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x+b*(-
d*(c^2*x^2-1))^(1/2)*g/c^2/d/(c^2*x^2-1)*arccos(c*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(agx+af+\left(bgx+bf\right)\arccos\left(cx\right)\right)}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arccos(c*x))/(c
^2*d*x^2 - d), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx+f\right)\left(b\arccos\left(cx\right)+a\right)}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((g*x + f)*(b*arccos(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

3.17 $\int \frac{a+b\cos^{-1}(cx)}{(f+gx)\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=370

$$\frac{b\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}}-\frac{b\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}}+\frac{i\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\log\left(1+\right.}{\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}}$$

```
[Out] (I*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f
 - Sqrt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) - (I*Sq
rt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sq
rt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[1
- c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))]
)/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*PolyLog[
2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(Sqrt[c^2*f^2 - g
^2]*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.606007, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4778, 4774, 3321, 2264, 2190, 2279, 2391}

$$\frac{b\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}}-\frac{b\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}}+\frac{i\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\log\left(1+\right.}{\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])/((f + g*x)*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (I*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f
 - Sqrt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) - (I*Sq
rt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sq
rt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[1
- c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))]
)/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*PolyLog[
2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(Sqrt[c^2*f^2 - g
^2]*Sqrt[d - c^2*d*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4774

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst
[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &&
 (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cos^{-1}(cx)}{(f+gx)\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{a+b\cos^{-1}(cx)}{(f+gx)\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{a+bx}{cf+g\cos(x)}\,dx, x, \cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{2ce^{ix}f+g+e^{2ix}g}\,dx, x, \cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(2g\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{2cf+2e^{ix}g-2\sqrt{c^2f^2-g^2}}\,dx, x, \cos^{-1}(cx)\right)}{\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}} + \frac{\left(2g\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\right.}{\sqrt{c^2f^2}}$$

$$= \frac{i\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}} - \frac{i\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\log\left(1+\right.}{\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}}$$

$$= \frac{i\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}} - \frac{i\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\log\left(1+\right.}{\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}}$$

$$= \frac{i\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}} - \frac{i\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\log\left(1+\right.}{\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}}$$

Mathematica [B] time = 2.056, size = 930, normalized size = 2.51

$$\frac{\frac{a\log(f+gx)}{\sqrt{d}} - \frac{a\log\left(d\left(fxc^2+g\right)+\sqrt{d}\sqrt{g^2-c^2f^2}\sqrt{d-c^2dx^2}\right)}{\sqrt{d}} - \frac{b\sqrt{1-c^2x^2}\left(2\cos^{-1}(cx)\tanh^{-1}\left(\frac{(cf+g)\cot\left(\frac{1}{2}\cos^{-1}(cx)\right)}{\sqrt{g^2-c^2f^2}}\right)-2\cos^{-1}\left(-\frac{cf}{g}\right)\tanh^{-1}\left(\frac{(g-cf)\tan\left(\frac{1}{2}\cos\right.}{\sqrt{g^2-c^2f^2}}\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])/((f + g*x)*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] ((a*Log[f + g*x])/Sqrt[d] - (a*Log[d*(g + c^2*f*x) + Sqrt[d]*Sqrt[-(c^2*f^2
) + g^2]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] - (b*Sqrt[1 - c^2*x^2]*(2*ArcCos[c*x
]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - 2*ArcCos
[-((c*f)/g)]*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^
2]] + (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sq
rt[-(c^2*f^2) + g^2]] + (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqr
t[-(c^2*f^2) + g^2]])*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^((I/2)*ArcCos[c
*x])*Sqrt[g]*Sqrt[c*(f + g*x)])] + (ArcCos[-((c*f)/g)] + (2*I)*(ArcTanh[((c
*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - ArcTanh[((-(c*f) + g)
*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^((I/2)*ArcCos[c*x])*S
qrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*
f)/g)] - (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) +
g^2]])*Log[((c*f + g)*((-I)*c*f + I*g + Sqrt[-(c^2*f^2) + g^2])*(-I + Tan[A
rcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))]
- (ArcCos[-((c*f)/g)] + (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqr
t[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(I*c*f - I*g + Sqrt[-(c^2*f^2) + g^2])
*(I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[
c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - Sqr
t[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^
2]*Tan[ArcCos[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^2])*(c*
f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^
2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))])))/Sqrt[d - c^2*d*x^2])/Sqrt[-(c^2*f^2)
 + g^2]
```

Maple [A] time = 0.196, size = 487, normalized size = 1.3

$$-\frac{a}{g}\ln\left(\left(\left(-2\,\frac{d\left(c^2f^2-g^2\right)}{g^2}+2\,\frac{c^2fd}{g}\left(x+\frac{f}{g}\right)+2\,\sqrt{-\frac{d\left(c^2f^2-g^2\right)}{g^2}}\sqrt{-dc^2\left(x+\frac{f}{g}\right)^2+2\,\frac{c^2fd}{g}\left(x+\frac{f}{g}\right)-\frac{d\left(c^2f^2-g^2\right)}{g^2}}\right)\right(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/g/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x
+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-
d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-b*(-d*(c^2*x^2-1))^(1/2)/(c^2*f^2-g^2)
^(1/2)*(-c^2*x^2+1)^(1/2)*(I*arccos(c*x)*ln((-(c*x+I*(-c^2*x^2+1)^(1/2))*g-
c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))-I*arccos(c*x)*ln(((c*x
```

```
+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2))
)+dilog((-(c*x+I*(-c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*
f^2-g^2)^(1/2)))-dilog(((c*x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2
))/(c*f+(c^2*f^2-g^2)^(1/2)))))/d/(c^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arccos{(cx)} + a}{\sqrt{-c^2dx^2 + d}\left(gx + f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*arccos(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2 + d}(b \arccos{(cx)} + a)}{c^2dgx^3 + c^2dfx^2 - dgx - df}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^2*d*g*x^3 + c^2*d*f*x
^2 - d*g*x - d*f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acos}{(cx)}}{\sqrt{-d(cx-1)(cx+1)}\left(f + gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))/(g*x+f)/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acos(c*x))/(sqrt(-d*(c*x - 1)*(c*x + 1))*(f + g*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arccos(cx) + a}{\sqrt{-c^2dx^2 + d}(gx + f)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((b*arccos(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)), x)
```

3.18 $\int \frac{a+b\cos^{-1}(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=496

$$\frac{bc^2f\sqrt{1-c^2x^2}\text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)^{3/2}}-\frac{bc^2f\sqrt{1-c^2x^2}\text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)^{3/2}}+\frac{g\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)(f+g}$$

```
[Out] (g*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/((c^2*f^2 - g^2)*(f + g*x)*Sqrt[d - c
^2*d*x^2]) + (I*c^2*f*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*A
rcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d
- c^2*d*x^2]) - (I*c^2*f*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(
I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/((c^2*f^2 - g^2)^(3/2)*Sqrt
[d - c^2*d*x^2]) + (b*c*Sqrt[1 - c^2*x^2]*Log[f + g*x])/((c^2*f^2 - g^2)*Sq
rt[d - c^2*d*x^2]) + (b*c^2*f*Sqrt[1 - c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c
*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*
d*x^2]) - (b*c^2*f*Sqrt[1 - c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*
f + Sqrt[c^2*f^2 - g^2]))])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.721545, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {4778, 4774, 3324, 3321, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{bc^2f\sqrt{1-c^2x^2}\text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)^{3/2}}-\frac{bc^2f\sqrt{1-c^2x^2}\text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)^{3/2}}+\frac{g\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)(f+g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])/((f + g*x)^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (g*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/((c^2*f^2 - g^2)*(f + g*x)*Sqrt[d - c
^2*d*x^2]) + (I*c^2*f*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*A
rcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d
- c^2*d*x^2]) - (I*c^2*f*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(
I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/((c^2*f^2 - g^2)^(3/2)*Sqrt
[d - c^2*d*x^2]) + (b*c*Sqrt[1 - c^2*x^2]*Log[f + g*x])/((c^2*f^2 - g^2)*Sq
rt[d - c^2*d*x^2]) + (b*c^2*f*Sqrt[1 - c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c
*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*
d*x^2]) - (b*c^2*f*Sqrt[1 - c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*
```

```
f + Sqrt[c^2*f^2 - g^2]))])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2])
```

Rule 4778

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(1 - c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*Arc
Cos[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e,
 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4774

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst
[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &&
 (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+b\cos^{-1}(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{a+b\cos^{-1}(cx)}{(f+gx)^2\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(c\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{a+bx}{(cf+g\cos(x))^2}\,dx, x, \cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= \frac{g\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} - \frac{\left(c^2f\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{a+bx}{cf+g\cos(x)}\,dx, x, \cos^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)\sqrt{d-c^2dx^2}} - \Big($$

$$= \frac{g\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{cf+x}\,dx, x, cgx\right)}{\left(c^2f^2-g^2\right)\sqrt{d-c^2dx^2}} - \frac{\left(2c^2f\sqrt{1-c^2\right.}{}$$

$$= \frac{g\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{1-c^2x^2}\log(f+gx)}{\left(c^2f^2-g^2\right)\sqrt{d-c^2dx^2}} - \frac{\left(2c^2fg\sqrt{1-c^2x^2}\right)\mathrm{Subst}\Big(\int}{\left(c^2f^2-g\right.}$$

$$= \frac{g\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{ic^2f\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\left(c^2f^2-g^2\right)^{3/2}\sqrt{d-c^2dx^2}} -$$

$$= \frac{g\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{ic^2f\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\left(c^2f^2-g^2\right)^{3/2}\sqrt{d-c^2dx^2}} -$$

$$= \frac{g\left(1-c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{ic^2f\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\left(c^2f^2-g^2\right)^{3/2}\sqrt{d-c^2dx^2}} -$$

<u>**Mathematica**</u> **[B]** time = 4.98671, size = 1108, normalized size = 2.23

$$-\frac{af\log(f+gx)c^2}{\sqrt{d}\left(g^2-c^2f^2\right)^{3/2}} - \frac{af\log\left(d\left(fxc^2+g\right)+\sqrt{d}\sqrt{g^2-c^2f^2}\sqrt{d-c^2dx^2}\right)c^2}{\sqrt{d}(cf-g)(cf+g)\sqrt{g^2-c^2f^2}} - \frac{b\sqrt{1-c^2x^2}\left(-\frac{g\sqrt{1-c^2x^2}\cos^{-1}(cx)}{(cf-g)(cf+g)(cf+cgx)} - \frac{\log\left(\frac{gx}{f}+1\right)}{c^2f^2-g^2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])/((f + g*x)^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -((a*g*Sqrt[d - c^2*d*x^2])/(d*(-(c^2*f^2) + g^2)*(f + g*x))) - (a*c^2*f*Lo
g[f + g*x])/(Sqrt[d]*(-(c^2*f^2) + g^2)^(3/2)) - (a*c^2*f*Log[d*(g + c^2*f*
x) + Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]])/(Sqrt[d]*(c*f - g
)*(c*f + g)*Sqrt[-(c^2*f^2) + g^2]) - (b*c*Sqrt[1 - c^2*x^2]*(-((g*Sqrt[1 -
 c^2*x^2]*ArcCos[c*x])/((c*f - g)*(c*f + g)*(c*f + c*g*x))) - Log[1 + (g*x)
/f]/(c^2*f^2 - g^2) - (c*f*(2*ArcCos[c*x]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x
]/2])/Sqrt[-(c^2*f^2) + g^2]] - 2*ArcCos[-((c*f)/g)]*ArcTanh[((-(c*f) + g)*
Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] - (2*I)*A
rcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*ArcTa
nh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[Sqrt[-(c^
2*f^2) + g^2]/(Sqrt[2]*E^((I/2)*ArcCos[c*x])*Sqrt[g]*Sqrt[c*(f + g*x)])] +
(ArcCos[-((c*f)/g)] + (2*I)*(ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(
c^2*f^2) + g^2]] - ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2
) + g^2]]))*Log[(E^((I/2)*ArcCos[c*x])*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqr
t[g]*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((-(c*f) + g
)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*((-I)*c*f + I
*g + Sqrt[-(c^2*f^2) + g^2])*(-I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[
-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - (ArcCos[-((c*f)/g)] + (2*I)*ArcTa
nh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g
)*(I*c*f - I*g + Sqrt[-(c^2*f^2) + g^2])*(I + Tan[ArcCos[c*x]/2]))/(g*(c*f
+ g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] + I*(PolyLog[2, ((c*f -
I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]
/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - PolyLog[
2, ((c*f + I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[
ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))]
)))/(-(c^2*f^2) + g^2)^(3/2)))/Sqrt[d - c^2*d*x^2]
```

Maple [B] time = 0.305, size = 1622, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a/d/(c^2*f^2-g^2)/(x+f/g)*(-d*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-
g^2)/g^2)^(1/2)-a/g*c^2*f/(c^2*f^2-g^2)/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2
*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-d
*c^2*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))+b*(
-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*(-c^2
```

```
*x^2+1)*x*c^2*f+b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2
-g^2)/(g*x+f)*x^3*c^4*f+I*b*c^2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d
/(c^2*x^2-1)/(c^2*f^2-g^2)^(3/2)*ln(((c*x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*
f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*arccos(c*x)*f+b*(-d*(c^2*x^2-1))
^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*x^2*c^2*g-I*b*(-d*(c
^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*(-c^2*x^2+
1)^(1/2)*x*c*g-b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-
g^2)/(g*x+f)*x*c^2*f-b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(c^
2*f^2-g^2)/(g*x+f)*g-I*b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(
c^2*f^2-g^2)/(g*x+f)*(-c^2*x^2+1)^(1/2)*c*f-I*b*c^2*(-d*(c^2*x^2-1))^(1/2)*
(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)/(c^2*f^2-g^2)^(3/2)*ln((-(c*x+I*(-c^2*x^2+
1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*arccos(c*x
)*f-b*c^3*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)/(c^2*f^2-
g^2)^2*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2*g+2*c*f*(c*x+I*(-c^2*x^2+1)^(1/2))+g
)*f^2+2*b*c^3*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)/(c^2*
f^2-g^2)^2*ln(c*x+I*(-c^2*x^2+1)^(1/2))*f^2-b*c^2*(-d*(c^2*x^2-1))^(1/2)*(-
c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)/(c^2*f^2-g^2)^(3/2)*dilog((-(c*x+I*(-c^2*x^2
+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*f+b*c^2*(
-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)/(c^2*f^2-g^2)^(3/2)*
dilog(((c*x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-
g^2)^(1/2)))*f+b*c*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)/
(c^2*f^2-g^2)^2*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2*g+2*c*f*(c*x+I*(-c^2*x^2+1)
^(1/2))+g)*g^2-2*b*c*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1
)/(c^2*f^2-g^2)^2*ln(c*x+I*(-c^2*x^2+1)^(1/2))*g^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arccos(cx) + a}{\sqrt{-c^2dx^2 + d}\,(gx + f)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*arccos(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)}{c^2dg^2x^4+2\,c^2dfgx^3-2\,dfgx-df^2+\left(c^2df^2-dg^2\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^2*d*g^2*x^4 + 2*c^2*d
*f*g*x^3 - 2*d*f*g*x - d*f^2 + (c^2*d*f^2 - d*g^2)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\mathrm{acos}\,(cx)}{\sqrt{-d\,(cx-1)\,(cx+1)}\,\left(f+gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))/(g*x+f)**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acos(c*x))/(sqrt(-d*(c*x - 1)*(c*x + 1))*(f + g*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.19 $\int \frac{\left(a+b\cos^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\frac{\left(a+b\cos^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable[((a + b*ArcCos[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],
x]
```

Rubi [A] time = 0.192177, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*ArcCos[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] Defer[Int][((a + b*ArcCos[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2], x]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx=\int \frac{\left(a+b\cos^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Mathematica [A] time = 0.15798, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcCos[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] Integrate[((a + b*ArcCos[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2], x]
```

Maple [A] time = 5.372, size = 0, normalized size = 0.

$$\int (a+b\arccos(cx))^n \ln\left(h\left(gx+f\right)^m\right)\frac{1}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^n*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccos(c*x))^n*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)^n\log\left(\left(gx+f\right)^m h\right)}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)^n*log((g*x + f)^m*h)/(c^2*
x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**n*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arccos(cx)+a)^n\log\left(\left(gx+f\right)^m h\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

3.20 $\int \frac{\left(a+b\cos^{-1}(cx)\right)^2 \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=496

$$-\frac{im\left(a+b\cos^{-1}(cx)\right)^2 \text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{im\left(a+b\cos^{-1}(cx)\right)^2 \text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{c}+\frac{2bm\left(a+b\cos\right.}{}$$

```
[Out] ((-I/12)*m*(a + b*ArcCos[c*x])^4)/(b^2*c) + (m*(a + b*ArcCos[c*x])^3*Log[1
+ (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(3*b*c) + (m*(a + b*A
rcCos[c*x])^3*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(
3*b*c) - ((a + b*ArcCos[c*x])^3*Log[h*(f + g*x)^m])/(3*b*c) - (I*m*(a + b*A
rcCos[c*x])^2*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]
))])/c - (I*m*(a + b*ArcCos[c*x])^2*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f
 + Sqrt[c^2*f^2 - g^2]))])/c + (2*b*m*(a + b*ArcCos[c*x])*PolyLog[3, -((E^(
I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/c + (2*b*m*(a + b*ArcCos[c
*x])*PolyLog[3, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/c +
((2*I)*b^2*m*PolyLog[4, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])
)])/c + ((2*I)*b^2*m*PolyLog[4, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2
 - g^2]))])/c
```

Rubi [A] time = 0.791464, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4642, 4780, 4742, 4520, 2190, 2531, 6609, 2282, 6589}

$$-\frac{im\left(a+b\cos^{-1}(cx)\right)^2 \text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{im\left(a+b\cos^{-1}(cx)\right)^2 \text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{c}+\frac{2bm\left(a+b\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCos[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] ((-I/12)*m*(a + b*ArcCos[c*x])^4)/(b^2*c) + (m*(a + b*ArcCos[c*x])^3*Log[1
+ (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(3*b*c) + (m*(a + b*A
rcCos[c*x])^3*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(
3*b*c) - ((a + b*ArcCos[c*x])^3*Log[h*(f + g*x)^m])/(3*b*c) - (I*m*(a + b*A
rcCos[c*x])^2*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]
))])/c - (I*m*(a + b*ArcCos[c*x])^2*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f
 + Sqrt[c^2*f^2 - g^2]))])/c + (2*b*m*(a + b*ArcCos[c*x])*PolyLog[3, -((E^(
I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/c + (2*b*m*(a + b*ArcCos[c
```

```
*x])*PolyLog[3, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/c +
((2*I)*b^2*m*PolyLog[4, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])
)])/c + ((2*I)*b^2*m*PolyLog[4, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2
 - g^2]))])/c
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4780

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCos[(c_.)*(x_)]*(b_.
))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> -Simp[(Log[h*(f + g*x)^m]
*(a + b*ArcCos[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] + Dist[(g*m)/(b*c*S
qrt[d]*(n + 1)), Int[(a + b*ArcCos[c*x])^(n + 1)/(f + g*x), x], x] /; FreeQ
[{a, b, c, d, e, f, g, h, m}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && IGtQ[
n, 0]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[((a + b*x)^n*Sin[x])/(c*d + e*Cos[x]), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4520

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x))), x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a +
 Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = -\frac{\left(a+b\cos^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc}+\frac{(gm)\int\frac{\left(a+b\cos^{-1}(cx)\right)^3}{f+gx}\,dx}{3bc}$$

$$= -\frac{\left(a+b\cos^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc}-\frac{(gm)\,\text{Subst}\left(\int\frac{(a+bx)^3\sin(x)}{cf+g\cos(x)}\,dx,x,\cos\right.}{3bc}$$

$$= -\frac{im\left(a+b\cos^{-1}(cx)\right)^4}{12b^2c}-\frac{\left(a+b\cos^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc}+\frac{(igm)\,\text{Subs}}{}$$

$$= -\frac{im\left(a+b\cos^{-1}(cx)\right)^4}{12b^2c}+\frac{m\left(a+b\cos^{-1}(cx)\right)^3\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{3bc}+\frac{m\left(a\right.}{}$$

$$= -\frac{im\left(a+b\cos^{-1}(cx)\right)^4}{12b^2c}+\frac{m\left(a+b\cos^{-1}(cx)\right)^3\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{3bc}+\frac{m\left(a\right.}{}$$

$$= -\frac{im\left(a+b\cos^{-1}(cx)\right)^4}{12b^2c}+\frac{m\left(a+b\cos^{-1}(cx)\right)^3\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{3bc}+\frac{m\left(a\right.}{}$$

$$= -\frac{im\left(a+b\cos^{-1}(cx)\right)^4}{12b^2c}+\frac{m\left(a+b\cos^{-1}(cx)\right)^3\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{3bc}+\frac{m\left(a\right.}{}$$

$$= -\frac{im\left(a+b\cos^{-1}(cx)\right)^4}{12b^2c}+\frac{m\left(a+b\cos^{-1}(cx)\right)^3\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{3bc}+\frac{m\left(a\right.}{}$$

Mathematica [F] time = 26.654, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcCos[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] Integrate[((a + b*ArcCos[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2], x]
```

Maple [F] time = 4.708, size = 0, normalized size = 0.

$$\int (a+b\arccos(cx))^2 \ln\left(h\left(gx+f\right)^m\right)\frac{1}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^2*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccos(c*x))^2*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algori
thm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\left(b^2\arccos(cx)^2+2\,ab\arccos(cx)+a^2\right)\log\left(\left(gx+f\right)^m h\right)}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*
log((g*x + f)^m*h)/(c^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acos}(cx))^2 \log\left(h\left(f + gx\right)^m\right)}{\sqrt{-\left(cx - 1\right)\left(cx + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**2*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acos(c*x))**2*log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arccos(cx) + a)^2 \log\left(\left(gx + f\right)^m h\right)}{\sqrt{-c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^2*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

3.21 $\int \frac{\left(a+b\cos^{-1}(cx)\right)\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=374

$$-\frac{im\left(a+b\cos^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{im\left(a+b\cos^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{c}+\frac{bm\text{PolyLog}\left(3,\right.}{c}$$

```
[Out] ((-I/6)*m*(a + b*ArcCos[c*x])^3)/(b^2*c) + (m*(a + b*ArcCos[c*x])^2*Log[1 +
 (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(2*b*c) + (m*(a + b*Ar
cCos[c*x])^2*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(2
*b*c) - ((a + b*ArcCos[c*x])^2*Log[h*(f + g*x)^m])/(2*b*c) - (I*m*(a + b*Ar
cCos[c*x])*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))]
)/c - (I*m*(a + b*ArcCos[c*x])*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sq
rt[c^2*f^2 - g^2]))])/c + (b*m*PolyLog[3, -((E^(I*ArcCos[c*x])*g)/(c*f - Sq
rt[c^2*f^2 - g^2]))])/c + (b*m*PolyLog[3, -((E^(I*ArcCos[c*x])*g)/(c*f + Sq
rt[c^2*f^2 - g^2]))])/c
```

Rubi [A] time = 0.611272, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4642, 4780, 4742, 4520, 2190, 2531, 2282, 6589}

$$-\frac{im\left(a+b\cos^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{im\left(a+b\cos^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ge^{i\cos^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{c}+\frac{bm\text{PolyLog}\left(3,\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCos[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```


```
[Out] ((-I/6)*m*(a + b*ArcCos[c*x])^3)/(b^2*c) + (m*(a + b*ArcCos[c*x])^2*Log[1 +
 (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(2*b*c) + (m*(a + b*Ar
cCos[c*x])^2*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(2
*b*c) - ((a + b*ArcCos[c*x])^2*Log[h*(f + g*x)^m])/(2*b*c) - (I*m*(a + b*Ar
cCos[c*x])*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))]
)/c - (I*m*(a + b*ArcCos[c*x])*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sq
rt[c^2*f^2 - g^2]))])/c + (b*m*PolyLog[3, -((E^(I*ArcCos[c*x])*g)/(c*f - Sq
rt[c^2*f^2 - g^2]))])/c + (b*m*PolyLog[3, -((E^(I*ArcCos[c*x])*g)/(c*f + Sq
rt[c^2*f^2 - g^2]))])/c
```


Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4780

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCos[(c_.)*(x_)]*(b_.
))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> -Simp[(Log[h*(f + g*x)^m]
*(a + b*ArcCos[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] + Dist[(g*m)/(b*c*S
qrt[d]*(n + 1)), Int[(a + b*ArcCos[c*x])^(n + 1)/(f + g*x), x], x] /; FreeQ
[{a, b, c, d, e, f, g, h, m}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && IGtQ[
n, 0]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[((a + b*x)^n*Sin[x])/(c*d + e*Cos[x]), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4520

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x))), x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a +
 Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = -\frac{\left(a+b\cos^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc}+\frac{(gm)\int\frac{\left(a+b\cos^{-1}(cx)\right)^2}{f+gx}\,dx}{2bc}$$

$$= -\frac{\left(a+b\cos^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc}-\frac{(gm)\,\mathrm{Subst}\left(\int\frac{(a+bx)^2\sin(x)}{cf+g\cos(x)}\,dx,x,\mathrm{co}\ldots\right.}{2bc}$$

$$= -\frac{im\left(a+b\cos^{-1}(cx)\right)^3}{6b^2c}-\frac{\left(a+b\cos^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc}+\frac{(igm)\,\mathrm{Subs}\ldots}{\ldots}$$

$$= -\frac{im\left(a+b\cos^{-1}(cx)\right)^3}{6b^2c}+\frac{m\left(a+b\cos^{-1}(cx)\right)^2\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{2bc}+\frac{m\left(a\ldots\right.}{\ldots}$$

$$= -\frac{im\left(a+b\cos^{-1}(cx)\right)^3}{6b^2c}+\frac{m\left(a+b\cos^{-1}(cx)\right)^2\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{2bc}+\frac{m\left(a\ldots\right.}{\ldots}$$

$$= -\frac{im\left(a+b\cos^{-1}(cx)\right)^3}{6b^2c}+\frac{m\left(a+b\cos^{-1}(cx)\right)^2\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{2bc}+\frac{m\left(a\ldots\right.}{\ldots}$$

$$= -\frac{im\left(a+b\cos^{-1}(cx)\right)^3}{6b^2c}+\frac{m\left(a+b\cos^{-1}(cx)\right)^2\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{2bc}+\frac{m\left(a\ldots\right.}{\ldots}$$

Mathematica [B] time = 5.56597, size = 1248, normalized size = 3.34

$$-ibm\cos^{-1}(cx)^3-3iam\cos^{-1}(cx)^2+3bm\log\left(\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}+1\right)\cos^{-1}(cx)^2+3bm\log\left(\frac{e^{i\cos^{-1}(cx)}\left(cf-\sqrt{c^2f^2-g^2}\right)}{g}+1\right)\cos^{-1}(c$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcCos[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] ((-3*I)*a*m*ArcCos[c*x]^2 - I*b*m*ArcCos[c*x]^3 + (24*I)*a*m*ArcSin[Sqrt[1
+ (c*f)/g]/Sqrt[2]]*ArcTan[((c*f - g)*Tan[ArcCos[c*x]/2])/Sqrt[c^2*f^2 - g^
2]] + 3*b*m*ArcCos[c*x]^2*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2
 - g^2])] + 6*a*m*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*(c*f - Sqrt[c^2*f^
2 - g^2]))/g] + 3*b*m*ArcCos[c*x]^2*Log[1 + (E^(I*ArcCos[c*x])*(c*f - Sqrt[
c^2*f^2 - g^2]))/g] + 12*a*m*ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1 + (E^(
I*ArcCos[c*x])*(c*f - Sqrt[c^2*f^2 - g^2]))/g] + 12*b*m*ArcCos[c*x]*ArcSin[
Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1 + (E^(I*ArcCos[c*x])*(c*f - Sqrt[c^2*f^2 -
 g^2]))/g] + 3*b*m*ArcCos[c*x]^2*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[
c^2*f^2 - g^2])] + 6*a*m*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*(c*f + Sqrt
[c^2*f^2 - g^2]))/g] + 3*b*m*ArcCos[c*x]^2*Log[1 + (E^(I*ArcCos[c*x])*(c*f
+ Sqrt[c^2*f^2 - g^2]))/g] - 12*a*m*ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1
 + (E^(I*ArcCos[c*x])*(c*f + Sqrt[c^2*f^2 - g^2]))/g] - 12*b*m*ArcCos[c*x]*
ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1 + (E^(I*ArcCos[c*x])*(c*f + Sqrt[c^
2*f^2 - g^2]))/g] - 6*a*m*ArcCos[c*x]*Log[f + g*x] - 6*a*m*ArcSin[c*x]*Log[
f + g*x] - 3*b*ArcCos[c*x]^2*Log[h*(f + g*x)^m] + 6*a*ArcSin[c*x]*Log[h*(f
+ g*x)^m] - 3*b*m*ArcCos[c*x]^2*Log[1 + ((c*f - Sqrt[c^2*f^2 - g^2])*(c*x +
 I*Sqrt[1 - c^2*x^2]))/g] - 12*b*m*ArcCos[c*x]*ArcSin[Sqrt[1 + (c*f)/g]/Sqr
t[2]]*Log[1 + ((c*f - Sqrt[c^2*f^2 - g^2])*(c*x + I*Sqrt[1 - c^2*x^2]))/g]
- 3*b*m*ArcCos[c*x]^2*Log[1 + ((c*f + Sqrt[c^2*f^2 - g^2])*(c*x + I*Sqrt[1
- c^2*x^2]))/g] + 12*b*m*ArcCos[c*x]*ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[
1 + ((c*f + Sqrt[c^2*f^2 - g^2])*(c*x + I*Sqrt[1 - c^2*x^2]))/g] - (6*I)*b*
m*ArcCos[c*x]*PolyLog[2, (E^(I*ArcCos[c*x])*g)/(-(c*f) + Sqrt[c^2*f^2 - g^2
])] - (6*I)*a*m*PolyLog[2, (E^(I*ArcCos[c*x])*(-(c*f) + Sqrt[c^2*f^2 - g^2]
))/g] - (6*I)*b*m*ArcCos[c*x]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqr
t[c^2*f^2 - g^2]))] - (6*I)*a*m*PolyLog[2, -((E^(I*ArcCos[c*x])*(c*f + Sqrt
[c^2*f^2 - g^2]))/g)] + 6*b*m*PolyLog[3, (E^(I*ArcCos[c*x])*g)/(-(c*f) + Sq
rt[c^2*f^2 - g^2])] + 6*b*m*PolyLog[3, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[
c^2*f^2 - g^2]))])/(6*c)
```

Maple [F] time = 3.134, size = 0, normalized size = 0.

$$\int (a + b\arccos(cx))\ln\left(h\left(gx+f\right)^{m}\right)\frac{1}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccos(c*x))*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^{2}x^{2}+1}(b\arccos(cx)+a)\log\left(\left(gx+f\right)^{m}h\right)}{c^{2}x^{2}-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)*log((g*x + f)^m*h)/(c^2*x^
2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acos}\,(cx))\log \left(h\left(f + gx\right)^{m}\right)}{\sqrt{-\left(cx - 1\right)\left(cx + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acos(c*x))*log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arccos\,(cx) + a)\log\left(\left(gx + f\right)^{m}h\right)}{\sqrt{-c^{2}x^{2} + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

3.22 $\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=237

$$\frac{im\text{PolyLog}\left(2,\frac{ige^{i\sin^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{im\text{PolyLog}\left(2,\frac{ige^{i\sin^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{c}-\frac{m\sin^{-1}(cx)\log\left(1-\frac{ige^{i\sin^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{m\sin^{-1}(cx)\log\left(1-\right.}{c}$$

```
[Out] ((I/2)*m*ArcSin[c*x]^2)/c - (m*ArcSin[c*x]*Log[1 - (I*E^(I*ArcSin[c*x])*g)/
(c*f - Sqrt[c^2*f^2 - g^2])])/c - (m*ArcSin[c*x]*Log[1 - (I*E^(I*ArcSin[c*x
])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/c + (ArcSin[c*x]*Log[h*(f + g*x)^m])/c
+ (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/c +
 (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/c
```

Rubi [A] time = 0.338099, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {216, 2404, 4741, 4519, 2190, 2279, 2391}

$$\frac{im\text{PolyLog}\left(2,\frac{ige^{i\sin^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{im\text{PolyLog}\left(2,\frac{ige^{i\sin^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{c}-\frac{m\sin^{-1}(cx)\log\left(1-\frac{ige^{i\sin^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{m\sin^{-1}(cx)\log\left(1-\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Log[h*(f + g*x)^m]/Sqrt[1 - c^2*x^2],x]
```

```
[Out] ((I/2)*m*ArcSin[c*x]^2)/c - (m*ArcSin[c*x]*Log[1 - (I*E^(I*ArcSin[c*x])*g)/
(c*f - Sqrt[c^2*f^2 - g^2])])/c - (m*ArcSin[c*x]*Log[1 - (I*E^(I*ArcSin[c*x
])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/c + (ArcSin[c*x]*Log[h*(f + g*x)^m])/c
+ (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/c +
 (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/c
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
```

```
b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n, Int[SimplifyIntegrand[u/(d + e*x),
x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \frac{\sin^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\int \frac{\sin^{-1}(cx)}{cf+cgx}\,dx$$

$$= \frac{\sin^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\,\mathrm{Subst}\left(\int \frac{x\cos(x)}{c^2f+cg\sin(x)}\,dx, x, \sin^{-1}(cx)\right)$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} + \frac{\sin^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\,\mathrm{Subst}\left(\int \frac{e^{ix}x}{c^2f-ice^{ix}g-c\sqrt{c^2f^2-g^2}}\,dx,\right.$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c} + \frac{\sin^{-}}{}$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c} + \frac{\sin^{-}}{}$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c} + \frac{\sin^{-}}{}$$

Mathematica [A] time = 0.0218738, size = 246, normalized size = 1.04

$$\frac{im\,\mathrm{PolyLog}\left(2,\frac{ige^{i\sin^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} + \frac{im\,\mathrm{PolyLog}\left(2,\frac{ige^{i\sin^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{icge^{i\sin^{-1}(cx)}}{c^2f-c\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[h*(f + g*x)^m]/Sqrt[1 - c^2*x^2],x]
```

```
[Out] ((I/2)*m*ArcSin[c*x]^2)/c - (m*ArcSin[c*x]*Log[1 - (I*c*E^(I*ArcSin[c*x])*g
)/(c^2*f - c*Sqrt[c^2*f^2 - g^2])])/c - (m*ArcSin[c*x]*Log[1 - (I*c*E^(I*Ar
cSin[c*x])*g)/(c^2*f + c*Sqrt[c^2*f^2 - g^2])])/c + (ArcSin[c*x]*Log[h*(f +
 g*x)^m])/c + (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f - Sqrt[c^2*f^2 -
 g^2])])/c + (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f + Sqrt[c^2*f^2 -
g^2])])/c
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \ln\left(h\left(gx+f\right)^m\right)\frac{1}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\log\left(\left(gx+f\right)^{m}h\right)}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*log((g*x + f)^m*h)/(c^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(h\left(f+gx\right)^{m}\right)}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

3.23 $\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])), x]
```

Rubi [A] time = 0.200123, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

```
[Out] Defer[Int][Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])), x]
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}\,dx=\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.199132, size = 0, normalized size = 0.

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

```
[Out] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])), x]
```

Maple [A] time = 2.447, size = 0, normalized size = 0.

$$\int \frac{\ln\left(h\left(gx+f\right)^{m}\right)}{a+b\arccos\left(cx\right)}\frac{1}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)}{\sqrt{-c^{2}x^{2}+1}(b\arccos\left(cx\right)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^{2}x^{2}+1}\log\left(\left(gx+f\right)^{m}h\right)}{ac^{2}x^{2}+\left(bc^{2}x^{2}-b\right)\arccos\left(cx\right)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*log((g*x + f)^m*h)/(a*c^2*x^2 + (b*c^2*x^2 - b
)*arccos(c*x) - a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(h\left(f+gx\right)^{m}\right)}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acos}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(a+b*acos(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(log(h*(f + g*x)**m)/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x)))
, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)}{\sqrt{-c^{2}x^{2}+1}\left(b\arccos\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)), x)
```

3.24 $\int x^3 \cos^{-1}(a+bx)\,dx$

Optimal. Leaf size=137

$$\frac{\left(4a\left(19a^2+16\right)-\left(26a^2+9\right)(a+bx)\right)\sqrt{1-(a+bx)^2}}{96b^4}+\frac{\left(8a^4+24a^2+3\right)\sin^{-1}(a+bx)}{32b^4}+\frac{7ax^2\sqrt{1-(a+bx)^2}}{48b^2}-\frac{x^3\sqrt{1}}{}$$

```
[Out] (7*a*x^2*Sqrt[1 - (a + b*x)^2])/(48*b^2) - (x^3*Sqrt[1 - (a + b*x)^2])/(16*
b) + ((4*a*(16 + 19*a^2) - (9 + 26*a^2)*(a + b*x))*Sqrt[1 - (a + b*x)^2])/(
96*b^4) + (x^4*ArcCos[a + b*x])/4 + ((3 + 24*a^2 + 8*a^4)*ArcSin[a + b*x])/
(32*b^4)
```

Rubi [A] time = 0.194006, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4806, 4744, 743, 833, 780, 216}

$$\frac{\left(4a\left(19a^2+16\right)-\left(26a^2+9\right)(a+bx)\right)\sqrt{1-(a+bx)^2}}{96b^4}+\frac{\left(8a^4+24a^2+3\right)\sin^{-1}(a+bx)}{32b^4}+\frac{7ax^2\sqrt{1-(a+bx)^2}}{48b^2}-\frac{x^3\sqrt{1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCos[a + b*x],x]
```


```
[Out] (7*a*x^2*Sqrt[1 - (a + b*x)^2])/(48*b^2) - (x^3*Sqrt[1 - (a + b*x)^2])/(16*
b) + ((4*a*(16 + 19*a^2) - (9 + 26*a^2)*(a + b*x))*Sqrt[1 - (a + b*x)^2])/(
96*b^4) + (x^4*ArcCos[a + b*x])/4 + ((3 + 24*a^2 + 8*a^4)*ArcSin[a + b*x])/
(32*b^4)
```


Rule 4806

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(e*(m + 1)), x] +
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1
))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^3 \cos^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \cos^{-1}(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{4}x^4\cos^{-1}(a+bx) + \frac{1}{4}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{\sqrt{1-x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{x^3\sqrt{1-(a+bx)^2}}{16b} + \frac{1}{4}x^4\cos^{-1}(a+bx) - \frac{1}{16}\text{Subst}\left(\int \frac{\left(-\frac{3+4a^2}{b^2}+\frac{7ax}{b^2}\right)\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sqrt{1-x^2}}\,dx, x, a+b\right.$$

$$= \frac{7ax^2\sqrt{1-(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1-(a+bx)^2}}{16b} + \frac{1}{4}x^4\cos^{-1}(a+bx) + \frac{1}{48}\text{Subst}\left(\int \frac{\left(-\frac{a(23+12a^2)}{b^3}+ (\right.}{\sqrt{1}}\right.$$

$$= \frac{7ax^2\sqrt{1-(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1-(a+bx)^2}}{16b} + \frac{\left(4a\left(16+19a^2\right)-\left(9+26a^2\right)(a+bx)\right)\sqrt{1-(a+bx}}{96b^4}$$

$$= \frac{7ax^2\sqrt{1-(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1-(a+bx)^2}}{16b} + \frac{\left(4a\left(16+19a^2\right)-\left(9+26a^2\right)(a+bx)\right)\sqrt{1-(a+bx}}{96b^4}$$

Mathematica [A] time = 0.100011, size = 104, normalized size = 0.76

$$\frac{\sqrt{-a^2-2abx-b^2x^2+1}\left(-26a^2bx+50a^3+14ab^2x^2+55a-6b^3x^3-9bx\right)+3\left(8a^4+24a^2+3\right)\sin^{-1}(a+bx)+24b^4x^4}{96b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCos[a + b*x],x]
```

```
[Out] (Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*(55*a + 50*a^3 - 9*b*x - 26*a^2*b*x + 14
*a*b^2*x^2 - 6*b^3*x^3) + 24*b^4*x^4*ArcCos[a + b*x] + 3*(3 + 24*a^2 + 8*a^
4)*ArcSin[a + b*x])/(96*b^4)
```

Maple [A] time = 0.02, size = 235, normalized size = 1.7

$$\frac{1}{b^4}\left(\frac{\arccos(bx+a)(bx+a)^4}{4} - \arccos(bx+a)(bx+a)^3 a + \frac{3\arccos(bx+a)(bx+a)^2 a^2}{2} - \arccos(bx+a)(bx+a)\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccos(b*x+a),x)
```

```
[Out] 1/b^4*(1/4*arccos(b*x+a)*(b*x+a)^4-arccos(b*x+a)*(b*x+a)^3*a+3/2*arccos(b*x
+a)*(b*x+a)^2*a^2-arccos(b*x+a)*(b*x+a)*a^3+1/4*arccos(b*x+a)*a^4-1/16*(b*x
+a)^3*(1-(b*x+a)^2)^(1/2)-3/32*(b*x+a)*(1-(b*x+a)^2)^(1/2)+3/32*arcsin(b*x+
a)-a*(-1/3*(b*x+a)^2*(1-(b*x+a)^2)^(1/2)-2/3*(1-(b*x+a)^2)^(1/2))+3/2*a^2*(
-1/2*(b*x+a)*(1-(b*x+a)^2)^(1/2)+1/2*arcsin(b*x+a))+a^3*(1-(b*x+a)^2)^(1/2)
+1/4*arcsin(b*x+a)*a^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.47944, size = 219, normalized size = 1.6

$$\frac{3\left(8\,b^4x^4-8\,a^4-24\,a^2-3\right)\arccos\left(bx+a\right)-\left(6\,b^3x^3-14\,ab^2x^2-50\,a^3+\left(26\,a^2+9\right)bx-55\,a\right)\sqrt{-b^2x^2-2\,abx-a^2+1}}{96\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*b^4*x^4 - 8*a^4 - 24*a^2 - 3)*arccos(b*x + a) - (6*b^3*x^3 - 14*
a*b^2*x^2 - 50*a^3 + (26*a^2 + 9)*b*x - 55*a)*sqrt(-b^2*x^2 - 2*a*b*x - a^2
 + 1))/b^4
```

Sympy [A] time = 1.75661, size = 255, normalized size = 1.86

$$\begin{cases} -\frac{a^4\operatorname{acos}(a+bx)}{4b^4}+\frac{25a^3\sqrt{-a^2-2abx-b^2x^2+1}}{48b^4}-\frac{13a^2x\sqrt{-a^2-2abx-b^2x^2+1}}{48b^3}-\frac{3a^2\operatorname{acos}(a+bx)}{4b^4}+\frac{7ax^2\sqrt{-a^2-2abx-b^2x^2+1}}{48b^2}+\frac{55a\sqrt{-a^2-2abx-b^2x^2+1}}{96b^4}+ \\ \frac{x^4\operatorname{acos}(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acos(b*x+a),x)
```

```
[Out] Piecewise((-a**4*acos(a + b*x)/(4*b**4) + 25*a**3*sqrt(-a**2 - 2*a*b*x - b*
*2*x**2 + 1)/(48*b**4) - 13*a**2*x*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(4
8*b**3) - 3*a**2*acos(a + b*x)/(4*b**4) + 7*a*x**2*sqrt(-a**2 - 2*a*b*x - b
**2*x**2 + 1)/(48*b**2) + 55*a*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(96*b*
*4) + x**4*acos(a + b*x)/4 - x**3*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(16
*b) - 3*x*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(32*b**3) - 3*acos(a + b*x)
/(32*b**4), Ne(b, 0)), (x**4*acos(a)/4, True))
```

Giac [B] time = 1.31574, size = 327, normalized size = 2.39

$$\frac{(bx+a)^4 \arccos(bx+a)}{4\,b^4} - \frac{(bx+a)^3 a \arccos(bx+a)}{b^4} + \frac{3\,(bx+a)^2 a^2 \arccos(bx+a)}{2\,b^4} - \frac{(bx+a)a^3 \arccos(bx+a)}{b^4} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(b*x + a)^4*arccos(b*x + a)/b^4 - (b*x + a)^3*a*arccos(b*x + a)/b^4 + 3
/2*(b*x + a)^2*a^2*arccos(b*x + a)/b^4 - (b*x + a)*a^3*arccos(b*x + a)/b^4
- 1/16*sqrt(-(b*x + a)^2 + 1)*(b*x + a)^3/b^4 + 1/3*sqrt(-(b*x + a)^2 + 1)*
(b*x + a)^2*a/b^4 - 3/4*sqrt(-(b*x + a)^2 + 1)*(b*x + a)*a^2/b^4 + sqrt(-(b
*x + a)^2 + 1)*a^3/b^4 - 3/4*a^2*arccos(b*x + a)/b^4 - 3/32*sqrt(-(b*x + a)
^2 + 1)*(b*x + a)/b^4 + 2/3*sqrt(-(b*x + a)^2 + 1)*a/b^4 - 3/32*arccos(b*x
+ a)/b^4
```

3.25 $\int x^2 \cos^{-1}(a+bx)\,dx$

Optimal. Leaf size=94

$$-\frac{(11a^2-5abx+4)\sqrt{1-(a+bx)^2}}{18b^3}-\frac{a(2a^2+3)\sin^{-1}(a+bx)}{6b^3}-\frac{x^2\sqrt{1-(a+bx)^2}}{9b}+\frac{1}{3}x^3\cos^{-1}(a+bx)$$

```
[Out] -(x^2*Sqrt[1 - (a + b*x)^2])/(9*b) - ((4 + 11*a^2 - 5*a*b*x)*Sqrt[1 - (a +
b*x)^2])/(18*b^3) + (x^3*ArcCos[a + b*x])/3 - (a*(3 + 2*a^2)*ArcSin[a + b*x
])/(6*b^3)
```

Rubi [A] time = 0.114887, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4806, 4744, 743, 780, 216}

$$-\frac{(11a^2-5abx+4)\sqrt{1-(a+bx)^2}}{18b^3}-\frac{a(2a^2+3)\sin^{-1}(a+bx)}{6b^3}-\frac{x^2\sqrt{1-(a+bx)^2}}{9b}+\frac{1}{3}x^3\cos^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[a + b*x],x]
```

```
[Out] -(x^2*Sqrt[1 - (a + b*x)^2])/(9*b) - ((4 + 11*a^2 - 5*a*b*x)*Sqrt[1 - (a +
b*x)^2])/(18*b^3) + (x^3*ArcCos[a + b*x])/3 - (a*(3 + 2*a^2)*ArcSin[a + b*x
])/(6*b^3)
```

Rule 4806

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(e*(m + 1)), x] +
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1
))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^2\cos^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^2\cos^{-1}(x)\,dx,x,a+bx\right)}{b}$$

$$=\frac{1}{3}x^3\cos^{-1}(a+bx)+\frac{1}{3}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{\sqrt{1-x^2}}\,dx,x,a+bx\right)$$

$$=-\frac{x^2\sqrt{1-(a+bx)^2}}{9b}+\frac{1}{3}x^3\cos^{-1}(a+bx)-\frac{1}{9}\text{Subst}\left(\int\frac{\left(-\frac{2+3a^2}{b^2}+\frac{5ax}{b^2}\right)\left(-\frac{a}{b}+\frac{x}{b}\right)}{\sqrt{1-x^2}}\,dx,x,a+bx\right.$$

$$=-\frac{x^2\sqrt{1-(a+bx)^2}}{9b}-\frac{\left(4+11a^2-5abx\right)\sqrt{1-(a+bx)^2}}{18b^3}+\frac{1}{3}x^3\cos^{-1}(a+bx)-\frac{\left(a\left(3+2a^2\right)\right)\text{S}}{}$$

$$=-\frac{x^2\sqrt{1-(a+bx)^2}}{9b}-\frac{\left(4+11a^2-5abx\right)\sqrt{1-(a+bx)^2}}{18b^3}+\frac{1}{3}x^3\cos^{-1}(a+bx)-\frac{a\left(3+2a^2\right)\sin}{6b^3}$$

Mathematica [A] time = 0.0828341, size = 83, normalized size = 0.88

$$-\frac{\sqrt{-a^2-2abx-b^2x^2+1}\left(11a^2-5abx+2b^2x^2+4\right)+3a\left(2a^2+3\right)\sin^{-1}(a+bx)-6b^3x^3\cos^{-1}(a+bx)}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[a + b*x],x]
```

```
[Out] -(Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*(4 + 11*a^2 - 5*a*b*x + 2*b^2*x^2) - 6*
b^3*x^3*ArcCos[a + b*x] + 3*a*(3 + 2*a^2)*ArcSin[a + b*x])/(18*b^3)
```

Maple [A] time = 0.003, size = 161, normalized size = 1.7

$$\frac{1}{b^3}\left(\frac{\arccos(bx+a)(bx+a)^3}{3} - \arccos(bx+a)(bx+a)^2 a + \arccos(bx+a)(bx+a)a^2 - \frac{\arccos(bx+a)a^3}{3} - \frac{(bx+a)^2}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(b*x+a),x)
```

```
[Out] 1/b^3*(1/3*arccos(b*x+a)*(b*x+a)^3-arccos(b*x+a)*(b*x+a)^2*a+arccos(b*x+a)*
(b*x+a)*a^2-1/3*arccos(b*x+a)*a^3-1/9*(b*x+a)^2*(1-(b*x+a)^2)^(1/2)-2/9*(1-
(b*x+a)^2)^(1/2)-a*(-1/2*(b*x+a)*(1-(b*x+a)^2)^(1/2)+1/2*arcsin(b*x+a))-a^2
*(1-(b*x+a)^2)^(1/2)-1/3*arcsin(b*x+a)*a^3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.42065, size = 173, normalized size = 1.84

$$\frac{3\left(2\,b^3x^3 + 2\,a^3 + 3\,a\right)\arccos(bx+a) - \left(2\,b^2x^2 - 5\,abx + 11\,a^2 + 4\right)\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}}{18\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/18*(3*(2*b^3*x^3 + 2*a^3 + 3*a)*arccos(b*x + a) - (2*b^2*x^2 - 5*a*b*x +
11*a^2 + 4)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/b^3
```

Sympy [A] time = 0.819965, size = 170, normalized size = 1.81

$$\begin{cases} \frac{a^3 \operatorname{acos}(a+bx)}{3b^3} - \frac{11a^2\sqrt{-a^2-2abx-b^2x^2+1}}{18b^3} + \frac{5ax\sqrt{-a^2-2abx-b^2x^2+1}}{18b^2} + \frac{a \operatorname{acos}(a+bx)}{2b^3} + \frac{x^3 \operatorname{acos}(a+bx)}{3} - \frac{x^2\sqrt{-a^2-2abx-b^2x^2+1}}{9b} - \frac{2\sqrt{-a^2-2abx-b}}{9b^3} \\ \frac{x^3 \operatorname{acos}(a)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(b*x+a),x)
```

```
[Out] Piecewise((a**3*acos(a + b*x)/(3*b**3) - 11*a**2*sqrt(-a**2 - 2*a*b*x - b**
2*x**2 + 1)/(18*b**3) + 5*a*x*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(18*b**
2) + a*acos(a + b*x)/(2*b**3) + x**3*acos(a + b*x)/3 - x**2*sqrt(-a**2 - 2*
a*b*x - b**2*x**2 + 1)/(9*b) - 2*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(9*b
**3), Ne(b, 0)), (x**3*acos(a)/3, True))
```

Giac [A] time = 1.30621, size = 211, normalized size = 2.24

$$\frac{(bx+a)^3 \arccos(bx+a)}{3\,b^3} - \frac{(bx+a)^2 a \arccos(bx+a)}{b^3} + \frac{(bx+a)a^2 \arccos(bx+a)}{b^3} - \frac{\sqrt{-(bx+a)^2+1}(bx+a)^2}{9\,b^3} + \frac{\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*(b*x + a)^3*arccos(b*x + a)/b^3 - (b*x + a)^2*a*arccos(b*x + a)/b^3 + (
b*x + a)*a^2*arccos(b*x + a)/b^3 - 1/9*sqrt(-(b*x + a)^2 + 1)*(b*x + a)^2/b
^3 + 1/2*sqrt(-(b*x + a)^2 + 1)*(b*x + a)*a/b^3 - sqrt(-(b*x + a)^2 + 1)*a^
2/b^3 + 1/2*a*arccos(b*x + a)/b^3 - 2/9*sqrt(-(b*x + a)^2 + 1)/b^3
```

3.26 $\int x \cos^{-1}(a+bx)\,dx$

Optimal. Leaf size=80

$$\frac{(2a^2+1)\sin^{-1}(a+bx)}{4b^2}+\frac{3a\sqrt{1-(a+bx)^2}}{4b^2}+\frac{1}{2}x^2\cos^{-1}(a+bx)-\frac{x\sqrt{1-(a+bx)^2}}{4b}$$

```
[Out] (3*a*Sqrt[1 - (a + b*x)^2])/(4*b^2) - (x*Sqrt[1 - (a + b*x)^2])/(4*b) + (x^
2*ArcCos[a + b*x])/2 + ((1 + 2*a^2)*ArcSin[a + b*x])/(4*b^2)
```

Rubi [A] time = 0.0731415, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4806, 4744, 743, 641, 216}

$$\frac{(2a^2+1)\sin^{-1}(a+bx)}{4b^2}+\frac{3a\sqrt{1-(a+bx)^2}}{4b^2}+\frac{1}{2}x^2\cos^{-1}(a+bx)-\frac{x\sqrt{1-(a+bx)^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCos[a + b*x],x]
```

```
[Out] (3*a*Sqrt[1 - (a + b*x)^2])/(4*b^2) - (x*Sqrt[1 - (a + b*x)^2])/(4*b) + (x^
2*ArcCos[a + b*x])/2 + ((1 + 2*a^2)*ArcSin[a + b*x])/(4*b^2)
```

Rule 4806

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(e*(m + 1)), x] +
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1
))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
```

```
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x\cos^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\cos^{-1}(x)\,dx,x,a+bx\right)}{b}\\
&= \frac{1}{2}x^2\cos^{-1}(a+bx)+\frac{1}{2}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sqrt{1-x^2}}\,dx,x,a+bx\right)\\
&= -\frac{x\sqrt{1-(a+bx)^2}}{4b}+\frac{1}{2}x^2\cos^{-1}(a+bx)-\frac{1}{4}\text{Subst}\left(\int\frac{-\frac{1+2a^2}{b^2}+\frac{3ax}{b^2}}{\sqrt{1-x^2}}\,dx,x,a+bx\right)\\
&= \frac{3a\sqrt{1-(a+bx)^2}}{4b^2}-\frac{x\sqrt{1-(a+bx)^2}}{4b}+\frac{1}{2}x^2\cos^{-1}(a+bx)+\frac{\left(1+2a^2\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}}\,dx,x,a+\right.}{4b^2}\\
&= \frac{3a\sqrt{1-(a+bx)^2}}{4b^2}-\frac{x\sqrt{1-(a+bx)^2}}{4b}+\frac{1}{2}x^2\cos^{-1}(a+bx)+\frac{\left(1+2a^2\right)\sin^{-1}(a+bx)}{4b^2}
\end{aligned}$$

Mathematica [A] time = 0.0494883, size = 69, normalized size = 0.86

$$\frac{(3a-bx)\sqrt{-a^2-2abx-b^2x^2+1}+\left(2a^2+1\right)\sin^{-1}(a+bx)+2b^2x^2\cos^{-1}(a+bx)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCos[a + b*x],x]
```

```
[Out] ((3*a - b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] + 2*b^2*x^2*ArcCos[a + b*x]
+ (1 + 2*a^2)*ArcSin[a + b*x])/(4*b^2)
```

Maple [A] time = 0.004, size = 78, normalized size = 1.

$$\frac{1}{b^2}\left(\frac{\arccos(bx+a)(bx+a)^2}{2} - \arccos(bx+a)\,a\,(bx+a) - \frac{bx+a}{4}\sqrt{1-(bx+a)^2} + \frac{\arcsin(bx+a)}{4} + a\sqrt{1-(bx+a)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(b*x+a),x)
```

```
[Out] 1/b^2*(1/2*arccos(b*x+a)*(b*x+a)^2-arccos(b*x+a)*a*(b*x+a)-1/4*(b*x+a)*(1-(
b*x+a)^2)^(1/2)+1/4*arcsin(b*x+a)+a*(1-(b*x+a)^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.55125, size = 135, normalized size = 1.69

$$\frac{\left(2\,b^2x^2 - 2\,a^2 - 1\right)\arccos(bx+a) - \sqrt{-b^2x^2 - 2\,abx - a^2 + 1}(bx - 3\,a)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^2*x^2 - 2*a^2 - 1)*arccos(b*x + a) - sqrt(-b^2*x^2 - 2*a*b*x - a^
2 + 1)*(b*x - 3*a))/b^2
```

Sympy [A] time = 0.361235, size = 104, normalized size = 1.3

$$\begin{cases} -\frac{a^2 \operatorname{acos}(a+bx)}{2b^2} + \frac{3a\sqrt{-a^2-2abx-b^2x^2+1}}{4b^2} + \frac{x^2 \operatorname{acos}(a+bx)}{2} - \frac{x\sqrt{-a^2-2abx-b^2x^2+1}}{4b} - \frac{\operatorname{acos}(a+bx)}{4b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{acos}(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(b*x+a),x)
```

```
[Out] Piecewise((-a**2*acos(a + b*x)/(2*b**2) + 3*a*sqrt(-a**2 - 2*a*b*x - b**2*x
**2 + 1)/(4*b**2) + x**2*acos(a + b*x)/2 - x*sqrt(-a**2 - 2*a*b*x - b**2*x*
*2 + 1)/(4*b) - acos(a + b*x)/(4*b**2), Ne(b, 0)), (x**2*acos(a)/2, True))
```

Giac [A] time = 1.32168, size = 119, normalized size = 1.49

$$\frac{(bx+a)^2 \arccos(bx+a)}{2\,b^2} - \frac{(bx+a)a \arccos(bx+a)}{b^2} - \frac{\sqrt{-(bx+a)^2+1}(bx+a)}{4\,b^2} + \frac{\sqrt{-(bx+a)^2+1}a}{b^2} - \frac{\arccos(bx+a)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*x + a)^2*arccos(b*x + a)/b^2 - (b*x + a)*a*arccos(b*x + a)/b^2 - 1/4
*sqrt(-(b*x + a)^2 + 1)*(b*x + a)/b^2 + sqrt(-(b*x + a)^2 + 1)*a/b^2 - 1/4*
arccos(b*x + a)/b^2
```

3.27 $\int \cos^{-1}(a+bx)\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx)\cos^{-1}(a+bx)}{b}-\frac{\sqrt{1-(a+bx)^2}}{b}$$

```
[Out] -(Sqrt[1 - (a + b*x)^2]/b) + ((a + b*x)*ArcCos[a + b*x])/b
```

Rubi [A] time = 0.0168144, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4804, 4620, 261}

$$\frac{(a+bx)\cos^{-1}(a+bx)}{b}-\frac{\sqrt{1-(a+bx)^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x],x]
```

```
[Out] -(Sqrt[1 - (a + b*x)^2]/b) + ((a + b*x)*ArcCos[a + b*x])/b
```

Rule 4804

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \cos^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \cos^{-1}(x)\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\cos^{-1}(a+bx)}{b} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\sqrt{1-(a+bx)^2}}{b} + \frac{(a+bx)\cos^{-1}(a+bx)}{b}$$

Mathematica [A] time = 0.036752, size = 47, normalized size = 1.31

$$x\cos^{-1}(a+bx) - \frac{\sqrt{-a^2-2abx-b^2x^2+1} + a\sin^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x],x]
```

```
[Out] x*ArcCos[a + b*x] - (Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] + a*ArcSin[a + b*x])
/b
```

Maple [A] time = 0.002, size = 33, normalized size = 0.9

$$\frac{1}{b}\left((bx+a)\arccos(bx+a) - \sqrt{1-(bx+a)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a),x)
```

```
[Out] 1/b*((b*x+a)*arccos(b*x+a)-(1-(b*x+a)^2)^(1/2))
```

Maxima [A] time = 1.41026, size = 43, normalized size = 1.19

$$\frac{(bx+a)\arccos(bx+a) - \sqrt{-(bx+a)^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*x + a)*arccos(b*x + a) - sqrt(-(b*x + a)^2 + 1))/b
```

Fricas [A] time = 2.50697, size = 92, normalized size = 2.56

$$\frac{(bx+a)\arccos(bx+a)-\sqrt{-b^2x^2-2\,abx-a^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*x + a)*arccos(b*x + a) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/b
```

Sympy [A] time = 0.174177, size = 46, normalized size = 1.28

$$\begin{cases} \frac{a\operatorname{acos}(a+bx)}{b}+x\operatorname{acos}(a+bx)-\frac{\sqrt{-a^2-2abx-b^2x^2+1}}{b} & \text{for}\ b\neq 0\\ x\operatorname{acos}(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a),x)
```

```
[Out] Piecewise((a*acos(a + b*x)/b + x*acos(a + b*x) - sqrt(-a**2 - 2*a*b*x - b**
2*x**2 + 1)/b, Ne(b, 0)), (x*acos(a), True))
```

Giac [A] time = 1.2806, size = 43, normalized size = 1.19

$$\frac{(bx+a)\arccos(bx+a)-\sqrt{-(bx+a)^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a),x, algorithm="giac")
```

```
[Out] ((b*x + a)*arccos(b*x + a) - sqrt(-(b*x + a)^2 + 1))/b
```

3.28 $\int \frac{\cos^{-1}(a+bx)}{x}\,dx$

Optimal. Leaf size=177

$$-i\text{PolyLog}\left(2,\frac{e^{i\cos^{-1}(a+bx)}}{a-i\sqrt{1-a^2}}\right)-i\text{PolyLog}\left(2,\frac{e^{i\cos^{-1}(a+bx)}}{a+i\sqrt{1-a^2}}\right)+\cos^{-1}(a+bx)\log\left(1-\frac{e^{i\cos^{-1}(a+bx)}}{a-i\sqrt{1-a^2}}\right)+\cos^{-1}(a+bx)\log$$

```
[Out] (-I/2)*ArcCos[a + b*x]^2 + ArcCos[a + b*x]*Log[1 - E^(I*ArcCos[a + b*x])/(a
 - I*Sqrt[1 - a^2])] + ArcCos[a + b*x]*Log[1 - E^(I*ArcCos[a + b*x])/(a + I
*Sqrt[1 - a^2])] - I*PolyLog[2, E^(I*ArcCos[a + b*x])/(a - I*Sqrt[1 - a^2])
] - I*PolyLog[2, E^(I*ArcCos[a + b*x])/(a + I*Sqrt[1 - a^2])]
```

Rubi [A] time = 0.276321, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4806, 4742, 4522, 2190, 2279, 2391}

$$-i\text{PolyLog}\left(2,\frac{e^{i\cos^{-1}(a+bx)}}{a-i\sqrt{1-a^2}}\right)-i\text{PolyLog}\left(2,\frac{e^{i\cos^{-1}(a+bx)}}{a+i\sqrt{1-a^2}}\right)+\cos^{-1}(a+bx)\log\left(1-\frac{e^{i\cos^{-1}(a+bx)}}{a-i\sqrt{1-a^2}}\right)+\cos^{-1}(a+bx)\log$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]/x,x]
```

```
[Out] (-I/2)*ArcCos[a + b*x]^2 + ArcCos[a + b*x]*Log[1 - E^(I*ArcCos[a + b*x])/(a
 - I*Sqrt[1 - a^2])] + ArcCos[a + b*x]*Log[1 - E^(I*ArcCos[a + b*x])/(a + I
*Sqrt[1 - a^2])] - I*PolyLog[2, E^(I*ArcCos[a + b*x])/(a - I*Sqrt[1 - a^2])
] - I*PolyLog[2, E^(I*ArcCos[a + b*x])/(a + I*Sqrt[1 - a^2])]
```

Rule 4806

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[((a + b*x)^n*Sin[x])/(c*d + e*Cos[x]), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4522

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(I*a - Rt[-a^2 + b^2, 2] + I*b*E^
(I*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(I*a + Rt[-a^2 + b^2
, 2] + I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && NegQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cos^{-1}(a+bx)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int \frac{x\sin(x)}{-\frac{a}{b}+\frac{\cos(x)}{b}}\,dx, x, \cos^{-1}(a+bx)\right)}{b}$$

$$= -\frac{1}{2}i\cos^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int \frac{e^{ix}x}{-\frac{ia}{b}-\frac{\sqrt{1-a^2}}{b}+\frac{ie^{ix}}{b}}\,dx, x, \cos^{-1}(a+bx)\right)}{b} - \frac{\text{Subst}\left(\int \frac{e^{ix}x}{-\frac{ia}{b}+\frac{\sqrt{1-a^2}}{b}+\frac{ie^{ix}}{b}}\,dx,\right.}{b}$$

$$= -\frac{1}{2}i\cos^{-1}(a+bx)^2 + \cos^{-1}(a+bx)\log\left(1-\frac{e^{i\cos^{-1}(a+bx)}}{a-i\sqrt{1-a^2}}\right) + \cos^{-1}(a+bx)\log\left(1-\frac{e^{i\cos^{-1}(a+bx)}}{a+i\sqrt{1-a^2}}\right.$$

$$= -\frac{1}{2}i\cos^{-1}(a+bx)^2 + \cos^{-1}(a+bx)\log\left(1-\frac{e^{i\cos^{-1}(a+bx)}}{a-i\sqrt{1-a^2}}\right) + \cos^{-1}(a+bx)\log\left(1-\frac{e^{i\cos^{-1}(a+bx)}}{a+i\sqrt{1-a^2}}\right.$$

$$= -\frac{1}{2}i\cos^{-1}(a+bx)^2 + \cos^{-1}(a+bx)\log\left(1-\frac{e^{i\cos^{-1}(a+bx)}}{a-i\sqrt{1-a^2}}\right) + \cos^{-1}(a+bx)\log\left(1-\frac{e^{i\cos^{-1}(a+bx)}}{a+i\sqrt{1-a^2}}\right.$$

Mathematica [A] time = 0.191965, size = 228, normalized size = 1.29

$$-i\left(\text{PolyLog}\left(2,\left(a-\sqrt{a^2-1}\right)e^{i\cos^{-1}(a+bx)}\right)+\text{PolyLog}\left(2,\left(\sqrt{a^2-1}+a\right)e^{i\cos^{-1}(a+bx)}\right)\right)+\log\left(1+\left(\sqrt{a^2-1}-a\right)e^{i\cos^{-1}(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a + b*x]/x,x]
```

```
[Out] (-I/2)*ArcCos[a + b*x]^2 - (4*I)*ArcSin[Sqrt[1 - a]/Sqrt[2]]*ArcTan[((1 + a
)*Tan[ArcCos[a + b*x]/2])/Sqrt[-1 + a^2]] + (ArcCos[a + b*x] - 2*ArcSin[Sqr
t[1 - a]/Sqrt[2]])*Log[1 + (-a + Sqrt[-1 + a^2])*E^(I*ArcCos[a + b*x])] + (
ArcCos[a + b*x] + 2*ArcSin[Sqrt[1 - a]/Sqrt[2]])*Log[1 - (a + Sqrt[-1 + a^2
])*E^(I*ArcCos[a + b*x])] - I*(PolyLog[2, (a - Sqrt[-1 + a^2])*E^(I*ArcCos[
a + b*x])] + PolyLog[2, (a + Sqrt[-1 + a^2])*E^(I*ArcCos[a + b*x])])
```

Maple [A] time = 0.104, size = 199, normalized size = 1.1

$$-\frac{i}{2}(\arccos(bx+a))^2+\arccos(bx+a)\ln\left(\left(\sqrt{a^2-1}+bx+i\sqrt{1-(bx+a)^2}\right)\left(-a+\sqrt{a^2-1}\right)^{-1}\right)+\arccos(bx+a)\ln\left(\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)/x,x)
```

```
[Out] -1/2*I*arccos(b*x+a)^2+arccos(b*x+a)*ln(((a^2-1)^(1/2)+b*x+I*(1-(b*x+a)^2)^
(1/2))/(-a+(a^2-1)^(1/2)))+arccos(b*x+a)*ln(((a^2-1)^(1/2)-b*x-I*(1-(b*x+a)
^2)^(1/2))/(a+(a^2-1)^(1/2)))-I*dilog(((a^2-1)^(1/2)+b*x+I*(1-(b*x+a)^2)^(1
/2))/(-a+(a^2-1)^(1/2)))-I*dilog(((a^2-1)^(1/2)-b*x-I*(1-(b*x+a)^2)^(1/2))/
(a+(a^2-1)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arccos(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arccos(b*x + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)/x,x)
```

```
[Out] Integral(acos(a + b*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(b*x + a)/x, x)
```

3.29 $\int \frac{\cos^{-1}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{\sqrt{1-a^2}}-\frac{\cos^{-1}(a+bx)}{x}$$

```
[Out] -(ArcCos[a + b*x]/x) + (b*ArcTanh[(1 - a*(a + b*x))/(Sqrt[1 - a^2]*Sqrt[1 -
 (a + b*x)^2])])/Sqrt[1 - a^2]
```

Rubi [A] time = 0.075788, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4806, 4744, 725, 206}

$$\frac{b\tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{\sqrt{1-a^2}}-\frac{\cos^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]/x^2,x]
```

```
[Out] -(ArcCos[a + b*x]/x) + (b*ArcTanh[(1 - a*(a + b*x))/(Sqrt[1 - a^2]*Sqrt[1 -
 (a + b*x)^2])])/Sqrt[1 - a^2]
```

Rule 4806

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(e*(m + 1)), x] +
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1
))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(a+bx)}{x^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\cos^{-1}(a+bx)}{x} - \text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1-x^2}}\,dx, x, a+bx\right)\\
&= -\frac{\cos^{-1}(a+bx)}{x} + \text{Subst}\left(\int \frac{1}{\frac{1}{b^2}-\frac{a^2}{b^2}-x^2}\,dx, x, \frac{\frac{1}{b}-\frac{a(a+bx)}{b}}{\sqrt{1-(a+bx)^2}}\right)\\
&= -\frac{\cos^{-1}(a+bx)}{x} + \frac{b\tanh^{-1}\left(\frac{b\left(\frac{1}{b}-\frac{a(a+bx)}{b}\right)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{\sqrt{1-a^2}}
\end{aligned}$$

Mathematica [A] time = 0.0550105, size = 79, normalized size = 1.25

$$\frac{b\left(\log\left(\sqrt{1-a^2}\sqrt{-a^2-2abx-b^2x^2+1}-a^2-abx+1\right)-\log(x)\right)}{\sqrt{1-a^2}}-\frac{\cos^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]/x^2,x]
```

```
[Out] -(ArcCos[a + b*x]/x) + (b*(-Log[x] + Log[1 - a^2 - a*b*x + Sqrt[1 - a^2]*Sq
rt[1 - a^2 - 2*a*b*x - b^2*x^2]]))/Sqrt[1 - a^2]
```

Maple [A] time = 0.003, size = 77, normalized size = 1.2

$$-\frac{\arccos(bx+a)}{x}+b\ln\left(\frac{1}{bx}\left(-2a^2+2-2xab+2\sqrt{-a^2+1}\sqrt{-b^2x^2-2xab-a^2+1}\right)\right)\frac{1}{\sqrt{-a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)/x^2,x)
```

```
[Out] -arccos(b*x+a)/x+b/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b
^2*x^2-2*a*b*x-a^2+1)^(1/2))/b/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.01077, size = 857, normalized size = 13.6

$$\left[-\frac{\sqrt{-a^2+1}bx\log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx-2\sqrt{-b^2x^2-2abx-a^2+1}(abx+a^2-1)\sqrt{-a^2+1}-4a^2+2}{x^2}\right)+2(a^2-1)x\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2}}{b^2x^2+2abx+}\right.}{2(a^2-1)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + 1)*b*x*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*
x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1) -
 4*a^2 + 2)/x^2) + 2*(a^2 - 1)*x*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*
(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + 2*(a^2 - (a^2 - 1)*x - 1)*arccos
(b*x + a))/((a^2 - 1)*x), -(sqrt(a^2 - 1)*b*x*arctan(sqrt(-b^2*x^2 - 2*a*b*
x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(
```

```
a^3 - a)*b*x - 2*a^2 + 1)) + (a^2 - 1)*x*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (a^2 - (a^2 - 1)*x - 1)*
arccos(b*x + a))/((a^2 - 1)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)/x**2,x)
```

```
[Out] Integral(acos(a + b*x)/x**2, x)
```

Giac [A] time = 1.31559, size = 107, normalized size = 1.7

$$-\frac{2\,b^2\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}|b|}-\frac{\arccos(bx+a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] -2*b^2*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(b^2*x + a
*b) - 1)/sqrt(a^2 - 1))/(sqrt(a^2 - 1)*abs(b)) - arccos(b*x + a)/x
```

3.30 $\int \frac{\cos^{-1}(a+bx)}{x^3} dx$

Optimal. Leaf size=103

$$\frac{ab^2\tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{2\left(1-a^2\right)^{3/2}}+\frac{b\sqrt{1-(a+bx)^2}}{2\left(1-a^2\right)x}-\frac{\cos^{-1}(a+bx)}{2x^2}$$

```
[Out] (b*Sqrt[1 - (a + b*x)^2])/(2*(1 - a^2)*x) - ArcCos[a + b*x]/(2*x^2) + (a*b^
2*ArcTanh[(1 - a*(a + b*x))/(Sqrt[1 - a^2]*Sqrt[1 - (a + b*x)^2])])/(2*(1 -
 a^2)^(3/2))
```

Rubi [A] time = 0.111094, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4806, 4744, 731, 725, 206}

$$\frac{ab^2\tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{2\left(1-a^2\right)^{3/2}}+\frac{b\sqrt{1-(a+bx)^2}}{2\left(1-a^2\right)x}-\frac{\cos^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]/x^3,x]
```

```
[Out] (b*Sqrt[1 - (a + b*x)^2])/(2*(1 - a^2)*x) - ArcCos[a + b*x]/(2*x^2) + (a*b^
2*ArcTanh[(1 - a*(a + b*x))/(Sqrt[1 - a^2]*Sqrt[1 - (a + b*x)^2])])/(2*(1 -
 a^2)^(3/2))
```

Rule 4806

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(e*(m + 1)), x] +
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1
))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

```
&& NeQ[m, -1]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(a+bx)}{x^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\cos^{-1}(a+bx)}{2x^2}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1-x^2}}\,dx, x, a+bx\right)\\
&= \frac{b\sqrt{1-(a+bx)^2}}{2\left(1-a^2\right)x}-\frac{\cos^{-1}(a+bx)}{2x^2}-\frac{(ab)\,\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1-x^2}}\,dx, x, a+bx\right)}{2\left(1-a^2\right)}\\
&= \frac{b\sqrt{1-(a+bx)^2}}{2\left(1-a^2\right)x}-\frac{\cos^{-1}(a+bx)}{2x^2}+\frac{(ab)\,\text{Subst}\left(\int \frac{1}{\frac{1}{b^2}-\frac{a^2}{b^2}-x^2}\,dx, x, \frac{\frac{1}{b}-\frac{a(a+bx)}{b}}{\sqrt{1-(a+bx)^2}}\right)}{2\left(1-a^2\right)}\\
&= \frac{b\sqrt{1-(a+bx)^2}}{2\left(1-a^2\right)x}-\frac{\cos^{-1}(a+bx)}{2x^2}+\frac{ab^2\tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{2\left(1-a^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.174607, size = 126, normalized size = 1.22

$$-\frac{\cos^{-1}(a+bx)-\frac{bx\left(\sqrt{1-a^2}\sqrt{-a^2-2abx-b^2x^2+1}+abx\log\left(\sqrt{1-a^2}\sqrt{-a^2-2abx-b^2x^2+1}-a^2-abx+1\right)-abx\log(x)\right)}{\left(1-a^2\right)^{3/2}}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]/x^3,x]
```

```
[Out] -(ArcCos[a + b*x] - (b*x*(Sqrt[1 - a^2]*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] -
 a*b*x*Log[x] + a*b*x*Log[1 - a^2 - a*b*x + Sqrt[1 - a^2]*Sqrt[1 - a^2 - 2*
a*b*x - b^2*x^2]]))/(1 - a^2)^(3/2))/(2*x^2)
```

Maple [A] time = 0.004, size = 118, normalized size = 1.2

$$-\frac{\arccos(bx+a)}{2x^2}+\frac{b}{\left(-2a^2+2\right)x}\sqrt{-b^2x^2-2xab-a^2+1}+\frac{b^2a}{2}\ln\left(\frac{1}{bx}\left(-2a^2+2-2xab+2\sqrt{-a^2+1}\sqrt{-b^2x^2-2xab}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)/x^3,x)
```

```
[Out] -1/2*arccos(b*x+a)/x^2+1/2*b/(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/2*
b^2*a/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2))/b/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.94729, size = 1121, normalized size = 10.88

$$\left[-\frac{\sqrt{-a^2+1}ab^2x^2\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx+2\sqrt{-b^2x^2-2abx-a^2+1}\left(abx+a^2-1\right)\sqrt{-a^2+1}-4a^2+2}{x^2}\right)+2\left(a^4-2a^2+1\right)x^2\arctan\left(\cdots\right.}{4\left(a^4-2a\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a^2 + 1)*a*b^2*x^2*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 -
 a)*b*x + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2
+ 1) - 4*a^2 + 2)/x^2) + 2*(a^4 - 2*a^2 + 1)*x^2*arctan(sqrt(-b^2*x^2 - 2*a
*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + 2*sqrt(-b^2*x^2
- 2*a*b*x - a^2 + 1)*(a^2 - 1)*b*x + 2*(a^4 - (a^4 - 2*a^2 + 1)*x^2 - 2*a^2
 + 1)*arccos(b*x + a))/((a^4 - 2*a^2 + 1)*x^2), 1/2*(sqrt(a^2 - 1)*a*b^2*x^
2*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)
/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) - (a^4 - 2*a^2 +
1)*x^2*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b
*x + a^2 - 1)) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1)*b*x - (a^4 -
(a^4 - 2*a^2 + 1)*x^2 - 2*a^2 + 1)*arccos(b*x + a))/((a^4 - 2*a^2 + 1)*x^2)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)/x**3,x)
```

```
[Out] Integral(acos(a + b*x)/x**3, x)
```

Giac [B] time = 1.33583, size = 327, normalized size = 3.17

$$\left(\frac{ab^2\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\left(a^2|b|-|b|\right)\sqrt{a^2-1}}-\frac{ab^2-\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)b^2}{b^2x+ab}}{\left(a^3|b|-a|b|\right)\left(\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)^2a}{\left(b^2x+ab\right)^2}+a-\frac{2\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)}{b^2x+ab}\right)}\right)b-\frac{\arccos\left(b\right.}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] (a*b^2*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(b^2*x + a
*b) - 1)/sqrt(a^2 - 1))/((a^2*abs(b) - abs(b))*sqrt(a^2 - 1)) - (a*b^2 - (s
qrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*b^2/(b^2*x + a*b))/((a^3*abs(
b) - a*abs(b))*((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a/(b^2*x
+ a*b)^2 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a
*b))))*b - 1/2*arccos(b*x + a)/x^2
```

3.31 $\int \frac{\cos^{-1}(a+bx)}{x^4} dx$

Optimal. Leaf size=144

$$\frac{ab^2\sqrt{1-(a+bx)^2}}{2\left(1-a^2\right)^2 x}+\frac{\left(2a^2+1\right)b^3\tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{6\left(1-a^2\right)^{5/2}}+\frac{b\sqrt{1-(a+bx)^2}}{6\left(1-a^2\right)x^2}-\frac{\cos^{-1}(a+bx)}{3x^3}$$

```
[Out] (b*Sqrt[1 - (a + b*x)^2])/(6*(1 - a^2)*x^2) + (a*b^2*Sqrt[1 - (a + b*x)^2])
/(2*(1 - a^2)^2*x) - ArcCos[a + b*x]/(3*x^3) + ((1 + 2*a^2)*b^3*ArcTanh[(1
- a*(a + b*x))/(Sqrt[1 - a^2]*Sqrt[1 - (a + b*x)^2])])/(6*(1 - a^2)^(5/2))
```

Rubi [A] time = 0.179291, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4806, 4744, 745, 807, 725, 206}

$$\frac{ab^2\sqrt{1-(a+bx)^2}}{2\left(1-a^2\right)^2 x}+\frac{\left(2a^2+1\right)b^3\tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{6\left(1-a^2\right)^{5/2}}+\frac{b\sqrt{1-(a+bx)^2}}{6\left(1-a^2\right)x^2}-\frac{\cos^{-1}(a+bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]/x^4,x]
```

```
[Out] (b*Sqrt[1 - (a + b*x)^2])/(6*(1 - a^2)*x^2) + (a*b^2*Sqrt[1 - (a + b*x)^2])
/(2*(1 - a^2)^2*x) - ArcCos[a + b*x]/(3*x^3) + ((1 + 2*a^2)*b^3*ArcTanh[(1
- a*(a + b*x))/(Sqrt[1 - a^2]*Sqrt[1 - (a + b*x)^2])])/(6*(1 - a^2)^(5/2))
```

Rule 4806

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(e*(m + 1)), x] +
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1
))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

```
&& NeQ[m, -1]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{-1}(a+bx)}{x^4}\,dx = \frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\cos^{-1}(a+bx)}{3x^3} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\sqrt{1-x^2}}\,dx, x, a+bx\right)$$

$$= \frac{b\sqrt{1-(a+bx)^2}}{6\left(1-a^2\right)x^2} - \frac{\cos^{-1}(a+bx)}{3x^3} - \frac{b^2\,\text{Subst}\left(\int \frac{\frac{2a}{b}+\frac{x}{b}}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1-x^2}}\,dx, x, a+bx\right)}{6\left(1-a^2\right)}$$

$$= \frac{b\sqrt{1-(a+bx)^2}}{6\left(1-a^2\right)x^2} + \frac{ab^2\sqrt{1-(a+bx)^2}}{2\left(1-a^2\right)^2 x} - \frac{\cos^{-1}(a+bx)}{3x^3} - \frac{\left(\left(1+2a^2\right)b^2\right)\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1-x^2}}\,dx\right.}{6\left(1-a^2\right)^2}$$

$$= \frac{b\sqrt{1-(a+bx)^2}}{6\left(1-a^2\right)x^2} + \frac{ab^2\sqrt{1-(a+bx)^2}}{2\left(1-a^2\right)^2 x} - \frac{\cos^{-1}(a+bx)}{3x^3} + \frac{\left(\left(1+2a^2\right)b^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{b^2}-\frac{a^2}{b^2}-x^2}\,dx, x,\right.}{6\left(1-a^2\right)^2}$$

$$= \frac{b\sqrt{1-(a+bx)^2}}{6\left(1-a^2\right)x^2} + \frac{ab^2\sqrt{1-(a+bx)^2}}{2\left(1-a^2\right)^2 x} - \frac{\cos^{-1}(a+bx)}{3x^3} + \frac{\left(1+2a^2\right)b^3\tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{6\left(1-a^2\right)^{5/2}}$$

Mathematica [A] time = 0.184182, size = 168, normalized size = 1.17

$$\frac{\sqrt{1-a^2}bx\left(-a^2+3abx+1\right)\sqrt{-a^2-2abx-b^2x^2+1}-\left(2a^2+1\right)b^3x^3\log(x)+\left(2a^2+1\right)b^3x^3\log\left(\sqrt{1-a^2}\sqrt{-a^2-2abx\right.}{6\left(1-a^2\right)^{5/2}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]/x^4,x]
```

```
[Out] (Sqrt[1 - a^2]*b*x*(1 - a^2 + 3*a*b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] -
2*(1 - a^2)^(5/2)*ArcCos[a + b*x] - (1 + 2*a^2)*b^3*x^3*Log[x] + (1 + 2*a^2
)*b^3*x^3*Log[1 - a^2 - a*b*x + Sqrt[1 - a^2]*Sqrt[1 - a^2 - 2*a*b*x - b^2*
x^2]])/(6*(1 - a^2)^(5/2)*x^3)
```

Maple [A] time = 0.005, size = 227, normalized size = 1.6

$$-\frac{\arccos(bx+a)}{3x^3}+\frac{b}{(-6a^2+6)x^2}\sqrt{-b^2x^2-2xab-a^2+1}+\frac{b^2a}{2(-a^2+1)^2x}\sqrt{-b^2x^2-2xab-a^2+1}+\frac{b^3a^2}{2}\ln\left(\frac{1}{bx}\left(-2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)/x^4,x)
```

```
[Out] -1/3*arccos(b*x+a)/x^3+1/6*b/(-a^2+1)/x^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/
2*b^2*a/(-a^2+1)^2/x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/2*b^3*a^2/(-a^2+1)^(5
/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/b
/x)+1/6*b^3/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*x*a*b+2*(-a^2+1)^(1/2)*(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2))/b/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.54357, size = 1328, normalized size = 9.22

$$\left[-\frac{(2a^2+1)\sqrt{-a^2+1}b^3x^3\log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx-2\sqrt{-b^2x^2-2abx-a^2+1}(abx+a^2-1)\sqrt{-a^2+1}-4a^2+2}{x^2}\right)+4\left(a^6-3a^4+3a^2-1\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/12*((2*a^2 + 1)*sqrt(-a^2 + 1)*b^3*x^3*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4
 + 4*(a^3 - a)*b*x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)
```

```
*sqrt(-a^2 + 1) - 4*a^2 + 2)/x^2) + 4*(a^6 - 3*a^4 + 3*a^2 - 1)*x^3*arctan(
sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1))
 + 4*(a^6 - 3*a^4 - (a^6 - 3*a^4 + 3*a^2 - 1)*x^3 + 3*a^2 - 1)*arccos(b*x +
 a) - 2*(3*(a^3 - a)*b^2*x^2 - (a^4 - 2*a^2 + 1)*b*x)*sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1))/((a^6 - 3*a^4 + 3*a^2 - 1)*x^3), -1/6*((2*a^2 + 1)*sqrt(a^2
- 1)*b^3*x^3*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sq
rt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + 2*(a
^6 - 3*a^4 + 3*a^2 - 1)*x^3*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x
+ a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + 2*(a^6 - 3*a^4 - (a^6 - 3*a^4 + 3*a^2
 - 1)*x^3 + 3*a^2 - 1)*arccos(b*x + a) - (3*(a^3 - a)*b^2*x^2 - (a^4 - 2*a^
2 + 1)*b*x)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^6 - 3*a^4 + 3*a^2 - 1)*
x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}(a + bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)/x**4,x)
```

```
[Out] Integral(acos(a + b*x)/x**4, x)
```

Giac [B] time = 1.35558, size = 752, normalized size = 5.22

$$-\frac{1}{3}\,b\left(\frac{\left(2\,a^2b^3 + b^3\right)\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\left(a^4|b| - 2\,a^2|b| + |b|\right)\sqrt{a^2-1}} - \frac{\frac{4\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)^2a^4b^3}{\left(b^2x+ab\right)^2} + 4\,a^4b^3 - \frac{11\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)a^3b^3}{b^2x+ab}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*b*((2*a^2*b^3 + b^3)*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b
) + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/((a^4*abs(b) - 2*a^2*abs(b) + ab
```

```
s(b))*sqrt(a^2 - 1)) - (4*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2
*a^4*b^3/(b^2*x + a*b)^2 + 4*a^4*b^3 - 11*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
1)*abs(b) + b)*a^3*b^3/(b^2*x + a*b) - 5*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1
)*abs(b) + b)^3*a^3*b^3/(b^2*x + a*b)^3 + 7*(sqrt(-b^2*x^2 - 2*a*b*x - a^2
+ 1)*abs(b) + b)^2*a^2*b^3/(b^2*x + a*b)^2 - a^2*b^3 + 2*(sqrt(-b^2*x^2 - 2
*a*b*x - a^2 + 1)*abs(b) + b)*a*b^3/(b^2*x + a*b) + 2*(sqrt(-b^2*x^2 - 2*a*
b*x - a^2 + 1)*abs(b) + b)^3*a*b^3/(b^2*x + a*b)^3 - 2*(sqrt(-b^2*x^2 - 2*a
*b*x - a^2 + 1)*abs(b) + b)^2*b^3/(b^2*x + a*b)^2)/((a^6*abs(b) - 2*a^4*abs
(b) + a^2*abs(b))*((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a/(b^2
*x + a*b)^2 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x
+ a*b))^2)) - 1/3*arccos(b*x + a)/x^3
```

3.32 $\int \cos^{-1}(a+bx)^3\,dx$

Optimal. Leaf size=82

$$\frac{6\sqrt{1-(a+bx)^2}}{b}+\frac{(a+bx)\cos^{-1}(a+bx)^3}{b}-\frac{3\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)^2}{b}-\frac{6(a+bx)\cos^{-1}(a+bx)}{b}$$

```
[Out] (6*Sqrt[1 - (a + b*x)^2])/b - (6*(a + b*x)*ArcCos[a + b*x])/b - (3*Sqrt[1 -
 (a + b*x)^2]*ArcCos[a + b*x]^2)/b + ((a + b*x)*ArcCos[a + b*x]^3)/b
```

Rubi [A] time = 0.0820063, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4804, 4620, 4678, 261}

$$\frac{6\sqrt{1-(a+bx)^2}}{b}+\frac{(a+bx)\cos^{-1}(a+bx)^3}{b}-\frac{3\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)^2}{b}-\frac{6(a+bx)\cos^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]^3,x]
```


```
[Out] (6*Sqrt[1 - (a + b*x)^2])/b - (6*(a + b*x)*ArcCos[a + b*x])/b - (3*Sqrt[1 -
 (a + b*x)^2]*ArcCos[a + b*x]^2)/b + ((a + b*x)*ArcCos[a + b*x]^3)/b
```


Rule 4804

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
```

```
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^{-1}(a+bx)^3\,dx &= \frac{\text{Subst}\left(\int \cos^{-1}(x)^3\,dx,x,a+bx\right)}{b}\\ &= \frac{(a+bx)\cos^{-1}(a+bx)^3}{b}+\frac{3\,\text{Subst}\left(\int \frac{x\cos^{-1}(x)^2}{\sqrt{1-x^2}}\,dx,x,a+bx\right)}{b}\\ &= -\frac{3\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)^2}{b}+\frac{(a+bx)\cos^{-1}(a+bx)^3}{b}-\frac{6\,\text{Subst}\left(\int \cos^{-1}(x)\,dx,x,a+bx\right)}{b}\\ &= -\frac{6(a+bx)\cos^{-1}(a+bx)}{b}-\frac{3\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)^2}{b}+\frac{(a+bx)\cos^{-1}(a+bx)^3}{b}-\frac{6\,\text{Subst}}{}\\ &= \frac{6\sqrt{1-(a+bx)^2}}{b}-\frac{6(a+bx)\cos^{-1}(a+bx)}{b}-\frac{3\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)^2}{b}+\frac{(a+bx)\cos^{-1}(a-}{b}\end{aligned}$$

Mathematica [A] time = 0.0356647, size = 74, normalized size = 0.9

$$\frac{6\sqrt{1-(a+bx)^2}+(a+bx)\cos^{-1}(a+bx)^3-3\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)^2-6(a+bx)\cos^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]^3,x]
```

```
[Out] (6*Sqrt[1 - (a + b*x)^2] - 6*(a + b*x)*ArcCos[a + b*x] - 3*Sqrt[1 - (a + b*
x)^2]*ArcCos[a + b*x]^2 + (a + b*x)*ArcCos[a + b*x]^3)/b
```

Maple [A] time = 0.054, size = 71, normalized size = 0.9

$$\frac{1}{b}\left((\arccos(bx+a))^3(bx+a)-3(\arccos(bx+a))^2\sqrt{1-(bx+a)^2}+6\sqrt{1-(bx+a)^2}-6(bx+a)\arccos(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)^3,x)
```

```
[Out] 1/b*(arccos(b*x+a)^3*(b*x+a)-3*arccos(b*x+a)^2*(1-(b*x+a)^2)^(1/2)+6*(1-(b*
x+a)^2)^(1/2)-6*(b*x+a)*arccos(b*x+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.55929, size = 170, normalized size = 2.07

$$\frac{(bx+a)\arccos(bx+a)^3 - 6(bx+a)\arccos(bx+a) - 3\sqrt{-b^2x^2 - 2abx - a^2 + 1}\left(\arccos(bx+a)^2 - 2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*arccos(b*x + a)^3 - 6*(b*x + a)*arccos(b*x + a) - 3*sqrt(-b^2*x^
2 - 2*a*b*x - a^2 + 1)*(arccos(b*x + a)^2 - 2))/b
```

Sympy [A] time = 0.692653, size = 109, normalized size = 1.33

$$\begin{cases} \frac{a\operatorname{acos}^3(a+bx)}{b} - \frac{6a\operatorname{acos}(a+bx)}{b} + x\operatorname{acos}^3(a+bx) - 6x\operatorname{acos}(a+bx) - \frac{3\sqrt{-a^2-2abx-b^2x^2+1}\operatorname{acos}^2(a+bx)}{b} + \frac{6\sqrt{-a^2-2abx-b^2x^2+1}}{b} & \text{fo} \\ x\operatorname{acos}^3(a) & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)**3,x)
```

```
[Out] Piecewise((a*acos(a + b*x)**3/b - 6*a*acos(a + b*x)/b + x*acos(a + b*x)**3
- 6*x*acos(a + b*x) - 3*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)*acos(a + b*x)
**2/b + 6*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/b, Ne(b, 0)), (x*acos(a)**3
, True))
```

Giac [A] time = 1.42999, size = 105, normalized size = 1.28

$$\frac{(bx+a)\arccos(bx+a)^3}{b} - \frac{3\sqrt{-(bx+a)^2+1}\arccos(bx+a)^2}{b} - \frac{6(bx+a)\arccos(bx+a)}{b} + \frac{6\sqrt{-(bx+a)^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] (b*x + a)*arccos(b*x + a)^3/b - 3*sqrt(-(b*x + a)^2 + 1)*arccos(b*x + a)^2/
b - 6*(b*x + a)*arccos(b*x + a)/b + 6*sqrt(-(b*x + a)^2 + 1)/b
```

3.33 $\int \cos^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=47

$$\frac{(a+bx)\cos^{-1}(a+bx)^2}{b}-\frac{2\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)}{b}-2x$$

```
[Out] -2*x - (2*Sqrt[1 - (a + b*x)^2]*ArcCos[a + b*x])/b + ((a + b*x)*ArcCos[a +
b*x]^2)/b
```

Rubi [A] time = 0.0544189, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4804, 4620, 4678, 8}

$$\frac{(a+bx)\cos^{-1}(a+bx)^2}{b}-\frac{2\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)}{b}-2x$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]^2,x]
```

```
[Out] -2*x - (2*Sqrt[1 - (a + b*x)^2]*ArcCos[a + b*x])/b + ((a + b*x)*ArcCos[a +
b*x]^2)/b
```

Rule 4804

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
```

```
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{-1}(a+bx)^2\,dx &= \frac{\text{Subst}\left(\int \cos^{-1}(x)^2\,dx, x, a+bx\right)}{b}\\ &= \frac{(a+bx)\cos^{-1}(a+bx)^2}{b} + \frac{2\,\text{Subst}\left(\int \frac{x\cos^{-1}(x)}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{b}\\ &= -\frac{2\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)}{b} + \frac{(a+bx)\cos^{-1}(a+bx)^2}{b} - \frac{2\,\text{Subst}(\int 1\,dx, x, a+bx)}{b}\\ &= -2x - \frac{2\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)}{b} + \frac{(a+bx)\cos^{-1}(a+bx)^2}{b}\end{aligned}$$

Mathematica [A] time = 0.0228701, size = 49, normalized size = 1.04

$$\frac{-2(a+bx) + (a+bx)\cos^{-1}(a+bx)^2 - 2\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]^2,x]
```

```
[Out] (-2*(a + b*x) - 2*Sqrt[1 - (a + b*x)^2]*ArcCos[a + b*x] + (a + b*x)*ArcCos[
a + b*x]^2)/b
```

Maple [A] time = 0.049, size = 48, normalized size = 1.

$$\frac{1}{b}\left((\arccos(bx+a))^2(bx+a) - 2\,bx - 2\,a - 2\,\arccos(bx+a)\sqrt{1-(bx+a)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)^2,x)
```

```
[Out] 1/b*(arccos(b*x+a)^2*(b*x+a)-2*b*x-2*a-2*arccos(b*x+a)*(1-(b*x+a)^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.33554, size = 130, normalized size = 2.77

$$\frac{(bx+a)\arccos(bx+a)^2 - 2\,bx - 2\,\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}\arccos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*arccos(b*x + a)^2 - 2*b*x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
*arccos(b*x + a))/b
```

Sympy [A] time = 0.282811, size = 63, normalized size = 1.34

$$\begin{cases} \frac{a\,\mathrm{acos}^2(a+bx)}{b} + x\,\mathrm{acos}^2(a+bx) - 2x - \frac{2\sqrt{-a^2-2abx-b^2x^2+1}\,\mathrm{acos}(a+bx)}{b} & \text{for } b \neq 0 \\ x\,\mathrm{acos}^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)**2,x)
```

```
[Out] Piecewise((a*acos(a + b*x)**2/b + x*acos(a + b*x)**2 - 2*x - 2*sqrt(-a**2 -
 2*a*b*x - b**2*x**2 + 1)*acos(a + b*x)/b, Ne(b, 0)), (x*acos(a)**2, True))
```

Giac [A] time = 1.34207, size = 70, normalized size = 1.49

$$\frac{(bx+a)\arccos(bx+a)^2}{b} - \frac{2\sqrt{-(bx+a)^2+1}\arccos(bx+a)}{b} - \frac{2(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b*x + a)*arccos(b*x + a)^2/b - 2*sqrt(-(b*x + a)^2 + 1)*arccos(b*x + a)/b
- 2*(b*x + a)/b
```

3.34 $\int \frac{1}{\cos^{-1}(a+bx)}\, dx$

Optimal. Leaf size=12

$$-\frac{\text{Si}\left(\cos^{-1}(a+bx)\right)}{b}$$

```
[Out] -(SinIntegral[ArcCos[a + b*x]]/b)
```

Rubi [A] time = 0.0215678, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4804, 4624, 3299}

$$-\frac{\text{Si}\left(\cos^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]^(-1),x]
```

```
[Out] -(SinIntegral[ArcCos[a + b*x]]/b)
```

Rule 4804

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \cos^{-1}(a+bx)\right)}{b}$$
$$= -\frac{\text{Si}\left(\cos^{-1}(a+bx)\right)}{b}$$

Mathematica [A] time = 0.0259629, size = 12, normalized size = 1.

$$-\frac{\text{Si}\left(\cos^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]^(-1),x]
```

```
[Out] -(SinIntegral[ArcCos[a + b*x]]/b)
```

Maple [A] time = 0.046, size = 13, normalized size = 1.1

$$-\frac{\text{Si}\left(\arccos\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(b*x+a),x)
```

```
[Out] -Si(arccos(b*x+a))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\arccos\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(1/arccos(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\arccos(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(1/arccos(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acos}(a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(b*x+a),x)
```

```
[Out] Integral(1/acos(a + b*x), x)
```

Giac [A] time = 1.34309, size = 16, normalized size = 1.33

$$-\frac{\operatorname{Si}(\arccos(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a),x, algorithm="giac")
```

```
[Out] -sin_integral(arccos(b*x + a))/b
```

3.35 $\int \frac{1}{\cos^{-1}(a+bx)^2} \, dx$

Optimal. Leaf size=40

$$\frac{\sqrt{1-(a+bx)^2}}{b\cos^{-1}(a+bx)} - \frac{\text{CosIntegral}\left(\cos^{-1}(a+bx)\right)}{b}$$

```
[Out] Sqrt[1 - (a + b*x)^2]/(b*ArcCos[a + b*x]) - CosIntegral[ArcCos[a + b*x]]/b
```

Rubi [A] time = 0.0784772, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4804, 4622, 4724, 3302}

$$\frac{\sqrt{1-(a+bx)^2}}{b\cos^{-1}(a+bx)} - \frac{\text{CosIntegral}\left(\cos^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]^(-2),x]
```

```
[Out] Sqrt[1 - (a + b*x)^2]/(b*ArcCos[a + b*x]) - CosIntegral[ArcCos[a + b*x]]/b
```

Rule 4804

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 4622

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1))
, Int[(x*(a + b*ArcCos[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && LtQ[n, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
```

```
rQ[p] || GtQ[d, 0])
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{-1}(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)^2}\,dx, x, a+bx\right)}{b}\\
&= \frac{\sqrt{1-(a+bx)^2}}{b\cos^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\cos^{-1}(x)}\,dx, x, a+bx\right)}{b}\\
&= \frac{\sqrt{1-(a+bx)^2}}{b\cos^{-1}(a+bx)} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \cos^{-1}(a+bx)\right)}{b}\\
&= \frac{\sqrt{1-(a+bx)^2}}{b\cos^{-1}(a+bx)} - \frac{\text{Ci}\left(\cos^{-1}(a+bx)\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0508103, size = 40, normalized size = 1.

$$\frac{\sqrt{1-(a+bx)^2}}{b\cos^{-1}(a+bx)} - \frac{\text{CosIntegral}\left(\cos^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]^(-2),x]
```

```
[Out] Sqrt[1 - (a + b*x)^2]/(b*ArcCos[a + b*x]) - CosIntegral[ArcCos[a + b*x]]/b
```

Maple [A] time = 0.049, size = 37, normalized size = 0.9

$$\frac{1}{b}\left(\frac{1}{\arccos(bx+a)}\sqrt{1-(bx+a)^2} - \text{Ci}\left(\arccos(bx+a)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(b*x+a)^2,x)
```

```
[Out] 1/b*(1/arccos(b*x+a)*(1-(b*x+a)^2)^(1/2)-Ci(arccos(b*x+a)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\arccos\left(bx+a\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(arccos(b*x + a)^(-2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acos}^2\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(b*x+a)**2,x)
```

```
[Out] Integral(acos(a + b*x)**(-2), x)
```

Giac [A] time = 1.32966, size = 51, normalized size = 1.27

$$-\frac{\operatorname{Ci}\left(\arccos\left(bx+a\right)\right)}{b}+\frac{\sqrt{-(bx+a)^{2}+1}}{b\arccos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -cos_integral(arccos(b*x + a))/b + sqrt(-(b*x + a)^2 + 1)/(b*arccos(b*x + a
))
```

3.36 $\int \frac{1}{\cos^{-1}(a+bx)^3}\,dx$

Optimal. Leaf size=65

$$\frac{\operatorname{Si}\left(\cos^{-1}(a+bx)\right)}{2b}+\frac{a+bx}{2b\cos^{-1}(a+bx)}+\frac{\sqrt{1-(a+bx)^2}}{2b\cos^{-1}(a+bx)^2}$$

```
[Out] Sqrt[1 - (a + b*x)^2]/(2*b*ArcCos[a + b*x]^2) + (a + b*x)/(2*b*ArcCos[a + b
*x]) + SinIntegral[ArcCos[a + b*x]]/(2*b)
```

Rubi [A] time = 0.08065, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4804, 4622, 4720, 4624, 3299}

$$\frac{\operatorname{Si}\left(\cos^{-1}(a+bx)\right)}{2b}+\frac{a+bx}{2b\cos^{-1}(a+bx)}+\frac{\sqrt{1-(a+bx)^2}}{2b\cos^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]^(-3),x]
```

```
[Out] Sqrt[1 - (a + b*x)^2]/(2*b*ArcCos[a + b*x]^2) + (a + b*x)/(2*b*ArcCos[a + b
*x]) + SinIntegral[ArcCos[a + b*x]]/(2*b)
```

Rule 4804

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 4622

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1))
, Int[(x*(a + b*ArcCos[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && LtQ[n, -1]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
```

```
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{-1}(a+bx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)^3}\,dx, x, a+bx\right)}{b}\\
&= \frac{\sqrt{1-(a+bx)^2}}{2b\cos^{-1}(a+bx)^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\cos^{-1}(x)^2}\,dx, x, a+bx\right)}{2b}\\
&= \frac{\sqrt{1-(a+bx)^2}}{2b\cos^{-1}(a+bx)^2} + \frac{a+bx}{2b\cos^{-1}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)}\,dx, x, a+bx\right)}{2b}\\
&= \frac{\sqrt{1-(a+bx)^2}}{2b\cos^{-1}(a+bx)^2} + \frac{a+bx}{2b\cos^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \cos^{-1}(a+bx)\right)}{2b}\\
&= \frac{\sqrt{1-(a+bx)^2}}{2b\cos^{-1}(a+bx)^2} + \frac{a+bx}{2b\cos^{-1}(a+bx)} + \frac{\text{Si}\left(\cos^{-1}(a+bx)\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.0501086, size = 65, normalized size = 1.

$$\frac{\text{Si}\left(\cos^{-1}(a+bx)\right)}{2b} + \frac{a+bx}{2b\cos^{-1}(a+bx)} + \frac{\sqrt{1-(a+bx)^2}}{2b\cos^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]^(-3),x]
```

```
[Out] Sqrt[1 - (a + b*x)^2]/(2*b*ArcCos[a + b*x]^2) + (a + b*x)/(2*b*ArcCos[a + b
*x]) + SinIntegral[ArcCos[a + b*x]]/(2*b)
```

Maple [A] time = 0.051, size = 53, normalized size = 0.8

$$\frac{1}{b}\left(\frac{1}{2\,(\arccos(bx+a))^2}\sqrt{1-(bx+a)^2}+\frac{bx+a}{2\,\arccos(bx+a)}+\frac{\mathrm{Si}\,(\arccos(bx+a))}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(b*x+a)^3,x)
```

```
[Out] 1/b*(1/2/arccos(b*x+a)^2*(1-(b*x+a)^2)^(1/2)+1/2/arccos(b*x+a)*(b*x+a)+1/2*
Si(arccos(b*x+a)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\arccos(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(arccos(b*x + a)^(-3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acos}^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(b*x+a)**3,x)
```

```
[Out] Integral(acos(a + b*x)**(-3), x)
```

Giac [A] time = 1.29833, size = 77, normalized size = 1.18

$$\frac{\operatorname{Si}(\arccos(bx+a))}{2b}+\frac{bx+a}{2b\arccos(bx+a)}+\frac{\sqrt{-(bx+a)^2+1}}{2b\arccos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*sin_integral(arccos(b*x + a))/b + 1/2*(b*x + a)/(b*arccos(b*x + a)) + 1
/2*sqrt(-(b*x + a)^2 + 1)/(b*arccos(b*x + a)^2)
```

3.37 $\int \cos^{-1}(a+bx)^{5/2}\,dx$

Optimal. Leaf size=111

$$\frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{4b}+\frac{(a+bx)\cos^{-1}(a+bx)^{5/2}}{b}-\frac{5\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)^{3/2}}{2b}-\frac{15(a+bx)\sqrt{\cos}}{4b}$$

```
[Out] (-15*(a + b*x)*Sqrt[ArcCos[a + b*x]])/(4*b) - (5*Sqrt[1 - (a + b*x)^2]*ArcC
os[a + b*x]^(3/2))/(2*b) + ((a + b*x)*ArcCos[a + b*x]^(5/2))/b + (15*Sqrt[P
i/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a + b*x]]])/(4*b)
```

Rubi [A] time = 0.14736, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4804, 4620, 4678, 4724, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{4b}+\frac{(a+bx)\cos^{-1}(a+bx)^{5/2}}{b}-\frac{5\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)^{3/2}}{2b}-\frac{15(a+bx)\sqrt{\cos}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]^(5/2),x]
```


```
[Out] (-15*(a + b*x)*Sqrt[ArcCos[a + b*x]])/(4*b) - (5*Sqrt[1 - (a + b*x)^2]*ArcC
os[a + b*x]^(3/2))/(2*b) + ((a + b*x)*ArcCos[a + b*x]^(5/2))/b + (15*Sqrt[P
i/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a + b*x]]])/(4*b)
```


Rule 4804

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \cos^{-1}(a+bx)^{5/2}\,dx = \frac{\text{Subst}\left(\int \cos^{-1}(x)^{5/2}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\cos^{-1}(a+bx)^{5/2}}{b} + \frac{5\,\text{Subst}\left(\int \frac{x\cos^{-1}(x)^{3/2}}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{2b}$$

$$= -\frac{5\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)^{3/2}}{2b} + \frac{(a+bx)\cos^{-1}(a+bx)^{5/2}}{b} - \frac{15\,\text{Subst}\left(\int \sqrt{\cos^{-1}(x)}\,dx, x, a\right.}{4b}$$

$$= -\frac{15(a+bx)\sqrt{\cos^{-1}(a+bx)}}{4b} - \frac{5\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)^{3/2}}{2b} + \frac{(a+bx)\cos^{-1}(a+bx)^{5/2}}{b} - \frac{15}{}$$

$$= -\frac{15(a+bx)\sqrt{\cos^{-1}(a+bx)}}{4b} - \frac{5\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)^{3/2}}{2b} + \frac{(a+bx)\cos^{-1}(a+bx)^{5/2}}{b} + \frac{15}{}$$

$$= -\frac{15(a+bx)\sqrt{\cos^{-1}(a+bx)}}{4b} - \frac{5\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)^{3/2}}{2b} + \frac{(a+bx)\cos^{-1}(a+bx)^{5/2}}{b} + \frac{15}{}$$

$$= -\frac{15(a+bx)\sqrt{\cos^{-1}(a+bx)}}{4b} - \frac{5\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)^{3/2}}{2b} + \frac{(a+bx)\cos^{-1}(a+bx)^{5/2}}{b} + \frac{15}{}$$

Mathematica [C] time = 0.0488174, size = 90, normalized size = 0.81

$$-\frac{\frac{\sqrt{\cos^{-1}(a+bx)}\text{Gamma}\left(\frac{7}{2},-i\cos^{-1}(a+bx)\right)}{2\sqrt{-i\cos^{-1}(a+bx)}} + \frac{\sqrt{\cos^{-1}(a+bx)}\text{Gamma}\left(\frac{7}{2},i\cos^{-1}(a+bx)\right)}{2\sqrt{i\cos^{-1}(a+bx)}}}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a + b*x]^(5/2),x]
```

```
[Out] -(((Sqrt[ArcCos[a + b*x]]*Gamma[7/2, (-I)*ArcCos[a + b*x]])/(2*Sqrt[(-I)*Ar
cCos[a + b*x]]) + (Sqrt[ArcCos[a + b*x]]*Gamma[7/2, I*ArcCos[a + b*x]])/(2*
Sqrt[I*ArcCos[a + b*x]]))/b)
```

Maple [A] time = 0.092, size = 140, normalized size = 1.3

$$-\frac{\sqrt{2}}{8b\sqrt{\pi}}\left(-4\,(\arccos(bx+a))^{5/2}\sqrt{2}\sqrt{\pi}xb - 4\,(\arccos(bx+a))^{5/2}\sqrt{2}\sqrt{\pi}a + 10\,(\arccos(bx+a))^{3/2}\sqrt{2}\sqrt{\pi}\sqrt{-b^2x^2-2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)^(5/2),x)
```

```
[Out] -1/8/b*2^(1/2)*(-4*arccos(b*x+a)^(5/2)*2^(1/2)*Pi^(1/2)*x*b-4*arccos(b*x+a)
^(5/2)*2^(1/2)*Pi^(1/2)*a+10*arccos(b*x+a)^(3/2)*2^(1/2)*Pi^(1/2)*(-b^2*x^2
-2*a*b*x-a^2+1)^(1/2)+15*2^(1/2)*arccos(b*x+a)^(1/2)*Pi^(1/2)*x*b+15*2^(1/2
)*arccos(b*x+a)^(1/2)*Pi^(1/2)*a-15*Pi*FresnelC(2^(1/2)/Pi^(1/2)*arccos(b*x
+a)^(1/2)))/Pi^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.57023, size = 282, normalized size = 2.54

$$\frac{5\,i\arccos\left(bx+a\right)^{\frac{3}{2}}e^{\left(i\arccos\left(bx+a\right)\right)}}{4\,b}+\frac{\arccos\left(bx+a\right)^{\frac{5}{2}}e^{\left(i\arccos\left(bx+a\right)\right)}}{2\,b}-\frac{5\,i\arccos\left(bx+a\right)^{\frac{3}{2}}e^{\left(-i\arccos\left(bx+a\right)\right)}}{4\,b}+\frac{\arccos\left(bx-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5/4*i*arccos(b*x + a)^(3/2)*e^(i*arccos(b*x + a))/b + 1/2*arccos(b*x + a)^(
5/2)*e^(i*arccos(b*x + a))/b - 5/4*i*arccos(b*x + a)^(3/2)*e^(-i*arccos(b*x
 + a))/b + 1/2*arccos(b*x + a)^(5/2)*e^(-i*arccos(b*x + a))/b - 15/16*sqrt(
2)*sqrt(pi)*i*erf(sqrt(2)*sqrt(arccos(b*x + a))/(i - 1))/(b*(i - 1)) - 15/8
*sqrt(arccos(b*x + a))*e^(i*arccos(b*x + a))/b - 15/8*sqrt(arccos(b*x + a))
*e^(-i*arccos(b*x + a))/b + 15/16*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*i*sqrt(arcc
os(b*x + a))/(i - 1))/(b*(i - 1))
```

3.38 $\int \cos^{-1}(a+bx)^{3/2}\,dx$

Optimal. Leaf size=89

$$\frac{3\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{2b}+\frac{(a+bx)\cos^{-1}(a+bx)^{3/2}}{b}-\frac{3\sqrt{1-(a+bx)^2}\sqrt{\cos^{-1}(a+bx)}}{2b}$$

```
[Out] (-3*Sqrt[1 - (a + b*x)^2]*Sqrt[ArcCos[a + b*x]])/(2*b) + ((a + b*x)*ArcCos[
a + b*x]^(3/2))/b + (3*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a + b*x]]
])/(2*b)
```

Rubi [A] time = 0.09211, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4804, 4620, 4678, 4624, 3305, 3351}

$$\frac{3\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{2b}+\frac{(a+bx)\cos^{-1}(a+bx)^{3/2}}{b}-\frac{3\sqrt{1-(a+bx)^2}\sqrt{\cos^{-1}(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]^(3/2),x]
```


```
[Out] (-3*Sqrt[1 - (a + b*x)^2]*Sqrt[ArcCos[a + b*x]])/(2*b) + ((a + b*x)*ArcCos[
a + b*x]^(3/2))/b + (3*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a + b*x]]
])/(2*b)
```


Rule 4804

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \cos^{-1}(a+bx)^{3/2}\,dx = \frac{\text{Subst}\left(\int \cos^{-1}(x)^{3/2}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\cos^{-1}(a+bx)^{3/2}}{b} + \frac{3\,\text{Subst}\left(\int \frac{x\sqrt{\cos^{-1}(x)}}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{2b}$$

$$= -\frac{3\sqrt{1-(a+bx)^2}\sqrt{\cos^{-1}(a+bx)}}{2b} + \frac{(a+bx)\cos^{-1}(a+bx)^{3/2}}{b} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{\cos^{-1}(x)}}\,dx, x, a+b\right.}{4b}$$

$$= -\frac{3\sqrt{1-(a+bx)^2}\sqrt{\cos^{-1}(a+bx)}}{2b} + \frac{(a+bx)\cos^{-1}(a+bx)^{3/2}}{b} + \frac{3\,\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(a\right.}{4b}$$

$$= -\frac{3\sqrt{1-(a+bx)^2}\sqrt{\cos^{-1}(a+bx)}}{2b} + \frac{(a+bx)\cos^{-1}(a+bx)^{3/2}}{b} + \frac{3\,\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\cos}\right.}{2b}$$

$$= -\frac{3\sqrt{1-(a+bx)^2}\sqrt{\cos^{-1}(a+bx)}}{2b} + \frac{(a+bx)\cos^{-1}(a+bx)^{3/2}}{b} + \frac{3\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{2b}$$

Mathematica [C] time = 0.0353963, size = 76, normalized size = 0.85

$$-\frac{\sqrt{-i\cos^{-1}(a+bx)}\text{Gamma}\left(\frac{5}{2},-i\cos^{-1}(a+bx)\right)+\sqrt{i\cos^{-1}(a+bx)}\text{Gamma}\left(\frac{5}{2},i\cos^{-1}(a+bx)\right)}{2b\sqrt{\cos^{-1}(a+bx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a + b*x]^(3/2),x]
```

```
[Out] -(Sqrt[(-I)*ArcCos[a + b*x]]*Gamma[5/2, (-I)*ArcCos[a + b*x]] + Sqrt[I*ArcC
os[a + b*x]]*Gamma[5/2, I*ArcCos[a + b*x]])/(2*b*Sqrt[ArcCos[a + b*x]])
```

Maple [A] time = 0.082, size = 105, normalized size = 1.2

$$\frac{\sqrt{2}}{4\,b\sqrt{\pi}}\left(2\,\left(\arccos\left(bx+a\right)\right)^{3/2}\sqrt{2}\sqrt{\pi}xb+2\,\left(\arccos\left(bx+a\right)\right)^{3/2}\sqrt{2}\sqrt{\pi}a-3\,\sqrt{2}\sqrt{\pi}\sqrt{\arccos\left(bx+a\right)}\sqrt{-b^{2}x^{2}-2\,xab-a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)^(3/2),x)
```

```
[Out] 1/4/b*2^(1/2)/Pi^(1/2)*(2*arccos(b*x+a)^(3/2)*2^(1/2)*Pi^(1/2)*x*b+2*arccos
(b*x+a)^(3/2)*2^(1/2)*Pi^(1/2)*a-3*2^(1/2)*Pi^(1/2)*arccos(b*x+a)^(1/2)*(-b
^2*x^2-2*a*b*x-a^2+1)^(1/2)+3*Pi*FresnelS(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1
/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acos}^{\frac{3}{2}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)**(3/2),x)
```

```
[Out] Integral(acos(a + b*x)**(3/2), x)
```

Giac [B] time = 1.62439, size = 221, normalized size = 2.48

$$\frac{3\,i\sqrt{\arccos(bx+a)}e^{(i\arccos(bx+a))}}{4\,b}+\frac{\arccos(bx+a)^{\frac{3}{2}}e^{(i\arccos(bx+a))}}{2\,b}-\frac{3\,i\sqrt{\arccos(bx+a)}e^{(-i\arccos(bx+a))}}{4\,b}+\frac{\arccos(bx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 3/4*i*sqrt(arccos(b*x + a))*e^(i*arccos(b*x + a))/b + 1/2*arccos(b*x + a)^(
3/2)*e^(i*arccos(b*x + a))/b - 3/4*i*sqrt(arccos(b*x + a))*e^(-i*arccos(b*x
 + a))/b + 1/2*arccos(b*x + a)^(3/2)*e^(-i*arccos(b*x + a))/b - 3/8*sqrt(2)
*sqrt(pi)*i*erf(-sqrt(2)*i*sqrt(arccos(b*x + a))/(i - 1))/(b*(i - 1)) + 3/8
*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(arccos(b*x + a))/(i - 1))/(b*(i - 1))
```

3.39 $\int \sqrt{\cos^{-1}(a+bx)}\,dx$

Optimal. Leaf size=55

$$\frac{(a+bx)\sqrt{\cos^{-1}(a+bx)}}{b}-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{b}$$

```
[Out] ((a + b*x)*Sqrt[ArcCos[a + b*x]])/b - (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[
ArcCos[a + b*x]]])/b
```

Rubi [A] time = 0.0806047, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4804, 4620, 4724, 3304, 3352}

$$\frac{(a+bx)\sqrt{\cos^{-1}(a+bx)}}{b}-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcCos[a + b*x]],x]
```

```
[Out] ((a + b*x)*Sqrt[ArcCos[a + b*x]])/b - (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[
ArcCos[a + b*x]]])/b
```

Rule 4804

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
```

```
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\cos^{-1}(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{\cos^{-1}(x)}\,dx, x, a+bx\right)}{b}\\
&= \frac{(a+bx)\sqrt{\cos^{-1}(a+bx)}}{b} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{\cos^{-1}(x)}}\,dx, x, a+bx\right)}{2b}\\
&= \frac{(a+bx)\sqrt{\cos^{-1}(a+bx)}}{b} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(a+bx)\right)}{2b}\\
&= \frac{(a+bx)\sqrt{\cos^{-1}(a+bx)}}{b} - \frac{\text{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{\cos^{-1}(a+bx)}\right)}{b}\\
&= \frac{(a+bx)\sqrt{\cos^{-1}(a+bx)}}{b} - \frac{\sqrt{\frac{\pi}{2}}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{b}
\end{aligned}
$$

Mathematica [C] time = 0.0390328, size = 90, normalized size = 1.64

$$
-\frac{-\frac{\sqrt{\cos^{-1}(a+bx)}\text{Gamma}\left(\frac{3}{2},-i\cos^{-1}(a+bx)\right)}{2\sqrt{-i\cos^{-1}(a+bx)}}-\frac{\sqrt{\cos^{-1}(a+bx)}\text{Gamma}\left(\frac{3}{2},i\cos^{-1}(a+bx)\right)}{2\sqrt{i\cos^{-1}(a+bx)}}}{b}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[ArcCos[a + b*x]],x]
```

```
[Out] -((-(Sqrt[ArcCos[a + b*x]]*Gamma[3/2, (-I)*ArcCos[a + b*x]])/(2*Sqrt[(-I)*A
rcCos[a + b*x]]) - (Sqrt[ArcCos[a + b*x]]*Gamma[3/2, I*ArcCos[a + b*x]])/(2
```

```
*Sqrt[I*ArcCos[a + b*x]]))/b)
```

Maple [A] time = 0.073, size = 66, normalized size = 1.2

$$\frac{1}{2b}\left(-\sqrt{2}\sqrt{\arccos(bx+a)}\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\sqrt{\arccos(bx+a)}\right)+2\arccos(bx+a)\,xb+2\arccos(bx+a)\,a\right)\frac{1}{\sqrt{\arccos(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)^(1/2),x)
```

```
[Out] 1/2/b/arccos(b*x+a)^(1/2)*(-2^(1/2)*arccos(b*x+a)^(1/2)*Pi^(1/2)*FresnelC(2
^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2))+2*arccos(b*x+a)*x*b+2*arccos(b*x+a)*a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{acos}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(acos(a + b*x)), x)
```

Giac [B] time = 1.37766, size = 158, normalized size = 2.87

$$\frac{\sqrt{2}\sqrt{\pi}i\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\arccos(bx+a)}}{i-1}\right)}{4\,b(i-1)}+\frac{\sqrt{\arccos\left(bx+a\right)}e^{(i\arccos(bx+a))}}{2\,b}+\frac{\sqrt{\arccos\left(bx+a\right)}e^{(-i\arccos(bx+a))}}{2\,b}-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{\sqrt{2}i\sqrt{\text{arc}}}{i}\right.}{4\,b(i-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*sqrt(pi)*i*erf(sqrt(2)*sqrt(arccos(b*x + a))/(i - 1))/(b*(i - 1
)) + 1/2*sqrt(arccos(b*x + a))*e^(i*arccos(b*x + a))/b + 1/2*sqrt(arccos(b*
x + a))*e^(-i*arccos(b*x + a))/b - 1/4*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*i*sqrt
(arccos(b*x + a))/(i - 1))/(b*(i - 1))
```

3.40 $\int \frac{1}{\sqrt{\cos^{-1}(a+bx)}}\,dx$

Optimal. Leaf size=33

$$-\frac{\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{b}$$

```
[Out] -((Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a + b*x]]])/b)
```

Rubi [A] time = 0.0297642, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4804, 4624, 3305, 3351}

$$-\frac{\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[ArcCos[a + b*x]],x]
```

```
[Out] -((Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a + b*x]]])/b)
```

Rule 4804

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\cos^{-1}(a+bx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cos^{-1}(x)}}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(a+bx)\right)}{b}\\
&= -\frac{2\,\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\cos^{-1}(a+bx)}\right)}{b}\\
&= -\frac{\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{b}
\end{aligned}$$

Mathematica [C] time = 0.0316839, size = 78, normalized size = 2.36

$$-\frac{-\sqrt{-i\cos^{-1}(a+bx)}\text{Gamma}\left(\frac{1}{2},-i\cos^{-1}(a+bx)\right)-\sqrt{i\cos^{-1}(a+bx)}\text{Gamma}\left(\frac{1}{2},i\cos^{-1}(a+bx)\right)}{2b\sqrt{\cos^{-1}(a+bx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[ArcCos[a + b*x]],x]
```

```
[Out] -(-(Sqrt[(-I)*ArcCos[a + b*x]]*Gamma[1/2, (-I)*ArcCos[a + b*x]]) - Sqrt[I*A
rcCos[a + b*x]]*Gamma[1/2, I*ArcCos[a + b*x]])/(2*b*Sqrt[ArcCos[a + b*x]])
```

Maple [A] time = 0.055, size = 28, normalized size = 0.9

$$-\frac{\sqrt{2}\sqrt{\pi}}{b}\text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\sqrt{\arccos(bx+a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(b*x+a)^(1/2),x)
```

```
[Out] -FresnelS(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2))*2^(1/2)*Pi^(1/2)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\operatorname{acos}(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(acos(a + b*x)), x)
```

<u>Giac</u> [B] time = 1.57481, size = 97, normalized size = 2.94

$$\frac{\sqrt{2}\sqrt{\pi}i\,\mathrm{erf}\left(-\frac{\sqrt{2}i\sqrt{\arccos(bx+a)}}{i-1}\right)}{2\,b(i-1)}-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{\arccos(bx+a)}}{i-1}\right)}{2\,b(i-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(pi)*i*erf(-sqrt(2)*i*sqrt(arccos(b*x + a))/(i - 1))/(b*(i
- 1)) - 1/2*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(arccos(b*x + a))/(i - 1))/(b*
(i - 1))
```

3.41 $\int \frac{1}{\cos^{-1}(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}}-\frac{2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{b}$$

```
[Out] (2*Sqrt[1 - (a + b*x)^2])/(b*Sqrt[ArcCos[a + b*x]]) - (2*Sqrt[2*Pi]*Fresnel
C[Sqrt[2/Pi]*Sqrt[ArcCos[a + b*x]]])/b
```

Rubi [A] time = 0.0860013, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4804, 4622, 4724, 3304, 3352}

$$\frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}}-\frac{2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]^(-3/2),x]
```

```
[Out] (2*Sqrt[1 - (a + b*x)^2])/(b*Sqrt[ArcCos[a + b*x]]) - (2*Sqrt[2*Pi]*Fresnel
C[Sqrt[2/Pi]*Sqrt[ArcCos[a + b*x]]])/b
```

Rule 4804

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 4622

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1))
, Int[(x*(a + b*ArcCos[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && LtQ[n, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{-1}(a+bx)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)^{3/2}}\,dx, x, a+bx\right)}{b}\\
&= \frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{\cos^{-1}(x)}}\,dx, x, a+bx\right)}{b}\\
&= \frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}} - \frac{2\,\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(a+bx)\right)}{b}\\
&= \frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}} - \frac{4\,\text{Subst}\left(\int \cos\left(x^2\right)dx, x, \sqrt{\cos^{-1}(a+bx)}\right)}{b}\\
&= \frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}} - \frac{2\sqrt{2\pi}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{b}
\end{aligned}$$

Mathematica [C] time = 0.0535818, size = 97, normalized size = 1.52

$$-\frac{-i\sqrt{-i\cos^{-1}(a+bx)}\,\text{Gamma}\left(\frac{1}{2},-i\cos^{-1}(a+bx)\right)+i\sqrt{i\cos^{-1}(a+bx)}\,\text{Gamma}\left(\frac{1}{2},i\cos^{-1}(a+bx)\right)-2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a + b*x]^(-3/2),x]
```

```
[Out] -((-2*Sqrt[1 - (a + b*x)^2] - I*Sqrt[(-I)*ArcCos[a + b*x]]*Gamma[1/2, (-I)*
ArcCos[a + b*x]] + I*Sqrt[I*ArcCos[a + b*x]]*Gamma[1/2, I*ArcCos[a + b*x]])
/(b*Sqrt[ArcCos[a + b*x]]))
```

Maple [A] time = 0.075, size = 84, normalized size = 1.3

$$-\frac{\sqrt{2}}{b\sqrt{\pi}\arccos(bx+a)}\left(2\arccos(bx+a)\,\pi\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{\arccos(bx+a)}}{\sqrt{\pi}}\right)-\sqrt{2}\sqrt{\pi}\sqrt{\arccos(bx+a)}\sqrt{-b^2x^2-2\,xa b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(b*x+a)^(3/2),x)
```

```
[Out] -1/b*2^(1/2)/Pi^(1/2)/arccos(b*x+a)*(2*arccos(b*x+a)*Pi*FresnelC(2^(1/2)/Pi
^(1/2)*arccos(b*x+a)^(1/2))-2^(1/2)*Pi^(1/2)*arccos(b*x+a)^(1/2)*(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acos}^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(b*x+a)**(3/2),x)
```

```
[Out] Integral(acos(a + b*x)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\arccos(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arccos(b*x + a)^(-3/2), x)
```

3.42 $\int \frac{1}{\cos^{-1}(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=90

$$\frac{4\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{3b}+\frac{4(a+bx)}{3b\sqrt{\cos^{-1}(a+bx)}}+\frac{2\sqrt{1-(a+bx)^2}}{3b\cos^{-1}(a+bx)^{3/2}}$$

```
[Out] (2*Sqrt[1 - (a + b*x)^2])/(3*b*ArcCos[a + b*x]^(3/2)) + (4*(a + b*x))/(3*b*
Sqrt[ArcCos[a + b*x]]) + (4*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a +
b*x]]])/(3*b)
```

Rubi [A] time = 0.0920133, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4804, 4622, 4720, 4624, 3305, 3351}

$$\frac{4\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{3b}+\frac{4(a+bx)}{3b\sqrt{\cos^{-1}(a+bx)}}+\frac{2\sqrt{1-(a+bx)^2}}{3b\cos^{-1}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]^(-5/2),x]
```

```
[Out] (2*Sqrt[1 - (a + b*x)^2])/(3*b*ArcCos[a + b*x]^(3/2)) + (4*(a + b*x))/(3*b*
Sqrt[ArcCos[a + b*x]]) + (4*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a +
b*x]]])/(3*b)
```

Rule 4804

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 4622

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1))
, Int[(x*(a + b*ArcCos[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && LtQ[n, -1]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{-1}(a+bx)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)^{5/2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{2\sqrt{1-(a+bx)^2}}{3b\cos^{-1}(a+bx)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\cos^{-1}(x)^{3/2}}\,dx, x, a+bx\right)}{3b}$$

$$= \frac{2\sqrt{1-(a+bx)^2}}{3b\cos^{-1}(a+bx)^{3/2}} + \frac{4(a+bx)}{3b\sqrt{\cos^{-1}(a+bx)}} - \frac{4\,\text{Subst}\left(\int \frac{1}{\sqrt{\cos^{-1}(x)}}\,dx, x, a+bx\right)}{3b}$$

$$= \frac{2\sqrt{1-(a+bx)^2}}{3b\cos^{-1}(a+bx)^{3/2}} + \frac{4(a+bx)}{3b\sqrt{\cos^{-1}(a+bx)}} + \frac{4\,\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(a+bx)\right)}{3b}$$

$$= \frac{2\sqrt{1-(a+bx)^2}}{3b\cos^{-1}(a+bx)^{3/2}} + \frac{4(a+bx)}{3b\sqrt{\cos^{-1}(a+bx)}} + \frac{8\,\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\cos^{-1}(a+bx)}\right)}{3b}$$

$$= \frac{2\sqrt{1-(a+bx)^2}}{3b\cos^{-1}(a+bx)^{3/2}} + \frac{4(a+bx)}{3b\sqrt{\cos^{-1}(a+bx)}} + \frac{4\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{3b}$$

Mathematica [C] time = 0.291454, size = 139, normalized size = 1.54

$$-\frac{2\left(i\left(-i\cos^{-1}(a+bx)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-i\cos^{-1}(a+bx)\right)-i\left(i\cos^{-1}(a+bx)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},i\cos^{-1}(a+bx)\right)-\sqrt{1-(a\right.}{3b\cos^{-1}(a+bx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a + b*x]^(-5/2),x]
```

```
[Out] (-2*(-Sqrt[1 - (a + b*x)^2] - ArcCos[a + b*x]/E^(I*ArcCos[a + b*x]) - E^(I*
ArcCos[a + b*x])*ArcCos[a + b*x] + I*((-I)*ArcCos[a + b*x])^(3/2)*Gamma[1/2
, (-I)*ArcCos[a + b*x]] - I*(I*ArcCos[a + b*x])^(3/2)*Gamma[1/2, I*ArcCos[a
 + b*x]]))/(3*b*ArcCos[a + b*x]^(3/2))
```

Maple [A] time = 0.079, size = 120, normalized size = 1.3

$$\frac{\sqrt{2}}{3\,b\sqrt{\pi}\,(\arccos(bx+a))^2}\left(4\,\pi\,(\arccos(bx+a))^2\,\text{FresnelS}\left(\frac{\sqrt{2}\sqrt{\arccos(bx+a)}}{\sqrt{\pi}}\right)+2\,(\arccos(bx+a))^{3/2}\sqrt{2}\sqrt{\pi}xb+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(b*x+a)^(5/2),x)
```

```
[Out] 1/3/b*2^(1/2)/Pi^(1/2)*(4*Pi*arccos(b*x+a)^2*FresnelS(2^(1/2)/Pi^(1/2)*arcc
os(b*x+a)^(1/2))+2*arccos(b*x+a)^(3/2)*2^(1/2)*Pi^(1/2)*x*b+2*arccos(b*x+a)
^(3/2)*2^(1/2)*Pi^(1/2)*a+2^(1/2)*Pi^(1/2)*arccos(b*x+a)^(1/2)*(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2))/arccos(b*x+a)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acos}^{\frac{5}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(b*x+a)**(5/2),x)
```

```
[Out] Integral(acos(a + b*x)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\arccos(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arccos(b*x + a)^(-5/2), x)
```

3.43 $\int \frac{1}{\sqrt{a+b\cos^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{\sqrt{2\pi}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}-\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}$$

```
[Out] -((Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c + d*x]])/Sq
rt[b]])/(Sqrt[b]*d)) + (Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c
 + d*x]])/Sqrt[b]]*Sin[a/b])/(Sqrt[b]*d)
```

Rubi [A] time = 0.130424, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4804, 4624, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{2\pi}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}-\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCos[c + d*x]],x]
```

```
[Out] -((Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c + d*x]])/Sq
rt[b]])/(Sqrt[b]*d)) + (Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c
 + d*x]])/Sqrt[b]]*Sin[a/b])/(Sqrt[b]*d)
```

Rule 4804

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\cos^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cos^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cos^{-1}(c+dx)\right)}{bd} + \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\right.}{bd}$$

$$= -\frac{\left(2\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right)dx, x, \sqrt{a+b\cos^{-1}(c+dx)}\right)}{bd} + \frac{\left(2\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \cos\left(\frac{x}{b}\right.\right.}{}$$

$$= -\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d} + \frac{\sqrt{2\pi}C\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(c+dx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{\sqrt{b}d}$$

Mathematica [C] time = 0.104934, size = 128, normalized size = 1.21

$$\frac{e^{-\frac{ia}{b}}\left(\sqrt{-\frac{i\left(a+b\cos^{-1}(c+dx)\right)}{b}}\text{Gamma}\left(\frac{1}{2}, -\frac{i\left(a+b\cos^{-1}(c+dx)\right)}{b}\right) + e^{\frac{2ia}{b}}\sqrt{\frac{i\left(a+b\cos^{-1}(c+dx)\right)}{b}}\text{Gamma}\left(\frac{1}{2}, \frac{i\left(a+b\cos^{-1}(c+dx)\right)}{b}\right)\right)}{2d\sqrt{a+b\cos^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcCos[c + d*x]],x]
```

```
[Out] (Sqrt[((-I)*(a + b*ArcCos[c + d*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c +
d*x]))/b] + E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c + d*x]))/b]*Gamma[1/2,
(I*(a + b*ArcCos[c + d*x]))/b])/(2*d*E^((I*a)/b)*Sqrt[a + b*ArcCos[c + d*x]
])
```

Maple [A] time = 0.071, size = 89, normalized size = 0.8

$$-\frac{\sqrt{2}\sqrt{\pi}}{d}\sqrt{b^{-1}}\left(\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}b}\sqrt{a+b\arccos(dx+c)}\frac{1}{\sqrt{b^{-1}}}\right) - \sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}b}\sqrt{a+b\arccos(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x+c))^(1/2),x)
```

```
[Out] -2^(1/2)*Pi^(1/2)*(1/b)^(1/2)*(cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(1/b)^(1/
2)*(a+b*arccos(d*x+c))^(1/2)/b)-sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(1/b)^(1
/2)*(a+b*arccos(d*x+c))^(1/2)/b))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \arccos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccos(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acos}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acos(c + d*x)), x)
```

Giac [A] time = 2.34393, size = 231, normalized size = 2.18

$$\frac{\sqrt{\pi}i\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{b\arccos(dx+c)+a}i}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(dx+c)+a}\sqrt{|b|}}{2b}\right)e^{\left(\frac{ai}{b}\right)}}{\left(\frac{\sqrt{2}bi}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)d}+\frac{\sqrt{\pi}i\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b\arccos(dx+c)+a}i}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(dx+c)+a}\sqrt{|b|}}{2b}\right)e^{\left(-\frac{ai}{b}\right)}}{\left(\frac{\sqrt{2}bi}{\sqrt{|b|}}-\sqrt{2}\sqrt{|b|}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(pi)*i*erf(-1/2*sqrt(2)*sqrt(b*arccos(d*x + c) + a)*i/sqrt(abs(b)) - 1/
2*sqrt(2)*sqrt(b*arccos(d*x + c) + a)*sqrt(abs(b))/b)*e^(a*i/b)/((sqrt(2)*b
*i/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))*d) + sqrt(pi)*i*erf(1/2*sqrt(2)*sqr
t(b*arccos(d*x + c) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(d*x + c
) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((sqrt(2)*b*i/sqrt(abs(b)) - sqrt(2)*sqrt
(abs(b)))*d)
```

3.44 $\int \frac{1}{\sqrt{a-b\cos^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{2\pi}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-b\cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}-\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-b\cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}$$

```
[Out] -((Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a - b*ArcCos[c + d*x]])/Sq
rt[b]])/(Sqrt[b]*d)) + (Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a - b*ArcCos[c
 + d*x]])/Sqrt[b]]*Sin[a/b])/(Sqrt[b]*d)
```

Rubi [A] time = 0.115801, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {4804, 4624, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{2\pi}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-b\cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}-\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-b\cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - b*ArcCos[c + d*x]],x]
```


```
[Out] -((Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a - b*ArcCos[c + d*x]])/Sq
rt[b]])/(Sqrt[b]*d)) + (Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a - b*ArcCos[c
 + d*x]])/Sqrt[b]]*Sin[a/b])/(Sqrt[b]*d)
```


Rule 4804

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-b\cos^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-b\cos^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a-b\cos^{-1}(c+dx)\right)}{bd}$$
$$= -\frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a-b\cos^{-1}(c+dx)\right)}{bd} + \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a-b\right.}{bd}$$
$$= -\frac{\left(2\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right)dx, x, \sqrt{a-b\cos^{-1}(c+dx)}\right)}{bd} + \frac{\left(2\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \cos\left(\right.\right.}{}$$
$$= -\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-b\cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d} + \frac{\sqrt{2\pi}C\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-b\cos^{-1}(c+dx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{\sqrt{b}d}$$

Mathematica [C] time = 0.101873, size = 133, normalized size = 1.23

$$\frac{e^{-\frac{ia}{b}}\left(\sqrt{-\frac{i\left(a-b\cos^{-1}(c+dx)\right)}{b}}\text{Gamma}\left(\frac{1}{2}, -\frac{i\left(a-b\cos^{-1}(c+dx)\right)}{b}\right) + e^{\frac{2ia}{b}}\sqrt{\frac{i\left(a-b\cos^{-1}(c+dx)\right)}{b}}\text{Gamma}\left(\frac{1}{2}, \frac{i\left(a-b\cos^{-1}(c+dx)\right)}{b}\right)\right)}{2d\sqrt{a-b\cos^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a - b*ArcCos[c + d*x]],x]
```

```
[Out] (Sqrt[((-I)*(a - b*ArcCos[c + d*x]))/b]*Gamma[1/2, ((-I)*(a - b*ArcCos[c +
d*x]))/b] + E^(((2*I)*a)/b)*Sqrt[(I*(a - b*ArcCos[c + d*x]))/b]*Gamma[1/2,
(I*(a - b*ArcCos[c + d*x]))/b])/(2*d*E^((I*a)/b)*Sqrt[a - b*ArcCos[c + d*x]
])
```

Maple [A] time = 0.064, size = 90, normalized size = 0.8

$$\frac{\sqrt{2}\sqrt{\pi}}{d}\sqrt{b^{-1}}\left(\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}b}\sqrt{a-b\arccos(dx+c)}\frac{1}{\sqrt{b^{-1}}}\right) - \cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}b}\sqrt{a-b\arccos(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*arccos(d*x+c))^(1/2),x)
```

```
[Out] 2^(1/2)*Pi^(1/2)*(1/b)^(1/2)*(sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(1/b)^(1/2
)*(a-b*arccos(d*x+c))^(1/2)/b)-cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(1/b)^(1/
2)*(a-b*arccos(d*x+c))^(1/2)/b))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b \arccos(dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*arccos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-b*arccos(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*arccos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a - b \operatorname{acos}(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*acos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a - b*acos(c + d*x)), x)
```

Giac [A] time = 2.94386, size = 236, normalized size = 2.19

$$\frac{\sqrt{\pi} i \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{-b\arccos(dx+c)+a}i}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{-b\arccos(dx+c)+a}\sqrt{|b|}}{2b}\right)e^{\left(\frac{ai}{b}\right)}}{\left(\frac{\sqrt{2}bi}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)d}+\frac{\sqrt{\pi} i \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{-b\arccos(dx+c)+a}i}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{-b\arccos(dx+c)+a}\sqrt{|b|}}{2b}\right)e}{\left(\frac{\sqrt{2}bi}{\sqrt{|b|}}-\sqrt{2}\sqrt{|b|}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*arccos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(pi)*i*erf(-1/2*sqrt(2)*sqrt(-b*arccos(d*x + c) + a)*i/sqrt(abs(b)) - 1
/2*sqrt(2)*sqrt(-b*arccos(d*x + c) + a)*sqrt(abs(b))/b)*e^(a*i/b)/((sqrt(2)
*b*i/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))*d) + sqrt(pi)*i*erf(1/2*sqrt(2)*s
qrt(-b*arccos(d*x + c) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(-b*arccos(d*x
 + c) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((sqrt(2)*b*i/sqrt(abs(b)) - sqrt(2)*
sqrt(abs(b)))*d)
```

3.45 $\int \frac{\cos^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal. Leaf size=68

$$-\frac{i\text{PolyLog}\left(2,-e^{2i\cos^{-1}(a+bx)}\right)}{2d}-\frac{i\cos^{-1}(a+bx)^2}{2d}+\frac{\cos^{-1}(a+bx)\log\left(1+e^{2i\cos^{-1}(a+bx)}\right)}{d}$$

```
[Out] ((-I/2)*ArcCos[a + b*x]^2)/d + (ArcCos[a + b*x]*Log[1 + E^((2*I)*ArcCos[a +
 b*x])])/d - ((I/2)*PolyLog[2, -E^((2*I)*ArcCos[a + b*x])])/d
```

Rubi [A] time = 0.0813552, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {4806, 12, 4626, 3719, 2190, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,-e^{2i\cos^{-1}(a+bx)}\right)}{2d}-\frac{i\cos^{-1}(a+bx)^2}{2d}+\frac{\cos^{-1}(a+bx)\log\left(1+e^{2i\cos^{-1}(a+bx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((-I/2)*ArcCos[a + b*x]^2)/d + (ArcCos[a + b*x]*Log[1 + E^((2*I)*ArcCos[a +
 b*x])])/d - ((I/2)*PolyLog[2, -E^((2*I)*ArcCos[a + b*x])])/d
```

Rule 4806

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cos^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{\text{Subst}\left(\int \frac{b\cos^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}(a+bx)\right)}{d}$$
$$= -\frac{i\cos^{-1}(a+bx)^2}{2d} + \frac{(2i)\,\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}(a+bx)\right)}{d}$$
$$= -\frac{i\cos^{-1}(a+bx)^2}{2d} + \frac{\cos^{-1}(a+bx)\log\left(1+e^{2i\cos^{-1}(a+bx)}\right)}{d} - \frac{\text{Subst}\left(\int \log\left(1+e^{2ix}\right)dx, x, \cos^{-1}(a\right.}{d}$$
$$= -\frac{i\cos^{-1}(a+bx)^2}{2d} + \frac{\cos^{-1}(a+bx)\log\left(1+e^{2i\cos^{-1}(a+bx)}\right)}{d} + \frac{i\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\cos^{-1}(a+bx)}\right)}{2d}$$
$$= -\frac{i\cos^{-1}(a+bx)^2}{2d} + \frac{\cos^{-1}(a+bx)\log\left(1+e^{2i\cos^{-1}(a+bx)}\right)}{d} - \frac{i\text{Li}_2\left(-e^{2i\cos^{-1}(a+bx)}\right)}{2d}$$

Mathematica [A] time = 0.0414374, size = 59, normalized size = 0.87

$$-\frac{i\left(\text{PolyLog}\left(2,-e^{2i\cos^{-1}(a+bx)}\right)+\cos^{-1}(a+bx)\left(\cos^{-1}(a+bx)+2i\log\left(1+e^{2i\cos^{-1}(a+bx)}\right)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((-I/2)*(ArcCos[a + b*x]*(ArcCos[a + b*x] + (2*I)*Log[1 + E^((2*I)*ArcCos[a
 + b*x])]) + PolyLog[2, -E^((2*I)*ArcCos[a + b*x])]))/d
```

Maple [A] time = 0.089, size = 85, normalized size = 1.3

$$\frac{-\frac{i}{2}\left(\arccos(bx+a)\right)^2}{d} + \frac{\arccos(bx+a)}{d}\ln\left(1+\left(bx+a+i\sqrt{1-(bx+a)^2}\right)^2\right) - \frac{\frac{i}{2}}{d}\text{polylog}\left(2,-\left(bx+a+i\sqrt{1-(bx+a)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] -1/2*I*arccos(b*x+a)^2/d+arccos(b*x+a)*ln(1+(b*x+a+I*(1-(b*x+a)^2)^(1/2))^2
)/d-1/2*I*polylog(2,-(b*x+a+I*(1-(b*x+a)^2)^(1/2))^2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \arccos\left(bx+a\right)}{bdx+ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arccos(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b \int \frac{\operatorname{acos}\left(a+bx\right)}{a+bx}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(acos(a + b*x)/(a + b*x), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos\left(bx+a\right)}{dx+\frac{ad}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] integrate(arccos(b*x + a)/(d*x + a*d/b), x)
```

3.46 $\int \sqrt{1-x^2} \cos^{-1}(x)\, dx$

Optimal. Leaf size=34

$$\frac{x^2}{4}+\frac{1}{2}\sqrt{1-x^2}x\cos^{-1}(x)-\frac{1}{4}\cos^{-1}(x)^2$$

```
[Out] x^2/4 + (x*Sqrt[1 - x^2]*ArcCos[x])/2 - ArcCos[x]^2/4
```

Rubi [A] time = 0.0297441, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4648, 4642, 30}

$$\frac{x^2}{4}+\frac{1}{2}\sqrt{1-x^2}x\cos^{-1}(x)-\frac{1}{4}\cos^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]*ArcCos[x],x]
```

```
[Out] x^2/4 + (x*Sqrt[1 - x^2]*ArcCos[x])/2 - ArcCos[x]^2/4
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{1-x^2}\cos^{-1}(x)\,dx = \frac{1}{2}x\sqrt{1-x^2}\cos^{-1}(x)+\frac{\int x\,dx}{2}+\frac{1}{2}\int \frac{\cos^{-1}(x)}{\sqrt{1-x^2}}\,dx$$
$$= \frac{x^2}{4}+\frac{1}{2}x\sqrt{1-x^2}\cos^{-1}(x)-\frac{1}{4}\cos^{-1}(x)^2$$

Mathematica [A] time = 0.0157414, size = 30, normalized size = 0.88

$$\frac{1}{4}\left(x^2+2\sqrt{1-x^2}x\cos^{-1}(x)-\cos^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]*ArcCos[x],x]
```

```
[Out] (x^2 + 2*x*Sqrt[1 - x^2]*ArcCos[x] - ArcCos[x]^2)/4
```

Maple [A] time = 0.083, size = 33, normalized size = 1.

$$-\frac{\arccos(x)}{2}\left(-x\sqrt{-x^2+1}+\arccos(x)\right)+\frac{(\arccos(x))^2}{4}+\frac{x^2}{4}-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x)*(-x^2+1)^(1/2),x)
```

```
[Out] -1/2*arccos(x)*(-x*(-x^2+1)^(1/2)+arccos(x))+1/4*arccos(x)^2+1/4*x^2-1/4
```

Maxima [A] time = 1.49029, size = 41, normalized size = 1.21

$$\frac{1}{4}x^2+\frac{1}{2}\left(\sqrt{-x^2+1}x+\arcsin(x)\right)\arccos(x)+\frac{1}{4}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arccos(x) + 1/4*arcsin(x)^2
```

Fricas [A] time = 2.38116, size = 81, normalized size = 2.38

$$\frac{1}{2}\sqrt{-x^2+1}x\arccos(x)+\frac{1}{4}x^2-\frac{1}{4}\arccos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2
```

Sympy [A] time = 21.5284, size = 48, normalized size = 1.41

$$\left(\begin{cases} \frac{x\sqrt{1-x^2}}{2}+\frac{\operatorname{asin}(x)}{2} & \text{for } x>-1\wedge x<1 \end{cases}\right)\operatorname{acos}(x)+\begin{cases} \text{NaN} & \text{for } x<-1 \\ \frac{x^2}{4}+\frac{\operatorname{asin}^2(x)}{4}-\frac{\pi^2}{16}-\frac{1}{4} & \text{for } x<1 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x)*(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((x*sqrt(1 - x**2)/2 + asin(x)/2, (x > -1) & (x < 1)))*acos(x) + P
iecewise((nan, x < -1), (x**2/4 + asin(x)**2/4 - pi**2/16 - 1/4, x < 1), (n
an, True))
```

Giac [A] time = 1.32261, size = 36, normalized size = 1.06

$$\frac{1}{2}\sqrt{-x^2+1}x\arccos(x)+\frac{1}{4}x^2-\frac{1}{4}\arccos(x)^2-\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2 - 1/8
```

3.47 $\int x^3 \cos^{-1}(ax^2)\,dx$

Optimal. Leaf size=51

$$-\frac{x^2\sqrt{1-a^2x^4}}{8a}+\frac{\sin^{-1}(ax^2)}{8a^2}+\frac{1}{4}x^4\cos^{-1}(ax^2)$$

```
[Out] -(x^2*Sqrt[1 - a^2*x^4])/(8*a) + (x^4*ArcCos[a*x^2])/4 + ArcSin[a*x^2]/(8*a
^2)
```

Rubi [A] time = 0.0385854, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4843, 12, 275, 321, 216}

$$-\frac{x^2\sqrt{1-a^2x^4}}{8a}+\frac{\sin^{-1}(ax^2)}{8a^2}+\frac{1}{4}x^4\cos^{-1}(ax^2)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCos[a*x^2],x]
```

```
[Out] -(x^2*Sqrt[1 - a^2*x^4])/(8*a) + (x^4*ArcCos[a*x^2])/4 + ArcSin[a*x^2]/(8*a
^2)
```

Rule 4843

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcCos[u]))/(d*(m + 1)), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 - u^2], x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int x^3 \cos^{-1}\left(ax^2\right) dx &= \frac{1}{4}x^4 \cos^{-1}\left(ax^2\right) + \frac{1}{4}\int \frac{2ax^5}{\sqrt{1-a^2x^4}}\, dx \\ &= \frac{1}{4}x^4 \cos^{-1}\left(ax^2\right) + \frac{1}{2}a\int \frac{x^5}{\sqrt{1-a^2x^4}}\, dx \\ &= \frac{1}{4}x^4 \cos^{-1}\left(ax^2\right) + \frac{1}{4}a\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-a^2x^2}}\, dx, x, x^2\right) \\ &= -\frac{x^2\sqrt{1-a^2x^4}}{8a} + \frac{1}{4}x^4 \cos^{-1}\left(ax^2\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-a^2x^2}}\, dx, x, x^2\right)}{8a} \\ &= -\frac{x^2\sqrt{1-a^2x^4}}{8a} + \frac{1}{4}x^4 \cos^{-1}\left(ax^2\right) + \frac{\sin^{-1}\left(ax^2\right)}{8a^2}\end{aligned}$$

Mathematica [A] time = 0.0275862, size = 48, normalized size = 0.94

$$\frac{-ax^2\sqrt{1-a^2x^4} + 2a^2x^4 \cos^{-1}\left(ax^2\right) + \sin^{-1}\left(ax^2\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCos[a*x^2],x]
```

```
[Out] (-(a*x^2*Sqrt[1 - a^2*x^4]) + 2*a^2*x^4*ArcCos[a*x^2] + ArcSin[a*x^2])/(8*a
^2)
```

Maple [A] time = 0.053, size = 65, normalized size = 1.3

$$\frac{x^4 \arccos\left(a x^2\right)}{4} - \frac{x^2}{8 a}\sqrt{-a^2 x^4 + 1} + \frac{1}{8 a}\arctan\left(x^2\sqrt{a^2}\frac{1}{\sqrt{-a^2 x^4 + 1}}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccos(a*x^2),x)
```

```
[Out] 1/4*x^4*arccos(a*x^2)-1/8*x^2*(-a^2*x^4+1)^(1/2)/a+1/8/a/(a^2)^(1/2)*arctan
((a^2)^(1/2)*x^2/(-a^2*x^4+1)^(1/2))
```

Maxima [A] time = 1.47624, size = 107, normalized size = 2.1

$$\frac{1}{4}x^4 \arccos\left(a x^2\right) - \frac{1}{8}a\left(\frac{\arctan\left(\frac{\sqrt{-a^2 x^4 + 1}}{a x^2}\right)}{a^3} + \frac{\sqrt{-a^2 x^4 + 1}}{\left(a^4 - \frac{(a^2 x^4 - 1)a^2}{x^4}\right)x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccos(a*x^2) - 1/8*a*(arctan(sqrt(-a^2*x^4 + 1)/(a*x^2))/a^3 + sqr
t(-a^2*x^4 + 1)/((a^4 - (a^2*x^4 - 1)*a^2/x^4)*x^2))
```

Fricas [A] time = 2.52952, size = 93, normalized size = 1.82

$$-\frac{\sqrt{-a^2 x^4 + 1}a x^2 - \left(2 a^2 x^4 - 1\right)\arccos\left(a x^2\right)}{8 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x^2),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(-a^2*x^4 + 1)*a*x^2 - (2*a^2*x^4 - 1)*arccos(a*x^2))/a^2
```

Sympy [A] time = 1.03189, size = 48, normalized size = 0.94

$$\begin{cases} \frac{x^4 \operatorname{acos}(ax^2)}{4} - \frac{x^2\sqrt{-a^2x^4+1}}{8a} - \frac{\operatorname{acos}(ax^2)}{8a^2} & \text{for } a \neq 0 \\ \frac{\pi x^4}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acos(a*x**2),x)
```

```
[Out] Piecewise((x**4*acos(a*x**2)/4 - x**2*sqrt(-a**2*x**4 + 1)/(8*a) - acos(a*x
**2)/(8*a**2), Ne(a, 0)), (pi*x**4/8, True))
```

Giac [A] time = 1.34345, size = 62, normalized size = 1.22

$$\frac{2\,a^2x^4 \arccos\left(ax^2\right) - \sqrt{-a^2x^4+1}ax^2 - \arccos\left(ax^2\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x^2),x, algorithm="giac")
```

```
[Out] 1/8*(2*a^2*x^4*arccos(a*x^2) - sqrt(-a^2*x^4 + 1)*a*x^2 - arccos(a*x^2))/a^
2
```

3.48 $\int x^2 \cos^{-1}\left(ax^2\right) dx$

Optimal. Leaf size=55

$$\frac{2\text{EllipticF}\left(\sin^{-1}\left(\sqrt{a}x\right),-1\right)}{9a^{3/2}}-\frac{2x\sqrt{1-a^2x^4}}{9a}+\frac{1}{3}x^3\cos^{-1}\left(ax^2\right)$$

```
[Out] (-2*x*Sqrt[1 - a^2*x^4])/(9*a) + (x^3*ArcCos[a*x^2])/3 + (2*EllipticF[ArcSi
n[Sqrt[a]*x], -1])/(9*a^(3/2))
```

Rubi [A] time = 0.0284355, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4843, 12, 321, 221}

$$-\frac{2x\sqrt{1-a^2x^4}}{9a}+\frac{2F\left(\sin^{-1}\left(\sqrt{a}x\right)\middle|-1\right)}{9a^{3/2}}+\frac{1}{3}x^3\cos^{-1}\left(ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[a*x^2],x]
```

```
[Out] (-2*x*Sqrt[1 - a^2*x^4])/(9*a) + (x^3*ArcCos[a*x^2])/3 + (2*EllipticF[ArcSi
n[Sqrt[a]*x], -1])/(9*a^(3/2))
```

Rule 4843

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcCos[u]))/(d*(m + 1)), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 - u^2], x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \cos^{-1}\left(ax^2\right) dx &= \frac{1}{3}x^3 \cos^{-1}\left(ax^2\right) + \frac{1}{3}\int \frac{2ax^4}{\sqrt{1-a^2x^4}}\,dx \\ &= \frac{1}{3}x^3 \cos^{-1}\left(ax^2\right) + \frac{1}{3}(2a)\int \frac{x^4}{\sqrt{1-a^2x^4}}\,dx \\ &= -\frac{2x\sqrt{1-a^2x^4}}{9a} + \frac{1}{3}x^3 \cos^{-1}\left(ax^2\right) + \frac{2\int \frac{1}{\sqrt{1-a^2x^4}}\,dx}{9a} \\ &= -\frac{2x\sqrt{1-a^2x^4}}{9a} + \frac{1}{3}x^3 \cos^{-1}\left(ax^2\right) + \frac{2F\left(\sin^{-1}\left(\sqrt{a}x\right)\middle|-1\right)}{9a^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.155284, size = 63, normalized size = 1.15

$$\frac{1}{9}\left(\frac{2i\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-a}x\right),-1\right)}{(-a)^{3/2}} - \frac{2x\sqrt{1-a^2x^4}}{a} + 3x^3\cos^{-1}\left(ax^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[a*x^2],x]
```

```
[Out] ((-2*x*Sqrt[1 - a^2*x^4])/a + 3*x^3*ArcCos[a*x^2] + ((2*I)*EllipticF[I*ArcS
inh[Sqrt[-a]*x], -1])/(-a)^(3/2))/9
```

Maple [A] time = 0.007, size = 79, normalized size = 1.4

$$\frac{x^3\arccos\left(ax^2\right)}{3} + \frac{2\,a}{3}\left(-\frac{x}{3\,a^2}\sqrt{-a^2x^4+1} + \frac{1}{3}\sqrt{-ax^2+1}\sqrt{ax^2+1}\text{EllipticF}\left(x\sqrt{a},i\right)a^{-\frac{5}{2}}\frac{1}{\sqrt{-a^2x^4+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(a*x^2),x)
```

```
[Out] 1/3*x^3*arccos(a*x^2)+2/3*a*(-1/3/a^2*x*(-a^2*x^4+1)^(1/2)+1/3/a^(5/2)*(-a*
x^2+1)^(1/2)*(a*x^2+1)^(1/2)/(-a^2*x^4+1)^(1/2)*EllipticF(x*a^(1/2),I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\arccos\left(ax^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^2*arccos(a*x^2), x)
```

Sympy [A] time = 1.42681, size = 48, normalized size = 0.87

$$\frac{ax^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle| a^2x^4e^{2i\pi}\right)}{6\Gamma\left(\frac{9}{4}\right)}+\frac{x^3\operatorname{acos}\left(ax^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(a*x**2),x)
```

```
[Out] a*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), a**2*x**4*exp_polar(2*I*pi))/(6
*gamma(9/4)) + x**3*acos(a*x**2)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \arccos\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccos(a*x^2), x)
```

3.49 $\int x \cos^{-1}(ax^2)\,dx$

Optimal. Leaf size=35

$$\frac{1}{2}x^2\cos^{-1}\left(ax^2\right)-\frac{\sqrt{1-a^2x^4}}{2a}$$

```
[Out] -Sqrt[1 - a^2*x^4]/(2*a) + (x^2*ArcCos[a*x^2])/2
```

Rubi [A] time = 0.0236618, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6715, 4620, 261}

$$\frac{1}{2}x^2\cos^{-1}\left(ax^2\right)-\frac{\sqrt{1-a^2x^4}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCos[a*x^2],x]
```

```
[Out] -Sqrt[1 - a^2*x^4]/(2*a) + (x^2*ArcCos[a*x^2])/2
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\cos^{-1}\left(ax^2\right)\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \cos^{-1}(ax)\,dx,x,x^2\right)\\ &= \frac{1}{2}x^2\cos^{-1}\left(ax^2\right)+\frac{1}{2}a\operatorname{Subst}\left(\int \frac{x}{\sqrt{1-a^2x^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{1-a^2x^4}}{2a}+\frac{1}{2}x^2\cos^{-1}\left(ax^2\right)\end{aligned}$$

Mathematica [A] time = 0.0161599, size = 35, normalized size = 1.

$$\frac{1}{2}x^2\cos^{-1}\left(ax^2\right)-\frac{\sqrt{1-a^2x^4}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCos[a*x^2],x]
```

```
[Out] -Sqrt[1 - a^2*x^4]/(2*a) + (x^2*ArcCos[a*x^2])/2
```

Maple [A] time = 0.002, size = 32, normalized size = 0.9

$$\frac{1}{2a}\left(x^2a\arccos\left(ax^2\right)-\sqrt{-a^2x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(a*x^2),x)
```

```
[Out] 1/2/a*(x^2*a*arccos(a*x^2)-(-a^2*x^4+1)^(1/2))
```

Maxima [A] time = 1.44838, size = 42, normalized size = 1.2

$$\frac{ax^2\arccos\left(ax^2\right)-\sqrt{-a^2x^4+1}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*x^2*arccos(a*x^2) - sqrt(-a^2*x^4 + 1))/a
```

Fricas [A] time = 2.37848, size = 68, normalized size = 1.94

$$\frac{ax^2 \arccos\left(ax^2\right) - \sqrt{-a^2x^4 + 1}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*x^2*arccos(a*x^2) - sqrt(-a^2*x^4 + 1))/a
```

Sympy [A] time = 0.229752, size = 32, normalized size = 0.91

$$\begin{cases} \frac{x^2 \operatorname{acos}\left(ax^2\right)}{2} - \frac{\sqrt{-a^2x^4+1}}{2a} & \text{for } a \neq 0 \\ \frac{\pi x^2}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(a*x**2),x)
```

```
[Out] Piecewise((x**2*acos(a*x**2)/2 - sqrt(-a**2*x**4 + 1)/(2*a), Ne(a, 0)), (pi
*x**2/4, True))
```

Giac [A] time = 1.32775, size = 42, normalized size = 1.2

$$\frac{ax^2 \arccos\left(ax^2\right) - \sqrt{-a^2x^4 + 1}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*x^2*arccos(a*x^2) - sqrt(-a^2*x^4 + 1))/a
```

3.50 $\int \cos^{-1}\left(ax^2\right) dx$

Optimal. Leaf size=43

$$-\frac{2\text{EllipticF}\left(\sin^{-1}\left(\sqrt{a}x\right),-1\right)}{\sqrt{a}}+x\cos^{-1}\left(ax^2\right)+\frac{2E\left(\sin^{-1}\left(\sqrt{a}x\right)\middle|-1\right)}{\sqrt{a}}$$

```
[Out] x*ArcCos[a*x^2] + (2*EllipticE[ArcSin[Sqrt[a]*x], -1])/Sqrt[a] - (2*Ellipti
cF[ArcSin[Sqrt[a]*x], -1])/Sqrt[a]
```

Rubi [A] time = 0.0324736, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4841, 12, 307, 221, 1199, 424}

$$x\cos^{-1}\left(ax^2\right)-\frac{2F\left(\sin^{-1}\left(\sqrt{a}x\right)\middle|-1\right)}{\sqrt{a}}+\frac{2E\left(\sin^{-1}\left(\sqrt{a}x\right)\middle|-1\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x^2],x]
```



```
[Out] x*ArcCos[a*x^2] + (2*EllipticE[ArcSin[Sqrt[a]*x], -1])/Sqrt[a] - (2*Ellipti
cF[ArcSin[Sqrt[a]*x], -1])/Sqrt[a]
```


Rule 4841

```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^{-1}\left(ax^2\right)\,dx &= x\cos^{-1}\left(ax^2\right)+\int \frac{2ax^2}{\sqrt{1-a^2x^4}}\,dx\\
&= x\cos^{-1}\left(ax^2\right)+(2a)\int \frac{x^2}{\sqrt{1-a^2x^4}}\,dx\\
&= x\cos^{-1}\left(ax^2\right)-2\int \frac{1}{\sqrt{1-a^2x^4}}\,dx+2\int \frac{1+ax^2}{\sqrt{1-a^2x^4}}\,dx\\
&= x\cos^{-1}\left(ax^2\right)-\frac{2F\left(\sin^{-1}\left(\sqrt{a}x\right)\middle|-1\right)}{\sqrt{a}}+2\int \frac{\sqrt{1+ax^2}}{\sqrt{1-ax^2}}\,dx\\
&= x\cos^{-1}\left(ax^2\right)+\frac{2E\left(\sin^{-1}\left(\sqrt{a}x\right)\middle|-1\right)}{\sqrt{a}}-\frac{2F\left(\sin^{-1}\left(\sqrt{a}x\right)\middle|-1\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [C] time = 0.0047055, size = 34, normalized size = 0.79

$$\frac{2}{3}ax^3\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},a^2x^4\right)+x\cos^{-1}\left(ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x^2],x]
```

```
[Out] x*ArcCos[a*x^2] + (2*a*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, a^2*x^4])/3
```

Maple [A] time = 0.007, size = 65, normalized size = 1.5

$$x\arccos\left(ax^2\right) - 2\,\frac{\sqrt{-ax^2+1}\sqrt{ax^2+1}\left(\mathrm{EllipticF}\left(x\sqrt{a},i\right) - \mathrm{EllipticE}\left(x\sqrt{a},i\right)\right)}{\sqrt{a}\sqrt{-a^2x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x^2),x)
```

```
[Out] x*arccos(a*x^2)-2/a^(1/2)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)/(-a^2*x^4+1)^(1/
2)*(EllipticF(x*a^(1/2),I)-EllipticE(x*a^(1/2),I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\arccos\left(ax^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2),x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x^2), x)
```

Sympy [A] time = 1.11102, size = 44, normalized size = 1.02

$$\frac{ax^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| a^2x^4e^{2i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)}+x\operatorname{acos}\left(ax^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x**2),x)
```

```
[Out] a*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), a**2*x**4*exp_polar(2*I*pi))/(2
*gamma(7/4)) + x*acos(a*x**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arccos\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2),x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x^2), x)
```

3.51 $\int \frac{\cos^{-1}(ax^2)}{x} dx$

Optimal. Leaf size=62

$$-\frac{1}{4}i\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax^2)}\right)-\frac{1}{4}i\cos^{-1}\left(ax^2\right)^2+\frac{1}{2}\cos^{-1}\left(ax^2\right)\log\left(1+e^{2i\cos^{-1}(ax^2)}\right)$$

```
[Out] (-I/4)*ArcCos[a*x^2]^2 + (ArcCos[a*x^2]*Log[1 + E^((2*I)*ArcCos[a*x^2])])/2
 - (I/4)*PolyLog[2, -E^((2*I)*ArcCos[a*x^2])]
```

Rubi [A] time = 0.0584569, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4831, 3719, 2190, 2279, 2391}

$$-\frac{1}{4}i\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax^2)}\right)-\frac{1}{4}i\cos^{-1}\left(ax^2\right)^2+\frac{1}{2}\cos^{-1}\left(ax^2\right)\log\left(1+e^{2i\cos^{-1}(ax^2)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x^2]/x,x]
```

```
[Out] (-I/4)*ArcCos[a*x^2]^2 + (ArcCos[a*x^2]*Log[1 + E^((2*I)*ArcCos[a*x^2])])/2
 - (I/4)*PolyLog[2, -E^((2*I)*ArcCos[a*x^2])]
```

Rule 4831

```
Int[ArcCos[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> -Dist[p^(-1), Subst[In
t[x^n*Tan[x], x], x, ArcCos[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}\left(ax^2\right)}{x}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}\left(ax^2\right)\right)\right)\\
&= -\frac{1}{4}i\cos^{-1}\left(ax^2\right)^2 + i\,\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(ax^2\right)\right)\\
&= -\frac{1}{4}i\cos^{-1}\left(ax^2\right)^2 + \frac{1}{2}\cos^{-1}\left(ax^2\right)\log\left(1+e^{2i\cos^{-1}\left(ax^2\right)}\right) - \frac{1}{2}\,\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, \cos^{-1}\left(ax^2\right.\right.\\
&= -\frac{1}{4}i\cos^{-1}\left(ax^2\right)^2 + \frac{1}{2}\cos^{-1}\left(ax^2\right)\log\left(1+e^{2i\cos^{-1}\left(ax^2\right)}\right) + \frac{1}{4}i\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\cos^{-1}\left(ax^2\right)}\right.\\
&= -\frac{1}{4}i\cos^{-1}\left(ax^2\right)^2 + \frac{1}{2}\cos^{-1}\left(ax^2\right)\log\left(1+e^{2i\cos^{-1}\left(ax^2\right)}\right) - \frac{1}{4}i\text{Li}_2\left(-e^{2i\cos^{-1}\left(ax^2\right)}\right)
\end{aligned}$$

Mathematica [A] time = 0.0309198, size = 56, normalized size = 0.9

$$-\frac{1}{4}i\left(\text{PolyLog}\left(2,-e^{2i\cos^{-1}\left(ax^2\right)}\right)+\cos^{-1}\left(ax^2\right)\left(\cos^{-1}\left(ax^2\right)+2i\log\left(1+e^{2i\cos^{-1}\left(ax^2\right)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x^2]/x,x]
```

```
[Out] (-I/4)*(ArcCos[a*x^2]*(ArcCos[a*x^2] + (2*I)*Log[1 + E^((2*I)*ArcCos[a*x^2]
)]) + PolyLog[2, -E^((2*I)*ArcCos[a*x^2])])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{\arccos\left(a x^{2}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x^2)/x,x)
```

```
[Out] int(arccos(a*x^2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos\left(a x^{2}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(arccos(a*x^2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arccos\left(a x^{2}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}\left(a x^{2}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x**2)/x,x)
```

```
[Out] Integral(acos(a*x**2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos\left(ax^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x^2)/x, x)
```

3.52 $\int \frac{\cos^{-1}(ax^2)}{x^2} dx$

Optimal. Leaf size=29

$$-2\sqrt{a}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{a}x\right),-1\right)-\frac{\cos^{-1}\left(ax^2\right)}{x}$$

```
[Out] -(ArcCos[a*x^2]/x) - 2*Sqrt[a]*EllipticF[ArcSin[Sqrt[a]*x], -1]
```

Rubi [A] time = 0.014442, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4843, 12, 221}

$$-\frac{\cos^{-1}\left(ax^2\right)}{x}-2\sqrt{a}F\left(\sin^{-1}\left(\sqrt{a}x\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x^2]/x^2,x]
```

```
[Out] -(ArcCos[a*x^2]/x) - 2*Sqrt[a]*EllipticF[ArcSin[Sqrt[a]*x], -1]
```

Rule 4843

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcCos[u]))/(d*(m + 1)), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 - u^2], x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{-1}\left(ax^2\right)}{x^2}\,dx &= -\frac{\cos^{-1}\left(ax^2\right)}{x} - \int \frac{2a}{\sqrt{1-a^2x^4}}\,dx\\ &= -\frac{\cos^{-1}\left(ax^2\right)}{x} - (2a)\int \frac{1}{\sqrt{1-a^2x^4}}\,dx\\ &= -\frac{\cos^{-1}\left(ax^2\right)}{x} - 2\sqrt{a}F\left(\sin^{-1}\left(\sqrt{a}x\right)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.0385381, size = 40, normalized size = 1.38

$$-\frac{\cos^{-1}\left(ax^2\right)+2i\sqrt{-a}x\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-a}x\right),-1\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x^2]/x^2,x]
```

```
[Out] -((ArcCos[a*x^2] + (2*I)*Sqrt[-a]*x*EllipticF[I*ArcSinh[Sqrt[-a]*x], -1])/x
)
```

Maple [B] time = 0.006, size = 57, normalized size = 2.

$$-\frac{\arccos\left(ax^2\right)}{x}-2\,\frac{\sqrt{a}\sqrt{-ax^2+1}\sqrt{ax^2+1}\text{EllipticF}\left(x\sqrt{a},i\right)}{\sqrt{-a^2x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x^2)/x^2,x)
```

```
[Out] -arccos(a*x^2)/x-2*a^(1/2)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)/(-a^2*x^4+1)^(1
/2)*EllipticF(x*a^(1/2),I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arccos\left(ax^2\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x^2)/x^2, x)
```

Sympy [A] time = 1.21139, size = 44, normalized size = 1.52

$$-\frac{ax\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right| a^2x^4e^{2i\pi}\right)}{2\Gamma\left(\frac{5}{4}\right)} - \frac{\mathrm{acos}\left(ax^2\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x**2)/x**2,x)
```

```
[Out] -a*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), a**2*x**4*exp_polar(2*I*pi))/(2*g
amma(5/4)) - acos(a*x**2)/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos\left(ax^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x^2)/x^2, x)
```

3.53 $\int x^2 \cos^{-1}\left(\frac{a}{x}\right) dx$

Optimal. Leaf size=58

$$-\frac{1}{6}ax^2\sqrt{1-\frac{a^2}{x^2}}-\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1-\frac{a^2}{x^2}}\right)+\frac{1}{3}x^3\sec^{-1}\left(\frac{x}{a}\right)$$

```
[Out] -(a*Sqrt[1 - a^2/x^2]*x^2)/6 + (x^3*ArcSec[x/a])/3 - (a^3*ArcTanh[Sqrt[1 -
a^2/x^2]])/6
```

Rubi [A] time = 0.036197, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4833, 5220, 266, 51, 63, 208}

$$-\frac{1}{6}ax^2\sqrt{1-\frac{a^2}{x^2}}-\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1-\frac{a^2}{x^2}}\right)+\frac{1}{3}x^3\sec^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[a/x],x]
```

```
[Out] -(a*Sqrt[1 - a^2/x^2]*x^2)/6 + (x^3*ArcSec[x/a])/3 - (a^3*ArcTanh[Sqrt[1 -
a^2/x^2]])/6
```

Rule 4833

```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 5220

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcSec[c*x]))/(d*(m + 1)), x] - Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \cos^{-1}\left(\frac{a}{x}\right) dx &= \int x^2 \sec^{-1}\left(\frac{x}{a}\right) dx \\
&= \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{3}a \int \frac{x}{\sqrt{1-\frac{a^2}{x^2}}} dx \\
&= \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right) + \frac{1}{6}a \operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}} dx, x, \frac{1}{x^2}\right) \\
&= -\frac{1}{6}a\sqrt{1-\frac{a^2}{x^2}}x^2 + \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right) + \frac{1}{12}a^3 \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}} dx, x, \frac{1}{x^2}\right) \\
&= -\frac{1}{6}a\sqrt{1-\frac{a^2}{x^2}}x^2 + \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{6}a \operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}} dx, x, \sqrt{1-\frac{a^2}{x^2}}\right) \\
&= -\frac{1}{6}a\sqrt{1-\frac{a^2}{x^2}}x^2 + \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{6}a^3 \tanh^{-1}\left(\sqrt{1-\frac{a^2}{x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0505763, size = 61, normalized size = 1.05

$$\frac{1}{3}x^3\cos^{-1}\left(\frac{a}{x}\right)-\frac{1}{6}a\left(x^2\sqrt{1-\frac{a^2}{x^2}}+a^2\log\left(x\left(\sqrt{1-\frac{a^2}{x^2}}+1\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[a/x],x]
```

```
[Out] (x^3*ArcCos[a/x])/3 - (a*(Sqrt[1 - a^2/x^2]*x^2 + a^2*Log[(1 + Sqrt[1 - a^2
/x^2])*x]))/6
```

Maple [A] time = 0.01, size = 56, normalized size = 1.

$$-a^3\left(-\frac{x^3}{3\,a^3}\arccos\left(\frac{a}{x}\right)+\frac{x^2}{6\,a^2}\sqrt{1-\frac{a^2}{x^2}}+\frac{1}{6}\mathrm{Artanh}\left(\frac{1}{\sqrt{1-\frac{a^2}{x^2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(a/x),x)
```

```
[Out] -a^3*(-1/3/a^3*x^3*arccos(a/x)+1/6/a^2*x^2*(1-a^2/x^2)^(1/2)+1/6*arctanh(1/
(1-a^2/x^2)^(1/2)))
```

Maxima [A] time = 1.46236, size = 97, normalized size = 1.67

$$\frac{1}{3}\,x^3\arccos\left(\frac{a}{x}\right)-\frac{1}{12}\left(a^2\log\left(\sqrt{-\frac{a^2}{x^2}+1}+1\right)-a^2\log\left(\sqrt{-\frac{a^2}{x^2}+1}-1\right)+2\,x^2\sqrt{-\frac{a^2}{x^2}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a/x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccos(a/x) - 1/12*(a^2*log(sqrt(-a^2/x^2 + 1) + 1) - a^2*log(sqrt(
-a^2/x^2 + 1) - 1) + 2*x^2*sqrt(-a^2/x^2 + 1))*a
```

Fricas [A] time = 2.53943, size = 207, normalized size = 3.57

$$\frac{1}{6}\,a^{3}\log\left(x\sqrt{-\frac{a^{2}-x^{2}}{x^{2}}}-x\right)-\frac{1}{6}\,ax^{2}\sqrt{-\frac{a^{2}-x^{2}}{x^{2}}}+\frac{1}{3}\left(x^{3}-1\right)\arccos\left(\frac{a}{x}\right)+\frac{2}{3}\,\arctan\left(\frac{x\sqrt{-\frac{a^{2}-x^{2}}{x^{2}}}-x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a/x),x, algorithm="fricas")
```

```
[Out] 1/6*a^3*log(x*sqrt(-(a^2 - x^2)/x^2) - x) - 1/6*a*x^2*sqrt(-(a^2 - x^2)/x^2
) + 1/3*(x^3 - 1)*arccos(a/x) + 2/3*arctan((x*sqrt(-(a^2 - x^2)/x^2) - x)/a
)
```

Sympy [C] time = 5.29622, size = 99, normalized size = 1.71

$$-\frac{a\left(\begin{cases}\frac{a^{2}\operatorname{acosh}\left(\frac{x}{a}\right)}{2}+\frac{ax\sqrt{-1+\frac{x^{2}}{a^{2}}}}{2} & \text{for}\ \frac{|x^{2}|}{|a^{2}|}>1\\ -\frac{ia^{2}\operatorname{asin}\left(\frac{x}{a}\right)}{2}+\frac{iax}{2\sqrt{1-\frac{x^{2}}{a^{2}}}}-\frac{ix^{3}}{2a\sqrt{1-\frac{x^{2}}{a^{2}}}} & \text{otherwise}\end{cases}\right)}{3}+\frac{x^{3}\operatorname{acos}\left(\frac{a}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(a/x),x)
```

```
[Out] -a*Piecewise((a**2*acosh(x/a)/2 + a*x*sqrt(-1 + x**2/a**2)/2, Abs(x**2)/Abs
(a**2) > 1), (-I*a**2*asin(x/a)/2 + I*a*x/(2*sqrt(1 - x**2/a**2)) - I*x**3/
(2*a*sqrt(1 - x**2/a**2)), True))/3 + x**3*acos(a/x)/3
```

Giac [A] time = 1.29039, size = 95, normalized size = 1.64

$$\frac{1}{3}\,x^{3}\arccos\left(\frac{a}{x}\right)-\frac{1}{12}\left(a^{2}\log\left(a^{2}\right)\operatorname{sgn}\left(x\right)-\frac{2\,a^{2}\log\left(\left|-x+\sqrt{-a^{2}+x^{2}}\right|\right)}{\operatorname{sgn}\left(x\right)}+\frac{2\,\sqrt{-a^{2}+x^{2}}x}{\operatorname{sgn}\left(x\right)}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a/x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arccos(a/x) - 1/12*(a^2*log(a^2)*sgn(x) - 2*a^2*log(abs(-x + sqrt(-
a^2 + x^2)))/sgn(x) + 2*sqrt(-a^2 + x^2)*x/sgn(x))*a
```

3.54 $\int x \cos^{-1}\left(\frac{a}{x}\right) dx$

Optimal. Leaf size=34

$$\frac{1}{2}x^2 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{2}ax\sqrt{1 - \frac{a^2}{x^2}}$$

```
[Out] -(a*Sqrt[1 - a^2/x^2]*x)/2 + (x^2*ArcSec[x/a])/2
```

Rubi [A] time = 0.0165922, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4833, 5220, 191}

$$\frac{1}{2}x^2 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{2}ax\sqrt{1 - \frac{a^2}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCos[a/x],x]
```

```
[Out] -(a*Sqrt[1 - a^2/x^2]*x)/2 + (x^2*ArcSec[x/a])/2
```

Rule 4833

```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 5220

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcSec[c*x]))/(d*(m + 1)), x] - Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\cos^{-1}\left(\frac{a}{x}\right)dx &= \int x\sec^{-1}\left(\frac{x}{a}\right)dx\\ &= \frac{1}{2}x^2\sec^{-1}\left(\frac{x}{a}\right)-\frac{1}{2}a\int\frac{1}{\sqrt{1-\frac{a^2}{x^2}}}dx\\ &= -\frac{1}{2}a\sqrt{1-\frac{a^2}{x^2}}x+\frac{1}{2}x^2\sec^{-1}\left(\frac{x}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.021294, size = 33, normalized size = 0.97

$$\frac{1}{2}\left(x^2\cos^{-1}\left(\frac{a}{x}\right)-ax\sqrt{1-\frac{a^2}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCos[a/x],x]
```

```
[Out] (-(a*Sqrt[1 - a^2/x^2]*x) + x^2*ArcCos[a/x])/2
```

Maple [A] time = 0.004, size = 39, normalized size = 1.2

$$-a^2\left(-\frac{x^2}{2a^2}\arccos\left(\frac{a}{x}\right)+\frac{x}{2a}\sqrt{1-\frac{a^2}{x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(a/x),x)
```

```
[Out] -a^2*(-1/2/a^2*x^2*arccos(a/x)+1/2*(1-a^2/x^2)^(1/2)/a*x)
```

Maxima [A] time = 1.44162, size = 38, normalized size = 1.12

$$\frac{1}{2}x^2\arccos\left(\frac{a}{x}\right)-\frac{1}{2}ax\sqrt{-\frac{a^2}{x^2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a/x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccos(a/x) - 1/2*a*x*sqrt(-a^2/x^2 + 1)
```

Fricas [A] time = 2.56326, size = 73, normalized size = 2.15

$$\frac{1}{2}x^2\arccos\left(\frac{a}{x}\right) - \frac{1}{2}ax\sqrt{-\frac{a^2 - x^2}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a/x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*arccos(a/x) - 1/2*a*x*sqrt(-(a^2 - x^2)/x^2)
```

Sympy [A] time = 2.68407, size = 49, normalized size = 1.44

$$-\frac{a\left(\begin{cases} a\sqrt{-1+\frac{x^2}{a^2}} & \text{for}\ \frac{|x^2|}{|a^2|} > 1 \\ ia\sqrt{1-\frac{x^2}{a^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x^2\operatorname{acos}\left(\frac{a}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(a/x),x)
```

```
[Out] -a*Piecewise((a*sqrt(-1 + x**2/a**2), Abs(x**2)/Abs(a**2) > 1), (I*a*sqrt(1
 - x**2/a**2), True))/2 + x**2*acos(a/x)/2
```

Giac [A] time = 1.24789, size = 58, normalized size = 1.71

$$\frac{1}{2}x^2\arccos\left(\frac{a}{x}\right) + \frac{1}{2}\left(\sqrt{-a^2}\operatorname{sgn}(x) - \frac{\sqrt{-a^2+x^2}}{\operatorname{sgn}(x)}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a/x),x, algorithm="giac")

[Out] 1/2*x^2*arccos(a/x) + 1/2*(sqrt(-a^2)*sgn(x) - sqrt(-a^2 + x^2)/sgn(x))*a
```

3.55 $\int \cos^{-1}\left(\frac{a}{x}\right) dx$

Optimal. Leaf size=27

$$x\sec^{-1}\left(\frac{x}{a}\right) - a\tanh^{-1}\left(\sqrt{1-\frac{a^2}{x^2}}\right)$$

```
[Out] x*ArcSec[x/a] - a*ArcTanh[Sqrt[1 - a^2/x^2]]
```

Rubi [A] time = 0.0169255, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {4833, 5214, 266, 63, 208}

$$x\sec^{-1}\left(\frac{x}{a}\right) - a\tanh^{-1}\left(\sqrt{1-\frac{a^2}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a/x],x]
```

```
[Out] x*ArcSec[x/a] - a*ArcTanh[Sqrt[1 - a^2/x^2]]
```

Rule 4833

```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 5214

```
Int[ArcSec[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSec[c*x], x] - Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \cos^{-1}\left(\frac{a}{x}\right)dx &= \int \sec^{-1}\left(\frac{x}{a}\right)dx \\
&= x\sec^{-1}\left(\frac{x}{a}\right) - a\int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}x}\,dx \\
&= x\sec^{-1}\left(\frac{x}{a}\right) + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, \frac{1}{x^2}\right) \\
&= x\sec^{-1}\left(\frac{x}{a}\right) - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-\frac{a^2}{x^2}}\right)}{a} \\
&= x\sec^{-1}\left(\frac{x}{a}\right) - a\tanh^{-1}\left(\sqrt{1-\frac{a^2}{x^2}}\right)
\end{aligned}
$$

Mathematica [B] time = 0.1011, size = 84, normalized size = 3.11

$$
x\cos^{-1}\left(\frac{a}{x}\right) - \frac{a\sqrt{x^2-a^2}\left(\log\left(\frac{x}{\sqrt{x^2-a^2}}+1\right) - \log\left(1-\frac{x}{\sqrt{x^2-a^2}}\right)\right)}{2x\sqrt{1-\frac{a^2}{x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a/x],x]
```

```
[Out] x*ArcCos[a/x] - (a*Sqrt[-a^2 + x^2]*(-Log[1 - x/Sqrt[-a^2 + x^2]] + Log[1 +
 x/Sqrt[-a^2 + x^2]]))/(2*Sqrt[1 - a^2/x^2]*x)
```

Maple [A] time = 0.006, size = 30, normalized size = 1.1

$$-a\left(-\frac{x}{a}\arccos\left(\frac{a}{x}\right)+\operatorname{Artanh}\left(\frac{1}{\sqrt{1-\frac{a^2}{x^2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a/x),x)
```

```
[Out] -a*(-1/a*x*arccos(a/x)+arctanh(1/(1-a^2/x^2)^(1/2)))
```

Maxima [A] time = 1.45402, size = 61, normalized size = 2.26

$$-\frac{1}{2}a\left(\log\left(\sqrt{-\frac{a^2}{x^2}+1}+1\right)-\log\left(\sqrt{-\frac{a^2}{x^2}+1}-1\right)\right)+x\arccos\left(\frac{a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(sqrt(-a^2/x^2 + 1) + 1) - log(sqrt(-a^2/x^2 + 1) - 1)) + x*arcc
os(a/x)
```

Fricas [B] time = 2.52093, size = 140, normalized size = 5.19

$$(x-1)\arccos\left(\frac{a}{x}\right)+a\log\left(x\sqrt{-\frac{a^2-x^2}{x^2}}-x\right)+2\arctan\left(\frac{x\sqrt{-\frac{a^2-x^2}{x^2}}-x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x),x, algorithm="fricas")
```

```
[Out] (x - 1)*arccos(a/x) + a*log(x*sqrt(-(a^2 - x^2)/x^2) - x) + 2*arctan((x*sqr
t(-(a^2 - x^2)/x^2) - x)/a)
```

Sympy [A] time = 2.27124, size = 29, normalized size = 1.07

$$-a\left(\begin{cases}\operatorname{acosh}\left(\frac{x}{a}\right) & \text{for}\ \frac{|x^2|}{|a^2|}>1\\ -i\operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise}\end{cases}\right)+x\operatorname{acos}\left(\frac{a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a/x),x)
```

```
[Out] -a*Piecewise((acosh(x/a), Abs(x**2)/Abs(a**2) > 1), (-I*asin(x/a), True)) +
 x*acos(a/x)
```

Giac [A] time = 1.26154, size = 58, normalized size = 2.15

$$-\frac{1}{2}\left(\log\left(a^2\right)\operatorname{sgn}(x)-\frac{2\log\left(\left|-x+\sqrt{-a^2+x^2}\right|\right)}{\operatorname{sgn}(x)}\right)a+x\arccos\left(\frac{a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x),x, algorithm="giac")
```

```
[Out] -1/2*(log(a^2)*sgn(x) - 2*log(abs(-x + sqrt(-a^2 + x^2)))/sgn(x))*a + x*arc
cos(a/x)
```

3.56 $\int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x} dx$

Optimal. Leaf size=60

$$\frac{1}{2}i\text{PolyLog}\left(2,-e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right)+\frac{1}{2}i\cos^{-1}\left(\frac{a}{x}\right)^2-\cos^{-1}\left(\frac{a}{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right)$$

```
[Out] (I/2)*ArcCos[a/x]^2 - ArcCos[a/x]*Log[1 + E^((2*I)*ArcCos[a/x])] + (I/2)*Po
lyLog[2, -E^((2*I)*ArcCos[a/x])]
```

Rubi [A] time = 0.0546249, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4831, 3719, 2190, 2279, 2391}

$$\frac{1}{2}i\text{PolyLog}\left(2,-e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right)+\frac{1}{2}i\cos^{-1}\left(\frac{a}{x}\right)^2-\cos^{-1}\left(\frac{a}{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a/x]/x,x]
```

```
[Out] (I/2)*ArcCos[a/x]^2 - ArcCos[a/x]*Log[1 + E^((2*I)*ArcCos[a/x])] + (I/2)*Po
lyLog[2, -E^((2*I)*ArcCos[a/x])]
```

Rule 4831

```
Int[ArcCos[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> -Dist[p^(-1), Subst[In
t[x^n*Tan[x], x], x, ArcCos[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x}\,dx &= \text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}\left(\frac{a}{x}\right)\right)\\
&= \frac{1}{2}i\cos^{-1}\left(\frac{a}{x}\right)^2 - 2i\,\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\frac{a}{x}\right)\right)\\
&= \frac{1}{2}i\cos^{-1}\left(\frac{a}{x}\right)^2 - \cos^{-1}\left(\frac{a}{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right) + \text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, \cos^{-1}\left(\frac{a}{x}\right)\right)\\
&= \frac{1}{2}i\cos^{-1}\left(\frac{a}{x}\right)^2 - \cos^{-1}\left(\frac{a}{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right)\\
&= \frac{1}{2}i\cos^{-1}\left(\frac{a}{x}\right)^2 - \cos^{-1}\left(\frac{a}{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right) + \frac{1}{2}i\text{Li}_2\left(-e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right)
\end{aligned}$$

Mathematica [A] time = 0.0208701, size = 60, normalized size = 1.

$$\frac{1}{2}i\text{PolyLog}\left(2,-e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right)+\frac{1}{2}i\cos^{-1}\left(\frac{a}{x}\right)^2-\cos^{-1}\left(\frac{a}{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a/x]/x,x]
```

```
[Out] (I/2)*ArcCos[a/x]^2 - ArcCos[a/x]*Log[1 + E^((2*I)*ArcCos[a/x])] + (I/2)*Po
lyLog[2, -E^((2*I)*ArcCos[a/x])]
```

Maple [A] time = 0.055, size = 77, normalized size = 1.3

$$\frac{i}{2}\left(\arccos\left(\frac{a}{x}\right)\right)^{2}-\arccos\left(\frac{a}{x}\right)\ln\left(1+\left(\frac{a}{x}+i\sqrt{1-\frac{a^{2}}{x^{2}}}\right)^{2}\right)+\frac{i}{2}\mathrm{polylog}\left(2,-\left(\frac{a}{x}+i\sqrt{1-\frac{a^{2}}{x^{2}}}\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a/x)/x,x)
```

```
[Out] 1/2*I*arccos(a/x)^2-arccos(a/x)*ln(1+(a/x+I*(1-a^2/x^2)^(1/2))^2)+1/2*I*pol
ylog(2,-(a/x+I*(1-a^2/x^2)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-i\,a^{2}\int-\frac{\log(x)}{a^{2}x-x^{3}}\,dx-a\int-\frac{\sqrt{a+x}\sqrt{-a+x}\log(x)}{a^{2}x-x^{3}}\,dx+\arctan\left(\sqrt{a+x}\sqrt{-a+x},a\right)\log(x)-\frac{1}{2}i\,\log\left(x^{2}\right)\log(x)+\frac{1}{2}i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x,x, algorithm="maxima")
```

```
[Out] -I*a^2*integrate(-log(x)/(a^2*x - x^3), x) - a*integrate(-sqrt(a + x)*sqrt(
-a + x)*log(x)/(a^2*x - x^3), x) + arctan2(sqrt(a + x)*sqrt(-a + x), a)*log
(x) - 1/2*I*log(x^2)*log(x) + 1/2*I*log(x)^2 + 1/2*I*log(x)*log((a + x)/a)
+ 1/2*I*log(x)*log((a - x)/a) + 1/2*I*dilog(x/a) + 1/2*I*dilog(-x/a)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arccos\left(\frac{a}{x}\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x,x, algorithm="fricas")
```

```
[Out] integral(arccos(a/x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}\left(\frac{a}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a/x)/x,x)
```

```
[Out] Integral(acos(a/x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(a/x)/x, x)
```

3.57 $\int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=30

$$\frac{\sqrt{1-\frac{a^2}{x^2}}}{a}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{x}$$

```
[Out] Sqrt[1 - a^2/x^2]/a - ArcSec[x/a]/x
```

Rubi [A] time = 0.0218937, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4833, 5220, 261}

$$\frac{\sqrt{1-\frac{a^2}{x^2}}}{a}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a/x]/x^2,x]
```

```
[Out] Sqrt[1 - a^2/x^2]/a - ArcSec[x/a]/x
```

Rule 4833

```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 5220

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcSec[c*x]))/(d*(m + 1)), x] - Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx &= \int \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x^2}\,dx\\ &= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{x} + a\int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}x^3}\,dx\\ &= \frac{\sqrt{1-\frac{a^2}{x^2}}}{a} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.0185694, size = 30, normalized size = 1.

$$\frac{\sqrt{1-\frac{a^2}{x^2}}}{a} - \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a/x]/x^2,x]
```

```
[Out] Sqrt[1 - a^2/x^2]/a - ArcCos[a/x]/x
```

Maple [A] time = 0.001, size = 32, normalized size = 1.1

$$-\frac{1}{a}\left(\frac{a}{x}\arccos\left(\frac{a}{x}\right) - \sqrt{1-\frac{a^2}{x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a/x)/x^2,x)
```

```
[Out] -1/a*(1/x*a*arccos(a/x)-(1-a^2/x^2)^(1/2))
```

Maxima [A] time = 1.43795, size = 42, normalized size = 1.4

$$-\frac{\frac{a\arccos\left(\frac{a}{x}\right)}{x} - \sqrt{-\frac{a^2}{x^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^2,x, algorithm="maxima")
```

```
[Out] -(a*arccos(a/x)/x - sqrt(-a^2/x^2 + 1))/a
```

Fricas [A] time = 2.39408, size = 69, normalized size = 2.3

$$-\frac{a\arccos\left(\frac{a}{x}\right)-x\sqrt{-\frac{a^2-x^2}{x^2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*arccos(a/x) - x*sqrt(-(a^2 - x^2)/x^2))/(a*x)
```

Sympy [A] time = 2.34016, size = 26, normalized size = 0.87

$$\begin{cases} -\frac{\operatorname{acos}\left(\frac{a}{x}\right)}{x}+\frac{\sqrt{-\frac{a^2}{x^2}+1}}{a} & \text{for } a\neq 0\\ -\frac{\pi}{2x} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a/x)/x**2,x)
```

```
[Out] Piecewise((-acos(a/x)/x + sqrt(-a**2/x**2 + 1)/a, Ne(a, 0)), (-pi/(2*x), Tr
ue))
```

Giac [A] time = 1.2283, size = 42, normalized size = 1.4

$$-\frac{\frac{a\arccos\left(\frac{a}{x}\right)}{x}-\sqrt{-\frac{a^2}{x^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^2,x, algorithm="giac")
```

```
[Out] -(a*arccos(a/x)/x - sqrt(-a^2/x^2 + 1))/a
```

3.58 $\int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{1-\frac{a^2}{x^2}}}{4ax}-\frac{\csc^{-1}\left(\frac{x}{a}\right)}{4a^2}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

```
[Out] Sqrt[1 - a^2/x^2]/(4*a*x) - ArcCsc[x/a]/(4*a^2) - ArcSec[x/a]/(2*x^2)
```

Rubi [A] time = 0.0325045, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4833, 5220, 335, 321, 216}

$$\frac{\sqrt{1-\frac{a^2}{x^2}}}{4ax}-\frac{\csc^{-1}\left(\frac{x}{a}\right)}{4a^2}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a/x]/x^3,x]
```

```
[Out] Sqrt[1 - a^2/x^2]/(4*a*x) - ArcCsc[x/a]/(4*a^2) - ArcSec[x/a]/(2*x^2)
```

Rule 4833

```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 5220

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcSec[c*x]))/(d*(m + 1)), x] - Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx &= \int \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x^3}\,dx\\
&= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2} + \frac{1}{2}a\int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}\,x^4}\,dx\\
&= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2} - \frac{1}{2}a\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-a^2x^2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\sqrt{1-\frac{a^2}{x^2}}}{4ax} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-a^2x^2}}\,dx, x, \frac{1}{x}\right)}{4a}\\
&= \frac{\sqrt{1-\frac{a^2}{x^2}}}{4ax} - \frac{\csc^{-1}\left(\frac{x}{a}\right)}{4a^2} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2}
\end{aligned}$$

Mathematica [A] time = 0.0244137, size = 50, normalized size = 0.98

$$\frac{ax\sqrt{1-\frac{a^2}{x^2}} - 2a^2\cos^{-1}\left(\frac{a}{x}\right) - x^2\sin^{-1}\left(\frac{a}{x}\right)}{4a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a/x]/x^3,x]
```

```
[Out] (a*Sqrt[1 - a^2/x^2]*x - 2*a^2*ArcCos[a/x] - x^2*ArcSin[a/x])/(4*a^2*x^2)
```

Maple [A] time = 0.006, size = 47, normalized size = 0.9

$$-\frac{1}{a^2}\left(\frac{a^2}{2\,x^2}\arccos\left(\frac{a}{x}\right)-\frac{a}{4\,x}\sqrt{1-\frac{a^2}{x^2}}+\frac{1}{4}\arcsin\left(\frac{a}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a/x)/x^3,x)
```

```
[Out] -1/a^2*(1/2*arccos(a/x)*a^2/x^2-1/4*a/x*(1-a^2/x^2)^(1/2)+1/4*arcsin(a/x))
```

Maxima [A] time = 1.46544, size = 104, normalized size = 2.04

$$-\frac{1}{4}\,a\left(\frac{x\sqrt{-\frac{a^2}{x^2}+1}}{a^2x^2\left(\frac{a^2}{x^2}-1\right)-a^4}-\frac{\arctan\left(\frac{x\sqrt{-\frac{a^2}{x^2}+1}}{a}\right)}{a^3}\right)-\frac{\arccos\left(\frac{a}{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*(x*sqrt(-a^2/x^2 + 1)/(a^2*x^2*(a^2/x^2 - 1) - a^4) - arctan(x*sqrt(
-a^2/x^2 + 1)/a)/a^3) - 1/2*arccos(a/x)/x^2
```

Fricas [A] time = 2.47003, size = 97, normalized size = 1.9

$$\frac{ax\sqrt{-\frac{a^2-x^2}{x^2}}-\left(2\,a^2-x^2\right)\arccos\left(\frac{a}{x}\right)}{4\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a*x*sqrt(-(a^2 - x^2)/x^2) - (2*a^2 - x^2)*arccos(a/x))/(a^2*x^2)
```

Sympy [C] time = 5.30487, size = 102, normalized size = 2.

$$\frac{a\left(\begin{cases}\frac{i\sqrt{\frac{a^2}{x^2}-1}}{2a^2x}+\frac{i\operatorname{acosh}\left(\frac{a}{x}\right)}{2a^3} & \text{for } \frac{|a^2|}{|x^2|}>1\\ -\frac{1}{2x^3\sqrt{-\frac{a^2}{x^2}+1}}+\frac{1}{2a^2x\sqrt{-\frac{a^2}{x^2}+1}}-\frac{\operatorname{asin}\left(\frac{a}{x}\right)}{2a^3} & \text{otherwise}\end{cases}\right)}{2}-\frac{\operatorname{acos}\left(\frac{a}{x}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a/x)/x**3,x)
```

```
[Out] a*Piecewise((I*sqrt(a**2/x**2 - 1)/(2*a**2*x) + I*acosh(a/x)/(2*a**3), Abs(
a**2)/Abs(x**2) > 1), (-1/(2*x**3*sqrt(-a**2/x**2 + 1)) + 1/(2*a**2*x*sqrt(
-a**2/x**2 + 1)) - asin(a/x)/(2*a**3), True))/2 - acos(a/x)/(2*x**2)
```

Giac [A] time = 1.28026, size = 84, normalized size = 1.65

$$\frac{1}{4}\,a\left(\frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a^3\operatorname{sgn}(x)}+\frac{\sqrt{-a^2+x^2}}{a^2x^2\operatorname{sgn}(x)}\right)-\frac{\arccos\left(\frac{a}{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*a*(arctan(sqrt(-a^2 + x^2)/a)/(a^3*sgn(x)) + sqrt(-a^2 + x^2)/(a^2*x^2*
sgn(x))) - 1/2*arccos(a/x)/x^2
```

3.59 $\int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx$

Optimal. Leaf size=56

$$-\frac{\left(1-\frac{a^2}{x^2}\right)^{3/2}}{9a^3}+\frac{\sqrt{1-\frac{a^2}{x^2}}}{3a^3}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

```
[Out] Sqrt[1 - a^2/x^2]/(3*a^3) - (1 - a^2/x^2)^(3/2)/(9*a^3) - ArcSec[x/a]/(3*x^
3)
```

Rubi [A] time = 0.0383783, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4833, 5220, 266, 43}

$$-\frac{\left(1-\frac{a^2}{x^2}\right)^{3/2}}{9a^3}+\frac{\sqrt{1-\frac{a^2}{x^2}}}{3a^3}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a/x]/x^4,x]
```

```
[Out] Sqrt[1 - a^2/x^2]/(3*a^3) - (1 - a^2/x^2)^(3/2)/(9*a^3) - ArcSec[x/a]/(3*x^
3)
```

Rule 4833

```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 5220

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcSec[c*x]))/(d*(m + 1)), x] - Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx &= \int \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x^4}\,dx\\
&= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3} + \frac{1}{3}a\int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}x^5}\,dx\\
&= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3} - \frac{1}{6}a\,\text{Subst}\left(\int \frac{x}{\sqrt{1-a^2x}}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3} - \frac{1}{6}a\,\text{Subst}\left(\int \left(\frac{1}{a^2\sqrt{1-a^2x}} - \frac{\sqrt{1-a^2x}}{a^2}\right)dx, x, \frac{1}{x^2}\right)\\
&= \frac{\sqrt{1-\frac{a^2}{x^2}}}{3a^3} - \frac{\left(1-\frac{a^2}{x^2}\right)^{3/2}}{9a^3} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.0308231, size = 47, normalized size = 0.84

$$\frac{x\sqrt{1-\frac{a^2}{x^2}}\left(a^2+2x^2\right)-3a^3\cos^{-1}\left(\frac{a}{x}\right)}{9a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a/x]/x^4,x]
```

```
[Out] (Sqrt[1 - a^2/x^2]*x*(a^2 + 2*x^2) - 3*a^3*ArcCos[a/x])/(9*a^3*x^3)
```

Maple [A] time = 0.006, size = 55, normalized size = 1.

$$-\frac{1}{a^3}\left(\frac{a^3}{3x^3}\arccos\left(\frac{a}{x}\right)-\frac{a^2}{9x^2}\sqrt{1-\frac{a^2}{x^2}}-\frac{2}{9}\sqrt{1-\frac{a^2}{x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a/x)/x^4,x)
```

```
[Out] -1/a^3*(1/3*arccos(a/x)*a^3/x^3-1/9*a^2/x^2*(1-a^2/x^2)^(1/2)-2/9*(1-a^2/x^
2)^(1/2))
```

Maxima [A] time = 1.44467, size = 66, normalized size = 1.18

$$-\frac{1}{9}a\left(\frac{\left(-\frac{a^2}{x^2}+1\right)^{\frac{3}{2}}}{a^4}-\frac{3\sqrt{-\frac{a^2}{x^2}+1}}{a^4}\right)-\frac{\arccos\left(\frac{a}{x}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/9*a*((-a^2/x^2 + 1)^(3/2)/a^4 - 3*sqrt(-a^2/x^2 + 1)/a^4) - 1/3*arccos(a
/x)/x^3
```

Fricas [A] time = 2.46169, size = 104, normalized size = 1.86

$$-\frac{3a^3\arccos\left(\frac{a}{x}\right)-\left(a^2x+2x^3\right)\sqrt{-\frac{a^2-x^2}{x^2}}}{9a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(3*a^3*arccos(a/x) - (a^2*x + 2*x^3)*sqrt(-(a^2 - x^2)/x^2))/(a^3*x^3)
```

Sympy [A] time = 6.3656, size = 102, normalized size = 1.82

$$\frac{a\left(\begin{cases}\frac{\sqrt{-1+\frac{x^2}{a^2}}}{3ax^3}+\frac{2\sqrt{-1+\frac{x^2}{a^2}}}{3a^3x} & \text{for } \frac{|x^2|}{|a^2|}>1\\ \frac{i\sqrt{1-\frac{x^2}{a^2}}}{3ax^3}+\frac{2i\sqrt{1-\frac{x^2}{a^2}}}{3a^3x} & \text{otherwise}\end{cases}\right)}{3}-\frac{\operatorname{acos}\left(\frac{a}{x}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a/x)/x**4,x)
```

```
[Out] a*Piecewise((sqrt(-1 + x**2/a**2)/(3*a*x**3) + 2*sqrt(-1 + x**2/a**2)/(3*a*
*3*x), Abs(x**2)/Abs(a**2) > 1), (I*sqrt(1 - x**2/a**2)/(3*a*x**3) + 2*I*sq
rt(1 - x**2/a**2)/(3*a**3*x), True))/3 - acos(a/x)/(3*x**3)
```

Giac [A] time = 1.31389, size = 88, normalized size = 1.57

$$-\frac{\arccos\left(\frac{a}{x}\right)}{3\,x^3}+\frac{4\left(a^2+3\left(x-\sqrt{-a^2+x^2}\right)^2\right)a}{9\left(a^2+\left(x-\sqrt{-a^2+x^2}\right)^2\right)^3\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*arccos(a/x)/x^3 + 4/9*(a^2 + 3*(x - sqrt(-a^2 + x^2))^2)*a/((a^2 + (x
- sqrt(-a^2 + x^2))^2)^3*sgn(x))
```

3.60 $\int x^2 \cos^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=78

$$-\frac{1}{18}\sqrt{1-x}x^{5/2}-\frac{5}{72}\sqrt{1-x}x^{3/2}+\frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)-\frac{5}{48}\sqrt{1-x}\sqrt{x}-\frac{5}{96}\sin^{-1}(1-2x)$$

```
[Out] (-5*Sqrt[1 - x]*Sqrt[x])/48 - (5*Sqrt[1 - x]*x^(3/2))/72 - (Sqrt[1 - x]*x^(
5/2))/18 + (x^3*ArcCos[Sqrt[x]])/3 - (5*ArcSin[1 - 2*x])/96
```

Rubi [A] time = 0.028212, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4843, 12, 50, 53, 619, 216}

$$-\frac{1}{18}\sqrt{1-x}x^{5/2}-\frac{5}{72}\sqrt{1-x}x^{3/2}+\frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)-\frac{5}{48}\sqrt{1-x}\sqrt{x}-\frac{5}{96}\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[Sqrt[x]],x]
```


```
[Out] (-5*Sqrt[1 - x]*Sqrt[x])/48 - (5*Sqrt[1 - x]*x^(3/2))/72 - (Sqrt[1 - x]*x^(
5/2))/18 + (x^3*ArcCos[Sqrt[x]])/3 - (5*ArcSin[1 - 2*x])/96
```


Rule 4843

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcCos[u]))/(d*(m + 1)), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 - u^2], x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^2\cos^{-1}\left(\sqrt{x}\right)dx &= \frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\int\frac{x^{5/2}}{2\sqrt{1-x}}\,dx\\
&= \frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)+\frac{1}{6}\int\frac{x^{5/2}}{\sqrt{1-x}}\,dx\\
&= -\frac{1}{18}\sqrt{1-x}x^{5/2}+\frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)+\frac{5}{36}\int\frac{x^{3/2}}{\sqrt{1-x}}\,dx\\
&= -\frac{5}{72}\sqrt{1-x}x^{3/2}-\frac{1}{18}\sqrt{1-x}x^{5/2}+\frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)+\frac{5}{48}\int\frac{\sqrt{x}}{\sqrt{1-x}}\,dx\\
&= -\frac{5}{48}\sqrt{1-x}\sqrt{x}-\frac{5}{72}\sqrt{1-x}x^{3/2}-\frac{1}{18}\sqrt{1-x}x^{5/2}+\frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)+\frac{5}{96}\int\frac{1}{\sqrt{1-x}\sqrt{x}}\,dx\\
&= -\frac{5}{48}\sqrt{1-x}\sqrt{x}-\frac{5}{72}\sqrt{1-x}x^{3/2}-\frac{1}{18}\sqrt{1-x}x^{5/2}+\frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)+\frac{5}{96}\int\frac{1}{\sqrt{x-x^2}}\,dx\\
&= -\frac{5}{48}\sqrt{1-x}\sqrt{x}-\frac{5}{72}\sqrt{1-x}x^{3/2}-\frac{1}{18}\sqrt{1-x}x^{5/2}+\frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)-\frac{5}{96}\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}}\,dx,x,\right.\\
&= -\frac{5}{48}\sqrt{1-x}\sqrt{x}-\frac{5}{72}\sqrt{1-x}x^{3/2}-\frac{1}{18}\sqrt{1-x}x^{5/2}+\frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)-\frac{5}{96}\sin^{-1}(1-2x)
\end{aligned}
$$

Mathematica [A] time = 0.0375544, size = 46, normalized size = 0.59

$$\frac{1}{144}\left(-\sqrt{-(x-1)x}\left(8x^2+10x+15\right)+48x^3\cos^{-1}\left(\sqrt{x}\right)+15\sin^{-1}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[Sqrt[x]],x]
```

```
[Out] (-(Sqrt[-((-1 + x)*x)]*(15 + 10*x + 8*x^2)) + 48*x^3*ArcCos[Sqrt[x]] + 15*A
rcSin[Sqrt[x]])/144
```

Maple [A] time = 0.004, size = 53, normalized size = 0.7

$$\frac{x^3}{3}\arccos\left(\sqrt{x}\right)-\frac{1}{18}x^{\frac{5}{2}}\sqrt{1-x}-\frac{5}{72}x^{\frac{3}{2}}\sqrt{1-x}-\frac{5}{48}\sqrt{1-x}\sqrt{x}+\frac{5}{48}\arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(x^(1/2)),x)
```

```
[Out] 1/3*x^3*arccos(x^(1/2))-1/18*x^(5/2)*(1-x)^(1/2)-5/72*x^(3/2)*(1-x)^(1/2)-5
/48*(1-x)^(1/2)*x^(1/2)+5/48*arcsin(x^(1/2))
```

Maxima [A] time = 1.43558, size = 70, normalized size = 0.9

$$\frac{1}{3}x^3\arccos\left(\sqrt{x}\right)-\frac{1}{18}x^{\frac{5}{2}}\sqrt{-x+1}-\frac{5}{72}x^{\frac{3}{2}}\sqrt{-x+1}-\frac{5}{48}\sqrt{x}\sqrt{-x+1}+\frac{5}{48}\arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccos(sqrt(x)) - 1/18*x^(5/2)*sqrt(-x + 1) - 5/72*x^(3/2)*sqrt(-x
+ 1) - 5/48*sqrt(x)*sqrt(-x + 1) + 5/48*arcsin(sqrt(x))
```

Fricas [A] time = 2.63229, size = 115, normalized size = 1.47

$$-\frac{1}{144}\left(8x^2+10x+15\right)\sqrt{x}\sqrt{-x+1}+\frac{1}{48}\left(16x^3-5\right)\arccos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/144*(8*x^2 + 10*x + 15)*sqrt(x)*sqrt(-x + 1) + 1/48*(16*x^3 - 5)*arccos(
sqrt(x))
```

Sympy [A] time = 10.1623, size = 73, normalized size = 0.94

$$\frac{x^3 \operatorname{acos}\left(\sqrt{x}\right)}{3} + \frac{\begin{cases} \frac{x^{\frac{3}{2}}(1-x)^{\frac{3}{2}}}{6} + \frac{3\sqrt{x}(1-2x)\sqrt{1-x}}{16} - \frac{\sqrt{x}\sqrt{1-x}}{2} + \frac{5\operatorname{asin}\left(\sqrt{x}\right)}{16} & \text{for } x \geq 0 \wedge x < 1 \end{cases}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(x**(1/2)),x)
```

```
[Out] x**3*acos(sqrt(x))/3 + Piecewise((x**(3/2)*(1 - x)**(3/2)/6 + 3*sqrt(x)*(1
- 2*x)*sqrt(1 - x)/16 - sqrt(x)*sqrt(1 - x)/2 + 5*asin(sqrt(x))/16, (x >= 0
) & (x < 1)))/3
```

Giac [A] time = 1.12811, size = 70, normalized size = 0.9

$$\frac{1}{3}\,x^3 \arccos\left(\sqrt{x}\right) - \frac{1}{18}\,x^{\frac{5}{2}}\sqrt{-x+1} - \frac{5}{72}\,x^{\frac{3}{2}}\sqrt{-x+1} - \frac{5}{48}\,\sqrt{x}\sqrt{-x+1} - \frac{5}{48}\,\arccos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arccos(sqrt(x)) - 1/18*x^(5/2)*sqrt(-x + 1) - 5/72*x^(3/2)*sqrt(-x
+ 1) - 5/48*sqrt(x)*sqrt(-x + 1) - 5/48*arccos(sqrt(x))
```

3.61 $\int x \cos^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=60

$$-\frac{1}{8}\sqrt{1-x}x^{3/2}+\frac{1}{2}x^2\cos^{-1}\left(\sqrt{x}\right)-\frac{3}{16}\sqrt{1-x}\sqrt{x}-\frac{3}{32}\sin^{-1}(1-2x)$$

```
[Out] (-3*Sqrt[1 - x]*Sqrt[x])/16 - (Sqrt[1 - x]*x^(3/2))/8 + (x^2*ArcCos[Sqrt[x]
])/2 - (3*ArcSin[1 - 2*x])/32
```

Rubi [A] time = 0.0191523, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {4843, 12, 50, 53, 619, 216}

$$-\frac{1}{8}\sqrt{1-x}x^{3/2}+\frac{1}{2}x^2\cos^{-1}\left(\sqrt{x}\right)-\frac{3}{16}\sqrt{1-x}\sqrt{x}-\frac{3}{32}\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCos[Sqrt[x]],x]
```

```
[Out] (-3*Sqrt[1 - x]*Sqrt[x])/16 - (Sqrt[1 - x]*x^(3/2))/8 + (x^2*ArcCos[Sqrt[x]
])/2 - (3*ArcSin[1 - 2*x])/32
```

Rule 4843

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcCos[u]))/(d*(m + 1)), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 - u^2], x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x\cos^{-1}\left(\sqrt{x}\right)\,dx &= \frac{1}{2}x^2\cos^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\int\frac{x^{3/2}}{2\sqrt{1-x}}\,dx\\
&= \frac{1}{2}x^2\cos^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int\frac{x^{3/2}}{\sqrt{1-x}}\,dx\\
&= -\frac{1}{8}\sqrt{1-x}x^{3/2}+\frac{1}{2}x^2\cos^{-1}\left(\sqrt{x}\right)+\frac{3}{16}\int\frac{\sqrt{x}}{\sqrt{1-x}}\,dx\\
&= -\frac{3}{16}\sqrt{1-x}\sqrt{x}-\frac{1}{8}\sqrt{1-x}x^{3/2}+\frac{1}{2}x^2\cos^{-1}\left(\sqrt{x}\right)+\frac{3}{32}\int\frac{1}{\sqrt{1-x}\sqrt{x}}\,dx\\
&= -\frac{3}{16}\sqrt{1-x}\sqrt{x}-\frac{1}{8}\sqrt{1-x}x^{3/2}+\frac{1}{2}x^2\cos^{-1}\left(\sqrt{x}\right)+\frac{3}{32}\int\frac{1}{\sqrt{x-x^2}}\,dx\\
&= -\frac{3}{16}\sqrt{1-x}\sqrt{x}-\frac{1}{8}\sqrt{1-x}x^{3/2}+\frac{1}{2}x^2\cos^{-1}\left(\sqrt{x}\right)-\frac{3}{32}\,\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}}\,dx,x,1-2x\right)\\
&= -\frac{3}{16}\sqrt{1-x}\sqrt{x}-\frac{1}{8}\sqrt{1-x}x^{3/2}+\frac{1}{2}x^2\cos^{-1}\left(\sqrt{x}\right)-\frac{3}{32}\sin^{-1}(1-2x)
\end{aligned}$$

Mathematica [A] time = 0.0293534, size = 41, normalized size = 0.68

$$\frac{1}{16}\left(8x^2\cos^{-1}\left(\sqrt{x}\right)-\sqrt{-(x-1)x}(2x+3)+3\sin^{-1}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCos[Sqrt[x]],x]
```

```
[Out] (-(Sqrt[-((-1 + x)*x)]*(3 + 2*x)) + 8*x^2*ArcCos[Sqrt[x]] + 3*ArcSin[Sqrt[x
]])/16
```

Maple [A] time = 0.003, size = 41, normalized size = 0.7

$$\frac{x^2}{2}\arccos\left(\sqrt{x}\right)-\frac{1}{8}x^{\frac{3}{2}}\sqrt{1-x}-\frac{3}{16}\sqrt{1-x}\sqrt{x}+\frac{3}{16}\arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(x^(1/2)),x)
```

```
[Out] 1/2*x^2*arccos(x^(1/2))-1/8*x^(3/2)*(1-x)^(1/2)-3/16*(1-x)^(1/2)*x^(1/2)+3/
16*arcsin(x^(1/2))
```

Maxima [A] time = 1.46528, size = 54, normalized size = 0.9

$$\frac{1}{2}x^2\arccos\left(\sqrt{x}\right)-\frac{1}{8}x^{\frac{3}{2}}\sqrt{-x+1}-\frac{3}{16}\sqrt{x}\sqrt{-x+1}+\frac{3}{16}\arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccos(sqrt(x)) - 1/8*x^(3/2)*sqrt(-x + 1) - 3/16*sqrt(x)*sqrt(-x +
 1) + 3/16*arcsin(sqrt(x))
```

Fricas [A] time = 2.5852, size = 99, normalized size = 1.65

$$-\frac{1}{16}(2x+3)\sqrt{x}\sqrt{-x+1}+\frac{1}{16}\left(8x^2-3\right)\arccos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/16*(2*x + 3)*sqrt(x)*sqrt(-x + 1) + 1/16*(8*x^2 - 3)*arccos(sqrt(x))
```

Sympy [A] time = 3.90069, size = 58, normalized size = 0.97

$$\frac{x^2 \operatorname{acos}\left(\sqrt{x}\right)}{2} + \frac{\begin{cases} \frac{\sqrt{x}(1-2x)\sqrt{1-x}}{8} - \frac{\sqrt{x}\sqrt{1-x}}{2} + \frac{3\operatorname{asin}\left(\sqrt{x}\right)}{8} & \text{for } x \geq 0 \wedge x < 1 \end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(x**(1/2)),x)
```

```
[Out] x**2*acos(sqrt(x))/2 + Piecewise((sqrt(x)*(1 - 2*x)*sqrt(1 - x)/8 - sqrt(x)
*sqrt(1 - x)/2 + 3*asin(sqrt(x))/8, (x >= 0) & (x < 1)))/2
```

Giac [A] time = 1.2774, size = 54, normalized size = 0.9

$$\frac{1}{2} x^2 \arccos\left(\sqrt{x}\right) - \frac{1}{8} x^{\frac{3}{2}} \sqrt{-x+1} - \frac{3}{16} \sqrt{x}\sqrt{-x+1} - \frac{3}{16} \arccos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arccos(sqrt(x)) - 1/8*x^(3/2)*sqrt(-x + 1) - 3/16*sqrt(x)*sqrt(-x +
 1) - 3/16*arccos(sqrt(x))
```

3.62 $\int \cos^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=37

$$-\frac{1}{2}\sqrt{1-x}\sqrt{x}-\frac{1}{4}\sin^{-1}(1-2x)+x\cos^{-1}\left(\sqrt{x}\right)$$

```
[Out] -(Sqrt[1 - x]*Sqrt[x])/2 + x*ArcCos[Sqrt[x]] - ArcSin[1 - 2*x]/4
```

Rubi [A] time = 0.0107611, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4841, 12, 50, 53, 619, 216}

$$-\frac{1}{2}\sqrt{1-x}\sqrt{x}-\frac{1}{4}\sin^{-1}(1-2x)+x\cos^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[x]],x]
```

```
[Out] -(Sqrt[1 - x]*Sqrt[x])/2 + x*ArcCos[Sqrt[x]] - ArcSin[1 - 2*x]/4
```

Rule 4841

```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \cos^{-1}\left(\sqrt{x}\right) dx &= x\cos^{-1}\left(\sqrt{x}\right) + \int \frac{\sqrt{x}}{2\sqrt{1-x}}\,dx \\
&= x\cos^{-1}\left(\sqrt{x}\right) + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx \\
&= -\frac{1}{2}\sqrt{1-x}\sqrt{x} + x\cos^{-1}\left(\sqrt{x}\right) + \frac{1}{4}\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx \\
&= -\frac{1}{2}\sqrt{1-x}\sqrt{x} + x\cos^{-1}\left(\sqrt{x}\right) + \frac{1}{4}\int \frac{1}{\sqrt{x-x^2}}\,dx \\
&= -\frac{1}{2}\sqrt{1-x}\sqrt{x} + x\cos^{-1}\left(\sqrt{x}\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1-2x\right) \\
&= -\frac{1}{2}\sqrt{1-x}\sqrt{x} + x\cos^{-1}\left(\sqrt{x}\right) - \frac{1}{4}\sin^{-1}(1-2x)
\end{aligned}$$

Mathematica [A] time = 0.0150251, size = 38, normalized size = 1.03

$$\frac{1}{2}\left(-\sqrt{-(x-1)x} - \sin^{-1}\left(\sqrt{1-x}\right)\right) + x\cos^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x]],x]
```

```
[Out] x*ArcCos[Sqrt[x]] + (-Sqrt[-((-1 + x)*x)] - ArcSin[Sqrt[1 - x]])/2
```

Maple [A] time = 0.002, size = 26, normalized size = 0.7

$$x\arccos\left(\sqrt{x}\right)-\frac{1}{2}\sqrt{1-x}\sqrt{x}+\frac{1}{2}\arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x^(1/2)),x)
```

```
[Out] x*arccos(x^(1/2))-1/2*(1-x)^(1/2)*x^(1/2)+1/2*arcsin(x^(1/2))
```

Maxima [A] time = 1.52363, size = 34, normalized size = 0.92

$$x\arccos\left(\sqrt{x}\right)-\frac{1}{2}\sqrt{x}\sqrt{-x+1}+\frac{1}{2}\arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arccos(sqrt(x)) - 1/2*sqrt(x)*sqrt(-x + 1) + 1/2*arcsin(sqrt(x))
```

Fricas [A] time = 2.61165, size = 78, normalized size = 2.11

$$\frac{1}{2}(2x-1)\arccos\left(\sqrt{x}\right)-\frac{1}{2}\sqrt{x}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x - 1)*arccos(sqrt(x)) - 1/2*sqrt(x)*sqrt(-x + 1)
```

Sympy [A] time = 0.337239, size = 29, normalized size = 0.78

$$-\frac{\sqrt{x}\sqrt{1-x}}{2}+x\operatorname{acos}\left(\sqrt{x}\right)-\frac{\operatorname{acos}\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x**(1/2)),x)
```

```
[Out] -sqrt(x)*sqrt(1 - x)/2 + x*acos(sqrt(x)) - acos(sqrt(x))/2
```

Giac [A] time = 1.19294, size = 34, normalized size = 0.92

$$x\arccos\left(\sqrt{x}\right)-\frac{1}{2}\sqrt{x}\sqrt{-x+1}-\frac{1}{2}\arccos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2)),x, algorithm="giac")
```

```
[Out] x*arccos(sqrt(x)) - 1/2*sqrt(x)*sqrt(-x + 1) - 1/2*arccos(sqrt(x))
```

3.63 $\int \frac{\cos^{-1}\left(\sqrt{x}\right)}{x}\,dx$

Optimal. Leaf size=56

$$-i\text{PolyLog}\left(2,-e^{2i\cos^{-1}\left(\sqrt{x}\right)}\right)-i\cos^{-1}\left(\sqrt{x}\right)^2+2\cos^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\sqrt{x}\right)}\right)$$

```
[Out] (-I)*ArcCos[Sqrt[x]]^2 + 2*ArcCos[Sqrt[x]]*Log[1 + E^((2*I)*ArcCos[Sqrt[x]]
)] - I*PolyLog[2, -E^((2*I)*ArcCos[Sqrt[x]])]
```

Rubi [A] time = 0.0558409, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4831, 3719, 2190, 2279, 2391}

$$-i\text{PolyLog}\left(2,-e^{2i\cos^{-1}\left(\sqrt{x}\right)}\right)-i\cos^{-1}\left(\sqrt{x}\right)^2+2\cos^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\sqrt{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[x]]/x,x]
```

```
[Out] (-I)*ArcCos[Sqrt[x]]^2 + 2*ArcCos[Sqrt[x]]*Log[1 + E^((2*I)*ArcCos[Sqrt[x]]
)] - I*PolyLog[2, -E^((2*I)*ArcCos[Sqrt[x]])]
```

Rule 4831

```
Int[ArcCos[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> -Dist[p^(-1), Subst[In
t[x^n*Tan[x], x], x, ArcCos[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}\left(\sqrt{x}\right)}{x}\,dx &= -\left(2\,\mathrm{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}\left(\sqrt{x}\right)\right)\right)\\
&= -i\cos^{-1}\left(\sqrt{x}\right)^2 + 4i\,\mathrm{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\sqrt{x}\right)\right)\\
&= -i\cos^{-1}\left(\sqrt{x}\right)^2 + 2\cos^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\sqrt{x}\right)}\right) - 2\,\mathrm{Subst}\left(\int \log\left(1+e^{2ix}\right)dx, x, \cos^{-1}\left(\sqrt{x}\right)\right)\\
&= -i\cos^{-1}\left(\sqrt{x}\right)^2 + 2\cos^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\sqrt{x}\right)}\right) + i\,\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\cos^{-1}\left(\sqrt{x}\right)}\right)\\
&= -i\cos^{-1}\left(\sqrt{x}\right)^2 + 2\cos^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\sqrt{x}\right)}\right) - i\mathrm{Li}_2\left(-e^{2i\cos^{-1}\left(\sqrt{x}\right)}\right)
\end{aligned}$$

Mathematica [A] time = 0.0260489, size = 54, normalized size = 0.96

$$-i\left(\mathrm{PolyLog}\left(2,-e^{2i\cos^{-1}\left(\sqrt{x}\right)}\right)+\cos^{-1}\left(\sqrt{x}\right)\left(\cos^{-1}\left(\sqrt{x}\right)+2i\log\left(1+e^{2i\cos^{-1}\left(\sqrt{x}\right)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x]]/x,x]
```

```
[Out] (-I)*(ArcCos[Sqrt[x]]*(ArcCos[Sqrt[x]] + (2*I)*Log[1 + E^((2*I)*ArcCos[Sqrt
[x]])]) + PolyLog[2, -E^((2*I)*ArcCos[Sqrt[x]])])
```

Maple [A] time = 0.002, size = 59, normalized size = 1.1

$$-i\left(\arccos\left(\sqrt{x}\right)\right)^{2}+2\arccos\left(\sqrt{x}\right)\ln\left(1+\left(\sqrt{x}+i\sqrt{1-x}\right)^{2}\right)-i\mathrm{polylog}\left(2,-\left(\sqrt{x}+i\sqrt{1-x}\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x^(1/2))/x,x)
```

```
[Out] -I*arccos(x^(1/2))^2+2*arccos(x^(1/2))*ln(1+(x^(1/2)+I*(1-x)^(1/2))^2)-I*po
lylog(2,-(x^(1/2)+I*(1-x)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\arccos\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccos(sqrt(x))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arccos\left(\sqrt{x}\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arccos(sqrt(x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acos}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x**(1/2))/x,x)
```

```
[Out] Integral(acos(sqrt(x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(sqrt(x))/x, x)
```

3.64 $\int \frac{\cos^{-1}\left(\sqrt{x}\right)}{x^2}\,dx$

Optimal. Leaf size=27

$$\frac{\sqrt{1-x}}{\sqrt{x}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{x}$$

```
[Out] Sqrt[1 - x]/Sqrt[x] - ArcCos[Sqrt[x]]/x
```

Rubi [A] time = 0.0119337, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4843, 12, 37}

$$\frac{\sqrt{1-x}}{\sqrt{x}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[x]]/x^2,x]
```

```
[Out] Sqrt[1 - x]/Sqrt[x] - ArcCos[Sqrt[x]]/x
```

Rule 4843

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcCos[u]))/(d*(m + 1)), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 - u^2], x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{-1}\left(\sqrt{x}\right)}{x^2}\,dx &= -\frac{\cos^{-1}\left(\sqrt{x}\right)}{x}-\int \frac{1}{2\sqrt{1-x}x^{3/2}}\,dx\\ &= -\frac{\cos^{-1}\left(\sqrt{x}\right)}{x}-\frac{1}{2}\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx\\ &= \frac{\sqrt{1-x}}{\sqrt{x}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.0156122, size = 24, normalized size = 0.89

$$\frac{\sqrt{x-x^2}-\cos^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x]]/x^2,x]
```

```
[Out] (Sqrt[x - x^2] - ArcCos[Sqrt[x]])/x
```

Maple [A] time = 0.001, size = 22, normalized size = 0.8

$$-\frac{1}{x}\arccos\left(\sqrt{x}\right)+\sqrt{1-x}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x^(1/2))/x^2,x)
```

```
[Out] -arccos(x^(1/2))/x+(1-x)^(1/2)/x^(1/2)
```

Maxima [A] time = 1.49926, size = 28, normalized size = 1.04

$$\frac{\sqrt{-x+1}}{\sqrt{x}} - \frac{\arccos\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] sqrt(-x + 1)/sqrt(x) - arccos(sqrt(x))/x
```

Fricas [A] time = 2.56133, size = 59, normalized size = 2.19

$$\frac{\sqrt{x}\sqrt{-x+1} - \arccos\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(x)*sqrt(-x + 1) - arccos(sqrt(x)))/x
```

Sympy [C] time = 4.22227, size = 44, normalized size = 1.63

$$-\frac{\begin{cases} -\frac{2i\sqrt{x-1}}{\sqrt{x}} & \text{for } |x| > 1 \\ -\frac{2\sqrt{1-x}}{\sqrt{x}} & \text{otherwise} \end{cases}}{2} - \frac{\operatorname{acos}\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x**(1/2))/x**2,x)
```

```
[Out] -Piecewise((-2*I*sqrt(x - 1)/sqrt(x), Abs(x) > 1), (-2*sqrt(1 - x)/sqrt(x),
 True))/2 - acos(sqrt(x))/x
```

Giac [A] time = 1.17879, size = 54, normalized size = 2.

$$\frac{\sqrt{-x+1}-1}{2\sqrt{x}}-\frac{\arccos\left(\sqrt{x}\right)}{x}-\frac{\sqrt{x}}{2\left(\sqrt{-x+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(-x + 1) - 1)/sqrt(x) - arccos(sqrt(x))/x - 1/2*sqrt(x)/(sqrt(-x +
 1) - 1)
```

3.65 $\int \frac{\cos^{-1}(\sqrt{x})}{x^3} \, dx$

Optimal. Leaf size=50

$$\frac{\sqrt{1-x}}{6x^{3/2}} - \frac{\cos^{-1}(\sqrt{x})}{2x^2} + \frac{\sqrt{1-x}}{3\sqrt{x}}$$

```
[Out] Sqrt[1 - x]/(6*x^(3/2)) + Sqrt[1 - x]/(3*Sqrt[x]) - ArcCos[Sqrt[x]]/(2*x^2)
```

Rubi [A] time = 0.0176729, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4843, 12, 45, 37}

$$\frac{\sqrt{1-x}}{6x^{3/2}} - \frac{\cos^{-1}(\sqrt{x})}{2x^2} + \frac{\sqrt{1-x}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[x]]/x^3,x]
```

```
[Out] Sqrt[1 - x]/(6*x^(3/2)) + Sqrt[1 - x]/(3*Sqrt[x]) - ArcCos[Sqrt[x]]/(2*x^2)
```

Rule 4843

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcCos[u]))/(d*(m + 1)), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 - u^2], x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
```

```
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{-1}\left(\sqrt{x}\right)}{x^3}\,dx &= -\frac{\cos^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{2}\int \frac{1}{2\sqrt{1-x}x^{5/2}}\,dx\\ &= -\frac{\cos^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{4}\int \frac{1}{\sqrt{1-x}x^{5/2}}\,dx\\ &= \frac{\sqrt{1-x}}{6x^{3/2}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{6}\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx\\ &= \frac{\sqrt{1-x}}{6x^{3/2}}+\frac{\sqrt{1-x}}{3\sqrt{x}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0214374, size = 43, normalized size = 0.86

$$\left(\frac{1}{6x^{3/2}}+\frac{1}{3\sqrt{x}}\right)\sqrt{1-x}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x]]/x^3,x]
```

```
[Out] (1/(6*x^(3/2)) + 1/(3*Sqrt[x]))*Sqrt[1 - x] - ArcCos[Sqrt[x]]/(2*x^2)
```

Maple [A] time = 0.004, size = 35, normalized size = 0.7

$$-\frac{1}{2\,x^2}\arccos\left(\sqrt{x}\right)+\frac{1}{6}\sqrt{1-x}x^{-\frac{3}{2}}+\frac{1}{3}\sqrt{1-x}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x^(1/2))/x^3,x)
```

```
[Out] -1/2*arccos(x^(1/2))/x^2+1/6*(1-x)^(1/2)/x^(3/2)+1/3*(1-x)^(1/2)/x^(1/2)
```

Maxima [A] time = 1.47328, size = 46, normalized size = 0.92

$$\frac{\sqrt{-x+1}}{3\sqrt{x}} + \frac{\sqrt{-x+1}}{6\,x^{\frac{3}{2}}} - \frac{\arccos\left(\sqrt{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-x + 1)/sqrt(x) + 1/6*sqrt(-x + 1)/x^(3/2) - 1/2*arccos(sqrt(x))/x
^2
```

Fricas [A] time = 2.6279, size = 84, normalized size = 1.68

$$\frac{(2\,x+1)\sqrt{x}\sqrt{-x+1} - 3\,\arccos\left(\sqrt{x}\right)}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*((2*x + 1)*sqrt(x)*sqrt(-x + 1) - 3*arccos(sqrt(x)))/x^2
```

Sympy [A] time = 24.9142, size = 44, normalized size = 0.88

$$-\frac{\begin{cases} -\frac{\sqrt{1-x}}{\sqrt{x}} - \frac{(1-x)^{\frac{3}{2}}}{3x^{\frac{3}{2}}} & \text{for}\ x \geq 0 \wedge x < 1 \end{cases}}{2} - \frac{\operatorname{acos}\left(\sqrt{x}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x**(1/2))/x**3,x)
```

```
[Out] -Piecewise((-sqrt(1 - x)/sqrt(x) - (1 - x)**(3/2)/(3*x**(3/2)), (x >= 0) &
(x < 1)))/2 - acos(sqrt(x))/(2*x**2)
```

Giac [B] time = 1.28093, size = 100, normalized size = 2.

$$\frac{\left(\sqrt{-x+1}-1\right)^{3}}{48\,x^{\frac{3}{2}}}+\frac{3\left(\sqrt{-x+1}-1\right)}{16\,\sqrt{x}}-\frac{x^{\frac{3}{2}}\left(\frac{9\left(\sqrt{-x+1}-1\right)^{2}}{x}+1\right)}{48\left(\sqrt{-x+1}-1\right)^{3}}-\frac{\arccos\left(\sqrt{x}\right)}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(-x + 1) - 1)^3/x^(3/2) + 3/16*(sqrt(-x + 1) - 1)/sqrt(x) - 1/48*
x^(3/2)*(9*(sqrt(-x + 1) - 1)^2/x + 1)/(sqrt(-x + 1) - 1)^3 - 1/2*arccos(sq
rt(x))/x^2
```

3.66 $\int \frac{\cos^{-1}(\sqrt{x})}{x^4} dx$

Optimal. Leaf size=68

$$\frac{4\sqrt{1-x}}{45x^{3/2}}+\frac{\sqrt{1-x}}{15x^{5/2}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{8\sqrt{1-x}}{45\sqrt{x}}$$

```
[Out] Sqrt[1 - x]/(15*x^(5/2)) + (4*Sqrt[1 - x])/(45*x^(3/2)) + (8*Sqrt[1 - x])/(
45*Sqrt[x]) - ArcCos[Sqrt[x]]/(3*x^3)
```

Rubi [A] time = 0.0218177, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4843, 12, 45, 37}

$$\frac{4\sqrt{1-x}}{45x^{3/2}}+\frac{\sqrt{1-x}}{15x^{5/2}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{8\sqrt{1-x}}{45\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[x]]/x^4,x]
```

```
[Out] Sqrt[1 - x]/(15*x^(5/2)) + (4*Sqrt[1 - x])/(45*x^(3/2)) + (8*Sqrt[1 - x])/(
45*Sqrt[x]) - ArcCos[Sqrt[x]]/(3*x^3)
```

Rule 4843

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcCos[u]))/(d*(m + 1)), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 - u^2], x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}\left(\sqrt{x}\right)}{x^4}\,dx &= -\frac{\cos^{-1}\left(\sqrt{x}\right)}{3x^3}-\frac{1}{3}\int \frac{1}{2\sqrt{1-x}x^{7/2}}\,dx\\
&= -\frac{\cos^{-1}\left(\sqrt{x}\right)}{3x^3}-\frac{1}{6}\int \frac{1}{\sqrt{1-x}x^{7/2}}\,dx\\
&= \frac{\sqrt{1-x}}{15x^{5/2}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{3x^3}-\frac{2}{15}\int \frac{1}{\sqrt{1-x}x^{5/2}}\,dx\\
&= \frac{\sqrt{1-x}}{15x^{5/2}}+\frac{4\sqrt{1-x}}{45x^{3/2}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{3x^3}-\frac{4}{45}\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx\\
&= \frac{\sqrt{1-x}}{15x^{5/2}}+\frac{4\sqrt{1-x}}{45x^{3/2}}+\frac{8\sqrt{1-x}}{45\sqrt{x}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.0376777, size = 37, normalized size = 0.54

$$\frac{\sqrt{-(x-1)x}\left(8x^2+4x+3\right)-15\cos^{-1}\left(\sqrt{x}\right)}{45x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x]]/x^4,x]
```

```
[Out] (Sqrt[-((-1 + x)*x)]*(3 + 4*x + 8*x^2) - 15*ArcCos[Sqrt[x]])/(45*x^3)
```

Maple [A] time = 0.003, size = 47, normalized size = 0.7

$$-\frac{1}{3x^3}\arccos\left(\sqrt{x}\right)+\frac{1}{15}\sqrt{1-x}x^{-\frac{5}{2}}+\frac{4}{45}\sqrt{1-x}x^{-\frac{3}{2}}+\frac{8}{45}\sqrt{1-x}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x^(1/2))/x^4,x)
```

```
[Out] -1/3*arccos(x^(1/2))/x^3+1/15*(1-x)^(1/2)/x^(5/2)+4/45*(1-x)^(1/2)/x^(3/2)+
8/45*(1-x)^(1/2)/x^(1/2)
```

Maxima [A] time = 1.4762, size = 62, normalized size = 0.91

$$\frac{8\sqrt{-x+1}}{45\sqrt{x}}+\frac{4\sqrt{-x+1}}{45x^{\frac{3}{2}}}+\frac{\sqrt{-x+1}}{15x^{\frac{5}{2}}}-\frac{\arccos\left(\sqrt{x}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] 8/45*sqrt(-x + 1)/sqrt(x) + 4/45*sqrt(-x + 1)/x^(3/2) + 1/15*sqrt(-x + 1)/x
^(5/2) - 1/3*arccos(sqrt(x))/x^3
```

Fricas [A] time = 2.63136, size = 97, normalized size = 1.43

$$\frac{\left(8x^2+4x+3\right)\sqrt{x}\sqrt{-x+1}-15\arccos\left(\sqrt{x}\right)}{45x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] 1/45*((8*x^2 + 4*x + 3)*sqrt(x)*sqrt(-x + 1) - 15*arccos(sqrt(x)))/x^3
```

Sympy [A] time = 150.038, size = 60, normalized size = 0.88

$$-\frac{\begin{cases} -\frac{\sqrt{1-x}}{\sqrt{x}} - \frac{2(1-x)^{\frac{3}{2}}}{3x^{\frac{3}{2}}} - \frac{(1-x)^{\frac{5}{2}}}{5x^{\frac{5}{2}}} & \text{for } x \geq 0 \wedge x < 1 \end{cases}}{3} - \frac{\operatorname{acos}\left(\sqrt{x}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x**(1/2))/x**4,x)
```

```
[Out] -Piecewise((-sqrt(1 - x)/sqrt(x) - 2*(1 - x)**(3/2)/(3*x**(3/2)) - (1 - x)*
*(5/2)/(5*x**(5/2)), (x >= 0) & (x < 1)))/3 - acos(sqrt(x))/(3*x**3)
```

Giac [B] time = 1.31196, size = 143, normalized size = 2.1

$$\frac{\left(\sqrt{-x+1}-1\right)^5}{480\,x^{\frac{5}{2}}} + \frac{5\left(\sqrt{-x+1}-1\right)^3}{288\,x^{\frac{3}{2}}} + \frac{5\left(\sqrt{-x+1}-1\right)}{48\,\sqrt{x}} - \frac{\left(\frac{150\left(\sqrt{-x+1}-1\right)^4}{x^2} + \frac{25\left(\sqrt{-x+1}-1\right)^2}{x} + 3\right)x^{\frac{5}{2}}}{1440\left(\sqrt{-x+1}-1\right)^5} - \frac{\arccos\left(\sqrt{x}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] 1/480*(sqrt(-x + 1) - 1)^5/x^(5/2) + 5/288*(sqrt(-x + 1) - 1)^3/x^(3/2) + 5
/48*(sqrt(-x + 1) - 1)/sqrt(x) - 1/1440*(150*(sqrt(-x + 1) - 1)^4/x^2 + 25*
(sqrt(-x + 1) - 1)^2/x + 3)*x^(5/2)/(sqrt(-x + 1) - 1)^5 - 1/3*arccos(sqrt(
x))/x^3
```

3.67 $\int \frac{\cos^{-1}(\sqrt{x})}{x^5} \, dx$

Optimal. Leaf size=86

$$\frac{2\sqrt{1-x}}{35x^{3/2}}+\frac{3\sqrt{1-x}}{70x^{5/2}}+\frac{\sqrt{1-x}}{28x^{7/2}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{4x^4}+\frac{4\sqrt{1-x}}{35\sqrt{x}}$$

```
[Out] Sqrt[1 - x]/(28*x^(7/2)) + (3*Sqrt[1 - x])/(70*x^(5/2)) + (2*Sqrt[1 - x])/(
35*x^(3/2)) + (4*Sqrt[1 - x])/(35*Sqrt[x]) - ArcCos[Sqrt[x]]/(4*x^4)
```

Rubi [A] time = 0.0277524, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4843, 12, 45, 37}

$$\frac{2\sqrt{1-x}}{35x^{3/2}}+\frac{3\sqrt{1-x}}{70x^{5/2}}+\frac{\sqrt{1-x}}{28x^{7/2}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{4x^4}+\frac{4\sqrt{1-x}}{35\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[x]]/x^5,x]
```

```
[Out] Sqrt[1 - x]/(28*x^(7/2)) + (3*Sqrt[1 - x])/(70*x^(5/2)) + (2*Sqrt[1 - x])/(
35*x^(3/2)) + (4*Sqrt[1 - x])/(35*Sqrt[x]) - ArcCos[Sqrt[x]]/(4*x^4)
```

Rule 4843

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcCos[u]))/(d*(m + 1)), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 - u^2], x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^{-1}\left(\sqrt{x}\right)}{x^5}\,dx &= -\frac{\cos^{-1}\left(\sqrt{x}\right)}{4x^4}-\frac{1}{4}\int \frac{1}{2\sqrt{1-x}x^{9/2}}\,dx\\
&= -\frac{\cos^{-1}\left(\sqrt{x}\right)}{4x^4}-\frac{1}{8}\int \frac{1}{\sqrt{1-x}x^{9/2}}\,dx\\
&= \frac{\sqrt{1-x}}{28x^{7/2}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{4x^4}-\frac{3}{28}\int \frac{1}{\sqrt{1-x}x^{7/2}}\,dx\\
&= \frac{\sqrt{1-x}}{28x^{7/2}}+\frac{3\sqrt{1-x}}{70x^{5/2}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{4x^4}-\frac{3}{35}\int \frac{1}{\sqrt{1-x}x^{5/2}}\,dx\\
&= \frac{\sqrt{1-x}}{28x^{7/2}}+\frac{3\sqrt{1-x}}{70x^{5/2}}+\frac{2\sqrt{1-x}}{35x^{3/2}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{4x^4}-\frac{2}{35}\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx\\
&= \frac{\sqrt{1-x}}{28x^{7/2}}+\frac{3\sqrt{1-x}}{70x^{5/2}}+\frac{2\sqrt{1-x}}{35x^{3/2}}+\frac{4\sqrt{1-x}}{35\sqrt{x}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{4x^4}
\end{aligned}
$$

Mathematica [A] time = 0.0436597, size = 42, normalized size = 0.49

$$
\frac{\sqrt{-(x-1)x}\left(16x^3+8x^2+6x+5\right)-35\cos^{-1}\left(\sqrt{x}\right)}{140x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x]]/x^5,x]
```

```
[Out] (Sqrt[-((-1 + x)*x)]*(5 + 6*x + 8*x^2 + 16*x^3) - 35*ArcCos[Sqrt[x]])/(140*
x^4)
```

Maple [A] time = 0.003, size = 59, normalized size = 0.7

$$-\frac{1}{4x^4}\arccos\left(\sqrt{x}\right)+\frac{1}{28}\sqrt{1-x}x^{-\frac{7}{2}}+\frac{3}{70}\sqrt{1-x}x^{-\frac{5}{2}}+\frac{2}{35}\sqrt{1-x}x^{-\frac{3}{2}}+\frac{4}{35}\sqrt{1-x}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x^(1/2))/x^5,x)
```

```
[Out] -1/4*arccos(x^(1/2))/x^4+1/28*(1-x)^(1/2)/x^(7/2)+3/70*(1-x)^(1/2)/x^(5/2)+
2/35*(1-x)^(1/2)/x^(3/2)+4/35*(1-x)^(1/2)/x^(1/2)
```

Maxima [A] time = 1.49298, size = 78, normalized size = 0.91

$$\frac{4\sqrt{-x+1}}{35\sqrt{x}}+\frac{2\sqrt{-x+1}}{35x^{\frac{3}{2}}}+\frac{3\sqrt{-x+1}}{70x^{\frac{5}{2}}}+\frac{\sqrt{-x+1}}{28x^{\frac{7}{2}}}-\frac{\arccos\left(\sqrt{x}\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] 4/35*sqrt(-x + 1)/sqrt(x) + 2/35*sqrt(-x + 1)/x^(3/2) + 3/70*sqrt(-x + 1)/x
^(5/2) + 1/28*sqrt(-x + 1)/x^(7/2) - 1/4*arccos(sqrt(x))/x^4
```

Fricas [A] time = 2.32641, size = 111, normalized size = 1.29

$$\frac{\left(16x^3+8x^2+6x+5\right)\sqrt{x}\sqrt{-x+1}-35\arccos\left(\sqrt{x}\right)}{140x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] 1/140*((16*x^3 + 8*x^2 + 6*x + 5)*sqrt(x)*sqrt(-x + 1) - 35*arccos(sqrt(x))
)/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x**(1/2))/x**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28499, size = 186, normalized size = 2.16

$$\frac{\left(\sqrt{-x+1}-1\right)^{7}}{3584\,x^{\frac{7}{2}}}+\frac{7\left(\sqrt{-x+1}-1\right)^{5}}{2560\,x^{\frac{5}{2}}}+\frac{7\left(\sqrt{-x+1}-1\right)^{3}}{512\,x^{\frac{3}{2}}}+\frac{35\left(\sqrt{-x+1}-1\right)}{512\,\sqrt{x}}-\frac{\left(\frac{1225\left(\sqrt{-x+1}-1\right)^{6}}{x^{3}}+\frac{245\left(\sqrt{-x+1}-1\right)^{4}}{x^{2}}+\frac{49\left(\sqrt{-\right.}{}\right.}{}}{17920\left(\sqrt{-x+1}-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] 1/3584*(sqrt(-x + 1) - 1)^7/x^(7/2) + 7/2560*(sqrt(-x + 1) - 1)^5/x^(5/2) +
 7/512*(sqrt(-x + 1) - 1)^3/x^(3/2) + 35/512*(sqrt(-x + 1) - 1)/sqrt(x) - 1
/17920*(1225*(sqrt(-x + 1) - 1)^6/x^3 + 245*(sqrt(-x + 1) - 1)^4/x^2 + 49*(
sqrt(-x + 1) - 1)^2/x + 5)*x^(7/2)/(sqrt(-x + 1) - 1)^7 - 1/4*arccos(sqrt(x
))/x^4
```

3.68 $\int \frac{\cos^{-1}(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=25

$$2\sqrt{x}\cos^{-1}\left(\sqrt{x}\right)-2\sqrt{1-x}$$

```
[Out] -2*Sqrt[1 - x] + 2*Sqrt[x]*ArcCos[Sqrt[x]]
```

Rubi [A] time = 0.0187569, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6715, 4620, 261}

$$2\sqrt{x}\cos^{-1}\left(\sqrt{x}\right)-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] -2*Sqrt[1 - x] + 2*Sqrt[x]*ArcCos[Sqrt[x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cos^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \cos^{-1}(x)\,dx, x, \sqrt{x}\right)$$
$$= 2\sqrt{x}\cos^{-1}\left(\sqrt{x}\right) + 2\,\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)$$
$$= -2\sqrt{1-x} + 2\sqrt{x}\cos^{-1}\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.0078178, size = 25, normalized size = 1.

$$2\sqrt{x}\cos^{-1}\left(\sqrt{x}\right) - 2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] -2*Sqrt[1 - x] + 2*Sqrt[x]*ArcCos[Sqrt[x]]
```

Maple [A] time = 0.003, size = 20, normalized size = 0.8

$$-2\,\sqrt{1-x} + 2\,\arccos\left(\sqrt{x}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x^(1/2))/x^(1/2),x)
```

```
[Out] -2*(1-x)^(1/2)+2*arccos(x^(1/2))*x^(1/2)
```

Maxima [A] time = 1.48198, size = 26, normalized size = 1.04

$$2\,\sqrt{x}\arccos\left(\sqrt{x}\right) - 2\,\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*arccos(sqrt(x)) - 2*sqrt(-x + 1)
```

Fricas [A] time = 2.53294, size = 59, normalized size = 2.36

$$2\sqrt{x}\arccos\left(\sqrt{x}\right) - 2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)*arccos(sqrt(x)) - 2*sqrt(-x + 1)
```

Sympy [A] time = 0.365094, size = 20, normalized size = 0.8

$$2\sqrt{x}\operatorname{acos}\left(\sqrt{x}\right) - 2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*sqrt(x)*acos(sqrt(x)) - 2*sqrt(1 - x)
```

Giac [A] time = 1.2737, size = 26, normalized size = 1.04

$$2\sqrt{x}\arccos\left(\sqrt{x}\right) - 2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*arccos(sqrt(x)) - 2*sqrt(-x + 1)
```

3.69 $\int \frac{\cos^{-1}(ax^n)}{x} dx$

Optimal. Leaf size=68

$$-\frac{i\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax^n)}\right)}{2n}-\frac{i\cos^{-1}(ax^n)^2}{2n}+\frac{\cos^{-1}(ax^n)\log\left(1+e^{2i\cos^{-1}(ax^n)}\right)}{n}$$

```
[Out] ((-I/2)*ArcCos[a*x^n]^2)/n + (ArcCos[a*x^n]*Log[1 + E^((2*I)*ArcCos[a*x^n])
])/n - ((I/2)*PolyLog[2, -E^((2*I)*ArcCos[a*x^n])])/n
```

Rubi [A] time = 0.0614399, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4831, 3719, 2190, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax^n)}\right)}{2n}-\frac{i\cos^{-1}(ax^n)^2}{2n}+\frac{\cos^{-1}(ax^n)\log\left(1+e^{2i\cos^{-1}(ax^n)}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x^n]/x,x]
```


```
[Out] ((-I/2)*ArcCos[a*x^n]^2)/n + (ArcCos[a*x^n]*Log[1 + E^((2*I)*ArcCos[a*x^n])
])/n - ((I/2)*PolyLog[2, -E^((2*I)*ArcCos[a*x^n])])/n
```


Rule 4831

```
Int[ArcCos[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> -Dist[p^(-1), Subst[In
t[x^n*Tan[x], x], x, ArcCos[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(ax^n)}{x}\,dx &= -\frac{\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}(ax^n)\right)}{n}\\
&= -\frac{i\cos^{-1}(ax^n)^2}{2n} + \frac{(2i)\,\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}(ax^n)\right)}{n}\\
&= -\frac{i\cos^{-1}(ax^n)^2}{2n} + \frac{\cos^{-1}(ax^n)\log\left(1+e^{2i\cos^{-1}(ax^n)}\right)}{n} - \frac{\text{Subst}\left(\int \log\left(1+e^{2ix}\right)dx, x, \cos^{-1}(ax^n)\right)}{n}\\
&= -\frac{i\cos^{-1}(ax^n)^2}{2n} + \frac{\cos^{-1}(ax^n)\log\left(1+e^{2i\cos^{-1}(ax^n)}\right)}{n} + \frac{i\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\cos^{-1}(ax^n)}\right)}{2n}\\
&= -\frac{i\cos^{-1}(ax^n)^2}{2n} + \frac{\cos^{-1}(ax^n)\log\left(1+e^{2i\cos^{-1}(ax^n)}\right)}{n} - \frac{i\text{Li}_2\left(-e^{2i\cos^{-1}(ax^n)}\right)}{2n}
\end{aligned}$$

Mathematica [B] time = 0.137089, size = 141, normalized size = 2.07

$$\frac{a\left(\text{PolyLog}\left(2, e^{-2\sinh^{-1}\left(\sqrt{-a^2}x^n\right)}\right) + 2n\log(x)\log\left(\sqrt{-a^2}x^n + \sqrt{1-a^2x^{2n}}\right) - \sinh^{-1}\left(\sqrt{-a^2}x^n\right)^2 - 2\sinh^{-1}\left(\sqrt{-a^2}x^n\right)\log\left(\right.\right.}{2\sqrt{-a^2}n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a*x^n]/x,x]
```

```
[Out] ArcCos[a*x^n]*Log[x] + (a*(-ArcSinh[Sqrt[-a^2]*x^n]^2 - 2*ArcSinh[Sqrt[-a^2
]*x^n]*Log[1 - E^(-2*ArcSinh[Sqrt[-a^2]*x^n])] + 2*n*Log[x]*Log[Sqrt[-a^2]*
```

```
x^n + Sqrt[1 - a^2*x^(2*n)]] + PolyLog[2, E^(-2*ArcSinh[Sqrt[-a^2]*x^n])]))
/(2*Sqrt[-a^2]*n)
```

Maple [A] time = 0.04, size = 89, normalized size = 1.3

$$\frac{-\frac{i}{2}\left(\arccos\left(ax^{n}\right)\right)^{2}}{n}+\frac{\arccos\left(ax^{n}\right)}{n}\ln\left(1+\left(ax^{n}+i\sqrt{1-a^{2}\left(x^{n}\right)^{2}}\right)^{2}\right)-\frac{\frac{i}{2}}{n}\mathrm{polylog}\left(2,-\left(ax^{n}+i\sqrt{1-a^{2}\left(x^{n}\right)^{2}}\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x^n)/x,x)
```

```
[Out] -1/2*I*arccos(a*x^n)^2/n+arccos(a*x^n)*ln(1+(a*x^n+I*(1-a^2*(x^n)^2)^(1/2))
^2)/n-1/2*I*polylog(2,-(a*x^n+I*(1-a^2*(x^n)^2)^(1/2))^2)/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-an\int\frac{\sqrt{ax^{n}+1}\sqrt{-ax^{n}+1}x^{n}\log\left(x\right)}{a^{2}xx^{2\,n}-x}\,dx+\arctan\left(\sqrt{ax^{n}+1}\sqrt{-ax^{n}+1},ax^{n}\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -a*n*integrate(sqrt(a*x^n + 1)*sqrt(-a*x^n + 1)*x^n*log(x)/(a^2*x*x^(2*n) -
 x), x) + arctan2(sqrt(a*x^n + 1)*sqrt(-a*x^n + 1), a*x^n)*log(x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{acos}\,(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x**n)/x,x)
```

```
[Out] Integral(acos(a*x**n)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos\,(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x^n)/x, x)
```

3.70 $\int \frac{\cos^{-1}(ax^5)}{x} dx$

Optimal. Leaf size=62

$$-\frac{1}{10}i\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax^5)}\right)-\frac{1}{10}i\cos^{-1}\left(ax^5\right)^2+\frac{1}{5}\cos^{-1}\left(ax^5\right)\log\left(1+e^{2i\cos^{-1}(ax^5)}\right)$$

```
[Out] (-I/10)*ArcCos[a*x^5]^2 + (ArcCos[a*x^5]*Log[1 + E^((2*I)*ArcCos[a*x^5])])/
5 - (I/10)*PolyLog[2, -E^((2*I)*ArcCos[a*x^5])]
```

Rubi [A] time = 0.0574571, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4831, 3719, 2190, 2279, 2391}

$$-\frac{1}{10}i\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax^5)}\right)-\frac{1}{10}i\cos^{-1}\left(ax^5\right)^2+\frac{1}{5}\cos^{-1}\left(ax^5\right)\log\left(1+e^{2i\cos^{-1}(ax^5)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x^5]/x,x]
```

```
[Out] (-I/10)*ArcCos[a*x^5]^2 + (ArcCos[a*x^5]*Log[1 + E^((2*I)*ArcCos[a*x^5])])/
5 - (I/10)*PolyLog[2, -E^((2*I)*ArcCos[a*x^5])]
```

Rule 4831

```
Int[ArcCos[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> -Dist[p^(-1), Subst[In
t[x^n*Tan[x], x], x, ArcCos[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}\left(ax^5\right)}{x}\,dx &= -\left(\frac{1}{5}\,\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}\left(ax^5\right)\right)\right)\\
&= -\frac{1}{10}i\cos^{-1}\left(ax^5\right)^2 + \frac{2}{5}i\,\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(ax^5\right)\right)\\
&= -\frac{1}{10}i\cos^{-1}\left(ax^5\right)^2 + \frac{1}{5}\cos^{-1}\left(ax^5\right)\log\left(1+e^{2i\cos^{-1}\left(ax^5\right)}\right) - \frac{1}{5}\,\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, \cos^{-1}\left(ax^5\right)\right)\\
&= -\frac{1}{10}i\cos^{-1}\left(ax^5\right)^2 + \frac{1}{5}\cos^{-1}\left(ax^5\right)\log\left(1+e^{2i\cos^{-1}\left(ax^5\right)}\right) + \frac{1}{10}i\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\cos^{-1}\left(ax^5\right)}\right)\\
&= -\frac{1}{10}i\cos^{-1}\left(ax^5\right)^2 + \frac{1}{5}\cos^{-1}\left(ax^5\right)\log\left(1+e^{2i\cos^{-1}\left(ax^5\right)}\right) - \frac{1}{10}i\text{Li}_2\left(-e^{2i\cos^{-1}\left(ax^5\right)}\right)
\end{aligned}$$

Mathematica [A] time = 0.0313788, size = 56, normalized size = 0.9

$$-\frac{1}{10}i\left(\text{PolyLog}\left(2,-e^{2i\cos^{-1}\left(ax^5\right)}\right)+\cos^{-1}\left(ax^5\right)\left(\cos^{-1}\left(ax^5\right)+2i\log\left(1+e^{2i\cos^{-1}\left(ax^5\right)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x^5]/x,x]
```

```
[Out] (-I/10)*(ArcCos[a*x^5]*(ArcCos[a*x^5] + (2*I)*Log[1 + E^((2*I)*ArcCos[a*x^5
])]) + PolyLog[2, -E^((2*I)*ArcCos[a*x^5])])
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{\arccos\left(a x^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x^5)/x,x)
```

```
[Out] int(arccos(a*x^5)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos\left(a x^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] integrate(arccos(a*x^5)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arccos\left(a x^5\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x^5)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}\left(a x^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x**5)/x,x)
```

```
[Out] Integral(acos(a*x**5)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x^5)/x, x)
```

3.71 $\int x^3 \cos^{-1}\left(a+bx^4\right) dx$

Optimal. Leaf size=47

$$\frac{\left(a+bx^4\right)\cos^{-1}\left(a+bx^4\right)}{4b}-\frac{\sqrt{1-\left(a+bx^4\right)^2}}{4b}$$

```
[Out] -Sqrt[1 - (a + b*x^4)^2]/(4*b) + ((a + b*x^4)*ArcCos[a + b*x^4])/(4*b)
```

Rubi [A] time = 0.0514806, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6715, 4804, 4620, 261}

$$\frac{\left(a+bx^4\right)\cos^{-1}\left(a+bx^4\right)}{4b}-\frac{\sqrt{1-\left(a+bx^4\right)^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCos[a + b*x^4],x]
```

```
[Out] -Sqrt[1 - (a + b*x^4)^2]/(4*b) + ((a + b*x^4)*ArcCos[a + b*x^4])/(4*b)
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 4804

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\cos^{-1}\left(a+bx^4\right)dx&=\frac{1}{4}\,\text{Subst}\left(\int\cos^{-1}(a+bx)\,dx,x,x^4\right)\\&=\frac{\text{Subst}\left(\int\cos^{-1}(x)\,dx,x,a+bx^4\right)}{4b}\\&=\frac{\left(a+bx^4\right)\cos^{-1}\left(a+bx^4\right)}{4b}+\frac{\text{Subst}\left(\int\frac{x}{\sqrt{1-x^2}}\,dx,x,a+bx^4\right)}{4b}\\&=-\frac{\sqrt{1-\left(a+bx^4\right)^2}}{4b}+\frac{\left(a+bx^4\right)\cos^{-1}\left(a+bx^4\right)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0276867, size = 43, normalized size = 0.91

$$\frac{\left(a+bx^4\right)\cos^{-1}\left(a+bx^4\right)-\sqrt{1-\left(a+bx^4\right)^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCos[a + b*x^4],x]
```

```
[Out] (-Sqrt[1 - (a + b*x^4)^2] + (a + b*x^4)*ArcCos[a + b*x^4])/(4*b)
```

Maple [A] time = 0.003, size = 40, normalized size = 0.9

$$\frac{1}{4\,b}\left(\left(bx^4+a\right)\arccos\left(bx^4+a\right)-\sqrt{1-\left(bx^4+a\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccos(b*x^4+a),x)
```

```
[Out] 1/4/b*((b*x^4+a)*arccos(b*x^4+a)-(1-(b*x^4+a)^2)^(1/2))
```

Maxima [A] time = 1.61154, size = 53, normalized size = 1.13

$$\frac{\left(bx^4+a\right)\arccos\left(bx^4+a\right)-\sqrt{-\left(bx^4+a\right)^2+1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*((b*x^4 + a)*arccos(b*x^4 + a) - sqrt(-(b*x^4 + a)^2 + 1))/b
```

Fricas [A] time = 2.43861, size = 105, normalized size = 2.23

$$\frac{\left(bx^4+a\right)\arccos\left(bx^4+a\right)-\sqrt{-b^2x^8-2\,abx^4-a^2+1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*((b*x^4 + a)*arccos(b*x^4 + a) - sqrt(-b^2*x^8 - 2*a*b*x^4 - a^2 + 1))/
b
```

Sympy [A] time = 1.14307, size = 61, normalized size = 1.3

$$\begin{cases}\frac{a\operatorname{acos}\left(a+bx^4\right)}{4b}+\frac{x^4\operatorname{acos}\left(a+bx^4\right)}{4}-\frac{\sqrt{-a^2-2abx^4-b^2x^8+1}}{4b} & \text{for}\ b\neq 0\\ \frac{x^4\operatorname{acos}\left(a\right)}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acos(b*x**4+a),x)
```

```
[Out] Piecewise((a*acos(a + b*x**4)/(4*b) + x**4*acos(a + b*x**4)/4 - sqrt(-a**2
- 2*a*b*x**4 - b**2*x**8 + 1)/(4*b), Ne(b, 0)), (x**4*acos(a)/4, True))
```

Giac [A] time = 1.26132, size = 53, normalized size = 1.13

$$\frac{\left(bx^4+a\right)\arccos\left(bx^4+a\right)-\sqrt{-\left(bx^4+a\right)^2+1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*((b*x^4 + a)*arccos(b*x^4 + a) - sqrt(-(b*x^4 + a)^2 + 1))/b
```

3.72 $\int x^{-1+n} \cos^{-1}(a+bx^n)\,dx$

Optimal. Leaf size=48

$$\frac{(a+bx^n)\cos^{-1}(a+bx^n)}{bn}-\frac{\sqrt{1-(a+bx^n)^2}}{bn}$$

```
[Out] -(Sqrt[1 - (a + b*x^n)^2]/(b*n)) + ((a + b*x^n)*ArcCos[a + b*x^n])/(b*n)
```

Rubi [A] time = 0.0544105, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6715, 4804, 4620, 261}

$$\frac{(a+bx^n)\cos^{-1}(a+bx^n)}{bn}-\frac{\sqrt{1-(a+bx^n)^2}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcCos[a + b*x^n],x]
```

```
[Out] -(Sqrt[1 - (a + b*x^n)^2]/(b*n)) + ((a + b*x^n)*ArcCos[a + b*x^n])/(b*n)
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]
```

Rule 4804

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d, Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*ArcCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x^{-1+n}\cos^{-1}\left(a+bx^n\right)\,dx &= \frac{\text{Subst}\left(\int \cos^{-1}(a+bx)\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \cos^{-1}(x)\,dx,x,a+bx^n\right)}{bn}\\
&= \frac{\left(a+bx^n\right)\cos^{-1}\left(a+bx^n\right)}{bn}+\frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}}\,dx,x,a+bx^n\right)}{bn}\\
&= -\frac{\sqrt{1-\left(a+bx^n\right)^2}}{bn}+\frac{\left(a+bx^n\right)\cos^{-1}\left(a+bx^n\right)}{bn}
\end{aligned}$$

Mathematica [A] time = 0.0413817, size = 43, normalized size = 0.9

$$\frac{\left(a+bx^n\right)\cos^{-1}\left(a+bx^n\right)-\sqrt{1-\left(a+bx^n\right)^2}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcCos[a + b*x^n],x]
```

```
[Out] (-Sqrt[1 - (a + b*x^n)^2] + (a + b*x^n)*ArcCos[a + b*x^n])/(b*n)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int x^{n-1}\arccos\left(a+bx^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*arccos(a+b*x^n),x)
```

```
[Out] int(x^(n-1)*arccos(a+b*x^n),x)
```

Maxima [A] time = 1.60569, size = 55, normalized size = 1.15

$$\frac{(bx^n + a)\arccos(bx^n + a) - \sqrt{-(bx^n + a)^2 + 1}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccos(a+b*x^n),x, algorithm="maxima")
```

```
[Out] ((b*x^n + a)*arccos(b*x^n + a) - sqrt(-(b*x^n + a)^2 + 1))/(b*n)
```

Fricas [A] time = 2.64617, size = 132, normalized size = 2.75

$$\frac{bx^n \arccos(bx^n + a) + a\arccos(bx^n + a) - \sqrt{-b^2x^{2n} - 2abx^n - a^2 + 1}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccos(a+b*x^n),x, algorithm="fricas")
```

```
[Out] (b*x^n*arccos(b*x^n + a) + a*arccos(b*x^n + a) - sqrt(-b^2*x^(2*n) - 2*a*b*
x^n - a^2 + 1))/(b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*acos(a+b*x**n),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31185, size = 55, normalized size = 1.15

$$\frac{(bx^n + a)\arccos(bx^n + a) - \sqrt{-(bx^n + a)^2 + 1}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccos(a+b*x^n),x, algorithm="giac")
```

```
[Out] ((b*x^n + a)*arccos(b*x^n + a) - sqrt(-(b*x^n + a)^2 + 1))/(b*n)
```

3.73 $\int \left(a+b\cos^{-1}\left(1+dx^2\right)\right)^4\,dx$

Optimal. Leaf size=127

$$\frac{192b^3\sqrt{-d^2x^4-2dx^2}\left(a+b\cos^{-1}\left(dx^2+1\right)\right)}{dx}-48b^2x\left(a+b\cos^{-1}\left(dx^2+1\right)\right)^2-\frac{8b\sqrt{-d^2x^4-2dx^2}\left(a+b\cos^{-1}\left(dx^2+1\right.\right.}{dx}$$

```
[Out] 384*b^4*x + (192*b^3*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2]))/(d
*x) - 48*b^2*x*(a + b*ArcCos[1 + d*x^2])^2 - (8*b*Sqrt[-2*d*x^2 - d^2*x^4]*
(a + b*ArcCos[1 + d*x^2])^3)/(d*x) + x*(a + b*ArcCos[1 + d*x^2])^4
```

Rubi [A] time = 0.0287647, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4815, 8}

$$\frac{192b^3\sqrt{-d^2x^4-2dx^2}\left(a+b\cos^{-1}\left(dx^2+1\right)\right)}{dx}-48b^2x\left(a+b\cos^{-1}\left(dx^2+1\right)\right)^2-\frac{8b\sqrt{-d^2x^4-2dx^2}\left(a+b\cos^{-1}\left(dx^2+1\right.\right.}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^4,x]
```

```
[Out] 384*b^4*x + (192*b^3*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2]))/(d
*x) - 48*b^2*x*(a + b*ArcCos[1 + d*x^2])^2 - (8*b*Sqrt[-2*d*x^2 - d^2*x^4]*
(a + b*ArcCos[1 + d*x^2])^3)/(d*x) + x*(a + b*ArcCos[1 + d*x^2])^4
```

Rule 4815

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcCos[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(1+dx^2\right)\right)^4 dx = -\frac{8b\sqrt{-2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^3}{dx} + x\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^4 - \left(48b^2\right)\int \left(a\right.$$
$$= \frac{192b^3\sqrt{-2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(1+dx^2\right)\right)}{dx} - 48b^2x\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2 - \frac{8b\sqrt{-2}}{}$$
$$= 384b^4x + \frac{192b^3\sqrt{-2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(1+dx^2\right)\right)}{dx} - 48b^2x\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2$$

Mathematica [A] time = 0.232636, size = 249, normalized size = 1.96

$$\frac{dx^2\left(-48a^2b^2+a^4+384b^4\right) - 8ab\left(a^2-24b^2\right)\sqrt{-dx^2\left(dx^2+2\right)} + 6b^2\cos^{-1}\left(dx^2+1\right)^2\left(a^2dx^2-4ab\sqrt{-dx^2\left(dx^2+2\right)}-8\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^4,x]
```

```
[Out] ((a^4 - 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*a*b*(a^2 - 24*b^2)*Sqrt[-(d*x^2*(2
+ d*x^2))] + 4*b*(a^3*d*x^2 - 24*a*b^2*d*x^2 - 6*a^2*b*Sqrt[-(d*x^2*(2 + d*
x^2))] + 48*b^3*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^2] + 6*b^2*(a^2*
d*x^2 - 8*b^2*d*x^2 - 4*a*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^2]^2
 + 4*b^3*(a*d*x^2 - 2*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^2]^3 + b
^4*d*x^2*ArcCos[1 + d*x^2]^4)/(d*x)
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^2+1\right)\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2+1))^4,x)
```

```
[Out] int((a+b*arccos(d*x^2+1))^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.60548, size = 474, normalized size = 3.73

$$\frac{b^4dx^2\arccos\left(dx^2+1\right)^4+4\,ab^3dx^2\arccos\left(dx^2+1\right)^3+6\left(a^2b^2-8\,b^4\right)dx^2\arccos\left(dx^2+1\right)^2+4\left(a^3b-24\,ab^3\right)dx^2\arcc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x^2*arccos(d*x^2 + 1)^4 + 4*a*b^3*d*x^2*arccos(d*x^2 + 1)^3 + 6*(a^2
*b^2 - 8*b^4)*d*x^2*arccos(d*x^2 + 1)^2 + 4*(a^3*b - 24*a*b^3)*d*x^2*arccos
(d*x^2 + 1) + (a^4 - 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*(b^4*arccos(d*x^2 + 1)
^3 + 3*a*b^3*arccos(d*x^2 + 1)^2 + a^3*b - 24*a*b^3 + 3*(a^2*b^2 - 8*b^4)*a
rccos(d*x^2 + 1))*sqrt(-d^2*x^4 - 2*d*x^2))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\,\mathrm{acos}\left(dx^2+1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2+1))**4,x)
```

```
[Out] Integral((a + b*acos(d*x**2 + 1))**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\arccos\left(dx^2+1\right)+a\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^4, x)
```

3.74 $\int \left(a + b\cos^{-1}\left(1 + dx^2\right)\right)^3 dx$

Optimal. Leaf size=110

$$-24ab^2x - \frac{6b\sqrt{-d^2x^4 - 2dx^2}\left(a + b\cos^{-1}\left(dx^2 + 1\right)\right)^2}{dx} + x\left(a + b\cos^{-1}\left(dx^2 + 1\right)\right)^3 + \frac{48b^3\sqrt{-d^2x^4 - 2dx^2}}{dx} - 24b^3x\cos^{-1}$$

```
[Out] -24*a*b^2*x + (48*b^3*Sqrt[-2*d*x^2 - d^2*x^4])/(d*x) - 24*b^3*x*ArcCos[1 +
 d*x^2] - (6*b*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2])^2)/(d*x)
+ x*(a + b*ArcCos[1 + d*x^2])^3
```

Rubi [A] time = 0.0571493, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4815, 4841, 12, 1588}

$$-24ab^2x - \frac{6b\sqrt{-d^2x^4 - 2dx^2}\left(a + b\cos^{-1}\left(dx^2 + 1\right)\right)^2}{dx} + x\left(a + b\cos^{-1}\left(dx^2 + 1\right)\right)^3 + \frac{48b^3\sqrt{-d^2x^4 - 2dx^2}}{dx} - 24b^3x\cos^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^3,x]
```

```
[Out] -24*a*b^2*x + (48*b^3*Sqrt[-2*d*x^2 - d^2*x^4])/(d*x) - 24*b^3*x*ArcCos[1 +
 d*x^2] - (6*b*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2])^2)/(d*x)
+ x*(a + b*ArcCos[1 + d*x^2])^3
```

Rule 4815

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcCos[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4841

```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(1+dx^2\right)\right)^3\,dx = -\frac{6b\sqrt{-2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2}{dx}+x\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^3-\left(24b^2\right)\int \left(a\right.$$

$$= -24ab^2x-\frac{6b\sqrt{-2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2}{dx}+x\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^3-\left(24\right.$$

$$= -24ab^2x-24b^3x\cos^{-1}\left(1+dx^2\right)-\frac{6b\sqrt{-2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2}{dx}+x\left(a+\right.$$

$$= -24ab^2x-24b^3x\cos^{-1}\left(1+dx^2\right)-\frac{6b\sqrt{-2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2}{dx}+x\left(a+\right.$$

$$= -24ab^2x+\frac{48b^3\sqrt{-2dx^2-d^2x^4}}{dx}-24b^3x\cos^{-1}\left(1+dx^2\right)-\frac{6b\sqrt{-2dx^2-d^2x^4}\left(a+b\cos\right.}{dx}$$

Mathematica [A] time = 0.125622, size = 162, normalized size = 1.47

$$\frac{adx^2\left(a^2-24b^2\right)-6b\left(a^2-8b^2\right)\sqrt{-dx^2\left(dx^2+2\right)}+3b\cos^{-1}\left(dx^2+1\right)\left(a^2dx^2-4ab\sqrt{-dx^2\left(dx^2+2\right)}-8b^2dx^2\right)+3b^2\cos}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^3,x]
```

```
[Out] (a*(a^2 - 24*b^2)*d*x^2 - 6*b*(a^2 - 8*b^2)*Sqrt[-(d*x^2*(2 + d*x^2))] + 3*
b*(a^2*d*x^2 - 8*b^2*d*x^2 - 4*a*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d
*x^2] + 3*b^2*(a*d*x^2 - 2*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^2]^
```

```
2 + b^3*d*x^2*ArcCos[1 + d*x^2]^3)/(d*x)
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^{2}+1\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2+1))^3,x)
```

```
[Out] int((a+b*arccos(d*x^2+1))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.36219, size = 328, normalized size = 2.98

$$\frac{b^{3}dx^{2}\arccos\left(dx^{2}+1\right)^{3}+3\,ab^{2}dx^{2}\arccos\left(dx^{2}+1\right)^{2}+3\left(a^{2}b-8\,b^{3}\right)dx^{2}\arccos\left(dx^{2}+1\right)+\left(a^{3}-24\,ab^{2}\right)dx^{2}-6\,\sqrt{-d^{2}}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x^2*arccos(d*x^2 + 1)^3 + 3*a*b^2*d*x^2*arccos(d*x^2 + 1)^2 + 3*(a^2
*b - 8*b^3)*d*x^2*arccos(d*x^2 + 1) + (a^3 - 24*a*b^2)*d*x^2 - 6*sqrt(-d^2*
x^4 - 2*d*x^2)*(b^3*arccos(d*x^2 + 1)^2 + 2*a*b^2*arccos(d*x^2 + 1) + a^2*b
 - 8*b^3))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\,\mathrm{acos}\left(dx^2+1\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2+1))**3,x)
```

```
[Out] Integral((a + b*acos(d*x**2 + 1))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\arccos\left(dx^2+1\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^3, x)
```

3.75 $\int \left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2 dx$

Optimal. Leaf size=63

$$-\frac{4b\sqrt{-d^2x^4-2dx^2}\left(a+b\cos^{-1}\left(dx^2+1\right)\right)}{dx}+x\left(a+b\cos^{-1}\left(dx^2+1\right)\right)^2-8b^2x$$

```
[Out] -8*b^2*x - (4*b*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2]))/(d*x) +
 x*(a + b*ArcCos[1 + d*x^2])^2
```

Rubi [A] time = 0.0115554, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4815, 8}

$$-\frac{4b\sqrt{-d^2x^4-2dx^2}\left(a+b\cos^{-1}\left(dx^2+1\right)\right)}{dx}+x\left(a+b\cos^{-1}\left(dx^2+1\right)\right)^2-8b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^2,x]
```

```
[Out] -8*b^2*x - (4*b*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2]))/(d*x) +
 x*(a + b*ArcCos[1 + d*x^2])^2
```

Rule 4815

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcCos[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2\,dx = -\frac{4b\sqrt{-2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(1+dx^2\right)\right)}{dx} + x\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2 - \left(8b^2\right)\int 1\,dx$$
$$= -8b^2x - \frac{4b\sqrt{-2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(1+dx^2\right)\right)}{dx} + x\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2$$

Mathematica [A] time = 0.0657708, size = 98, normalized size = 1.56

$$x\left(a^2-8b^2\right) - \frac{4ab\sqrt{-dx^2\left(dx^2+2\right)}}{dx} + \frac{2b\cos^{-1}\left(dx^2+1\right)\left(adx^2-2b\sqrt{-dx^2\left(dx^2+2\right)}\right)}{dx} + b^2x\cos^{-1}\left(dx^2+1\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^2,x]
```

```
[Out] (a^2 - 8*b^2)*x - (4*a*b*Sqrt[-(d*x^2*(2 + d*x^2))])/(d*x) + (2*b*(a*d*x^2
- 2*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^2])/(d*x) + b^2*x*ArcCos[1
 + d*x^2]^2
```

Maple [F] time = 0.117, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^2+1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2+1))^2,x)
```

```
[Out] int((a+b*arccos(d*x^2+1))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.50039, size = 205, normalized size = 3.25

$$\frac{b^2dx^2\arccos\left(dx^2+1\right)^2+2\,abdx^2\arccos\left(dx^2+1\right)+\left(a^2-8\,b^2\right)dx^2-4\,\sqrt{-d^2x^4-2\,dx^2}\left(b^2\arccos\left(dx^2+1\right)+ab\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x^2*arccos(d*x^2 + 1)^2 + 2*a*b*d*x^2*arccos(d*x^2 + 1) + (a^2 - 8*b
^2)*d*x^2 - 4*sqrt(-d^2*x^4 - 2*d*x^2)*(b^2*arccos(d*x^2 + 1) + a*b))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\operatorname{acos}\left(dx^2+1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2+1))**2,x)
```

```
[Out] Integral((a + b*acos(d*x**2 + 1))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\arccos\left(dx^2+1\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^2, x)
```

3.76 $\int \left(a+b\cos^{-1}\left(1+dx^2\right)\right)\,dx$

Optimal. Leaf size=43

$$ax-\frac{2b\sqrt{-d^2x^4-2dx^2}}{dx}+bx\cos^{-1}\left(dx^2+1\right)$$

```
[Out] a*x - (2*b*Sqrt[-2*d*x^2 - d^2*x^4])/(d*x) + b*x*ArcCos[1 + d*x^2]
```

Rubi [A] time = 0.0371819, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4841, 12, 1588}

$$ax-\frac{2b\sqrt{-d^2x^4-2dx^2}}{dx}+bx\cos^{-1}\left(dx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCos[1 + d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[-2*d*x^2 - d^2*x^4])/(d*x) + b*x*ArcCos[1 + d*x^2]
```

Rule 4841

```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] && !Functio
nOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\cos^{-1}\left(1+dx^2\right)\right)\,dx &= ax+b\int \cos^{-1}\left(1+dx^2\right)\,dx\\ &= ax+bx\cos^{-1}\left(1+dx^2\right)+b\int \frac{2dx^2}{\sqrt{-2dx^2-d^2x^4}}\,dx\\ &= ax+bx\cos^{-1}\left(1+dx^2\right)+(2bd)\int \frac{x^2}{\sqrt{-2dx^2-d^2x^4}}\,dx\\ &= ax-\frac{2b\sqrt{-2dx^2-d^2x^4}}{dx}+bx\cos^{-1}\left(1+dx^2\right)\end{aligned}$$

Mathematica [A] time = 0.0272371, size = 41, normalized size = 0.95

$$ax-\frac{2b\sqrt{-dx^2\left(dx^2+2\right)}}{dx}+bx\cos^{-1}\left(dx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCos[1 + d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[-(d*x^2*(2 + d*x^2))])/(d*x) + b*x*ArcCos[1 + d*x^2]
```

Maple [A] time = 0.005, size = 45, normalized size = 1.1

$$ax+b\left(x\arccos\left(dx^2+1\right)+2\,\frac{x\left(dx^2+2\right)}{\sqrt{-d^2x^4-2\,dx^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccos(d*x^2+1),x)
```

```
[Out] a*x+b*(x*arccos(d*x^2+1)+2/(-d^2*x^4-2*d*x^2)^(1/2)*x*(d*x^2+2))
```

Maxima [A] time = 1.56553, size = 61, normalized size = 1.42

$$\left(x\arccos\left(dx^2+1\right)+\frac{2\left(d^{\frac{3}{2}}x^2+2\sqrt{d}\right)}{\sqrt{-dx^2-2d}}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccos(d*x^2+1),x, algorithm="maxima")
```

```
[Out] (x*arccos(d*x^2 + 1) + 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(-d*x^2 - 2)*d))*b
+ a*x
```

Fricas [A] time = 2.30336, size = 103, normalized size = 2.4

$$\frac{bdx^2 \arccos\left(dx^2+1\right)+adx^2-2\sqrt{-d^2x^4-2\,dx^2}b}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccos(d*x^2+1),x, algorithm="fricas")
```

```
[Out] (b*d*x^2*arccos(d*x^2 + 1) + a*d*x^2 - 2*sqrt(-d^2*x^4 - 2*d*x^2)*b)/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\,\mathrm{acos}\left(dx^2+1\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acos(d*x**2+1),x)
```

```
[Out] Integral(a + b*acos(d*x**2 + 1), x)
```

Giac [A] time = 1.34852, size = 78, normalized size = 1.81

$$\left(2\,d\left(\frac{\sqrt{2}\sqrt{-d}\mathrm{sgn}\left(x\right)}{d^2}-\frac{\sqrt{-d^2x^2-2\,d}}{d^2\mathrm{sgn}\left(x\right)}\right)+x\arccos\left(dx^2+1\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccos(d*x^2+1),x, algorithm="giac")
```

```
[Out] (2*d*(sqrt(2)*sqrt(-d)*sgn(x)/d^2 - sqrt(-d^2*x^2 - 2*d)/(d^2*sgn(x))) + x*
arccos(d*x^2 + 1))*b + a*x
```

3.77 $\int \frac{1}{a+b\cos^{-1}(1+dx^2)}\,dx$

Optimal. Leaf size=99

$$\frac{x\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2}b\sqrt{-dx^2}}+\frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2}b\sqrt{-dx^2}}$$

```
[Out] (x*Cos[a/(2*b)]*CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)])/(Sqrt[2]*b*Sq
rt[-(d*x^2)]) + (x*Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)
])/(Sqrt[2]*b*Sqrt[-(d*x^2)])
```

Rubi [A] time = 0.0311988, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {4817}

$$\frac{x\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2}b\sqrt{-dx^2}}+\frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2}b\sqrt{-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(-1),x]
```

```
[Out] (x*Cos[a/(2*b)]*CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)])/(Sqrt[2]*b*Sq
rt[-(d*x^2)]) + (x*Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)
])/(Sqrt[2]*b*Sqrt[-(d*x^2)])
```

Rule 4817

```
Int[((a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(x*Cos
[a/(2*b)]*CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)])/(Sqrt[2]*b*Sqrt[-(d
*x^2)]), x] + Simp[(x*Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2
*b)])/(Sqrt[2]*b*Sqrt[-(d*x^2)]), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{a+b\cos^{-1}\left(1+dx^2\right)}\,dx = \frac{x\cos\left(\frac{a}{2b}\right)\mathrm{Ci}\left(\frac{a+b\cos^{-1}\left(1+dx^2\right)}{2b}\right)}{\sqrt{2}b\sqrt{-dx^2}} + \frac{x\sin\left(\frac{a}{2b}\right)\mathrm{Si}\left(\frac{a+b\cos^{-1}\left(1+dx^2\right)}{2b}\right)}{\sqrt{2}b\sqrt{-dx^2}}$$

Mathematica [A] time = 0.103861, size = 85, normalized size = 0.86

$$-\frac{\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\left(\cos\left(\frac{a}{2b}\right)\mathrm{CosIntegral}\left(\frac{a+b\cos^{-1}\left(dx^2+1\right)}{2b}\right)+\sin\left(\frac{a}{2b}\right)\mathrm{Si}\left(\frac{a+b\cos^{-1}\left(dx^2+1\right)}{2b}\right)\right)}{bdx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^(-1),x]
```

```
[Out] -((Sin[ArcCos[1 + d*x^2]/2]*(Cos[a/(2*b)]*CosIntegral[(a + b*ArcCos[1 + d*x
^2])/(2*b)] + Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]))/(
b*d*x))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^2+1\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2+1)),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2+1)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\arccos\left(dx^2+1\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccos(d*x^2 + 1) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b\arccos\left(dx^2+1\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arccos(d*x^2 + 1) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\,\mathrm{acos}\left(dx^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2+1)),x)
```

```
[Out] Integral(1/(a + b*acos(d*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\arccos\left(dx^2+1\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arccos(d*x^2 + 1) + a), x)
```

3.78 $\int \frac{1}{\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=151

$$\frac{x\sin\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}\left(dx^2+1\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{-dx^2}}-\frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}\left(dx^2+1\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{-dx^2}}+\frac{\sqrt{-d^2x^4-2dx^2}}{2bdx\left(a+b\cos^{-1}\left(dx^2+1\right)\right)}$$

```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcCos[1 + d*x^2])) + (x*CosIntegr
al[(a + b*ArcCos[1 + d*x^2])/(2*b)]*Sin[a/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[-(d*x
^2)]) - (x*Cos[a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)])/(2*Sq
rt[2]*b^2*Sqrt[-(d*x^2)])
```

Rubi [A] time = 0.0203403, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {4826}

$$\frac{x\sin\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}\left(dx^2+1\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{-dx^2}}-\frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}\left(dx^2+1\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{-dx^2}}+\frac{\sqrt{-d^2x^4-2dx^2}}{2bdx\left(a+b\cos^{-1}\left(dx^2+1\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(-2),x]
```

```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcCos[1 + d*x^2])) + (x*CosIntegr
al[(a + b*ArcCos[1 + d*x^2])/(2*b)]*Sin[a/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[-(d*x
^2)]) - (x*Cos[a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)])/(2*Sq
rt[2]*b^2*Sqrt[-(d*x^2)])
```

Rule 4826

```
Int[((a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> Simp[Sqrt[-
2*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcCos[1 + d*x^2])), x] + (Simp[(x*Sin[a
/(2*b)]*CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[(
-d)*x^2]), x] - Simp[(x*Cos[a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/
(2*b)])/(2*Sqrt[2]*b^2*Sqrt[(-d)*x^2]), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2}\,dx = \frac{\sqrt{-2dx^2-d^2x^4}}{2bdx\left(a+b\cos^{-1}\left(1+dx^2\right)\right)} + \frac{x\text{Ci}\left(\frac{a+b\cos^{-1}(1+dx^2)}{2b}\right)\sin\left(\frac{a}{2b}\right)}{2\sqrt{2}b^2\sqrt{-dx^2}} - \frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(1}{2b}\right.}{2\sqrt{2}b^2\sqrt{-dx^2}}$$

Mathematica [A] time = 0.332094, size = 133, normalized size = 0.88

$$\frac{\sqrt{-dx^2\left(dx^2+2\right)}\left(\frac{b}{a+b\cos^{-1}\left(dx^2+1\right)} - \frac{\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\left(\sin\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}\left(dx^2+1\right)}{2b}\right)-\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}\left(dx^2+1\right)}{2b}\right)\right)}{dx^2+2}\right)}{2b^2dx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^(-2),x]
```

```
[Out] (Sqrt[-(d*x^2*(2 + d*x^2))]*(b/(a + b*ArcCos[1 + d*x^2]) - (Cos[ArcCos[1 +
d*x^2]/2]*(CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]*Sin[a/(2*b)] - Cos[
a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]))/(2 + d*x^2)))/(2*b^
2*d*x)
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^2+1\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2+1))^2,x)
```

```
[Out] int(1/(a+b*arccos(d*x^2+1))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(b^2d\arctan\left(\sqrt{-dx^2-2}\sqrt{d}x, dx^2+1\right)+abd\right)\sqrt{d}\int \frac{\sqrt{-dx^2-2}x}{abdx^2+2\,ab+\left(b^2dx^2+2\,b^2\right)\arctan\left(\sqrt{-dx^2-2}\sqrt{d}x, dx^2+1\right)}\,dx - \sqrt{-dx^2-2}\sqrt{d}}{2\left(b^2d\arctan\left(\sqrt{-dx^2-2}\sqrt{d}x, dx^2+1\right)+abd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b^2*d*arctan2(sqrt(-d*x^2 - 2)*sqrt(d)*x, d*x^2 + 1) + a*b*d)*sqrt
(d)*integrate(1/2*sqrt(-d*x^2 - 2)*x/(a*b*d*x^2 + 2*a*b + (b^2*d*x^2 + 2*b^
2)*arctan2(sqrt(-d*x^2 - 2)*sqrt(d)*x, d*x^2 + 1)), x) - sqrt(-d*x^2 - 2)*s
qrt(d))/(b^2*d*arctan2(sqrt(-d*x^2 - 2)*sqrt(d)*x, d*x^2 + 1) + a*b*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{b^2\arccos\left(dx^2+1\right)^2+2\,ab\arccos\left(dx^2+1\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arccos(d*x^2 + 1)^2 + 2*a*b*arccos(d*x^2 + 1) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\,\mathrm{acos}\left(dx^2+1\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2+1))**2,x)
```

```
[Out] Integral((a + b*acos(d*x**2 + 1))**(-2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\arccos\left(dx^2+1\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(-2), x)
```

3.79 $\int \frac{1}{\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^3}\,dx$

Optimal. Leaf size=173

$$-\frac{x\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}\left(dx^2+1\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{-dx^2}}-\frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}\left(dx^2+1\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{-dx^2}}+\frac{x}{8b^2\left(a+b\cos^{-1}\left(dx^2+1\right)\right)}+\frac{\sqrt{-d\cdots}}{4bdx\left(a+b\cdots\right.}$$

```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(4*b*d*x*(a + b*ArcCos[1 + d*x^2])^2) + x/(8*b^2*(
a + b*ArcCos[1 + d*x^2])) - (x*Cos[a/(2*b)]*CosIntegral[(a + b*ArcCos[1 + d
*x^2])/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[-(d*x^2)]) - (x*Sin[a/(2*b)]*SinIntegral
[(a + b*ArcCos[1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[-(d*x^2)])
```

Rubi [A] time = 0.0362387, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4829, 4817}

$$-\frac{x\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}\left(dx^2+1\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{-dx^2}}-\frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}\left(dx^2+1\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{-dx^2}}+\frac{x}{8b^2\left(a+b\cos^{-1}\left(dx^2+1\right)\right)}+\frac{\sqrt{-d\cdots}}{4bdx\left(a+b\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(-3),x]
```

```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(4*b*d*x*(a + b*ArcCos[1 + d*x^2])^2) + x/(8*b^2*(
a + b*ArcCos[1 + d*x^2])) - (x*Cos[a/(2*b)]*CosIntegral[(a + b*ArcCos[1 + d
*x^2])/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[-(d*x^2)]) - (x*Sin[a/(2*b)]*SinIntegral
[(a + b*ArcCos[1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[-(d*x^2)])
```

Rule 4829

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcCos[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCos[c + d*x^2])^(n + 2), x], x] - Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcCos[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rule 4817

```
Int[((a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(x*Cos
[a/(2*b)]*CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)])/(Sqrt[2]*b*Sqrt[-(d
*x^2)]), x] + Simp[(x*Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2
*b)])/(Sqrt[2]*b*Sqrt[-(d*x^2)]), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^3}\,dx = \frac{\sqrt{-2dx^2-d^2x^4}}{4bdx\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2}+\frac{x}{8b^2\left(a+b\cos^{-1}\left(1+dx^2\right)\right)}-\frac{\int \frac{1}{a+b\cos^{-1}\left(1+dx^2\right)}\,dx}{8b^2}$$

$$= \frac{\sqrt{-2dx^2-d^2x^4}}{4bdx\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2}+\frac{x}{8b^2\left(a+b\cos^{-1}\left(1+dx^2\right)\right)}-\frac{x\cos\left(\frac{a}{2b}\right)\mathrm{Ci}\left(\frac{a+b\cos^{-1}(1+}{2b}\right.}{8\sqrt{2}b^3\sqrt{-dx^2}}$$

Mathematica [A] time = 0.246495, size = 147, normalized size = 0.85

$$\frac{\frac{2b^2\sqrt{-dx^2\left(dx^2+2\right)}}{d\left(a+b\cos^{-1}\left(dx^2+1\right)\right)^2}+\frac{\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\left(\cos\left(\frac{a}{2b}\right)\mathrm{CosIntegral}\left(\frac{a+b\cos^{-1}\left(dx^2+1\right)}{2b}\right)+\sin\left(\frac{a}{2b}\right)\mathrm{Si}\left(\frac{a+b\cos^{-1}\left(dx^2+1\right)}{2b}\right)\right)}{d}+\frac{bx^2}{a+b\cos^{-1}\left(dx^2+1\right)}}{8b^3x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^(-3),x]
```

```
[Out] ((2*b^2*Sqrt[-(d*x^2*(2 + d*x^2))])/(d*(a + b*ArcCos[1 + d*x^2])^2) + (b*x^
2)/(a + b*ArcCos[1 + d*x^2]) + (Sin[ArcCos[1 + d*x^2]/2]*(Cos[a/(2*b)]*CosI
ntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)] + Sin[a/(2*b)]*SinIntegral[(a + b*
ArcCos[1 + d*x^2])/(2*b)]))/d)/(8*b^3*x)
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^2+1\right)\right)^{-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2+1))^3,x)
```

```
[Out] int(1/(a+b*arccos(d*x^2+1))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bdx \arctan\left(\sqrt{-dx^2-2}\sqrt{d}x, dx^2+1\right) + adx + 2\sqrt{-dx^2-2}b\sqrt{d} - \left(b^4 d \arctan\left(\sqrt{-dx^2-2}\sqrt{d}x, dx^2+1\right)^2 + 2\,ab^3 d \,\mathrm{arct}}{8\left(b^4 d \arctan\left(\sqrt{-dx^2-2}\sqrt{d}x, dx^2+1\right)^2 + 2\,ab^3 d \arctan\left(\sqrt{-dx^2-2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b*d*x*arctan2(sqrt(-d*x^2 - 2)*sqrt(d)*x, d*x^2 + 1) + a*d*x + 2*sqrt(
-d*x^2 - 2)*b*sqrt(d) - 8*(b^4*d*arctan2(sqrt(-d*x^2 - 2)*sqrt(d)*x, d*x^2
+ 1)^2 + 2*a*b^3*d*arctan2(sqrt(-d*x^2 - 2)*sqrt(d)*x, d*x^2 + 1) + a^2*b^2
*d)*integrate(1/8/(b^3*arctan2(sqrt(-d*x^2 - 2)*sqrt(d)*x, d*x^2 + 1) + a*b
^2), x))/(b^4*d*arctan2(sqrt(-d*x^2 - 2)*sqrt(d)*x, d*x^2 + 1)^2 + 2*a*b^3*
d*arctan2(sqrt(-d*x^2 - 2)*sqrt(d)*x, d*x^2 + 1) + a^2*b^2*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{b^3 \arccos\left(dx^2+1\right)^3 + 3\,ab^2 \arccos\left(dx^2+1\right)^2 + 3\,a^2 b \arccos\left(dx^2+1\right) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arccos(d*x^2 + 1)^3 + 3*a*b^2*arccos(d*x^2 + 1)^2 + 3*a^2*b
*arccos(d*x^2 + 1) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b\,\mathrm{acos}\left(dx^2 + 1\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2+1))**3,x)
```

```
[Out] Integral((a + b*acos(d*x**2 + 1))**(-3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\arccos\left(dx^2 + 1\right) + a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(-3), x)
```

3.80 $\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^4 dx$

Optimal. Leaf size=127

$$\frac{192b^3\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(dx^2-1\right)\right)}{dx}-48b^2x\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^2-\frac{8b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^3}{dx}$$

```
[Out] 384*b^4*x + (192*b^3*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2]))/(d
*x) - 48*b^2*x*(a + b*ArcCos[-1 + d*x^2])^2 - (8*b*Sqrt[2*d*x^2 - d^2*x^4]*
(a + b*ArcCos[-1 + d*x^2])^3)/(d*x) + x*(a + b*ArcCos[-1 + d*x^2])^4
```

Rubi [A] time = 0.0283064, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4815, 8}

$$\frac{192b^3\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(dx^2-1\right)\right)}{dx}-48b^2x\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^2-\frac{8b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^3}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^4,x]
```

```
[Out] 384*b^4*x + (192*b^3*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2]))/(d
*x) - 48*b^2*x*(a + b*ArcCos[-1 + d*x^2])^2 - (8*b*Sqrt[2*d*x^2 - d^2*x^4]*
(a + b*ArcCos[-1 + d*x^2])^3)/(d*x) + x*(a + b*ArcCos[-1 + d*x^2])^4
```

Rule 4815

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcCos[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^4\,dx = -\frac{8b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^3}{dx} + x\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^4 - \left(48b^2\right)\int$$

$$= \frac{192b^3\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)}{dx} - 48b^2x\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2 - \frac{8b\sqrt{}}{}$$

$$= 384b^4x + \frac{192b^3\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)}{dx} - 48b^2x\left(a+b\cos^{-1}\left(-1+dx^2\right.\right.$$

Mathematica [A] time = 0.239112, size = 249, normalized size = 1.96

$$\frac{dx^2\left(-48a^2b^2+a^4+384b^4\right) - 8ab\left(a^2-24b^2\right)\sqrt{dx^2\left(2-dx^2\right)} + 6b^2\cos^{-1}\left(dx^2-1\right)^2\left(a^2dx^2-4ab\sqrt{-dx^2\left(dx^2-2\right)}-8b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^4,x]
```

```
[Out] ((a^4 - 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*a*b*(a^2 - 24*b^2)*Sqrt[d*x^2*(2 -
d*x^2)] + 4*b*(a^3*d*x^2 - 24*a*b^2*d*x^2 - 6*a^2*b*Sqrt[-(d*x^2*(-2 + d*x^
2))] + 48*b^3*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*x^2] + 6*b^2*(a^2*
d*x^2 - 8*b^2*d*x^2 - 4*a*b*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*x^2]
^2 + 4*b^3*(a*d*x^2 - 2*b*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*x^2]^3
 + b^4*d*x^2*ArcCos[-1 + d*x^2]^4)/(d*x)
```

Maple [F] time = 0.117, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^2-1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2-1))^4,x)
```

```
[Out] int((a+b*arccos(d*x^2-1))^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.65725, size = 474, normalized size = 3.73

$$\frac{b^4dx^2\arccos\left(dx^2-1\right)^4+4\,ab^3dx^2\arccos\left(dx^2-1\right)^3+6\left(a^2b^2-8\,b^4\right)dx^2\arccos\left(dx^2-1\right)^2+4\left(a^3b-24\,ab^3\right)dx^2\,\mathrm{arcc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x^2*arccos(d*x^2 - 1)^4 + 4*a*b^3*d*x^2*arccos(d*x^2 - 1)^3 + 6*(a^2
*b^2 - 8*b^4)*d*x^2*arccos(d*x^2 - 1)^2 + 4*(a^3*b - 24*a*b^3)*d*x^2*arccos
(d*x^2 - 1) + (a^4 - 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*(b^4*arccos(d*x^2 - 1)
^3 + 3*a*b^3*arccos(d*x^2 - 1)^2 + a^3*b - 24*a*b^3 + 3*(a^2*b^2 - 8*b^4)*a
rccos(d*x^2 - 1))*sqrt(-d^2*x^4 + 2*d*x^2))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\,\mathrm{acos}\left(dx^2-1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2-1))**4,x)
```

```
[Out] Integral((a + b*acos(d*x**2 - 1))**4, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arccos \left(dx^2 - 1\right) + a\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^4, x)
```

3.81 $\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^3\,dx$

Optimal. Leaf size=110

$$-24ab^2x-\frac{6b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^2}{dx}+x\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^3+\frac{48b^3\sqrt{2dx^2-d^2x^4}}{dx}-24b^3x\cos^{-1}\left(d\right.$$

```
[Out] -24*a*b^2*x + (48*b^3*Sqrt[2*d*x^2 - d^2*x^4])/(d*x) - 24*b^3*x*ArcCos[-1 +
 d*x^2] - (6*b*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2])^2)/(d*x)
+ x*(a + b*ArcCos[-1 + d*x^2])^3
```

Rubi [A] time = 0.055042, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4815, 4841, 12, 1588}

$$-24ab^2x-\frac{6b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^2}{dx}+x\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^3+\frac{48b^3\sqrt{2dx^2-d^2x^4}}{dx}-24b^3x\cos^{-1}\left(d\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^3,x]
```

```
[Out] -24*a*b^2*x + (48*b^3*Sqrt[2*d*x^2 - d^2*x^4])/(d*x) - 24*b^3*x*ArcCos[-1 +
 d*x^2] - (6*b*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2])^2)/(d*x)
+ x*(a + b*ArcCos[-1 + d*x^2])^3
```

Rule 4815

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcCos[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4841

```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^3\,dx = -\frac{6b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2}{dx}+x\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^3-\left(24b^2\right)\int$$

$$= -24ab^2x-\frac{6b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2}{dx}+x\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^3-$$

$$= -24ab^2x-24b^3x\cos^{-1}\left(-1+dx^2\right)-\frac{6b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2}{dx}+x\left(a\right.$$

$$= -24ab^2x-24b^3x\cos^{-1}\left(-1+dx^2\right)-\frac{6b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2}{dx}+x\left(a\right.$$

$$= -24ab^2x+\frac{48b^3\sqrt{2dx^2-d^2x^4}}{dx}-24b^3x\cos^{-1}\left(-1+dx^2\right)-\frac{6b\sqrt{2dx^2-d^2x^4}\left(a+b\cos\right.}{dx}$$

Mathematica [A] time = 0.135227, size = 162, normalized size = 1.47

$$\frac{adx^2\left(a^2-24b^2\right)-6b\left(a^2-8b^2\right)\sqrt{dx^2\left(2-dx^2\right)}+3b\cos^{-1}\left(dx^2-1\right)\left(a^2dx^2-4ab\sqrt{-dx^2\left(dx^2-2\right)}-8b^2dx^2\right)+3b^2\cos}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^3,x]
```

```
[Out] (a*(a^2 - 24*b^2)*d*x^2 - 6*b*(a^2 - 8*b^2)*Sqrt[d*x^2*(2 - d*x^2)] + 3*b*(
a^2*d*x^2 - 8*b^2*d*x^2 - 4*a*b*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*
x^2] + 3*b^2*(a*d*x^2 - 2*b*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*x^2]
```

```
^2 + b^3*d*x^2*ArcCos[-1 + d*x^2]^3)/(d*x)
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^2-1\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2-1))^3,x)
```

```
[Out] int((a+b*arccos(d*x^2-1))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.5729, size = 328, normalized size = 2.98

$$\frac{b^3dx^2\arccos\left(dx^2-1\right)^3+3\,ab^2dx^2\arccos\left(dx^2-1\right)^2+3\left(a^2b-8\,b^3\right)dx^2\arccos\left(dx^2-1\right)+\left(a^3-24\,ab^2\right)dx^2-6\,\sqrt{-d^2}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x^2*arccos(d*x^2 - 1)^3 + 3*a*b^2*d*x^2*arccos(d*x^2 - 1)^2 + 3*(a^2
*b - 8*b^3)*d*x^2*arccos(d*x^2 - 1) + (a^3 - 24*a*b^2)*d*x^2 - 6*sqrt(-d^2*
x^4 + 2*d*x^2)*(b^3*arccos(d*x^2 - 1)^2 + 2*a*b^2*arccos(d*x^2 - 1) + a^2*b
 - 8*b^3))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\mathrm{acos}\left(dx^2 - 1\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2-1))**3,x)
```

```
[Out] Integral((a + b*acos(d*x**2 - 1))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arccos\left(dx^2 - 1\right) + a\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^3, x)
```

3.82 $\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2\,dx$

Optimal. Leaf size=63

$$-\frac{4b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(dx^2-1\right)\right)}{dx}+x\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^2-8b^2x$$

```
[Out] -8*b^2*x - (4*b*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2]))/(d*x) +
 x*(a + b*ArcCos[-1 + d*x^2])^2
```

Rubi [A] time = 0.0115222, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4815, 8}

$$-\frac{4b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(dx^2-1\right)\right)}{dx}+x\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^2-8b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^2,x]
```

```
[Out] -8*b^2*x - (4*b*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2]))/(d*x) +
 x*(a + b*ArcCos[-1 + d*x^2])^2
```

Rule 4815

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcCos[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2\,dx = -\frac{4b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)}{dx} + x\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2 - \left(8b^2\right)\int 1$$
$$= -8b^2x - \frac{4b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)}{dx} + x\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2$$

Mathematica [A] time = 0.0658086, size = 98, normalized size = 1.56

$$x\left(a^2-8b^2\right) - \frac{4ab\sqrt{-dx^2\left(dx^2-2\right)}}{dx} + \frac{2b\cos^{-1}\left(dx^2-1\right)\left(adx^2-2b\sqrt{-dx^2\left(dx^2-2\right)}\right)}{dx} + b^2x\cos^{-1}\left(dx^2-1\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^2,x]
```

```
[Out] (a^2 - 8*b^2)*x - (4*a*b*Sqrt[-(d*x^2*(-2 + d*x^2))])/(d*x) + (2*b*(a*d*x^2
 - 2*b*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*x^2])/(d*x) + b^2*x*ArcCo
s[-1 + d*x^2]^2
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^2-1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2-1))^2,x)
```

```
[Out] int((a+b*arccos(d*x^2-1))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.50507, size = 205, normalized size = 3.25

$$\frac{b^2dx^2\arccos\left(dx^2-1\right)^2+2\,abdx^2\arccos\left(dx^2-1\right)+\left(a^2-8\,b^2\right)dx^2-4\,\sqrt{-d^2x^4+2\,dx^2}\left(b^2\arccos\left(dx^2-1\right)+ab\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x^2*arccos(d*x^2 - 1)^2 + 2*a*b*d*x^2*arccos(d*x^2 - 1) + (a^2 - 8*b
^2)*d*x^2 - 4*sqrt(-d^2*x^4 + 2*d*x^2)*(b^2*arccos(d*x^2 - 1) + a*b))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\operatorname{acos}\left(dx^2-1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2-1))**2,x)
```

```
[Out] Integral((a + b*acos(d*x**2 - 1))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\arccos\left(dx^2-1\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^2, x)
```

3.83 $\int \left(a + b\cos^{-1}\left(-1 + dx^2\right)\right) dx$

Optimal. Leaf size=43

$$ax - \frac{2b\sqrt{2dx^2 - d^2x^4}}{dx} + bx\cos^{-1}\left(dx^2 - 1\right)$$

```
[Out] a*x - (2*b*Sqrt[2*d*x^2 - d^2*x^4])/(d*x) + b*x*ArcCos[-1 + d*x^2]
```

Rubi [A] time = 0.0357681, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4841, 12, 1588}

$$ax - \frac{2b\sqrt{2dx^2 - d^2x^4}}{dx} + bx\cos^{-1}\left(dx^2 - 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCos[-1 + d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[2*d*x^2 - d^2*x^4])/(d*x) + b*x*ArcCos[-1 + d*x^2]
```

Rule 4841

```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)dx = ax+b\int \cos^{-1}\left(-1+dx^2\right)dx$$
$$= ax+bx\cos^{-1}\left(-1+dx^2\right)+b\int \frac{2dx^2}{\sqrt{2dx^2-d^2x^4}}\,dx$$
$$= ax+bx\cos^{-1}\left(-1+dx^2\right)+(2bd)\int \frac{x^2}{\sqrt{2dx^2-d^2x^4}}\,dx$$
$$= ax-\frac{2b\sqrt{2dx^2-d^2x^4}}{dx}+bx\cos^{-1}\left(-1+dx^2\right)$$

Mathematica [A] time = 0.0286509, size = 41, normalized size = 0.95

$$ax-\frac{2b\sqrt{-dx^2\left(dx^2-2\right)}}{dx}+bx\cos^{-1}\left(dx^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCos[-1 + d*x^2],x]

[Out] a*x - (2*b*Sqrt[-(d*x^2*(-2 + d*x^2))])/(d*x) + b*x*ArcCos[-1 + d*x^2]
```

Maple [A] time = 0.004, size = 45, normalized size = 1.1

$$ax+b\left(x\arccos\left(dx^2-1\right)+2\,\frac{x\left(dx^2-2\right)}{\sqrt{-d^2x^4+2\,dx^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccos(d*x^2-1),x)

[Out] a*x+b*(x*arccos(d*x^2-1)+2/(-d^2*x^4+2*d*x^2)^(1/2)*x*(d*x^2-2))
```

Maxima [A] time = 1.52665, size = 61, normalized size = 1.42

$$\left(x\arccos\left(dx^2-1\right)+\frac{2\left(d^{\frac{3}{2}}x^2-2\sqrt{d}\right)}{\sqrt{-dx^2+2d}}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccos(d*x^2-1),x, algorithm="maxima")
```

```
[Out] (x*arccos(d*x^2 - 1) + 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(-d*x^2 + 2)*d))*b
+ a*x
```

Fricas [A] time = 2.68217, size = 103, normalized size = 2.4

$$\frac{bdx^2 \arccos\left(dx^2 - 1\right) + adx^2 - 2\sqrt{-d^2x^4 + 2\,dx^2}b}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccos(d*x^2-1),x, algorithm="fricas")
```

```
[Out] (b*d*x^2*arccos(d*x^2 - 1) + a*d*x^2 - 2*sqrt(-d^2*x^4 + 2*d*x^2)*b)/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{acos}\left(dx^2 - 1\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acos(d*x**2-1),x)
```

```
[Out] Integral(a + b*acos(d*x**2 - 1), x)
```

Giac [A] time = 1.3, size = 72, normalized size = 1.67

$$\left(2\,d\left(\frac{\sqrt{2}\operatorname{sgn}(x)}{d^{\frac{3}{2}}} - \frac{\sqrt{-d^2x^2 + 2\,d}}{d^2\operatorname{sgn}(x)}\right) + x\arccos\left(dx^2 - 1\right)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccos(d*x^2-1),x, algorithm="giac")
```

```
[Out] (2*d*(sqrt(2)*sgn(x)/d^(3/2) - sqrt(-d^2*x^2 + 2*d)/(d^2*sgn(x))) + x*arcco
s(d*x^2 - 1))*b + a*x
```

3.84 $\int \frac{1}{a+b\cos^{-1}(-1+dx^2)}\,dx$

Optimal. Leaf size=98

$$\frac{x\sin\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}(dx^2-1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}-\frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(dx^2-1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}$$

```
[Out] (x*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]*Sin[a/(2*b)])/(Sqrt[2]*b*S
qrt[d*x^2]) - (x*Cos[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]
)/(Sqrt[2]*b*Sqrt[d*x^2])
```

Rubi [A] time = 0.0133217, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {4818}

$$\frac{x\sin\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}(dx^2-1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}-\frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(dx^2-1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(-1),x]
```

```
[Out] (x*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]*Sin[a/(2*b)])/(Sqrt[2]*b*S
qrt[d*x^2]) - (x*Cos[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]
)/(Sqrt[2]*b*Sqrt[d*x^2])
```

Rule 4818

```
Int[((a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(x*Si
n[a/(2*b)]*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)])/(Sqrt[2]*b*Sqrt[d
*x^2]), x] - Simp[(x*Cos[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2
*b)])/(Sqrt[2]*b*Sqrt[d*x^2]), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{a+b\cos^{-1}\left(-1+dx^2\right)}\,dx = \frac{x\mathrm{Ci}\left(\frac{a+b\cos^{-1}\left(-1+dx^2\right)}{2b}\right)\sin\left(\frac{a}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}} - \frac{x\cos\left(\frac{a}{2b}\right)\mathrm{Si}\left(\frac{a+b\cos^{-1}\left(-1+dx^2\right)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}$$

Mathematica [A] time = 0.0998995, size = 85, normalized size = 0.87

$$\frac{\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\left(\sin\left(\frac{a}{2b}\right)\mathrm{CosIntegral}\left(\frac{a+b\cos^{-1}\left(dx^2-1\right)}{2b}\right)-\cos\left(\frac{a}{2b}\right)\mathrm{Si}\left(\frac{a+b\cos^{-1}\left(dx^2-1\right)}{2b}\right)\right)}{bdx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^(-1),x]
```

```
[Out] (Cos[ArcCos[-1 + d*x^2]/2]*(CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]*S
in[a/(2*b)] - Cos[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]))/
(b*d*x)
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^2-1\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2-1)),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2-1)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\arccos\left(dx^2-1\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccos(d*x^2 - 1) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b\arccos\left(dx^2-1\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arccos(d*x^2 - 1) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{a+b\,\mathrm{acos}\left(dx^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2-1)),x)
```

```
[Out] Integral(1/(a + b*acos(d*x**2 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{b\arccos\left(dx^2-1\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arccos(d*x^2 - 1) + a), x)
```

3.85 $\int \frac{1}{\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=149

$$-\frac{x\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}\left(dx^2-1\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}-\frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}\left(dx^2-1\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}+\frac{\sqrt{2dx^2-d^2x^4}}{2bdx\left(a+b\cos^{-1}\left(dx^2-1\right)\right)}$$

```
[Out] Sqrt[2*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcCos[-1 + d*x^2])) - (x*Cos[a/(2*
b)]*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^
2]) - (x*Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)])/(2*Sqr
t[2]*b^2*Sqrt[d*x^2])
```

Rubi [A] time = 0.0176495, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {4827}

$$-\frac{x\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}\left(dx^2-1\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}-\frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}\left(dx^2-1\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}+\frac{\sqrt{2dx^2-d^2x^4}}{2bdx\left(a+b\cos^{-1}\left(dx^2-1\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(-2),x]
```

```
[Out] Sqrt[2*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcCos[-1 + d*x^2])) - (x*Cos[a/(2*
b)]*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^
2]) - (x*Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)])/(2*Sqr
t[2]*b^2*Sqrt[d*x^2])
```

Rule 4827

```
Int[((a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> Simp[Sqrt[
2*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcCos[-1 + d*x^2])), x] + (-Simp[(x*Cos
[a/(2*b)]*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)])/(2*Sqrt[2]*b^2*Sqr
t[d*x^2]), x] - Simp[(x*Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])
/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^2]), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2}\,dx = \frac{\sqrt{2dx^2-d^2x^4}}{2bdx\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)} - \frac{x\cos\left(\frac{a}{2b}\right)\text{Ci}\left(\frac{a+b\cos^{-1}\left(-1+dx^2\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}} - \frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos}{}\right.}{2\sqrt{2}b^2\sqrt{d}}$$

Mathematica [A] time = 0.335094, size = 131, normalized size = 0.88

$$\frac{\sqrt{-dx^2\left(dx^2-2\right)}\left(\frac{\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\left(\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}\left(dx^2-1\right)}{2b}\right)+\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}\left(dx^2-1\right)}{2b}\right)\right)}{dx^2-2}+\frac{b}{a+b\cos^{-1}\left(dx^2-1\right)}\right)}{2b^2dx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^(-2),x]
```

```
[Out] (Sqrt[-(d*x^2*(-2 + d*x^2))]*(b/(a + b*ArcCos[-1 + d*x^2]) + (Sin[ArcCos[-1
 + d*x^2]/2]*(Cos[a/(2*b)]*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)] +
Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]))/(-2 + d*x^2)))
/(2*b^2*d*x)
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^2-1\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2-1))^2,x)
```

```
[Out] int(1/(a+b*arccos(d*x^2-1))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(b^2d\arctan\left(\sqrt{-dx^2+2}\sqrt{d}x,dx^2-1\right)+abd\right)\sqrt{d}\int \frac{\sqrt{-dx^2+2}x}{abdx^2-2\,ab+\left(b^2dx^2-2\,b^2\right)\arctan\left(\sqrt{-dx^2+2}\sqrt{d}x,dx^2-1\right)}\,dx-\sqrt{-dx^2+2}\sqrt{d}}{2\left(b^2d\arctan\left(\sqrt{-dx^2+2}\sqrt{d}x,dx^2-1\right)+abd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b^2*d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a*b*d)*sqrt
(d)*integrate(1/2*sqrt(-d*x^2 + 2)*x/(a*b*d*x^2 - 2*a*b + (b^2*d*x^2 - 2*b^
2)*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1)), x) - sqrt(-d*x^2 + 2)*s
qrt(d))/(b^2*d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a*b*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2\arccos\left(dx^2-1\right)^2+2\,ab\arccos\left(dx^2-1\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arccos(d*x^2 - 1)^2 + 2*a*b*arccos(d*x^2 - 1) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a+b\,\mathrm{acos}\left(dx^2-1\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2-1))**2,x)
```

```
[Out] Integral((a + b*acos(d*x**2 - 1))**(-2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\arccos\left(dx^2-1\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(-2), x)
```

3.86 $\int \frac{1}{\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^3}\,dx$

Optimal. Leaf size=171

$$-\frac{x\sin\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}\left(dx^2-1\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}}+\frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}\left(dx^2-1\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}}+\frac{x}{8b^2\left(a+b\cos^{-1}\left(dx^2-1\right)\right)}+\frac{\sqrt{2a}\cdots}{4bdx\left(a+b\cdots\right.}$$

```
[Out] Sqrt[2*d*x^2 - d^2*x^4]/(4*b*d*x*(a + b*ArcCos[-1 + d*x^2])^2) + x/(8*b^2*(
a + b*ArcCos[-1 + d*x^2])) - (x*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b
)]*Sin[a/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[d*x^2]) + (x*Cos[a/(2*b)]*SinIntegral[
(a + b*ArcCos[-1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[d*x^2])
```

Rubi [A] time = 0.0328004, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4829, 4818}

$$-\frac{x\sin\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}\left(dx^2-1\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}}+\frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}\left(dx^2-1\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}}+\frac{x}{8b^2\left(a+b\cos^{-1}\left(dx^2-1\right)\right)}+\frac{\sqrt{2a}\cdots}{4bdx\left(a+b\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(-3),x]
```


```
[Out] Sqrt[2*d*x^2 - d^2*x^4]/(4*b*d*x*(a + b*ArcCos[-1 + d*x^2])^2) + x/(8*b^2*(
a + b*ArcCos[-1 + d*x^2])) - (x*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b
)]*Sin[a/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[d*x^2]) + (x*Cos[a/(2*b)]*SinIntegral[
(a + b*ArcCos[-1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[d*x^2])
```


Rule 4829

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcCos[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCos[c + d*x^2])^(n + 2), x], x] - Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcCos[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rule 4818

```
Int[((a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(x*Si
n[a/(2*b)]*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)])/(Sqrt[2]*b*Sqrt[d
*x^2]), x] - Simp[(x*Cos[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2
*b)])/(Sqrt[2]*b*Sqrt[d*x^2]), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^3}\,dx = \frac{\sqrt{2dx^2-d^2x^4}}{4bdx\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2}+\frac{x}{8b^2\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)}-\frac{\int \frac{1}{a+b\cos^{-1}\left(-1+dx^2\right)}\,dx}{8b^2}$$

$$= \frac{\sqrt{2dx^2-d^2x^4}}{4bdx\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2}+\frac{x}{8b^2\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)}-\frac{x\text{Ci}\left(\frac{a+b\cos^{-1}\left(-1+dx^2\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx}}$$

Mathematica [A] time = 0.212646, size = 149, normalized size = 0.87

$$\frac{\frac{2b^2\sqrt{-dx^2\left(dx^2-2\right)}}{d\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^2}-\frac{\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\left(\sin\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}\left(dx^2-1\right)}{2b}\right)-\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}\left(dx^2-1\right)}{2b}\right)\right)}{d}+\frac{bx^2}{a+b\cos^{-1}\left(dx^2-1\right)}}{8b^3x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^(-3),x]
```

```
[Out] ((2*b^2*Sqrt[-(d*x^2*(-2 + d*x^2))])/(d*(a + b*ArcCos[-1 + d*x^2])^2) + (b*
x^2)/(a + b*ArcCos[-1 + d*x^2]) - (Cos[ArcCos[-1 + d*x^2]/2]*(CosIntegral[(
a + b*ArcCos[-1 + d*x^2])/(2*b)]*Sin[a/(2*b)] - Cos[a/(2*b)]*SinIntegral[(a
 + b*ArcCos[-1 + d*x^2])/(2*b)]))/d)/(8*b^3*x)
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^2-1\right)\right)^{-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2-1))^3,x)
```

```
[Out] int(1/(a+b*arccos(d*x^2-1))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bdx\arctan\left(\sqrt{-dx^2+2}\sqrt{d}x,dx^2-1\right)+adx+2\sqrt{-dx^2+2}b\sqrt{d}-\left(b^4d\arctan\left(\sqrt{-dx^2+2}\sqrt{d}x,dx^2-1\right)^2+2ab^3d\operatorname{arct}\right.}{8\left(b^4d\arctan\left(\sqrt{-dx^2+2}\sqrt{d}x,dx^2-1\right)^2+2ab^3d\arctan\left(\sqrt{-dx^2+2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b*d*x*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a*d*x + 2*sqrt(
-d*x^2 + 2)*b*sqrt(d) - 8*(b^4*d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2
- 1)^2 + 2*a*b^3*d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a^2*b^2
*d)*integrate(1/8/(b^3*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a*b
^2), x))/(b^4*d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1)^2 + 2*a*b^3*
d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a^2*b^2*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^3\arccos\left(dx^2-1\right)^3+3\,ab^2\arccos\left(dx^2-1\right)^2+3\,a^2b\arccos\left(dx^2-1\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arccos(d*x^2 - 1)^3 + 3*a*b^2*arccos(d*x^2 - 1)^2 + 3*a^2*b
*arccos(d*x^2 - 1) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\operatorname{acos}\left(dx^2-1\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2-1))**3,x)
```

```
[Out] Integral((a + b*acos(d*x**2 - 1))**(-3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\arccos\left(dx^2-1\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(-3), x)
```

3.87 $\int \left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{5/2}\,dx$

Optimal. Leaf size=249

$$\frac{30b^2\sin^2\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}{dx}-\frac{5b\sqrt{-d^2x^4-2dx^2}\left(a+b\cos^{-1}\left(dx^2+1\right)\right)^{3/2}}{dx}+\frac{30\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\text{si}}{}$$

```
[Out] (-5*b*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2])^(3/2))/(d*x) + x*(
a + b*ArcCos[1 + d*x^2])^(5/2) - (30*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b
^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/(
(b^(-1))^(5/2)*d*x) + (30*Sqrt[Pi]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos
[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/((b^(-1))^(5
/2)*d*x) + (30*b^2*Sqrt[a + b*ArcCos[1 + d*x^2]]*Sin[ArcCos[1 + d*x^2]/2]^2
)/(d*x)
```

Rubi [A] time = 0.0949448, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4815, 4812}

$$\frac{30b^2\sin^2\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}{dx}-\frac{5b\sqrt{-d^2x^4-2dx^2}\left(a+b\cos^{-1}\left(dx^2+1\right)\right)^{3/2}}{dx}+\frac{30\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(5/2),x]
```


```
[Out] (-5*b*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2])^(3/2))/(d*x) + x*(
a + b*ArcCos[1 + d*x^2])^(5/2) - (30*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b
^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/(
(b^(-1))^(5/2)*d*x) + (30*Sqrt[Pi]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos
[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/((b^(-1))^(5
/2)*d*x) + (30*b^2*Sqrt[a + b*ArcCos[1 + d*x^2]]*Sin[ArcCos[1 + d*x^2]/2]^2
)/(d*x)
```


Rule 4815

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcCos[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4812

```
Int[Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-2*Sqr
t[a + b*ArcCos[1 + d*x^2]]*Sin[ArcCos[1 + d*x^2]/2]^2)/(d*x), x] + (-Simp[(
2*Sqrt[Pi]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sq
rt[a + b*ArcCos[1 + d*x^2]]])/(Sqrt[1/b]*d*x), x] + Simp[(2*Sqrt[Pi]*Cos[a/
(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1
 + d*x^2]]])/(Sqrt[1/b]*d*x), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{5/2}\,dx = -\frac{5b\sqrt{-2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}}{dx}+x\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{5/2}-\left(15b^2\right)\int$$

$$= -\frac{5b\sqrt{-2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}}{dx}+x\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{5/2}-\frac{30\sqrt{\pi}\,\mathrm{c}}{}$$

Mathematica [A] time = 2.61709, size = 256, normalized size = 1.03

$$-\frac{2\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\left(\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}\left(\left(a^2-15b^2\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)+5ab\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)+b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^(5/2),x]
```

```
[Out] (-2*Sin[ArcCos[1 + d*x^2]/2]*((15*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-
1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]])/(b^(-1))^(5/2) - (15*Sqrt[Pi]
*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b
```

```
)])/(b^(-1))^(5/2) + Sqrt[a + b*ArcCos[1 + d*x^2]]*(5*a*b*Cos[ArcCos[1 + d*
x^2]/2] + (a^2 - 15*b^2)*Sin[ArcCos[1 + d*x^2]/2] + b^2*ArcCos[1 + d*x^2]^2
*Sin[ArcCos[1 + d*x^2]/2] + b*ArcCos[1 + d*x^2]*(5*b*Cos[ArcCos[1 + d*x^2]/
2] + 2*a*Sin[ArcCos[1 + d*x^2]/2]))))/(d*x)
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^{2}+1\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2+1))^(5/2),x)
```

```
[Out] int((a+b*arccos(d*x^2+1))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\arccos\left(dx^{2}+1\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2+1))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arccos \left(dx^2+1\right)+a\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(5/2), x)
```

3.88 $\int \left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}\,dx$

Optimal. Leaf size=207

$$-\frac{3b\sqrt{-d^2x^4-2dx^2}\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}{dx}+\frac{6\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{3/2}dx}+\frac{6\ldots}{\ldots}$$

```
[Out] (-3*b*Sqrt[-2*d*x^2 - d^2*x^4]*Sqrt[a + b*ArcCos[1 + d*x^2]])/(d*x) + x*(a
+ b*ArcCos[1 + d*x^2])^(3/2) + (6*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-
1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/((b^
(-1))^(3/2)*d*x) + (6*Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 +
 d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/((b^(-1))^(3/2)*
d*x)
```

Rubi [A] time = 0.0663642, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4815, 4820}

$$-\frac{3b\sqrt{-d^2x^4-2dx^2}\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}{dx}+\frac{6\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{3/2}dx}+\frac{6\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[-2*d*x^2 - d^2*x^4]*Sqrt[a + b*ArcCos[1 + d*x^2]])/(d*x) + x*(a
+ b*ArcCos[1 + d*x^2])^(3/2) + (6*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-
1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/((b^
(-1))^(3/2)*d*x) + (6*Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 +
 d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/((b^(-1))^(3/2)*
d*x)
```

Rule 4815

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
```

```
c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcCos[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4820

```
Int[1/Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-2*S
qrt[Pi/b]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sqr
t[a + b*ArcCos[1 + d*x^2]]])/(d*x), x] - Simp[(2*Sqrt[Pi/b]*Sin[a/(2*b)]*Si
n[ArcCos[1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]
]])/(d*x), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}dx = -\frac{3b\sqrt{-2dx^2-d^2x^4}\sqrt{a+b\cos^{-1}\left(1+dx^2\right)}}{dx}+x\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}-\left(3b^2\right)\int -$$

$$= -\frac{3b\sqrt{-2dx^2-d^2x^4}\sqrt{a+b\cos^{-1}\left(1+dx^2\right)}}{dx}+x\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}+\frac{6\sqrt{\pi}\cos\left(\right.}{}$$

Mathematica [A] time = 0.713281, size = 200, normalized size = 0.97

$$-\frac{2\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\left(-3\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)-3\sqrt{\pi}\sin\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)+\left(\frac{1}{b}\right)^{3/}\right.}{\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^(3/2),x]
```

```
[Out] (-2*Sin[ArcCos[1 + d*x^2]/2]*(-3*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-1
)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]] - 3*Sqrt[Pi]*FresnelS[(Sqrt[b^(
-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)] + (b^(-1))^(3/2)
*Sqrt[a + b*ArcCos[1 + d*x^2]]*(3*b*Cos[ArcCos[1 + d*x^2]/2] + a*Sin[ArcCos
[1 + d*x^2]/2] + b*ArcCos[1 + d*x^2]*Sin[ArcCos[1 + d*x^2]/2])))/((b^(-1))^
(3/2)*d*x)
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^{2}+1\right)\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2+1))^(3/2),x)
```

```
[Out] int((a+b*arccos(d*x^2+1))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\arccos\left(dx^{2}+1\right)+a\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\operatorname{acos}\left(dx^{2}+1\right)\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2+1))**(3/2),x)
```

```
[Out] Integral((a + b*acos(d*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arccos \left(dx^2+1\right)+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(3/2), x)
```

3.89 $\int \sqrt{a+b\cos^{-1}\left(1+dx^2\right)}\,dx$

Optimal. Leaf size=184

$$-\frac{2\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}}dx}+\frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a\cdots}}{}\right.}{\sqrt{\frac{1}{b}}dx}$$

```
[Out] (2*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2
]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/(Sqrt[b^(-1)]*d*x) - (2*Sqrt[Pi]*Fr
esnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*
Sin[ArcCos[1 + d*x^2]/2])/(Sqrt[b^(-1)]*d*x) - (2*Sqrt[a + b*ArcCos[1 + d*x
^2]]*Sin[ArcCos[1 + d*x^2]/2]^2)/(d*x)
```

Rubi [A] time = 0.0215428, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4812}

$$-\frac{2\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}}dx}+\frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a\cdots}}{}\right.}{\sqrt{\frac{1}{b}}dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCos[1 + d*x^2]],x]
```

```
[Out] (2*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2
]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/(Sqrt[b^(-1)]*d*x) - (2*Sqrt[Pi]*Fr
esnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*
Sin[ArcCos[1 + d*x^2]/2])/(Sqrt[b^(-1)]*d*x) - (2*Sqrt[a + b*ArcCos[1 + d*x
^2]]*Sin[ArcCos[1 + d*x^2]/2]^2)/(d*x)
```

Rule 4812

```
Int[Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-2*Sqr
t[a + b*ArcCos[1 + d*x^2]]*Sin[ArcCos[1 + d*x^2]/2]^2)/(d*x), x] + (-Simp[(
2*Sqrt[Pi]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sq
rt[a + b*ArcCos[1 + d*x^2]]])/(Sqrt[1/b]*d*x), x] + Simp[(2*Sqrt[Pi]*Cos[a/
(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1
```

```
 + d*x^2]]])/(Sqrt[1/b]*d*x), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \sqrt{a+b\cos^{-1}\left(1+dx^2\right)}\,dx = \frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(1+dx^2\right)\right)}{\sqrt{\frac{1}{b}}dx} - \frac{2\sqrt{\pi}C\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(1+dx^2}}{\sqrt{\pi}}\right.}{}$$

Mathematica [A] time = 0.0848811, size = 157, normalized size = 0.85

$$-\frac{2\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\left(\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)-\sqrt{\pi}\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)+\sqrt{\frac{1}{b}}\sin\left(\right.\right.}{\sqrt{\frac{1}{b}}dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*ArcCos[1 + d*x^2]],x]
```

```
[Out] (-2*Sin[ArcCos[1 + d*x^2]/2]*(-(Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)
]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]) + Sqrt[Pi]*FresnelC[(Sqrt[b^(-1
)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)] + Sqrt[b^(-1)]*Sqr
t[a + b*ArcCos[1 + d*x^2]]*Sin[ArcCos[1 + d*x^2]/2]))/(Sqrt[b^(-1)]*d*x)
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \sqrt{a+b\arccos\left(dx^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2+1))^(1/2),x)
```

```
[Out] int((a+b*arccos(d*x^2+1))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \arccos\left(dx^2+1\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccos(d*x^2 + 1) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b \operatorname{acos}\left(dx^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2+1))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acos(d*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \arccos\left(dx^2+1\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arccos(d*x^2 + 1) + a), x)
```

3.90 $\int \frac{1}{\sqrt{a+b\cos^{-1}(1+dx^2)}}\,dx$

Optimal. Leaf size=145

$$-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)}{dx}-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)S}{dx}$$

```
[Out] (-2*Sqrt[b^(-1)]*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*Ar
cCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/(d*x) - (2*Sqrt[b^(-1
)]*Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]
*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/(d*x)
```

Rubi [A] time = 0.0175331, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4820}

$$-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)}{dx}-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)S}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCos[1 + d*x^2]],x]
```


```
[Out] (-2*Sqrt[b^(-1)]*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*Ar
cCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/(d*x) - (2*Sqrt[b^(-1
)]*Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]
*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/(d*x)
```


Rule 4820

```
Int[1/Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-2*S
qrt[Pi/b]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sqr
t[a + b*ArcCos[1 + d*x^2]]])/(d*x), x] - Simp[(2*Sqrt[Pi/b]*Sin[a/(2*b)]*Si
n[ArcCos[1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]
]])/(d*x), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos^{-1}\left(1+dx^2\right)}}\,dx = -\frac{2\sqrt{\frac{1}{b}}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)C\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(1+dx^2\right)\right)}{dx} - \frac{2\sqrt{\frac{1}{b}}\sqrt{\pi}S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}}}{}\right.}{}$$

Mathematica [A] time = 0.161629, size = 114, normalized size = 0.79

$$-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\left(\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)+\sin\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2+1)}}{\sqrt{\pi}}\right)\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*ArcCos[1 + d*x^2]],x]
```

```
[Out] (-2*Sqrt[b^(-1)]*Sqrt[Pi]*(Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*A
rcCos[1 + d*x^2]])/Sqrt[Pi]] + FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 +
 d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])*Sin[ArcCos[1 + d*x^2]/2])/(d*x)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\arccos\left(dx^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2+1))^(1/2),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2+1))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\arccos\left(dx^2+1\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccos(d*x^2 + 1) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \operatorname{acos}\left(dx^2 + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2+1))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acos(d*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \arccos\left(dx^2 + 1\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arccos(d*x^2 + 1) + a), x)
```

3.91 $\int \frac{1}{\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}}\,dx$

Optimal. Leaf size=190

$$\frac{\sqrt{-d^2x^4-2dx^2}}{bdx\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{dx}+\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\text{c}}{}$$

```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[1 + d*x^2]]) + (2*(b^(-1)
)^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 +
d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/(d*x) - (2*(b^(-1))^(3/2)*Sqrt
[Pi]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/
(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/(d*x)
```

Rubi [A] time = 0.0293162, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4823}

$$\frac{\sqrt{-d^2x^4-2dx^2}}{bdx\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{dx}+\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\text{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(-3/2),x]
```

```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[1 + d*x^2]]) + (2*(b^(-1)
)^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 +
d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/(d*x) - (2*(b^(-1))^(3/2)*Sqrt
[Pi]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/
(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/(d*x)
```

Rule 4823

```
Int[((a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[Sqrt
[-2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[1 + d*x^2]]), x] + (-Simp[(2*
(1/b)^(3/2)*Sqrt[Pi]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*FresnelC[Sqrt[1/
(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]]])/(d*x), x] + Simp[(2*(1/b)^(3/2)*Sqr
```

```
t[Pi]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a
+ b*ArcCos[1 + d*x^2]]])/(d*x), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}}\,dx=\frac{\sqrt{-2dx^2-d^2x^4}}{bdx\sqrt{a+b\cos^{-1}\left(1+dx^2\right)}}+\frac{2\left(\frac{1}{b}\right)^{3/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(1+dx^2\right)}}{\sqrt{\pi}}\right)\sin\left(\frac{1}{2}\cos^{-1}\right.}{dx}$$

Mathematica [A] time = 0.388768, size = 177, normalized size = 0.93

$$\frac{-2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)+2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)S}{bdx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^(-3/2),x]
```

```
[Out] (Sqrt[-(d*x^2*(2 + d*x^2))]/Sqrt[a + b*ArcCos[1 + d*x^2]] + 2*Sqrt[b^(-1)]*
Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])
/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2] - 2*Sqrt[b^(-1)]*Sqrt[Pi]*FresnelC[(Sqr
t[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[
1 + d*x^2]/2])/(b*d*x)
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^2+1\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2+1))^(3/2),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2+1))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \arccos\left(dx^2+1\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b \operatorname{acos}\left(dx^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2+1))**(3/2),x)
```

```
[Out] Integral((a + b*acos(d*x**2 + 1))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \arccos\left(dx^2+1\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(-3/2), x)
```

3.92 $\int \frac{1}{\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{5/2}}\,dx$

Optimal. Leaf size=221

$$\frac{x}{3b^2\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}+\frac{\sqrt{-d^2x^4-2dx^2}}{3bdx\left(a+b\cos^{-1}\left(dx^2+1\right)\right)^{3/2}}+\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{5/2}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\text{FresnelC}\left(\frac{\sqrt{}}{}\right.}{3dx}$$

```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(3*b*d*x*(a + b*ArcCos[1 + d*x^2])^(3/2)) + x/(3*b
^2*Sqrt[a + b*ArcCos[1 + d*x^2]]) + (2*(b^(-1))^(5/2)*Sqrt[Pi]*Cos[a/(2*b)]
*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos
[1 + d*x^2]/2])/(3*d*x) + (2*(b^(-1))^(5/2)*Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)]
*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2
]/2])/(3*d*x)
```

Rubi [A] time = 0.0500054, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4829, 4820}

$$\frac{x}{3b^2\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}+\frac{\sqrt{-d^2x^4-2dx^2}}{3bdx\left(a+b\cos^{-1}\left(dx^2+1\right)\right)^{3/2}}+\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{5/2}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\text{FresnelC}\left(\frac{\sqrt{}}{}\right.}{3dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(-5/2),x]
```


```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(3*b*d*x*(a + b*ArcCos[1 + d*x^2])^(3/2)) + x/(3*b
^2*Sqrt[a + b*ArcCos[1 + d*x^2]]) + (2*(b^(-1))^(5/2)*Sqrt[Pi]*Cos[a/(2*b)]
*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos
[1 + d*x^2]/2])/(3*d*x) + (2*(b^(-1))^(5/2)*Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)]
*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2
]/2])/(3*d*x)
```


Rule 4829

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcCos[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
```

```
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCos[c + d*x^2])^(n + 2), x], x] - Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcCos[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rule 4820

```
Int[1/Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-2*S
qrt[Pi/b]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sqr
t[a + b*ArcCos[1 + d*x^2]]])/(d*x), x] - Simp[(2*Sqrt[Pi/b]*Sin[a/(2*b)]*Si
n[ArcCos[1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]
]])/(d*x), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{5/2}}\,dx = \frac{\sqrt{-2dx^2-d^2x^4}}{3bdx\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}}+\frac{x}{3b^2\sqrt{a+b\cos^{-1}\left(1+dx^2\right)}}-\frac{\int \frac{1}{\sqrt{a+b\cos^{-1}\left(1+dx^2\right)}}\,dx}{3b^2}$$

$$= \frac{\sqrt{-2dx^2-d^2x^4}}{3bdx\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}}+\frac{x}{3b^2\sqrt{a+b\cos^{-1}\left(1+dx^2\right)}}+\frac{2\left(\frac{1}{b}\right)^{5/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)}{}$$

Mathematica [A] time = 0.72083, size = 234, normalized size = 1.06

$$\frac{2\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\left(\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\left(a+b\cos^{-1}\left(dx^2+1\right)\right)^{3/2}\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)+\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\left(\right.\right.}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^(-5/2),x]
```

```
[Out] (2*Sin[ArcCos[1 + d*x^2]/2]*(b*Cos[ArcCos[1 + d*x^2]/2] + Sqrt[b^(-1)]*Sqrt
[Pi]*(a + b*ArcCos[1 + d*x^2])^(3/2)*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-1)]*Sq
rt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]] + Sqrt[b^(-1)]*Sqrt[Pi]*(a + b*ArcCo
s[1 + d*x^2])^(3/2)*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/S
qrt[Pi]]*Sin[a/(2*b)] - a*Sin[ArcCos[1 + d*x^2]/2] - b*ArcCos[1 + d*x^2]*Si
n[ArcCos[1 + d*x^2]/2]))/(3*b^2*d*x*(a + b*ArcCos[1 + d*x^2])^(3/2))
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(d x^2+1\right)\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2+1))^(5/2),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2+1))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\arccos\left(d x^2+1\right)+a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\operatorname{acos}\left(d x^2+1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2+1))**(5/2),x)
```

```
[Out] Integral((a + b*acos(d*x**2 + 1))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \arccos \left(dx^2+1\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(-5/2), x)
```

3.93 $\int \frac{1}{\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{7/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{\sqrt{-d^2x^4-2dx^2}}{15b^3dx\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}+\frac{x}{15b^2\left(a+b\cos^{-1}\left(dx^2+1\right)\right)^{3/2}}+\frac{\sqrt{-d^2x^4-2dx^2}}{5bdx\left(a+b\cos^{-1}\left(dx^2+1\right)\right)^{5/2}}+\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{7/2}\sin\left(\frac{a}{2b}\right.}{}$$

```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(5*b*d*x*(a + b*ArcCos[1 + d*x^2])^(5/2)) + x/(15*
b^2*(a + b*ArcCos[1 + d*x^2])^(3/2)) - Sqrt[-2*d*x^2 - d^2*x^4]/(15*b^3*d*x
*Sqrt[a + b*ArcCos[1 + d*x^2]]) - (2*(b^(-1))^(7/2)*Sqrt[Pi]*Cos[a/(2*b)]*F
resnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1
 + d*x^2]/2])/(15*d*x) + (2*(b^(-1))^(7/2)*Sqrt[Pi]*FresnelC[(Sqrt[b^(-1)]*
Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]
/2])/(15*d*x)
```

Rubi [A] time = 0.060164, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4829, 4823}

$$-\frac{\sqrt{-d^2x^4-2dx^2}}{15b^3dx\sqrt{a+b\cos^{-1}\left(dx^2+1\right)}}+\frac{x}{15b^2\left(a+b\cos^{-1}\left(dx^2+1\right)\right)^{3/2}}+\frac{\sqrt{-d^2x^4-2dx^2}}{5bdx\left(a+b\cos^{-1}\left(dx^2+1\right)\right)^{5/2}}+\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{7/2}\sin\left(\frac{a}{2b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(-7/2),x]
```



```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(5*b*d*x*(a + b*ArcCos[1 + d*x^2])^(5/2)) + x/(15*
b^2*(a + b*ArcCos[1 + d*x^2])^(3/2)) - Sqrt[-2*d*x^2 - d^2*x^4]/(15*b^3*d*x
*Sqrt[a + b*ArcCos[1 + d*x^2]]) - (2*(b^(-1))^(7/2)*Sqrt[Pi]*Cos[a/(2*b)]*F
resnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1
 + d*x^2]/2])/(15*d*x) + (2*(b^(-1))^(7/2)*Sqrt[Pi]*FresnelC[(Sqrt[b^(-1)]*
Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]
/2])/(15*d*x)
```


Rule 4829

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcCos[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCos[c + d*x^2])^(n + 2), x], x] - Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcCos[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rule 4823

```
Int[((a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[Sqrt
[-2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[1 + d*x^2]]), x] + (-Simp[(2*
(1/b)^(3/2)*Sqrt[Pi]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*FresnelC[Sqrt[1/
(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]]])/(d*x), x] + Simp[(2*(1/b)^(3/2)*Sqr
t[Pi]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a
+ b*ArcCos[1 + d*x^2]]])/(d*x), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{7/2}}\,dx = \frac{\sqrt{-2dx^2-d^2x^4}}{5bdx\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{5/2}}+\frac{x}{15b^2\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}}-\frac{\int \frac{1}{\left(a+b\cos^{-1}\left(1+dx^2\right)\right.}}{15b^2}$$

$$= \frac{\sqrt{-2dx^2-d^2x^4}}{5bdx\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{5/2}}+\frac{x}{15b^2\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}}-\frac{\sqrt{-2dx^2-}}{15b^3dx\sqrt{a+b\,\mathrm{co}}}$$

Mathematica [A] time = 0.546504, size = 308, normalized size = 1.14

$$-\frac{2\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)\left(a^2\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2+1\right)\right)-\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\left(a+b\cos^{-1}\left(dx^2+1\right)\right)^{5/2}\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\,\mathrm{co}}}{\sqrt{}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^(-7/2),x]
```

```
[Out] (-2*Sin[ArcCos[1 + d*x^2]/2]*(a^2*Cos[ArcCos[1 + d*x^2]/2] - 3*b^2*Cos[ArcC
os[1 + d*x^2]/2] + 2*a*b*ArcCos[1 + d*x^2]*Cos[ArcCos[1 + d*x^2]/2] + b^2*A
rcCos[1 + d*x^2]^2*Cos[ArcCos[1 + d*x^2]/2] + Sqrt[b^(-1)]*Sqrt[Pi]*(a + b*
```

```
ArcCos[1 + d*x^2])^(5/2)*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*Arc
Cos[1 + d*x^2]])/Sqrt[Pi]] - Sqrt[b^(-1)]*Sqrt[Pi]*(a + b*ArcCos[1 + d*x^2]
)^(5/2)*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin
[a/(2*b)] + a*b*Sin[ArcCos[1 + d*x^2]/2] + b^2*ArcCos[1 + d*x^2]*Sin[ArcCos
[1 + d*x^2]/2]))/(15*b^3*d*x*(a + b*ArcCos[1 + d*x^2])^(5/2))
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^2+1\right)\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2+1))^(7/2),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2+1))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\arccos\left(dx^2+1\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(-7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2+1))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \arccos\left(dx^2+1\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(-7/2), x)
```

3.94 $\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{5/2}\,dx$

Optimal. Leaf size=249

$$-\frac{30b^2\cos^2\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}{dx}-\frac{5b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^{3/2}}{dx}+\frac{30\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos}{}$$

```
[Out] (-5*b*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2])^(3/2))/(d*x) + x*(
a + b*ArcCos[-1 + d*x^2])^(5/2) - (30*b^2*Sqrt[a + b*ArcCos[-1 + d*x^2]]*Co
s[ArcCos[-1 + d*x^2]/2]^2)/(d*x) + (30*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1
+ d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]
])/((b^(-1))^(5/2)*d*x) + (30*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(
Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/((b^(-
1))^(5/2)*d*x)
```

Rubi [A] time = 0.0552374, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4815, 4813}

$$-\frac{30b^2\cos^2\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}{dx}-\frac{5b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^{3/2}}{dx}+\frac{30\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(5/2),x]
```

```
[Out] (-5*b*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2])^(3/2))/(d*x) + x*(
a + b*ArcCos[-1 + d*x^2])^(5/2) - (30*b^2*Sqrt[a + b*ArcCos[-1 + d*x^2]]*Co
s[ArcCos[-1 + d*x^2]/2]^2)/(d*x) + (30*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1
+ d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]
])/((b^(-1))^(5/2)*d*x) + (30*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(
Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/((b^(-
1))^(5/2)*d*x)
```

Rule 4815

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcCos[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4813

```
Int[Sqrt[(a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(2*Sqr
t[a + b*ArcCos[-1 + d*x^2]]*Cos[(1/2)*ArcCos[-1 + d*x^2]]^2)/(d*x), x] + (-
Simp[(2*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[Sqrt[1/(Pi
*b)]*Sqrt[a + b*ArcCos[-1 + d*x^2]]])/(Sqrt[1/b]*d*x), x] - Simp[(2*Sqrt[Pi
]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b
*ArcCos[-1 + d*x^2]]])/(Sqrt[1/b]*d*x), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{5/2}\,dx = -\frac{5b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}}{dx}+x\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{5/2}-(15$$

$$= -\frac{5b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}}{dx}+x\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{5/2}-\frac{30b}{}$$

Mathematica [A] time = 2.14601, size = 256, normalized size = 1.03

$$2\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\left(\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}\left(\left(a^2-15b^2\right)\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)+b\cos^{-1}\left(dx^2-1\right)\left(2a\cos\left(\frac{1}{2}\mathrm{co}\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^(5/2),x]
```

```
[Out] (2*Cos[ArcCos[-1 + d*x^2]/2]*((15*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-
1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(b^(-1))^(5/2) + (15*Sqrt[Pi
]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2
```

```
*b)])/(b^(-1))^(5/2) + Sqrt[a + b*ArcCos[-1 + d*x^2]]*((a^2 - 15*b^2)*Cos[A
rcCos[-1 + d*x^2]/2] + b^2*ArcCos[-1 + d*x^2]^2*Cos[ArcCos[-1 + d*x^2]/2] -
 5*a*b*Sin[ArcCos[-1 + d*x^2]/2] + b*ArcCos[-1 + d*x^2]*(2*a*Cos[ArcCos[-1
+ d*x^2]/2] - 5*b*Sin[ArcCos[-1 + d*x^2]/2]))))/(d*x)
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(a + b \arccos\left(dx^2 - 1\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2-1))^(5/2),x)

[Out] int((a+b*arccos(d*x^2-1))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arccos\left(dx^2 - 1\right) + a\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(5/2),x, algorithm="maxima")

[Out] integrate((b*arccos(d*x^2 - 1) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2-1))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arccos\left(dx^2-1\right)+a\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(5/2), x)
```

3.95 $\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}\,dx$

Optimal. Leaf size=207

$$-\frac{3b\sqrt{2dx^2-d^2x^4}\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}{dx}-\frac{6\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{3/2}dx}+\frac{6\sqrt{\pi}}{}$$

```
[Out] (-3*b*Sqrt[2*d*x^2 - d^2*x^4]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/(d*x) + x*(a
+ b*ArcCos[-1 + d*x^2])^(3/2) + (6*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*
x^2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(
(b^(-1))^(3/2)*d*x) - (6*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[
b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/((b^(-1))^(
3/2)*d*x)
```

Rubi [A] time = 0.0440606, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4815, 4821}

$$-\frac{3b\sqrt{2dx^2-d^2x^4}\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}{dx}-\frac{6\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{3/2}dx}+\frac{6\sqrt{\pi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(3/2),x]
```



```
[Out] (-3*b*Sqrt[2*d*x^2 - d^2*x^4]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/(d*x) + x*(a
+ b*ArcCos[-1 + d*x^2])^(3/2) + (6*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*
x^2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(
(b^(-1))^(3/2)*d*x) - (6*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[
b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/((b^(-1))^(
3/2)*d*x)
```


Rule 4815

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
```

```
c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcCos[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4821

```
Int[1/Sqrt[(a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(2*S
qrt[Pi/b]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sq
rt[a + b*ArcCos[-1 + d*x^2]]])/(d*x), x] - Simp[(2*Sqrt[Pi/b]*Cos[a/(2*b)]*
Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[-1 + d*
x^2]]])/(d*x), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}\,dx=-\frac{3b\sqrt{2dx^2-d^2x^4}\sqrt{a+b\cos^{-1}\left(-1+dx^2\right)}}{dx}+x\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}-\left(3b^2\right)$$

$$=-\frac{3b\sqrt{2dx^2-d^2x^4}\sqrt{a+b\cos^{-1}\left(-1+dx^2\right)}}{dx}+x\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}+\frac{6\sqrt{\pi}}{}$$

Mathematica [A] time = 0.602761, size = 200, normalized size = 0.97

$$\frac{2\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\left(-3\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)+3\sqrt{\pi}\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)+\left(\frac{1}{b}\right)^{3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^(3/2),x]
```

```
[Out] (2*Cos[ArcCos[-1 + d*x^2]/2]*(3*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)
]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]] - 3*Sqrt[Pi]*FresnelC[(Sqrt[b^(
-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)] + (b^(-1))^(3/2
)*Sqrt[a + b*ArcCos[-1 + d*x^2]]*(a*Cos[ArcCos[-1 + d*x^2]/2] + b*ArcCos[-1
 + d*x^2]*Cos[ArcCos[-1 + d*x^2]/2] - 3*b*Sin[ArcCos[-1 + d*x^2]/2])))/((b^
(-1))^(3/2)*d*x)
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(a + b \arccos\left(dx^2 - 1\right)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2-1))^(3/2),x)
```

```
[Out] int((a+b*arccos(d*x^2-1))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arccos\left(dx^2 - 1\right) + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \operatorname{acos}\left(dx^2 - 1\right)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2-1))**(3/2),x)
```

```
[Out] Integral((a + b*acos(d*x**2 - 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arccos \left(dx^2-1\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(3/2), x)
```

3.96 $\int \sqrt{a+b\cos^{-1}\left(-1+dx^2\right)}\,dx$

Optimal. Leaf size=184

$$-\frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\mathrm{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}}dx}-\frac{2\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b}}{}\right.}{\sqrt{\frac{1}{b}}dx}$$

```
[Out] (2*Sqrt[a + b*ArcCos[-1 + d*x^2]]*Cos[ArcCos[-1 + d*x^2]/2]^2)/(d*x) - (2*S
qrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[
a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(Sqrt[b^(-1)]*d*x) - (2*Sqrt[Pi]*Cos[
ArcCos[-1 + d*x^2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]]
)/Sqrt[Pi]]*Sin[a/(2*b)])/(Sqrt[b^(-1)]*d*x)
```

Rubi [A] time = 0.0202655, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4813}

$$-\frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\mathrm{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}}dx}-\frac{2\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b}}{}\right.}{\sqrt{\frac{1}{b}}dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCos[-1 + d*x^2]],x]
```

```
[Out] (2*Sqrt[a + b*ArcCos[-1 + d*x^2]]*Cos[ArcCos[-1 + d*x^2]/2]^2)/(d*x) - (2*S
qrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[
a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(Sqrt[b^(-1)]*d*x) - (2*Sqrt[Pi]*Cos[
ArcCos[-1 + d*x^2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]]
)/Sqrt[Pi]]*Sin[a/(2*b)])/(Sqrt[b^(-1)]*d*x)
```

Rule 4813

```
Int[Sqrt[(a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(2*Sqr
t[a + b*ArcCos[-1 + d*x^2]]*Cos[(1/2)*ArcCos[-1 + d*x^2]]^2)/(d*x), x] + (-
Simp[(2*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[Sqrt[1/(Pi
*b)]*Sqrt[a + b*ArcCos[-1 + d*x^2]]])/(Sqrt[1/b]*d*x), x] - Simp[(2*Sqrt[Pi
]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b
```

```
*ArcCos[-1 + d*x^2]]])/(Sqrt[1/b]*d*x), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \sqrt{a+b\cos^{-1}\left(-1+dx^2\right)}\,dx = \frac{2\sqrt{a+b\cos^{-1}\left(-1+dx^2\right)}\cos^2\left(\frac{1}{2}\cos^{-1}\left(-1+dx^2\right)\right)}{dx} - \frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}\left(-1\right.\right.}{\sqrt{}}$$

Mathematica [A] time = 0.112897, size = 157, normalized size = 0.85

$$-\frac{2\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\left(\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)+\sqrt{\pi}\sin\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)-\sqrt{\frac{1}{b}}\cos\right.}{\sqrt{\frac{1}{b}}dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*ArcCos[-1 + d*x^2]],x]
```

```
[Out] (-2*Cos[ArcCos[-1 + d*x^2]/2]*(-(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]
]*Cos[ArcCos[-1 + d*x^2]/2]) + Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-1)]
*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]] + Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)
]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]))/(Sqrt[b^(-1)]*d*
x)
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \sqrt{a+b\arccos\left(dx^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2-1))^(1/2),x)
```

```
[Out] int((a+b*arccos(d*x^2-1))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \arccos \left(dx^2 - 1\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccos(d*x^2 - 1) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \operatorname{acos}\left(dx^2 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2-1))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acos(d*x**2 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \arccos \left(dx^2 - 1\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arccos(d*x^2 - 1) + a), x)
```

3.97 $\int \frac{1}{\sqrt{a+b\cos^{-1}(-1+dx^2)}}\,dx$

Optimal. Leaf size=145

$$\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{dx}-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)S\left(\sqrt{\right.}}{dx}$$

```
[Out] (-2*Sqrt[b^(-1)]*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(
Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(d*x) + (2*Sqrt[b^(
-1)]*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*A
rcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/(d*x)
```

Rubi [A] time = 0.0172358, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4821}

$$\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)}{dx}-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)S\left(\sqrt{\right.}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCos[-1 + d*x^2]],x]
```

```
[Out] (-2*Sqrt[b^(-1)]*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(
Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(d*x) + (2*Sqrt[b^(
-1)]*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*A
rcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/(d*x)
```

Rule 4821

```
Int[1/Sqrt[(a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(2*S
qrt[Pi/b]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sq
rt[a + b*ArcCos[-1 + d*x^2]]])/(d*x), x] - Simp[(2*Sqrt[Pi/b]*Cos[a/(2*b)]*
Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[-1 + d*
x^2]]])/(d*x), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos^{-1}(-1+dx^2)}}\,dx = -\frac{2\sqrt{\frac{1}{b}}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}(-1+dx^2)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(-1+dx^2)}}{\sqrt{\pi}}\right)}{dx} + \frac{2\sqrt{\frac{1}{b}}\sqrt{\pi}\cos\left(\right.}{}$$

Mathematica [A] time = 0.15789, size = 115, normalized size = 0.79

$$-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{1}{2}\cos^{-1}(dx^2-1)\right)\left(\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)-\sin\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(dx^2-1)}}{\sqrt{\pi}}\right)\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*ArcCos[-1 + d*x^2]],x]
```

```
[Out] (-2*Sqrt[b^(-1)]*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*(Cos[a/(2*b)]*FresnelS[
(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]] - FresnelC[(Sqrt[b^
(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]))/(d*x)
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\arccos(dx^2-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2-1))^(1/2),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2-1))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\arccos(dx^2-1)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccos(d*x^2 - 1) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \operatorname{acos}\left(dx^2 - 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2-1))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acos(d*x**2 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \arccos\left(dx^2 - 1\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arccos(d*x^2 - 1) + a), x)
```

3.98 $\int \frac{1}{\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}}\,dx$

Optimal. Leaf size=190

$$\frac{\sqrt{2dx^2-d^2x^4}}{bdx\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)}{dx}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}}{}$$

```
[Out] Sqrt[2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[-1 + d*x^2]]) - (2*(b^(-1)
)^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^(-
1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(d*x) - (2*(b^(-1))^(3/2)*Sq
rt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-
1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/(d*x)
```

Rubi [A] time = 0.0239368, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4824}

$$\frac{\sqrt{2dx^2-d^2x^4}}{bdx\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)}{dx}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(-3/2),x]
```



```
[Out] Sqrt[2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[-1 + d*x^2]]) - (2*(b^(-1)
)^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^(-
1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(d*x) - (2*(b^(-1))^(3/2)*Sq
rt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-
1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/(d*x)
```


Rule 4824

```
Int[((a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[Sqr
t[2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[-1 + d*x^2]]), x] + (-Simp[(2
*(1/b)^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[Sqrt[
1/(Pi*b)]*Sqrt[a + b*ArcCos[-1 + d*x^2]]])/(d*x), x] - Simp[(2*(1/b)^(3/2)*
```

```
Sqrt[Pi]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqr
t[a + b*ArcCos[-1 + d*x^2]]])/(d*x), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}}\,dx=\frac{\sqrt{2dx^2-d^2x^4}}{bdx\sqrt{a+b\cos^{-1}\left(-1+dx^2\right)}}-\frac{2\left(\frac{1}{b}\right)^{3/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}\left(-1+dx^2\right)\right)C\left(\frac{\sqrt{\frac{1}{b}}\sqrt{\ }}{\ }\right.}{dx}$$

Mathematica [A] time = 0.320724, size = 161, normalized size = 0.85

$$\frac{2\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\left(\sqrt{\pi}\left(-\sqrt{\frac{1}{b}}\right)\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)-\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)+\right.}{bdx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^(-3/2),x]
```

```
[Out] (2*Cos[ArcCos[-1 + d*x^2]/2]*(-(Sqrt[b^(-1)]*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC
[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]) - Sqrt[b^(-1)]*Sq
rt[Pi]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin
[a/(2*b)] + Sin[ArcCos[-1 + d*x^2]/2]/Sqrt[a + b*ArcCos[-1 + d*x^2]]))/(b*d
*x)
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \left(a+b\arccos\left(dx^2-1\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2-1))^(3/2),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2-1))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\arccos\left(dx^2-1\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\,\mathrm{acos}\left(dx^2-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2-1))**(3/2),x)
```

```
[Out] Integral((a + b*acos(d*x**2 - 1))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\arccos\left(dx^2-1\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(-3/2), x)
```

3.99 $\int \frac{1}{\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{5/2}}\,dx$

Optimal. Leaf size=221

$$\frac{x}{3b^2\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}+\frac{\sqrt{2dx^2-d^2x^4}}{3bdx\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^{3/2}}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{5/2}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\text{FresnelC}\left(\right.}{3dx}$$

```
[Out] Sqrt[2*d*x^2 - d^2*x^4]/(3*b*d*x*(a + b*ArcCos[-1 + d*x^2])^(3/2)) + x/(3*b
^2*Sqrt[a + b*ArcCos[-1 + d*x^2]]) + (2*(b^(-1))^(5/2)*Sqrt[Pi]*Cos[a/(2*b)
]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d
*x^2]])/Sqrt[Pi]])/(3*d*x) - (2*(b^(-1))^(5/2)*Sqrt[Pi]*Cos[ArcCos[-1 + d*x
^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin
[a/(2*b)])/(3*d*x)
```

Rubi [A] time = 0.0434584, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4829, 4821}

$$\frac{x}{3b^2\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}+\frac{\sqrt{2dx^2-d^2x^4}}{3bdx\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^{3/2}}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{5/2}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\text{FresnelC}\left(\right.}{3dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(-5/2),x]
```

```
[Out] Sqrt[2*d*x^2 - d^2*x^4]/(3*b*d*x*(a + b*ArcCos[-1 + d*x^2])^(3/2)) + x/(3*b
^2*Sqrt[a + b*ArcCos[-1 + d*x^2]]) + (2*(b^(-1))^(5/2)*Sqrt[Pi]*Cos[a/(2*b)
]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d
*x^2]])/Sqrt[Pi]])/(3*d*x) - (2*(b^(-1))^(5/2)*Sqrt[Pi]*Cos[ArcCos[-1 + d*x
^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin
[a/(2*b)])/(3*d*x)
```

Rule 4829

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcCos[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
```

```
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCos[c + d*x^2])^(n + 2), x], x] - Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcCos[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rule 4821

```
Int[1/Sqrt[(a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(2*S
qrt[Pi/b]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[Sqrt[1/(Pi*b)]*Sq
rt[a + b*ArcCos[-1 + d*x^2]]])/(d*x), x] - Simp[(2*Sqrt[Pi/b]*Cos[a/(2*b)]*
Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[-1 + d*
x^2]]])/(d*x), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{5/2}}\,dx = \frac{\sqrt{2dx^2-d^2x^4}}{3bdx\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}} + \frac{x}{3b^2\sqrt{a+b\cos^{-1}\left(-1+dx^2\right)}} - \frac{\int \frac{1}{\sqrt{a+b\cos^{-1}\left(-1+dx\right.}}}{3b^2}$$

$$= \frac{\sqrt{2dx^2-d^2x^4}}{3bdx\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}} + \frac{x}{3b^2\sqrt{a+b\cos^{-1}\left(-1+dx^2\right)}} + \frac{2\left(\frac{1}{b}\right)^{5/2}\sqrt{\pi}\cos\left(\right.}{}$$

Mathematica [A] time = 0.585468, size = 233, normalized size = 1.05

$$\frac{2\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\left(-\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^{3/2}\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)+\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\left(\right.}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^(-5/2),x]
```

```
[Out] (2*Cos[ArcCos[-1 + d*x^2]/2]*(a*Cos[ArcCos[-1 + d*x^2]/2] + b*ArcCos[-1 + d
*x^2]*Cos[ArcCos[-1 + d*x^2]/2] + Sqrt[b^(-1)]*Sqrt[Pi]*(a + b*ArcCos[-1 +
d*x^2])^(3/2)*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*
x^2]])/Sqrt[Pi]] - Sqrt[b^(-1)]*Sqrt[Pi]*(a + b*ArcCos[-1 + d*x^2])^(3/2)*F
resnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)
] + b*Sin[ArcCos[-1 + d*x^2]/2]))/(3*b^2*d*x*(a + b*ArcCos[-1 + d*x^2])^(3/
```

```
2))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \left(a + b \arccos\left(d x^2 - 1\right)\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2-1))^(5/2),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2-1))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \arccos\left(d x^2 - 1\right) + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b\operatorname{acos}\left(dx^2 - 1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2-1))**(5/2),x)
```

```
[Out] Integral((a + b*acos(d*x**2 - 1))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\arccos\left(dx^2 - 1\right) + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(-5/2), x)
```

3.100 $\int \frac{1}{\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{7/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{\sqrt{2dx^2-d^2x^4}}{15b^3dx\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}+\frac{x}{15b^2\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^{3/2}}+\frac{\sqrt{2dx^2-d^2x^4}}{5bdx\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^{5/2}}+\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{7/2}\cos\left(\right.}{}$$

```
[Out] Sqrt[2*d*x^2 - d^2*x^4]/(5*b*d*x*(a + b*ArcCos[-1 + d*x^2])^(5/2)) + x/(15*
b^2*(a + b*ArcCos[-1 + d*x^2])^(3/2)) - Sqrt[2*d*x^2 - d^2*x^4]/(15*b^3*d*x
*Sqrt[a + b*ArcCos[-1 + d*x^2]]) + (2*(b^(-1))^(7/2)*Sqrt[Pi]*Cos[a/(2*b)]*
Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x
^2]])/Sqrt[Pi]])/(15*d*x) + (2*(b^(-1))^(7/2)*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^
2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[
a/(2*b)])/(15*d*x)
```

Rubi [A] time = 0.0500292, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4829, 4824}

$$-\frac{\sqrt{2dx^2-d^2x^4}}{15b^3dx\sqrt{a+b\cos^{-1}\left(dx^2-1\right)}}+\frac{x}{15b^2\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^{3/2}}+\frac{\sqrt{2dx^2-d^2x^4}}{5bdx\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^{5/2}}+\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{7/2}\cos\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(-7/2),x]
```



```
[Out] Sqrt[2*d*x^2 - d^2*x^4]/(5*b*d*x*(a + b*ArcCos[-1 + d*x^2])^(5/2)) + x/(15*
b^2*(a + b*ArcCos[-1 + d*x^2])^(3/2)) - Sqrt[2*d*x^2 - d^2*x^4]/(15*b^3*d*x
*Sqrt[a + b*ArcCos[-1 + d*x^2]]) + (2*(b^(-1))^(7/2)*Sqrt[Pi]*Cos[a/(2*b)]*
Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x
^2]])/Sqrt[Pi]])/(15*d*x) + (2*(b^(-1))^(7/2)*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^
2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[
a/(2*b)])/(15*d*x)
```


Rule 4829

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcCos[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCos[c + d*x^2])^(n + 2), x], x] - Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcCos[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rule 4824

```
Int[((a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[Sqr
t[2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[-1 + d*x^2]]), x] + (-Simp[(2
*(1/b)^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[Sqrt[
1/(Pi*b)]*Sqrt[a + b*ArcCos[-1 + d*x^2]]])/(d*x), x] - Simp[(2*(1/b)^(3/2)*
Sqrt[Pi]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[Sqrt[1/(Pi*b)]*Sqr
t[a + b*ArcCos[-1 + d*x^2]]])/(d*x), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{7/2}}\,dx = \frac{\sqrt{2dx^2-d^2x^4}}{5bdx\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{5/2}} + \frac{x}{15b^2\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}} - \frac{\int \frac{1}{\left(a+b\cos^{-1}\left(-1+\right.\right.}}{15b^2}$$

$$= \frac{\sqrt{2dx^2-d^2x^4}}{5bdx\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{5/2}} + \frac{x}{15b^2\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}} - \frac{\sqrt{2d}}{15b^3dx\sqrt{a+b}}$$

Mathematica [A] time = 0.553429, size = 309, normalized size = 1.15

$$\frac{2\cos\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)\left(-a^2\sin\left(\frac{1}{2}\cos^{-1}\left(dx^2-1\right)\right)+\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\left(a+b\cos^{-1}\left(dx^2-1\right)\right)^{5/2}\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-}}}{\sqrt{\pi}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^(-7/2),x]
```

```
[Out] (2*Cos[ArcCos[-1 + d*x^2]/2]*(a*b*Cos[ArcCos[-1 + d*x^2]/2] + b^2*ArcCos[-1
 + d*x^2]*Cos[ArcCos[-1 + d*x^2]/2] + Sqrt[b^(-1)]*Sqrt[Pi]*(a + b*ArcCos[-
1 + d*x^2])^(5/2)*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1
```

```
+ d*x^2]])/Sqrt[Pi]] + Sqrt[b^(-1)]*Sqrt[Pi]*(a + b*ArcCos[-1 + d*x^2])^(5/
2)*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(
2*b)] - a^2*Sin[ArcCos[-1 + d*x^2]/2] + 3*b^2*Sin[ArcCos[-1 + d*x^2]/2] - 2
*a*b*ArcCos[-1 + d*x^2]*Sin[ArcCos[-1 + d*x^2]/2] - b^2*ArcCos[-1 + d*x^2]^
2*Sin[ArcCos[-1 + d*x^2]/2]))/(15*b^3*d*x*(a + b*ArcCos[-1 + d*x^2])^(5/2))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(a + b \arccos \left(dx^2 - 1\right)\right)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2-1))^(7/2),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2-1))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \arccos \left(dx^2 - 1\right) + a\right)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(-7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2-1))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\arccos\left(dx^2-1\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(-7/2), x)
```

3.101 $\int \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x
]
```

Rubi [A] time = 0.0437305, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Defer[Int][(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [A] time = 0.0972344, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Maple [A] time = 0.647, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\arccos\left(\sqrt{-cx+1}\,\frac{1}{\sqrt{cx+1}}\right)\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] -integrate((b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b \arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^n}{c^2x^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

3.102 $\int \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=279

$$-\frac{3b^2\text{PolyLog}\left(3,-e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3ib\text{PolyLog}\left(2,-e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-\frac{3ib}{}$$

```
[Out] ((I/4)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4)/(b*c) - ((a + b*ArcCo
s[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*Log[1 + E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqr
t[1 + c*x]])])/c + (((3*I)/2)*b*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])
^2*PolyLog[2, -E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c - (3*b^2*(
a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -E^((2*I)*ArcCos[Sqrt
[1 - c*x]/Sqrt[1 + c*x]])])/(2*c) - (((3*I)/4)*b^3*PolyLog[4, -E^((2*I)*Arc
Cos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c
```

Rubi [A] time = 0.207158, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6681, 4626, 3719, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3b^2\text{PolyLog}\left(3,-e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3ib\text{PolyLog}\left(2,-e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-\frac{3ib}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```


```
[Out] ((I/4)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4)/(b*c) - ((a + b*ArcCo
s[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*Log[1 + E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqr
t[1 + c*x]])])/c + (((3*I)/2)*b*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])
^2*PolyLog[2, -E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c - (3*b^2*(
a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -E^((2*I)*ArcCos[Sqrt
[1 - c*x]/Sqrt[1 + c*x]])])/(2*c) - (((3*I)/4)*b^3*PolyLog[4, -E^((2*I)*Arc
Cos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c
```


Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\cos^{-1}(x))^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= \frac{\text{Subst}\left(\int (a+bx)^3\tan(x)\,dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{(2i)\,\text{Subst}\left(\int \frac{e^{2ix}(a+bx)^3}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{(3b)\,\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{3ib\left(a+b\,\text{co}\cdots\right.}{\cdots}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{3ib\left(a+b\,\text{co}\cdots\right.}{\cdots}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{3ib\left(a+b\,\text{co}\cdots\right.}{\cdots}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{3ib\left(a+b\,\text{co}\cdots\right.}{\cdots}$$

Mathematica [F] time = 0.296082, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

Maple [B] time = 0.974, size = 707, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

```
[Out] 1/2*a^3/c*ln(c*x+1)-1/2*a^3/c*ln(c*x-1)+1/4*I*b^3/c*arccos((-c*x+1)^(1/2)/(
c*x+1)^(1/2))^4-b^3/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+((-c*x+1)
^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)+3/2*I*b^3/c*arccos((-
c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1
-(-c*x+1)/(c*x+1))^(1/2))^2)-3/2*b^3/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))
*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)-
3/4*I*b^3*polylog(4,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(
1/2))^2)/c+I*a*b^2/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3-3*a*b^2/c*arcco
s((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(
-c*x+1)/(c*x+1))^(1/2))^2)+3*I*a*b^2/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))
*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)-
3/2*a*b^2/c*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))
^(1/2))^2)+3/2*I*a^2*b/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2-3*a^2*b/c*a
rccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1
-(-c*x+1)/(c*x+1))^(1/2))^2)+3/2*I*a^2*b/c*polylog(2,-((-c*x+1)^(1/2)/(c*x+
1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^3\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)-\int\frac{b^3\arctan\left(\sqrt{2}\sqrt{c}\sqrt{x},\sqrt{-cx+1}\right)^3+3\,ab^2\arctan\left(\sqrt{2}\sqrt{c}\sqrt{x},\sqrt{-cx+1}\right)^2+3\,a^2b}{c^2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) - integrate((b^3*arctan2(sqrt(2)*
sqrt(c)*sqrt(x), sqrt(-c*x + 1))^3 + 3*a*b^2*arctan2(sqrt(2)*sqrt(c)*sqrt(x
), sqrt(-c*x + 1))^2 + 3*a^2*b*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x +
 1)))/(c^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^3 \arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3 + 3\,ab^2 \arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 3\,a^2 b \arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a^3}{c^2x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^3*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arccos(sqrt
(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1))
+ a^3)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.103 $\int \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=207

$$\frac{ib\text{PolyLog}\left(2,-e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{b^2\text{PolyLog}\left(3,-e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}+\frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}-\frac{\log}{}$$

```
[Out] ((I/3)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3)/(b*c) - ((a + b*ArcCo
s[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*Log[1 + E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqr
t[1 + c*x]])])/c + (I*b*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog
[2, -E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c - (b^2*PolyLog[3, -E
^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c)
```

Rubi [A] time = 0.174358, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {6681, 4626, 3719, 2190, 2531, 2282, 6589}

$$\frac{ib\text{PolyLog}\left(2,-e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{b^2\text{PolyLog}\left(3,-e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}+\frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}-\frac{\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] ((I/3)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3)/(b*c) - ((a + b*ArcCo
s[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*Log[1 + E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqr
t[1 + c*x]])])/c + (I*b*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog
[2, -E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c - (b^2*PolyLog[3, -E
^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
```

```
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(a+b\cos^{-1}(x)\right)^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\
&= \frac{\text{Subst}\left(\int (a+bx)^2\tan(x)\,dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c} \\
&= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{(2i)\,\text{Subst}\left(\int \frac{e^{2ix}(a+bx)^2}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c} \\
&= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1+e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{(2b)\,\text{Subst}\left(\cdots\right.}{\cdots} \\
&= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1+e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{ib\left(a+b\cos\cdots\right.}{\cdots} \\
&= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1+e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{ib\left(a+b\cos\cdots\right.}{\cdots} \\
&= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1+e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{ib\left(a+b\cos\cdots\right.}{\cdots}
\end{aligned}
$$

Mathematica [F] time = 0.695027, size = 0, normalized size = 0.

$$
\int \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx
$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

Maple [A] time = 0.007, size = 401, normalized size = 1.9

$$\frac{a^2\ln(cx+1)}{2c}-\frac{a^2\ln(cx-1)}{2c}+\frac{\frac{i}{3}b^2}{c}\left(\arccos\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^3-\frac{b^2}{c}\left(\arccos\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^2\ln\left(1+\left(\sqrt{-cx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

```
[Out] 1/2*a^2/c*ln(c*x+1)-1/2*a^2/c*ln(c*x-1)+1/3*I*b^2/c*arccos((-c*x+1)^(1/2)/(
c*x+1)^(1/2))^3-b^2/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+((-c*x+1)
^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)+I*b^2/c*arccos((-c*x+
1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x
+1)/(c*x+1))^(1/2))^2)-1/2*b^2*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(
1-(-c*x+1)/(c*x+1))^(1/2))^2)/c+I*a*b/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)
)^2-2*a*b/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+((-c*x+1)^(1/2)/(c*x+
1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)+I*a*b/c*polylog(2,-((-c*x+1)^(1/2
)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)-\int\frac{b^2\arctan\left(\sqrt{2}\sqrt{c}\sqrt{x},\sqrt{-cx+1}\right)^2+2\,ab\arctan\left(\sqrt{2}\sqrt{c}\sqrt{x},\sqrt{-cx+1}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) - integrate((b^2*arctan2(sqrt(2)*
sqrt(c)*sqrt(x), sqrt(-c*x + 1))^2 + 2*a*b*arctan2(sqrt(2)*sqrt(c)*sqrt(x),
 sqrt(-c*x + 1)))/(c^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^2\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+2\,ab\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a^2}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^2*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arccos(sqrt(-
c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.104 $\int \frac{a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=141

$$\frac{ib\text{PolyLog}\left(2,-e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}+\frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}-\frac{\log\left(1+e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}$$

```
[Out] ((I/2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)/(b*c) - ((a + b*ArcCo
s[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*Log[1 + E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[
1 + c*x]])])/c + ((I/2)*b*PolyLog[2, -E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1
+ c*x]])])/c
```

Rubi [A] time = 0.104312, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {206, 6681, 4626, 3719, 2190, 2279, 2391}

$$\frac{ib\text{PolyLog}\left(2,-e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}+\frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}-\frac{\log\left(1+e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] ((I/2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)/(b*c) - ((a + b*ArcCo
s[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*Log[1 + E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[
1 + c*x]])])/c + ((I/2)*b*PolyLog[2, -E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1
+ c*x]])])/c
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
```

```
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{a+b\cos^{-1}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$
$$= \frac{\text{Subst}\left(\int (a+bx)\tan(x)\,dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$
$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{(2i)\,\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$
$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b\,\text{Subst}\left(\int \log\right.}{}$$
$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{(ib)\,\text{Subst}\left(\int \right.}{}$$
$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{ib\text{Li}_2\left(-e^{2i\cos^{-1}}\right.}{2c}$$

Mathematica [F] time = 0.372158, size = 0, normalized size = 0.

$$\int \frac{a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2), x]
```

Maple [A] time = 0.005, size = 171, normalized size = 1.2

$$\frac{a\ln(cx+1)}{2c} - \frac{a\ln(cx-1)}{2c} + \frac{\frac{i}{2}b}{c}\left(\arccos\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^2 - \frac{b}{c}\arccos\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\ln\left(1+\left(\sqrt{-cx+1}\frac{}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

```
[Out] 1/2*a/c*ln(c*x+1)-1/2*a/c*ln(c*x-1)+1/2*I*b/c*arccos((-c*x+1)^(1/2)/(c*x+1)
^(1/2))^2-b/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+((-c*x+1)^(1/2)/(c*
x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)+1/2*I*b*polylog(2,-((-c*x+1)^(1
/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a\left(\frac{\log\left(cx+1\right)}{c}-\frac{\log\left(cx-1\right)}{c}\right)-b\int\frac{\arctan\left(\sqrt{2}\sqrt{c}\sqrt{x},\,\sqrt{-cx+1}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="maxima")
```

```
[Out] 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) - b*integrate(arctan2(sqrt(2)*sqrt(
c)*sqrt(x), sqrt(-c*x + 1))/(c^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="fricas")
```

```
[Out] integral(-(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="giac")
```

```
[Out] Exception raised: TypeError
```

3.105 $\int \frac{1}{(1-c^2x^2)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{1}{(1-c^2x^2)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)},x\right)$$

```
[Out] Unintegrable[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
 x]
```

Rubi [A] time = 0.0420346, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Mathematica [A] time = 0.100509, size = 0, normalized size = 0.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x]
```

Maple [A] time = 0.344, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\arccos\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^{-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(c^2x^2-1)\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="maxima")
```

```
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{ac^2x^2+(bc^2x^2-b)\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="fricas")
```

```
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1
)) - a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(c^2x^2-1\right)\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
x)
```

3.106 $\int \frac{1}{(1-c^2x^2)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{1}{(1-c^2x^2)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2
), x]
```

Rubi [A] time = 0.040541, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [A] time = 1.6925, size = 0, normalized size = 0.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x
]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

Maple [A] time = 0.342, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\arccos\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\frac{1}{2}\left(\sqrt{2}b^2c\arctan\left(\sqrt{2}\sqrt{c}\sqrt{x},\sqrt{-cx+1}\right)+\sqrt{2}abc-\left(\sqrt{2}b^2c^2\arctan\left(\sqrt{2}\sqrt{c}\sqrt{x},\sqrt{-cx+1}\right)+\sqrt{2}abc^2\right)x\right)\sqrt{c}\int\frac{}{(b^2c^3\arctan(\sqrt{}}}{b^2c\arctan\left(\sqrt{2}\sqrt{c}\sqrt{x},\sqrt{-cx+1}\right)+abc-\left(b^2c^2\,\mathrm{ar}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="maxima")
```

```
[Out] ((sqrt(2)*b^2*c*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1)) + sqrt(2)*
a*b*c - (sqrt(2)*b^2*c^2*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1)) +
 sqrt(2)*a*b*c^2)*x)*sqrt(c)*integrate(1/2*sqrt(-c*x + 1)*sqrt(x)/((b^2*c^3
*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1)) + a*b*c^3)*x^3 - 2*(b^2*c
^2*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1)) + a*b*c^2)*x^2 + (b^2*c
*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1)) + a*b*c)*x), x) - sqrt(2)
*sqrt(-c*x + 1)*sqrt(c)*sqrt(x))/(b^2*c*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sq
rt(-c*x + 1)) + a*b*c - (b^2*c^2*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x
```

```
+ 1)) + a*b*c^2)*x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{a^2c^2x^2+\left(b^2c^2x^2-b^2\right)\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2-a^2+2\left(abc^2x^2-ab\right)\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccos(sqrt(-c*x + 1)/sqrt(c
*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1
))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(c^2x^2-1\right)\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2)
, x)
```

3.107 $\int \cos^{-1}\left(ce^{a+bx}\right) dx$

Optimal. Leaf size=84

$$-\frac{i\text{PolyLog}\left(2,-e^{2i\cos^{-1}\left(ce^{a+bx}\right)}\right)}{2b}-\frac{i\cos^{-1}\left(ce^{a+bx}\right)^2}{2b}+\frac{\cos^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2i\cos^{-1}\left(ce^{a+bx}\right)}\right)}{b}$$

```
[Out] ((-I/2)*ArcCos[c*E^(a + b*x)]^2)/b + (ArcCos[c*E^(a + b*x)]*Log[1 + E^((2*I
)*ArcCos[c*E^(a + b*x)])])/b - ((I/2)*PolyLog[2, -E^((2*I)*ArcCos[c*E^(a +
b*x)])])/b
```

Rubi [A] time = 0.0663046, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {2282, 4626, 3719, 2190, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,-e^{2i\cos^{-1}\left(ce^{a+bx}\right)}\right)}{2b}-\frac{i\cos^{-1}\left(ce^{a+bx}\right)^2}{2b}+\frac{\cos^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2i\cos^{-1}\left(ce^{a+bx}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[c*E^(a + b*x)],x]
```

```
[Out] ((-I/2)*ArcCos[c*E^(a + b*x)]^2)/b + (ArcCos[c*E^(a + b*x)]*Log[1 + E^((2*I
)*ArcCos[c*E^(a + b*x)])])/b - ((I/2)*PolyLog[2, -E^((2*I)*ArcCos[c*E^(a +
b*x)])])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \cos^{-1}\left(ce^{a+bx}\right) dx = \frac{\text{Subst}\left(\int \frac{\cos^{-1}(cx)}{x} dx, x, e^{a+bx}\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int x\tan(x)\, dx, x, \cos^{-1}\left(ce^{a+bx}\right)\right)}{b}$$
$$= -\frac{i\cos^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{(2i)\,\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}} dx, x, \cos^{-1}\left(ce^{a+bx}\right)\right)}{b}$$
$$= -\frac{i\cos^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\cos^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2i\cos^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\text{Subst}\left(\int \log\left(1+e^{2ix}\right) dx, x, \cos^{-}\right.}{b}$$
$$= -\frac{i\cos^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\cos^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2i\cos^{-1}\left(ce^{a+bx}\right)}\right)}{b} + \frac{i\,\text{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^{2i\cos^{-1}\left(ce\right.}\right.}{2b}$$
$$= -\frac{i\cos^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\cos^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2i\cos^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{i\text{Li}_2\left(-e^{2i\cos^{-1}\left(ce^{a+bx}\right)}\right)}{2b}$$

Mathematica [F] time = 0.947438, size = 0, normalized size = 0.

$$\int \cos^{-1}\left(ce^{a+bx}\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCos[c*E^(a + b*x)],x]
```

```
[Out] Integrate[ArcCos[c*E^(a + b*x)], x]
```

Maple [A] time = 0.004, size = 107, normalized size = 1.3

$$\frac{-\frac{i}{2}\left(\arccos\left(c\mathrm{e}^{bx+a}\right)\right)^2}{b} + \frac{\arccos\left(c\mathrm{e}^{bx+a}\right)}{b}\ln\left(1+\left(c\mathrm{e}^{bx+a}+i\sqrt{1-c^2\left(\mathrm{e}^{bx+a}\right)^2}\right)^2\right) - \frac{\frac{i}{2}}{b}\text{polylog}\left(2, -\left(c\mathrm{e}^{bx+a}+i\sqrt{1-c^2\left(\mathrm{e}\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(c*exp(b*x+a)),x)
```

```
[Out] -1/2*I*arccos(c*exp(b*x+a))^2/b+arccos(c*exp(b*x+a))*ln(1+(c*exp(b*x+a)+I*(
1-c^2*exp(b*x+a)^2)^(1/2))^2)/b-1/2*I*polylog(2,-(c*exp(b*x+a)+I*(1-c^2*exp
```

```
(b*x+a)^2)^(1/2))^2)/b
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(c*exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(c*exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acos}\left(ce^{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(c*exp(b*x+a)),x)
```

```
[Out] Integral(acos(c*exp(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arccos\left(ce^{(bx+a)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(c*exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccos(c*e^(b*x + a)), x)
```

3.108 $\int e^{\cos^{-1}(ax)} x^3\, dx$

Optimal. Leaf size=81

$$\frac{e^{\cos^{-1}(ax)}\cos\left(2\cos^{-1}(ax)\right)}{10a^4}+\frac{e^{\cos^{-1}(ax)}\cos\left(4\cos^{-1}(ax)\right)}{34a^4}-\frac{e^{\cos^{-1}(ax)}\sin\left(2\cos^{-1}(ax)\right)}{20a^4}-\frac{e^{\cos^{-1}(ax)}\sin\left(4\cos^{-1}(ax)\right)}{136a^4}$$

```
[Out] (E^ArcCos[a*x]*Cos[2*ArcCos[a*x]])/(10*a^4) + (E^ArcCos[a*x]*Cos[4*ArcCos[a
*x]])/(34*a^4) - (E^ArcCos[a*x]*Sin[2*ArcCos[a*x]])/(20*a^4) - (E^ArcCos[a*
x]*Sin[4*ArcCos[a*x]])/(136*a^4)
```

Rubi [A] time = 0.0643541, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4837, 12, 4469, 4432}

$$\frac{e^{\cos^{-1}(ax)}\cos\left(2\cos^{-1}(ax)\right)}{10a^4}+\frac{e^{\cos^{-1}(ax)}\cos\left(4\cos^{-1}(ax)\right)}{34a^4}-\frac{e^{\cos^{-1}(ax)}\sin\left(2\cos^{-1}(ax)\right)}{20a^4}-\frac{e^{\cos^{-1}(ax)}\sin\left(4\cos^{-1}(ax)\right)}{136a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCos[a*x]*x^3,x]
```

```
[Out] (E^ArcCos[a*x]*Cos[2*ArcCos[a*x]])/(10*a^4) + (E^ArcCos[a*x]*Cos[4*ArcCos[a
*x]])/(34*a^4) - (E^ArcCos[a*x]*Sin[2*ArcCos[a*x]])/(20*a^4) - (E^ArcCos[a*
x]*Sin[4*ArcCos[a*x]])/(136*a^4)
```

Rule 4837

```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> -Dist
[b^(-1), Subst[Int[(u /. x -> -(a/b) + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, A
rcCos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
```

```
&& IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{\cos^{-1}(ax)}x^3\,dx &= -\frac{\text{Subst}\left(\int \frac{e^x\cos^3(x)\sin(x)}{a^3}\,dx, x, \cos^{-1}(ax)\right)}{a} \\
&= -\frac{\text{Subst}\left(\int e^x\cos^3(x)\sin(x)\,dx, x, \cos^{-1}(ax)\right)}{a^4} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{4}e^x\sin(2x) + \frac{1}{8}e^x\sin(4x)\right)dx, x, \cos^{-1}(ax)\right)}{a^4} \\
&= -\frac{\text{Subst}\left(\int e^x\sin(4x)\,dx, x, \cos^{-1}(ax)\right)}{8a^4} - \frac{\text{Subst}\left(\int e^x\sin(2x)\,dx, x, \cos^{-1}(ax)\right)}{4a^4} \\
&= \frac{e^{\cos^{-1}(ax)}\cos\left(2\cos^{-1}(ax)\right)}{10a^4} + \frac{e^{\cos^{-1}(ax)}\cos\left(4\cos^{-1}(ax)\right)}{34a^4} - \frac{e^{\cos^{-1}(ax)}\sin\left(2\cos^{-1}(ax)\right)}{20a^4} - \frac{e^{\cos^{-1}(ax)}\sin}{13}
\end{aligned}$$

Mathematica [A] time = 0.139171, size = 50, normalized size = 0.62

$$-\frac{e^{\cos^{-1}(ax)}\left(-68\cos\left(2\cos^{-1}(ax)\right) - 20\cos\left(4\cos^{-1}(ax)\right) + 34\sin\left(2\cos^{-1}(ax)\right) + 5\sin\left(4\cos^{-1}(ax)\right)\right)}{680a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCos[a*x]*x^3,x]
```

```
[Out] -(E^ArcCos[a*x]*(-68*Cos[2*ArcCos[a*x]] - 20*Cos[4*ArcCos[a*x]] + 34*Sin[2*
ArcCos[a*x]] + 5*Sin[4*ArcCos[a*x]]))/(680*a^4)
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int e^{\arccos(ax)}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccos(a*x))*x^3,x)
```

```
[Out] int(exp(arccos(a*x))*x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 e^{(\arccos(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*e^(arccos(a*x)), x)
```

Fricas [A] time = 2.61862, size = 130, normalized size = 1.6

$$\frac{\left(20\,a^4x^4 - 3\,a^2x^2 - \left(5\,a^3x^3 + 6\,ax\right)\sqrt{-a^2x^2 + 1} - 6\right)e^{(\arccos(ax))}}{85\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x^3,x, algorithm="fricas")
```

```
[Out] 1/85*(20*a^4*x^4 - 3*a^2*x^2 - (5*a^3*x^3 + 6*a*x)*sqrt(-a^2*x^2 + 1) - 6)*
e^(arccos(a*x))/a^4
```

Sympy [A] time = 4.83622, size = 105, normalized size = 1.3

$$\begin{cases} \frac{4x^4e^{\operatorname{acos}(ax)}}{17} - \frac{x^3\sqrt{-a^2x^2+1}e^{\operatorname{acos}(ax)}}{17a} - \frac{3x^2e^{\operatorname{acos}(ax)}}{85a^2} - \frac{6x\sqrt{-a^2x^2+1}e^{\operatorname{acos}(ax)}}{85a^3} - \frac{6e^{\operatorname{acos}(ax)}}{85a^4} & \text{for } a \neq 0 \\ \frac{x^4e^{\frac{\pi}{2}}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acos(a*x))*x**3,x)
```

```
[Out] Piecewise((4*x**4*exp(acos(a*x))/17 - x**3*sqrt(-a**2*x**2 + 1)*exp(acos(a*
x))/(17*a) - 3*x**2*exp(acos(a*x))/(85*a**2) - 6*x*sqrt(-a**2*x**2 + 1)*exp
(acos(a*x))/(85*a**3) - 6*exp(acos(a*x))/(85*a**4), Ne(a, 0)), (x**4*exp(pi
/2)/4, True))
```

Giac [A] time = 1.32475, size = 111, normalized size = 1.37

$$\frac{4}{17}x^4e^{(\arccos(ax))} - \frac{\sqrt{-a^2x^2+1}x^3e^{(\arccos(ax))}}{17a} - \frac{3x^2e^{(\arccos(ax))}}{85a^2} - \frac{6\sqrt{-a^2x^2+1}xe^{(\arccos(ax))}}{85a^3} - \frac{6e^{(\arccos(ax))}}{85a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x^3,x, algorithm="giac")
```

```
[Out] 4/17*x^4*e^(arccos(a*x)) - 1/17*sqrt(-a^2*x^2 + 1)*x^3*e^(arccos(a*x))/a -
3/85*x^2*e^(arccos(a*x))/a^2 - 6/85*sqrt(-a^2*x^2 + 1)*x*e^(arccos(a*x))/a^
3 - 6/85*e^(arccos(a*x))/a^4
```

3.109 $\int e^{\cos^{-1}(ax)} x^2 \, dx$

Optimal. Leaf size=82

$$-\frac{\sqrt{1-a^2x^2}e^{\cos^{-1}(ax)}}{8a^3}+\frac{xe^{\cos^{-1}(ax)}}{8a^2}+\frac{3e^{\cos^{-1}(ax)}\cos\left(3\cos^{-1}(ax)\right)}{40a^3}-\frac{e^{\cos^{-1}(ax)}\sin\left(3\cos^{-1}(ax)\right)}{40a^3}$$

```
[Out] (E^ArcCos[a*x]*x)/(8*a^2) - (E^ArcCos[a*x]*Sqrt[1 - a^2*x^2])/(8*a^3) + (3*
E^ArcCos[a*x]*Cos[3*ArcCos[a*x]])/(40*a^3) - (E^ArcCos[a*x]*Sin[3*ArcCos[a*
x]])/(40*a^3)
```

Rubi [A] time = 0.0613399, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4837, 12, 4469, 4432}

$$-\frac{\sqrt{1-a^2x^2}e^{\cos^{-1}(ax)}}{8a^3}+\frac{xe^{\cos^{-1}(ax)}}{8a^2}+\frac{3e^{\cos^{-1}(ax)}\cos\left(3\cos^{-1}(ax)\right)}{40a^3}-\frac{e^{\cos^{-1}(ax)}\sin\left(3\cos^{-1}(ax)\right)}{40a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCos[a*x]*x^2,x]
```

```
[Out] (E^ArcCos[a*x]*x)/(8*a^2) - (E^ArcCos[a*x]*Sqrt[1 - a^2*x^2])/(8*a^3) + (3*
E^ArcCos[a*x]*Cos[3*ArcCos[a*x]])/(40*a^3) - (E^ArcCos[a*x]*Sin[3*ArcCos[a*
x]])/(40*a^3)
```

Rule 4837

```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> -Dist
[b^(-1), Subst[Int[(u /. x -> -(a/b) + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, A
rcCos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
```

```
&& IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^{\cos^{-1}(ax)}x^2\,dx &= -\frac{\text{Subst}\left(\int \frac{e^x\cos^2(x)\sin(x)}{a^2}\,dx, x, \cos^{-1}(ax)\right)}{a}\\
&= -\frac{\text{Subst}\left(\int e^x\cos^2(x)\sin(x)\,dx, x, \cos^{-1}(ax)\right)}{a^3}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{4}e^x\sin(x)+\frac{1}{4}e^x\sin(3x)\right)dx, x, \cos^{-1}(ax)\right)}{a^3}\\
&= -\frac{\text{Subst}\left(\int e^x\sin(x)\,dx, x, \cos^{-1}(ax)\right)}{4a^3}-\frac{\text{Subst}\left(\int e^x\sin(3x)\,dx, x, \cos^{-1}(ax)\right)}{4a^3}\\
&= \frac{e^{\cos^{-1}(ax)}x}{8a^2}-\frac{e^{\cos^{-1}(ax)}\sqrt{1-a^2x^2}}{8a^3}+\frac{3e^{\cos^{-1}(ax)}\cos\left(3\cos^{-1}(ax)\right)}{40a^3}-\frac{e^{\cos^{-1}(ax)}\sin\left(3\cos^{-1}(ax)\right)}{40a^3}
\end{aligned}
$$

Mathematica [A] time = 0.106531, size = 50, normalized size = 0.61

$$
-\frac{e^{\cos^{-1}(ax)}\left(5\sqrt{1-a^2x^2}-5ax-3\cos\left(3\cos^{-1}(ax)\right)+\sin\left(3\cos^{-1}(ax)\right)\right)}{40a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCos[a*x]*x^2,x]
```

```
[Out] -(E^ArcCos[a*x]*(-5*a*x + 5*Sqrt[1 - a^2*x^2] - 3*Cos[3*ArcCos[a*x]] + Sin[
3*ArcCos[a*x]]))/(40*a^3)
```

Maple [F] time = 0.009, size = 0, normalized size = 0.

$$
\int e^{\arccos(ax)}x^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccos(a*x))*x^2,x)
```

```
[Out] int(exp(arccos(a*x))*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 e^{(\arccos(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*e^(arccos(a*x)), x)
```

Fricas [A] time = 2.5661, size = 107, normalized size = 1.3

$$\frac{\left(3\,a^3x^3 - ax - \left(a^2x^2 + 1\right)\sqrt{-a^2x^2 + 1}\right)e^{(\arccos(ax))}}{10\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x^2,x, algorithm="fricas")
```

```
[Out] 1/10*(3*a^3*x^3 - a*x - (a^2*x^2 + 1)*sqrt(-a^2*x^2 + 1))*e^(arccos(a*x))/a
^3
```

Sympy [A] time = 1.7158, size = 85, normalized size = 1.04

$$\begin{cases} \frac{3x^3e^{\operatorname{acos}(ax)}}{10} - \frac{x^2\sqrt{-a^2x^2+1}e^{\operatorname{acos}(ax)}}{10a} - \frac{xe^{\operatorname{acos}(ax)}}{10a^2} - \frac{\sqrt{-a^2x^2+1}e^{\operatorname{acos}(ax)}}{10a^3} & \text{for } a \neq 0 \\ \frac{x^3e^{\frac{\pi}{2}}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acos(a*x))*x**2,x)
```

```
[Out] Piecewise((3*x**3*exp(acos(a*x))/10 - x**2*sqrt(-a**2*x**2 + 1)*exp(acos(a*
x))/(10*a) - x*exp(acos(a*x))/(10*a**2) - sqrt(-a**2*x**2 + 1)*exp(acos(a*x
))/(10*a**3), Ne(a, 0)), (x**3*exp(pi/2)/3, True))
```

Giac [A] time = 1.37588, size = 93, normalized size = 1.13

$$\frac{3}{10}\,x^3e^{(\arccos(ax))} - \frac{\sqrt{-a^2x^2+1}x^2e^{(\arccos(ax))}}{10\,a} - \frac{xe^{(\arccos(ax))}}{10\,a^2} - \frac{\sqrt{-a^2x^2+1}e^{(\arccos(ax))}}{10\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x^2,x, algorithm="giac")
```

```
[Out] 3/10*x^3*e^(arccos(a*x)) - 1/10*sqrt(-a^2*x^2 + 1)*x^2*e^(arccos(a*x))/a -
1/10*x*e^(arccos(a*x))/a^2 - 1/10*sqrt(-a^2*x^2 + 1)*e^(arccos(a*x))/a^3
```

3.110 $\int e^{\cos^{-1}(ax)} x \, dx$

Optimal. Leaf size=41

$$\frac{e^{\cos^{-1}(ax)} \cos \left(2 \cos^{-1}(ax)\right)}{5a^2} - \frac{e^{\cos^{-1}(ax)} \sin \left(2 \cos^{-1}(ax)\right)}{10a^2}$$

```
[Out] (E^ArcCos[a*x]*Cos[2*ArcCos[a*x]])/(5*a^2) - (E^ArcCos[a*x]*Sin[2*ArcCos[a*
x]])/(10*a^2)
```

Rubi [A] time = 0.0340067, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4837, 12, 4469, 4432}

$$\frac{e^{\cos^{-1}(ax)} \cos \left(2 \cos^{-1}(ax)\right)}{5a^2} - \frac{e^{\cos^{-1}(ax)} \sin \left(2 \cos^{-1}(ax)\right)}{10a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCos[a*x]*x,x]
```

```
[Out] (E^ArcCos[a*x]*Cos[2*ArcCos[a*x]])/(5*a^2) - (E^ArcCos[a*x]*Sin[2*ArcCos[a*
x]])/(10*a^2)
```

Rule 4837

```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> -Dist
[b^(-1), Subst[Int[(u /. x -> -(a/b) + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, A
rcCos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{\cos^{-1}(ax)}x\,dx &= -\frac{\text{Subst}\left(\int \frac{e^x\cos(x)\sin(x)}{a}\,dx, x, \cos^{-1}(ax)\right)}{a}\\
&= -\frac{\text{Subst}\left(\int e^x\cos(x)\sin(x)\,dx, x, \cos^{-1}(ax)\right)}{a^2}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{2}e^x\sin(2x)\,dx, x, \cos^{-1}(ax)\right)}{a^2}\\
&= -\frac{\text{Subst}\left(\int e^x\sin(2x)\,dx, x, \cos^{-1}(ax)\right)}{2a^2}\\
&= \frac{e^{\cos^{-1}(ax)}\cos\left(2\cos^{-1}(ax)\right)}{5a^2} - \frac{e^{\cos^{-1}(ax)}\sin\left(2\cos^{-1}(ax)\right)}{10a^2}
\end{aligned}$$

Mathematica [A] time = 0.0366191, size = 30, normalized size = 0.73

$$-\frac{e^{\cos^{-1}(ax)}\left(\sin\left(2\cos^{-1}(ax)\right)-2\cos\left(2\cos^{-1}(ax)\right)\right)}{10a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCos[a*x]*x,x]
```

```
[Out] -(E^ArcCos[a*x]*(-2*Cos[2*ArcCos[a*x]] + Sin[2*ArcCos[a*x]]))/(10*a^2)
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int e^{\arccos(ax)}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccos(a*x))*x,x)
```

```
[Out] int(exp(arccos(a*x))*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{(\arccos(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x,x, algorithm="maxima")
```

```
[Out] integrate(x*e^(arccos(a*x)), x)
```

Fricas [A] time = 2.63661, size = 89, normalized size = 2.17

$$\frac{\left(2\,a^2x^2 - \sqrt{-a^2x^2+1}ax - 1\right)e^{(\arccos(ax))}}{5\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x,x, algorithm="fricas")
```

```
[Out] 1/5*(2*a^2*x^2 - sqrt(-a^2*x^2 + 1)*a*x - 1)*e^(arccos(a*x))/a^2
```

Sympy [A] time = 0.539324, size = 58, normalized size = 1.41

$$\begin{cases} \frac{2x^2e^{\operatorname{acos}(ax)}}{5} - \frac{x\sqrt{-a^2x^2+1}e^{\operatorname{acos}(ax)}}{5a} - \frac{e^{\operatorname{acos}(ax)}}{5a^2} & \text{for}\ a \neq 0 \\ \frac{x^2e^{\frac{\pi}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acos(a*x))*x,x)
```

```
[Out] Piecewise((2*x**2*exp(acos(a*x))/5 - x*sqrt(-a**2*x**2 + 1)*exp(acos(a*x))/
(5*a) - exp(acos(a*x))/(5*a**2), Ne(a, 0)), (x**2*exp(pi/2)/2, True))
```

Giac [A] time = 1.23812, size = 59, normalized size = 1.44

$$\frac{2}{5}\,x^2e^{(\arccos(ax))} - \frac{\sqrt{-a^2x^2+1}xe^{(\arccos(ax))}}{5\,a} - \frac{e^{(\arccos(ax))}}{5\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x,x, algorithm="giac")
```

```
[Out] 2/5*x^2*e^(arccos(a*x)) - 1/5*sqrt(-a^2*x^2 + 1)*x*e^(arccos(a*x))/a - 1/5*
e^(arccos(a*x))/a^2
```

3.111 $\int e^{\cos^{-1}(ax)}\,dx$

Optimal. Leaf size=39

$$\frac{1}{2}xe^{\cos^{-1}(ax)}-\frac{\sqrt{1-a^2x^2}e^{\cos^{-1}(ax)}}{2a}$$

```
[Out] (E^ArcCos[a*x]*x)/2 - (E^ArcCos[a*x]*Sqrt[1 - a^2*x^2])/(2*a)
```

Rubi [A] time = 0.0140558, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4837, 4432}

$$\frac{1}{2}xe^{\cos^{-1}(ax)}-\frac{\sqrt{1-a^2x^2}e^{\cos^{-1}(ax)}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCos[a*x],x]
```

```
[Out] (E^ArcCos[a*x]*x)/2 - (E^ArcCos[a*x]*Sqrt[1 - a^2*x^2])/(2*a)
```

Rule 4837

```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> -Dist
[b^(-1), Subst[Int[(u /. x -> -(a/b) + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, A
rcCos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{\cos^{-1}(ax)}\,dx = -\frac{\text{Subst}\left(\int e^x \sin(x)\,dx, x, \cos^{-1}(ax)\right)}{a}$$
$$= \frac{1}{2}e^{\cos^{-1}(ax)}x - \frac{e^{\cos^{-1}(ax)}\sqrt{1-a^2x^2}}{2a}$$

Mathematica [A] time = 0.0326375, size = 32, normalized size = 0.82

$$-\frac{\left(\sqrt{1-a^2x^2}-ax\right)e^{\cos^{-1}(ax)}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCos[a*x],x]
```

```
[Out] -(E^ArcCos[a*x]*(-(a*x) + Sqrt[1 - a^2*x^2]))/(2*a)
```

Maple [F] time = 0.004, size = 0, normalized size = 0.

$$\int \mathrm{e}^{\arccos(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccos(a*x)),x)
```

```
[Out] int(exp(arccos(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(\arccos(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(e^(arccos(a*x)), x)
```

Fricas [A] time = 2.59955, size = 68, normalized size = 1.74

$$\frac{\left(ax - \sqrt{-a^2x^2 + 1}\right)e^{(\arccos(ax))}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*x - sqrt(-a^2*x^2 + 1))*e^(arccos(a*x))/a
```

Sympy [A] time = 0.201251, size = 37, normalized size = 0.95

$$\begin{cases} \frac{xe^{\operatorname{acos}(ax)}}{2} - \frac{\sqrt{-a^2x^2+1}e^{\operatorname{acos}(ax)}}{2a} & \text{for } a \neq 0 \\ xe^{\frac{\pi}{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acos(a*x)),x)
```

```
[Out] Piecewise((x*exp(acos(a*x))/2 - sqrt(-a**2*x**2 + 1)*exp(acos(a*x))/(2*a),
Ne(a, 0)), (x*exp(pi/2), True))
```

Giac [A] time = 1.27696, size = 42, normalized size = 1.08

$$\frac{1}{2}\,xe^{(\arccos(ax))} - \frac{\sqrt{-a^2x^2 + 1}e^{(\arccos(ax))}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x)),x, algorithm="giac")
```

```
[Out] 1/2*x*e^(arccos(a*x)) - 1/2*sqrt(-a^2*x^2 + 1)*e^(arccos(a*x))/a
```

3.112 $\int \frac{e^{\cos^{-1}(ax)}}{x} dx$

Optimal. Leaf size=45

$$ie^{\cos^{-1}(ax)} - 2ie^{\cos^{-1}(ax)}\text{Hypergeometric2F1}\left(-\frac{i}{2}, 1, 1 - \frac{i}{2}, -e^{2i\cos^{-1}(ax)}\right)$$

```
[Out] I*E^ArcCos[a*x] - (2*I)*E^ArcCos[a*x]*Hypergeometric2F1[-I/2, 1, 1 - I/2, -
E^((2*I)*ArcCos[a*x])]
```

Rubi [A] time = 0.0543512, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4837, 12, 4442, 2194, 2251}

$$ie^{\cos^{-1}(ax)} - 2ie^{\cos^{-1}(ax)}\,{}_2F_1\left(-\frac{i}{2}, 1; 1 - \frac{i}{2}; -e^{2i\cos^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCos[a*x]/x,x]
```

```
[Out] I*E^ArcCos[a*x] - (2*I)*E^ArcCos[a*x]*Hypergeometric2F1[-I/2, 1, 1 - I/2, -
E^((2*I)*ArcCos[a*x])]
```

Rule 4837

```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> -Dist
[b^(-1), Subst[Int[(u /. x -> -(a/b) + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, A
rcCos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
```

```
&& IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\cos^{-1}(ax)}}{x}\,dx &= -\frac{\text{Subst}\left(\int ae^x\tan(x)\,dx, x, \cos^{-1}(ax)\right)}{a}\\
&= -\text{Subst}\left(\int e^x\tan(x)\,dx, x, \cos^{-1}(ax)\right)\\
&= -\left(i\,\text{Subst}\left(\int\left(-e^x+\frac{2e^x}{1+e^{2ix}}\right)dx, x, \cos^{-1}(ax)\right)\right)\\
&= i\,\text{Subst}\left(\int e^x\,dx, x, \cos^{-1}(ax)\right) - 2i\,\text{Subst}\left(\int\frac{e^x}{1+e^{2ix}}\,dx, x, \cos^{-1}(ax)\right)\\
&= ie^{\cos^{-1}(ax)} - 2ie^{\cos^{-1}(ax)}\,{}_2F_1\left(-\frac{i}{2},1;1-\frac{i}{2};-e^{2i\cos^{-1}(ax)}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0559199, size = 79, normalized size = 1.76

$$i\left(\left(\frac{1}{5}-\frac{2i}{5}\right)e^{(1+2i)\cos^{-1}(ax)}\text{Hypergeometric2F1}\left(1,1-\frac{i}{2},2-\frac{i}{2},-e^{2i\cos^{-1}(ax)}\right)-e^{\cos^{-1}(ax)}\text{Hypergeometric2F1}\left(-\frac{i}{2},1,1\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCos[a*x]/x,x]
```

```
[Out] I*(-(E^ArcCos[a*x]*Hypergeometric2F1[-I/2, 1, 1 - I/2, -E^((2*I)*ArcCos[a*x
])]) + (1/5 - (2*I)/5)*E^((1 + 2*I)*ArcCos[a*x])*Hypergeometric2F1[1, 1 - I
/2, 2 - I/2, -E^((2*I)*ArcCos[a*x])])
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{\arccos(ax)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccos(a*x))/x,x)
```

```
[Out] int(exp(arccos(a*x))/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arccos(ax))}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))/x,x, algorithm="maxima")
```

```
[Out] integrate(e^(arccos(a*x))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^{(\arccos(ax))}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))/x,x, algorithm="fricas")
```

```
[Out] integral(e^(arccos(a*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\operatorname{acos}(ax)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acos(a*x))/x,x)
```

```
[Out] Integral(exp(acos(a*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arccos(ax))}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))/x,x, algorithm="giac")
```

```
[Out] integrate(e^(arccos(a*x))/x, x)
```

3.113 $\int \frac{e^{\cos^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=87

$$(1+i)ae^{(1+i)\cos^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},1,\frac{3}{2}-\frac{i}{2},-e^{2i\cos^{-1}(ax)}\right)-(2+2i)ae^{(1+i)\cos^{-1}(ax)}\text{Hypergeometric2F1}$$

```
[Out] (1 + I)*a*E^((1 + I)*ArcCos[a*x])*Hypergeometric2F1[1/2 - I/2, 1, 3/2 - I/2
, -E^((2*I)*ArcCos[a*x])] - (2 + 2*I)*a*E^((1 + I)*ArcCos[a*x])*Hypergeomet
ric2F1[1/2 - I/2, 2, 3/2 - I/2, -E^((2*I)*ArcCos[a*x])]
```

Rubi [A] time = 0.105852, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4837, 12, 4471, 2251}

$$(1+i)ae^{(1+i)\cos^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},1;\frac{3}{2}-\frac{i}{2};-e^{2i\cos^{-1}(ax)}\right)-(2+2i)ae^{(1+i)\cos^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},2;\frac{3}{2}-\frac{i}{2};-e^{2i\cos^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCos[a*x]/x^2,x]
```

```
[Out] (1 + I)*a*E^((1 + I)*ArcCos[a*x])*Hypergeometric2F1[1/2 - I/2, 1, 3/2 - I/2
, -E^((2*I)*ArcCos[a*x])] - (2 + 2*I)*a*E^((1 + I)*ArcCos[a*x])*Hypergeomet
ric2F1[1/2 - I/2, 2, 3/2 - I/2, -E^((2*I)*ArcCos[a*x])]
```

Rule 4837

```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> -Dist
[b^(-1), Subst[Int[(u /. x -> -(a/b) + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, A
rcCos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4471

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
```

```
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && TrigQ[G] && TrigQ[H]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\cos^{-1}(ax)}}{x^2}\,dx &= -\frac{\text{Subst}\left(\int a^2 e^x \sec(x)\tan(x)\,dx, x, \cos^{-1}(ax)\right)}{a}\\
&= -\left(a\,\text{Subst}\left(\int e^x \sec(x)\tan(x)\,dx, x, \cos^{-1}(ax)\right)\right)\\
&= -\left(a\,\text{Subst}\left(\int\left(\frac{4ie^{(1+i)x}}{\left(1+e^{2ix}\right)^2}-\frac{2ie^{(1+i)x}}{1+e^{2ix}}\right)dx, x, \cos^{-1}(ax)\right)\right)\\
&= (2ia)\,\text{Subst}\left(\int\frac{e^{(1+i)x}}{1+e^{2ix}}\,dx, x, \cos^{-1}(ax)\right)-(4ia)\,\text{Subst}\left(\int\frac{e^{(1+i)x}}{\left(1+e^{2ix}\right)^2}\,dx, x, \cos^{-1}(ax)\right)\\
&= (1+i)ae^{(1+i)\cos^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},1;\frac{3}{2}-\frac{i}{2};-e^{2i\cos^{-1}(ax)}\right)-(2+2i)ae^{(1+i)\cos^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},2;\frac{3}{2}-\frac{i}{2};-e
\end{aligned}$$

Mathematica [A] time = 0.0618076, size = 55, normalized size = 0.63

$$-\frac{e^{\cos^{-1}(ax)}}{x}+(1-i)ae^{(1+i)\cos^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},1,\frac{3}{2}-\frac{i}{2},-e^{2i\cos^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCos[a*x]/x^2,x]
```

```
[Out] -(E^ArcCos[a*x]/x) + (1 - I)*a*E^((1 + I)*ArcCos[a*x])*Hypergeometric2F1[1/
2 - I/2, 1, 3/2 - I/2, -E^((2*I)*ArcCos[a*x])]
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{\arccos(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccos(a*x))/x^2,x)
```

```
[Out] int(exp(arccos(a*x))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arccos(ax))}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(arccos(a*x))/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^{(\arccos(ax))}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(e^(arccos(a*x))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\operatorname{acos}(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acos(a*x))/x**2,x)
```

```
[Out] Integral(exp(acos(a*x))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(\arccos(ax))}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate(e^(arccos(a*x))/x^2, x)
```

3.114 $\int \cos^{-1}\left(\frac{c}{a+bx}\right) dx$

Optimal. Leaf size=48

$$\frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{c\tanh^{-1}\left(\sqrt{1-\frac{c^2}{(a+bx)^2}}\right)}{b}$$

```
[Out] ((a + b*x)*ArcSec[a/c + (b*x)/c])/b - (c*ArcTanh[Sqrt[1 - c^2/(a + b*x)^2]]
)/b
```

Rubi [A] time = 0.0328311, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4833, 5250, 372, 266, 63, 206}

$$\frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{c\tanh^{-1}\left(\sqrt{1-\frac{c^2}{(a+bx)^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[c/(a + b*x)],x]
```

```
[Out] ((a + b*x)*ArcSec[a/c + (b*x)/c])/b - (c*ArcTanh[Sqrt[1 - c^2/(a + b*x)^2]]
)/b
```

Rule 4833

```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 5250

```
Int[ArcSec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[((c + d*x)*ArcSec[c + d*x]
)/d, x] - Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^{-1}\left(\frac{c}{a+bx}\right) dx = \int \sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right) dx$$
$$= \frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \int \frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)\sqrt{1-\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}} dx$$
$$= \frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \frac{c\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^2}}x} dx, x, \frac{a}{c}+\frac{bx}{c}\right)}{b}$$
$$= \frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} + \frac{c\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}x} dx, x, \frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}\right)}{2b}$$
$$= \frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \frac{c\,\mathrm{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{1-\frac{c^2}{(a+bx)^2}}\right)}{b}$$
$$= \frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \frac{c\tanh^{-1}\left(\sqrt{1-\frac{c^2}{(a+bx)^2}}\right)}{b}$$

Mathematica [B] time = 0.162596, size = 141, normalized size = 2.94

$$x\cos^{-1}\left(\frac{c}{a+bx}\right) - \frac{(a+bx)\sqrt{\frac{a^2+2abx+b^2x^2-c^2}{(a+bx)^2}}\left(c\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2+2abx+b^2x^2-c^2}}\right) - a\tan^{-1}\left(\frac{\sqrt{(a+bx)^2-c^2}}{c}\right)\right)}{b\sqrt{a^2+2abx+b^2x^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[c/(a + b*x)],x]
```

```
[Out] x*ArcCos[c/(a + b*x)] - ((a + b*x)*Sqrt[(a^2 - c^2 + 2*a*b*x + b^2*x^2)/(a
+ b*x)^2]*(-(a*ArcTan[Sqrt[-c^2 + (a + b*x)^2]/c]) + c*ArcTanh[(a + b*x)/Sq
rt[a^2 - c^2 + 2*a*b*x + b^2*x^2]]))/(b*Sqrt[a^2 - c^2 + 2*a*b*x + b^2*x^2]
)
```

Maple [A] time = 0.008, size = 45, normalized size = 0.9

$$-\frac{c}{b}\left(-\frac{bx+a}{c}\arccos\left(\frac{c}{bx+a}\right)+\operatorname{Artanh}\left(\frac{1}{\sqrt{1-\frac{c^2}{(bx+a)^2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(c/(b*x+a)),x)
```

```
[Out] -1/b*c*(-1/c*(b*x+a)*arccos(c/(b*x+a))+arctanh(1/(1-c^2/(b*x+a)^2)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x\arctan\left(\sqrt{bx+a+c}\sqrt{bx+a-c},c\right)-\int\frac{\left(b^2cx^2+abcx\right)e^{\left(\frac{1}{2}\log(bx+a+c)+\frac{1}{2}\log(bx+a-c)\right)}}{b^2c^2x^2+2\,abc^2x+a^2c^2-c^4+\left(b^2x^2+2\,abx+a^2-c^2\right)e^{(\log(bx+a+c)+\log(bx+a-c))}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(c/(b*x+a)),x, algorithm="maxima")
```

```
[Out] x*arctan2(sqrt(b*x + a + c)*sqrt(b*x + a - c), c) - integrate((b^2*c*x^2 +
a*b*c*x)*e^(1/2*log(b*x + a + c) + 1/2*log(b*x + a - c))/(b^2*c^2*x^2 + 2*a
*b*c^2*x + a^2*c^2 - c^4 + (b^2*x^2 + 2*a*b*x + a^2 - c^2)*e^(log(b*x + a +
 c) + log(b*x + a - c))), x)
```

Fricas [B] time = 2.52899, size = 305, normalized size = 6.35

$$\frac{bx\arccos\left(\frac{c}{bx+a}\right)+2\,a\arctan\left(-\frac{bx-(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}+a}{c}\right)+c\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(c/(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*x*arccos(c/(b*x + a)) + 2*a*arctan(-(b*x - (b*x + a)*sqrt((b^2*x^2 + 2*a
*b*x + a^2 - c^2)/(b^2*x^2 + 2*a*b*x + a^2)) + a)/c) + c*log(-b*x + (b*x +
a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 - c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acos}\left(\frac{c}{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(c/(b*x+a)),x)
```

```
[Out] Integral(acos(c/(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arccos\left(\frac{c}{bx+a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(c/(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccos(c/(b*x + a)), x)
```

3.115 $\int \frac{x}{\sqrt{1-x^2}\sqrt{\cos^{-1}(x)}} dx$

Optimal. Leaf size=26

$$-\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(x)}\right)$$

```
[Out] -(Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[x]]])
```

Rubi [A] time = 0.0706919, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4724, 3304, 3352}

$$-\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - x^2]*Sqrt[ArcCos[x]]),x]
```

```
[Out] -(Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[x]]])
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\cos^{-1}(x)}}\,dx = -\operatorname{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(x)\right)$$
$$= -\left(2\operatorname{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{\cos^{-1}(x)}\right)\right)$$
$$= -\sqrt{2\pi}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(x)}\right)$$

Mathematica [C] time = 0.0848823, size = 56, normalized size = 2.15

$$\frac{i\left(\sqrt{-i\cos^{-1}(x)}\text{Gamma}\left(\frac{1}{2},-i\cos^{-1}(x)\right)-\sqrt{i\cos^{-1}(x)}\text{Gamma}\left(\frac{1}{2},i\cos^{-1}(x)\right)\right)}{2\sqrt{\cos^{-1}(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 - x^2]*Sqrt[ArcCos[x]]),x]
```

```
[Out] ((I/2)*(Sqrt[(-I)*ArcCos[x]]*Gamma[1/2, (-I)*ArcCos[x]] - Sqrt[I*ArcCos[x]]
*Gamma[1/2, I*ArcCos[x]]))/Sqrt[ArcCos[x]]
```

Maple [A] time = 0.106, size = 21, normalized size = 0.8

$$-\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\sqrt{\arccos(x)}\right)\sqrt{2}\sqrt{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x)
```

```
[Out] -FresnelC(2^(1/2)/Pi^(1/2)*arccos(x)^(1/2))*2^(1/2)*Pi^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(x-1)(x+1)}\sqrt{\operatorname{acos}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(1/2)/acos(x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)*(x + 1))*sqrt(acos(x))), x)
```

Giac [B] time = 1.39764, size = 78, normalized size = 3.

$$\frac{\sqrt{2}\sqrt{\pi}i\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\arccos(x)}}{i-1}\right)}{2(i-1)}-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{\sqrt{2}i\sqrt{\arccos(x)}}{i-1}\right)}{2(i-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(pi)*i*erf(sqrt(2)*sqrt(arccos(x))/(i - 1))/(i - 1) - 1/2*s
qrt(2)*sqrt(pi)*erf(-sqrt(2)*i*sqrt(arccos(x))/(i - 1))/(i - 1)
```

3.116 $\int \frac{x}{\sqrt{1-x^2}\cos^{-1}(x)}\,dx$

Optimal. Leaf size=5

$$-\text{CosIntegral}\left(\cos^{-1}(x)\right)$$

```
[Out] -CosIntegral[ArcCos[x]]
```

Rubi [A] time = 0.0608316, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4724, 3302}

$$-\text{CosIntegral}\left(\cos^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - x^2]*ArcCos[x]),x]

[Out] -CosIntegral[ArcCos[x]]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{1-x^2}\cos^{-1}(x)}\,dx &= -\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \cos^{-1}(x)\right)\\ &= -\text{Ci}\left(\cos^{-1}(x)\right)\end{aligned}$$

Mathematica [A] time = 0.0400906, size = 5, normalized size = 1.

$$-\text{CosIntegral}\left(\cos^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 - x^2]*ArcCos[x]),x]
```

```
[Out] -CosIntegral[ArcCos[x]]
```

Maple [A] time = 0.067, size = 6, normalized size = 1.2

$$-\text{Ci}\left(\arccos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccos(x)/(-x^2+1)^(1/2),x)
```

```
[Out] -Ci(arccos(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^2+1}\arccos\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^2 + 1)*arccos(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^2+1}x}{\left(x^2-1\right)\arccos\left(x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*x/((x^2 - 1)*arccos(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-\left(x-1\right)\left(x+1\right)}\operatorname{acos}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acos(x)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)*(x + 1))*acos(x)), x)
```

Giac [A] time = 1.37826, size = 7, normalized size = 1.4

$$-\operatorname{Ci}\left(\arccos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -cos_integral(arccos(x))
```

3.117 $\int \frac{\cos^{-1}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{-bx^2}\cos^{-1}\left(\sqrt{bx^2+1}\right)^{n+1}}{b(n+1)x}$$

```
[Out] -((Sqrt[-(b*x^2)]*ArcCos[Sqrt[1 + b*x^2]]^(1 + n))/(b*(1 + n)*x))
```

Rubi [A] time = 0.0653671, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4835, 4642}

$$-\frac{\sqrt{-bx^2}\cos^{-1}\left(\sqrt{bx^2+1}\right)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[1 + b*x^2]]^n/Sqrt[1 + b*x^2],x]
```

```
[Out] -((Sqrt[-(b*x^2)]*ArcCos[Sqrt[1 + b*x^2]]^(1 + n))/(b*(1 + n)*x))
```

Rule 4835

```
Int[ArcCos[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
:> Dist[Sqrt[-(b*x^2)]/(b*x), Subst[Int[ArcCos[x]^n/Sqrt[1 - x^2], x], x, S
qrt[1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^{-1}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx = \frac{\sqrt{-bx^2}\,\text{Subst}\left(\int \frac{\cos^{-1}(x)^n}{\sqrt{1-x^2}}\,dx, x, \sqrt{1+bx^2}\right)}{bx}$$

$$= -\frac{\sqrt{-bx^2}\cos^{-1}\left(\sqrt{1+bx^2}\right)^{1+n}}{b(1+n)x}$$

Mathematica [A] time = 0.0458177, size = 39, normalized size = 1.

$$-\frac{\sqrt{-bx^2}\cos^{-1}\left(\sqrt{bx^2+1}\right)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[1 + b*x^2]]^n/Sqrt[1 + b*x^2],x]
```

```
[Out] -((Sqrt[-(b*x^2)]*ArcCos[Sqrt[1 + b*x^2]]^(1 + n))/(b*(1 + n)*x))
```

Maple [F] time = 0.193, size = 0, normalized size = 0.

$$\int \left(\arccos\left(\sqrt{bx^2+1}\right)\right)^n \frac{1}{\sqrt{bx^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x)
```

```
[Out] int(arccos((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.99523, size = 107, normalized size = 2.74

$$-\frac{\sqrt{-bx^2}\arccos\left(\sqrt{bx^2+1}\right)^n\arccos\left(\sqrt{bx^2+1}\right)}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x^2)*arccos(sqrt(b*x^2 + 1))^n*arccos(sqrt(b*x^2 + 1))/((b*n + b)*
x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos((b*x**2+1)**(1/2))**n/(b*x**2+1)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos\left(\sqrt{bx^2+1}\right)^n}{\sqrt{bx^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccos(sqrt(b*x^2 + 1))^n/sqrt(b*x^2 + 1), x)
```

3.118 $\int \frac{1}{\sqrt{1+bx^2}\cos^{-1}\left(\sqrt{1+bx^2}\right)}\,dx$

Optimal. Leaf size=31

$$-\frac{\sqrt{-bx^2}\log\left(\cos^{-1}\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

```
[Out] -((Sqrt[-(b*x^2)]*Log[ArcCos[Sqrt[1 + b*x^2]]])/(b*x))
```

Rubi [A] time = 0.0608216, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4835, 4640}

$$-\frac{\sqrt{-bx^2}\log\left(\cos^{-1}\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + b*x^2]*ArcCos[Sqrt[1 + b*x^2]]),x]

[Out] -((Sqrt[-(b*x^2)]*Log[ArcCos[Sqrt[1 + b*x^2]]])/(b*x))
```

Rule 4835

```
Int[ArcCos[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
:> Dist[Sqrt[-(b*x^2)]/(b*x), Subst[Int[ArcCos[x]^n/Sqrt[1 - x^2], x], x, S
qrt[1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

Rule 4640

```
Int[1/(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sym
bol] :> -Simp[Log[a + b*ArcCos[c*x]]/(b*c*Sqrt[d]), x] /; FreeQ[{a, b, c, d
, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+bx^2}\cos^{-1}\left(\sqrt{1+bx^2}\right)}\,dx = \frac{\sqrt{-bx^2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\cos^{-1}(x)}\,dx, x, \sqrt{1+bx^2}\right)}{bx}$$

$$= -\frac{\sqrt{-bx^2}\log\left(\cos^{-1}\left(\sqrt{1+bx^2}\right)\right)}{bx}$$

Mathematica [A] time = 0.0239597, size = 25, normalized size = 0.81

$$\frac{x\log\left(\cos^{-1}\left(\sqrt{bx^2+1}\right)\right)}{\sqrt{-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + b*x^2]*ArcCos[Sqrt[1 + b*x^2]]),x]
```

```
[Out] (x*Log[ArcCos[Sqrt[1 + b*x^2]]])/Sqrt[-(b*x^2)]
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int \left(\arccos\left(\sqrt{bx^2+1}\right)\right)^{-1}\frac{1}{\sqrt{bx^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x)
```

```
[Out] int(1/arccos((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+1}\arccos\left(\sqrt{bx^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + 1)*arccos(sqrt(b*x^2 + 1))), x)
```

Fricas [A] time = 2.57099, size = 68, normalized size = 2.19

$$-\frac{\sqrt{-bx^2}\log\left(\arccos\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x^2)*log(arccos(sqrt(b*x^2 + 1)))/(b*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+1}\,\mathrm{acos}\left(\sqrt{bx^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos((b*x**2+1)**(1/2))/(b*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x**2 + 1)*acos(sqrt(b*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+1}\arccos\left(\sqrt{bx^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + 1)*arccos(sqrt(b*x^2 + 1))), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=", ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```python


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```